Maths for cube.c

Chahyn Ettaghi

September 2025

I recommend checking the Donut script and the PDF file first for a better understanding of the concepts, before reading this one, which is only a small part of the Donut script applied to the moving torus.

Rotations applied to the cube

This short note presents two rotation matrices in R^3 and their composition. We work in a right-handed Cartesian coordinate system with positive angles following the right-hand rule.

Let

$$v = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

be a vector in Cartesian coordinates.

Rotation about the X-axis

The rotation matrix that rotates a vector by an angle A about the X-axis is

$$R_x(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos A & -\sin A \\ 0 & \sin A & \cos A \end{bmatrix}.$$

Rotation about the Y-axis

The rotation matrix that rotates a vector by an angle B about the Y-axis is

$$R_y(B) = \begin{bmatrix} \cos B & 0 & \sin B \\ 0 & 1 & 0 \\ -\sin B & 0 & \cos B \end{bmatrix}.$$

Composition: $R_y(B)\, R_x(A)$

We apply first a rotation by A about the X-axis, then a rotation by B about the Y-axis. The composed transformation is

$$v' = R_y(B)\, R_x(A)\, v.$$

So we obtain:

$$v' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x\cos B + (y\sin A + z\cos A)\sin B \\ y\cos A - z\sin A \\ -x\sin B + (y\sin A + z\cos A)\cos B \end{bmatrix}.$$