Segment trees

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Problems we would like to solve efficiently

A	**3**	**1**	**-2**	**4**	**6**	**13**	**2**	**0**
	0	1	2	3	4	5	6	7

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(Static) Prefix sums

- `sum`(i) reports the sum of the first i+1 integers
- `update`(i, x) sets A[i] to x

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`sum`(3) = 6
`sum`(5) = 25

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`update`(2, 5)

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`sum`(3) = 6
`sum`(5) = 25
`update`(2, 5)
`sum`(5) = 32

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Range `MIN` (`MAX`) queries

Report the `MIN` (`MAX`) in A[i,j]

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Range `MIN` (`MAX`) queries

Report the `MIN` (`MAX`) in A[i,j]

`min`(1,3) = -2

`max`(4,7) = 0

Problems we would like to solve efficiently

A	3	1	-2	4	6	13	2	0
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Range `MIN` (`MAX`) queries

Report the `MIN` (`MAX`) in A[i,j]

`min`(1,3) = -2
`max`(4,7) = 0

Range `SUM` queries

Report the sum of the elements in A[i,j]

Problems we would like to solve efficiently

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Range `MIN` (`MAX`) queries

Report the `MIN` (`MAX`) in A[i,j]

`min`(1,3) = -2
`max`(4,7) = 0

Range `SUM` queries

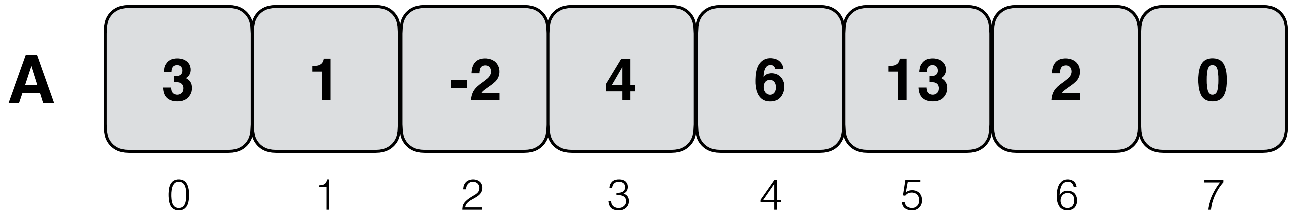
Report the sum of the elements in A[i,j]

`sum`(1,3) = 3
`sum`(4,7) = 21

Any solutions?

A	**3**	**1**	**-2**	**4**	**6**	**13**	**2**	**0**
	0	1	2	3	4	5	6	7

Any solutions?



1. Do nothing
2. Pre-calculate all queries

Any solutions?

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(Static) Prefix sums

1.
   `update`: O(1)
   `sum`: O(n)
   Space: no auxiliary space
2.
   `update`: O(n)
   `sum`: O(1)
   Space: no auxiliary space

3	4	2	6	12	25	27	27
0	1	2	3	4	5	6	7

Any solutions?

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Range `MIN` (`MAX`) and `SUM` queries

1.
   Query time: O(n)
   Space: no auxiliary space
2.
   Query time: O(1)
   Space: $O(n^2)$
   Building time: $O(n^2)$

0	1	2	3	4	5	6	7	
3	**1**	**-2**	**-2**	**-2**	**-2**	**-2**	**-2**	0
	1	**-2**	**-2**	**-2**	**-2**	**-2**	**-2**	1
		-2	**-2**	**-2**	**-2**	**-2**	**-2**	2
			4	**4**	**4**	**2**	**0**	3
				6	**6**	**2**	**0**	4
					13	**2**	**0**	5
						2	**0**	6
							0	7

An efficient solution

Remember
An efficient solution is the one that gives
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Idea
Impose a complete (static) binary tree over the array: a **segment tree**.

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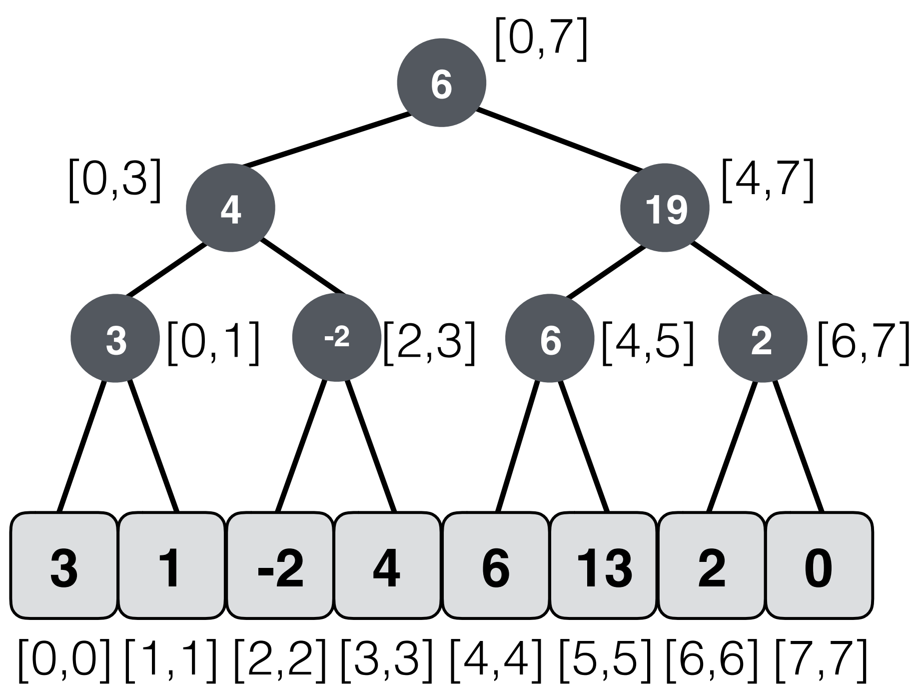

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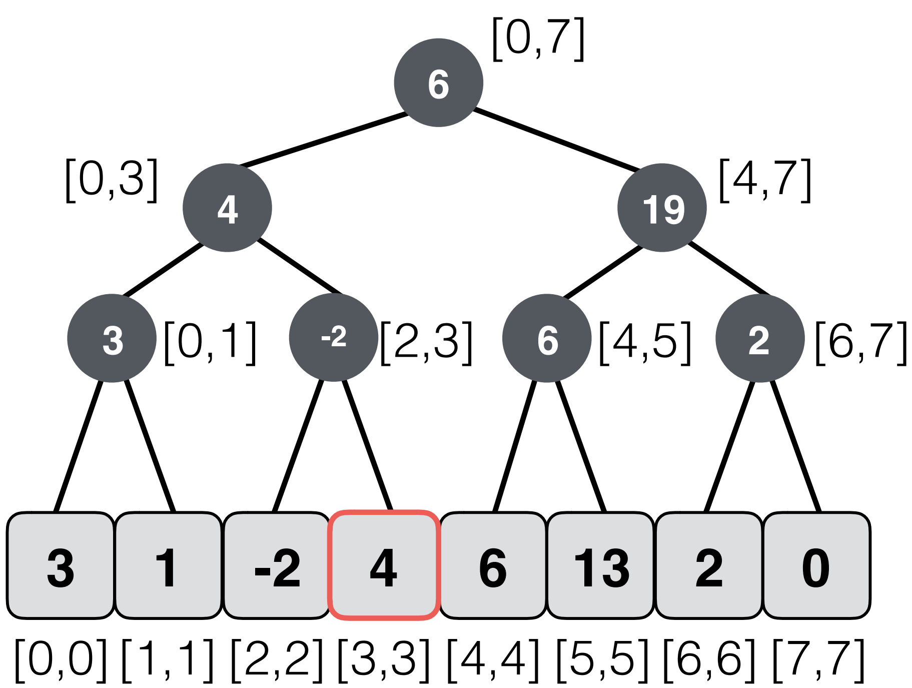

`sum`(3) = (4) +

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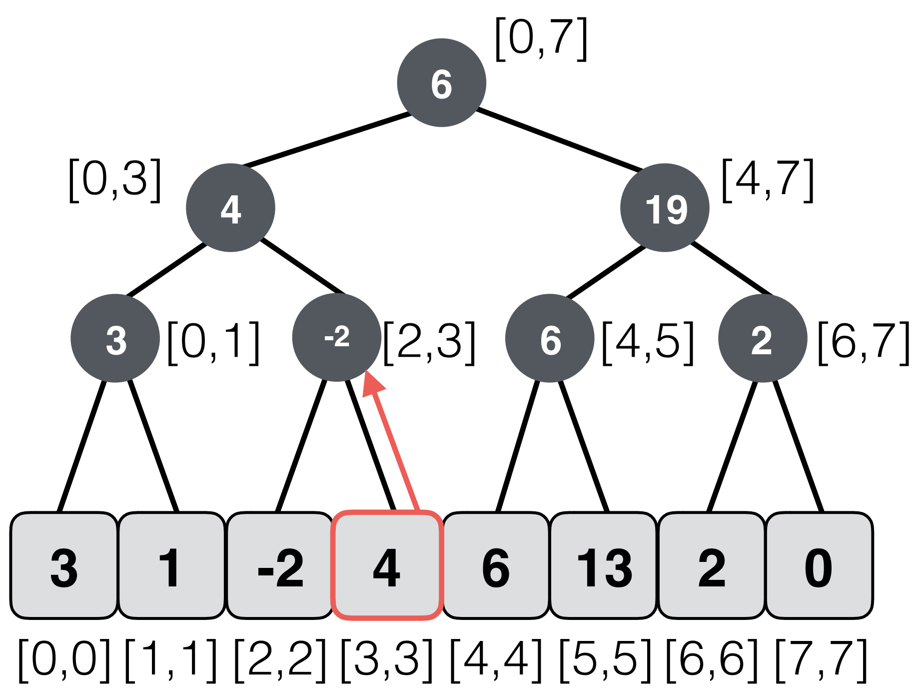

`sum`(3) = (4) + (-2) +

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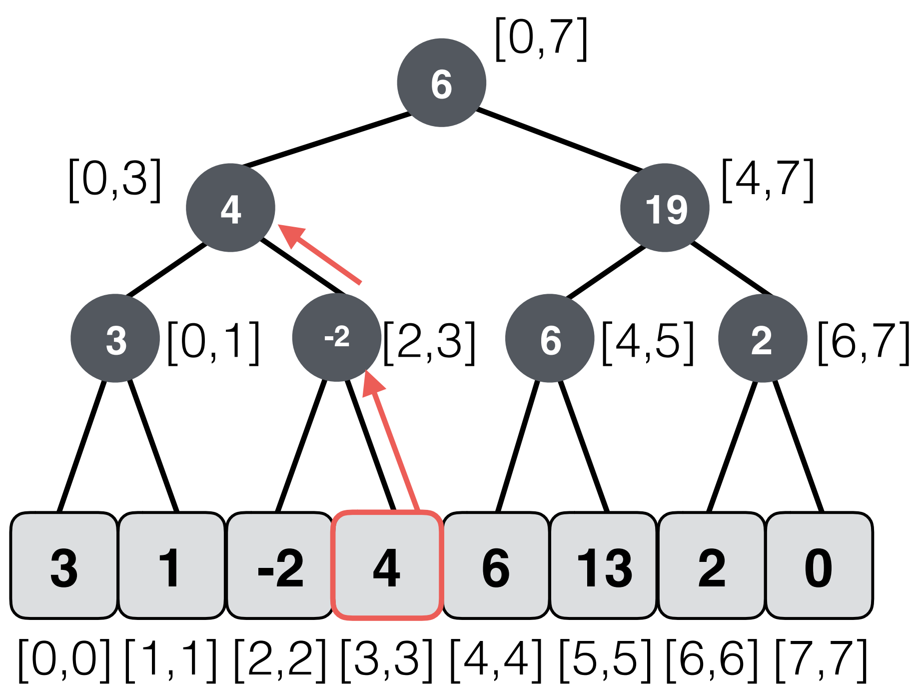

`sum`(3) = (4) + (-2) + (4) = 6

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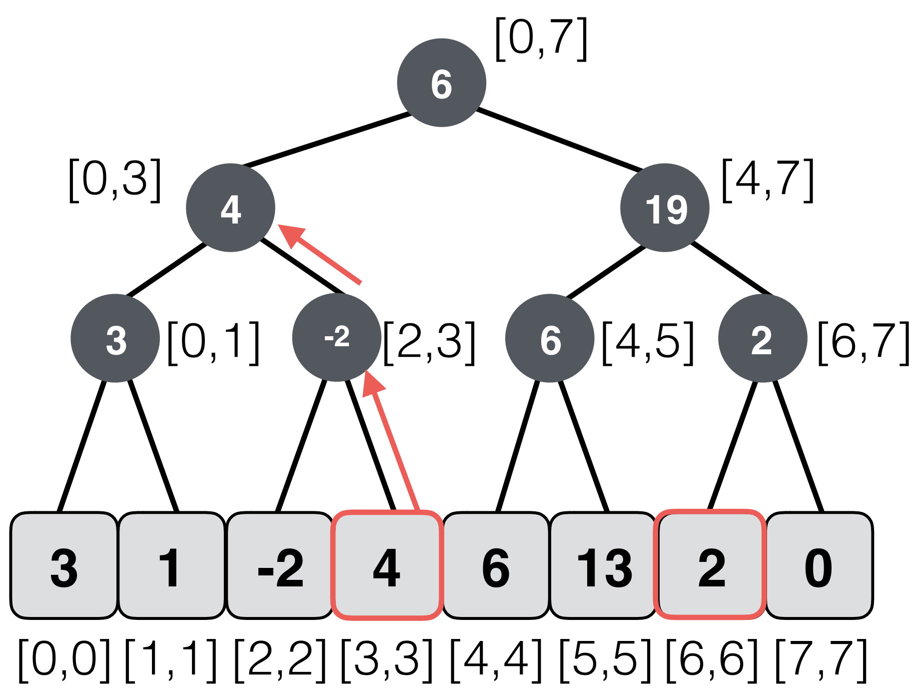

`sum`(3) = (4) + (-2) + (4) = 6

`sum`(6) = (2) +

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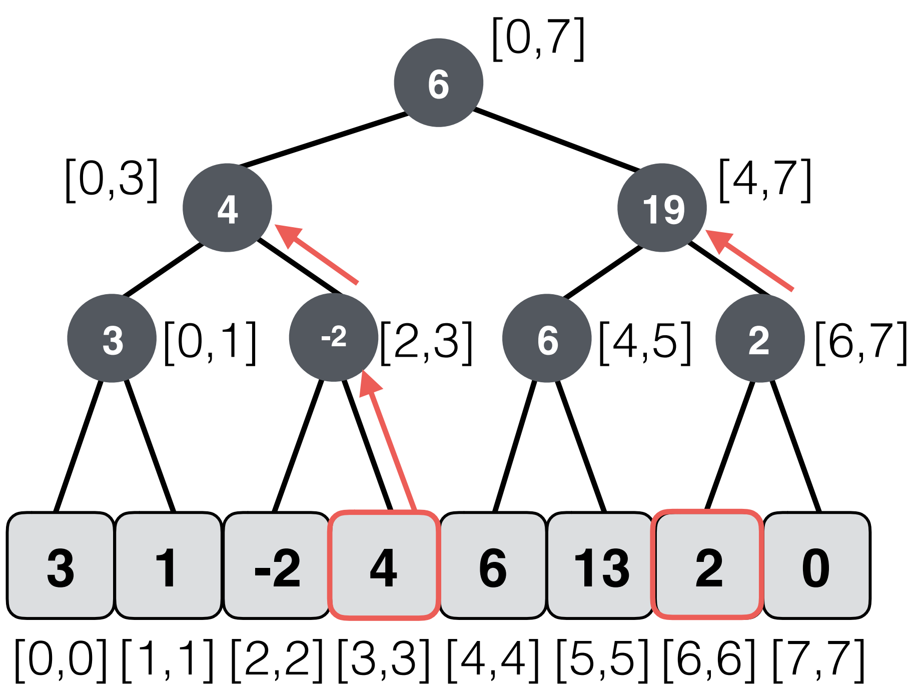

`sum`(3) = (4) + (-2) + (4) = 6

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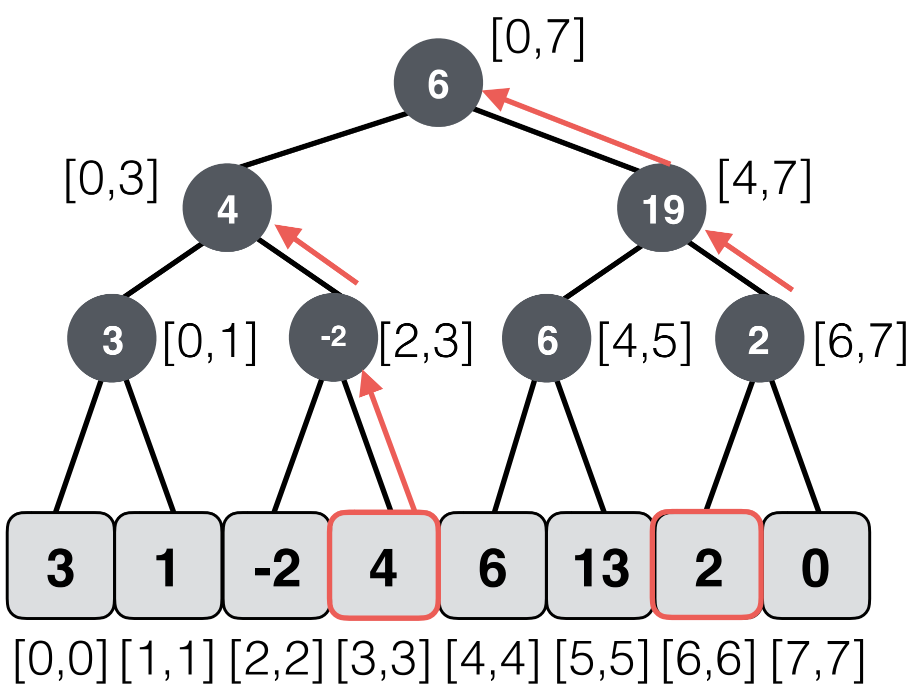

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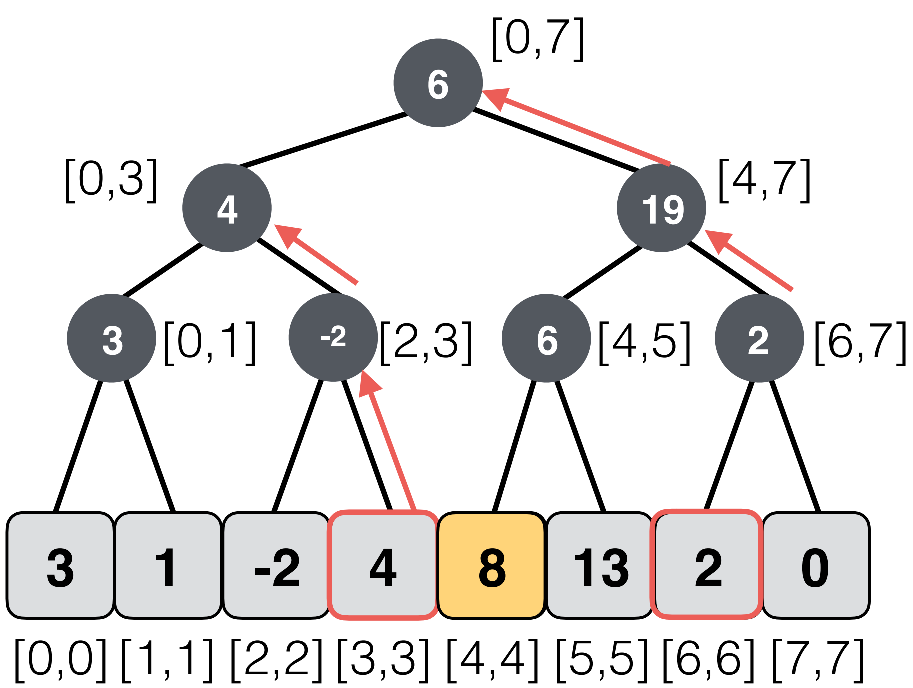

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`update`(4, 8)

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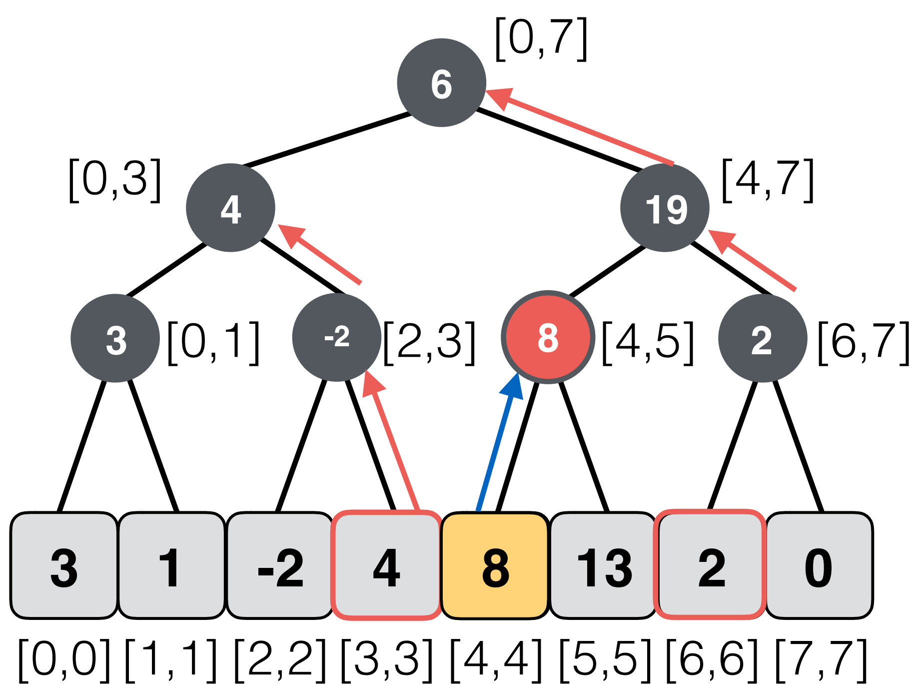

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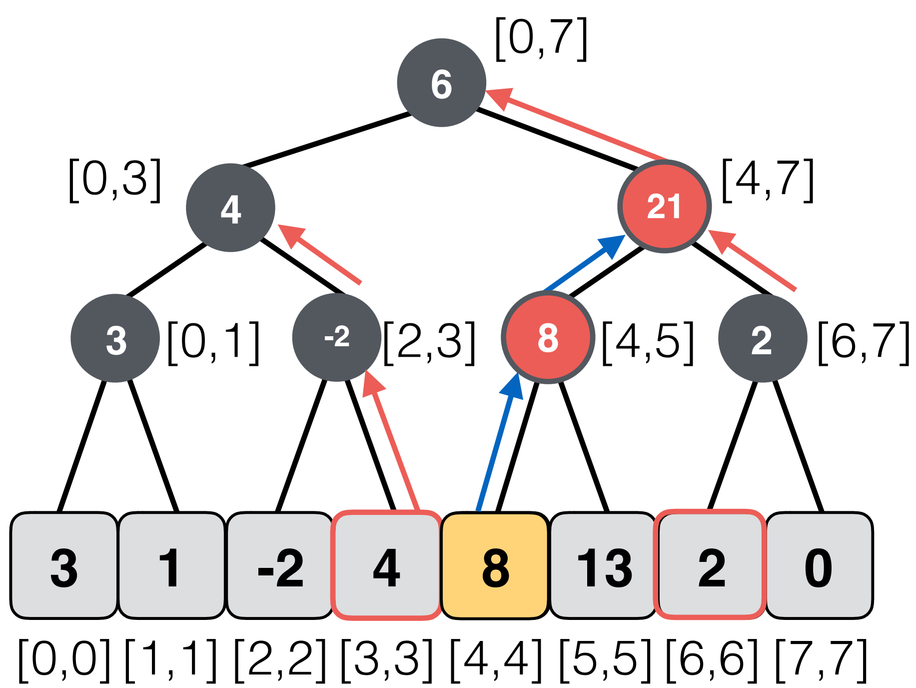

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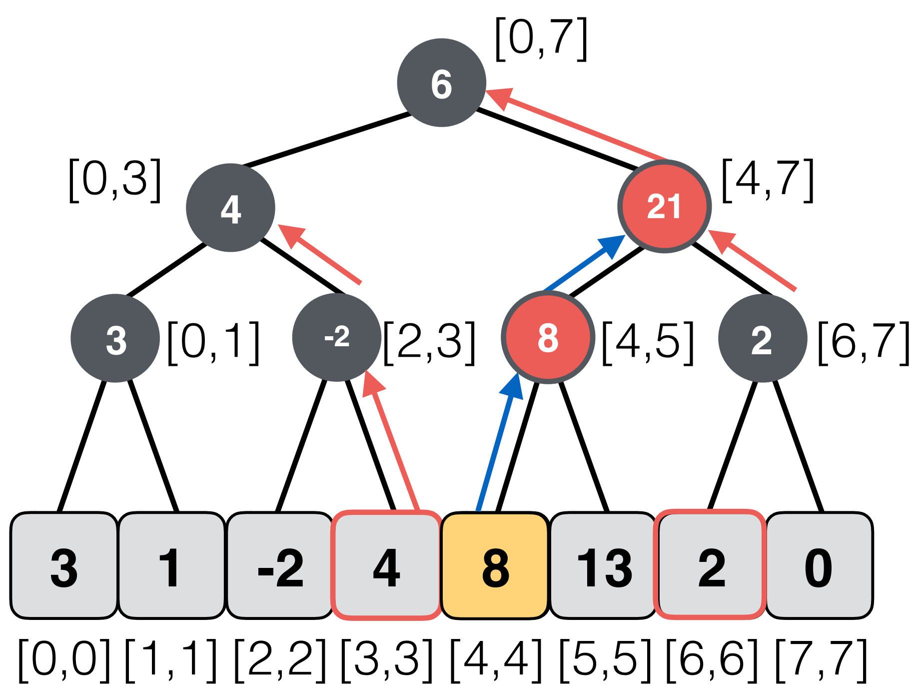

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`sum` and `update` in O(log n)

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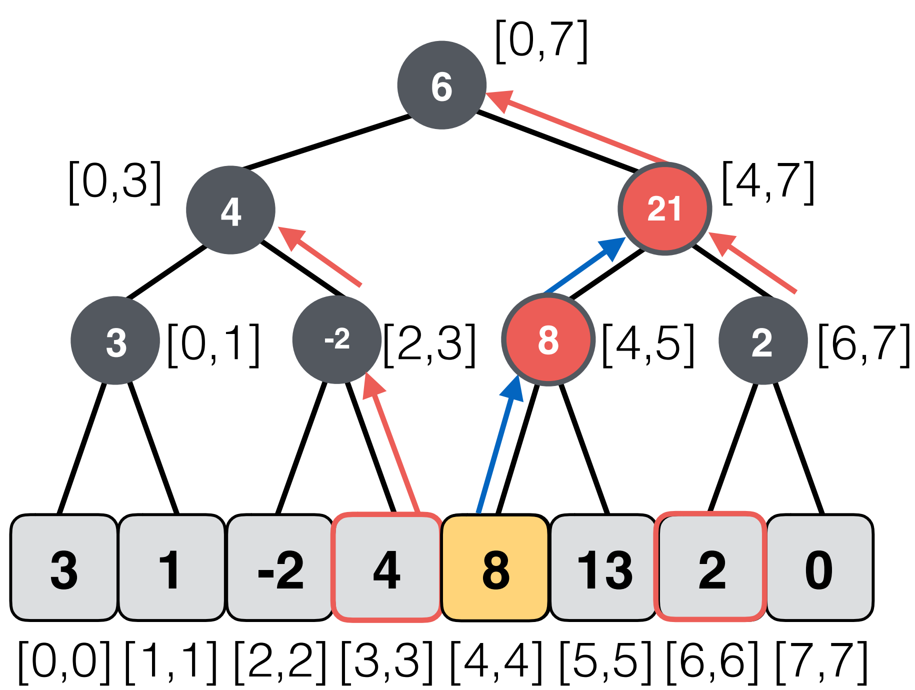

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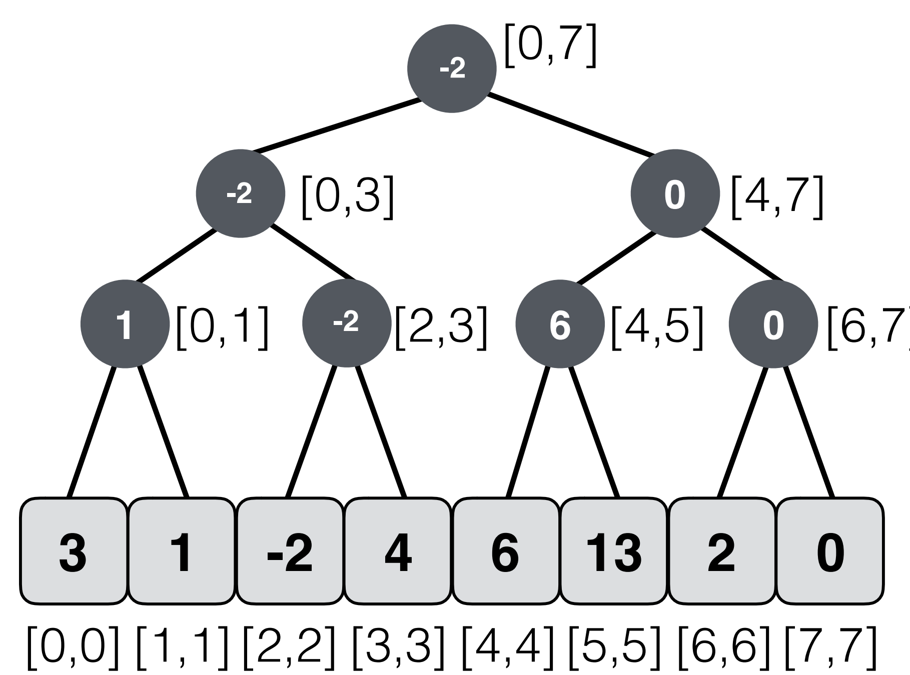
`sum` and `update` in O(log n)

What we consider next, stay tuned!

Range `MIN` queries

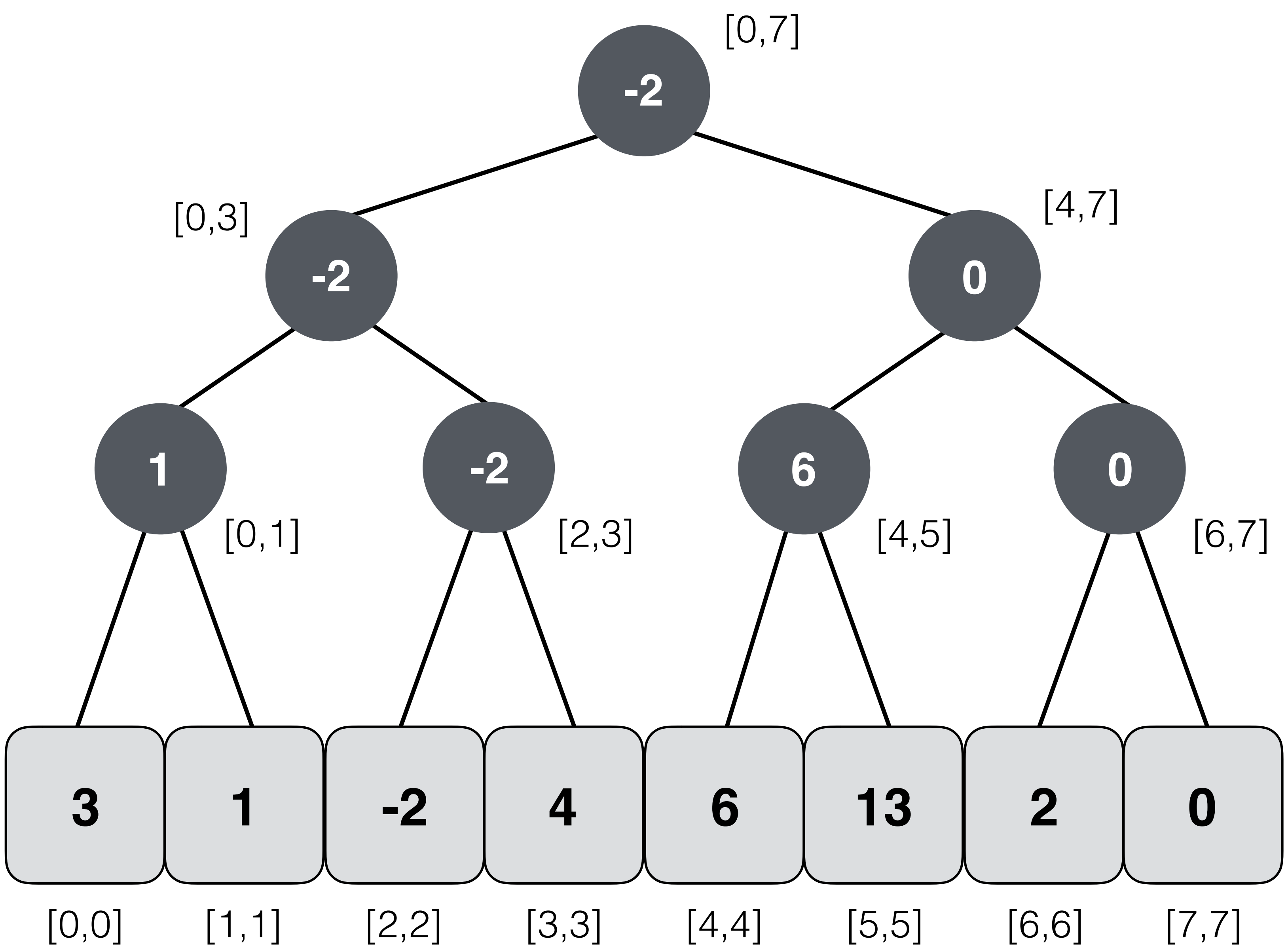


Range `MIN` Queries with Segment Trees

Consider a segment tree with n leaves (2n - 1 nodes in total).

Given an interval [i,j], search for it in the tree.

- **partial overlap**: search in both subtrees
- **total overlap**: stop and return value at node
- **no overlap**: stop and return `MAX` value



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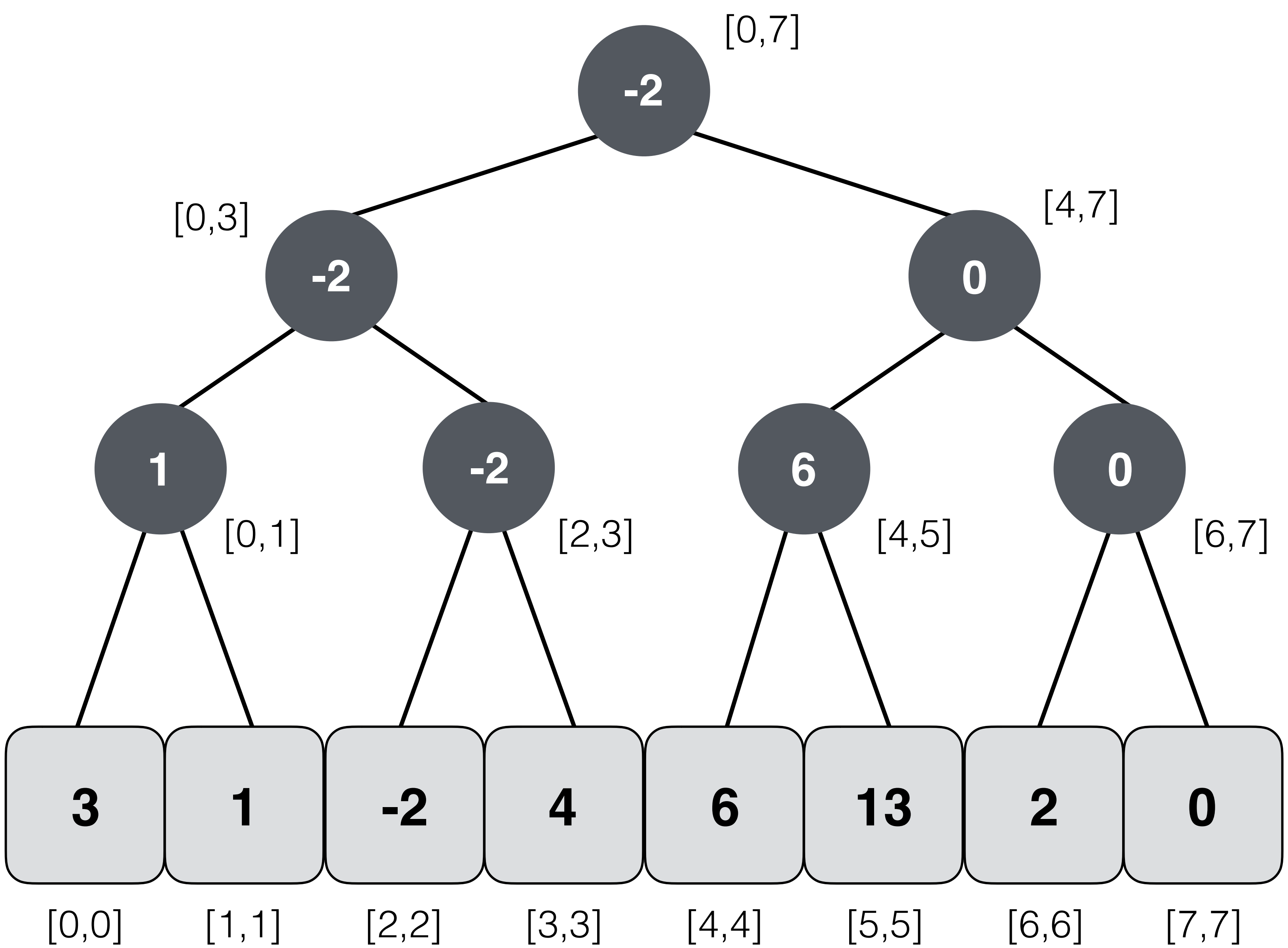

`min`(1,3)

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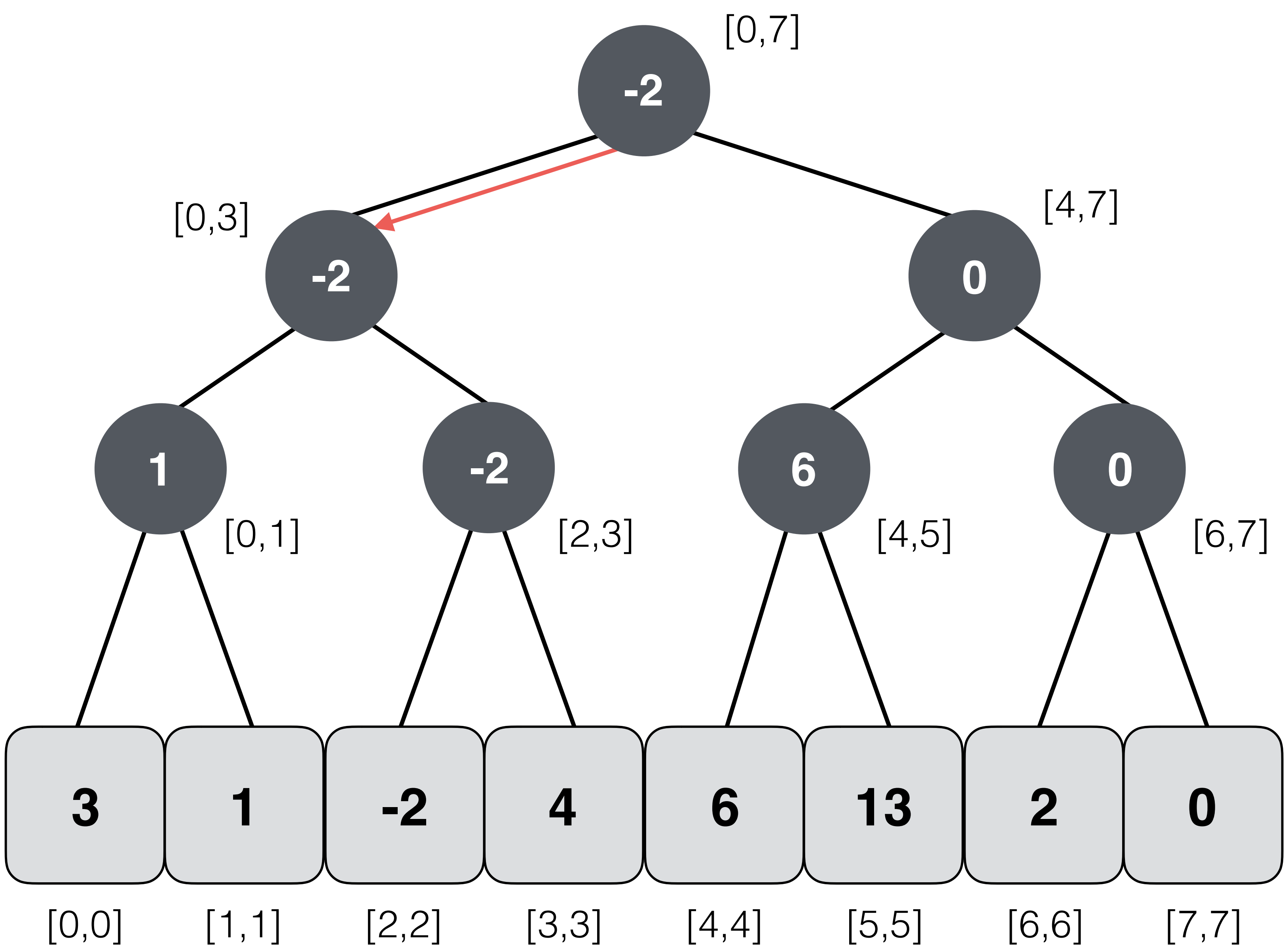

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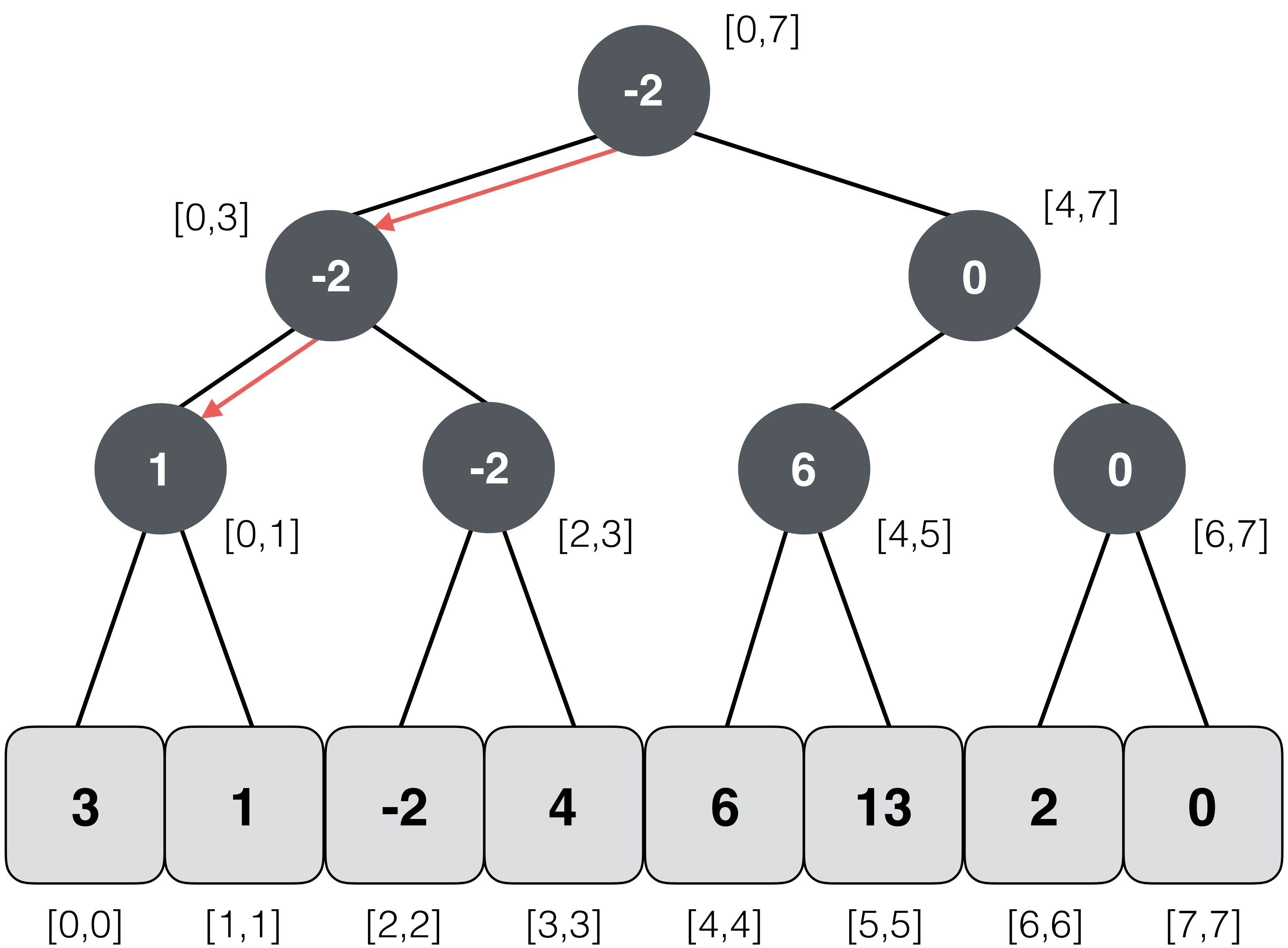

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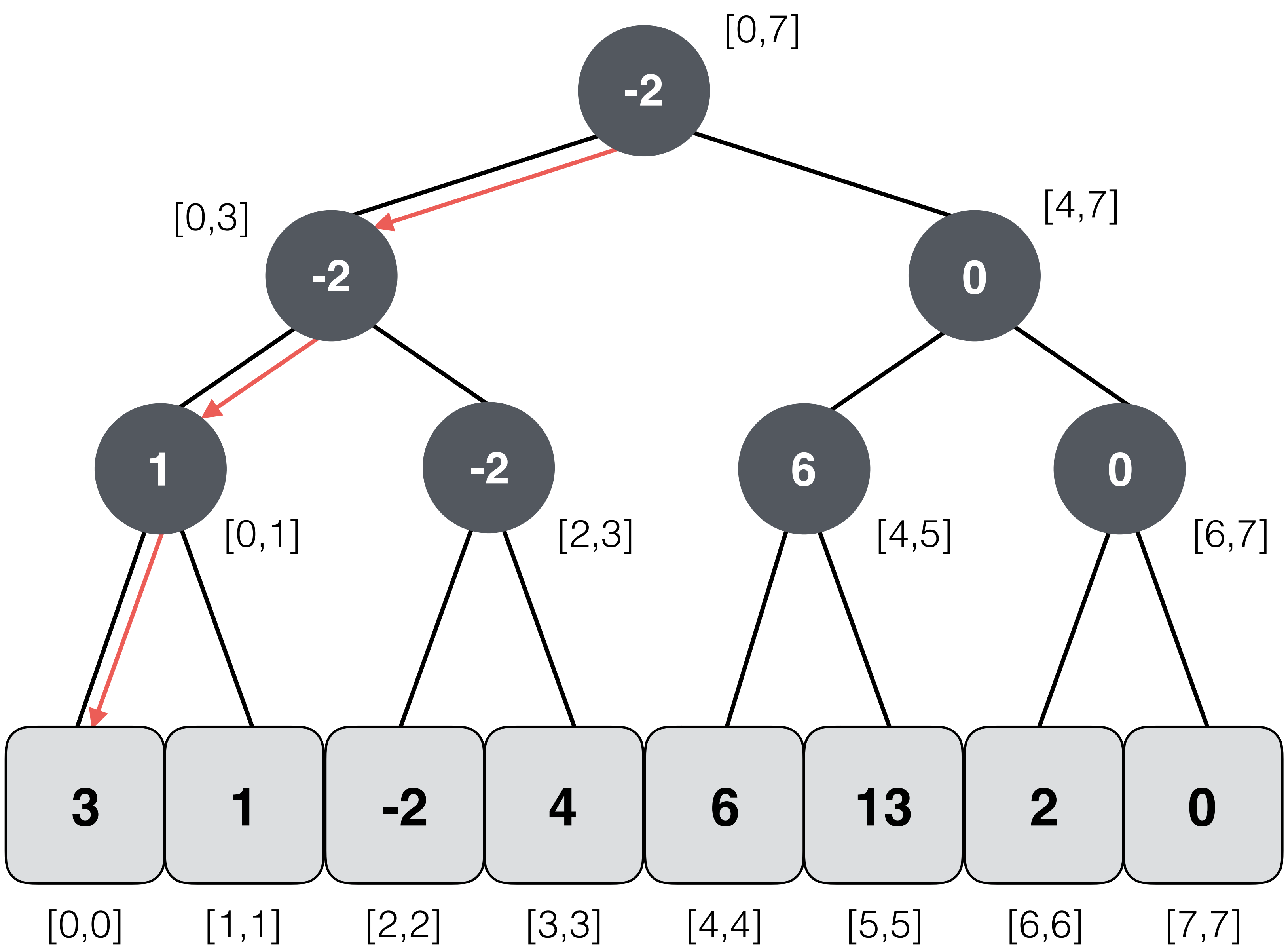

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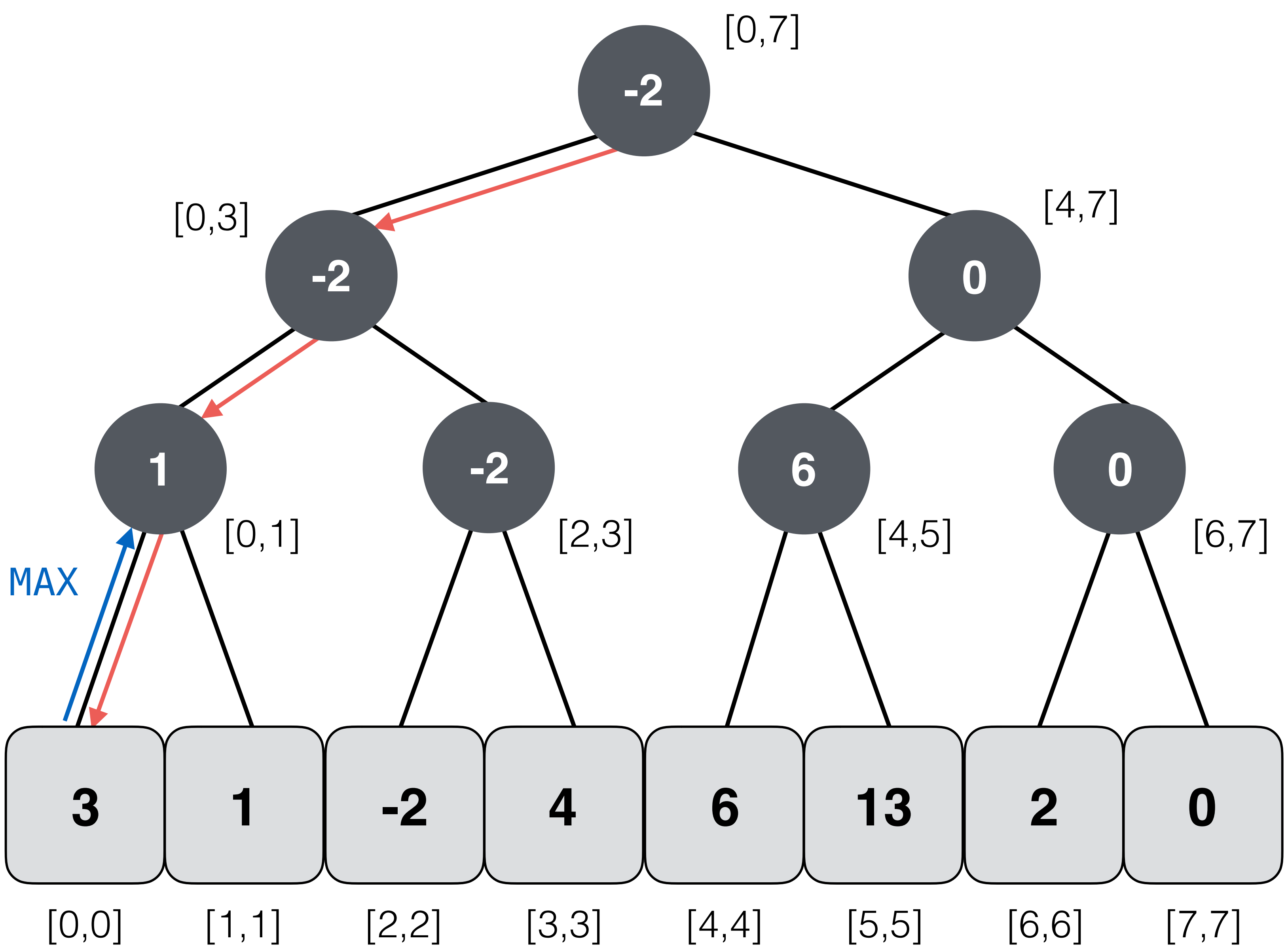

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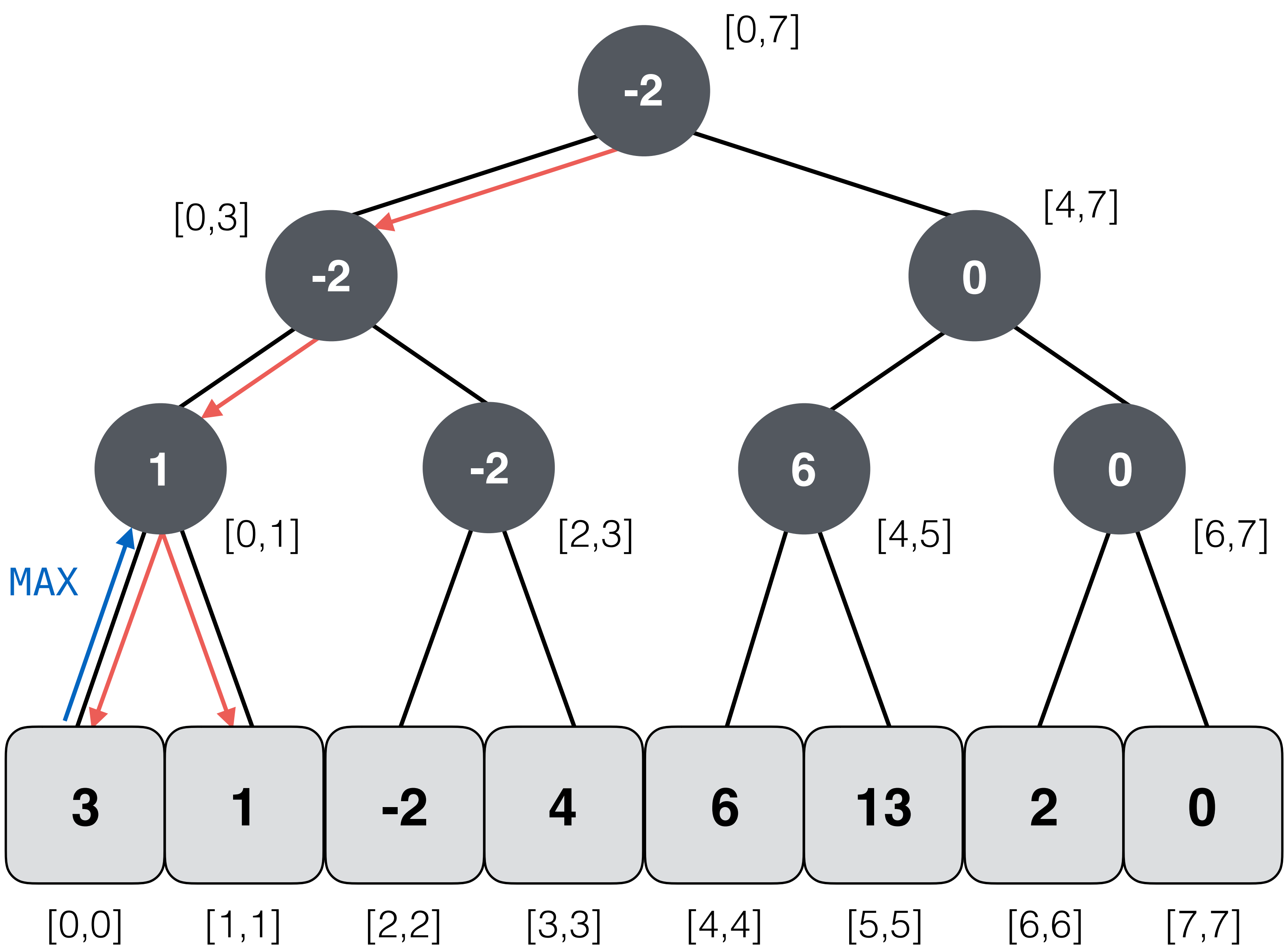

