CS/MATH111 ASSIGNMENT 1

due Tuesday, January 20(11AM)

Individual assignment: Problems 1 and 2.
Group assignment: Problems 1,2 and 3.

Problem 1: (a) Give the exact formula (as a function of n) for the number of times "bingo" is printed by Algorithm BingoPrint below. First express it as a summation formula and justify it. Then simplify it to obtain a closed-form expression. Show your derivation.
(b) Give the asymptotic value of the number of "bingo"s using the Θ-notation. Include a brief justification. You will need a formula for the sum of consecutive squares that you can find on the internet.

Algorithm BingoPrint (n : **integer**)
  for $i \leftarrow 1$ **to** $2n+1$ **do**
    for $j \leftarrow 1$ **to** $i^2 + 2i$ **do**
      print("bingo")

Problem 2: Use mathematical induction to prove that $3^n \geq n2^n$ for $n \geq 0$. (Note: dealing with the base case may require some thought.)

Base case:
First let's check P(2)/n=2.
$3^2 \geq 2 \cdot 2^2$ $9 \geq 8$
Which is always true.

Inductive step:
Let's assume P(k) / n=k (where k is also ¿=2) and is true.
$3^k \geq k2^k$
Then P(k+1) / n=k+1
$3^{k+1} \geq (k+1)2^{k+1}$
$3 \cdot 3^k \geq ...$
$3 \cdot k2^k \geq ...$ rewriting in terms of assumption
$3k \geq (k+1)2$ divided 2^k
$k \geq 2$ distribute and subtract $2k$
Which is always true.

Last cases: We have n=0 and n=1 left to check in order to satisfy all solutions n¿=0. Let n=0. $3^0 \geq 0 \cdot 2^0$ 1 >= 0 Which is always true. Let n=1. $3^1 \geq 1 \cdot 2^1$ $3 \geq 2$ Which is always true.

$\therefore$ P holds in all cases.

Problem 3: Give the asymptotic values of the following functions, using the Θ-notation:

(a) $9n^2 + n^3/2 + 29n + 13$

(b) $\sqrt{n} + 7\log^5 n + 2n\log n$

(c) $1 + n^3\log^3 n + 21n^2\log^4 n$

(d) $\log^7 n + n2^n + 13n\log^9 n$

(e) $n^5 3^n + 4^n$

Justify your answer. Give an informal explanation using asymptotic relations between the functions n^c, $\log n$, and c^n.

Submission. To submit the homework, you need to upload the pdf file into ilearn by 11AM on Tuesday, January 20, **and** turn-in a paper copy in class.

Reminders. Remember that only LaTeX papers are accepted.