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CS 111 ASSIGNMENT Assignment 4
due 3/7/2015

Problem 1: Give the asymptotic value (using the Θ-notation) for the number of letters that will be printed by the algorithms below. Your solution needs to consist of an appropriate recurrence equation and its solution, with a brief justification.

(a) **Algorithm** PRINTXS (n : **integer**)
  if $n < 3$
    print("X")
  else
    PRINTXS($\lceil n/3 \rceil$)
    PRINTXS($\lceil n/3 \rceil$)
    PRINTXS($\lceil n/3 \rceil$)
    PRINTXS($\lceil n/3 \rceil$)
    for $i \leftarrow 1$ **to** $2n$ **do** print("X")

Solution 1:

$$T(n) = aT(\frac{n}{b}) + cn^d$$

$$a = 4; b = 3;$$

$$cn^d = 2n; d = 1; c = 2$$

$$L(n) = 4L(\frac{n}{3}) + 2n$$

$$a > b^d$$

$$\theta(n^{log_3 4})$$

(b) **Algorithm** PRINTYS (n : **integer**)
  if $n < 2$
    print("Y")
  else
    for $j \leftarrow 1$ **to** 7 **do** PRINTYS($\lfloor n/2 \rfloor$)
    for $i \leftarrow 1$ **to** n^3 **do** print("Y")

Solution 2:

$$T(n) = aT(\frac{n}{b}) + cn^d$$

$$a = 7; b = 2;$$

$$cn^d = n^3; d = 3; c = 1$$

$$L(n) = 7L(\frac{n}{2}) + n^3$$
$$a < b^d$$
$$\theta(n^3)$$

(c) **Algorithm** PRINTZS (n : **integer**)

    if $n < 2$
        print("Z")
    else
        for $j \leftarrow 1$ **to** 8 **do** PRINTZS($\lfloor n/2 \rfloor$)
        for $i \leftarrow 1$ **to** n^3 **do** print("Z")

Solution 3:

$$T(n) = aT(\frac{n}{b}) + cn^d$$
$$a = 8; b = 2;$$
$$cn^d = n^3; d = 3; c = 1$$
$$L(n) = 8L(\frac{n}{2}) + n^3$$
$$a = b^d$$
$$\theta(n^3 logn)$$

(d) **Algorithm** PRINTUS (n : **integer**)

    if $n < 4$
        print("U")
    else
        PRINTUS($\lceil n/4 \rceil$)
        PRINTUS($\lfloor n/4 \rfloor$)
        for $i \leftarrow 1$ **to** 11 **do** print("U")

Solution 4:

$$T(n) = aT(\frac{n}{b}) + cn^d$$
$$a = 2; b = 4;$$
$$cn^d = 11; d = 0; c = 11$$
$$L(n) = 2L(\frac{n}{4}) + 11$$
$$a > b^d$$
$$\theta(n^{log_4 2})$$

(e) **Algorithm** $\textsc{PrintVs}$ (n : **integer**)

  if $n < 3$

    print("V")

  else

    for $j \leftarrow 1$ **to** 10 **do** $\textsc{PrintVs}(\lfloor n/3 \rfloor)$

    for $i \leftarrow 1$ **to** $2n^3$ **do** print("V")

Solution 5:

$$T(n) = aT(\frac{n}{b}) + cn^d$$

$$a = 10; b = 3;$$

$$cn^d = 2n^3; d = 3; c = 2$$

$$L(n) = 10L(\frac{n}{3}) + 2n^3$$

$$a < b^d$$

$$\theta(n^3)$$

Problem 2: Determine (using the inclusion-exclusion principle) the number of integer solutions of the equation:

$$x + y + z \quad = \quad 20,$$

under the constraints

$$\begin{aligned} 1 \ &\leq \ x \ \leq \ 5 \\ 3 \ &\leq \ y \ \leq \ 9 \\ 1 \ &\leq \ z \ \leq \ 8 \end{aligned}$$

Show your work.

Solution 6:

Re-writing the constraints, we get:

$$0 \leq x' \leq 4$$

$$0 \leq y' \leq 6$$

$$0 \leq z' \leq 7$$

Substituting these constraints into the original sum, we get:

$$(x' + 1) + (y' + 3) + (z' + 1) = 20$$
$$x' + y' + z' = 15$$

The inclusion-exclusion principle tells us that:

$$S(x' \leq 4 \wedge y' \leq 6 \wedge z' \leq 7)$$

Or

$$S - S(x' \geq 5 \vee y' \geq 7 \vee z' \geq 8)$$
$$= S - (S(x' \geq 5) + S(y' \geq 7) + S(z' \geq 8) - S(x' \geq 5 \wedge y' \geq 7) - S(x' \geq 5 \wedge z' \geq 8) - S(y' \geq 7 \wedge z' \geq 8) + S(x \geq 5 \wedge y' \geq 7 \wedge z$$

S is equal to $\binom{m+k-1}{k-1}$.

$$S = \binom{15+3-1}{3-1} = \binom{17}{2} = 136$$

For each induvidual inequality, we use $\binom{m-A+k-1}{k-1}$

$$= \binom{17}{2} - (\binom{12}{2} + \binom{10}{2} + \binom{9}{2} - \binom{5}{2} - \binom{4}{2} - \binom{2}{2} + \binom{0}{2})$$
$$= 136 - (66 + 45 + 36 - 10 - 6 - 1 + 0)$$
$$= 136 - 130$$
$$= 6$$

Problem 3: Determine (using the inclusion-exclusion principle) the number of integer solutions of the equation:

$$x + y + z \;=\; 20,$$

under the constraints

$$\begin{aligned} 1 \;\leq\; & x \;\leq\; 5 \\ 3 \;\leq\; & y \;\leq\; 9 \\ 1 \;\leq\; & z \;\leq\; 8 \end{aligned}$$

Show your work.

Submission. To submit the homework, you need to upload the pdf file into ilearn by 8AM on Thursday, March 7, and turn-in a paper copy in class.