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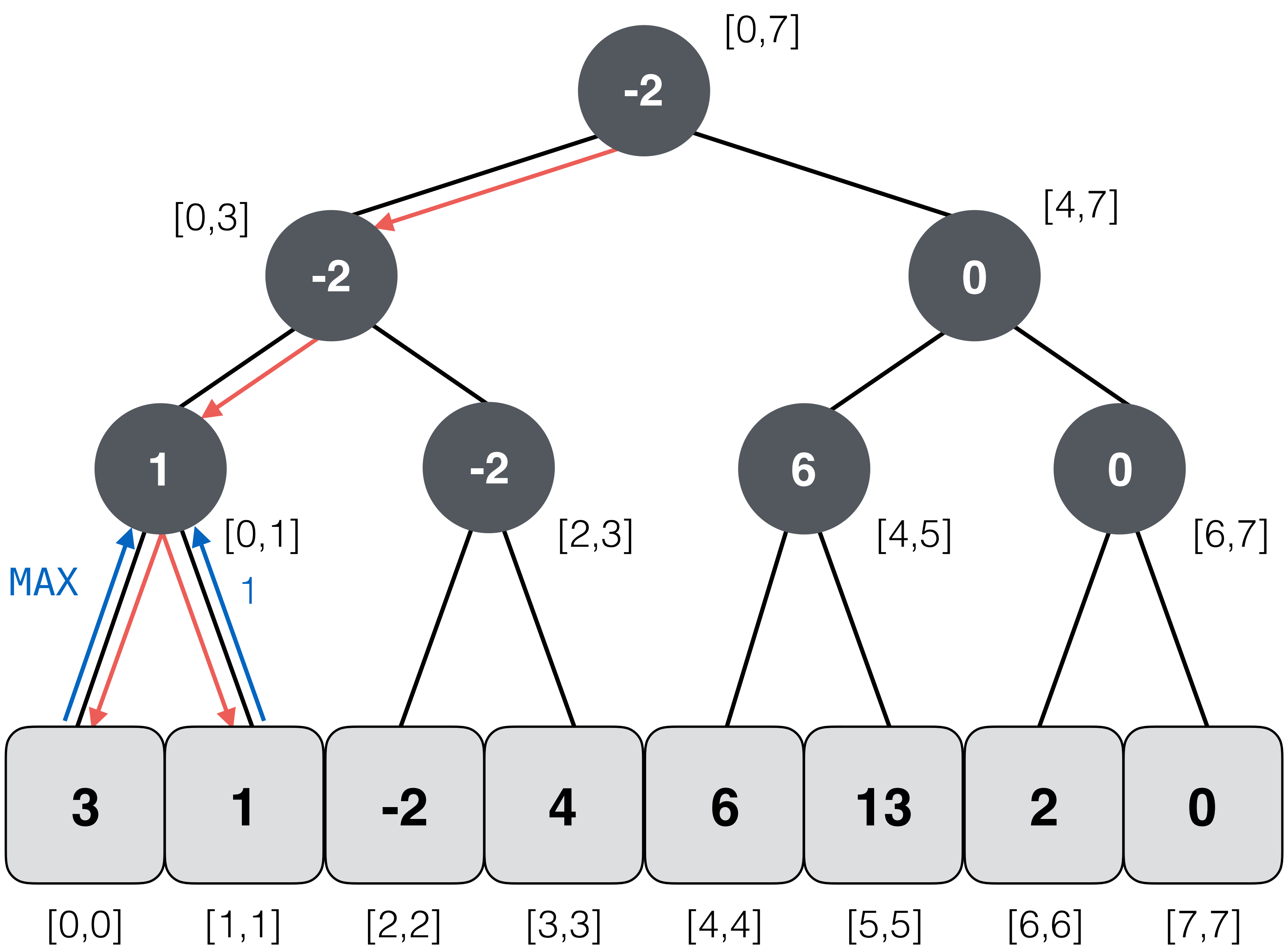

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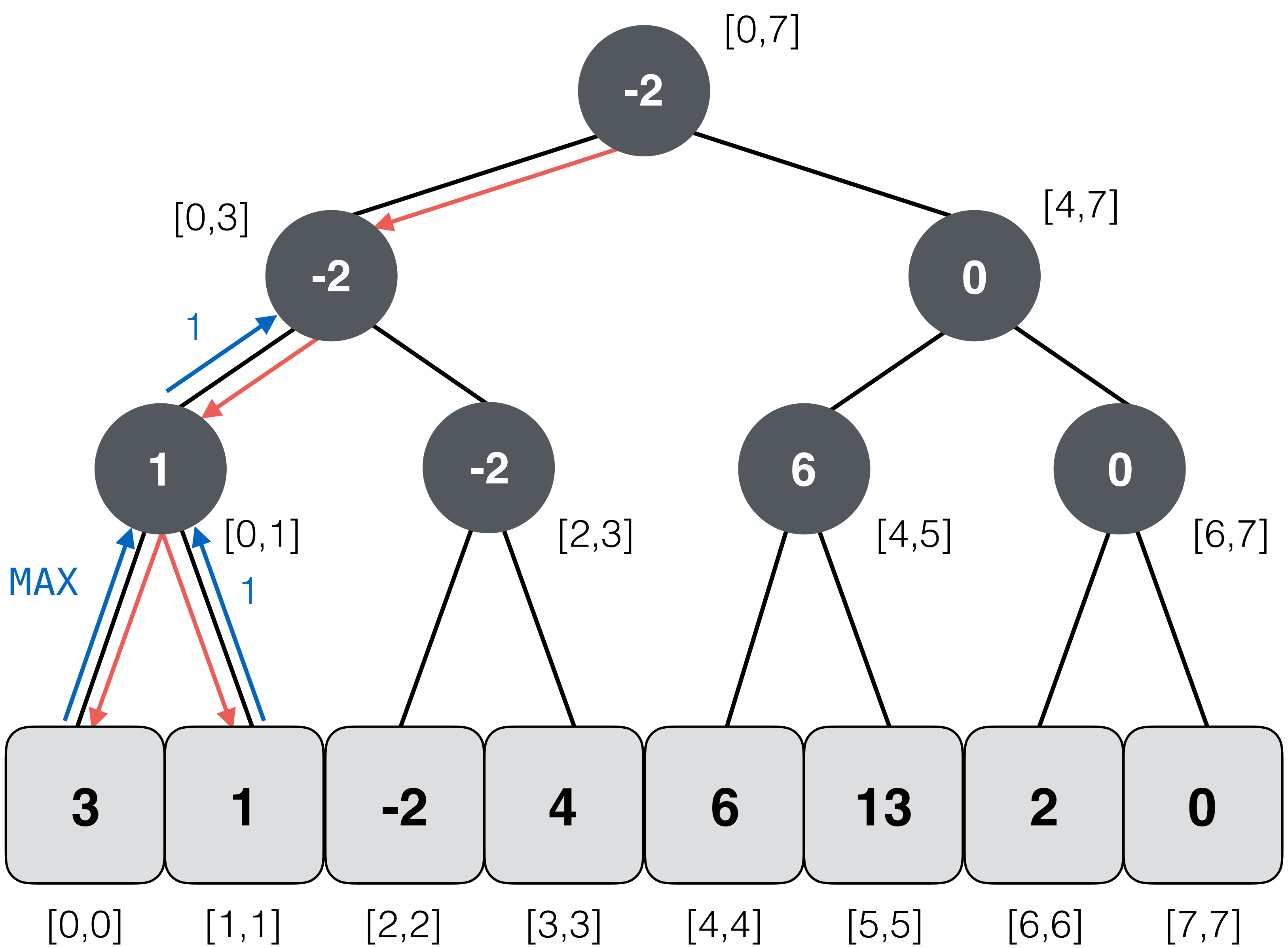

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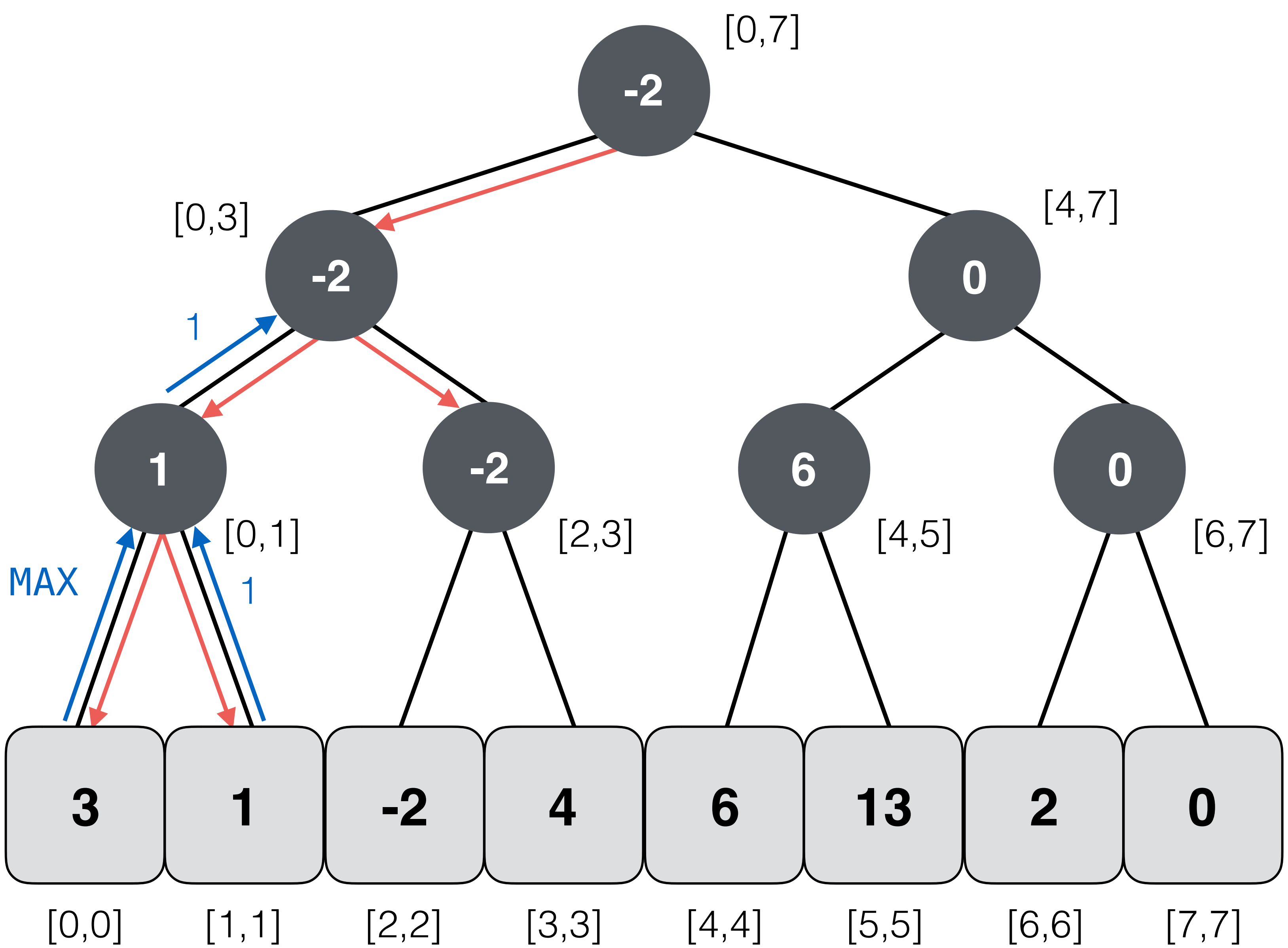

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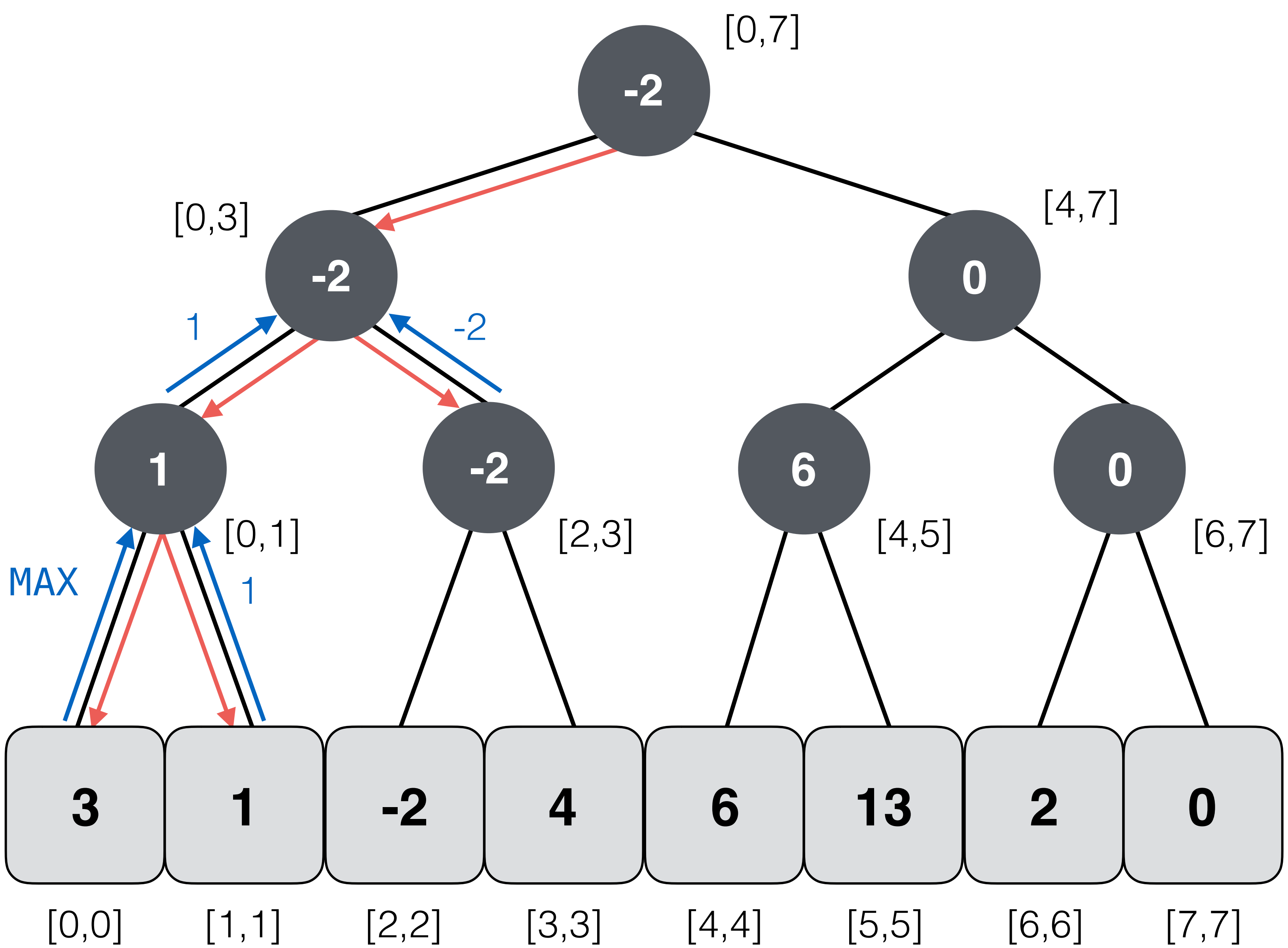

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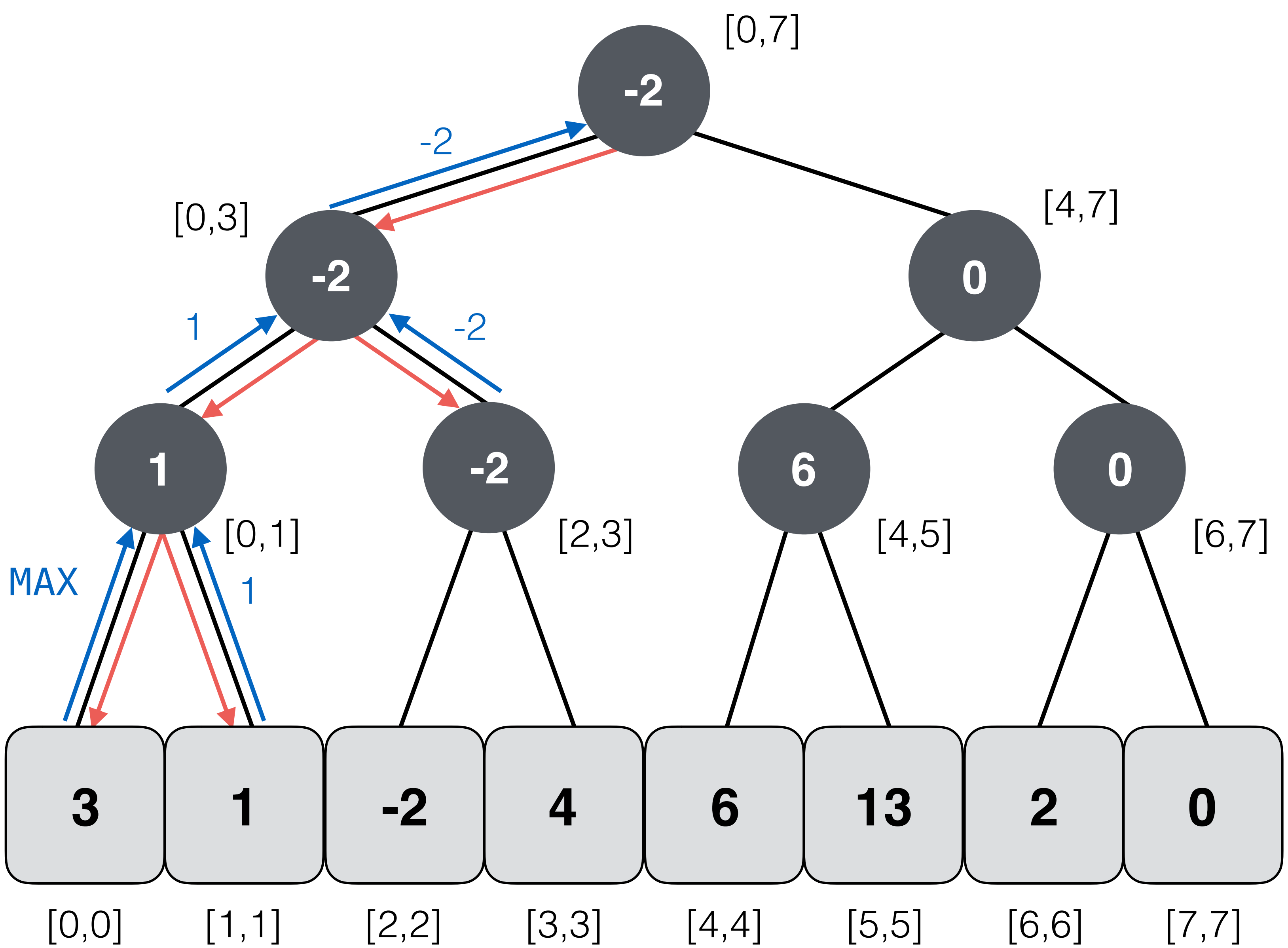

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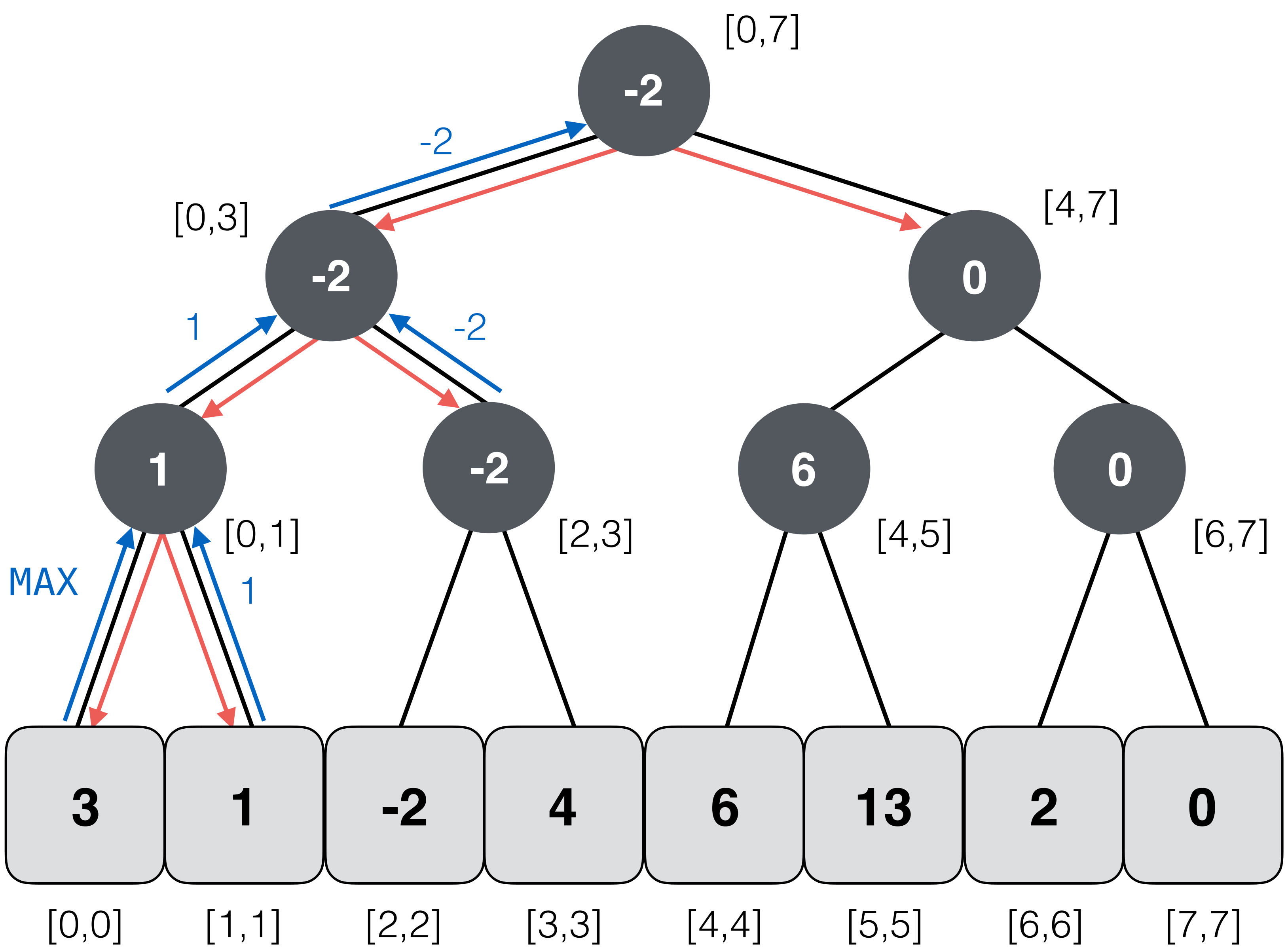

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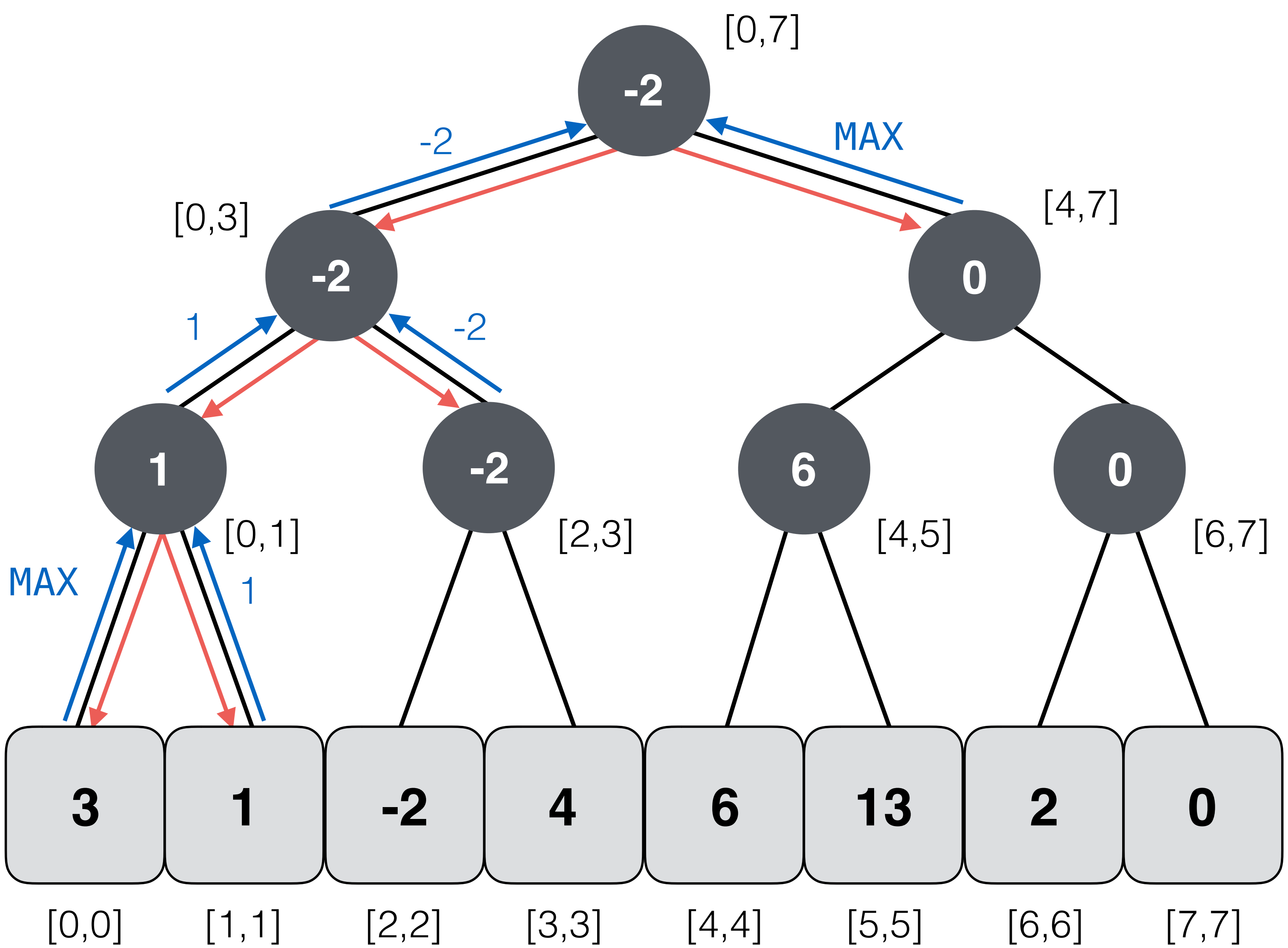

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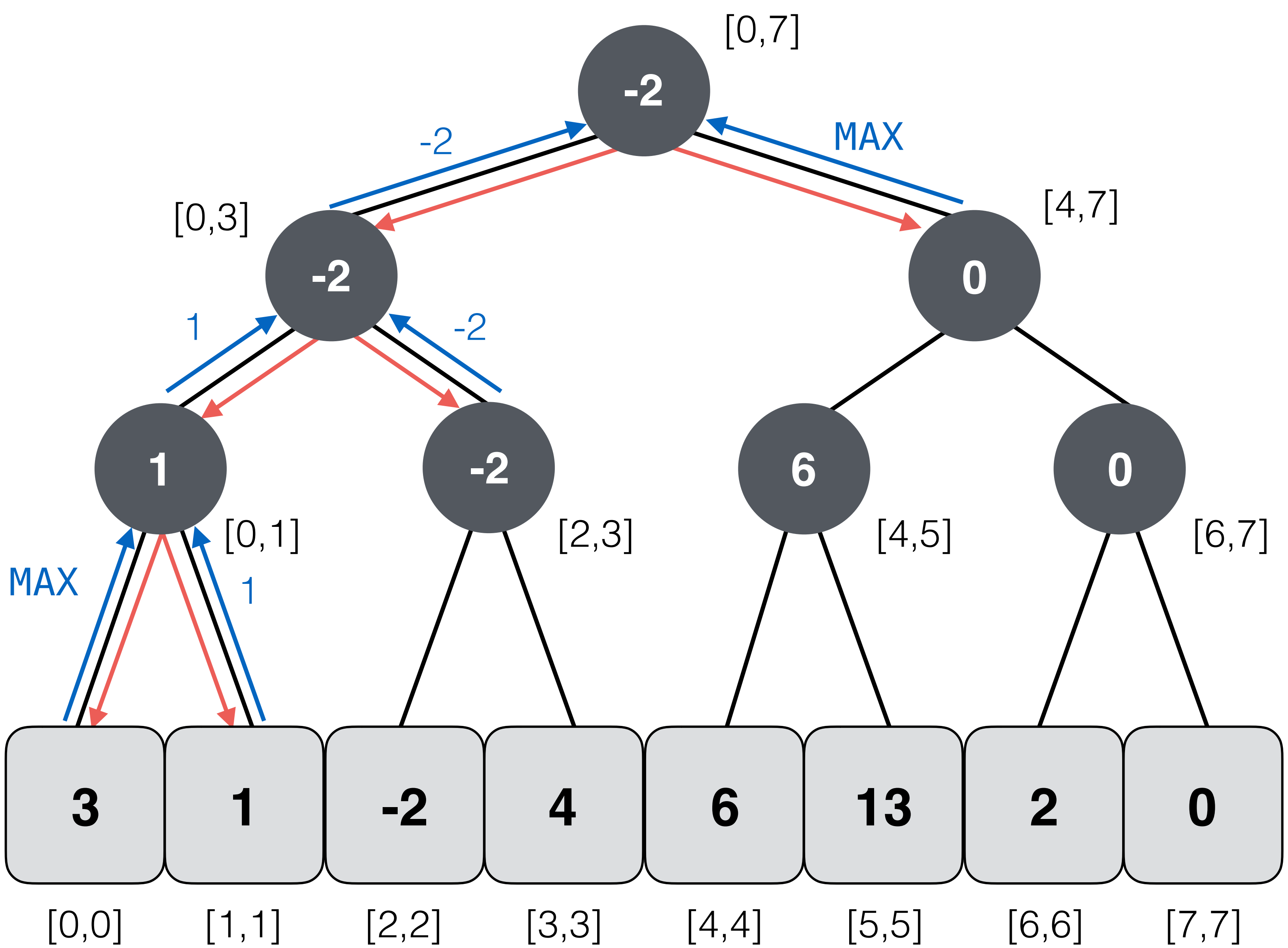

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Range `MIN` Queries with Segment Trees

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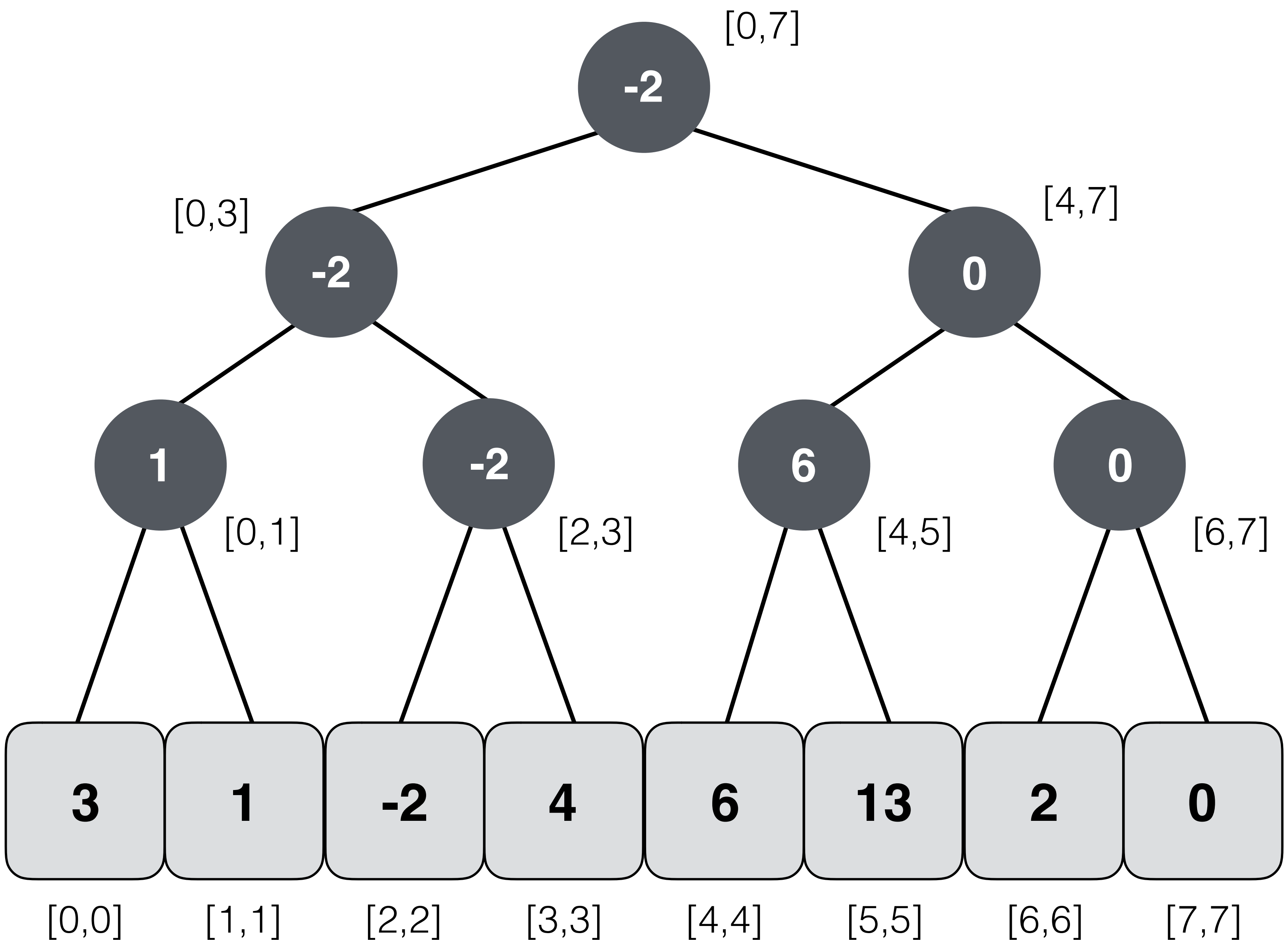

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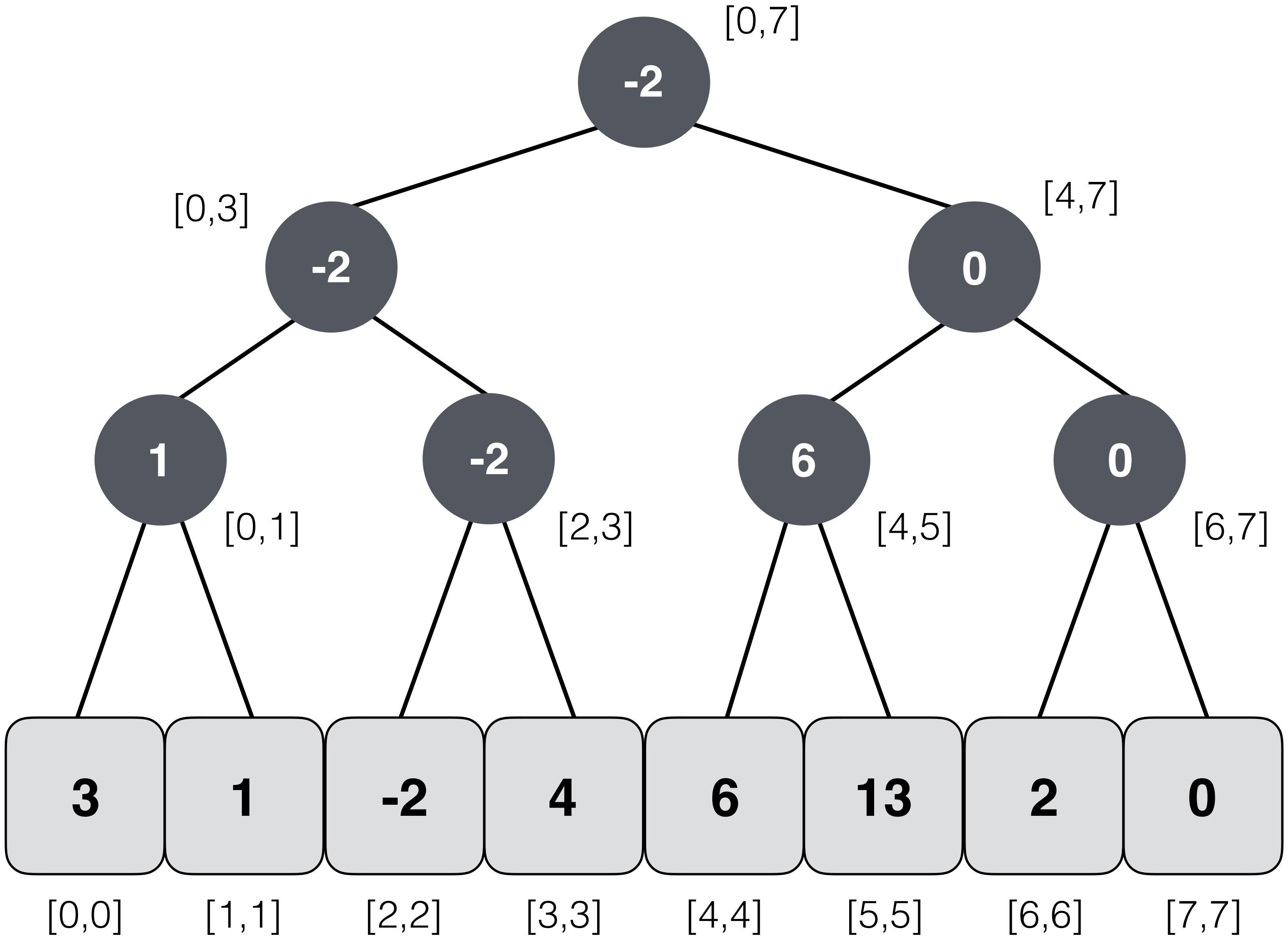

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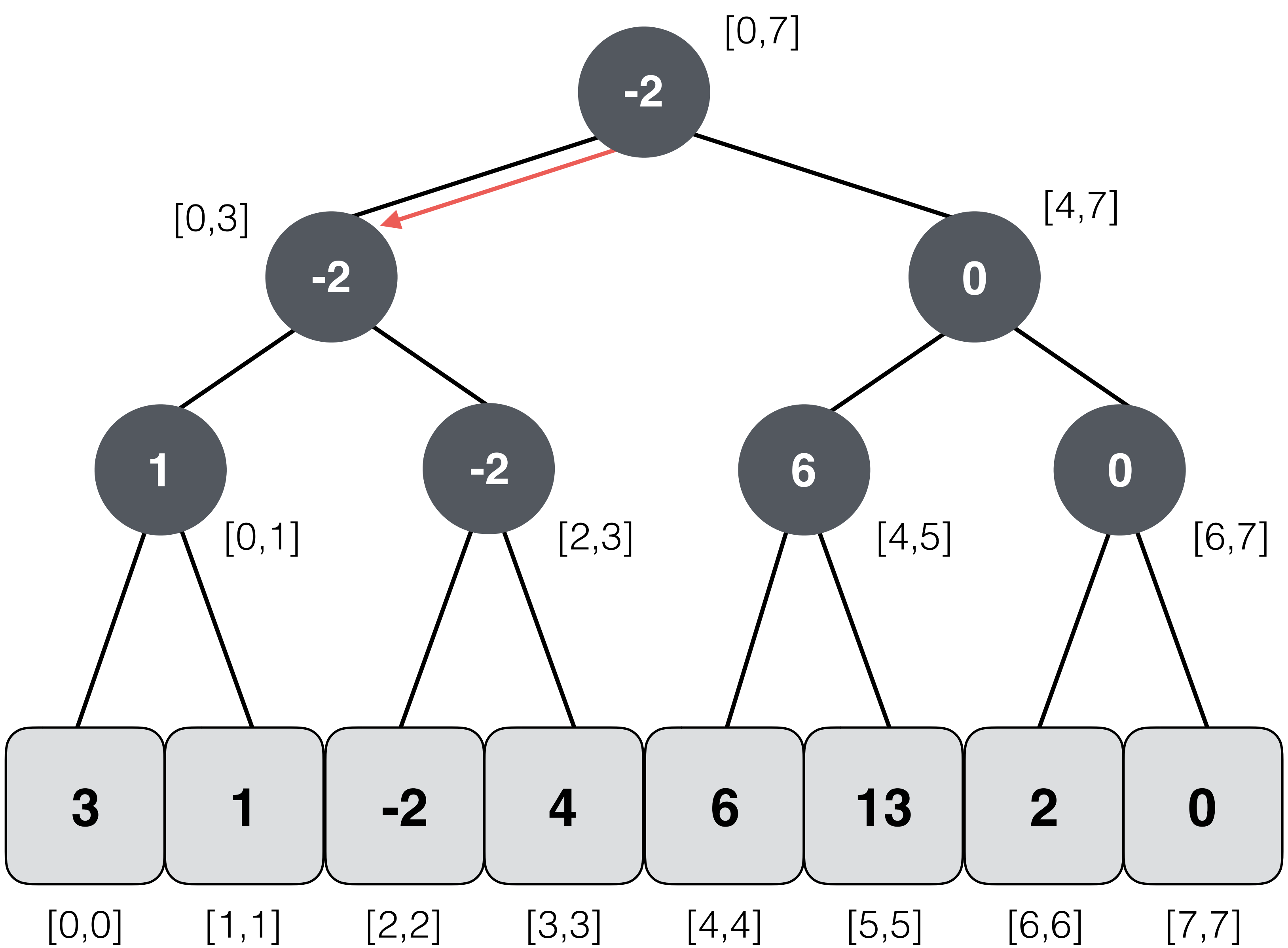

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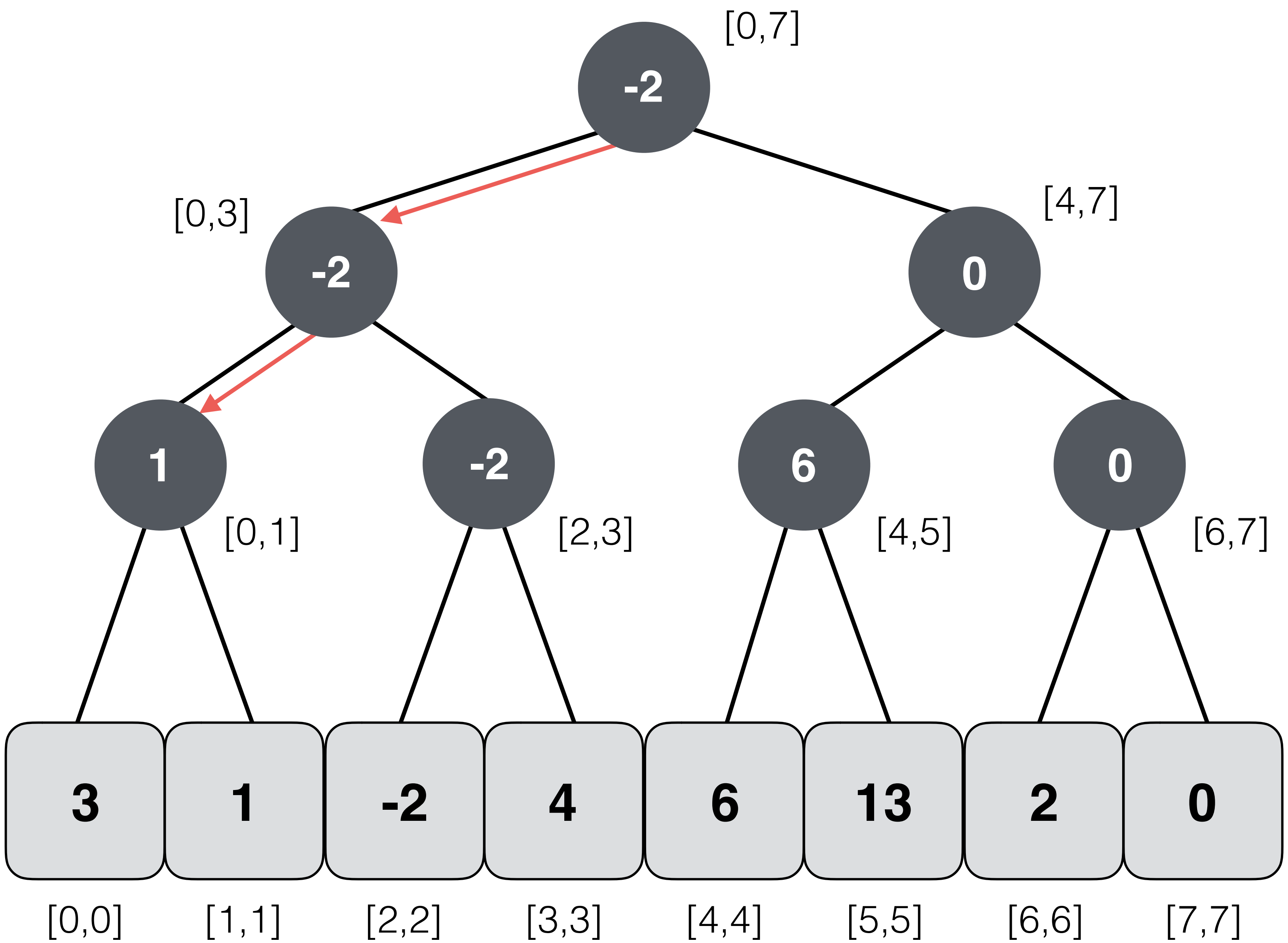

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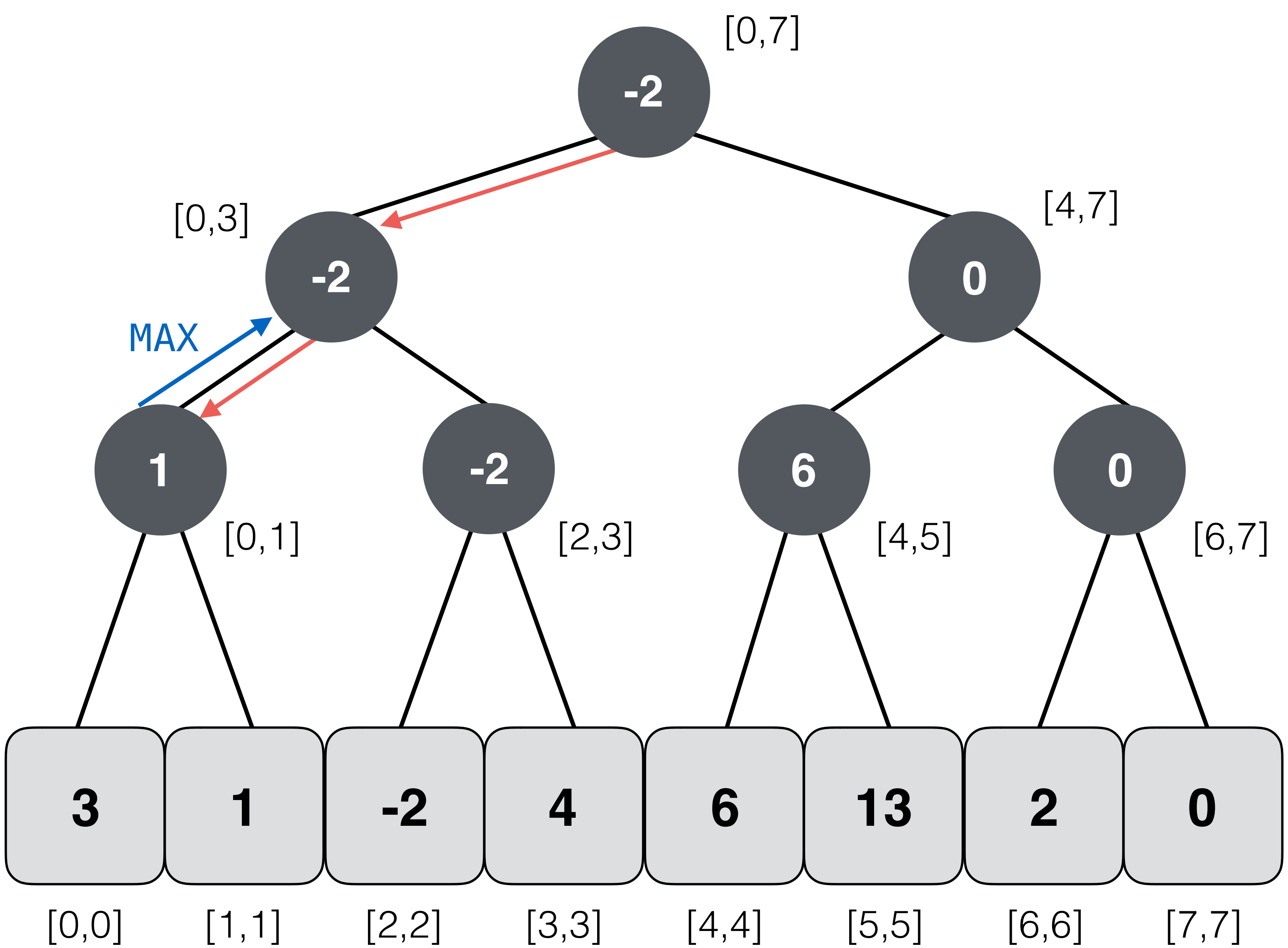

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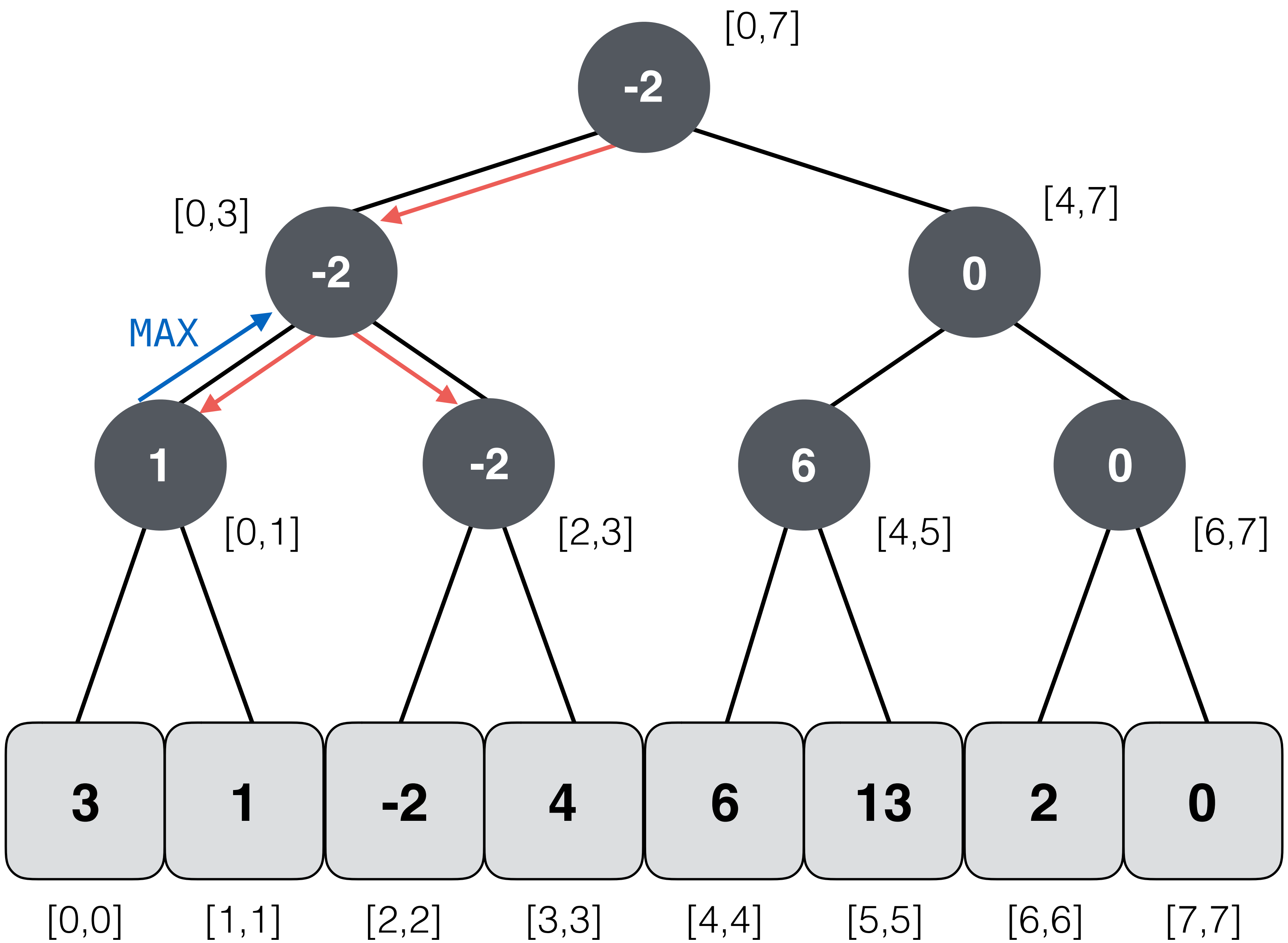

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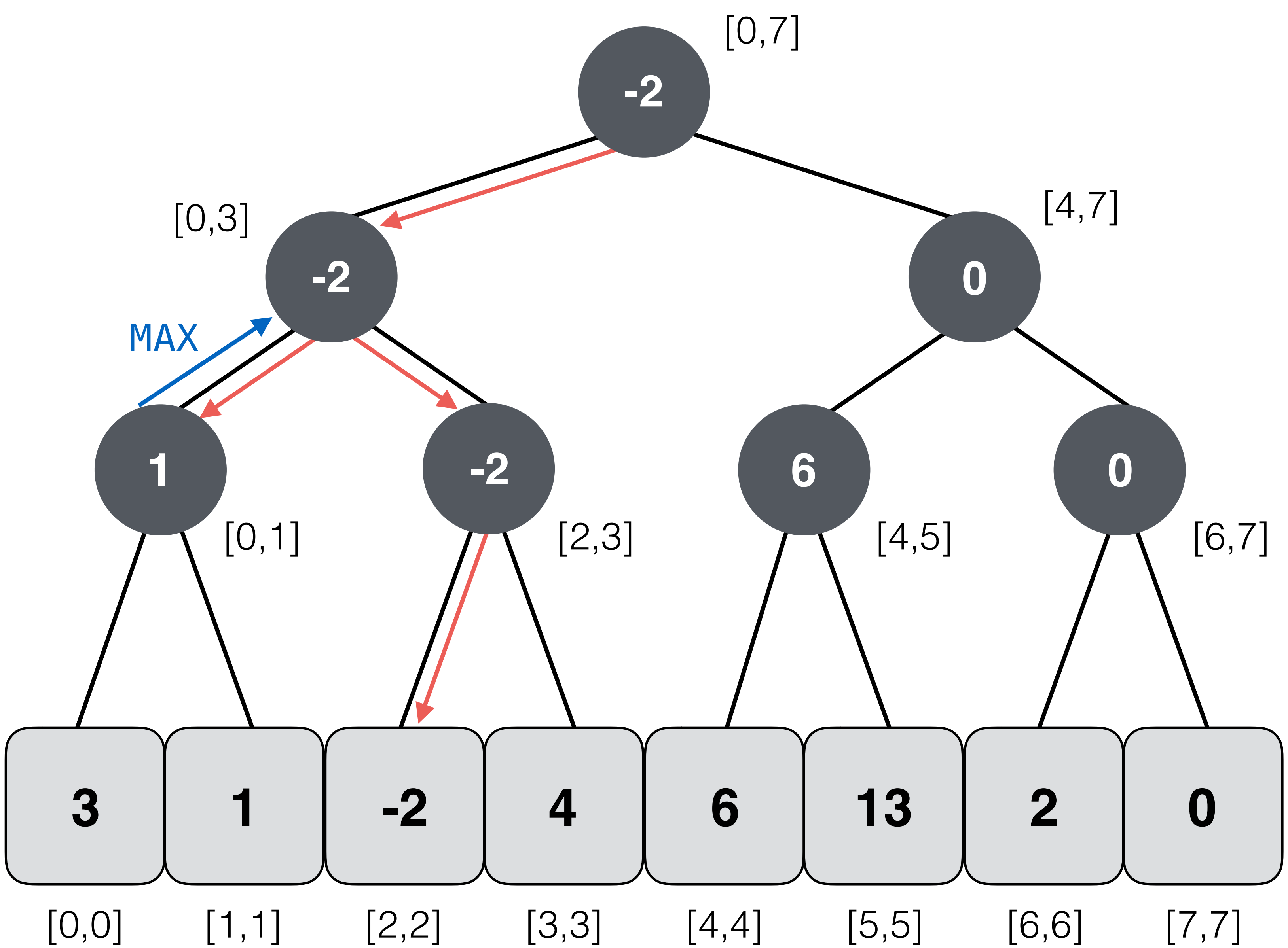

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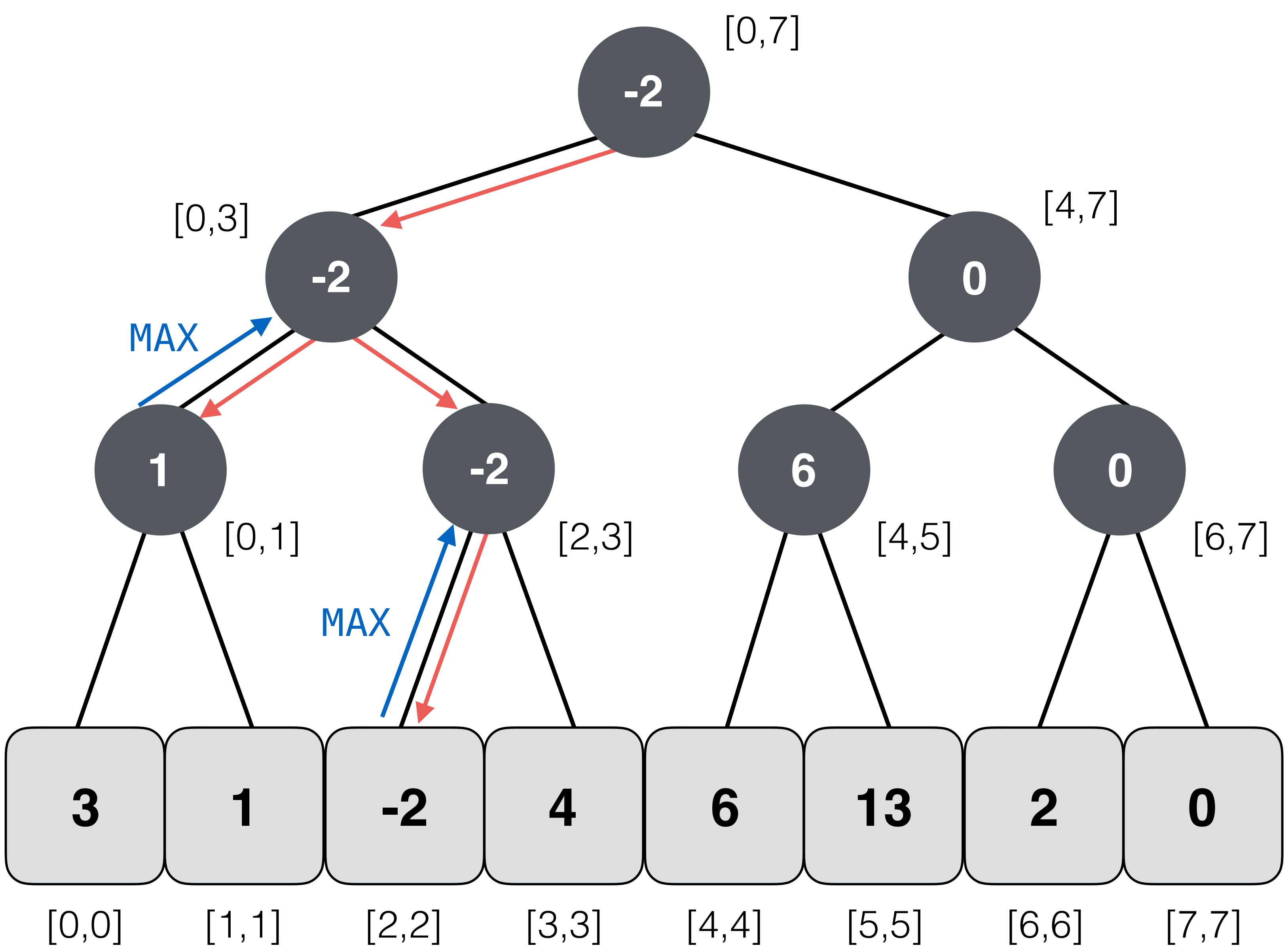

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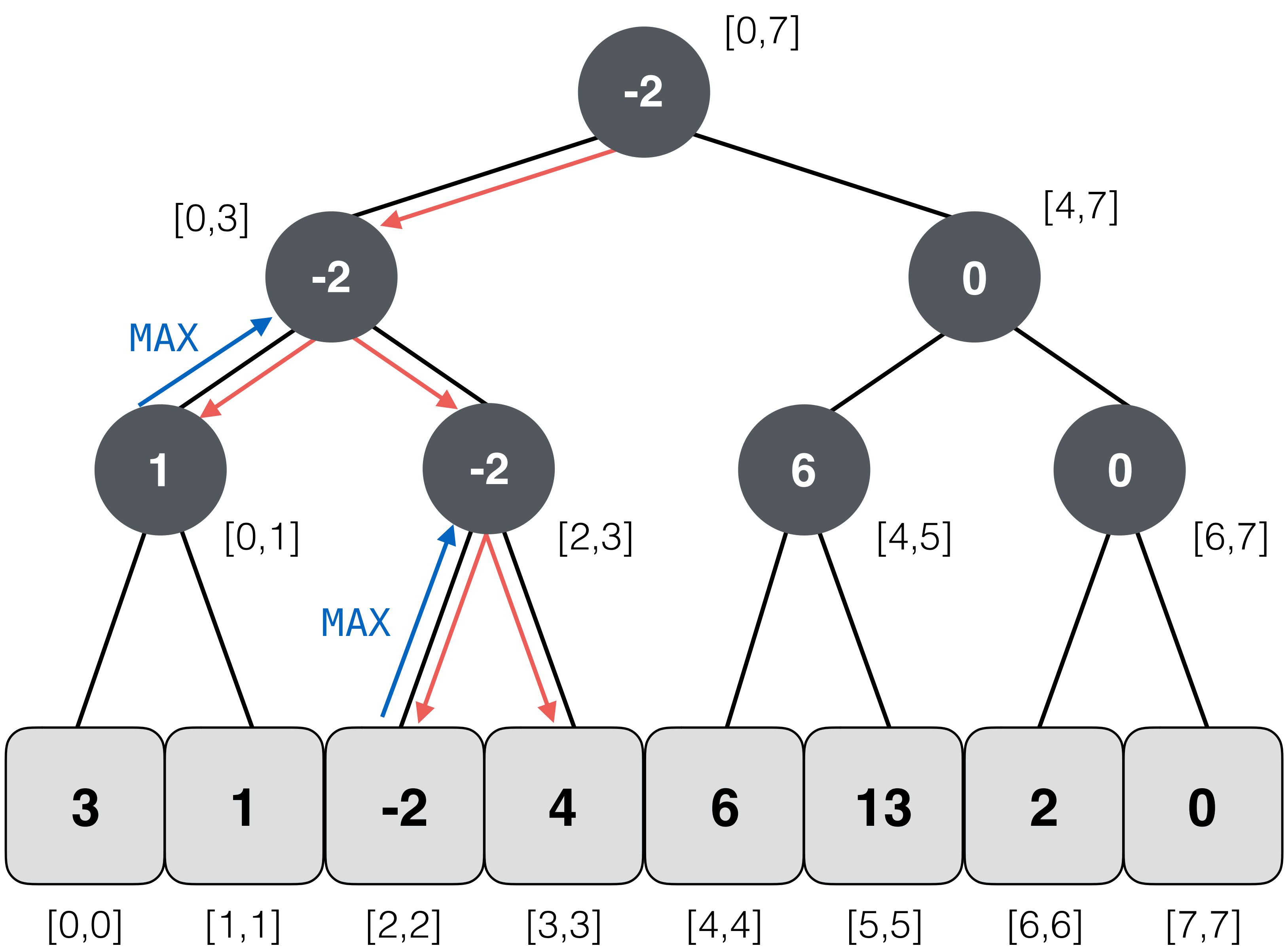

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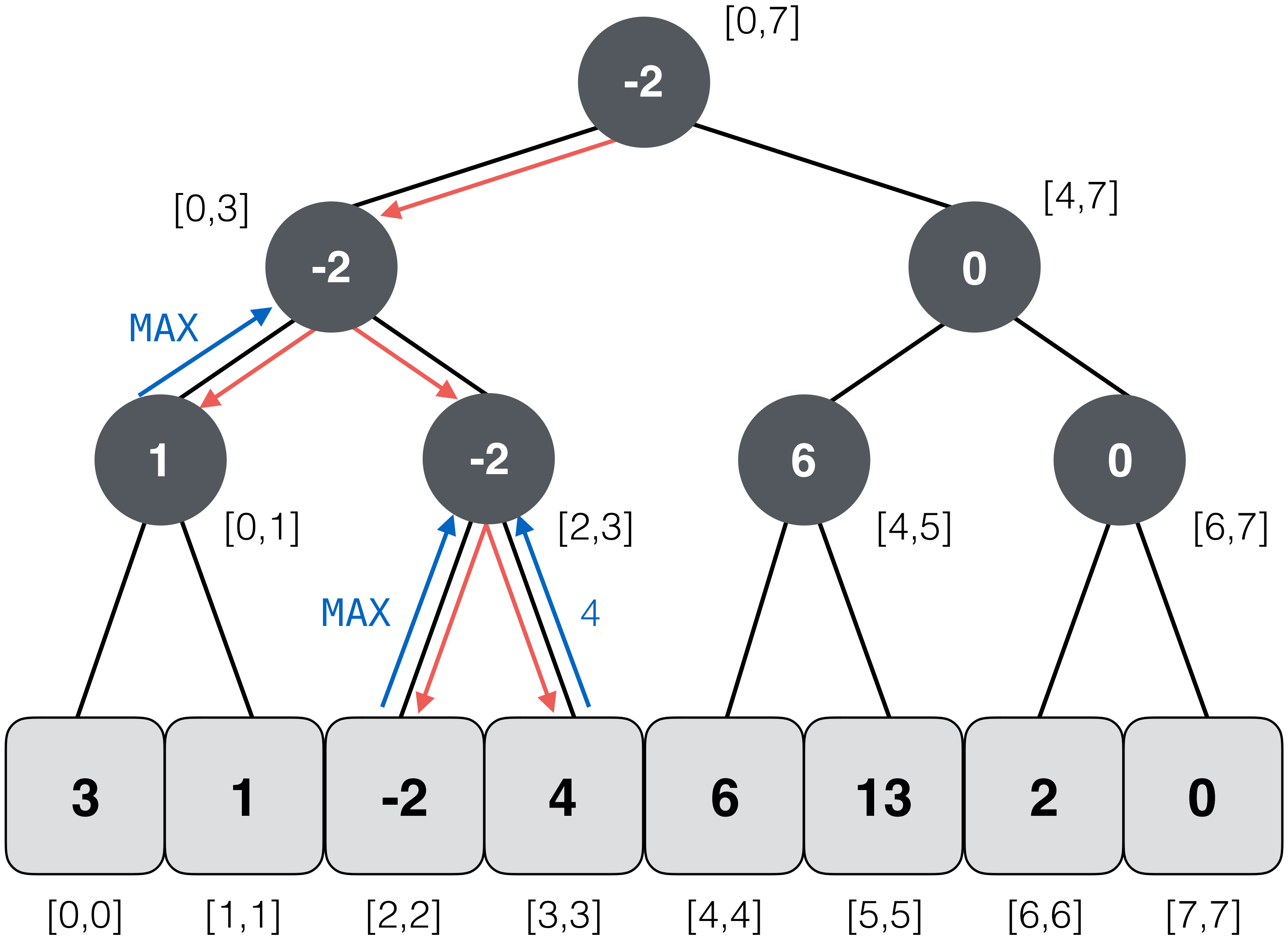

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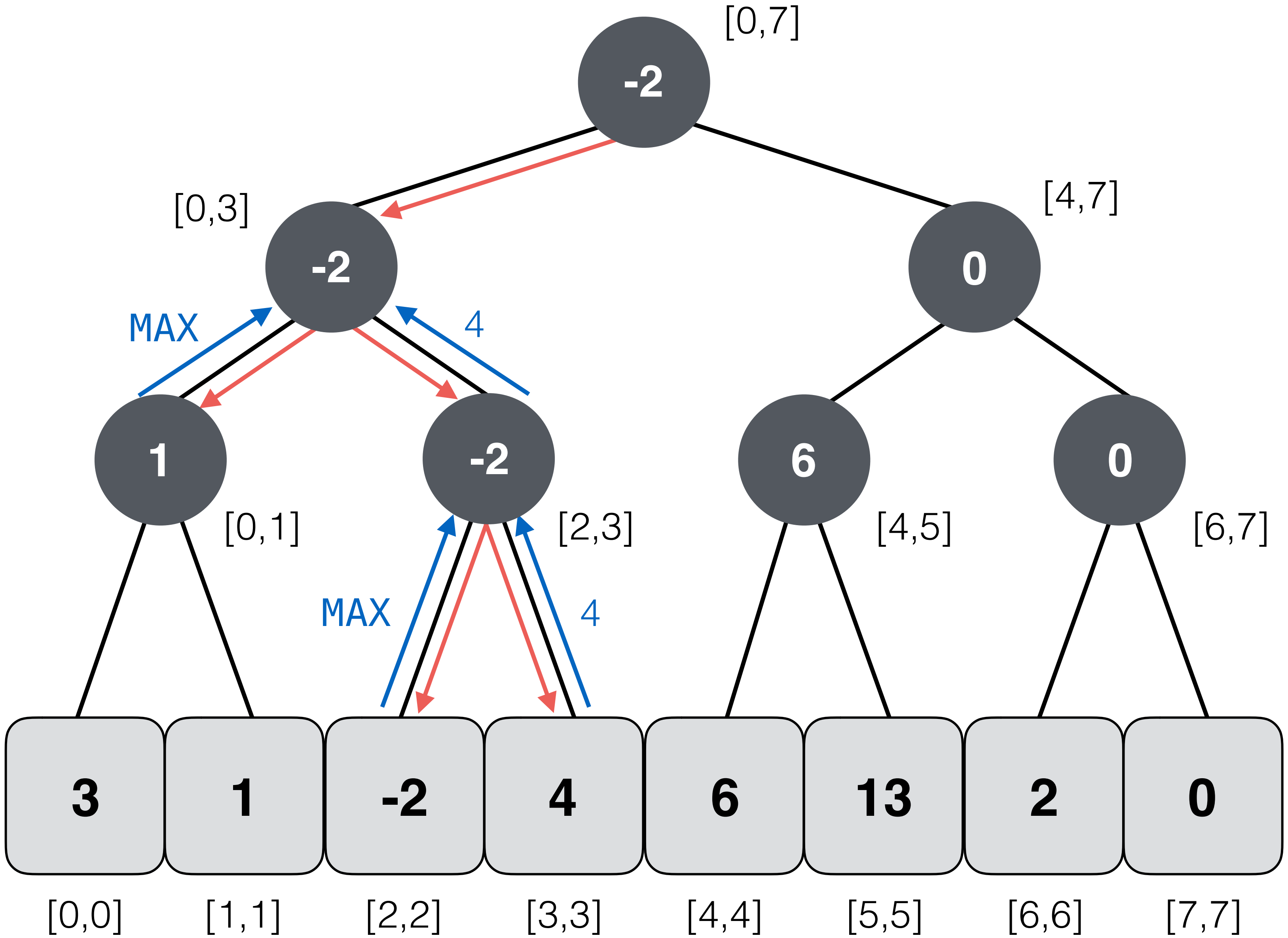

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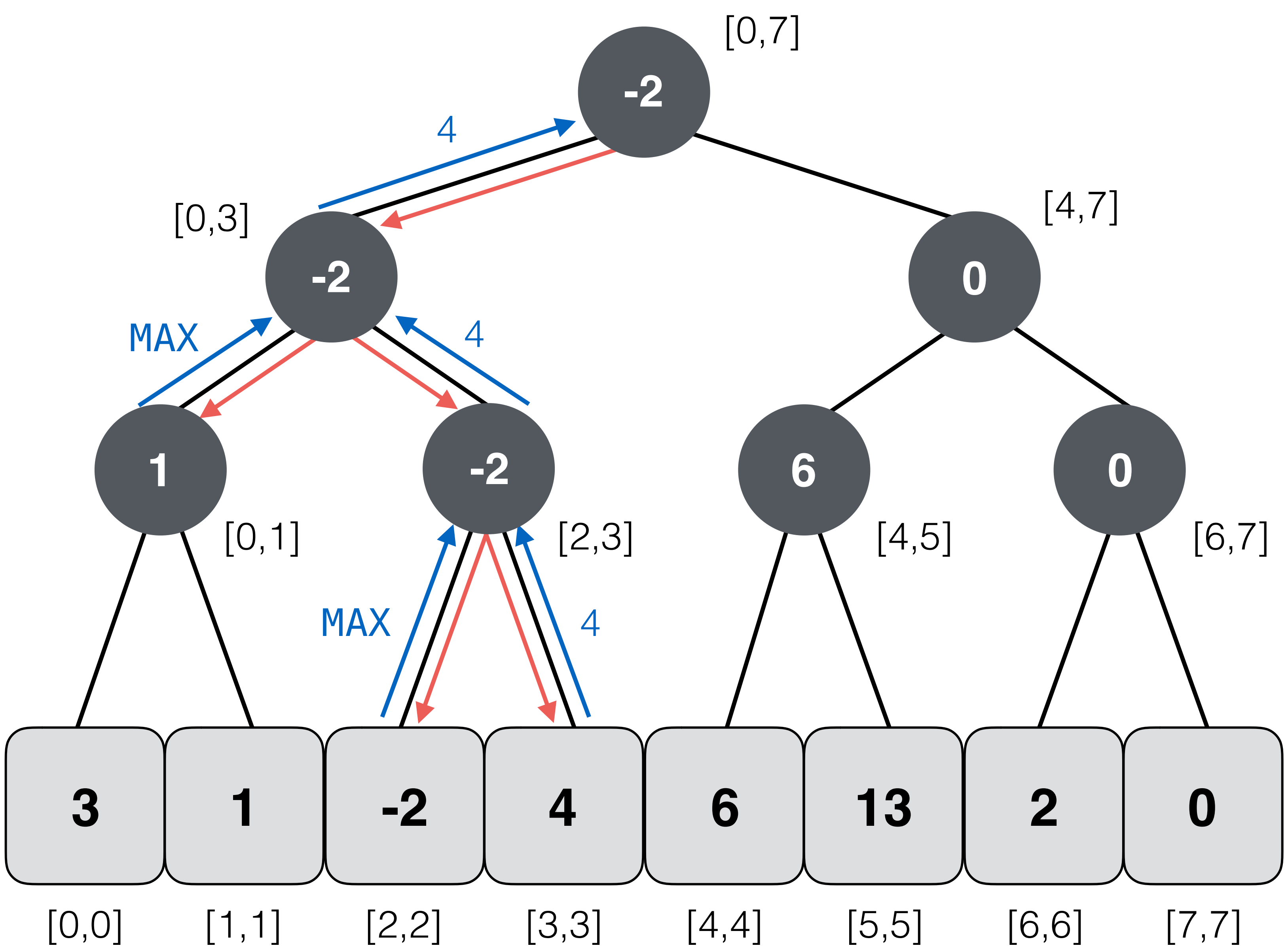

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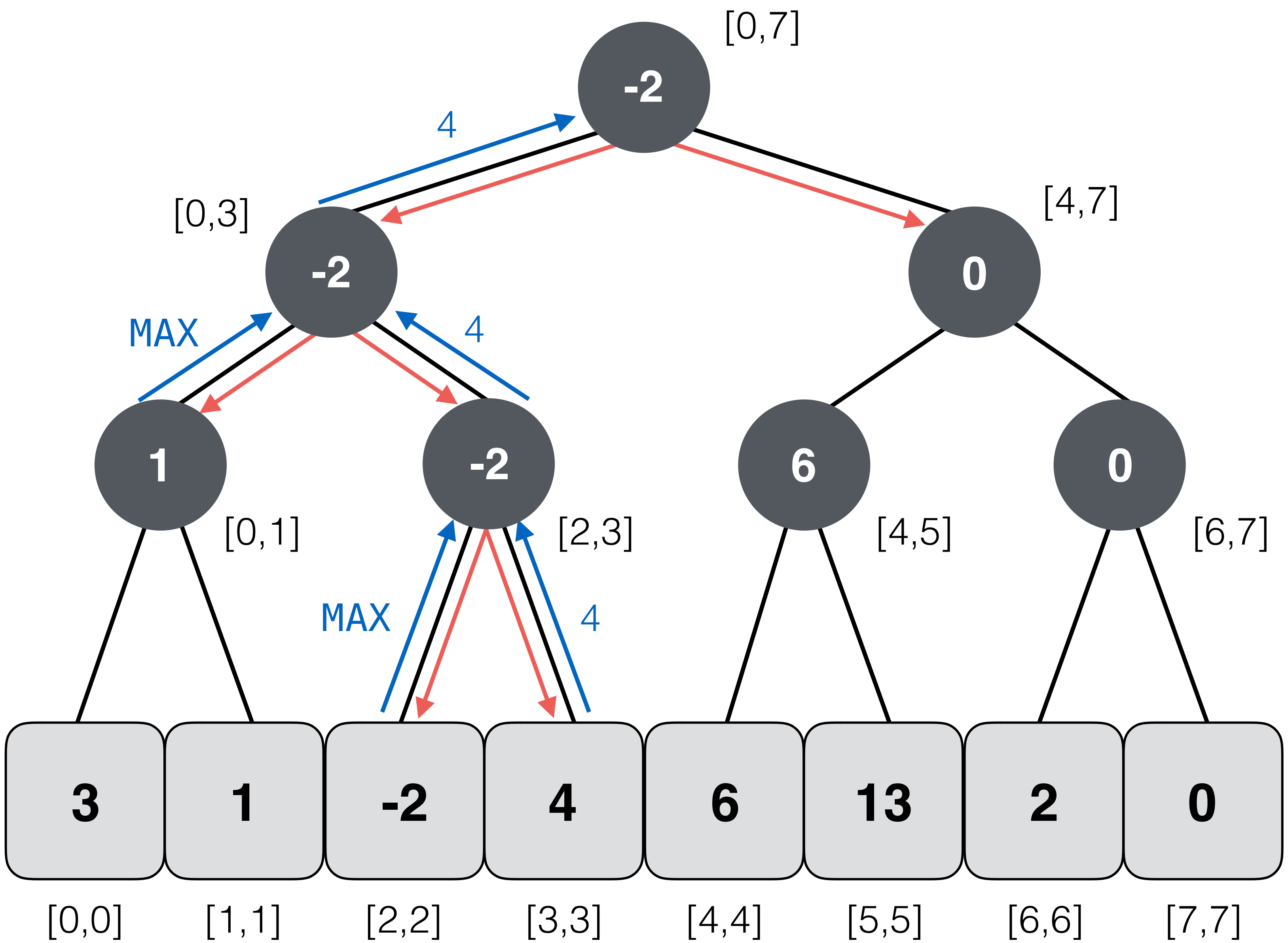

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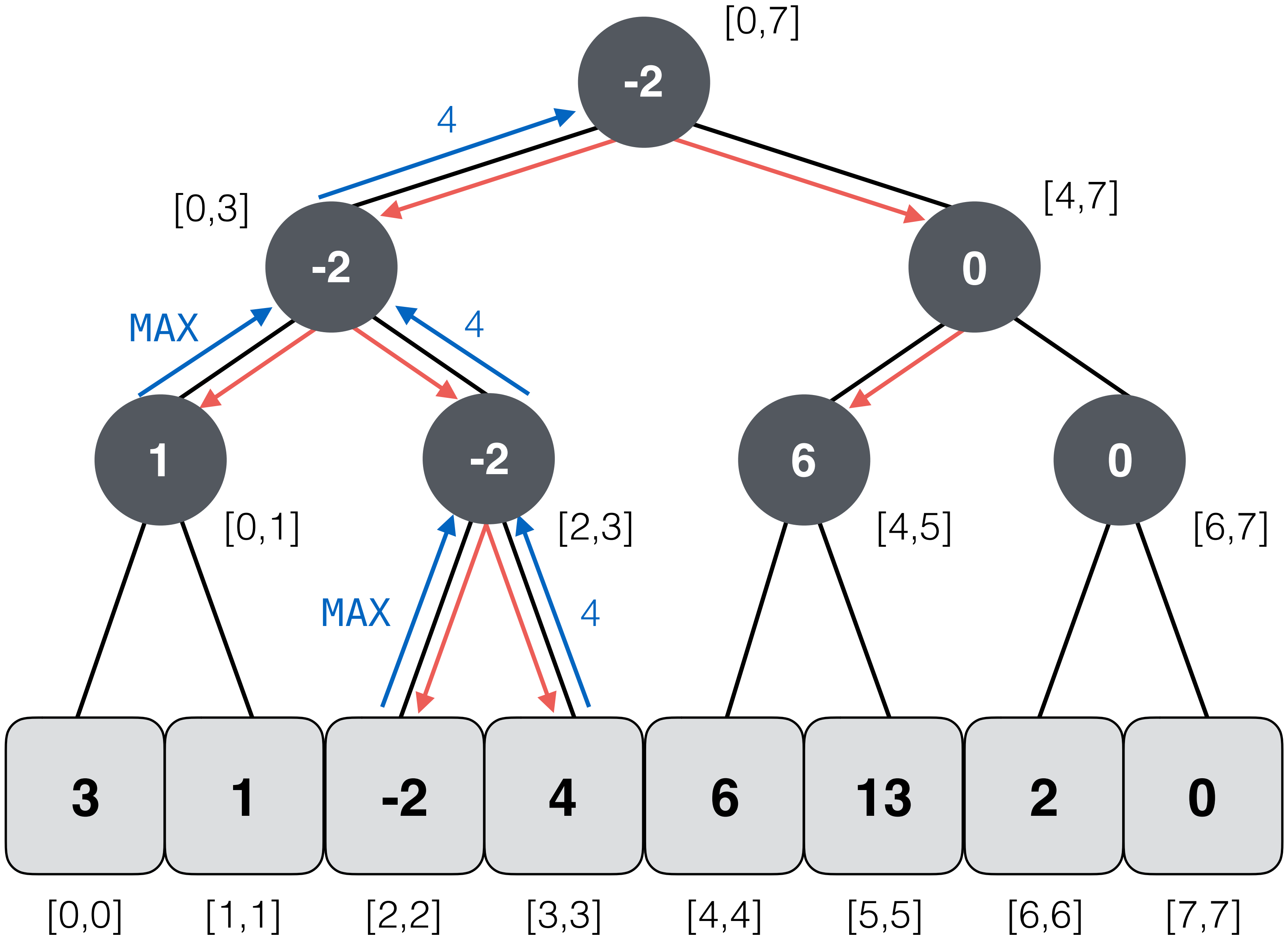

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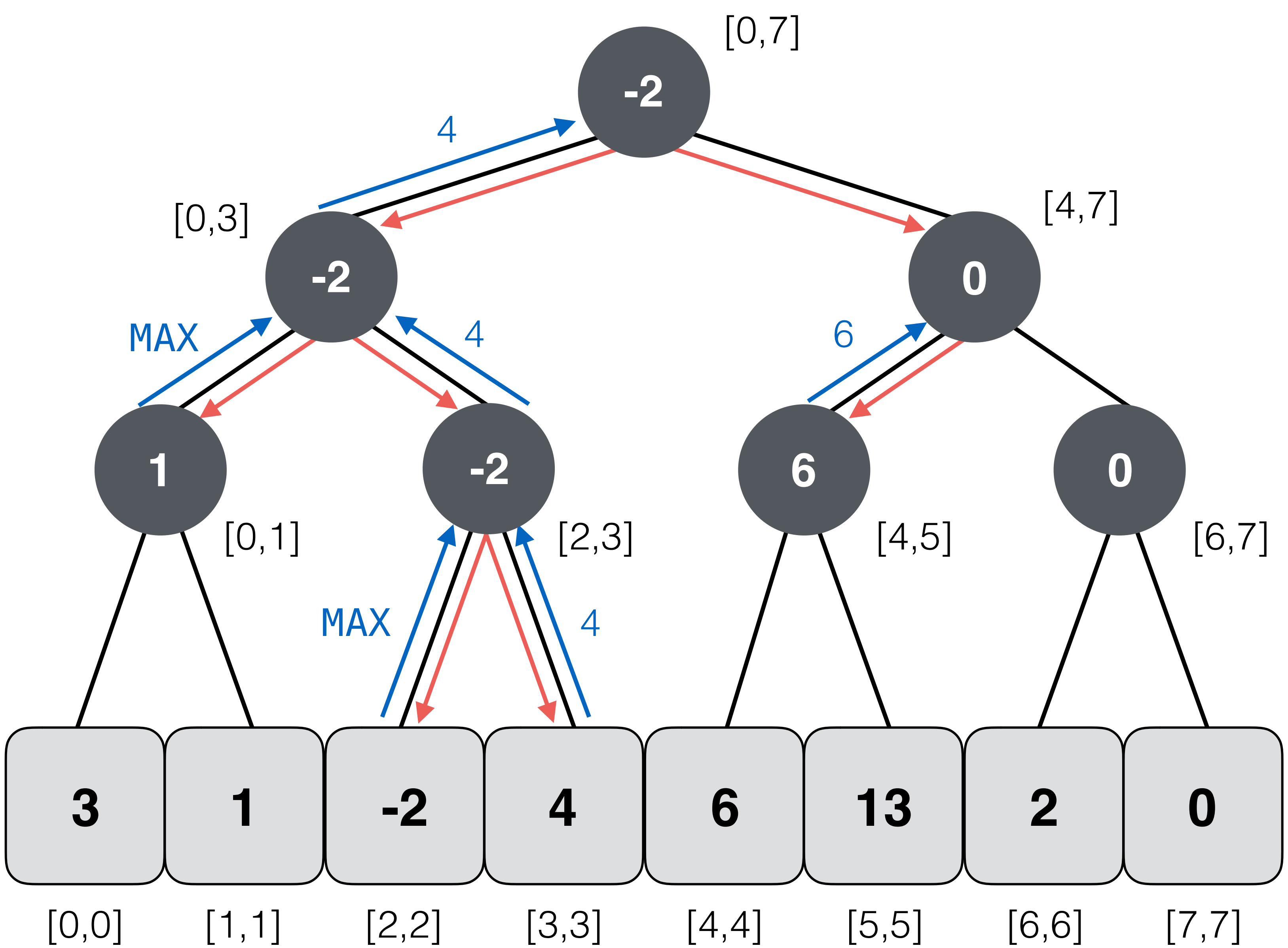

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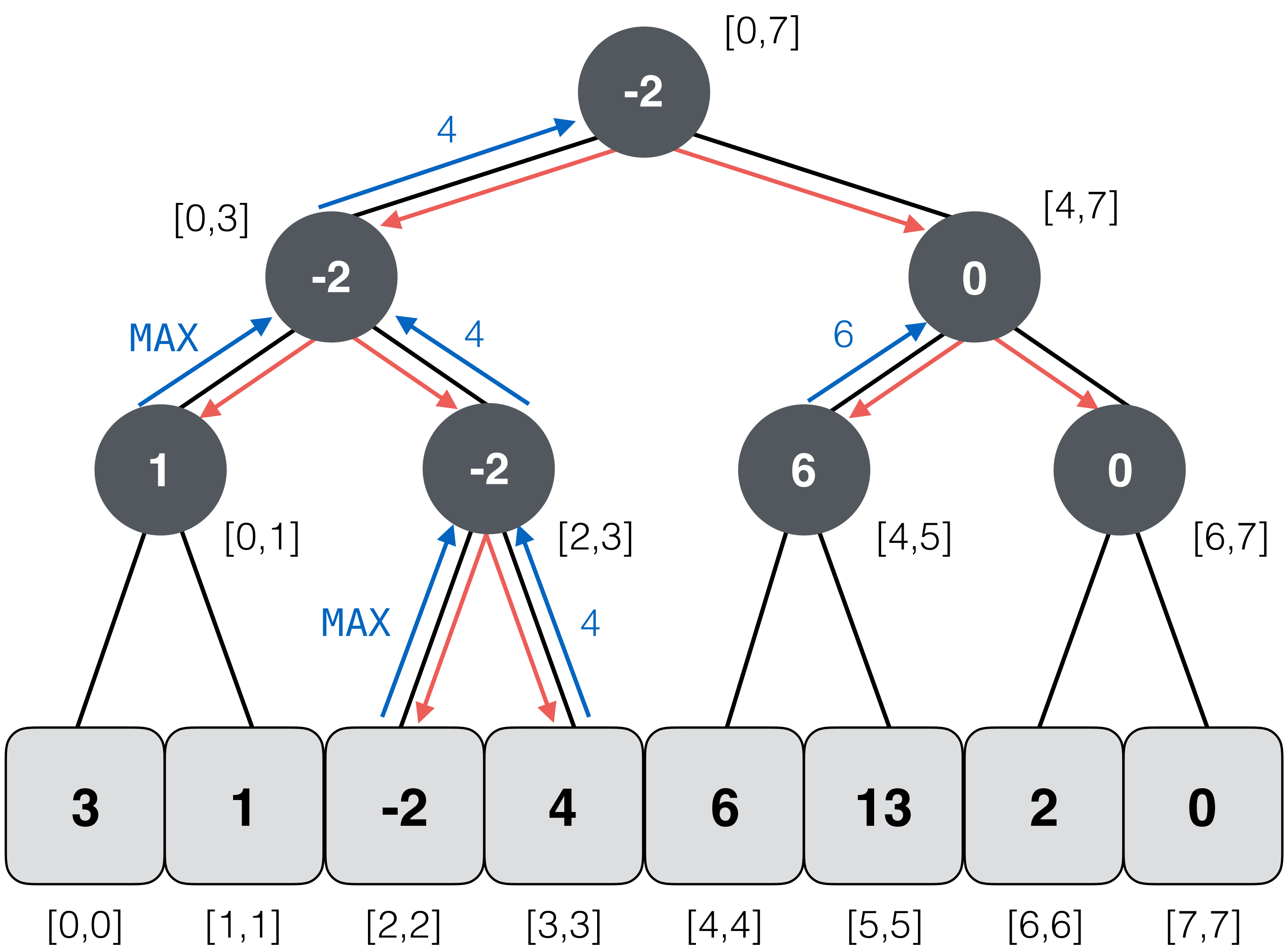

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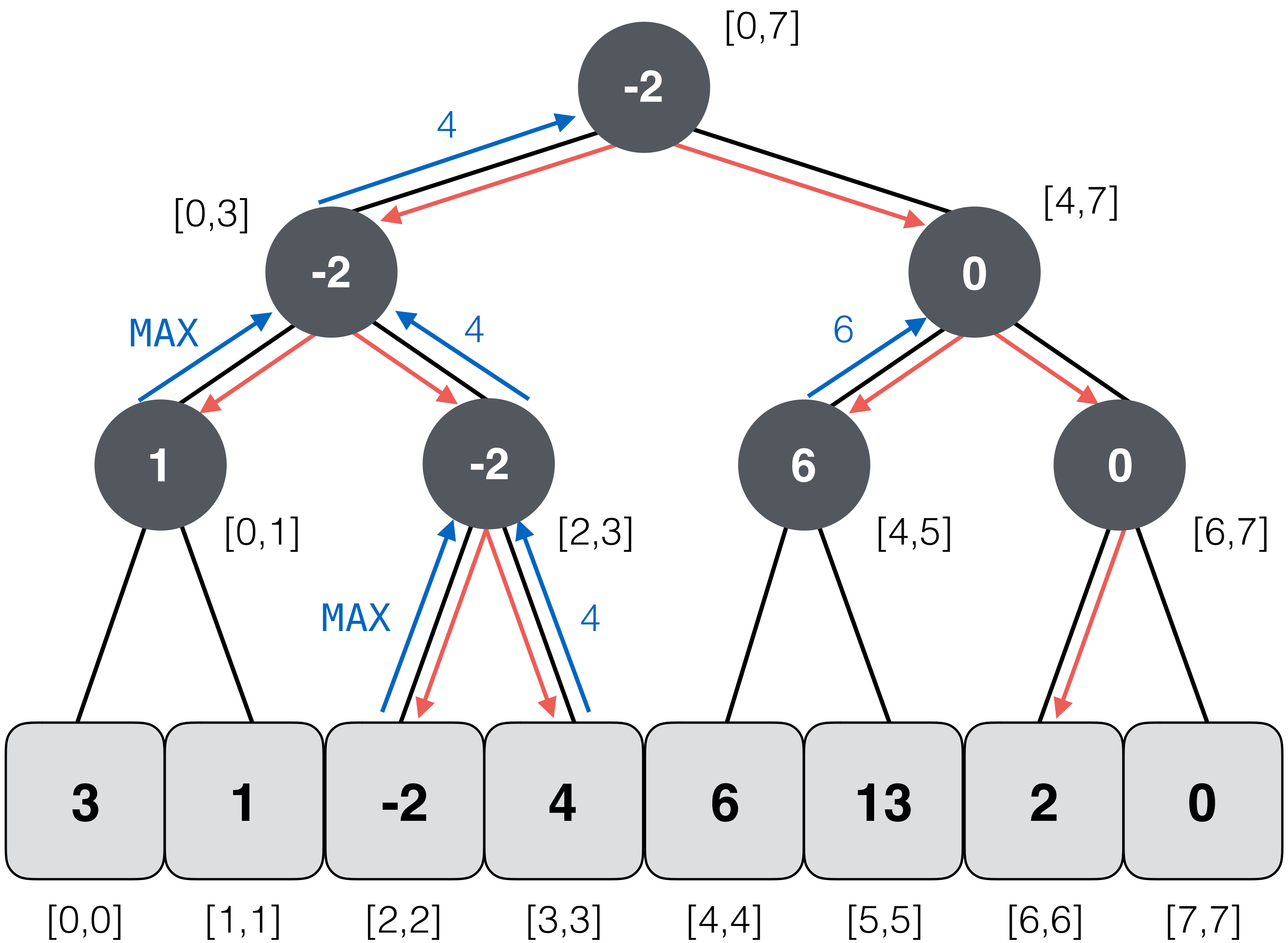

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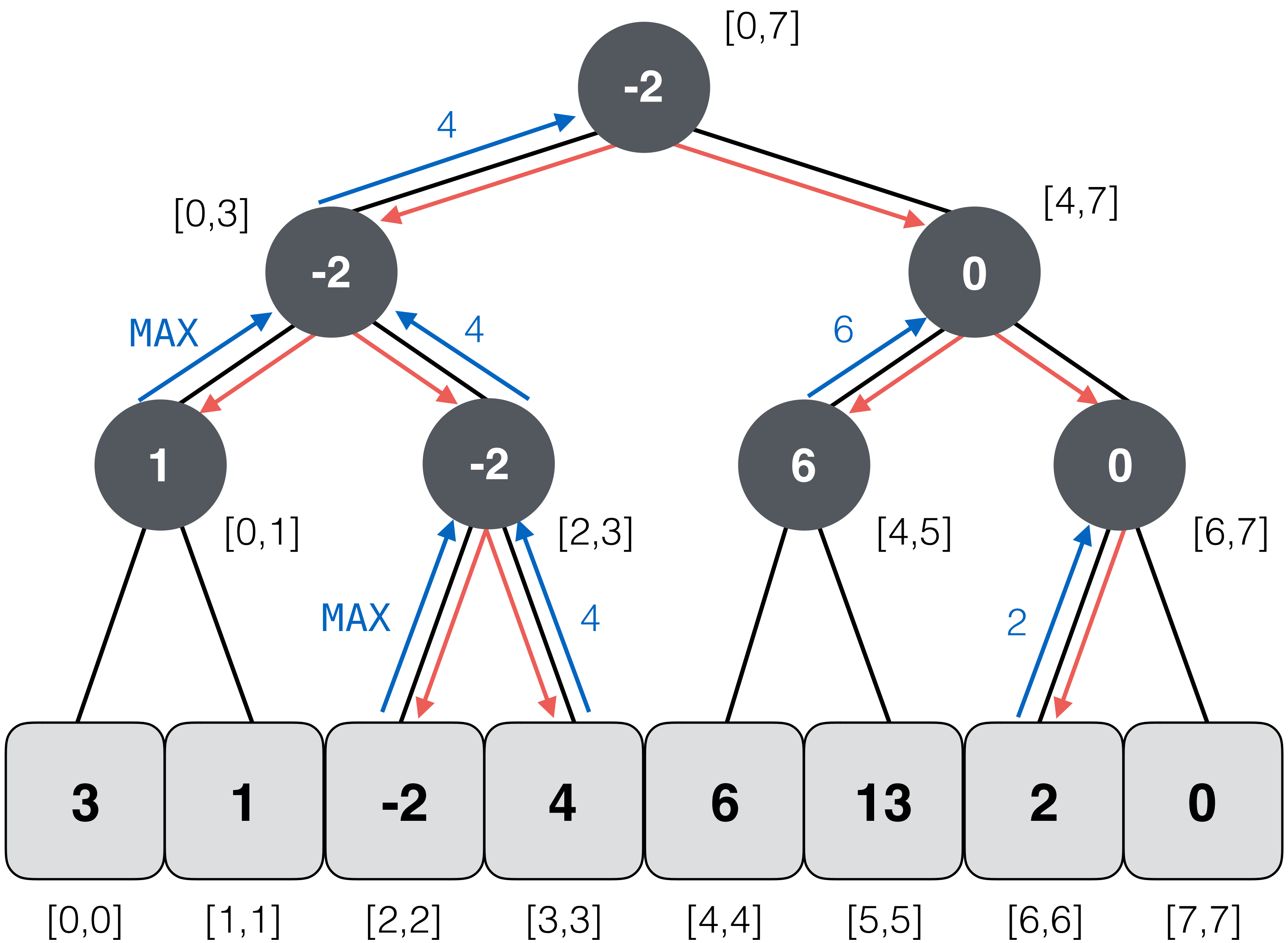

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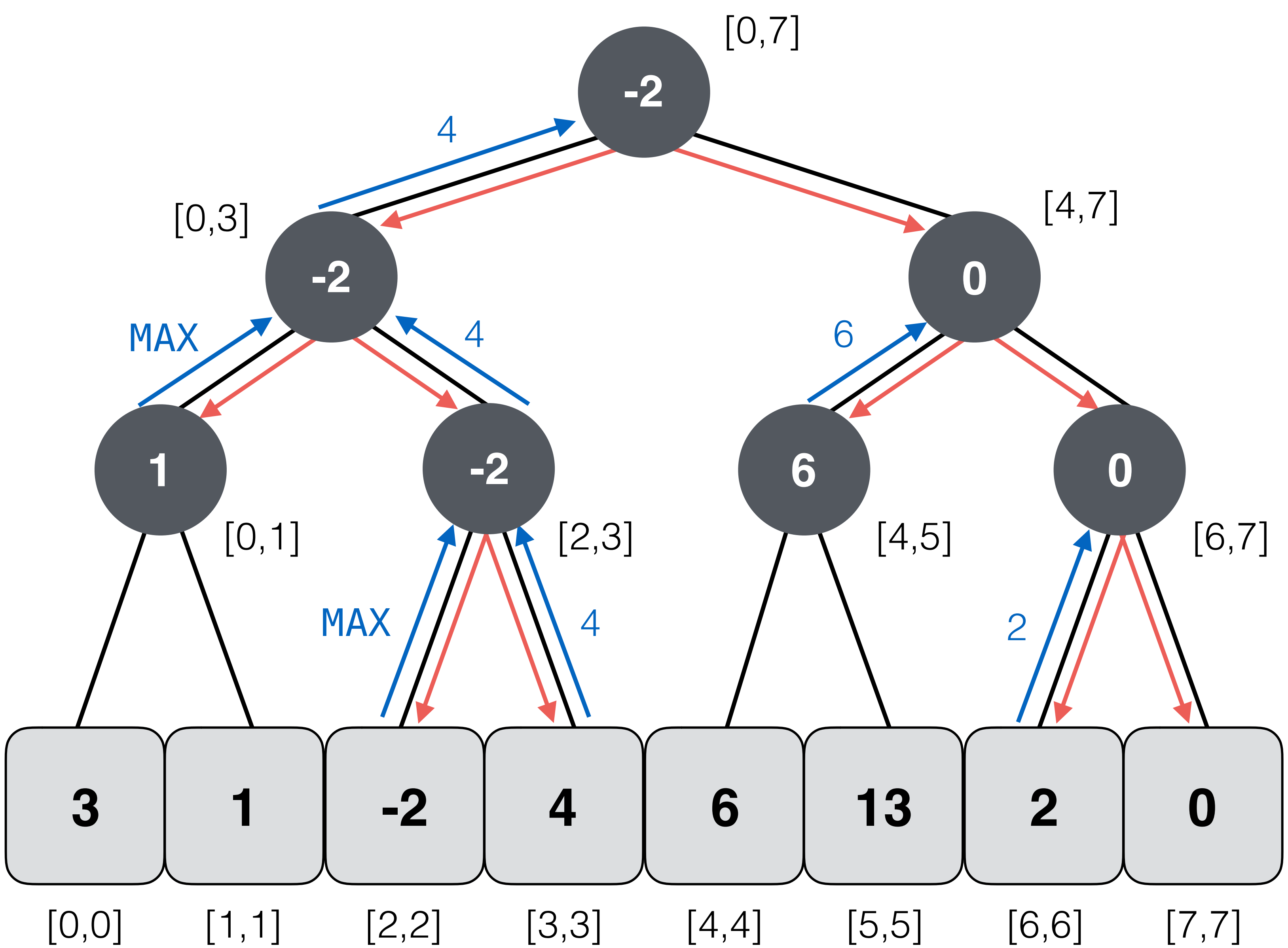

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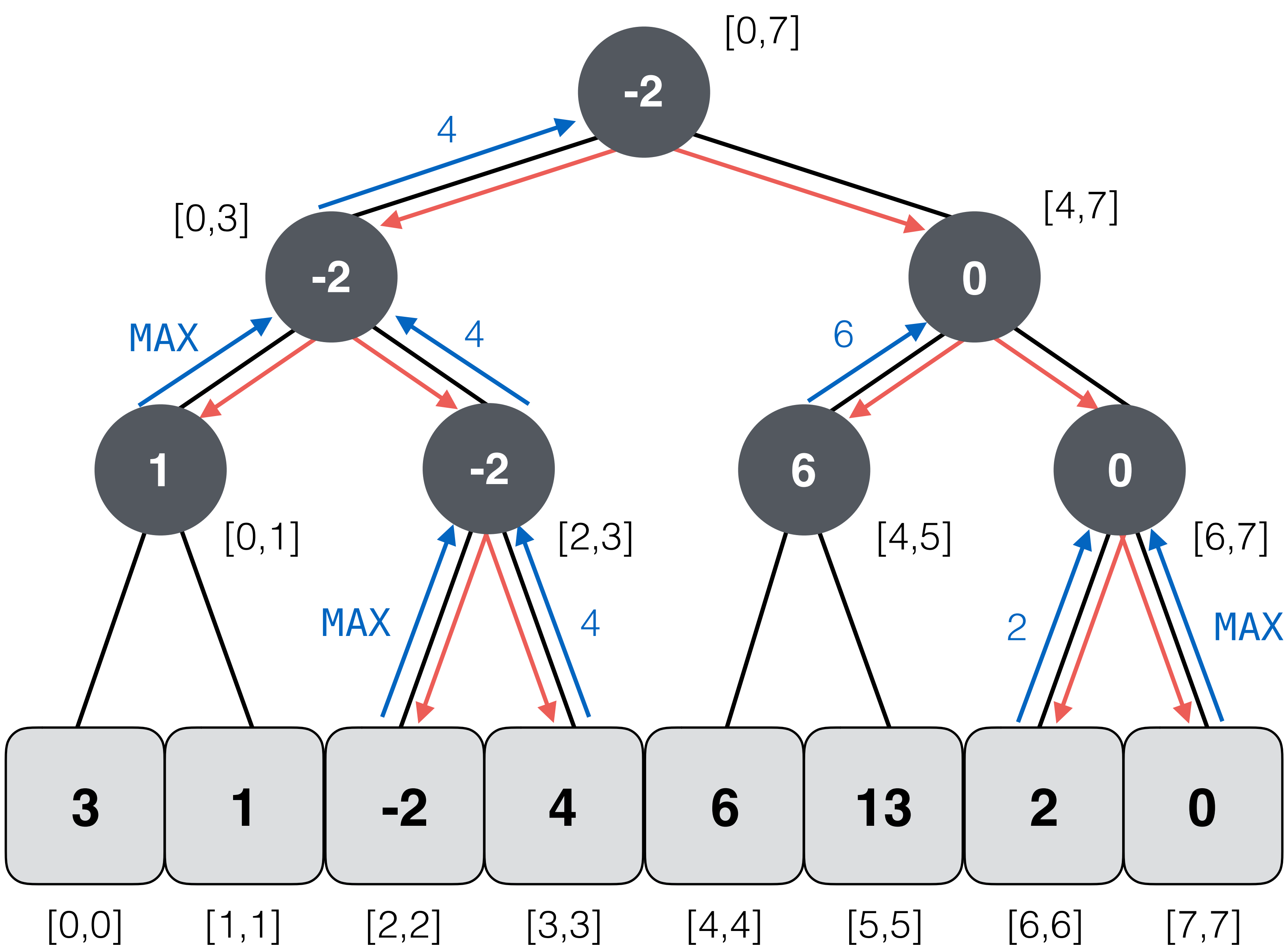

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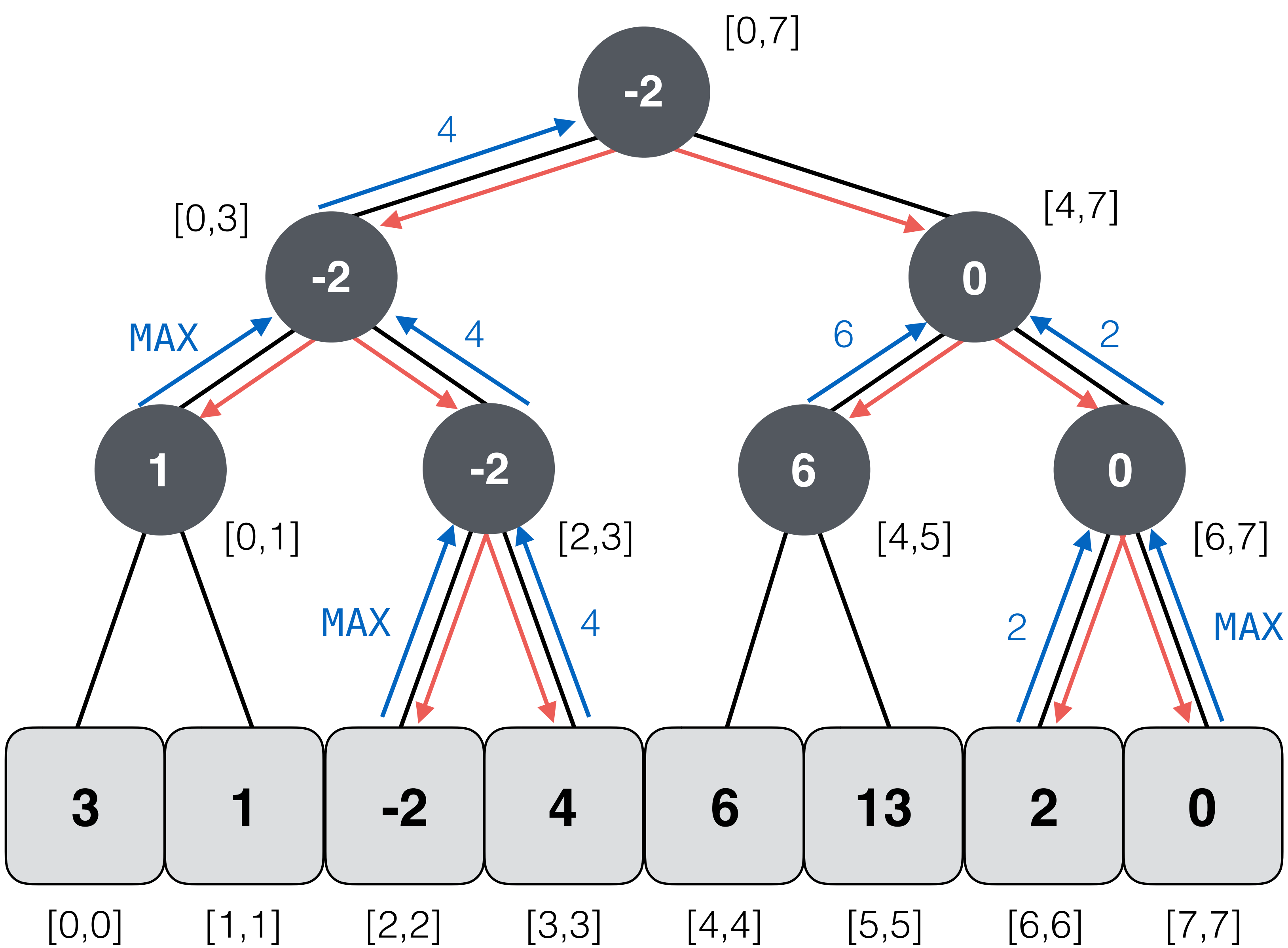

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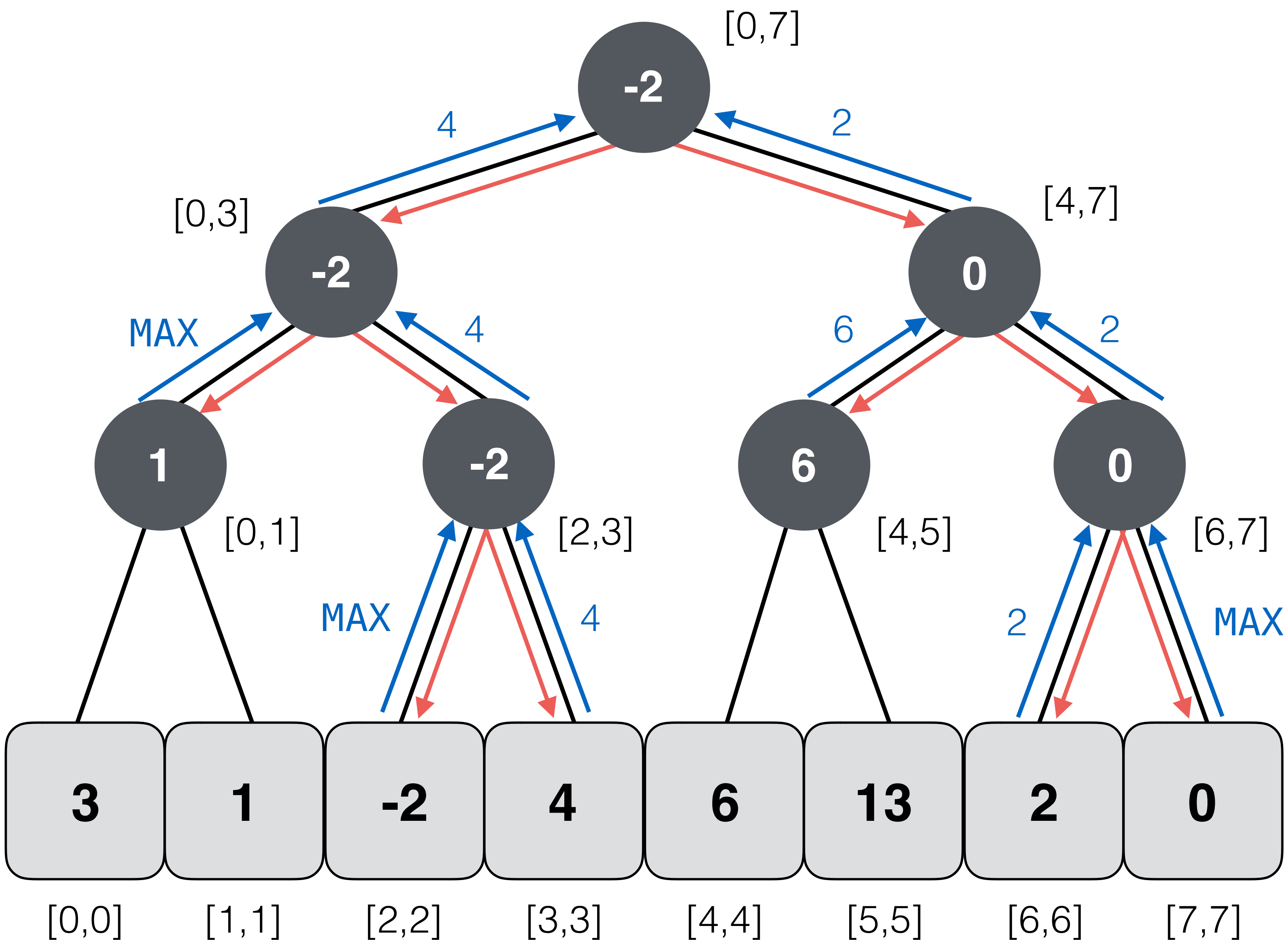

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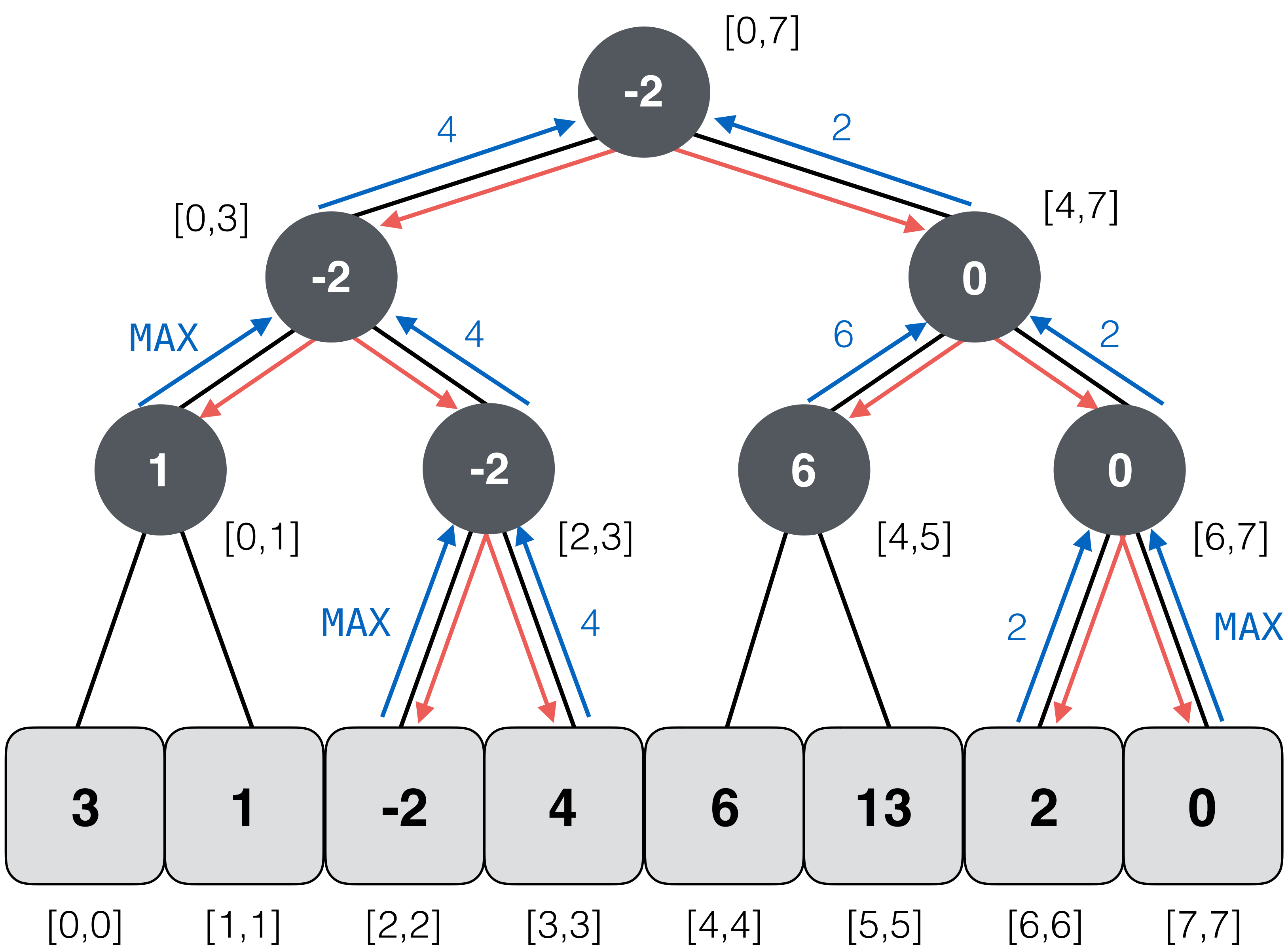

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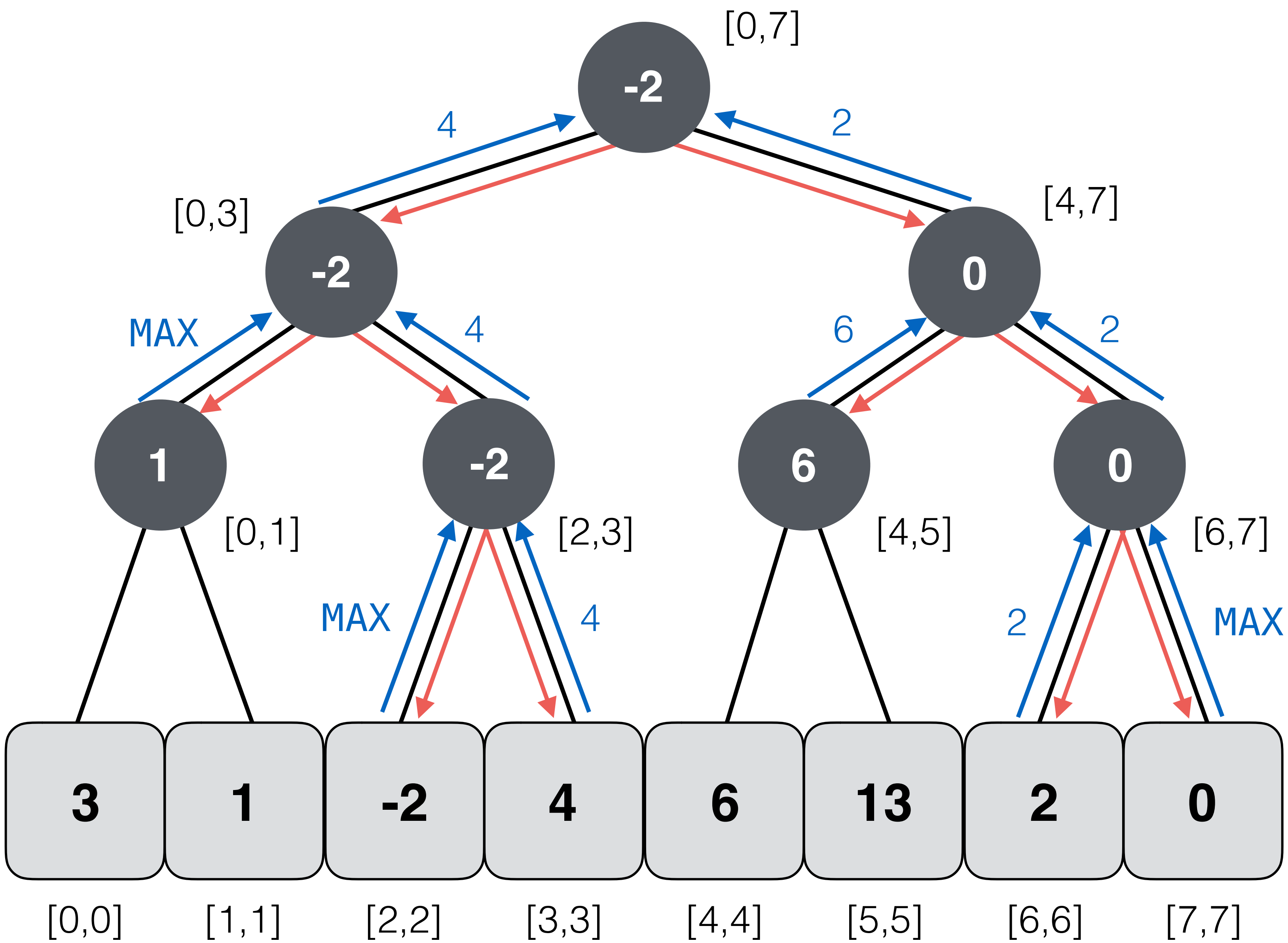

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Query time: O(log n)
Space: O(n)
Building time: O(n)

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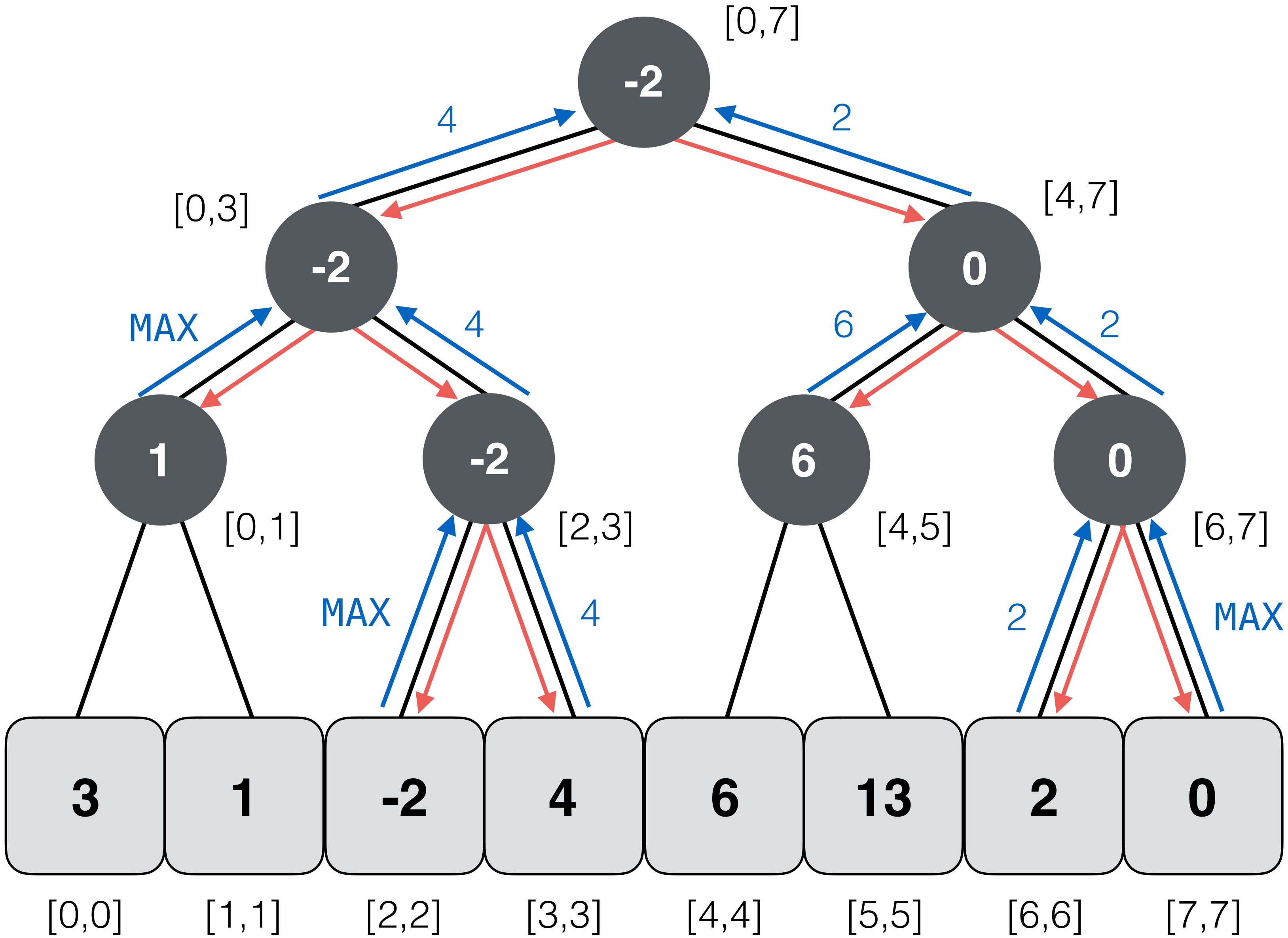

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Query time: O(log n)
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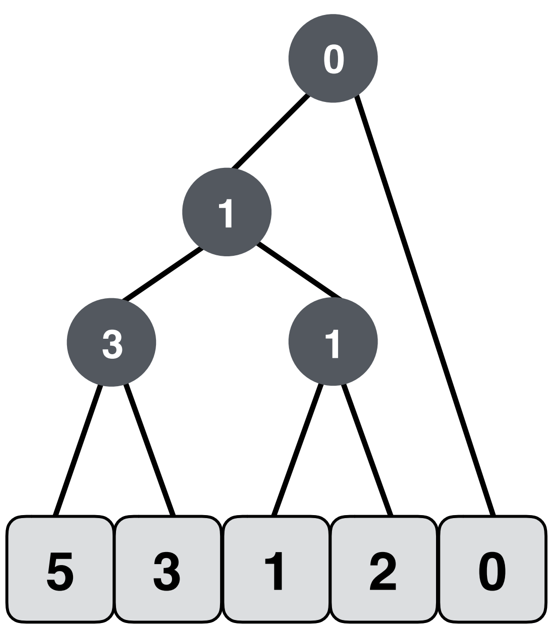
How do we represent trees?

```
node* root = nullptr;
std::deque<node*> q;
int n = 0;
std::cin >> n;

for (int i = 0; i < n; ++i) {
    int x = 0;
    std::cin >> x;
    node* n = new node(x);
    q.push_back(n);
}

node* last = nullptr;
if (n % 2) {
    last = q.back();
    q.pop_back();
}

auto min_parent = [&](node* left, node* right) {
    int min = std::min<int>(left->key, right->key);
    node* parent = new node(min, left, right);
    q.push_back(parent);
};

while (q.size() != 1) {
    min_parent(q[0], q[1]);
    q.pop_front();
    q.pop_front();
}

if (last != nullptr) {
    min_parent(q.front(), last);
    q.pop_front();
}

root = q.front();
```

Pointers

```
struct node {
    node(int k,
         node* l = nullptr,
         node* r = nullptr)
       : key(k), left(l), right(r)
    {}

    int key;
    node* left;
    node* right;
};
```

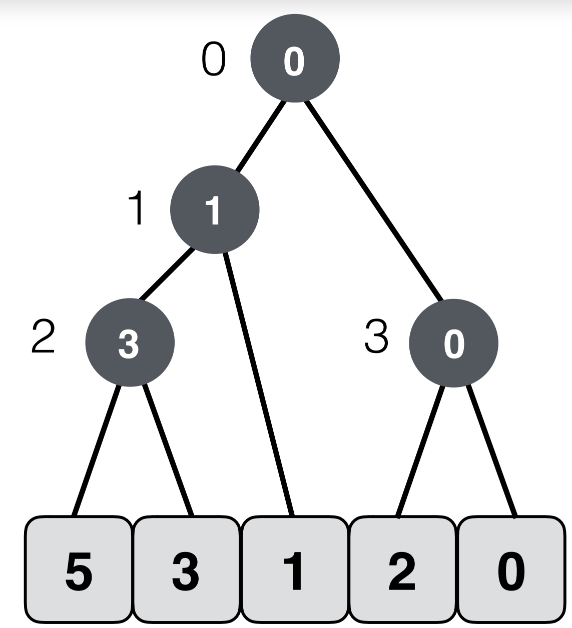


How do we represent trees?

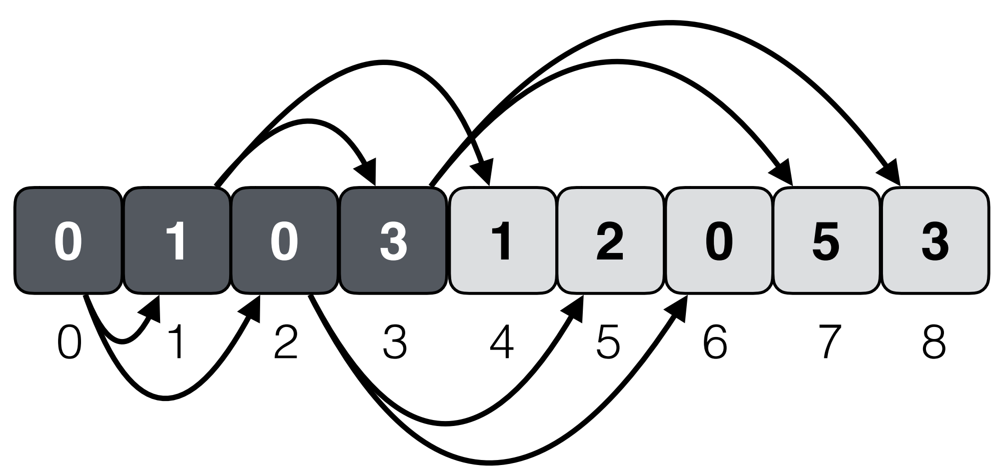
Arrays

```
std::vector<int> tree;

int n = 0;
std::cin >> n;
int tree_size = 2 * n - 1;
tree.resize(tree_size);

int h = ceil(log2(n));
 // left-most node id following level order
int left_most_node = (int(1) << (h - 1)) - 1;
int offset = 2 * left_most_node + 1;

// set leaves circularly

// 1. go forward
int i = 0;
for (int j = offset; j != tree_size; ++i, ++j) {
    int x = 0;
    std::cin >> x;
    tree[j] = x;
}

// 2. fall back
for (int j = 0; i != n; ++i, ++j) {
    int x = 0;
    std::cin >> x;
    tree[n - 1 + j] = x;
}

// set internal nodes
for (int i = tree_size - 1; i != 0; i -= 2) {
    int min = std::min<int>(tree[i], tree[i - 1]);
    tree[PARENT(i)] = min;
}
```

```
#define LEFT(i)     2 * i + 1
#define RIGHT(i)    2 * i + 2
#define PARENT(i) (i - 1) / 2
```



Pointers are implicit!



How do we represent trees?

Remember

Be skeptic: *measure* first and then conclude.

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Pointers vs. **Arrays**

How do we represent trees?

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Pointers VS. **Arrays**

Experiment over 5 million nodes

Visit the tree and compute the sum of all nodes.

```
→  segment_trees git:(master) ✗ g++ -std=c++11 -O3 trees_with_pointers.cpp -o trees_with_pointers
→  segment_trees git:(master) ✗ ./trees_with_pointers < input7
building took: 19.3895 [sec]
sum is: 3676408720
sum took: 0.118645 [sec]
→  segment_trees git:(master) ✗ g++ -std=c++11 -O3 trees_with_arrays.cpp -o trees_with_arrays
→  segment_trees git:(master) ✗ ./trees_with_arrays < input7
building took: 18.0724 [sec]
sum is: 3676408920
sum took: 0.0118351 [sec]
→  segment_trees git:(master) ✗
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10X

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10X

OK, we are going to adopt the array-based representation!

Building Segment Trees recursively

```
size_t n = leaves.size();
// round up to the next power of 2
size_t m = size_t(1) << static_cast<size_t>(ceil(log2(n)));
m_tree.resize(2 * m - 1, m_traits.invalid);

build(leaves, 0, m - 1, 0);
```

```
struct type_traits {
    IntType invalid;
    BinaryFunct funct;
};
```

```
void build(std::vector<IntType> const& leaves, size_t lo, size_t hi, size_t pos) {
    if (lo == hi) {
        m_tree[pos] = leaves[lo];
        return;
    }
    size_t mid = (lo + hi) / 2;
    build(leaves, lo, mid, LEFT(pos));
    build(leaves, mid + 1, hi, RIGHT(pos));
    m_tree[pos] = m_traits.funct(m_tree[LEFT(pos)], m_tree[RIGHT(pos)]);
}
```

```
#define LEFT(i)      2 * i + 1
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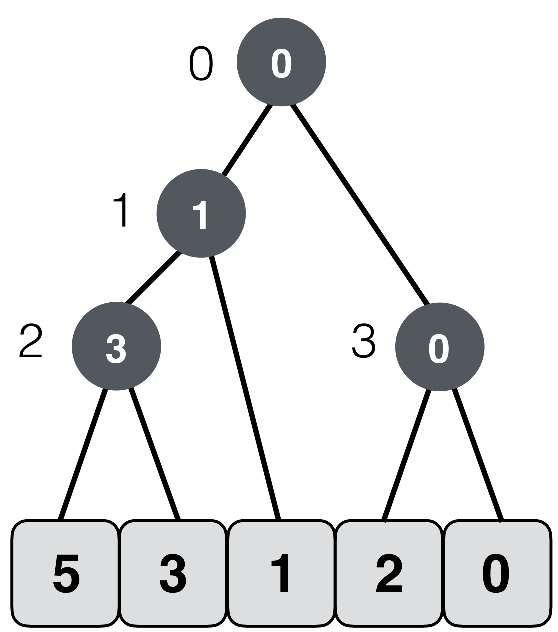
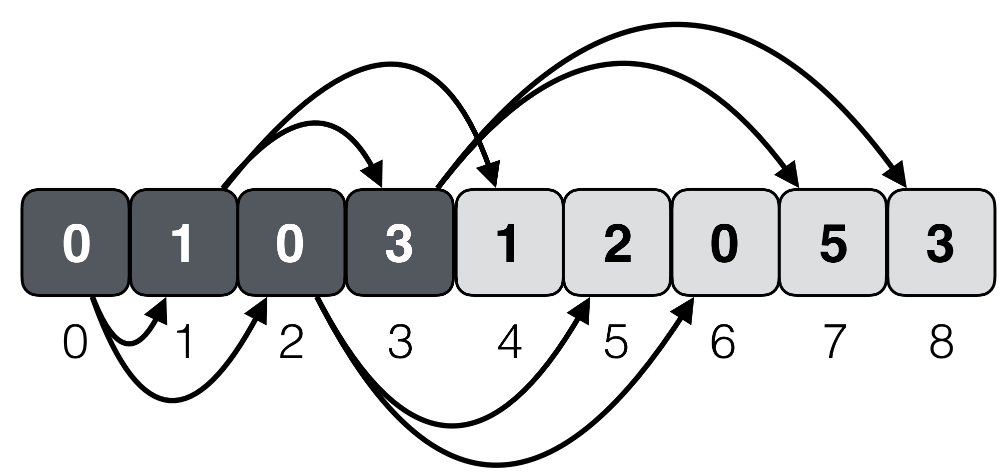
#define PARENT(i) (i - 1) / 2
```





Range (MIN) Queries with Segment Trees

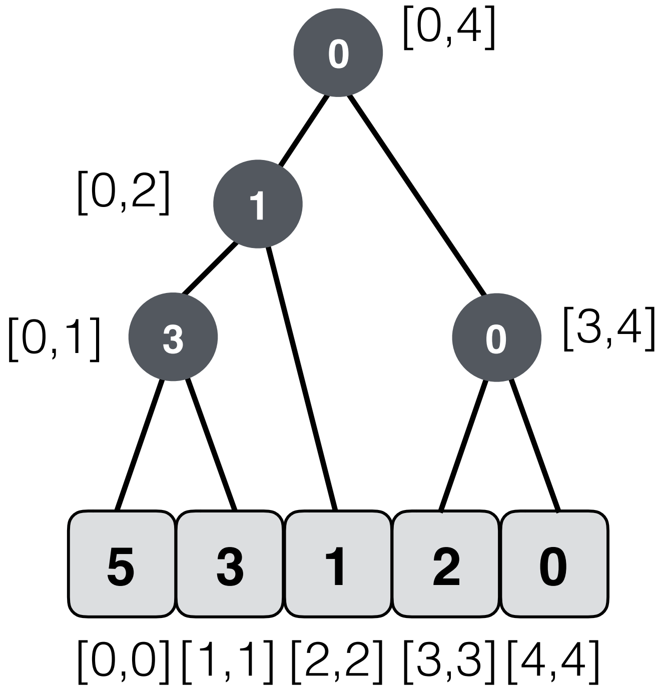
```
IntType rmq(range const& query, range node_segment, size_t pos) {

    if (query.lo <= node_segment.lo
    and query.hi >= node_segment.hi) { // total overlap
        return m_tree[pos];
    }
    if (query.lo > node_segment.hi
    or  query.hi < node_segment.lo) { // no overlap
        return m_traits.invalid;
    }

    // partial overlap
    size_t mid = (node_segment.lo + node_segment.hi) / 2;
    return m_traits.funct(
        rmq(query, {node_segment.lo, mid}, LEFT(pos)),
        rmq(query, {mid + 1, node_segment.hi}, RIGHT(pos))
    );
}
```

```
struct range {
    range(size_t l, size_t h)
        : lo(l), hi(h)
    {}

    size_t lo, hi;
};
```



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};
```
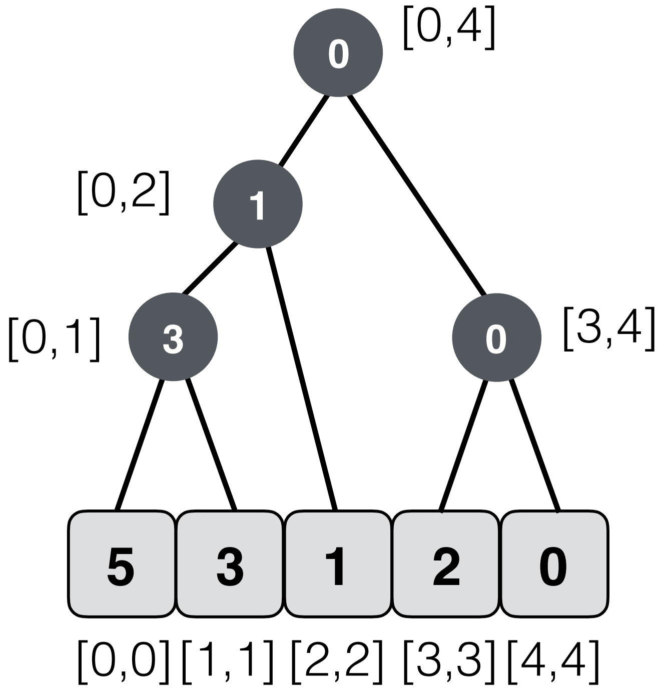

min(1,3) = ?

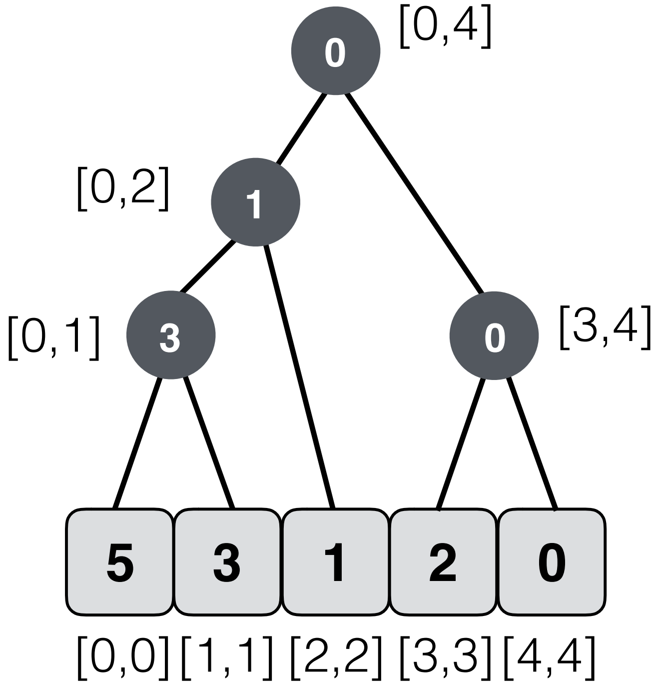


Updating Segment Trees

Let's add two new operations (updates):

- `update`(i, x) which increments the i-th leaf by x;
- `update_range`(i, j ,x) which increments all leaves from i to j by x.

```
void update(size_t i, IntType delta, range node_segment, size_t pos) {

    if (i > node_segment.hi
    or  i < node_segment.lo) {
        return;
    }

    if (node_segment.lo == node_segment.hi) { // leaf
        m_tree[pos] += delta;
        return;
    }

    size_t mid = (node_segment.lo + node_segment.hi) / 2;
    update(i, delta, {node_segment.lo, mid}, LEFT(pos));
    update(i, delta, {mid + 1, node_segment.hi}, RIGHT(pos));
    m_tree[pos] = m_traits.funct(m_tree[LEFT(pos)], m_tree[RIGHT(pos)]);
}
```



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```
         range const& query
void update(size_t i, IntType delta, range node_segment, size_t pos) {

    if (i > node_segment.hi
    or  i < node_segment.lo) {
        return;
    }
```

```
if (query.lo > node_segment.hi
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    return;
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```

```
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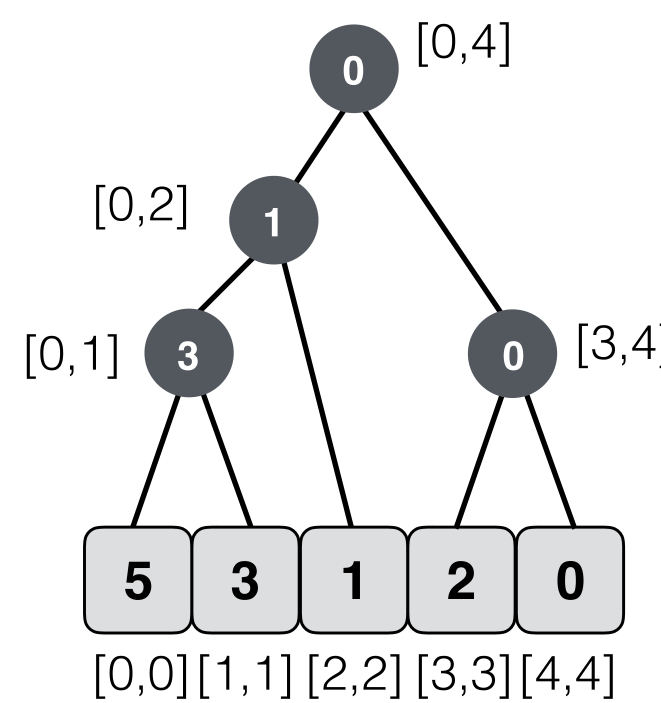

Benchmarking Segment Trees

```
→  segment_trees git:(master) ✗ python gen_data.py 5000000 100000 10000 > input13
→  segment_trees git:(master) ✗ ./rmq_segment_tree < input13
parsing the input took: 18.5193 [sec]
building tree with 5000000 leaves
building took: 0.314939 [sec]
executing 100000 range queries
average query time: 1.74382 [musec]
executing 10000 updates
average update time: 0.561733 [musec]
executing 10000 range updates
average range update time: 2.55461 [musec]
→  segment_trees git:(master) ✗
```

Lazy Propagation in Segment Trees

Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

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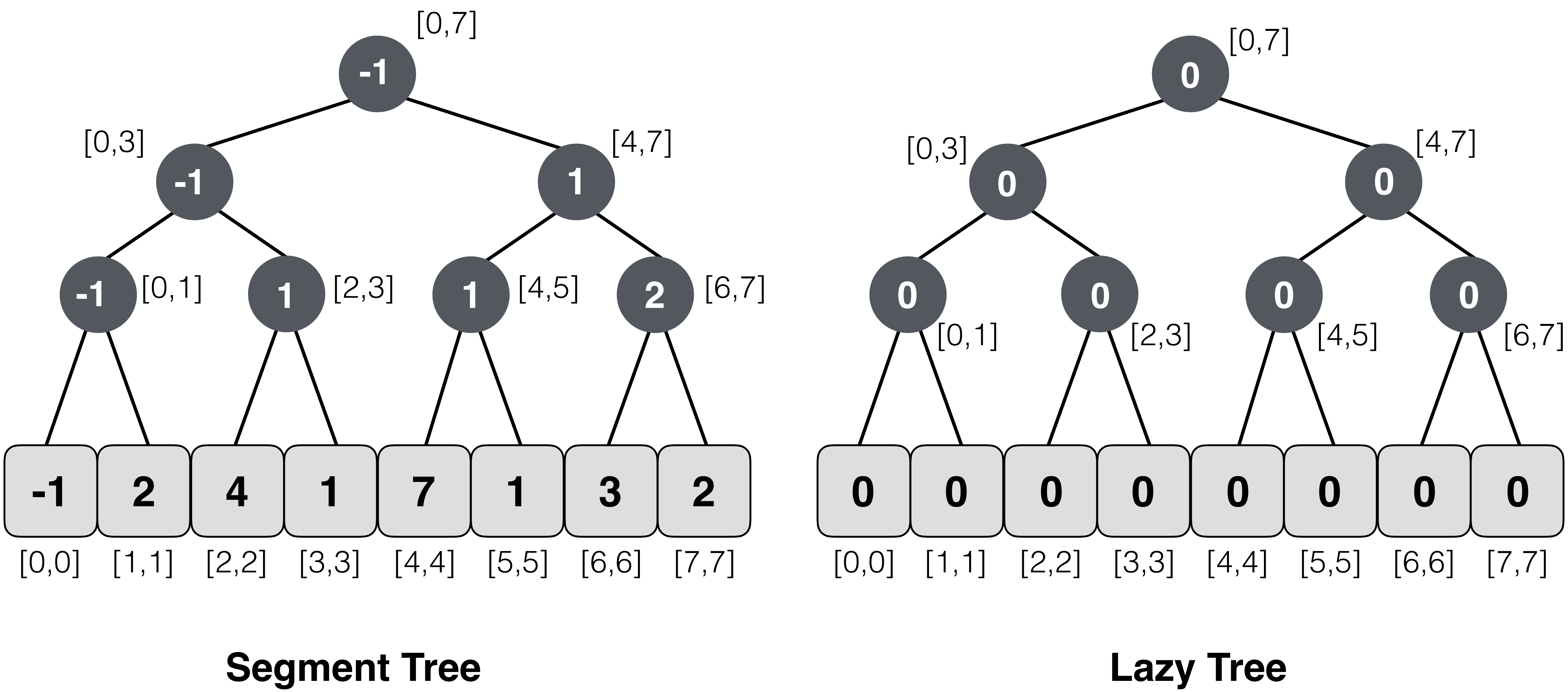

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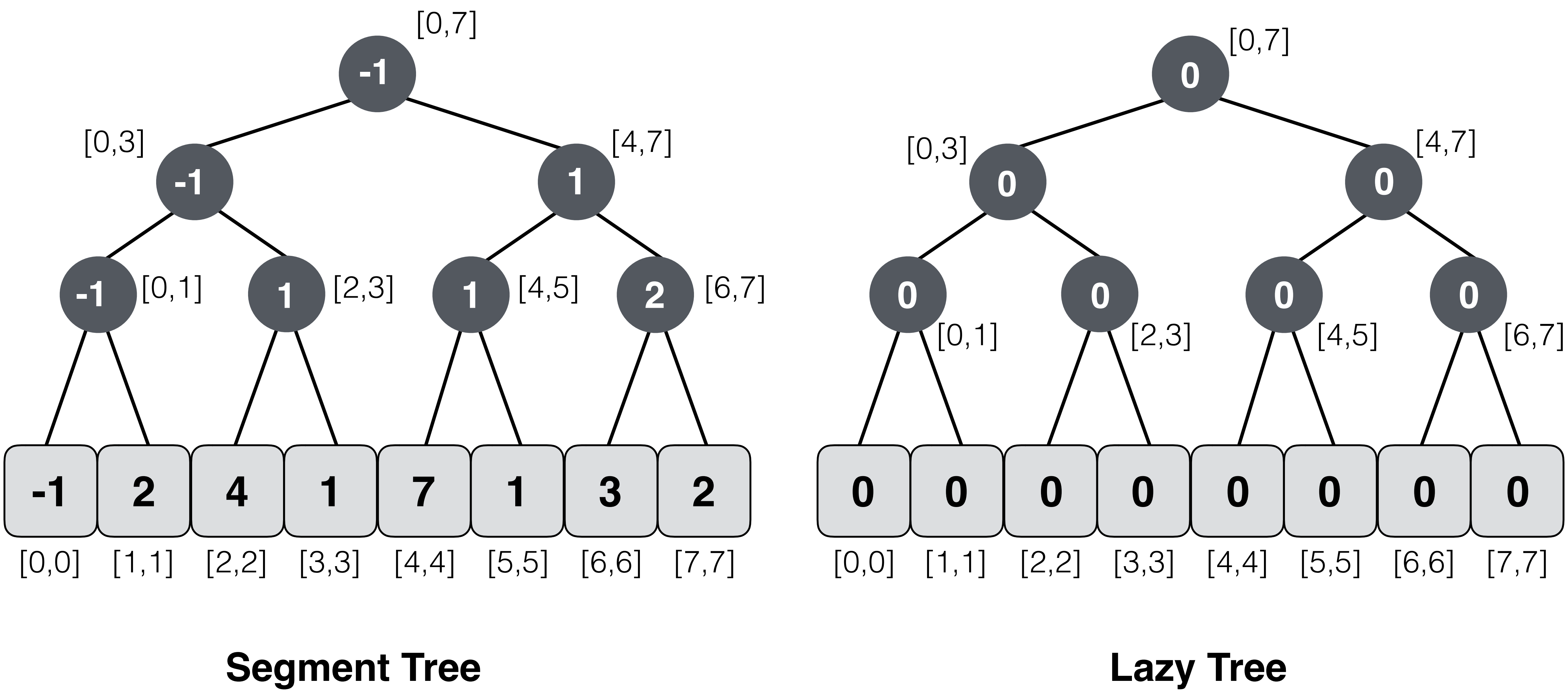

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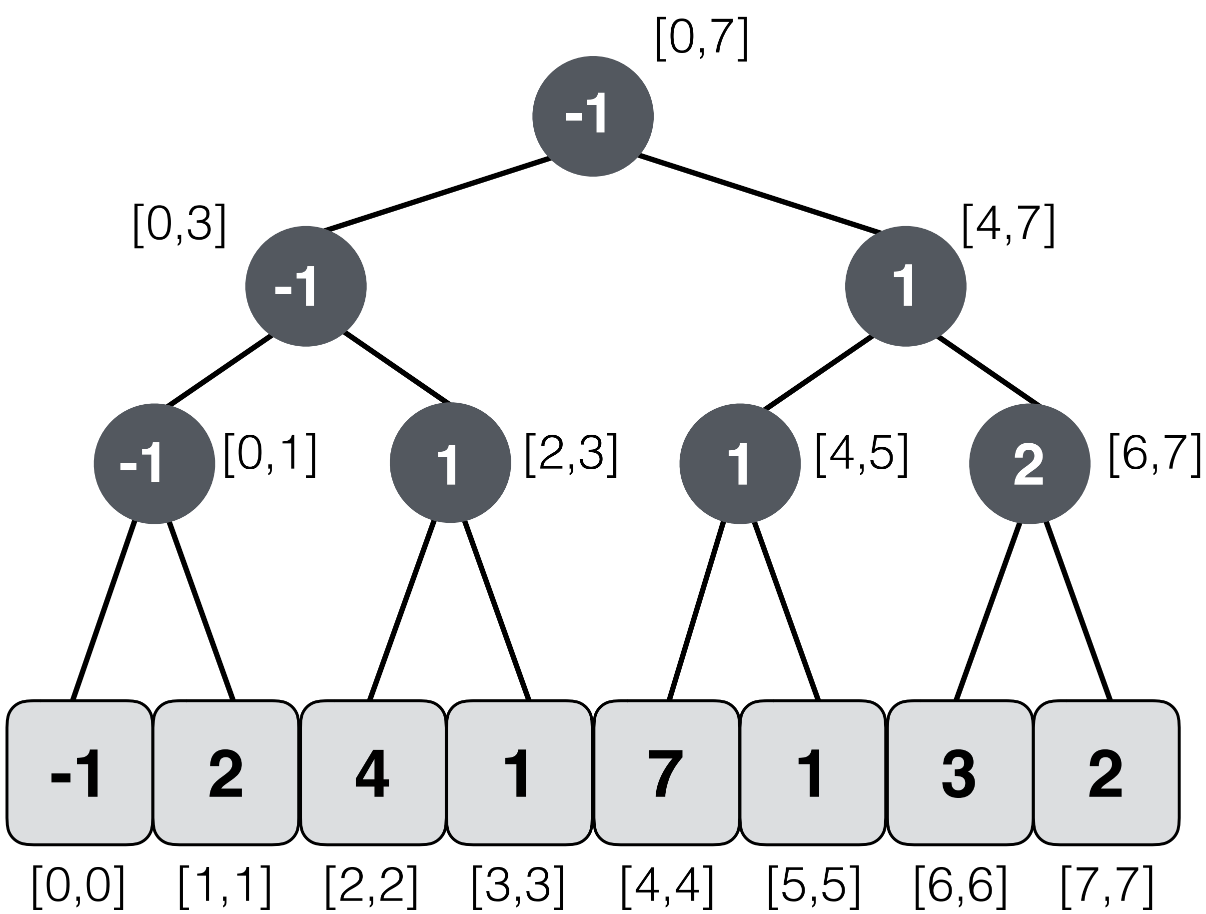

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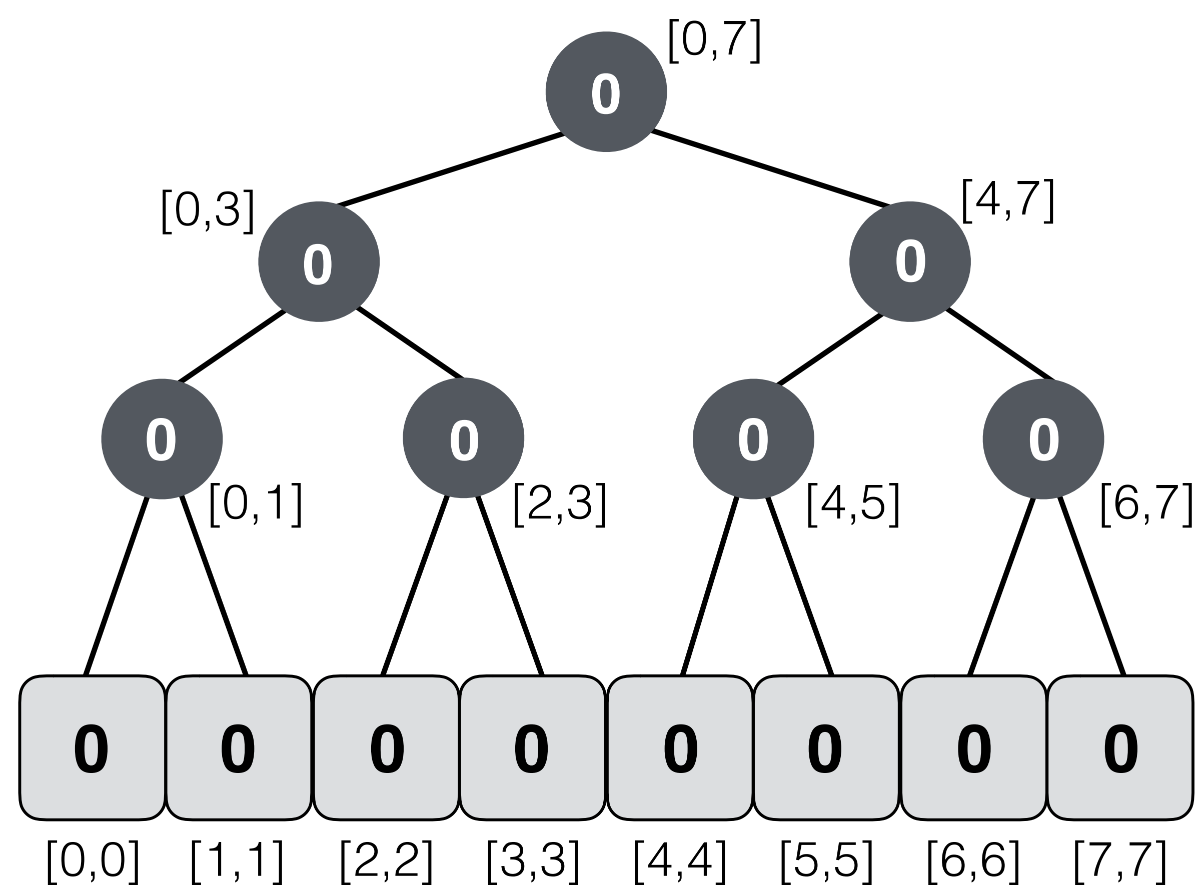

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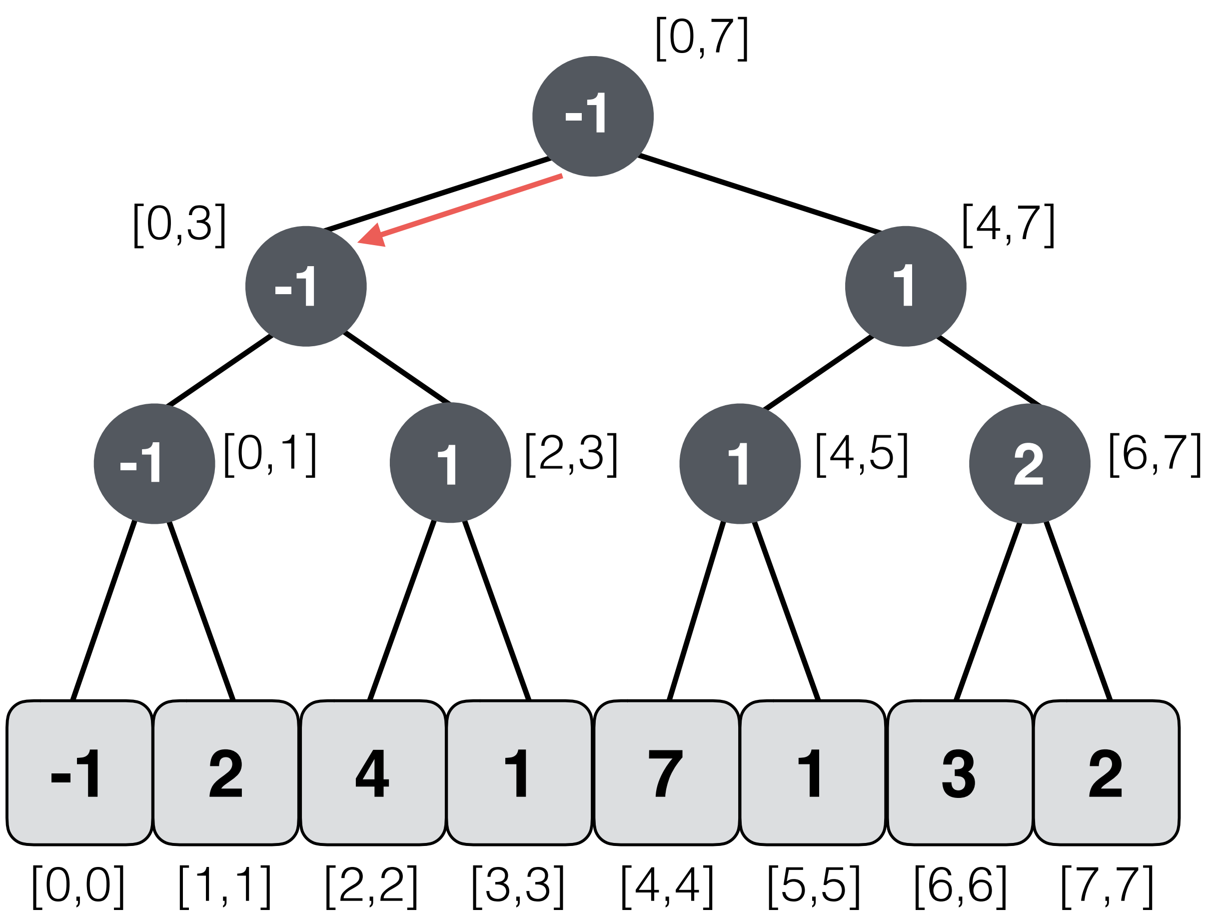

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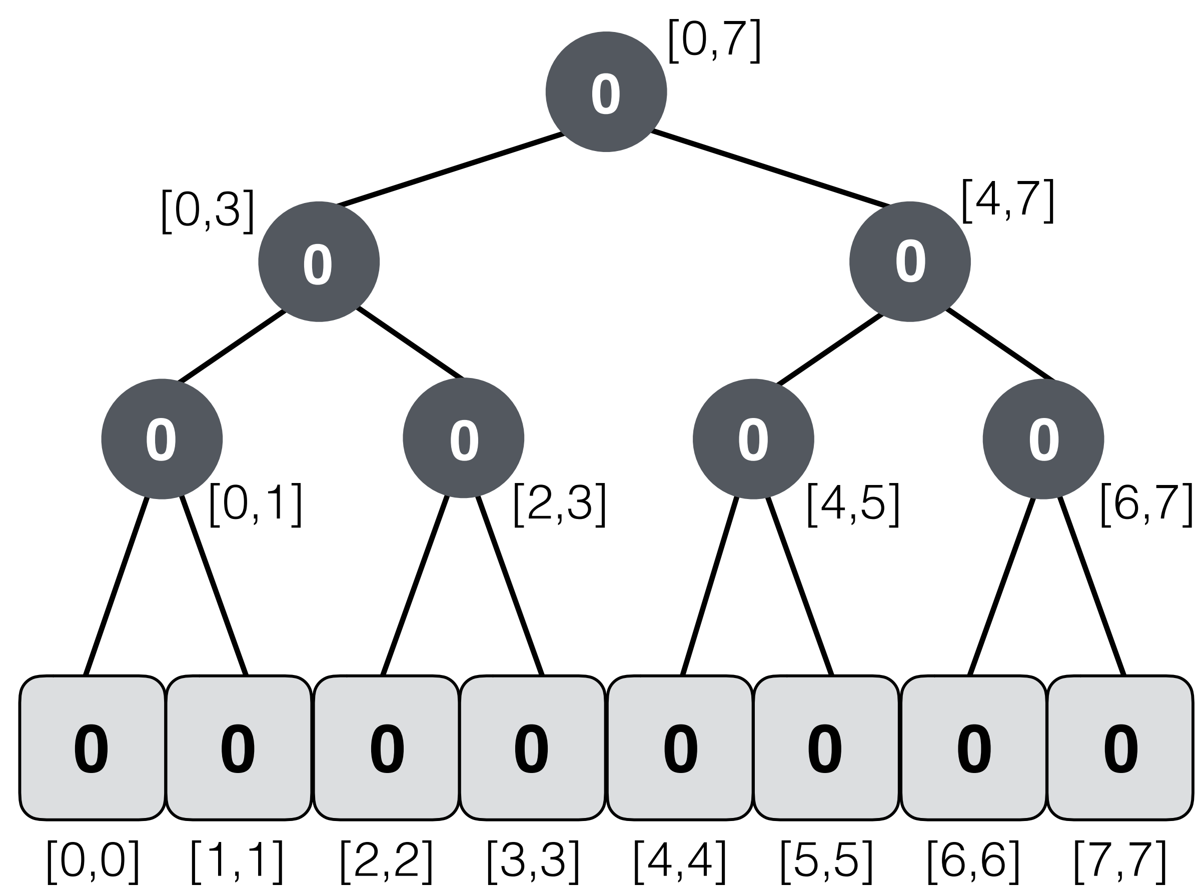

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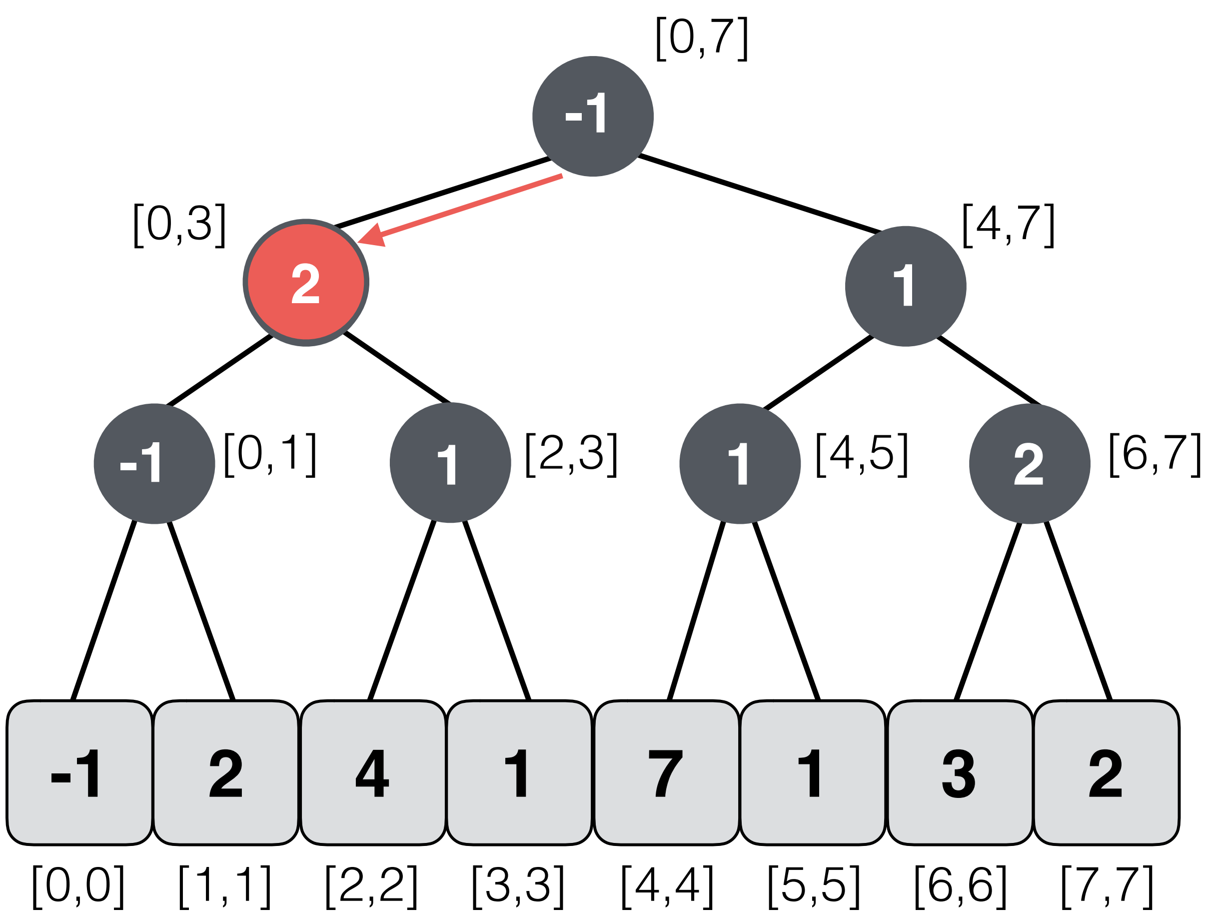

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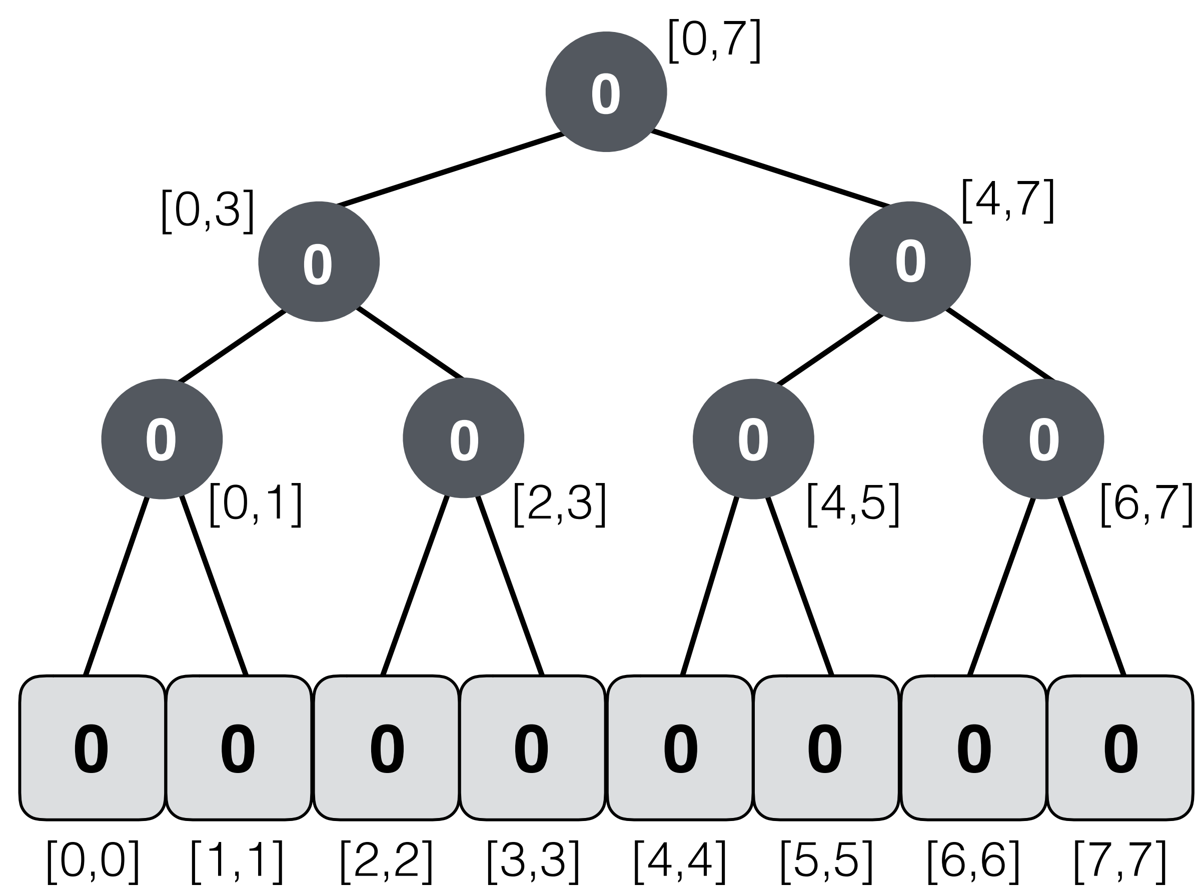

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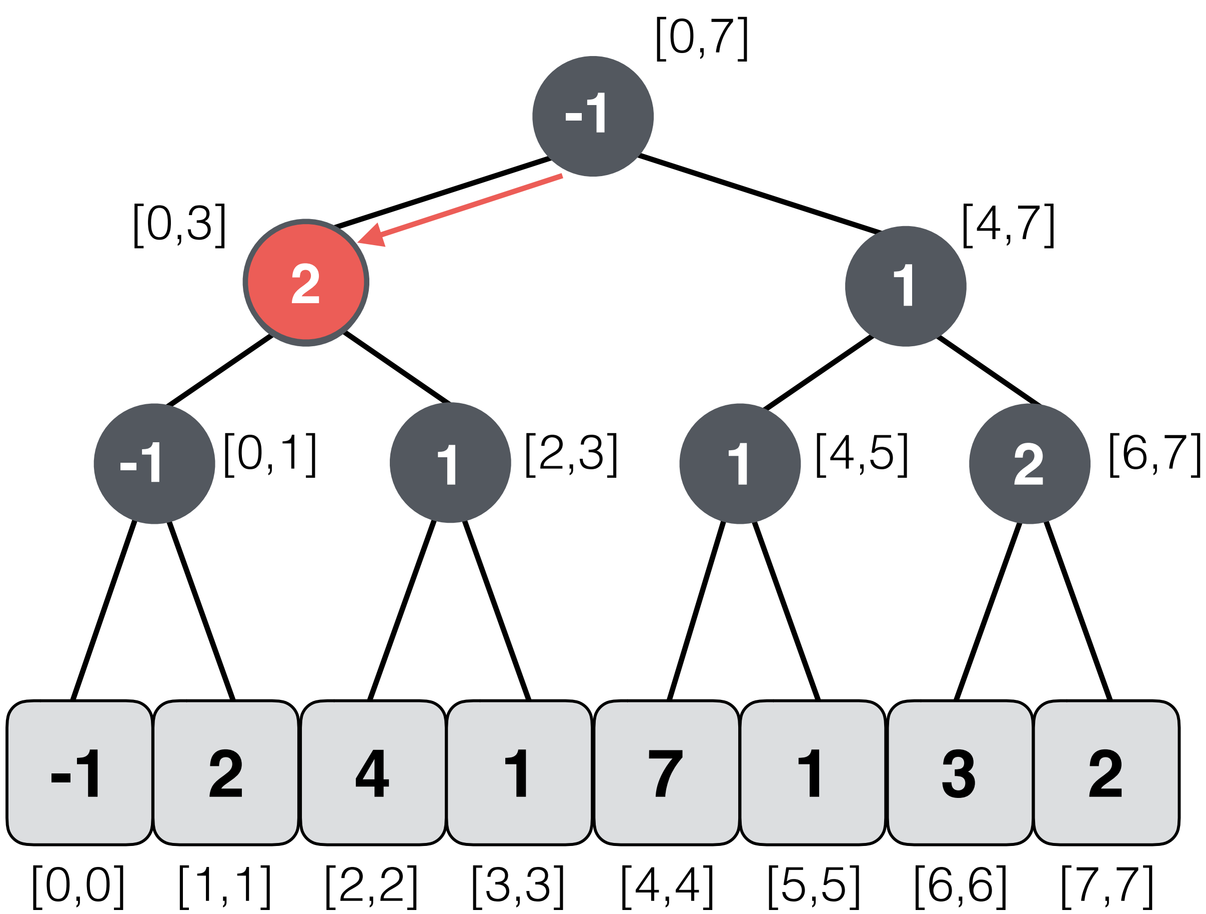

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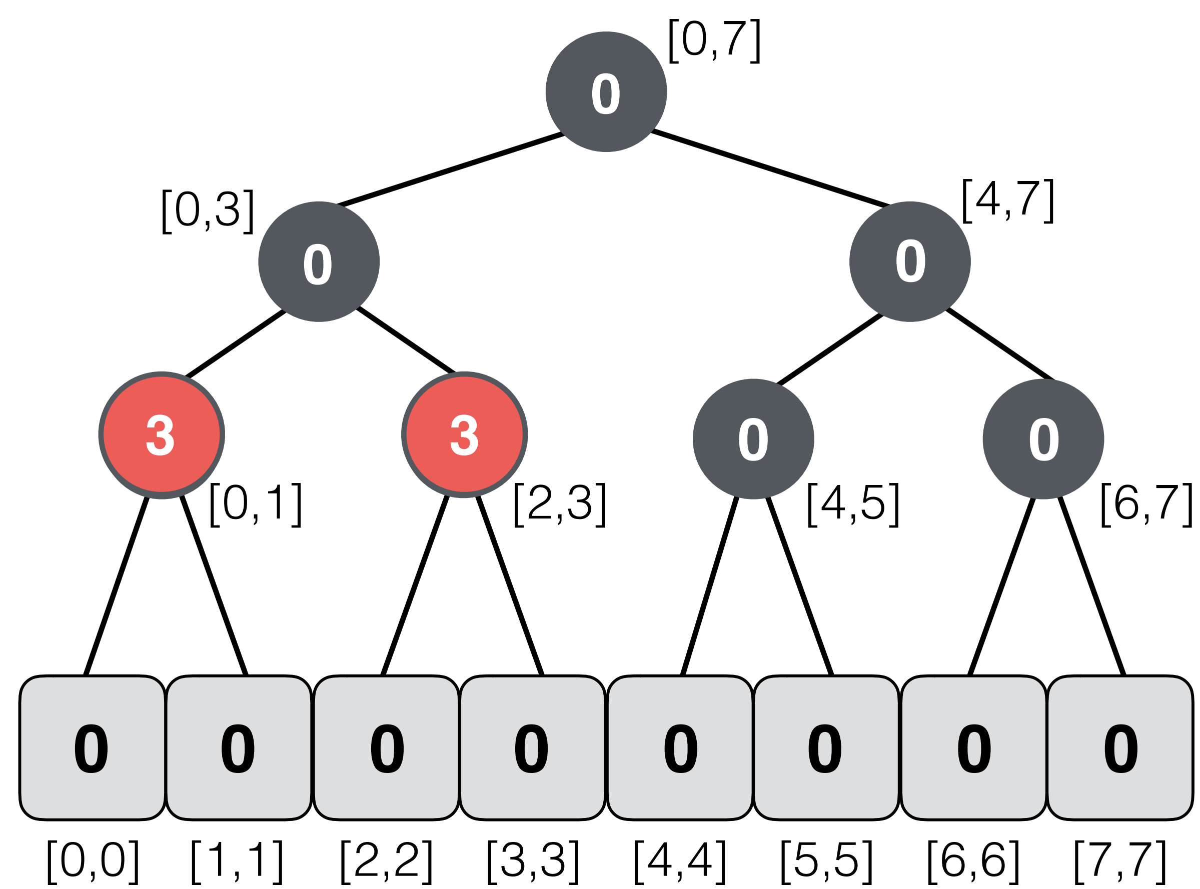

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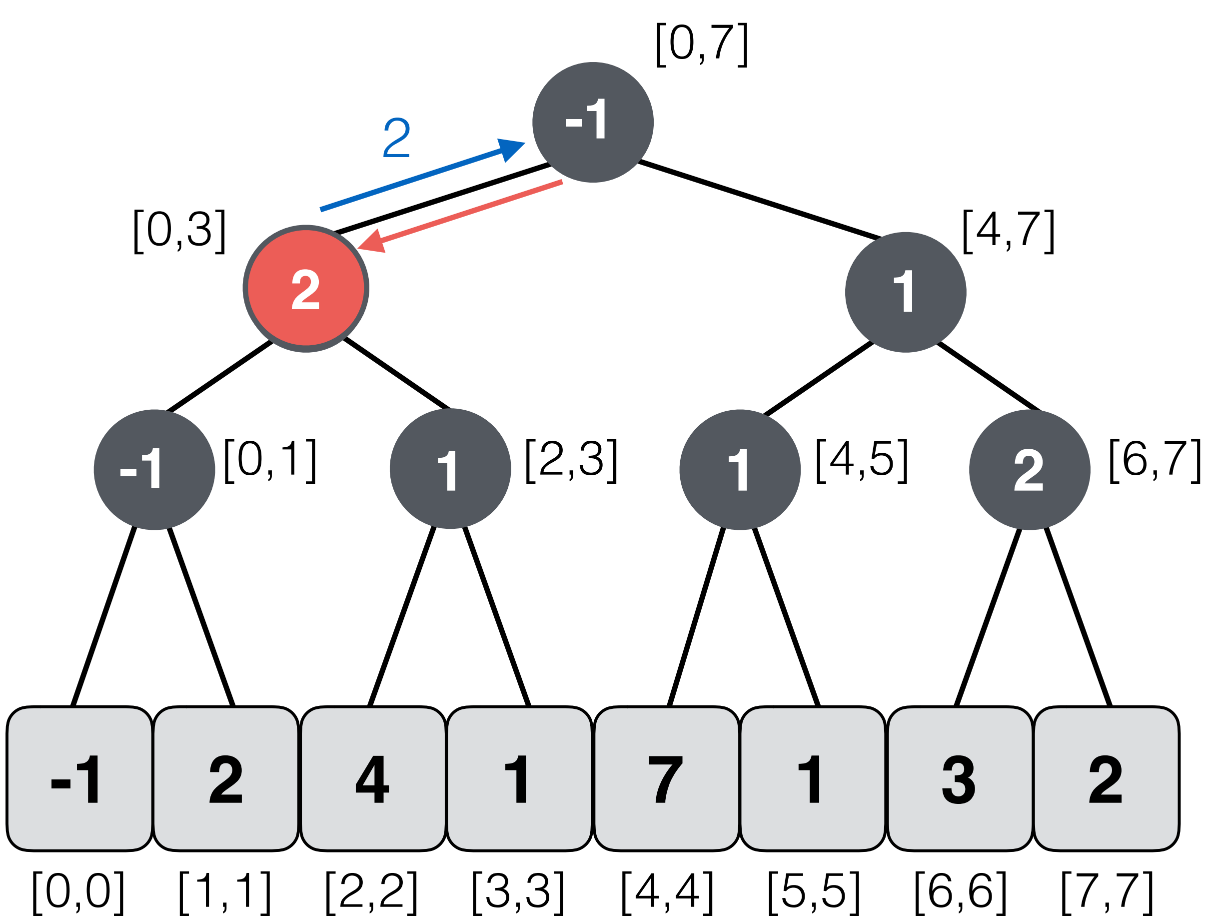

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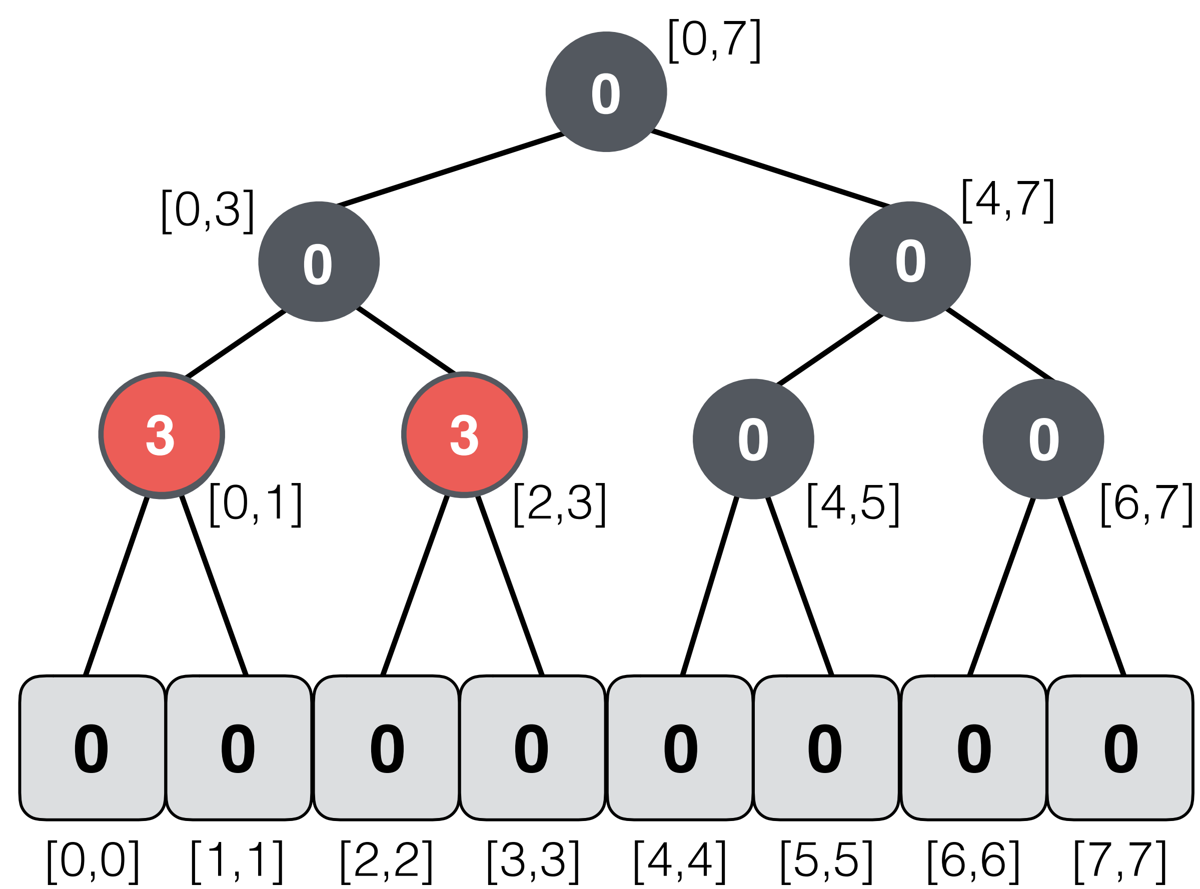

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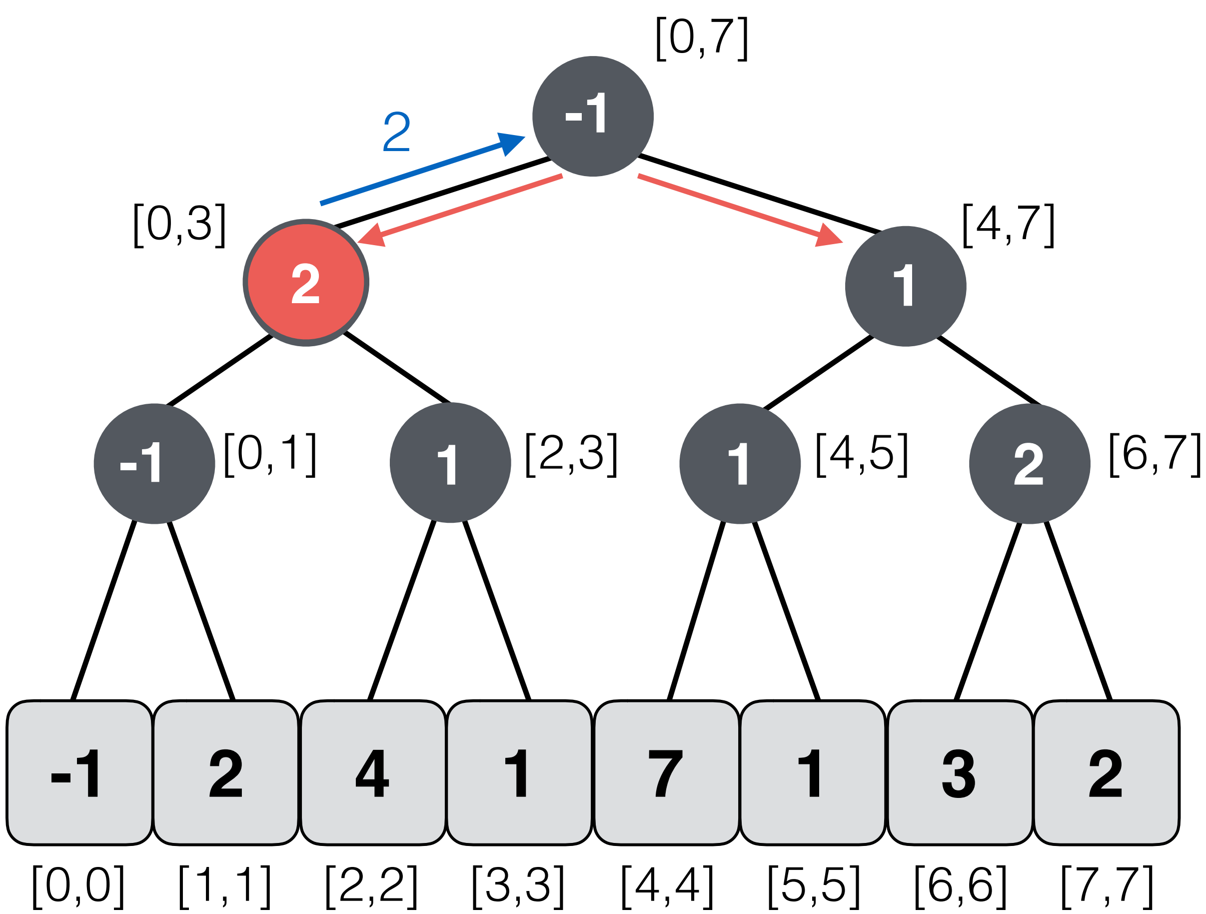

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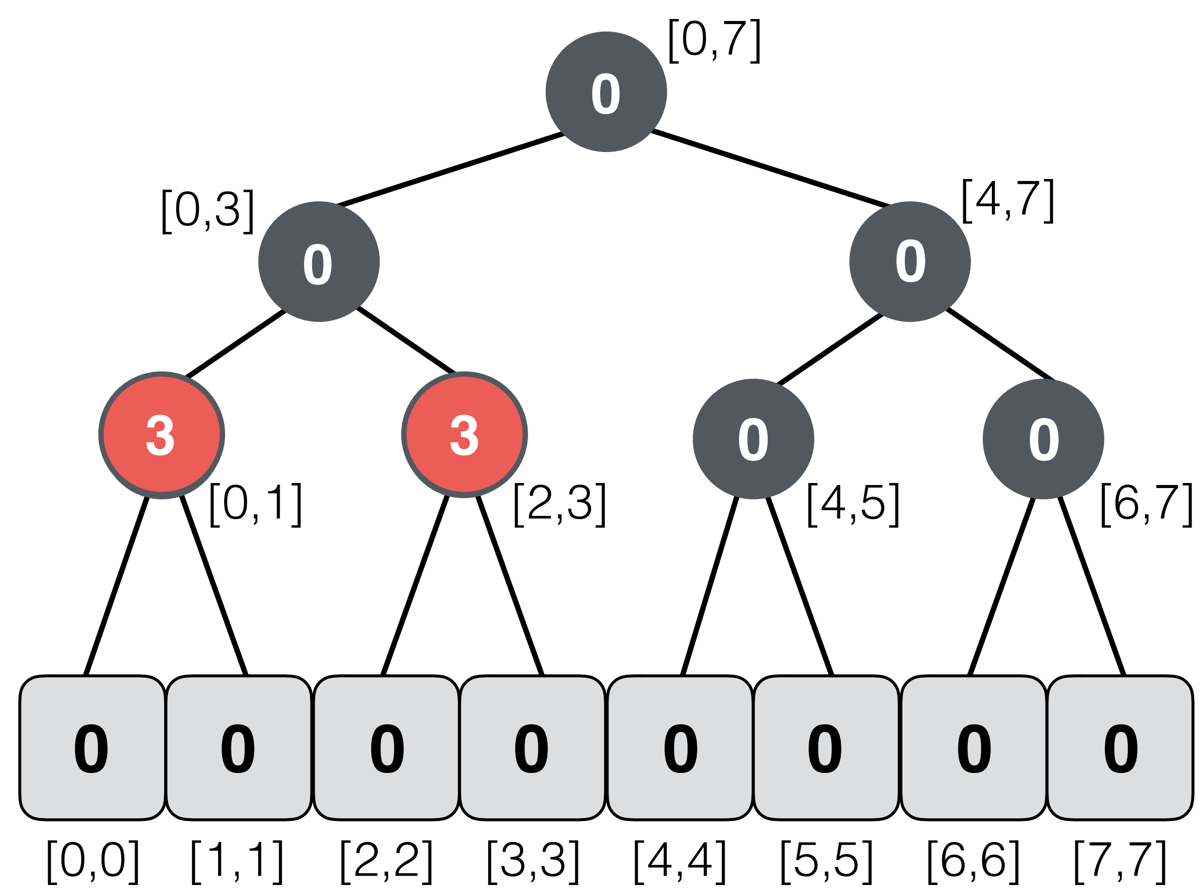

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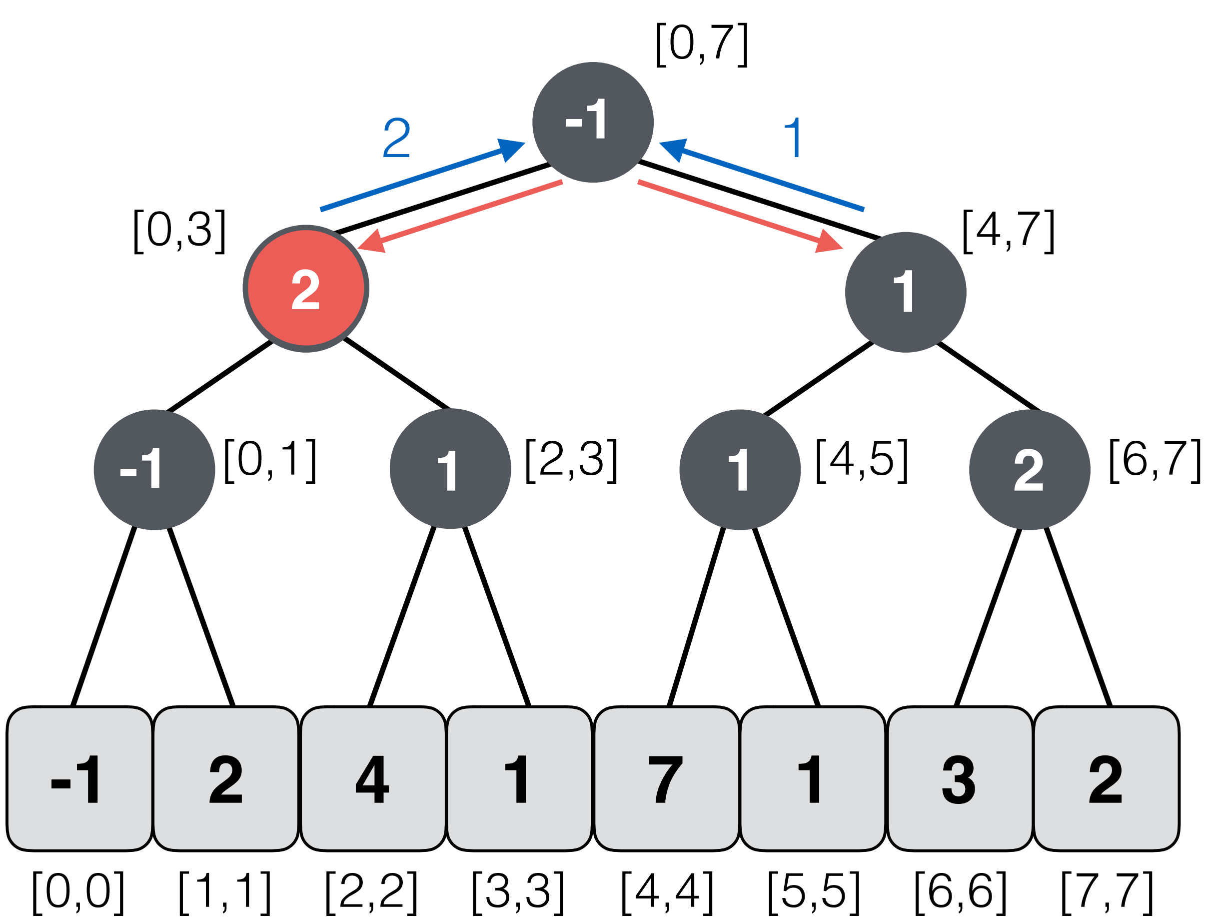

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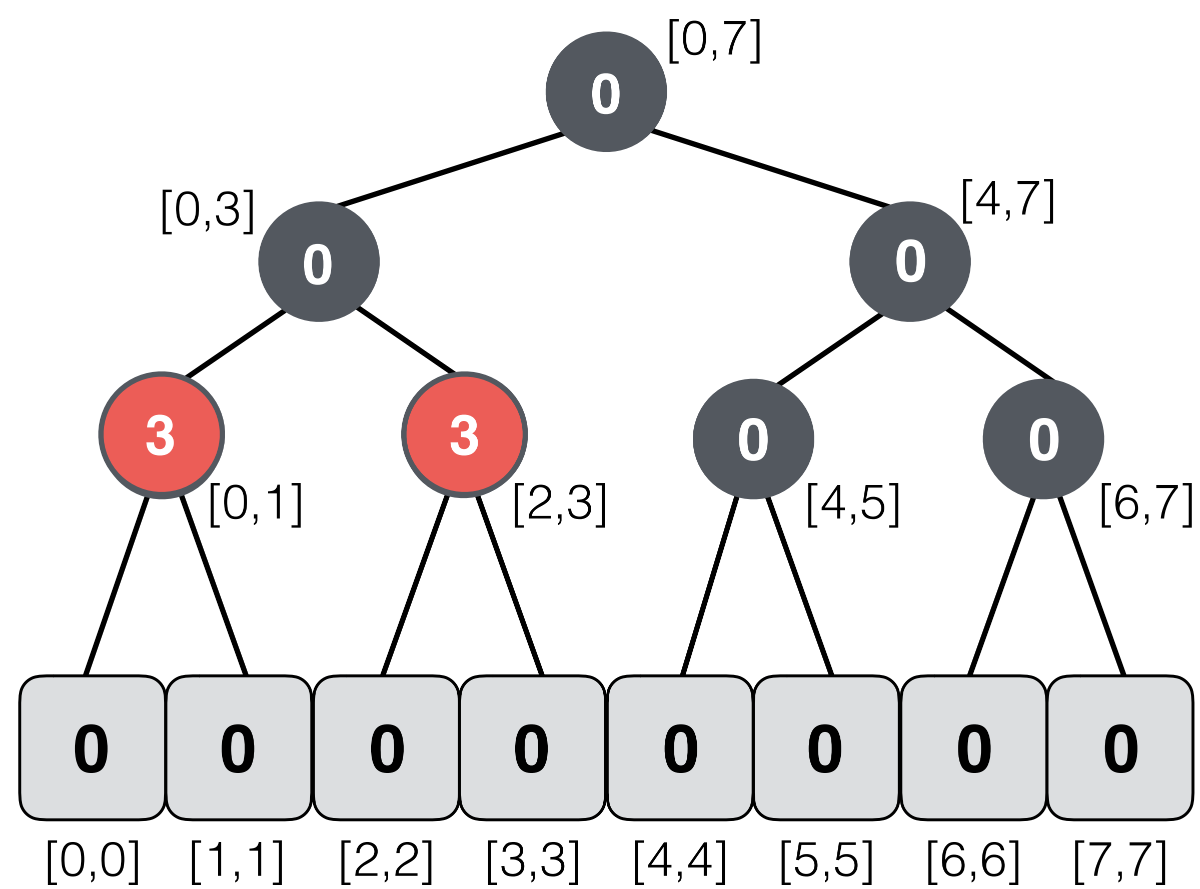

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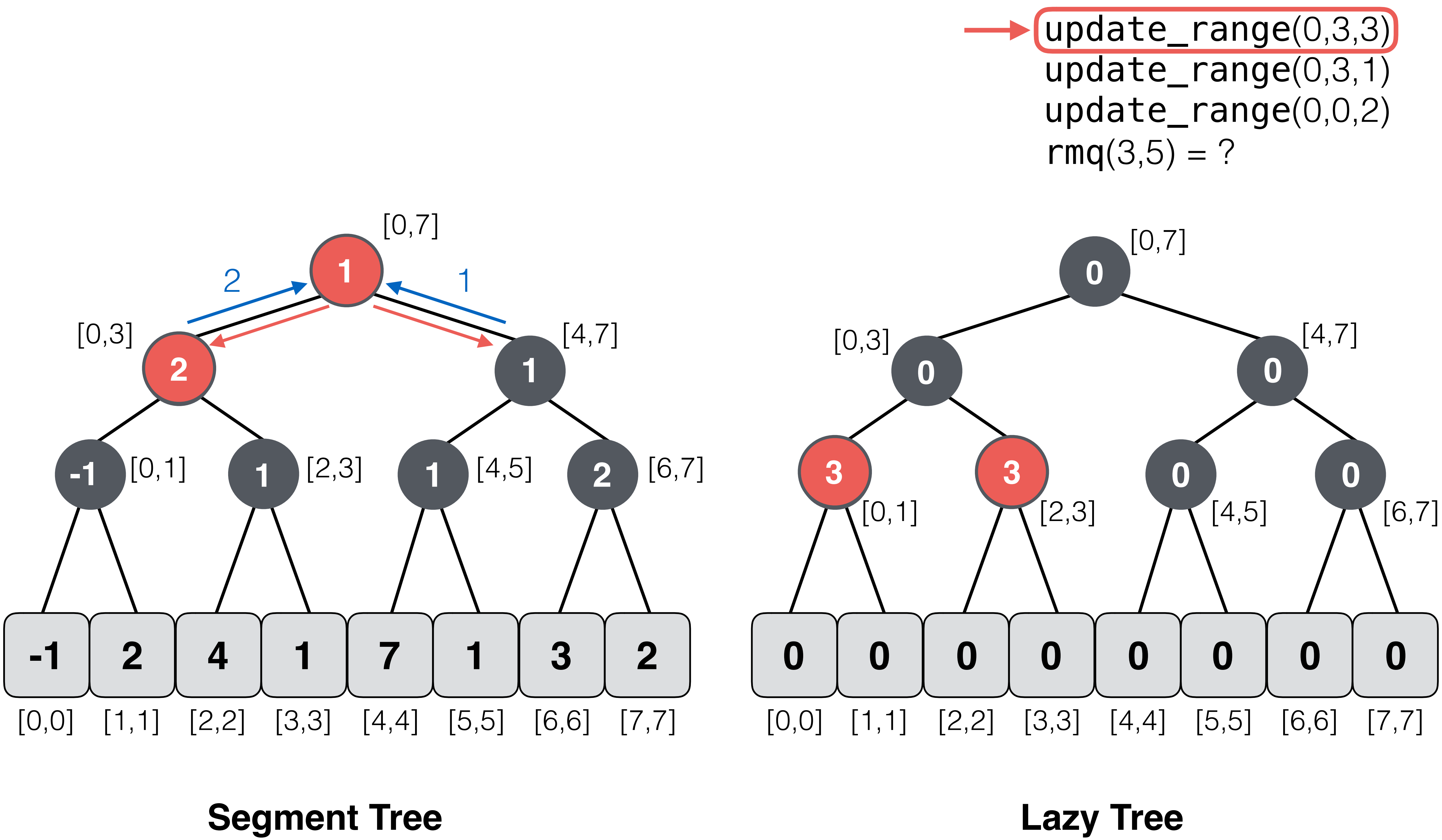

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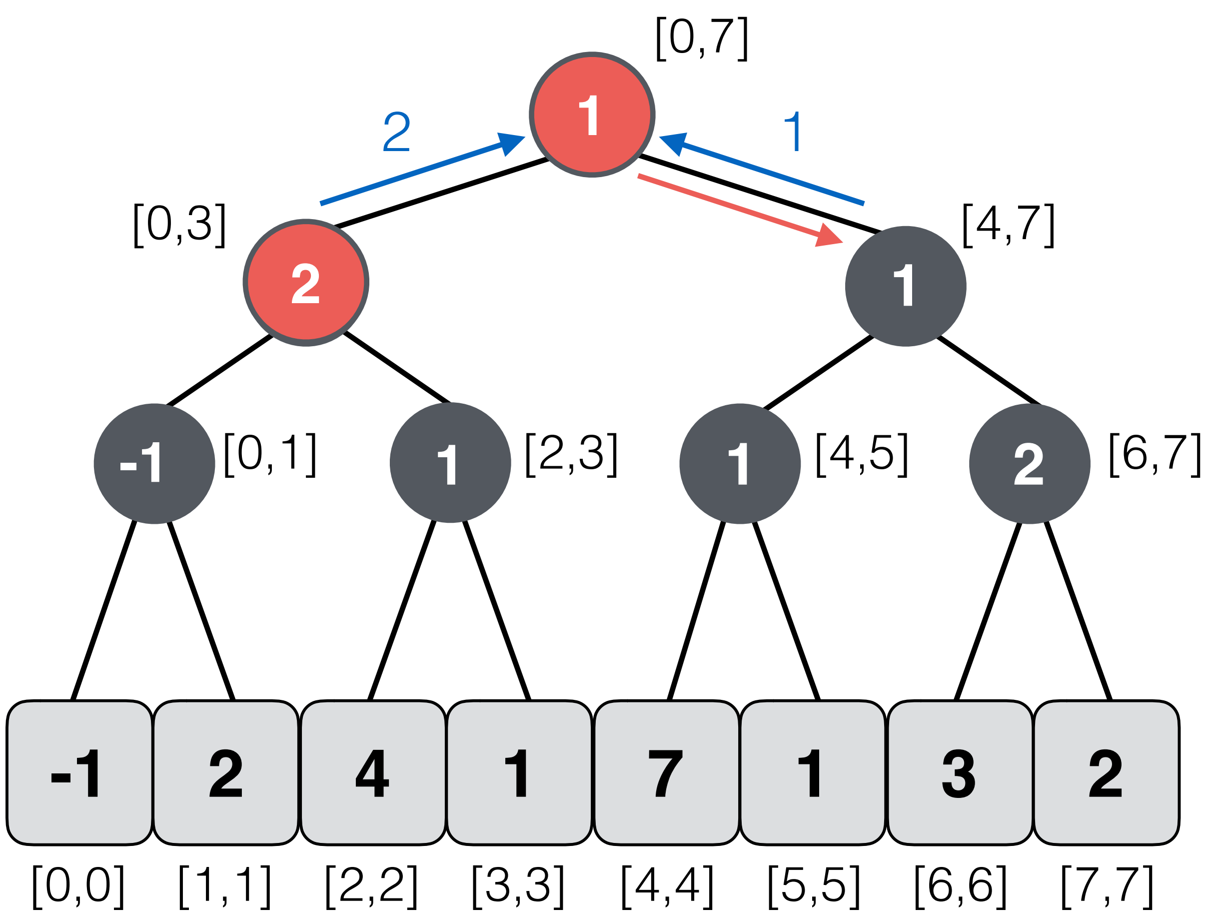

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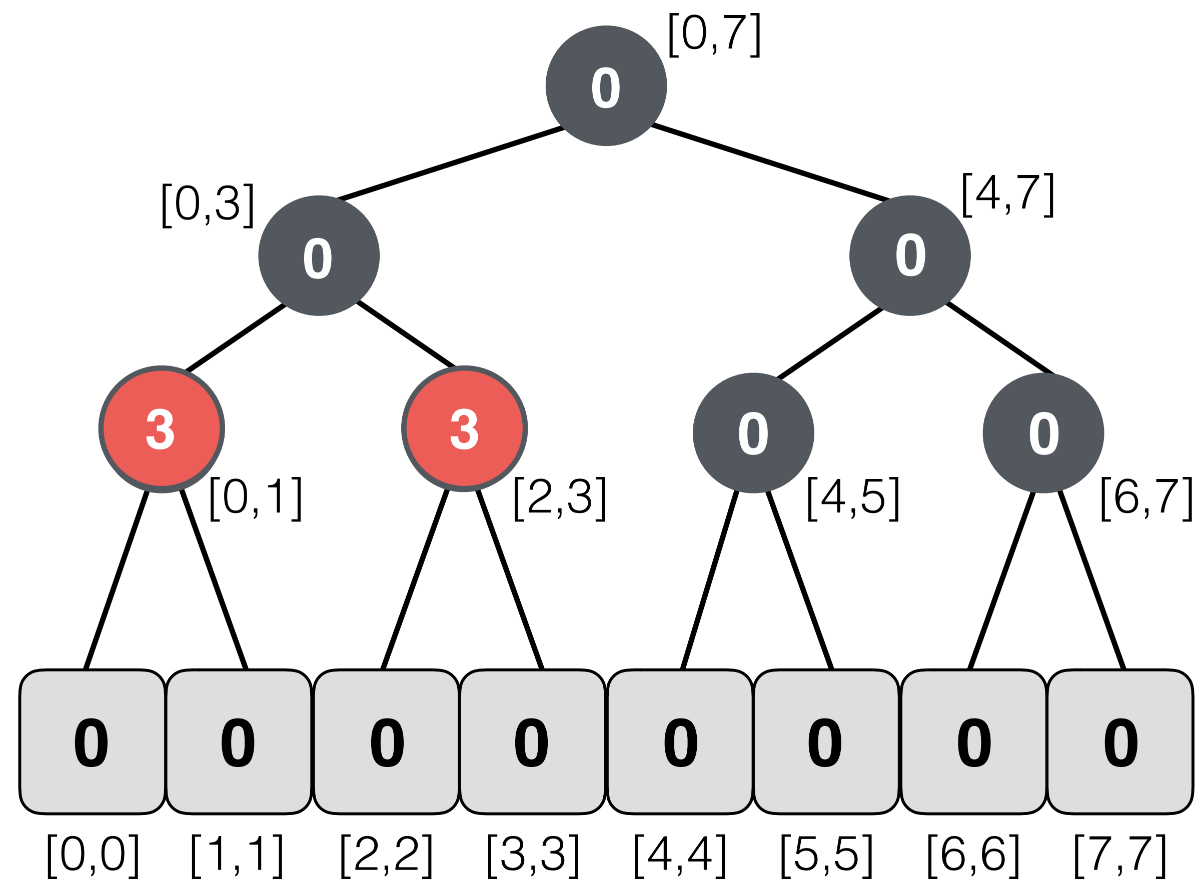

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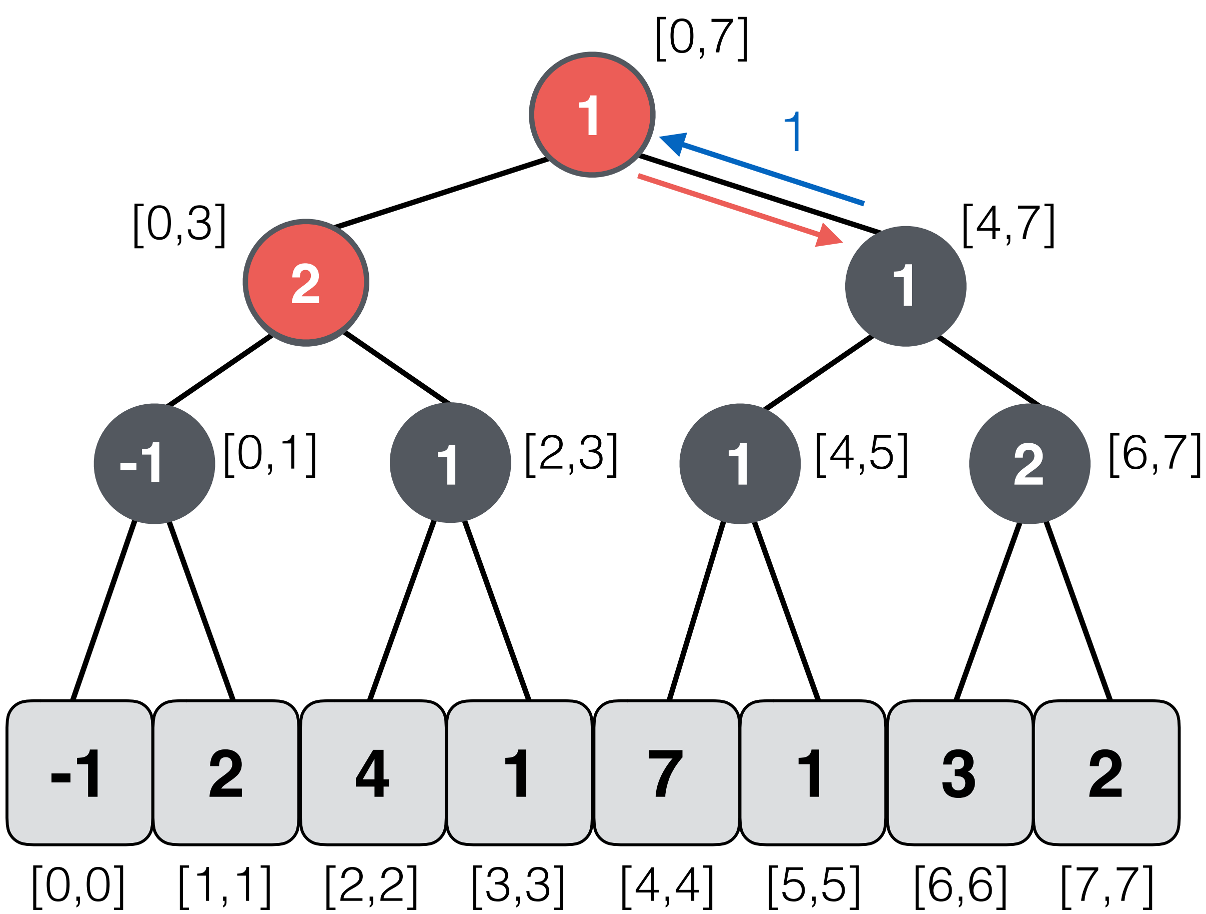

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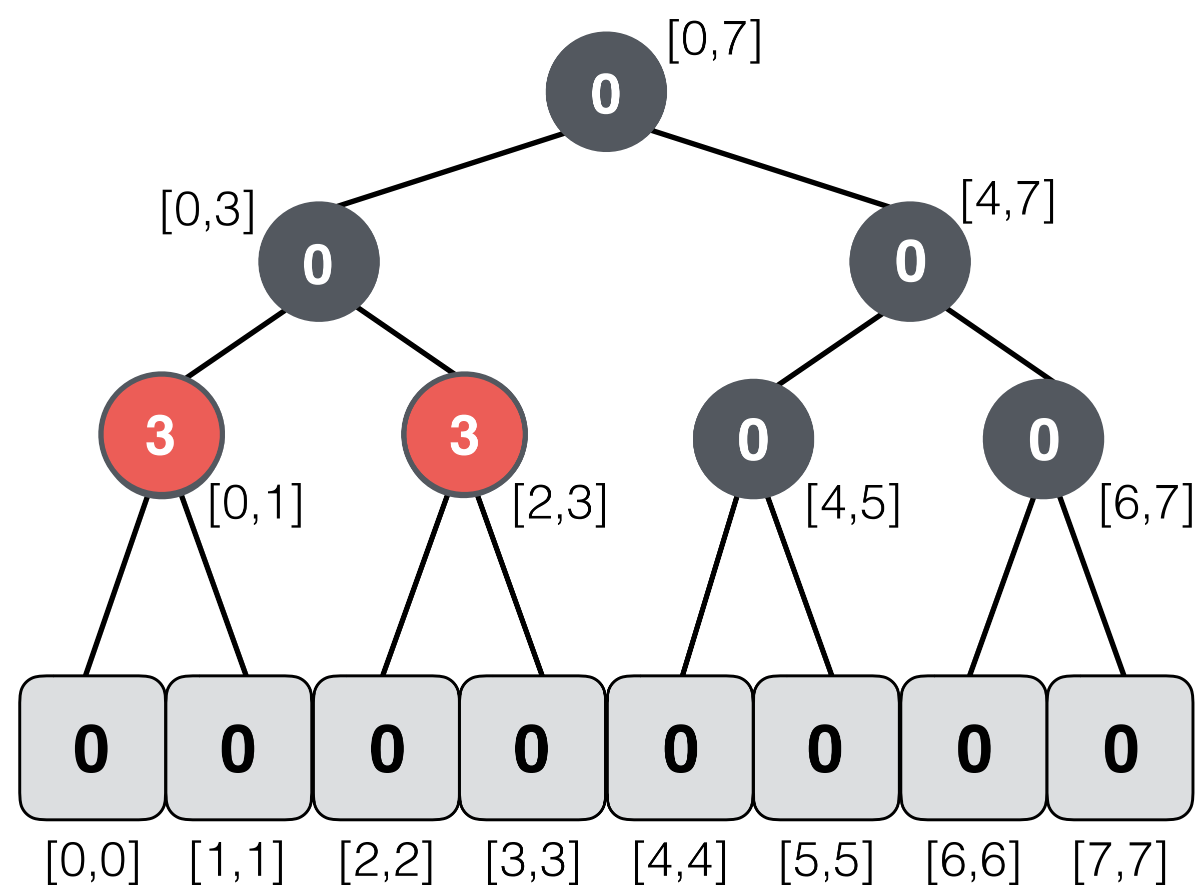

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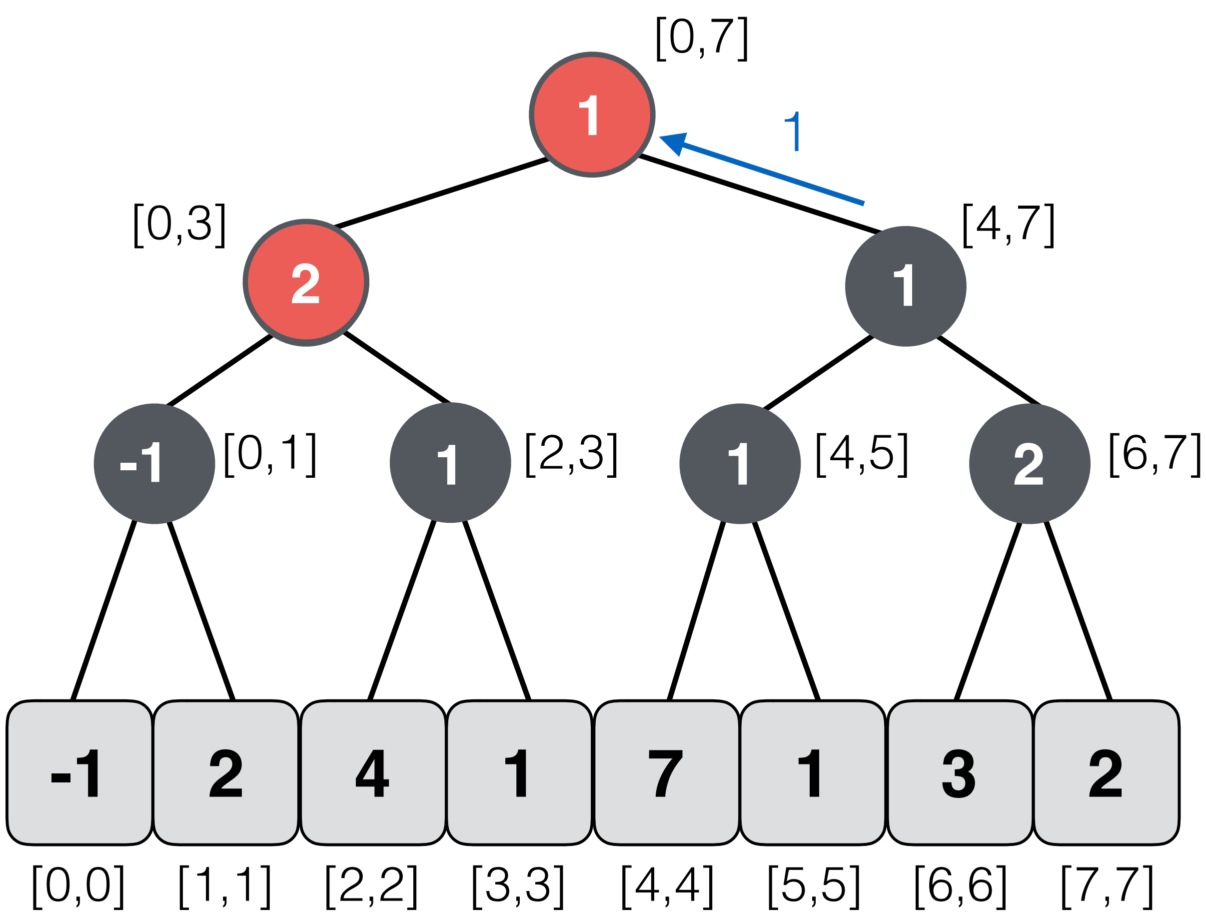

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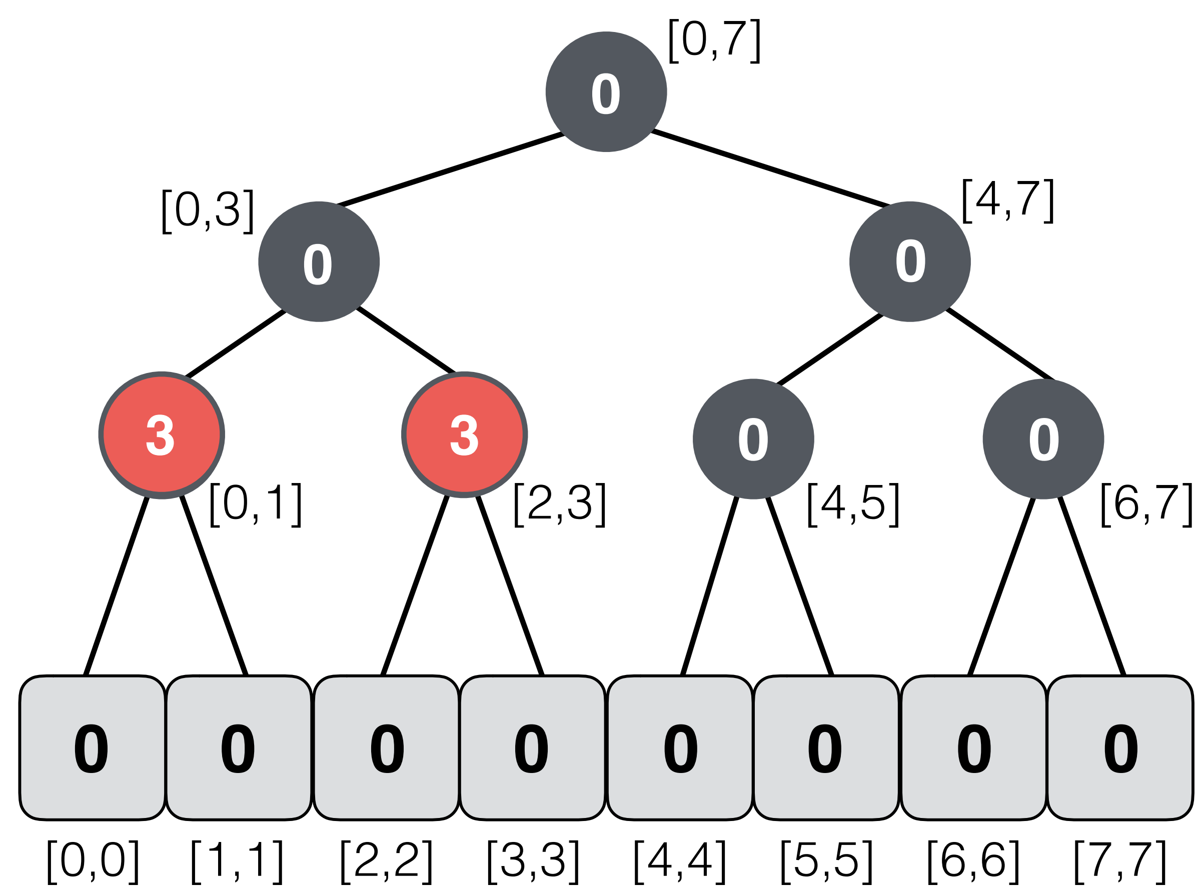

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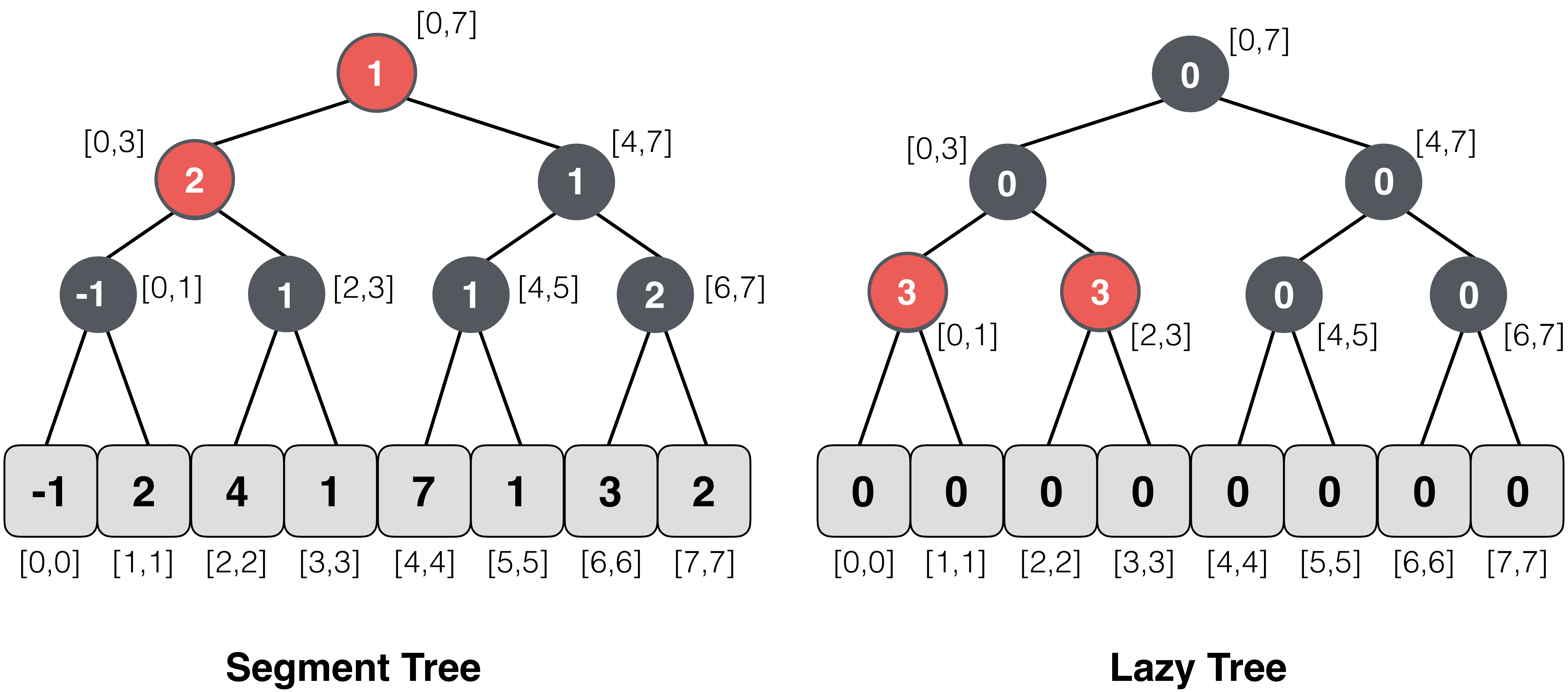

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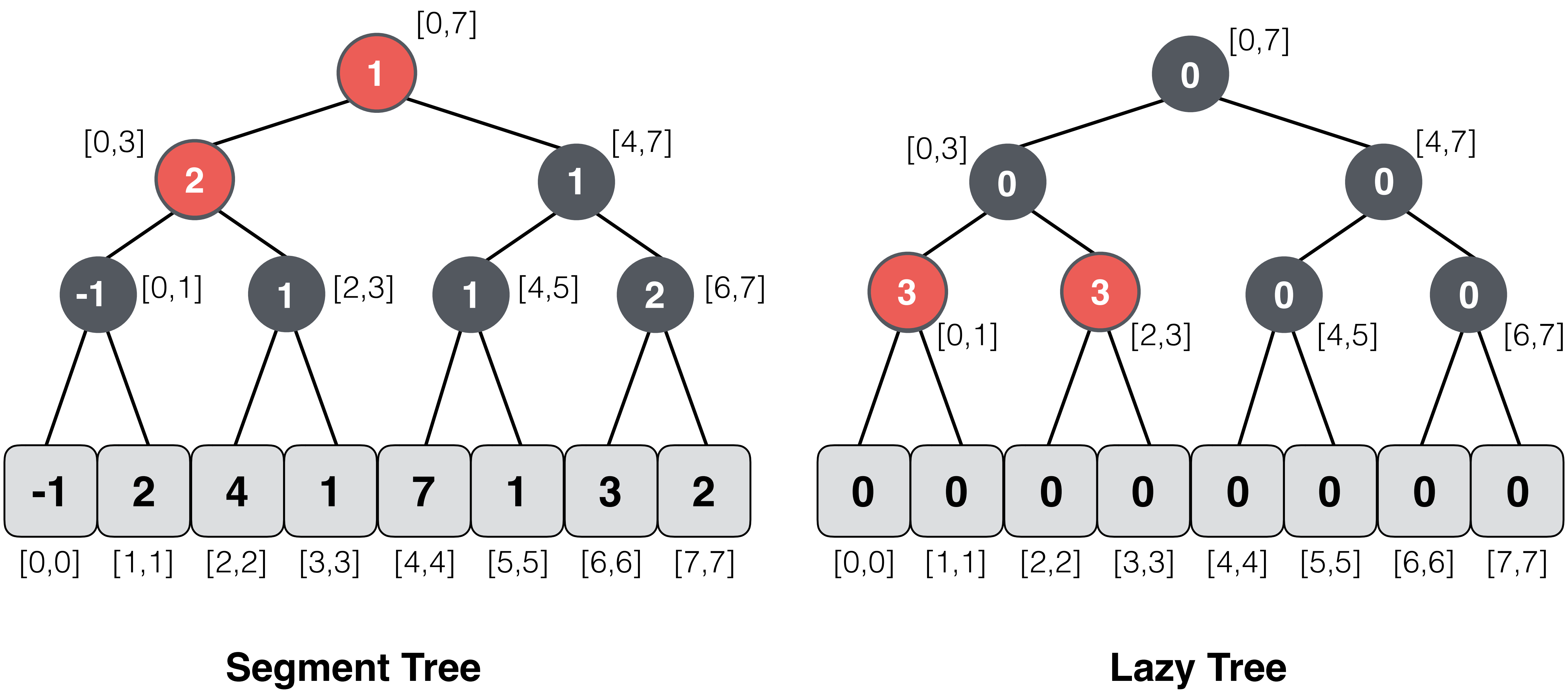

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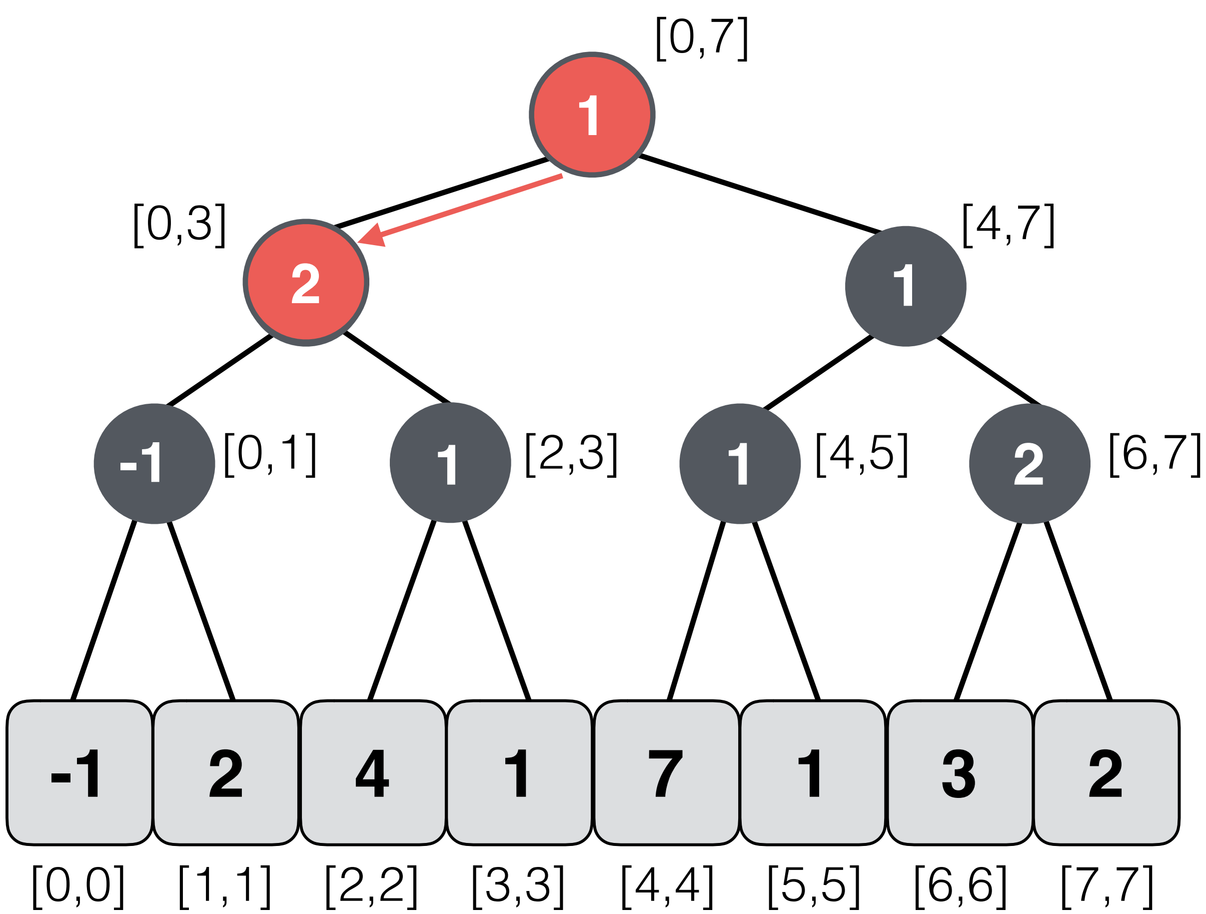

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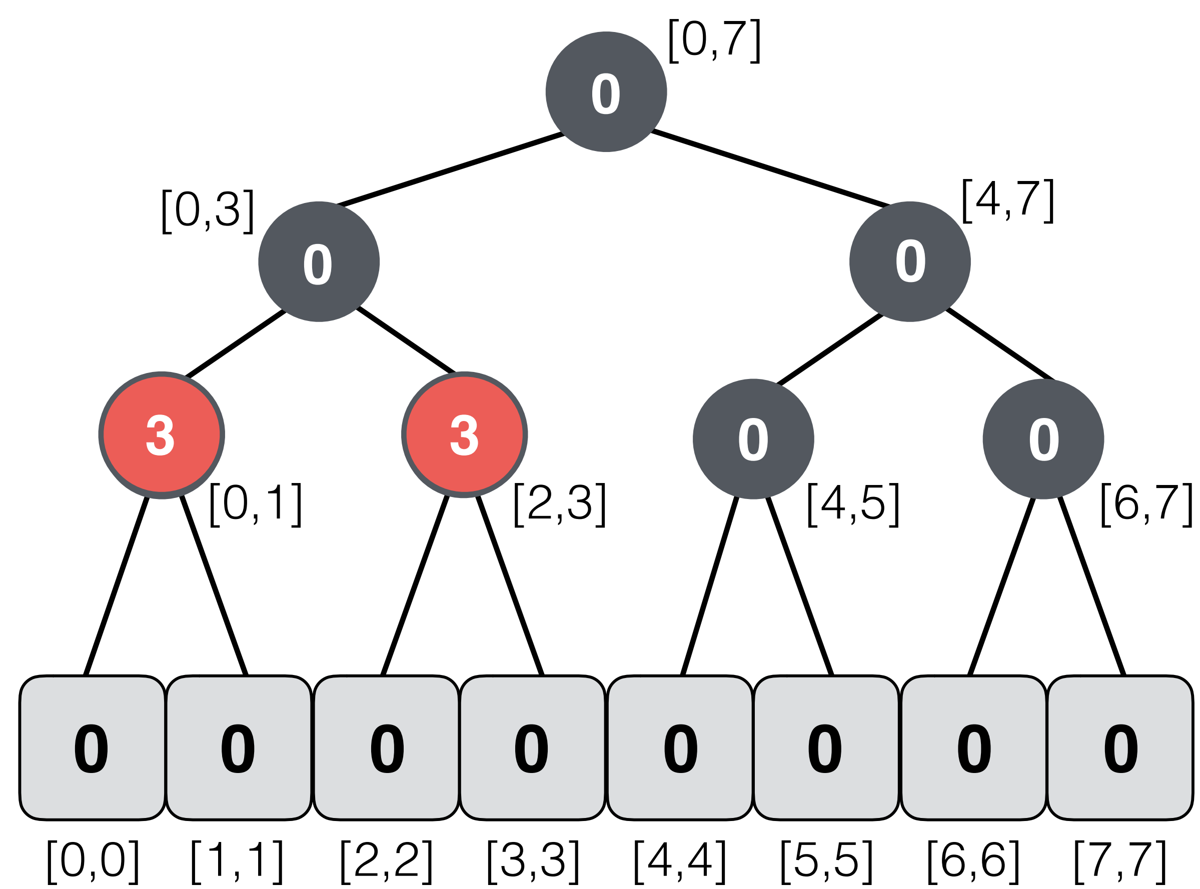

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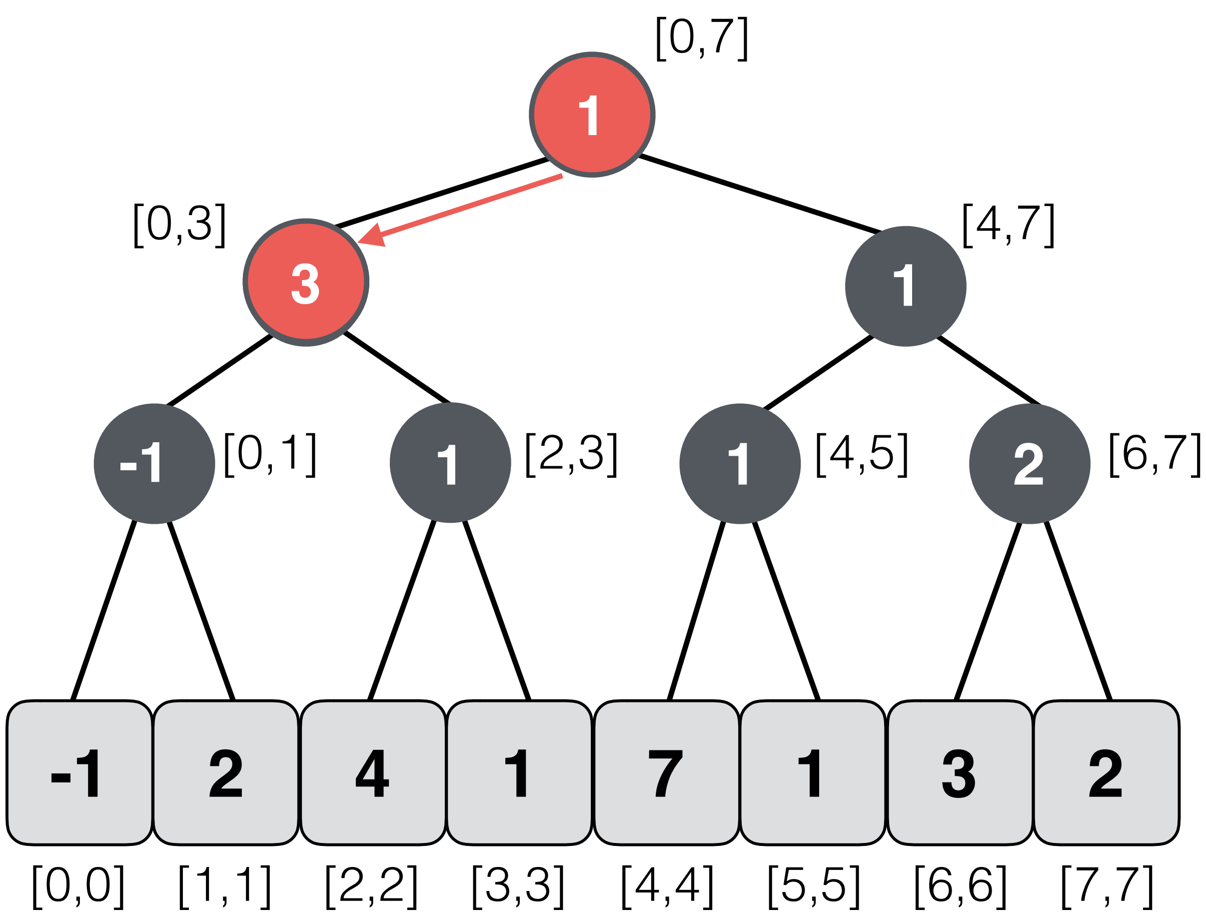

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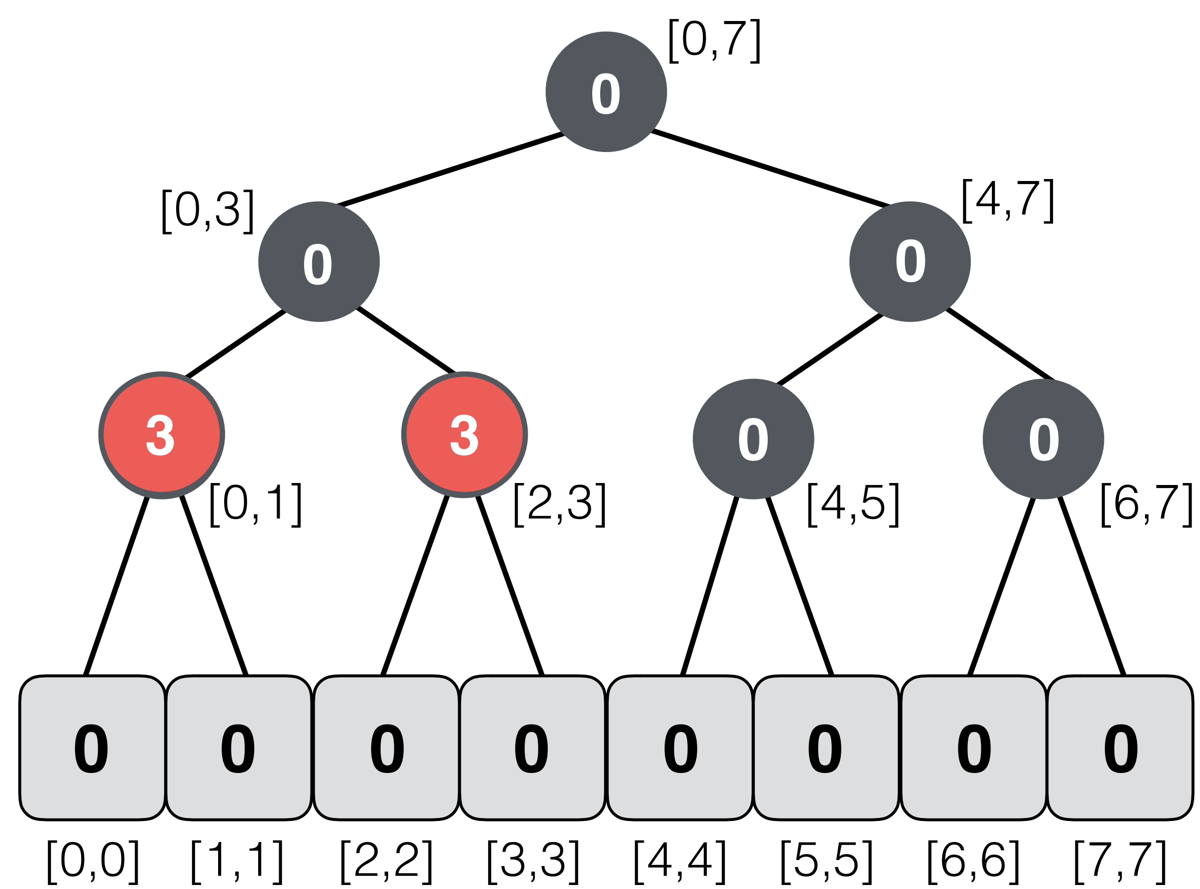

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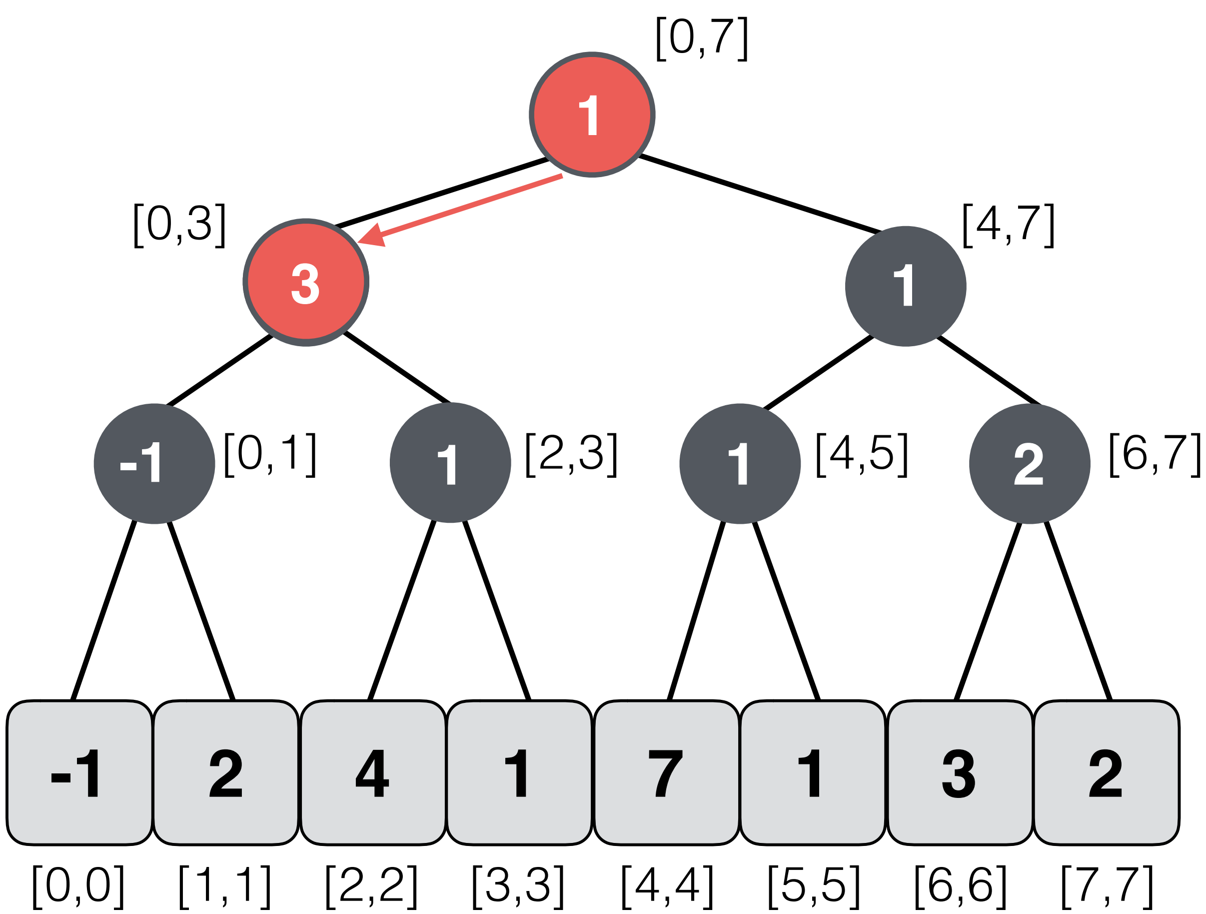

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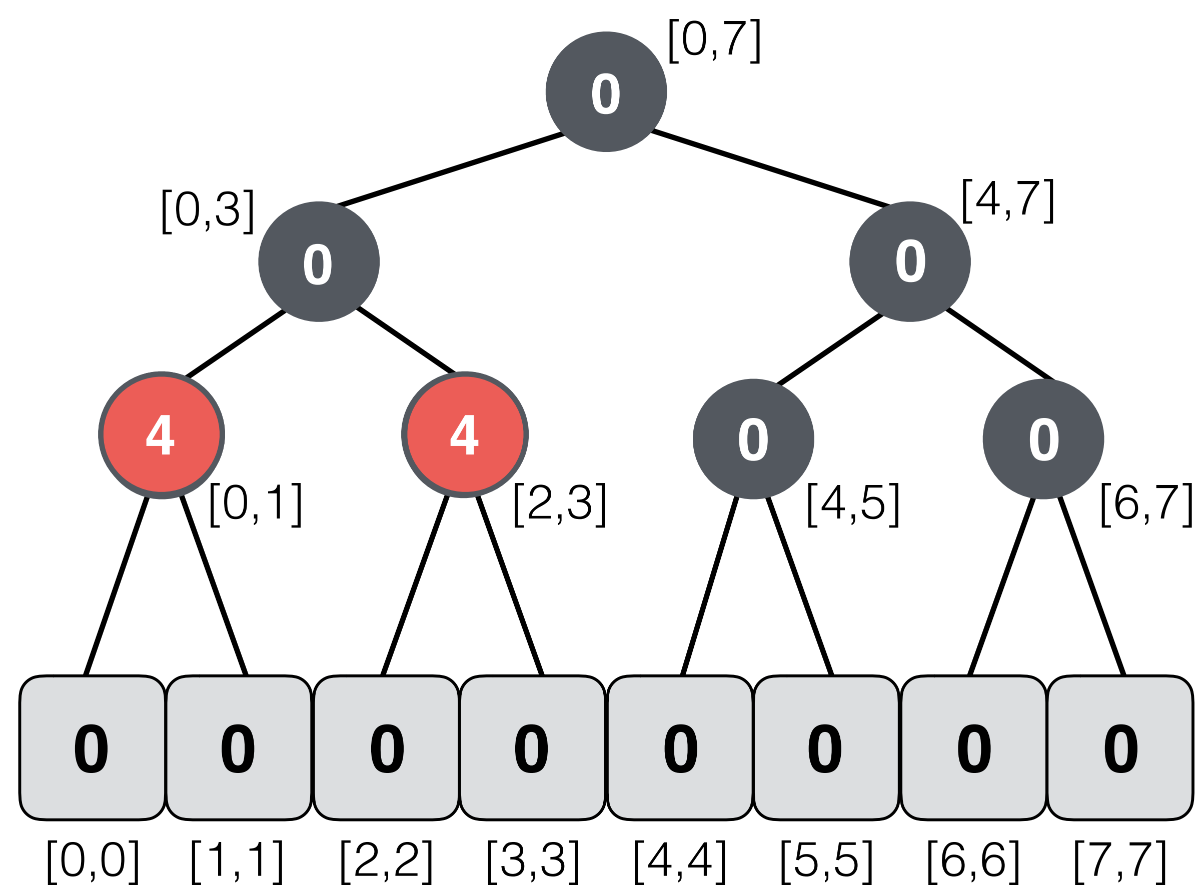

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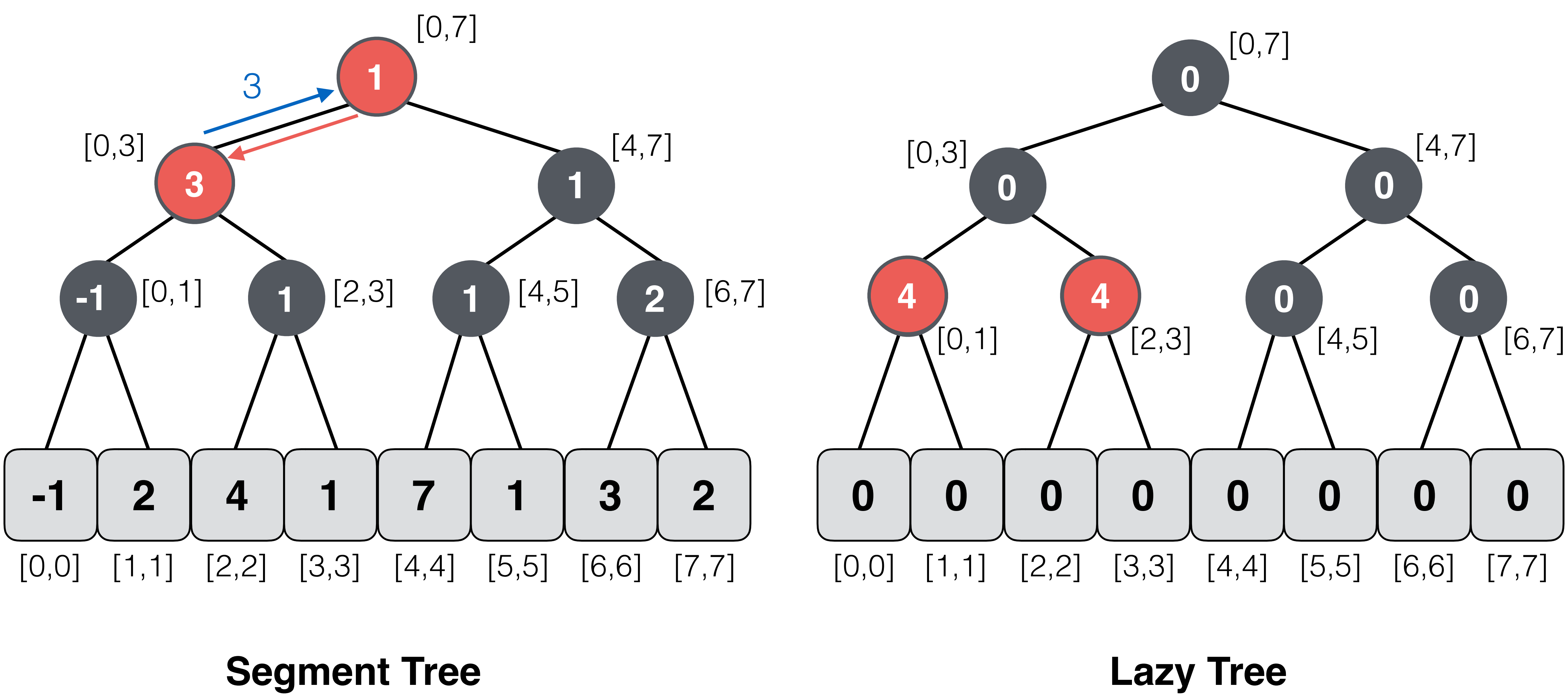

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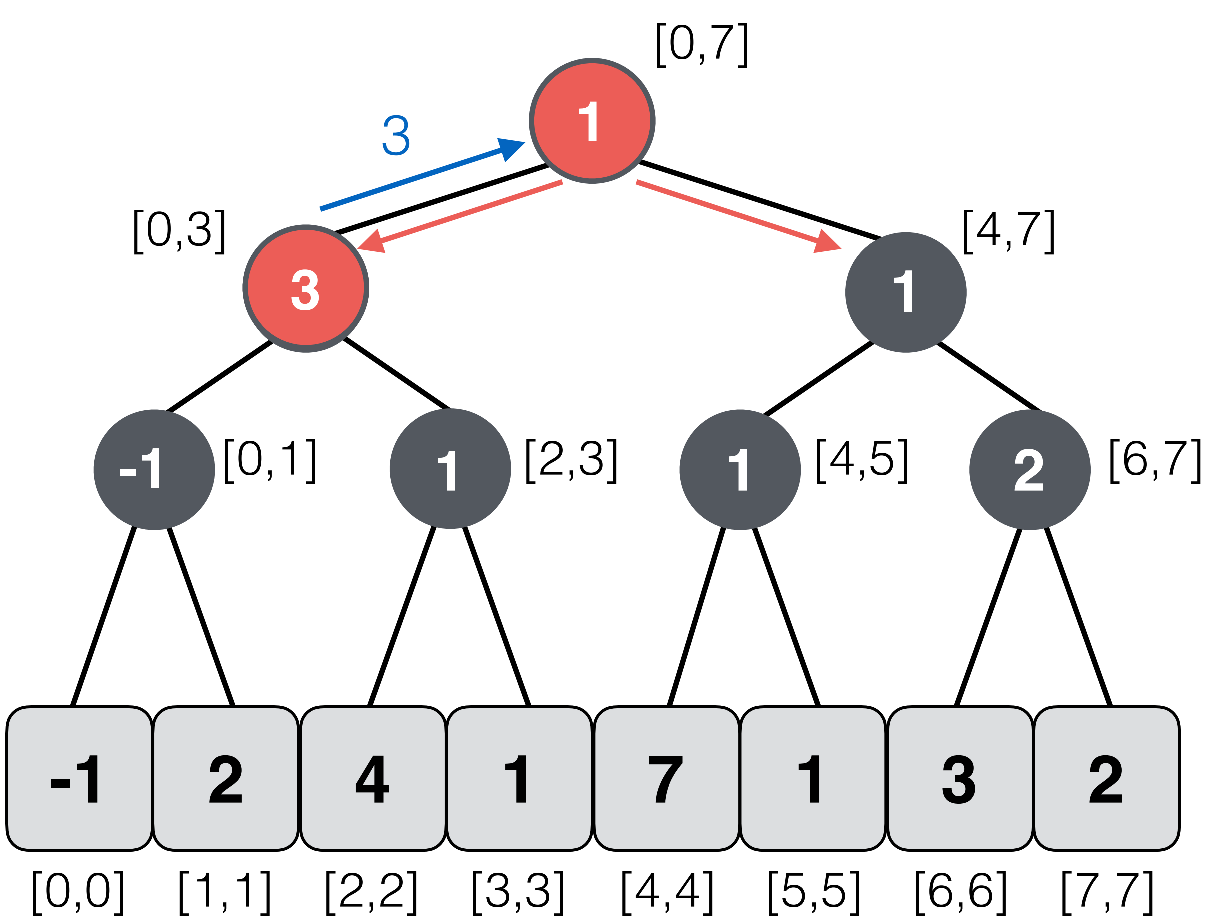

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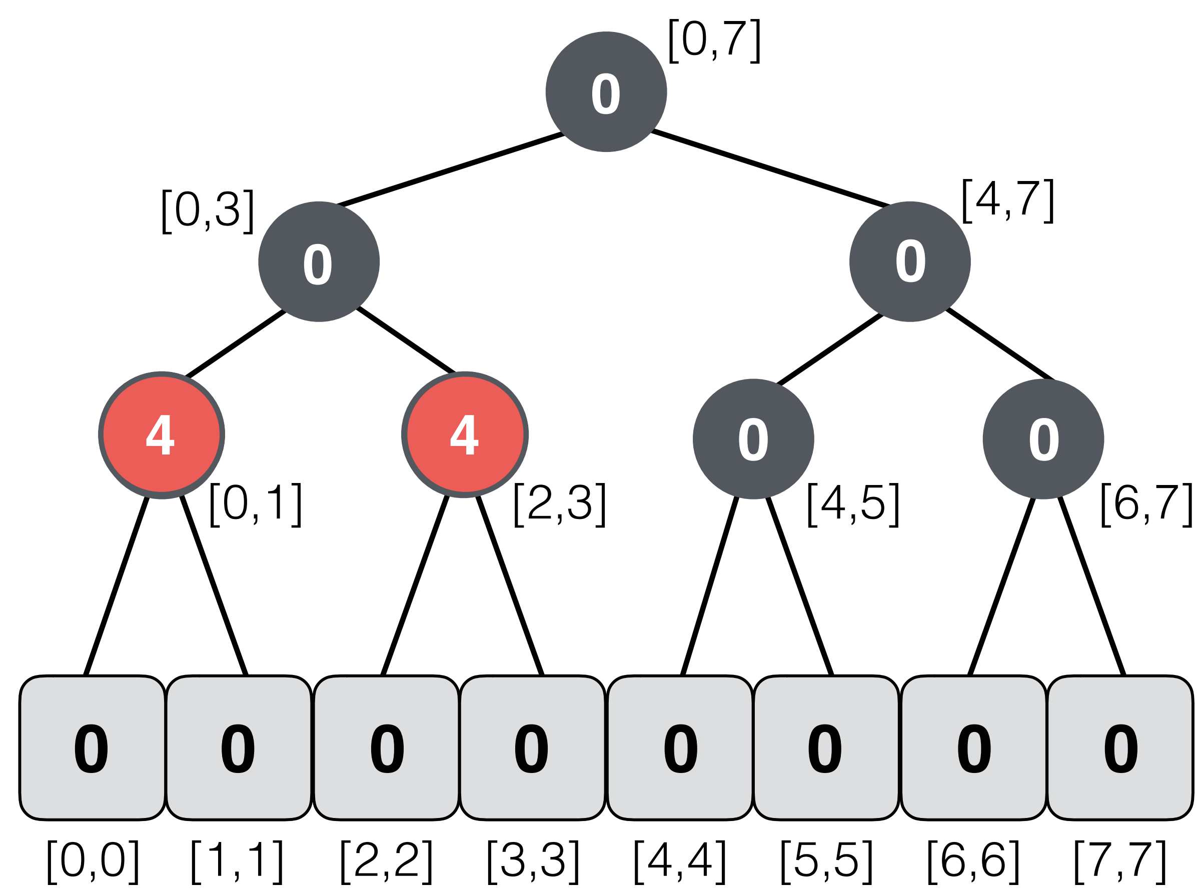

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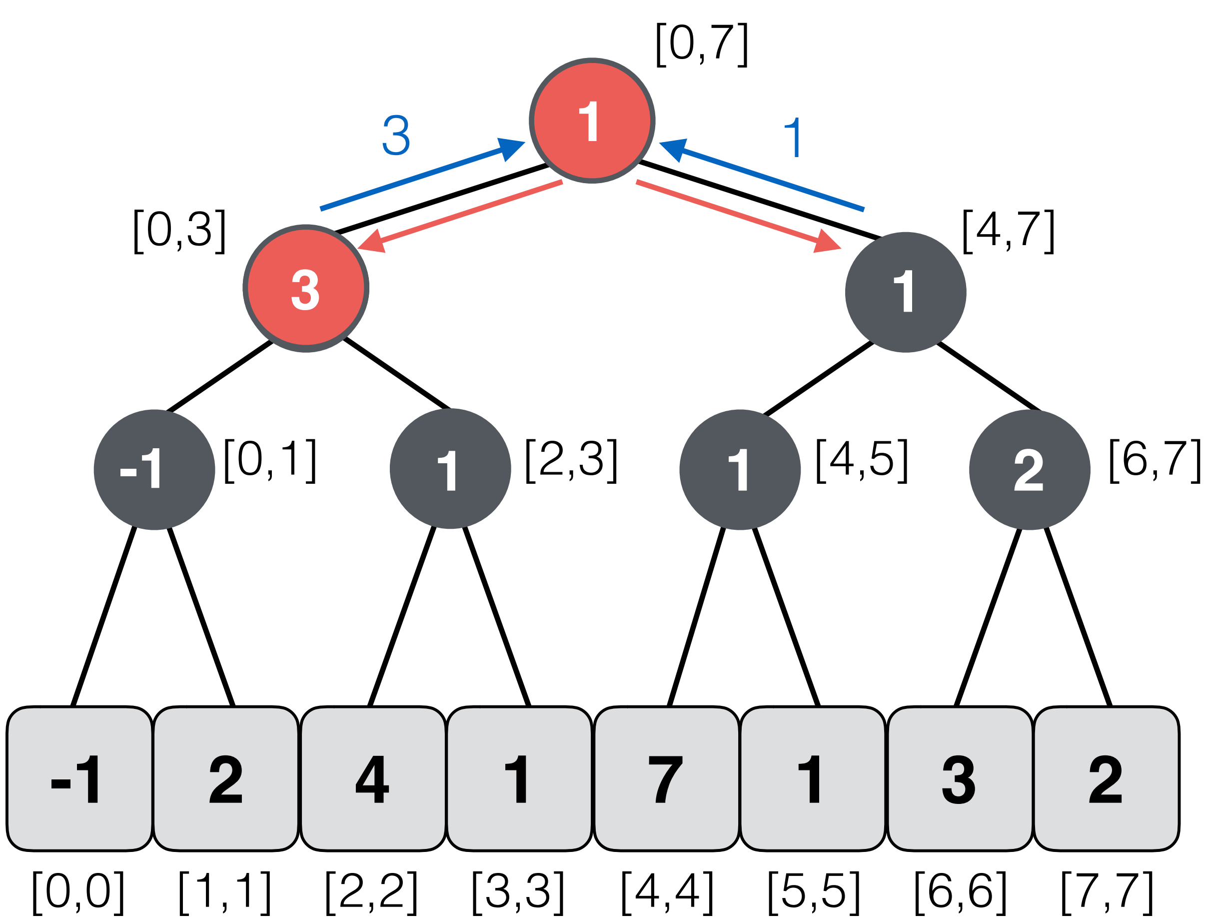

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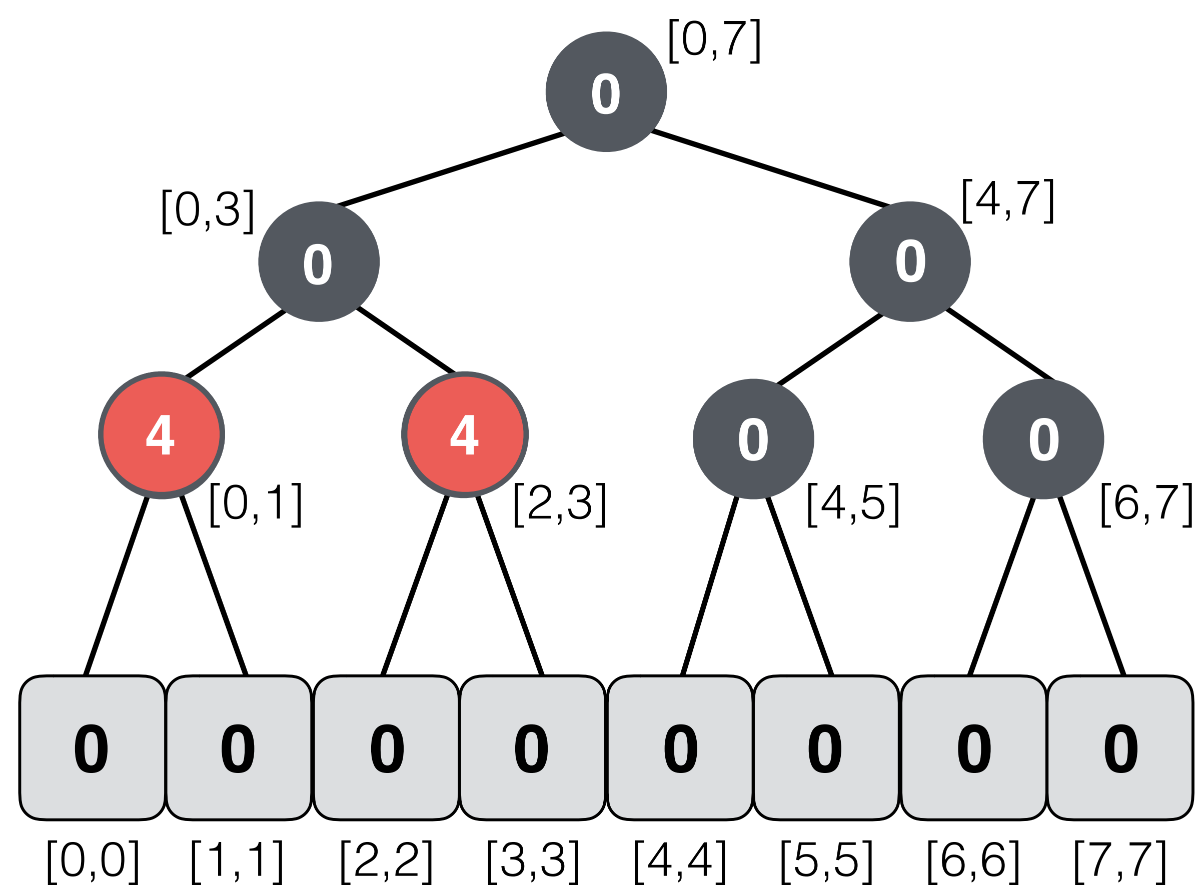

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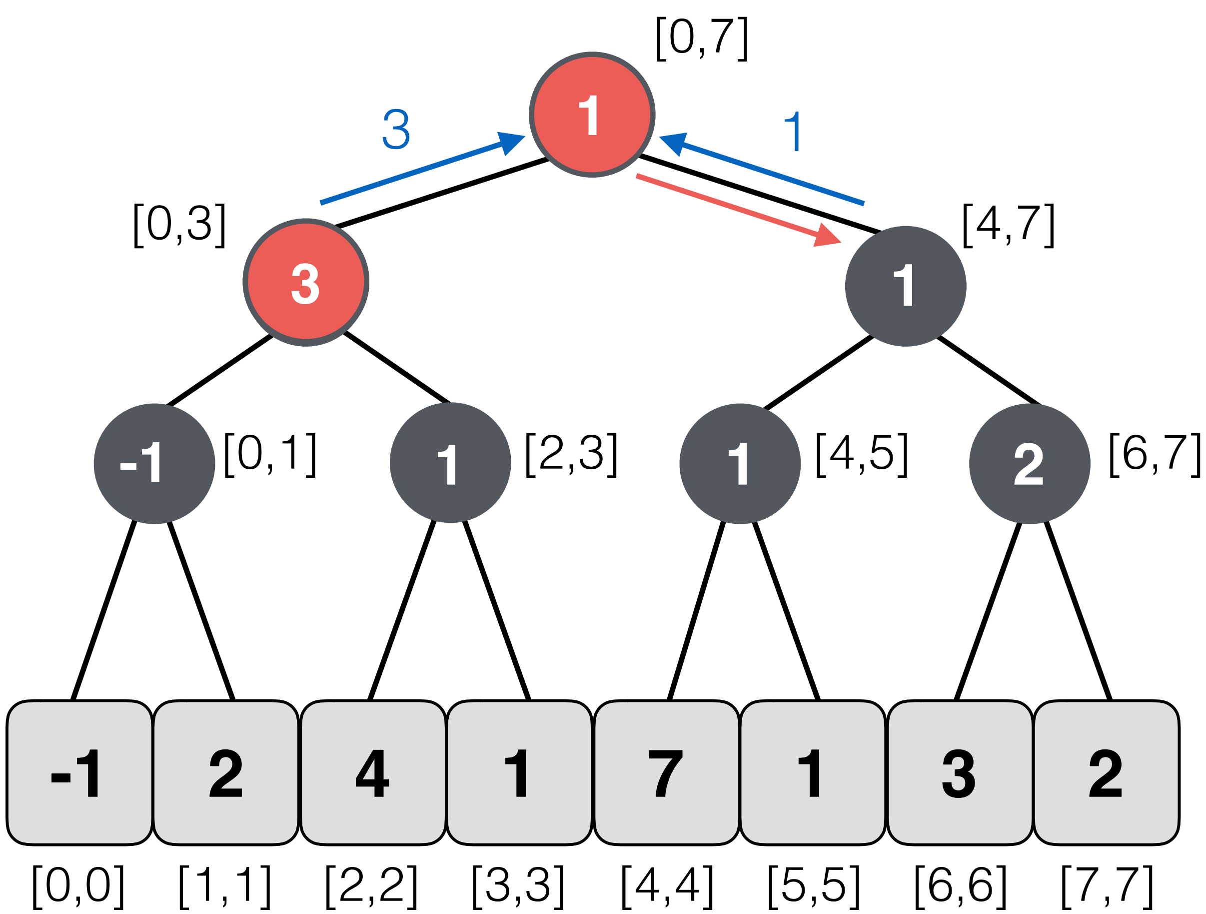

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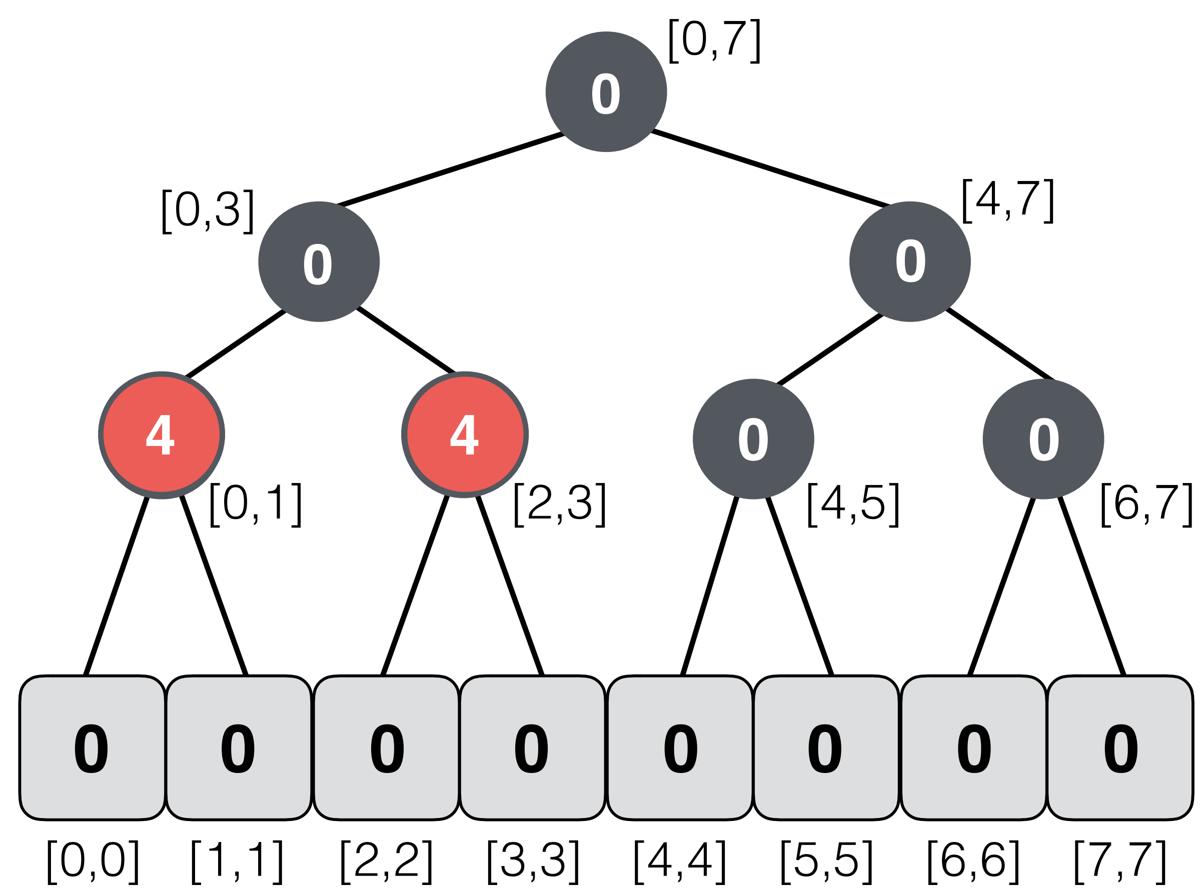

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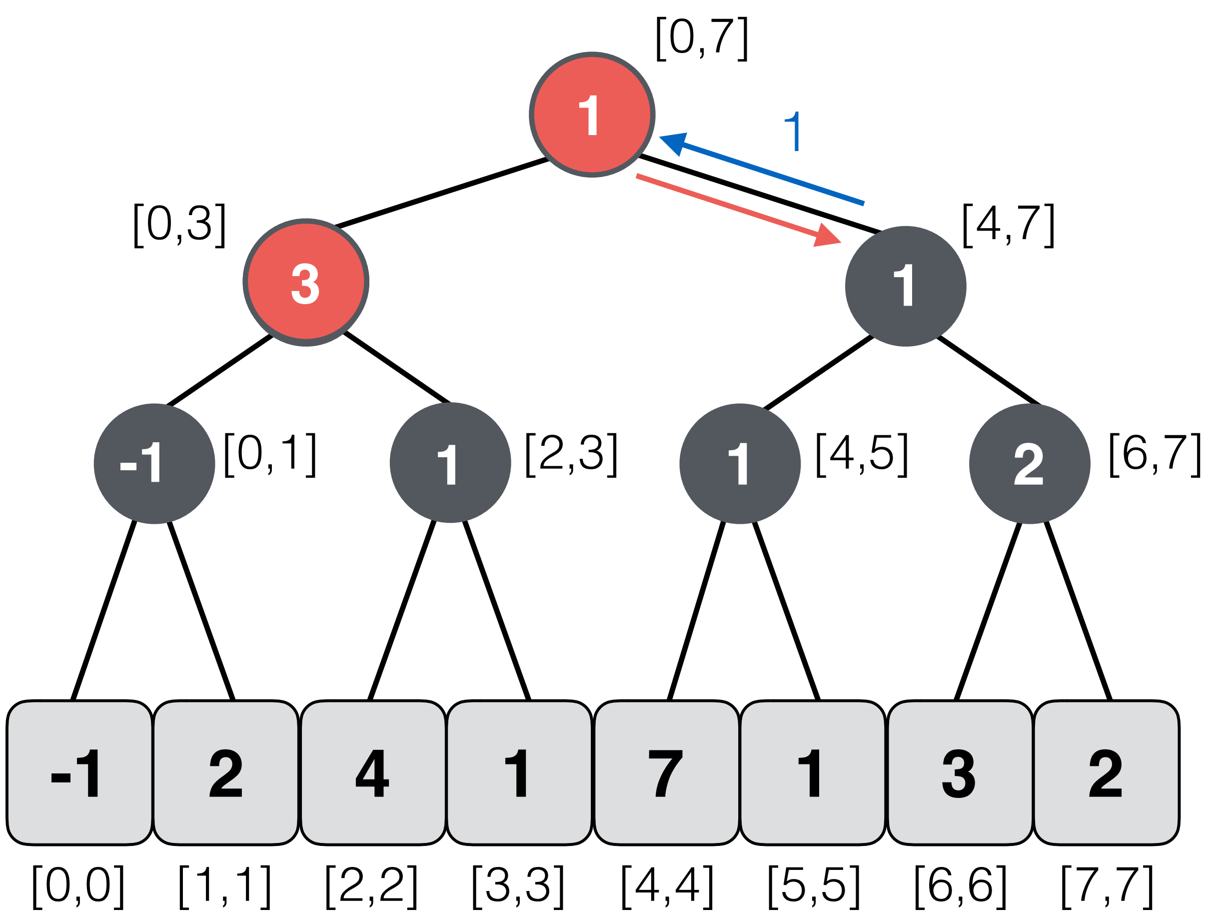
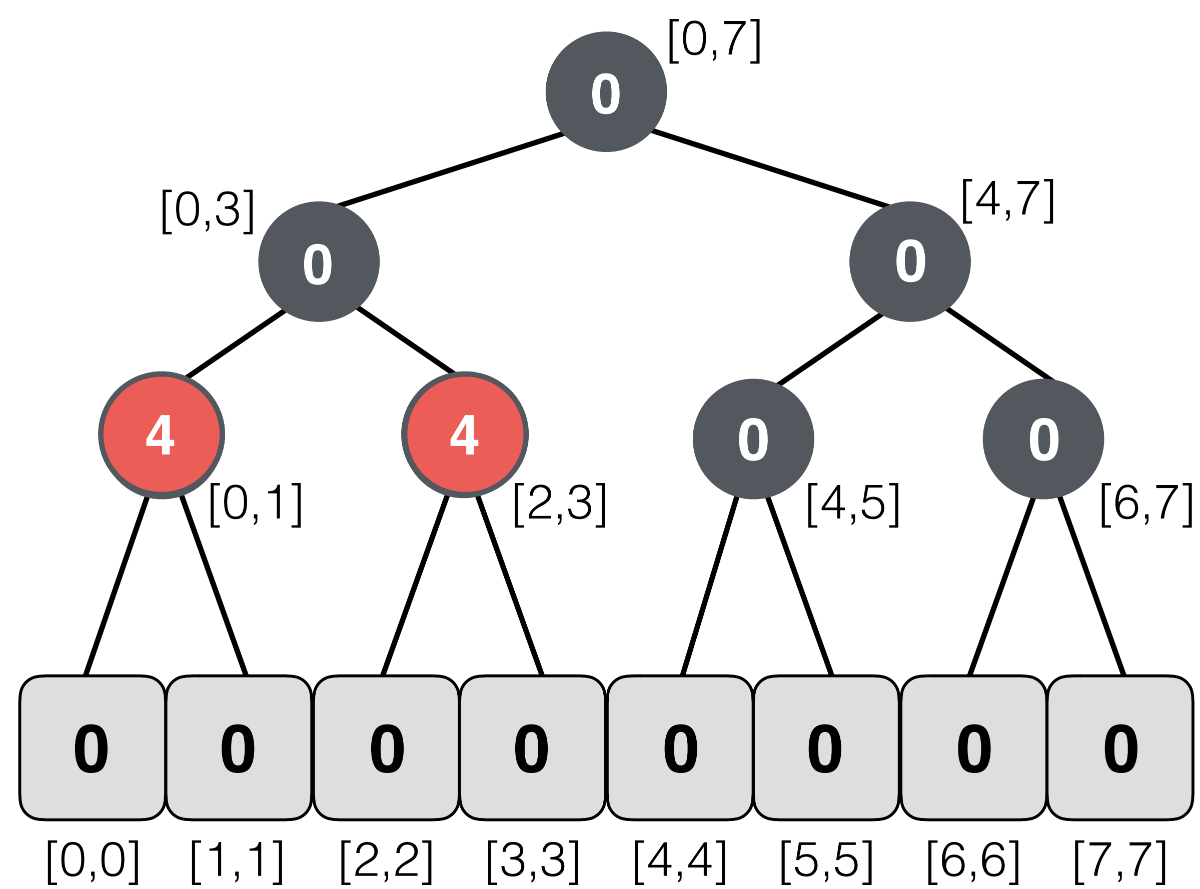


Segment Tree



Lazy Tree

Lazy Propagation in Segment Trees

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```
update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)
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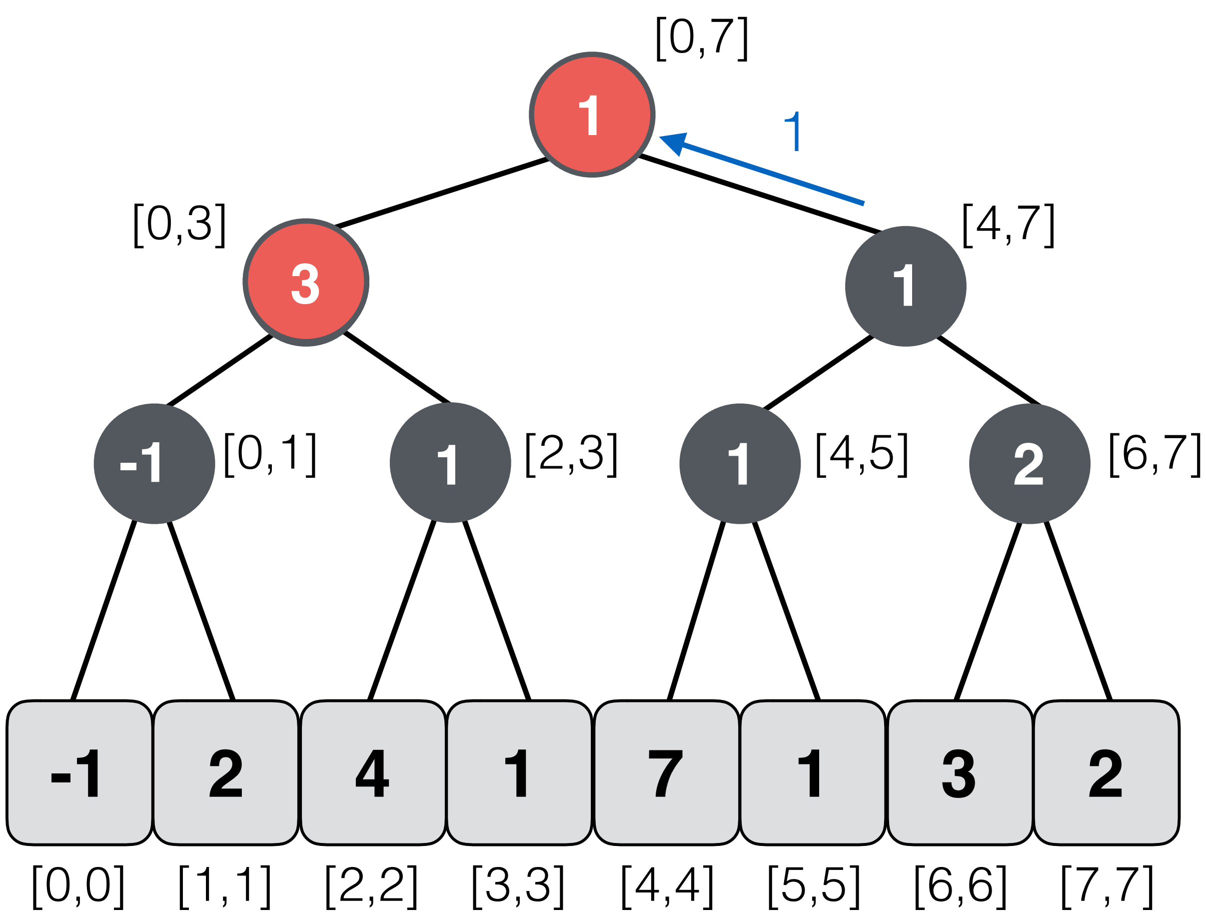
```



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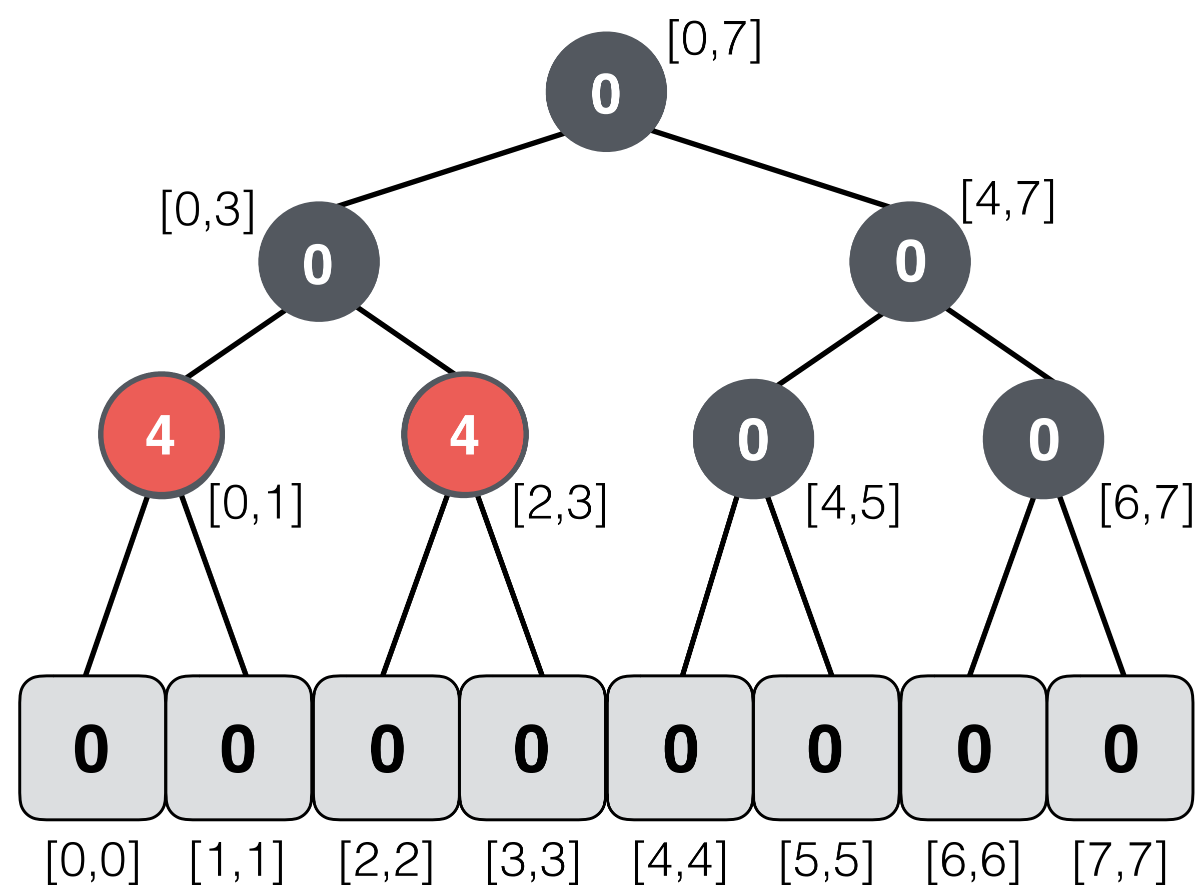

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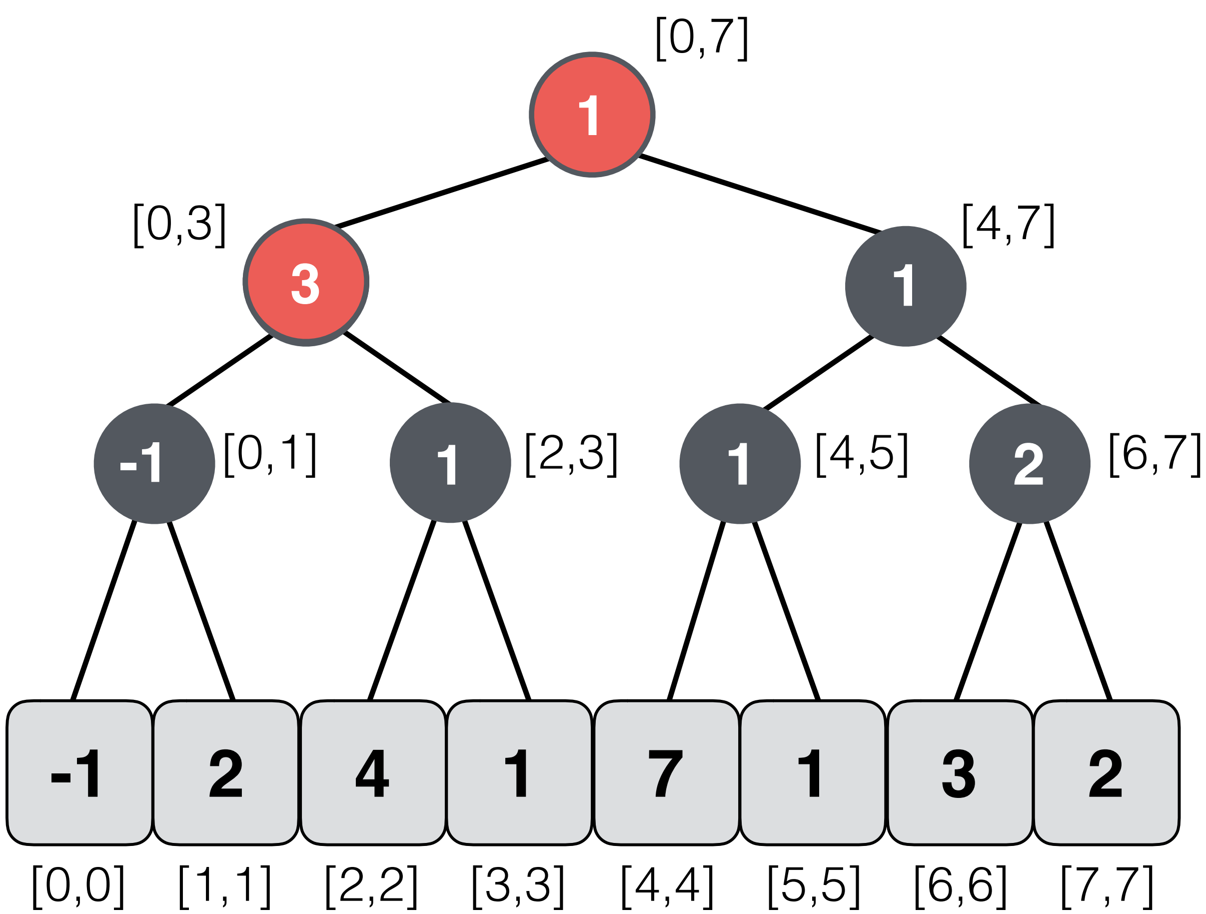

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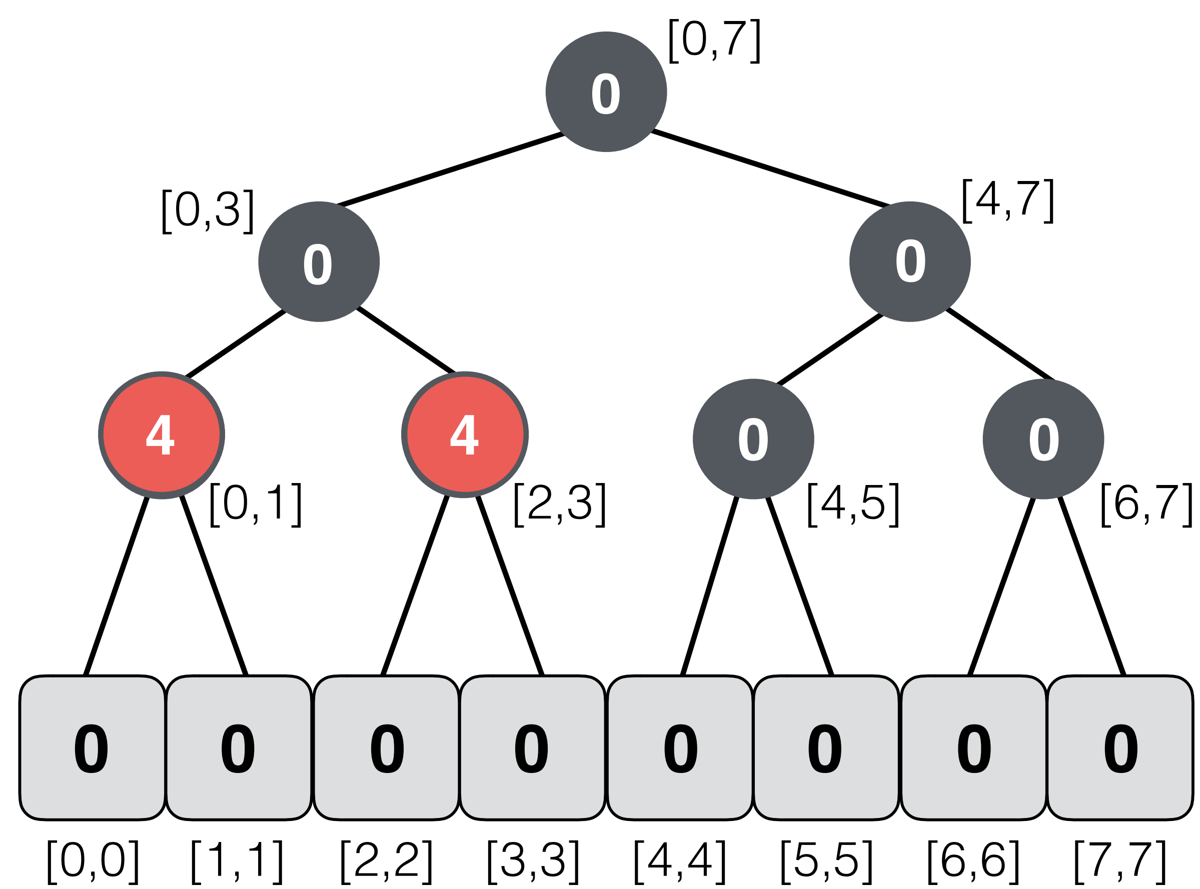

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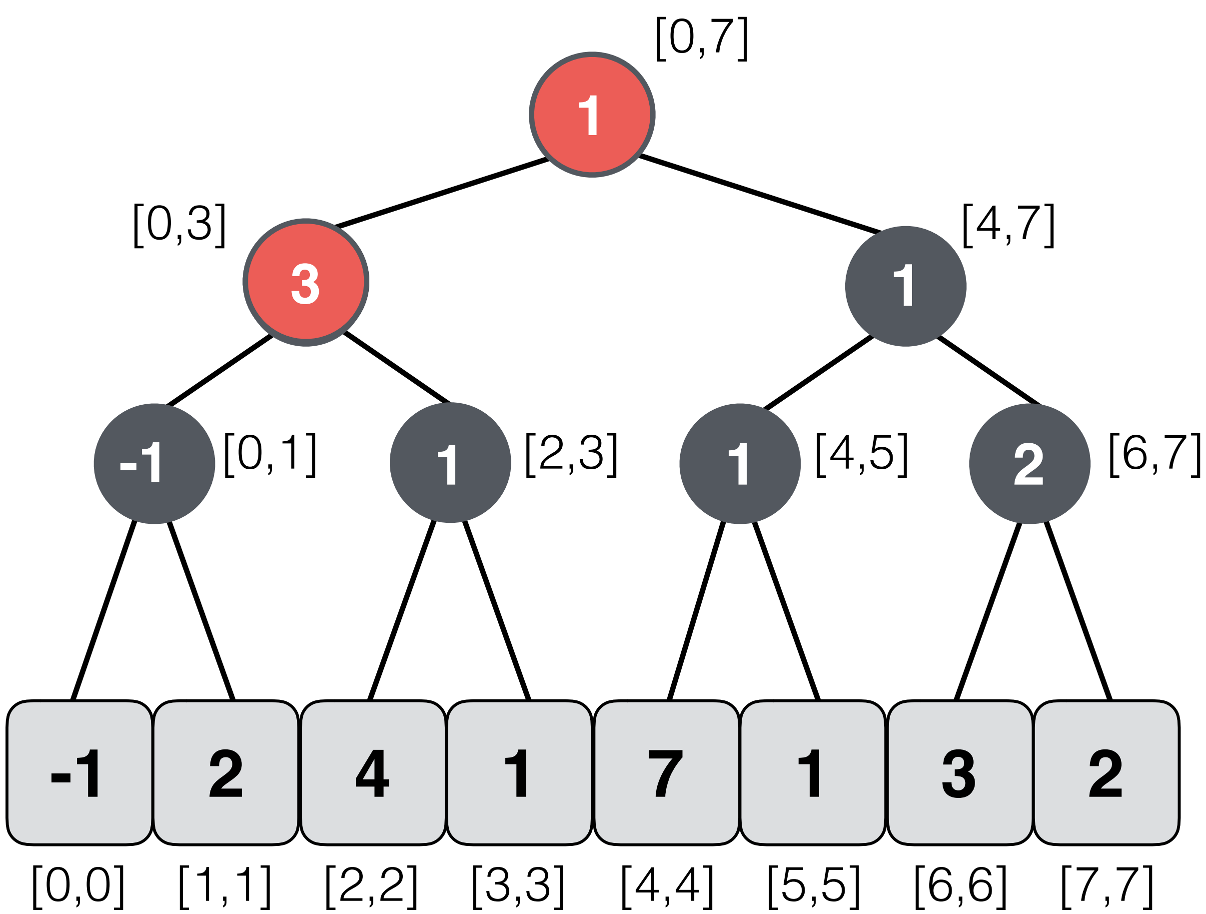

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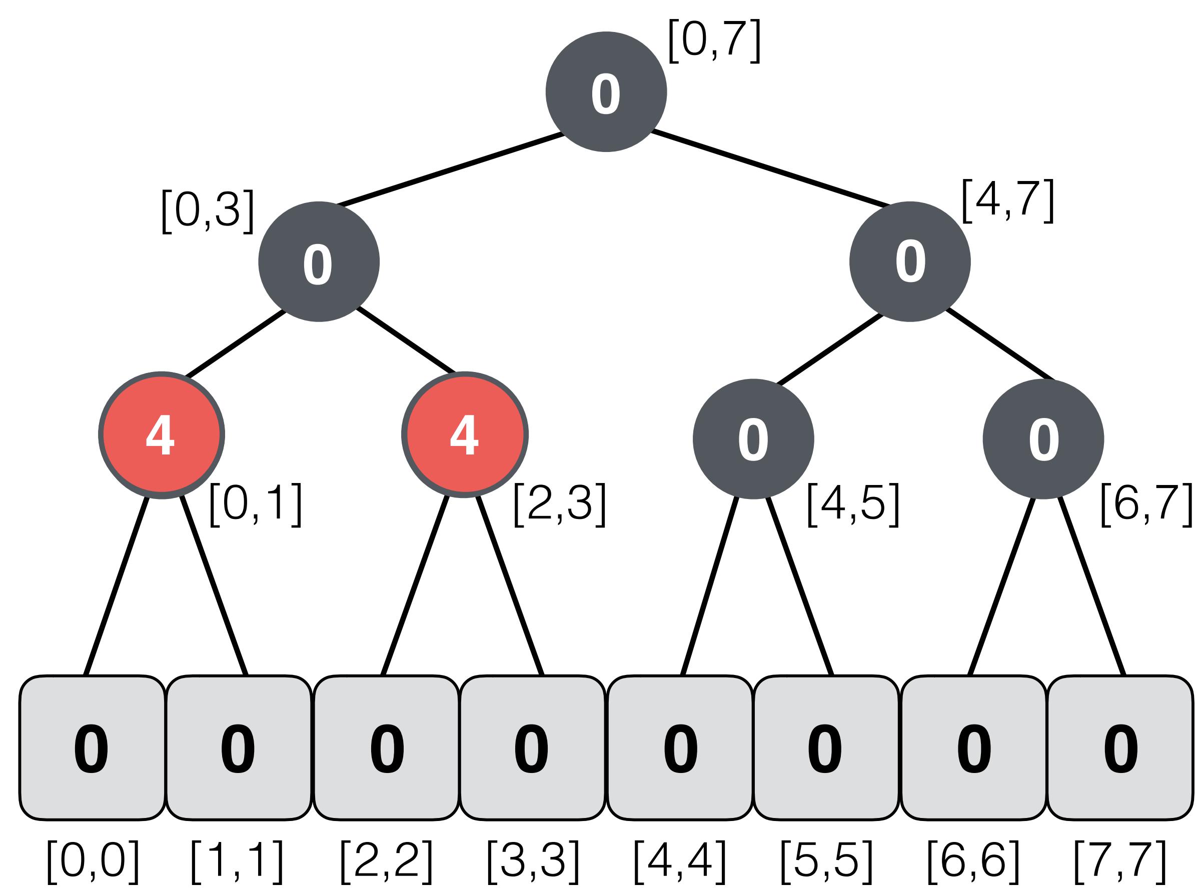

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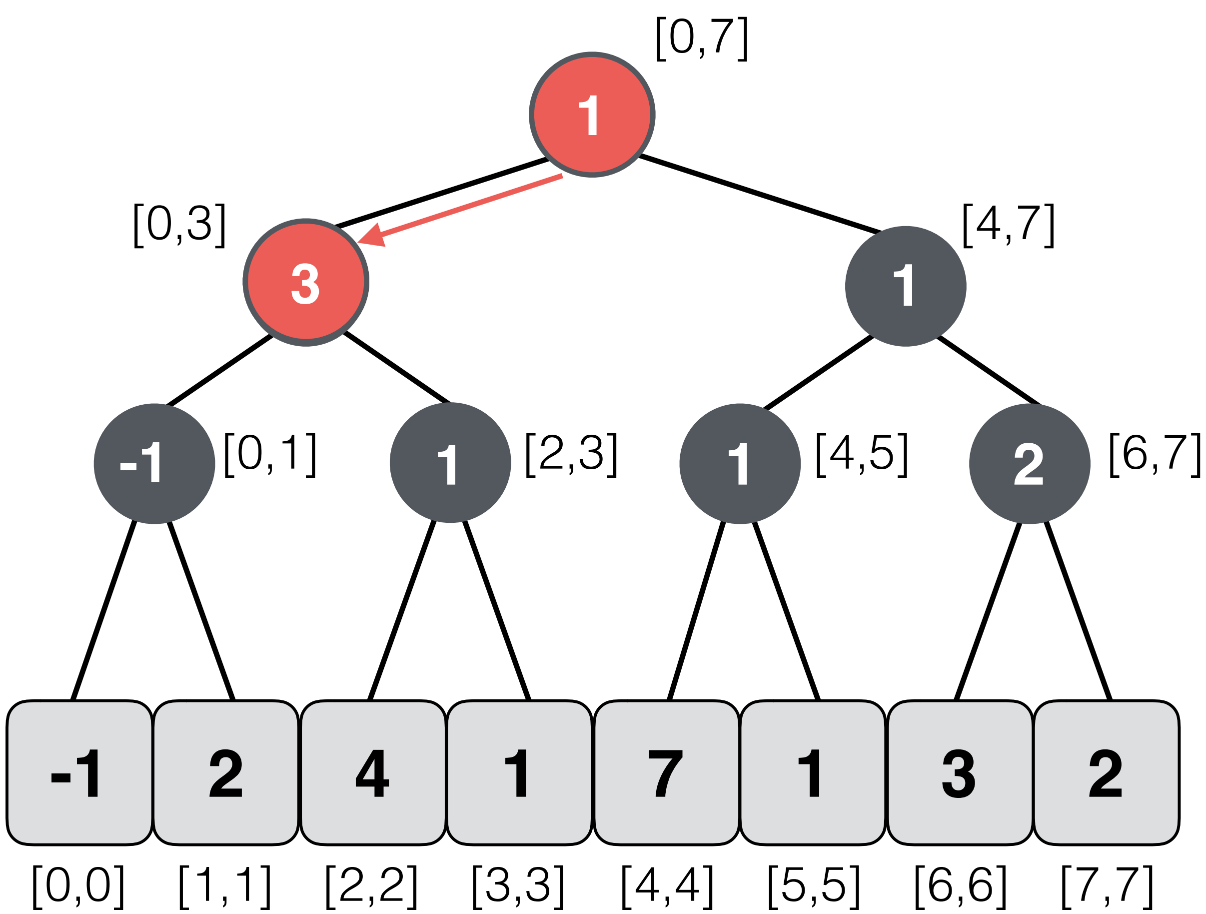

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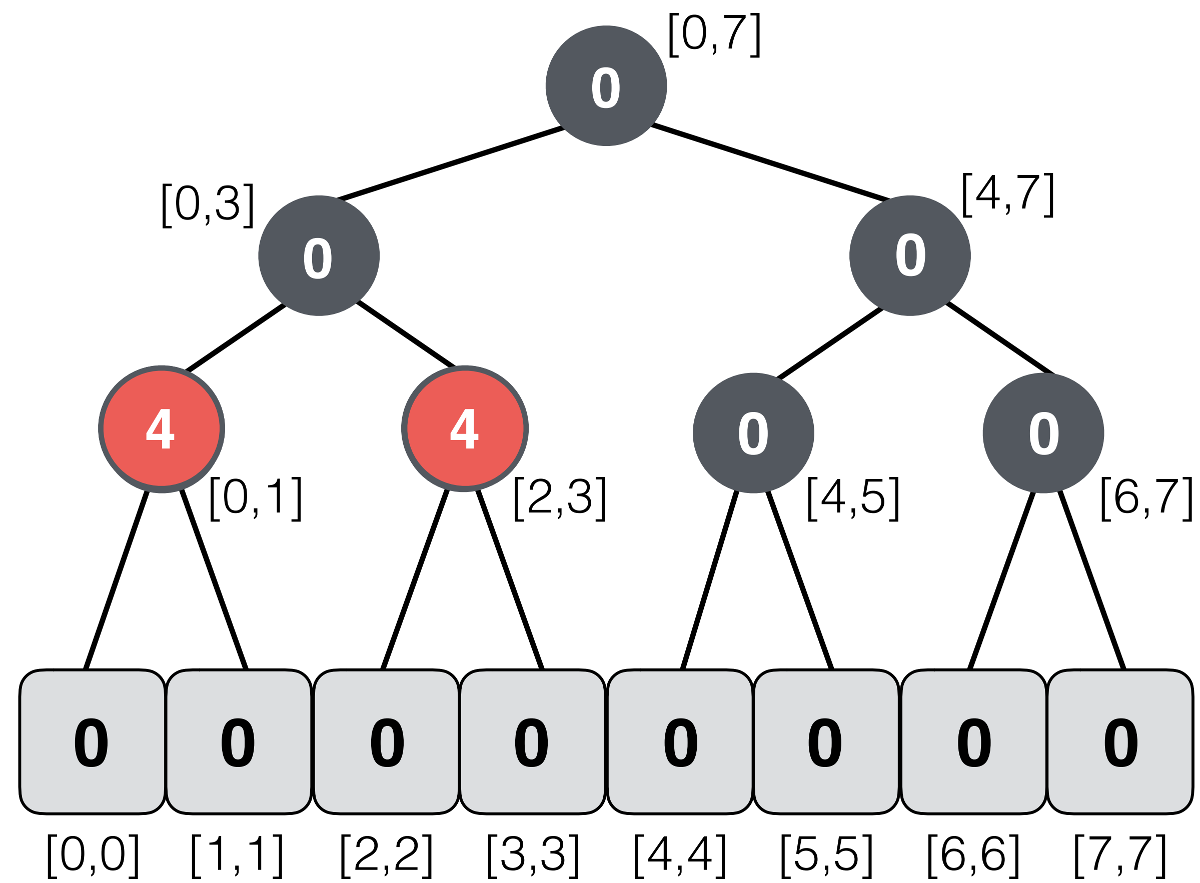

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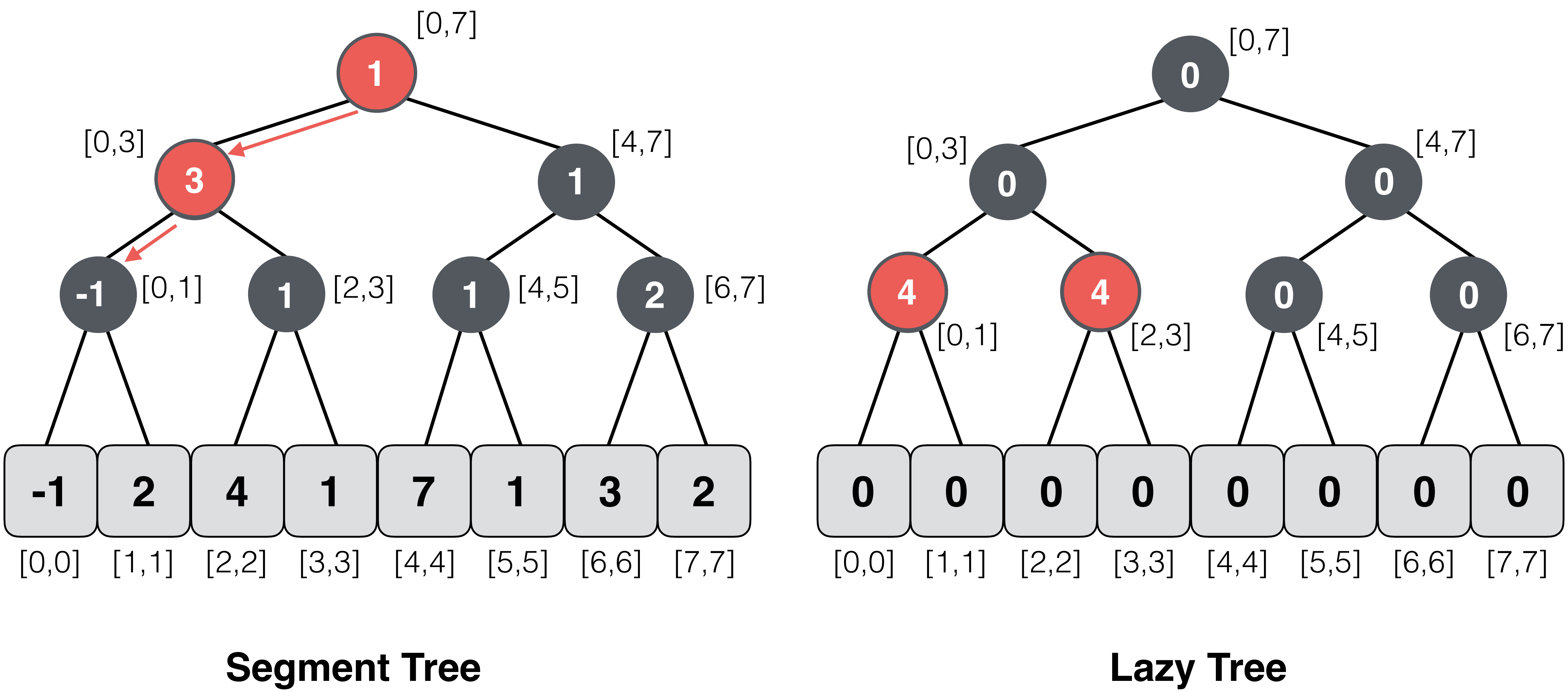

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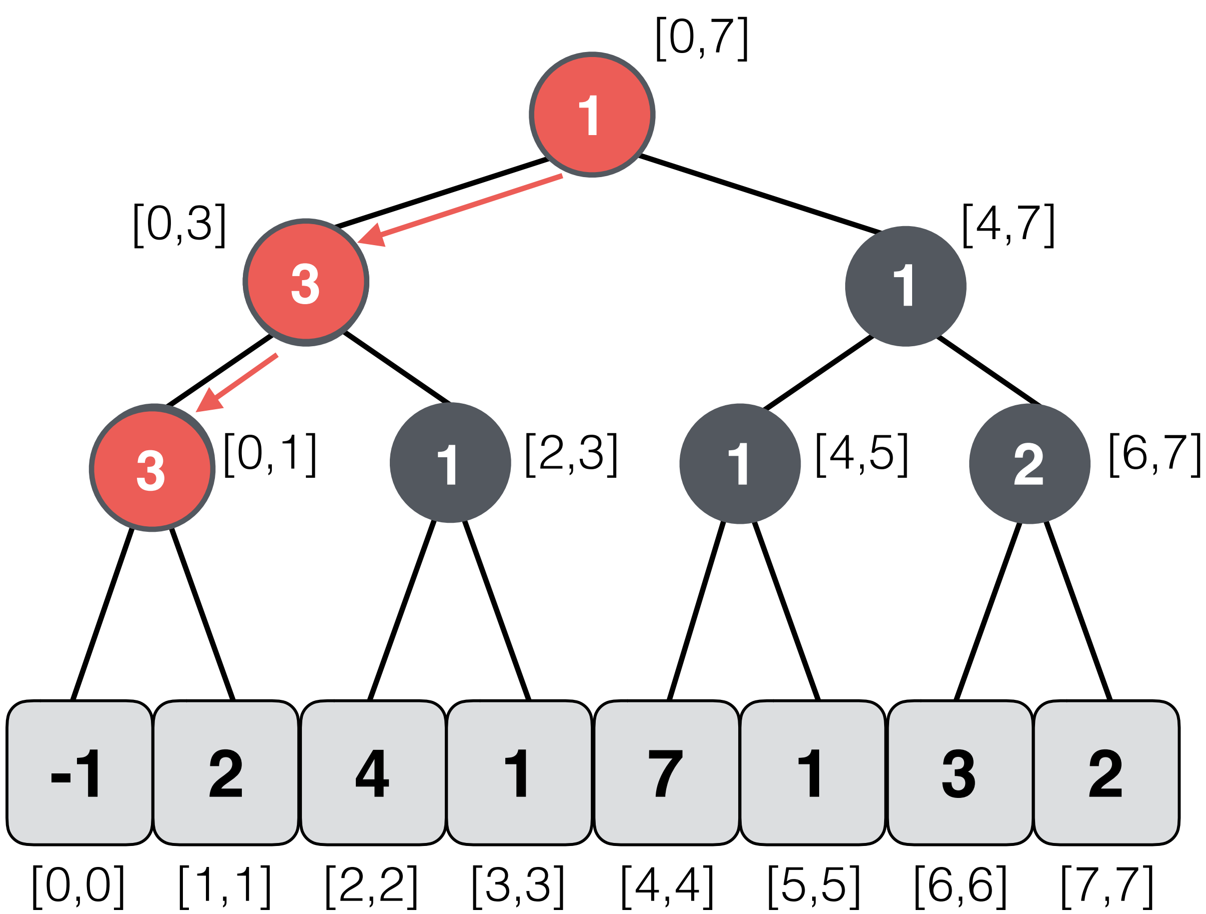

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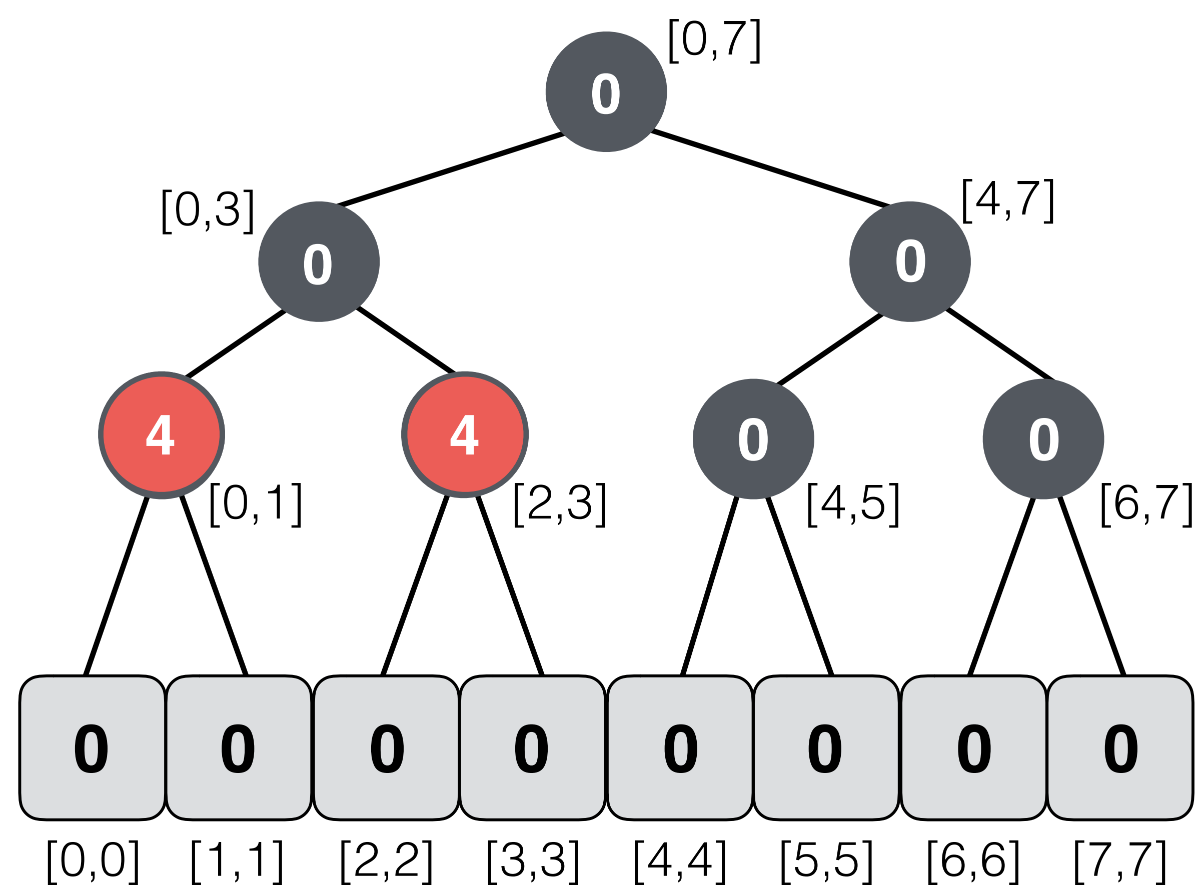

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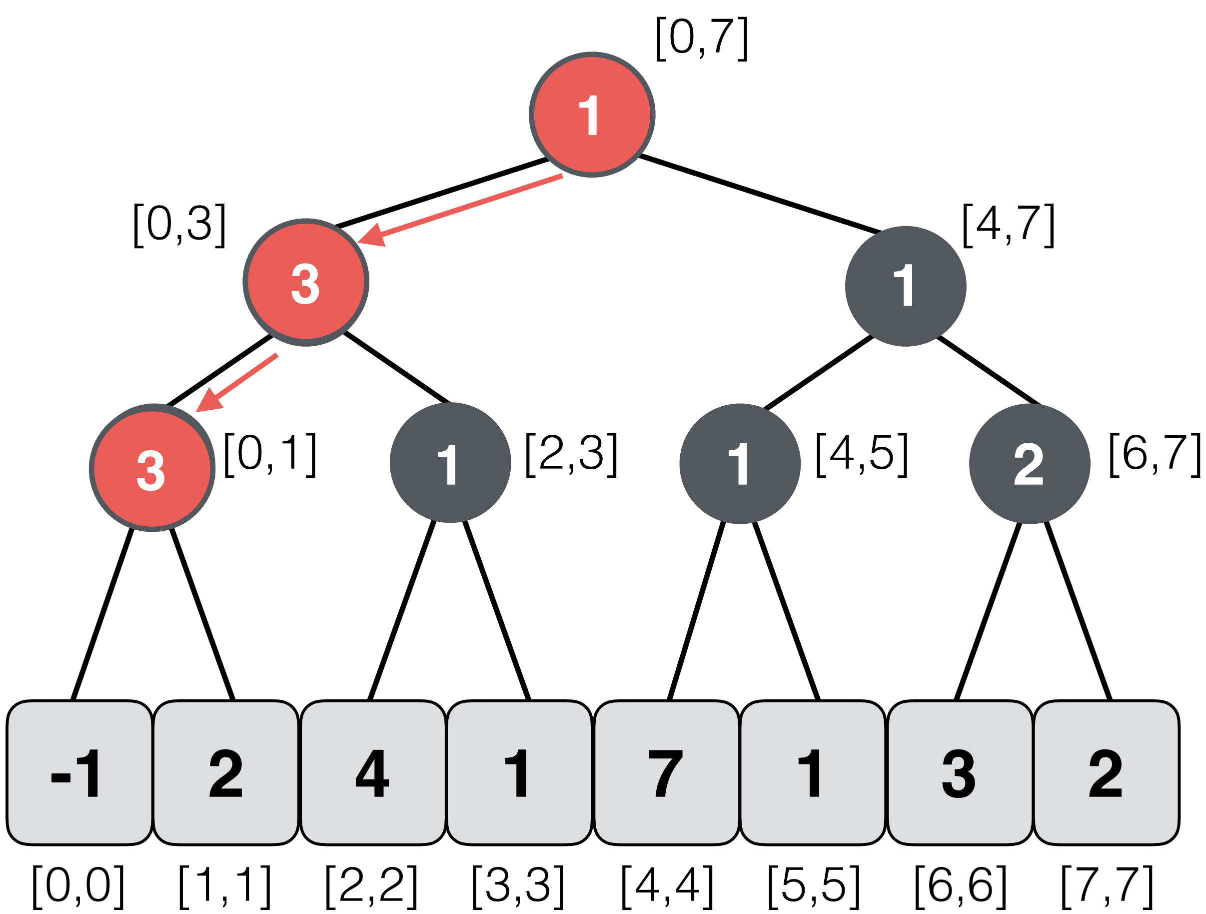

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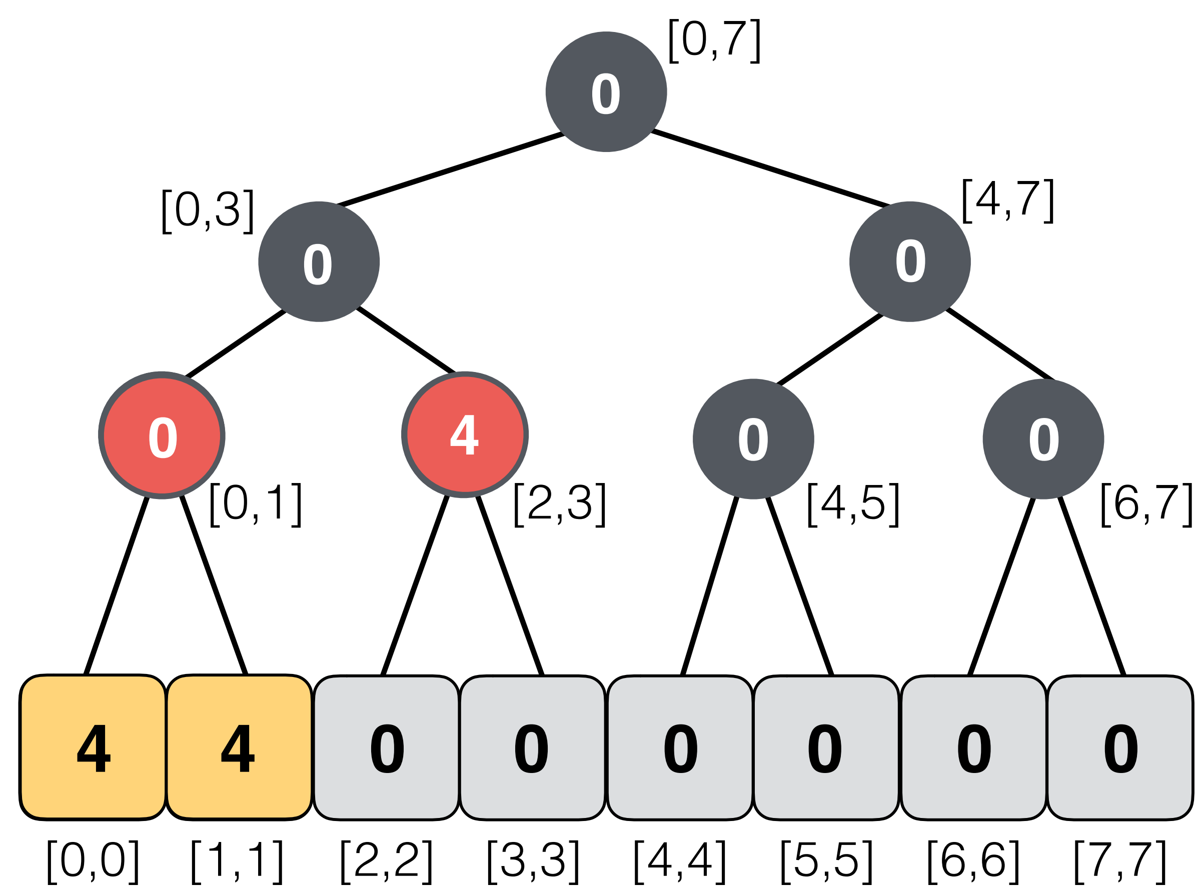

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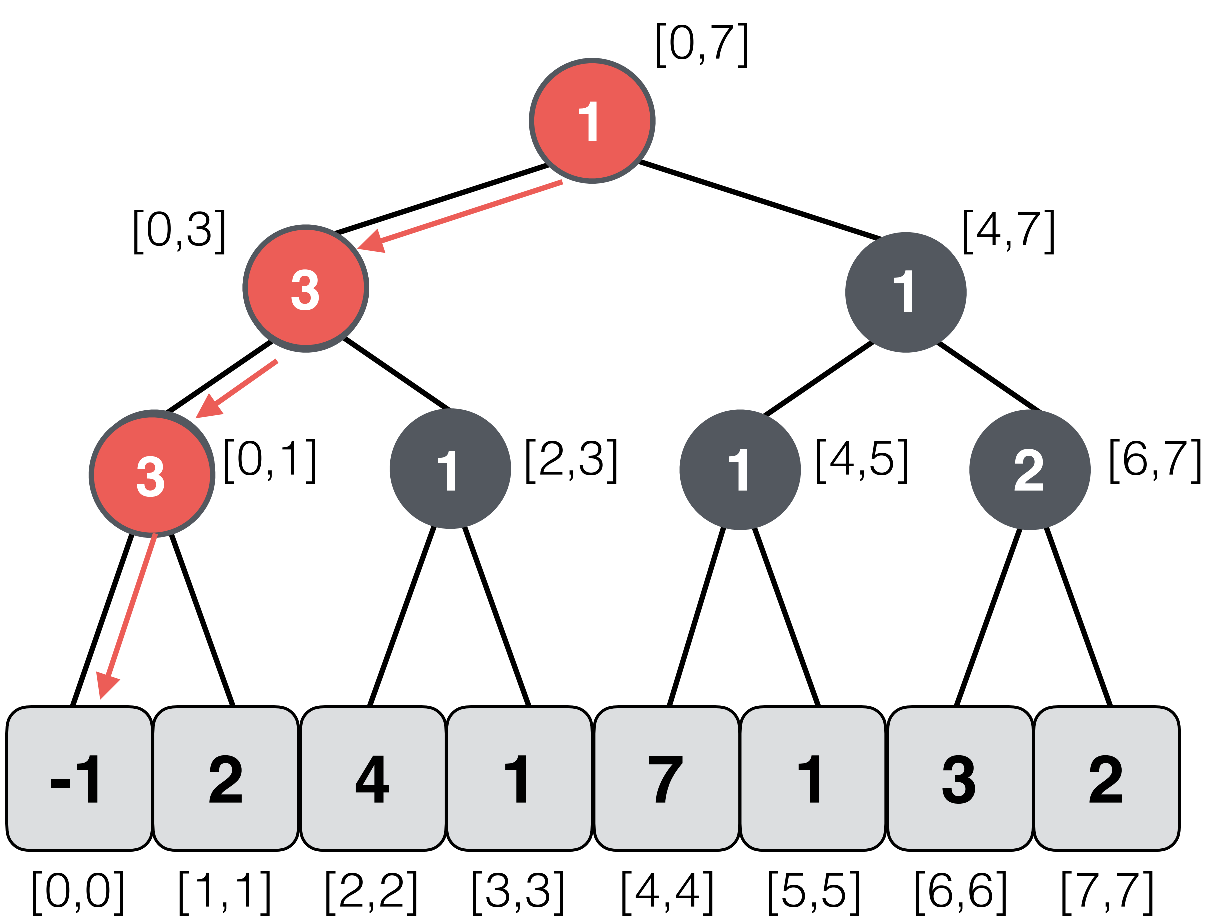

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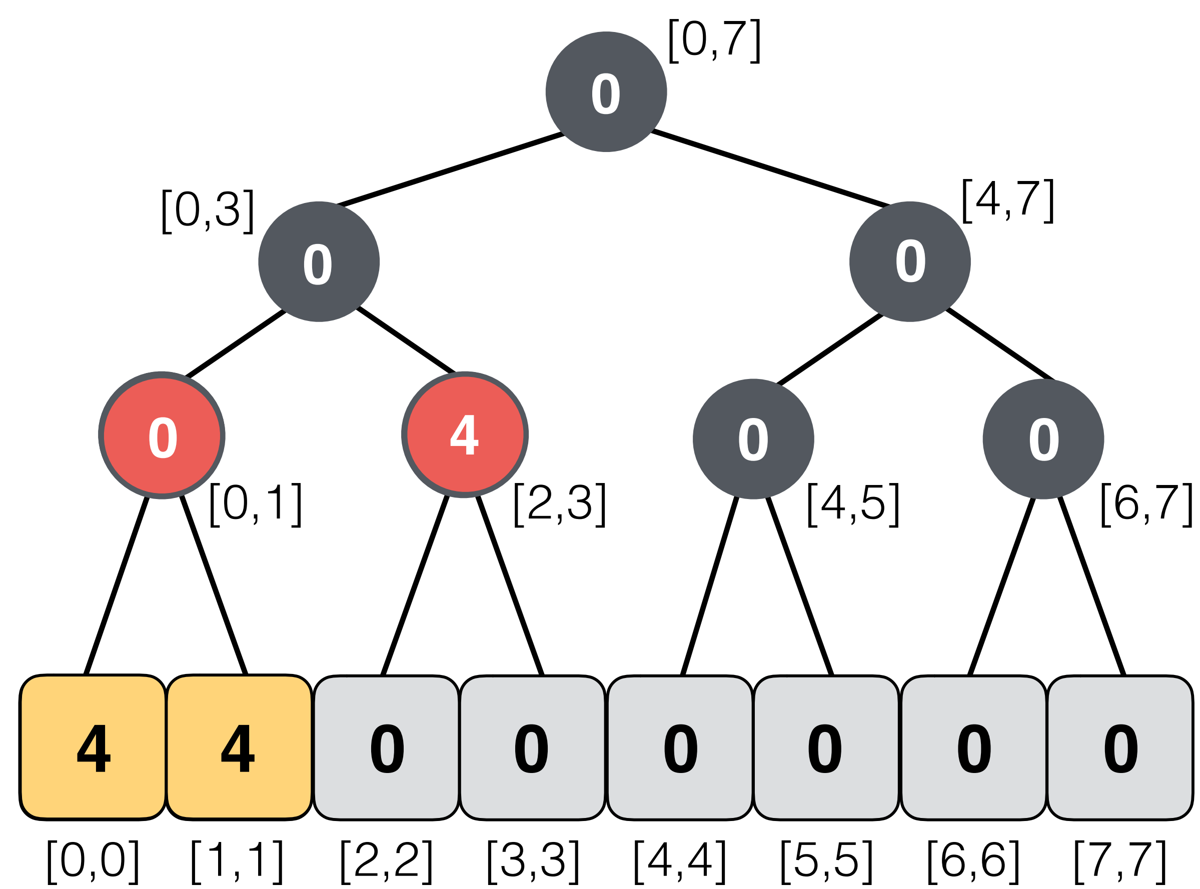

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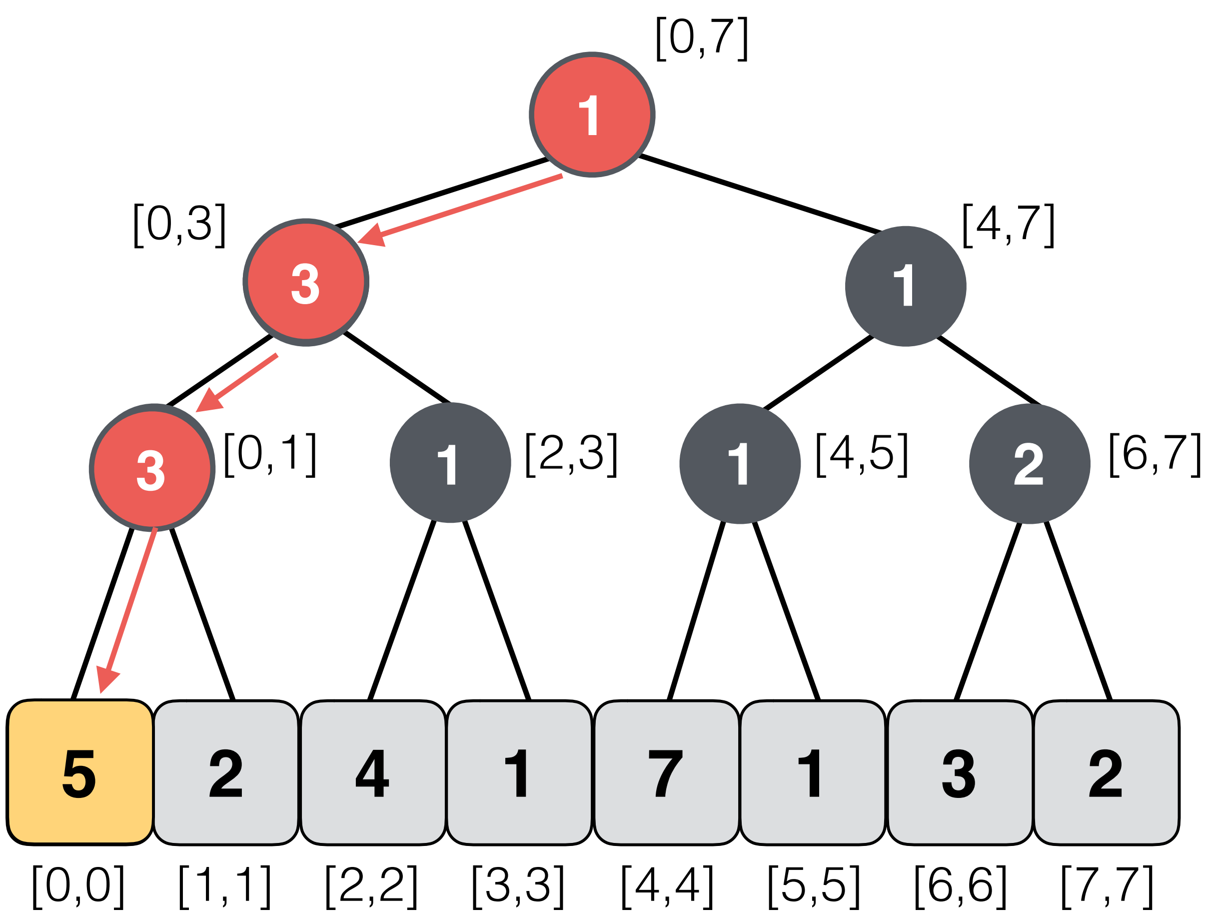

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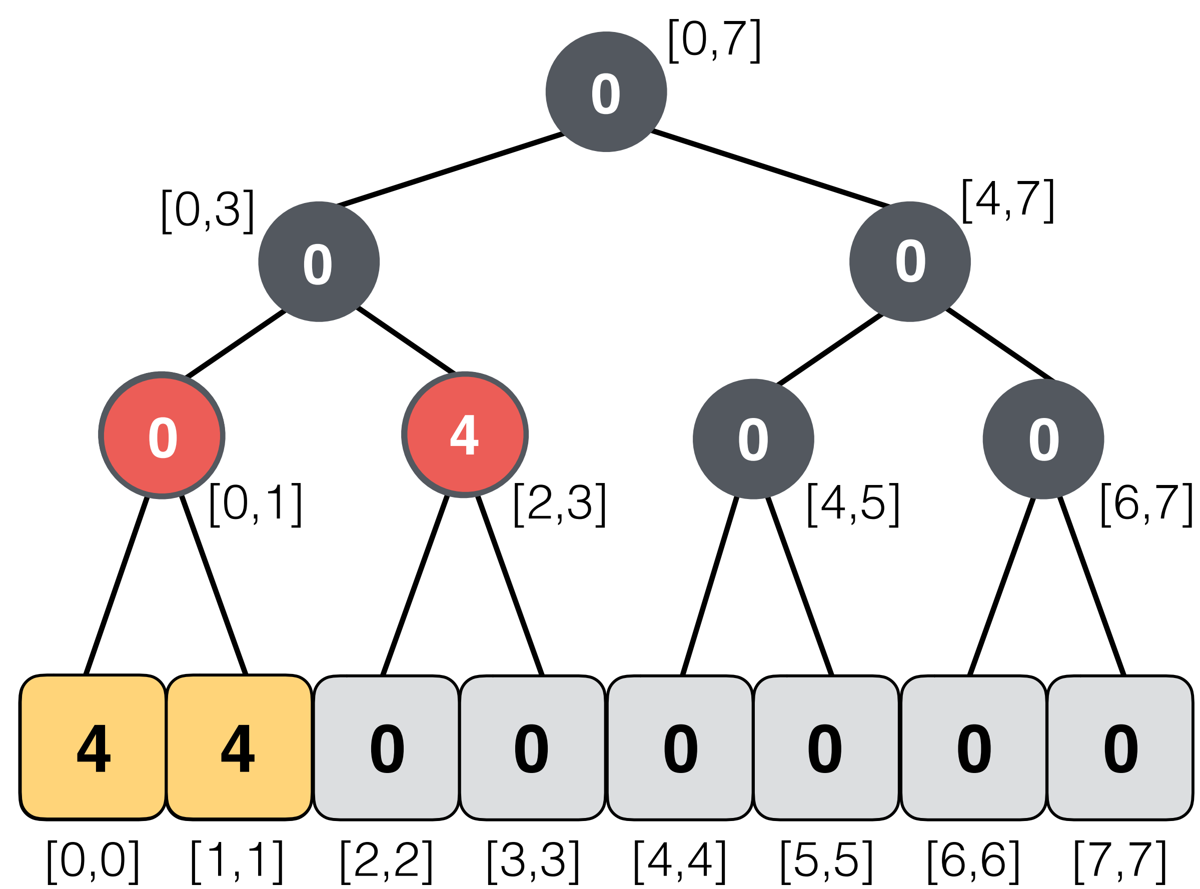

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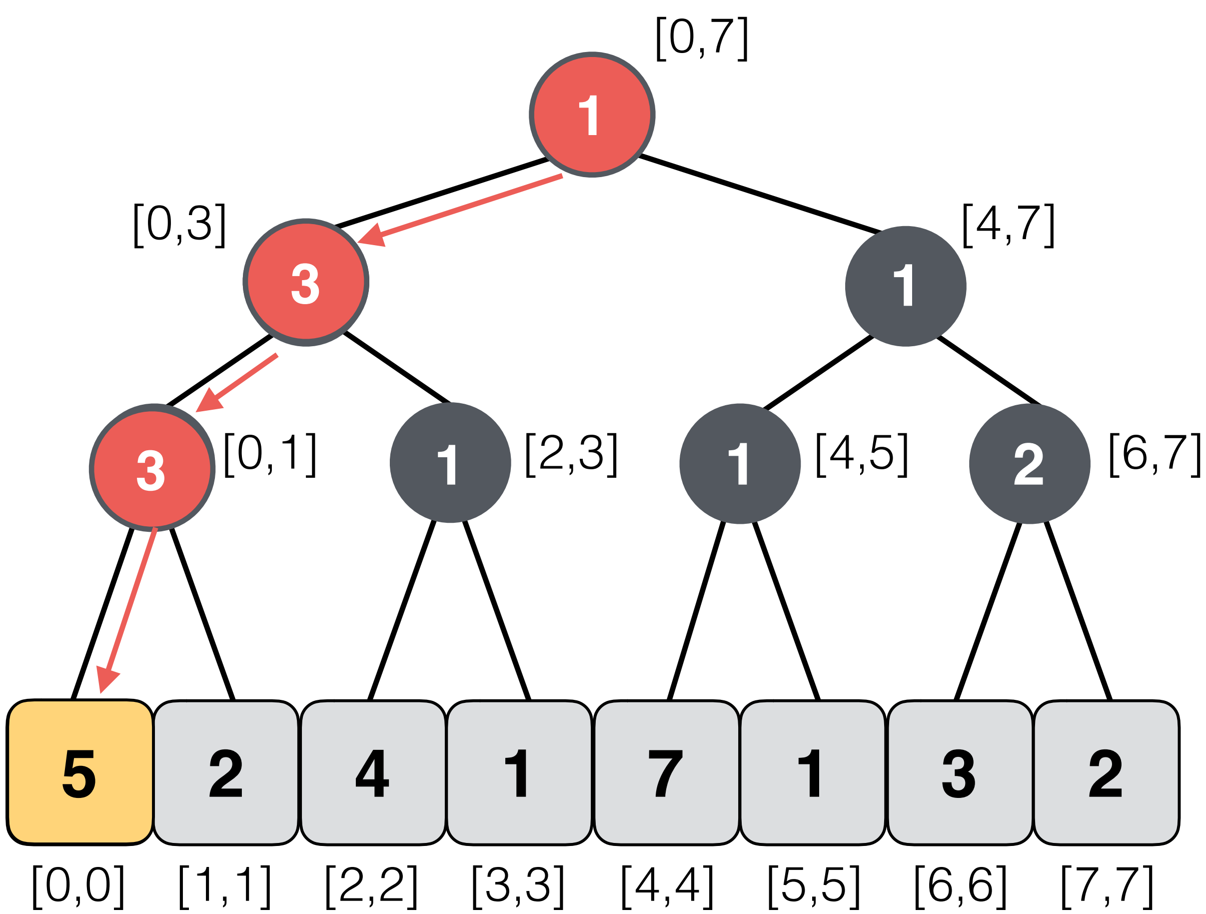
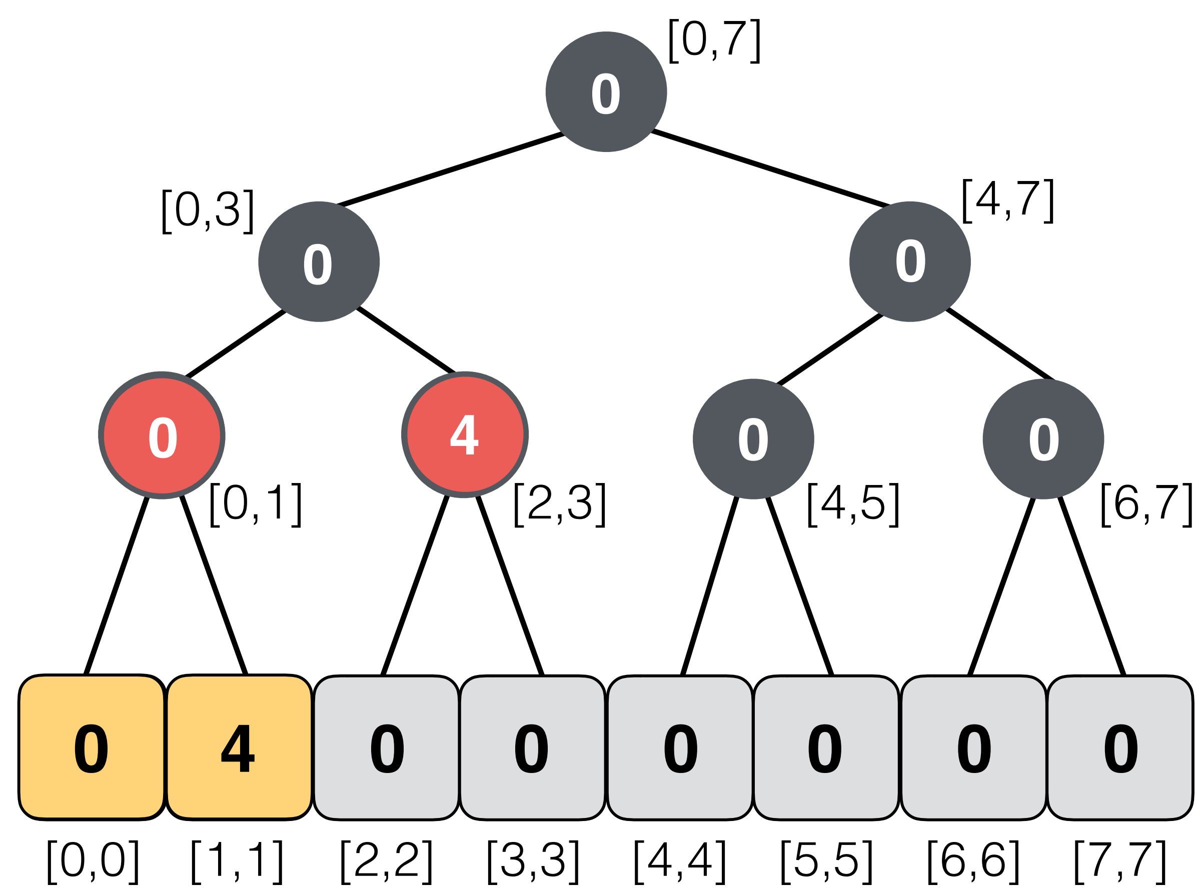


Segment Tree



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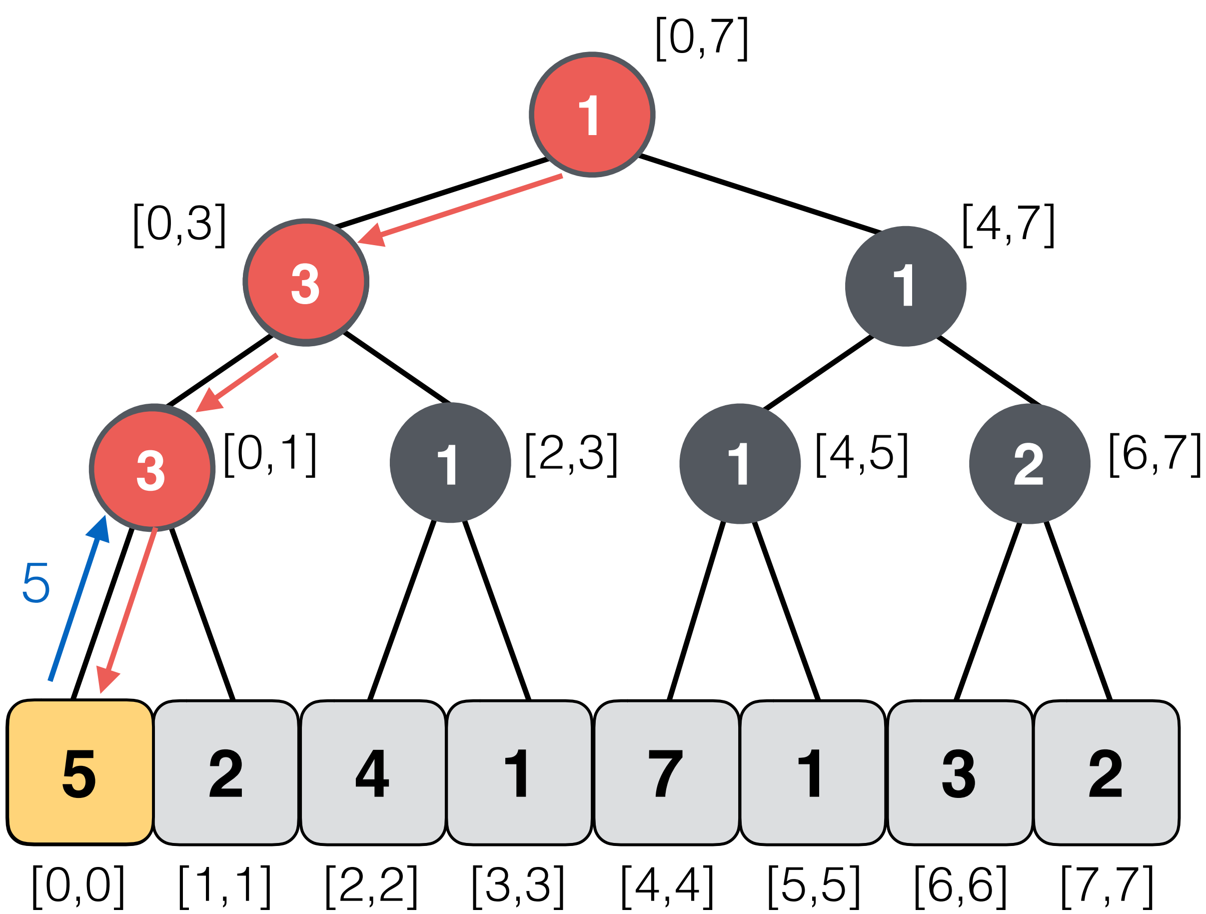

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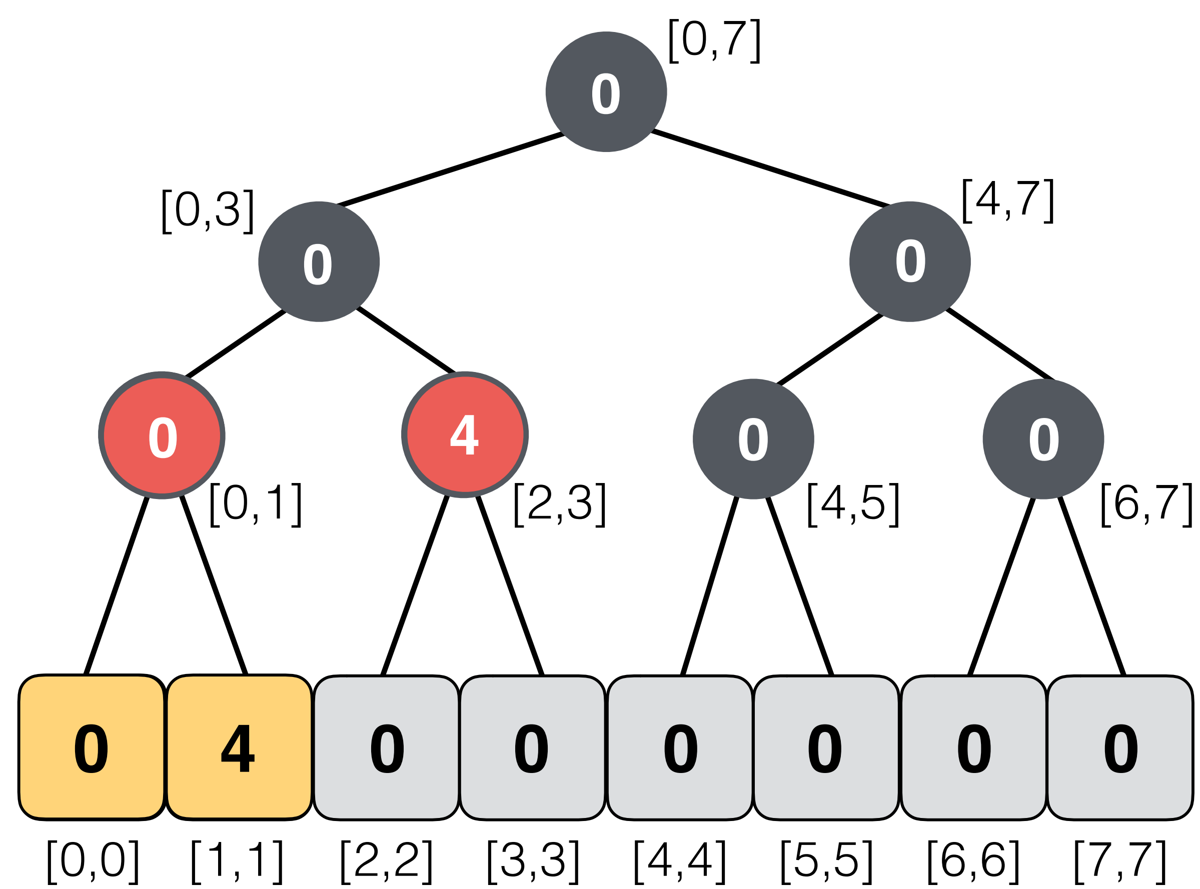

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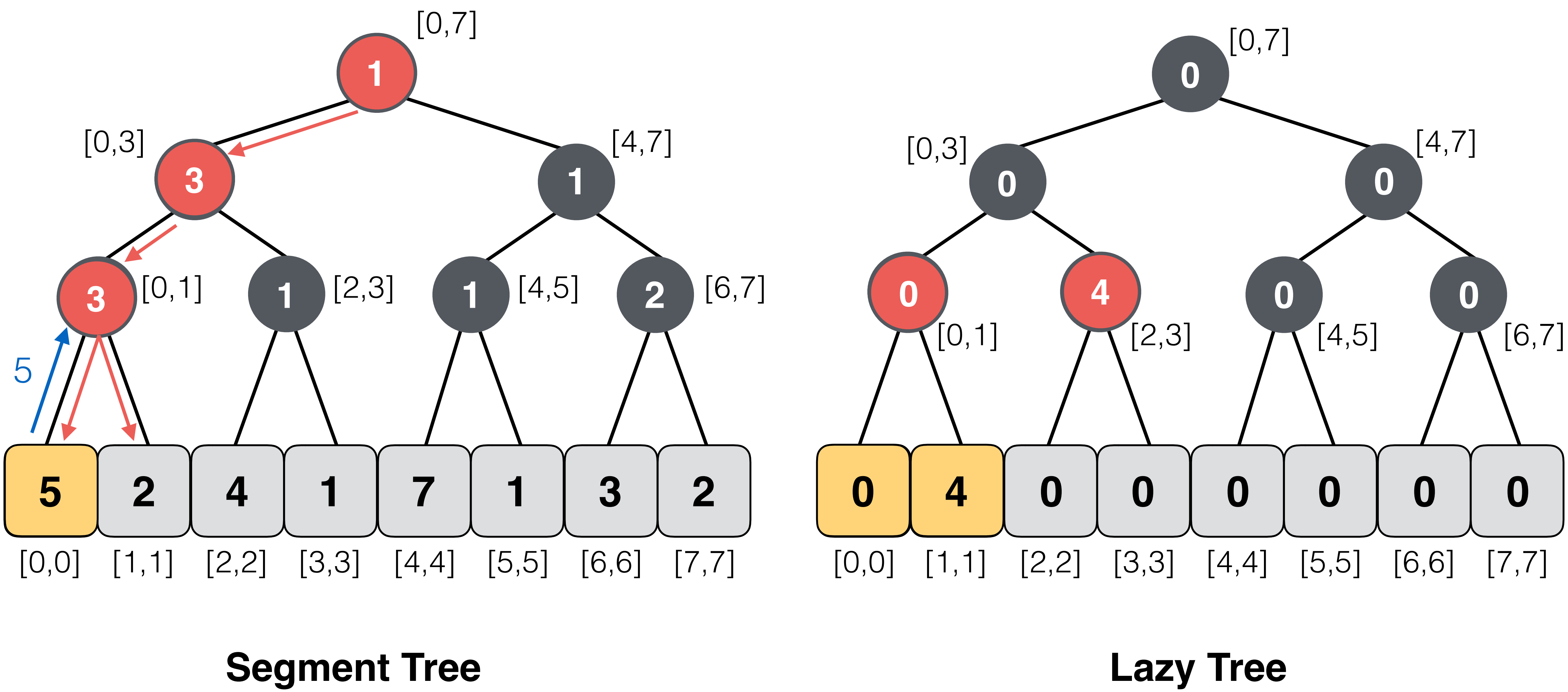

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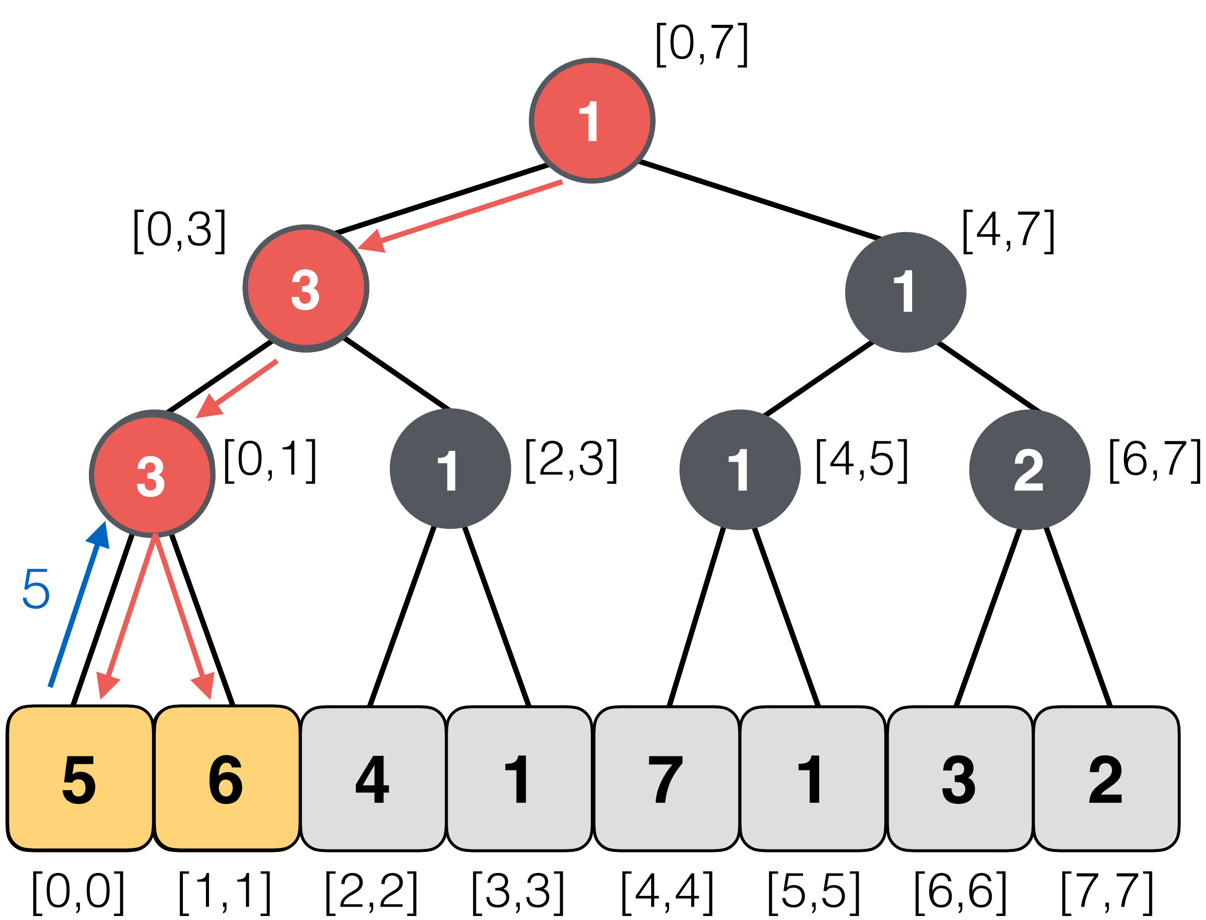

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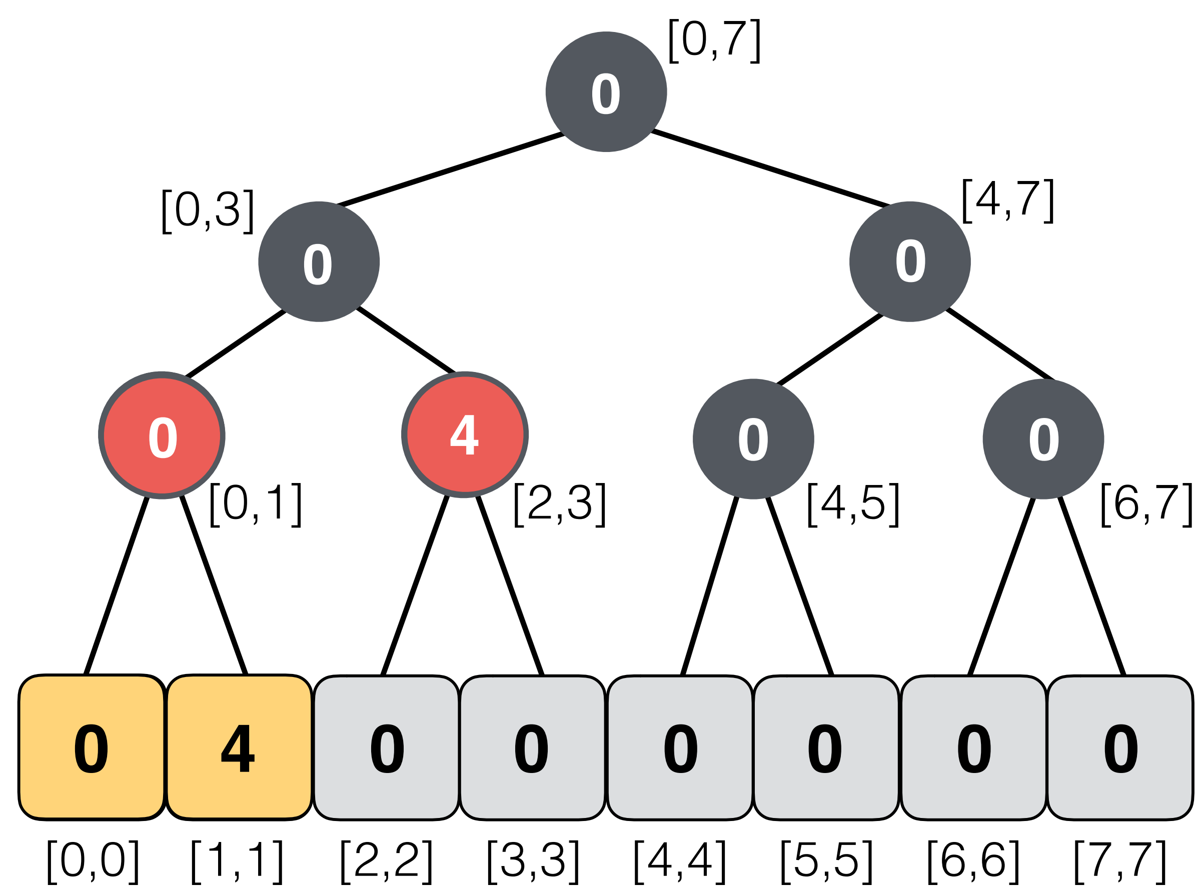

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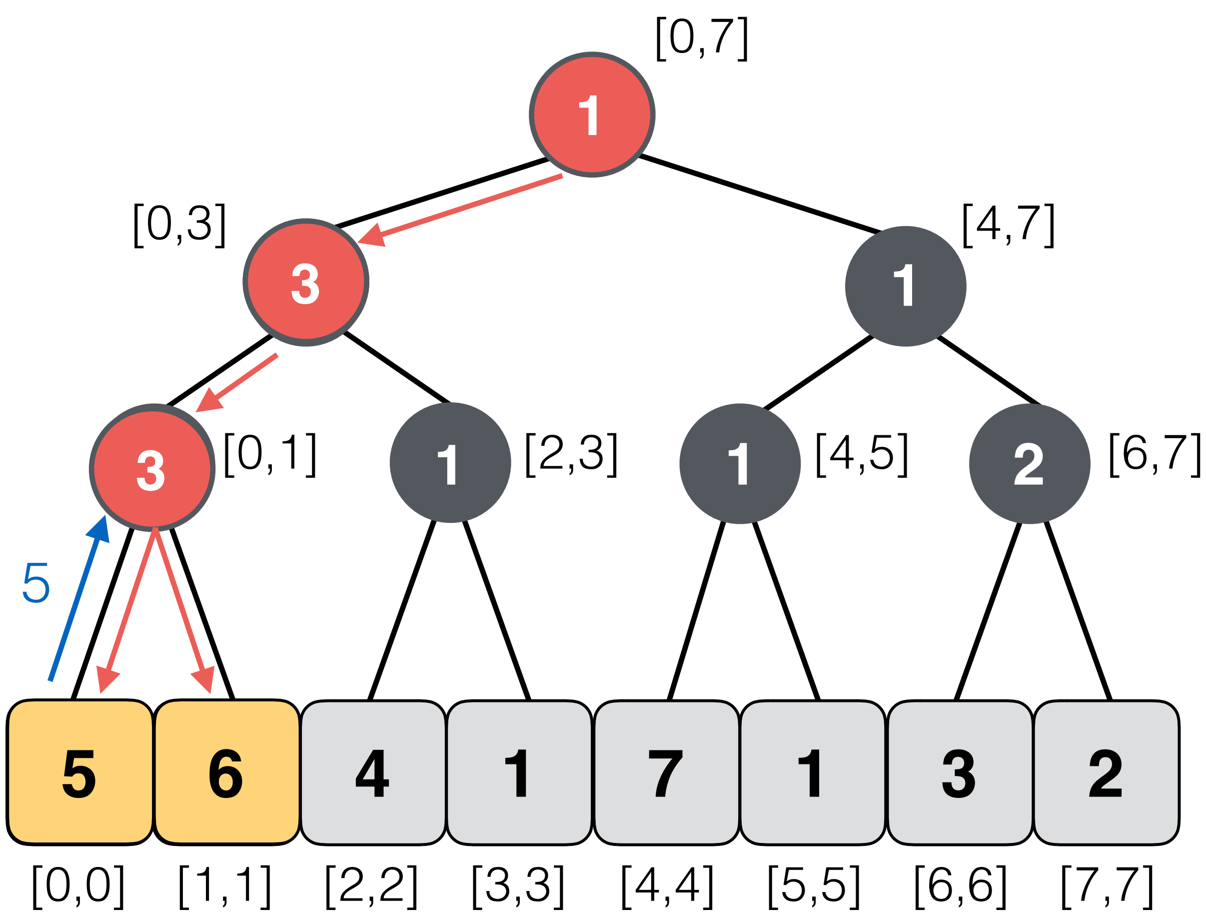

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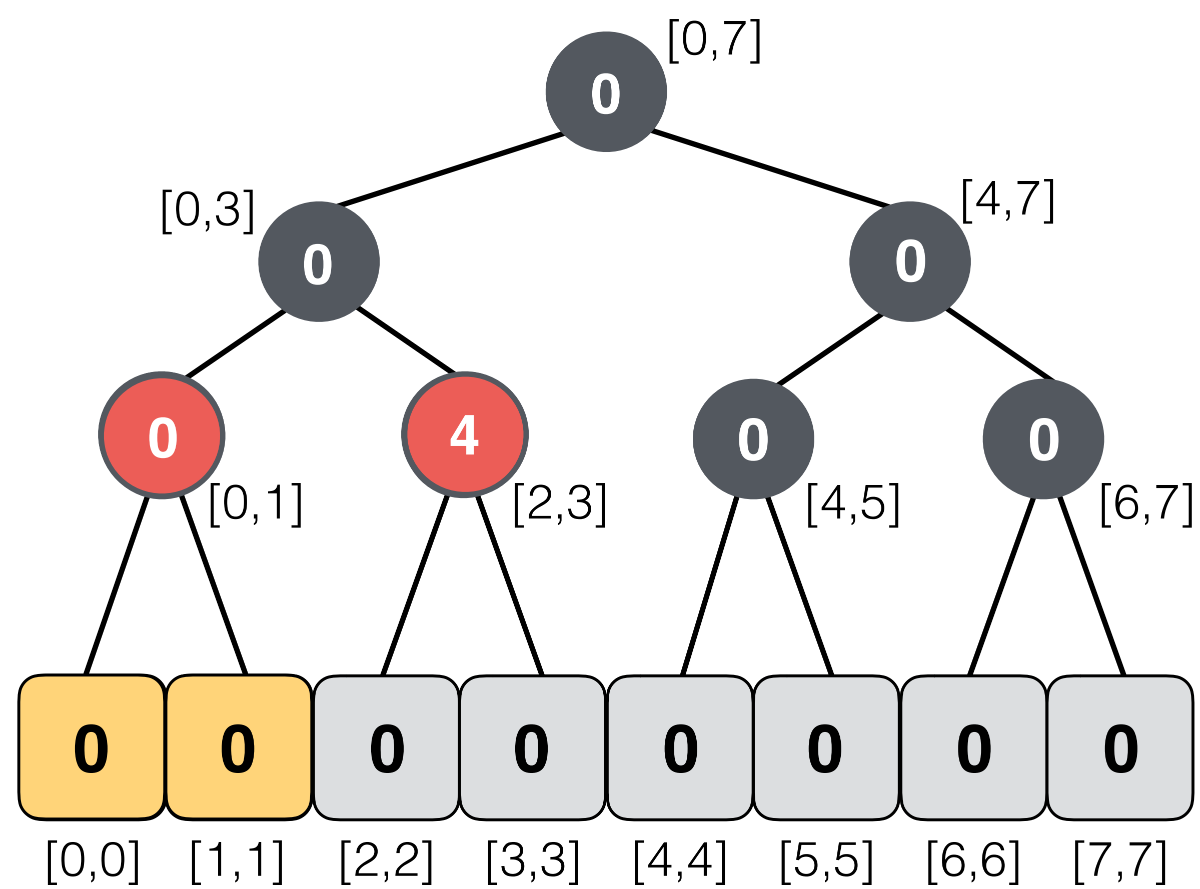

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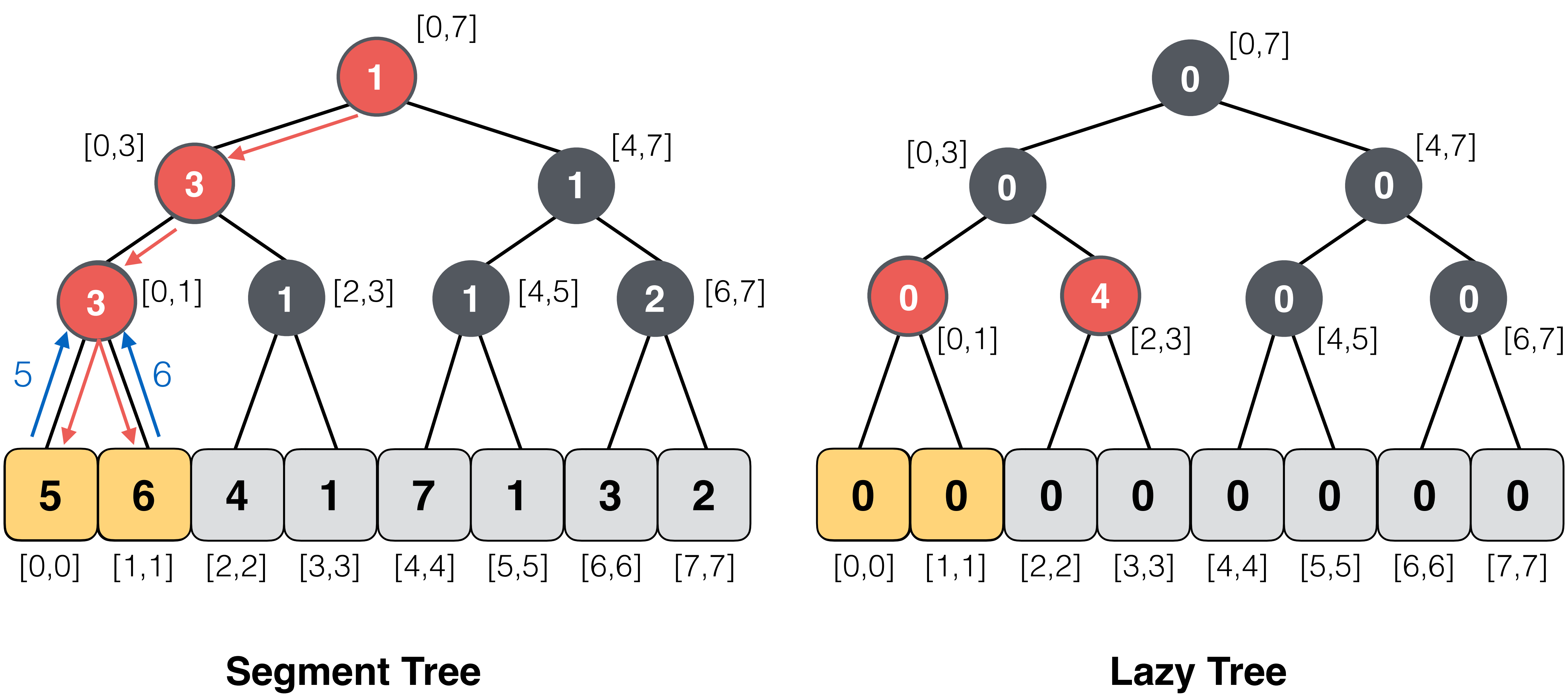

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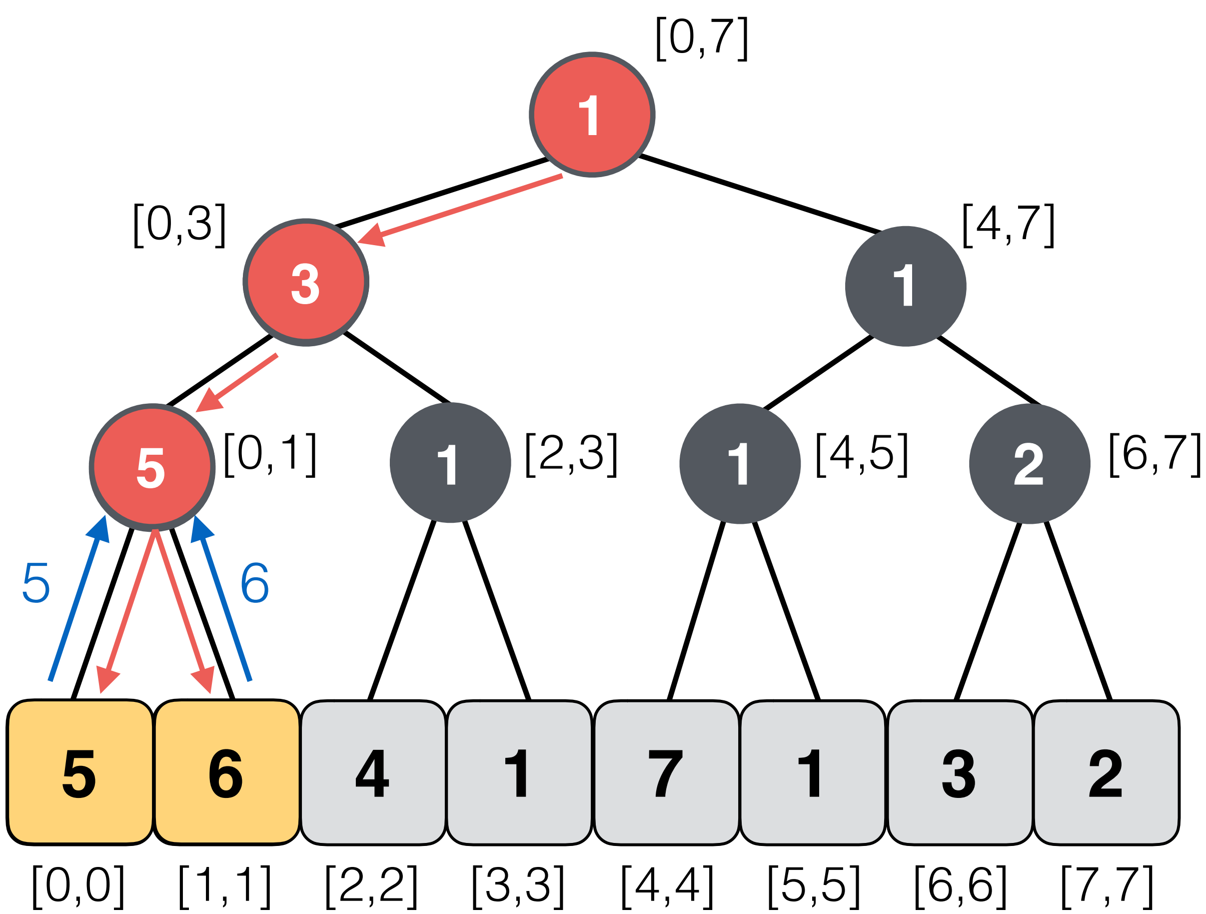

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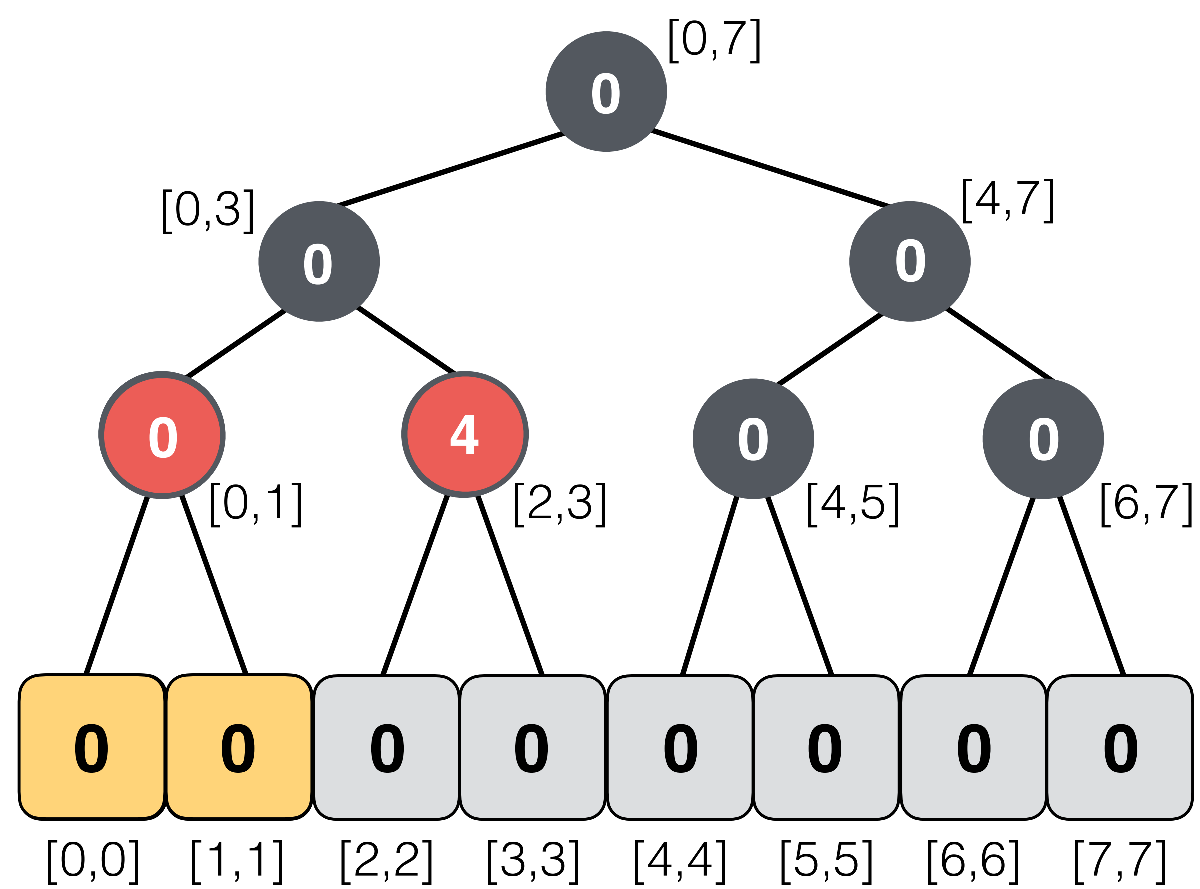

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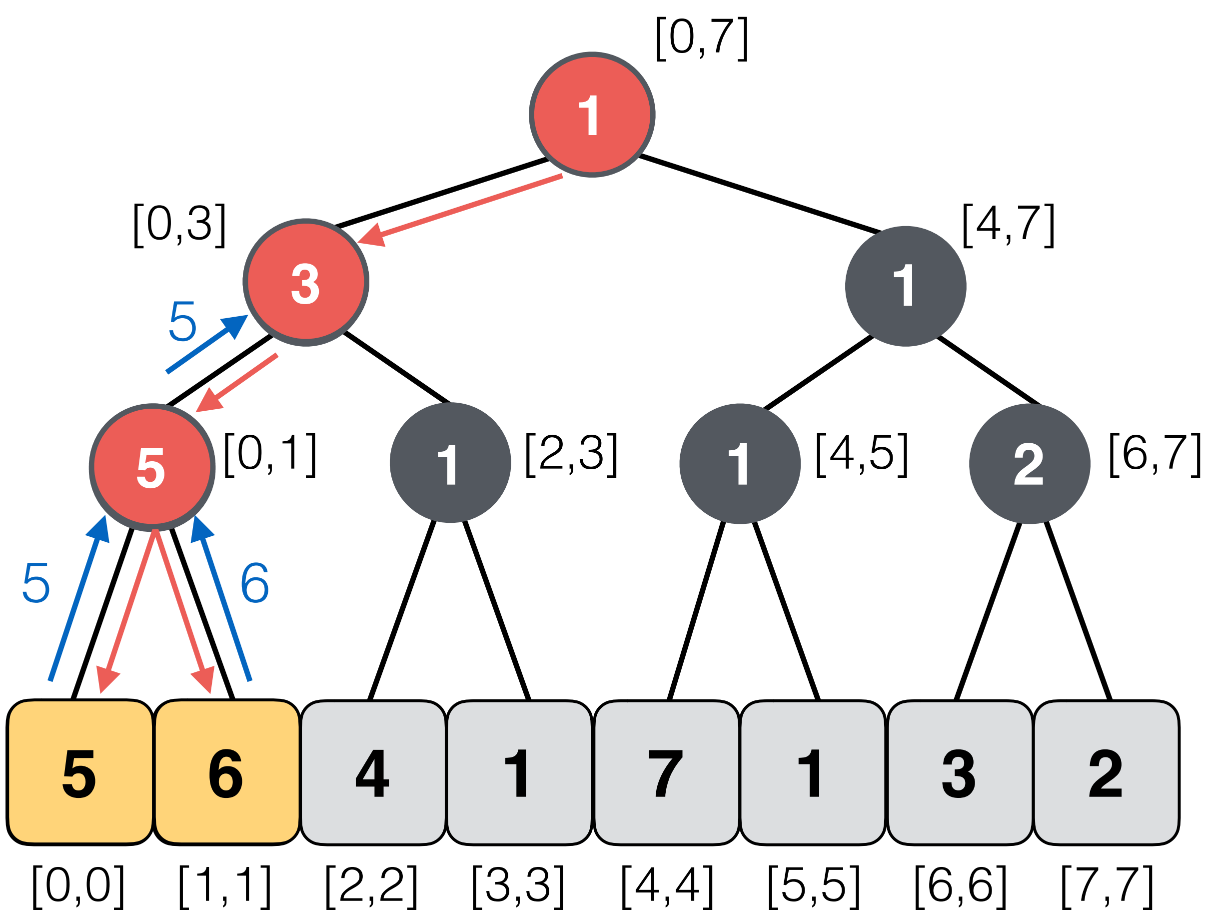

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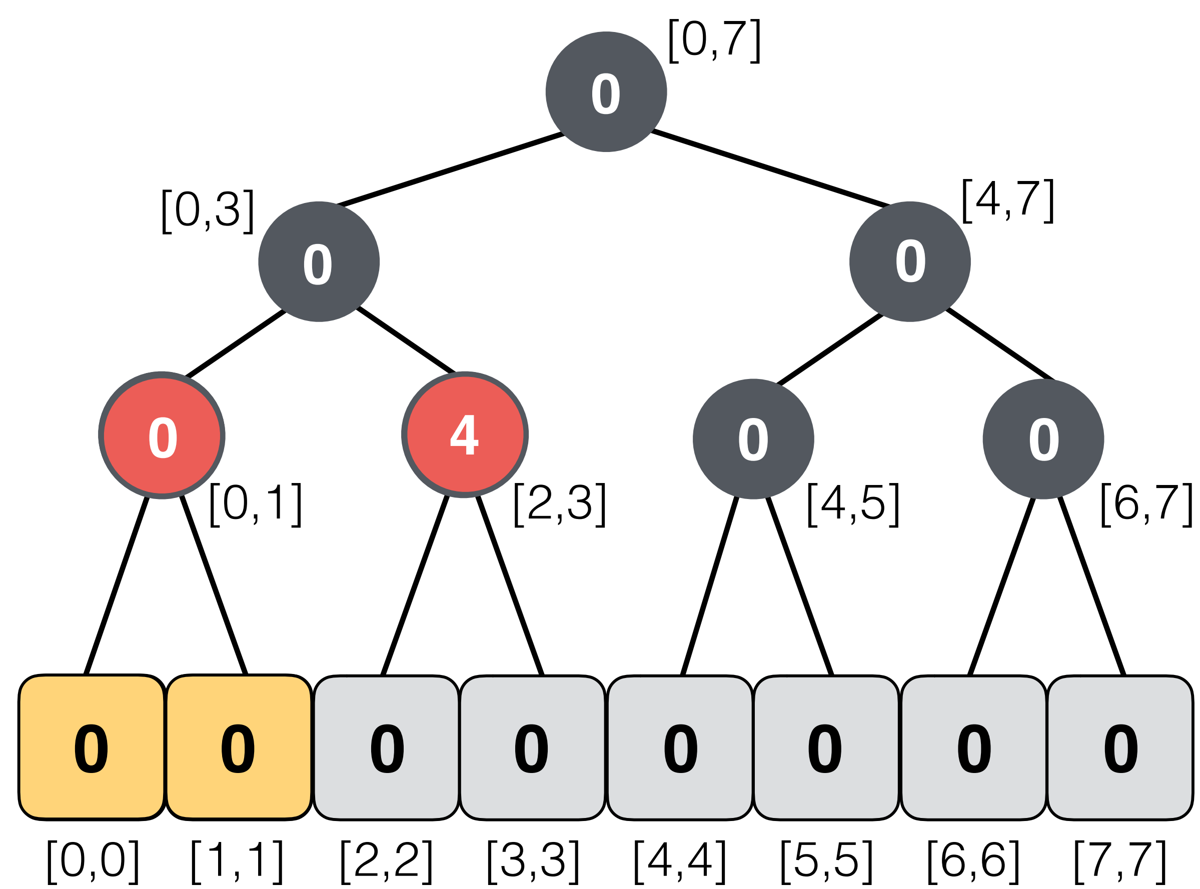

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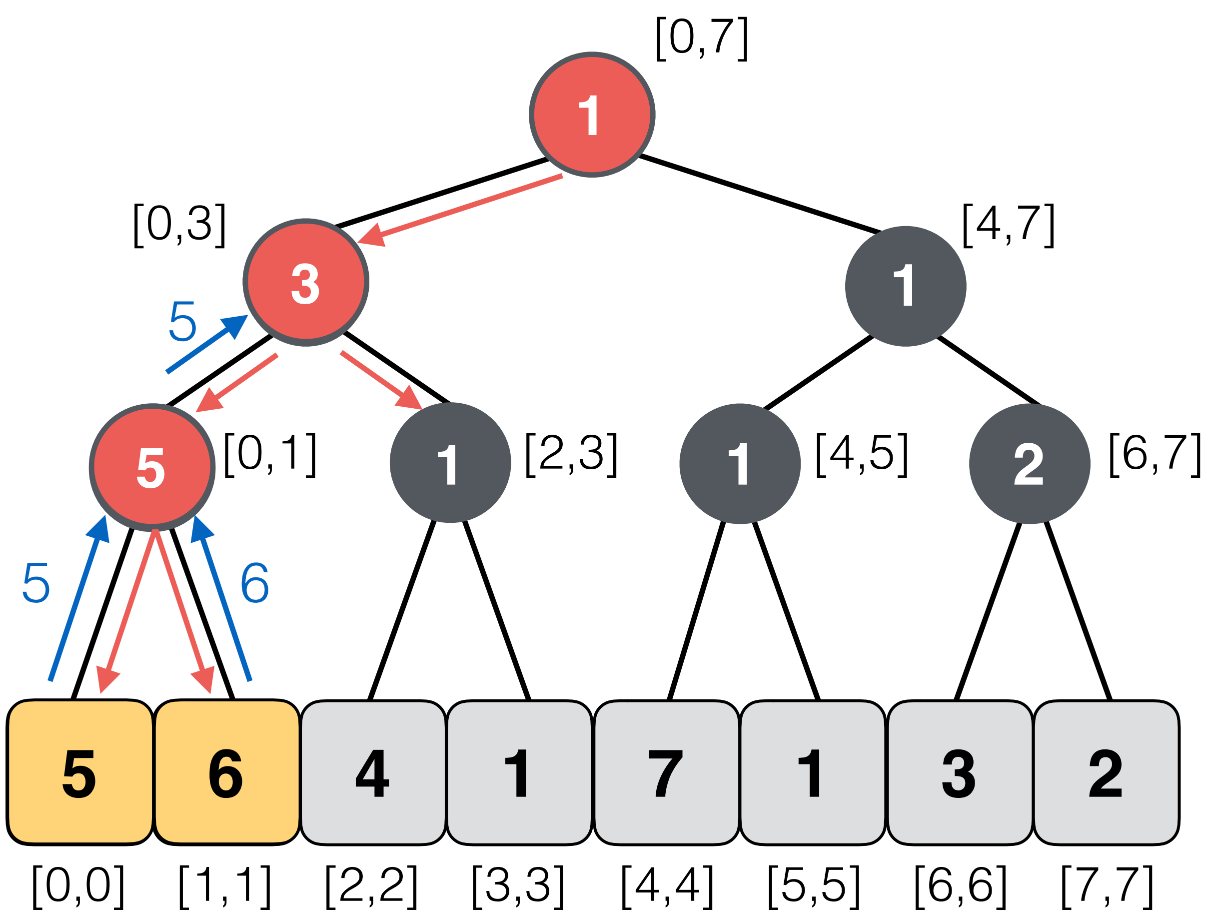

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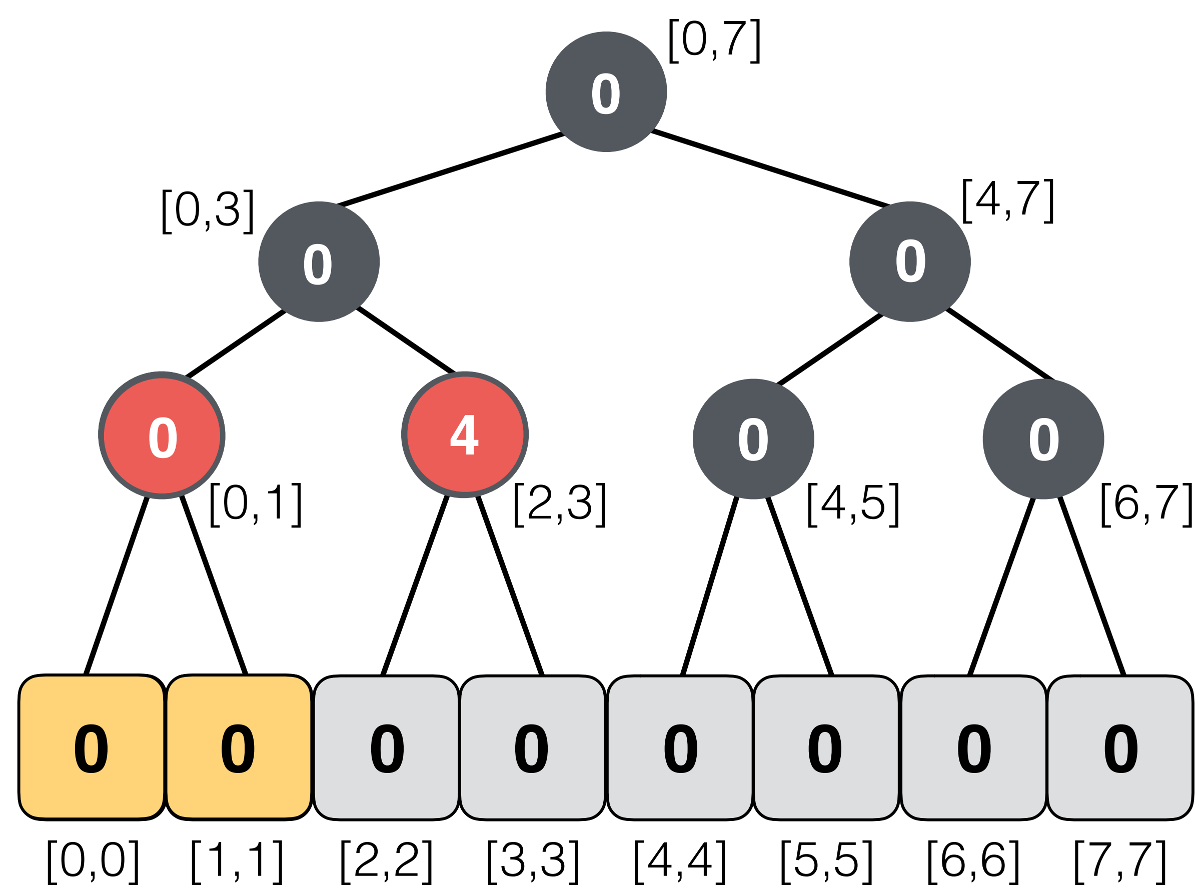

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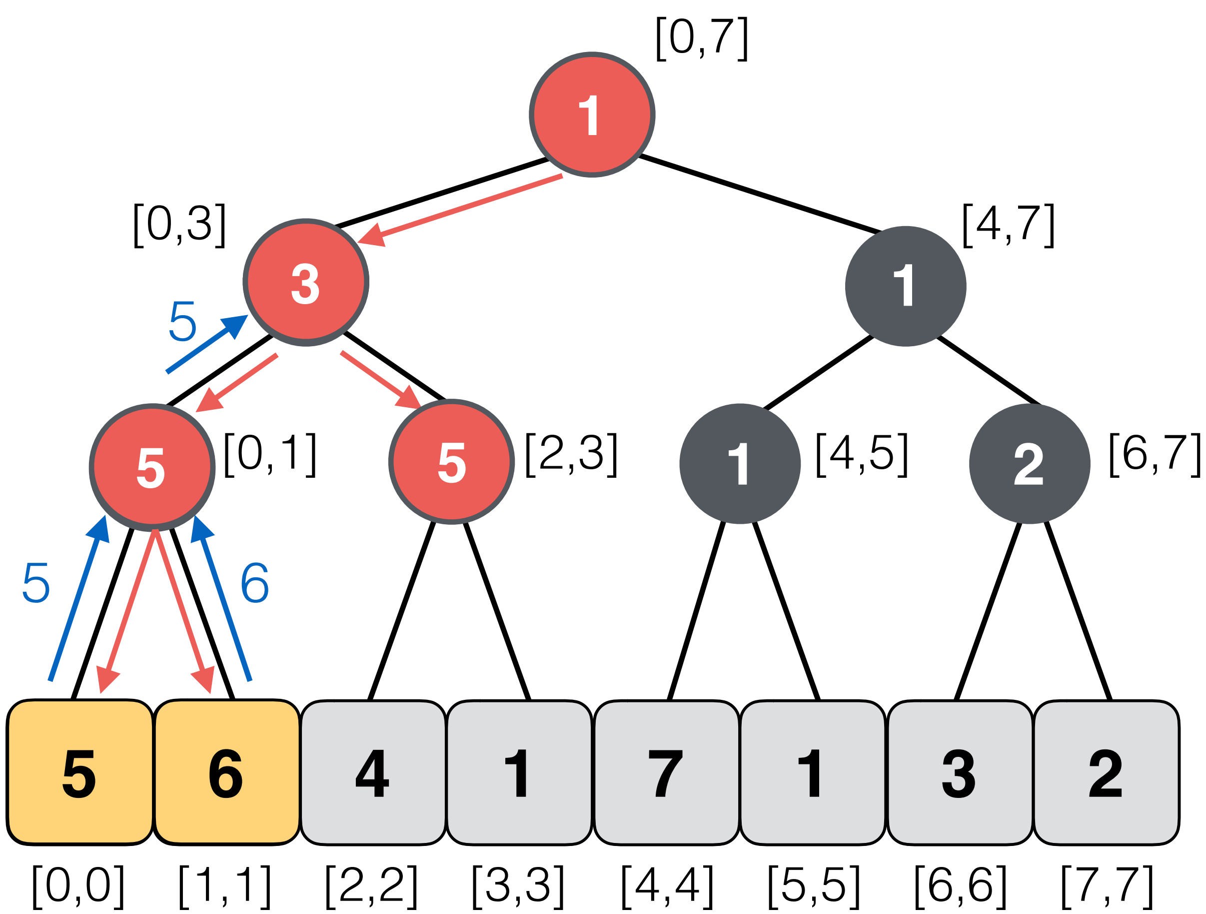

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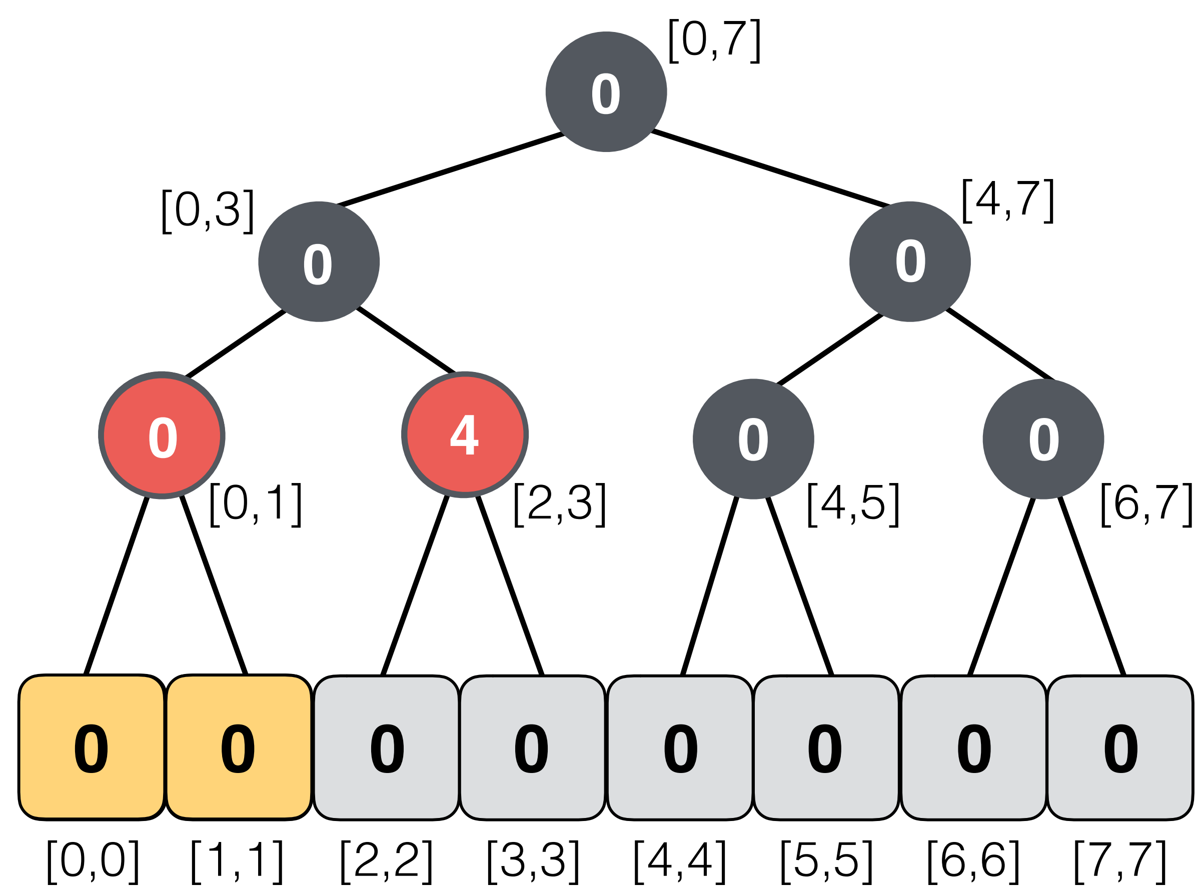

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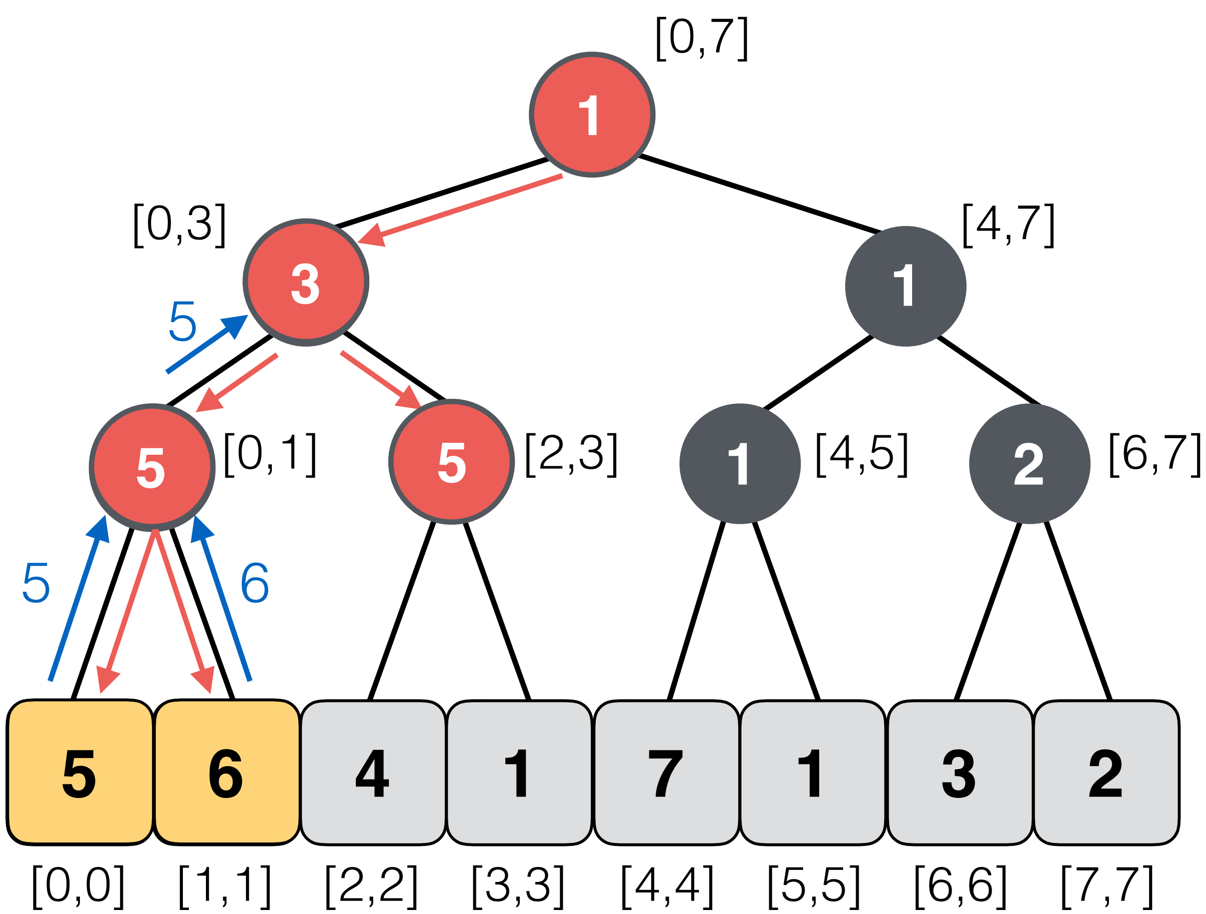

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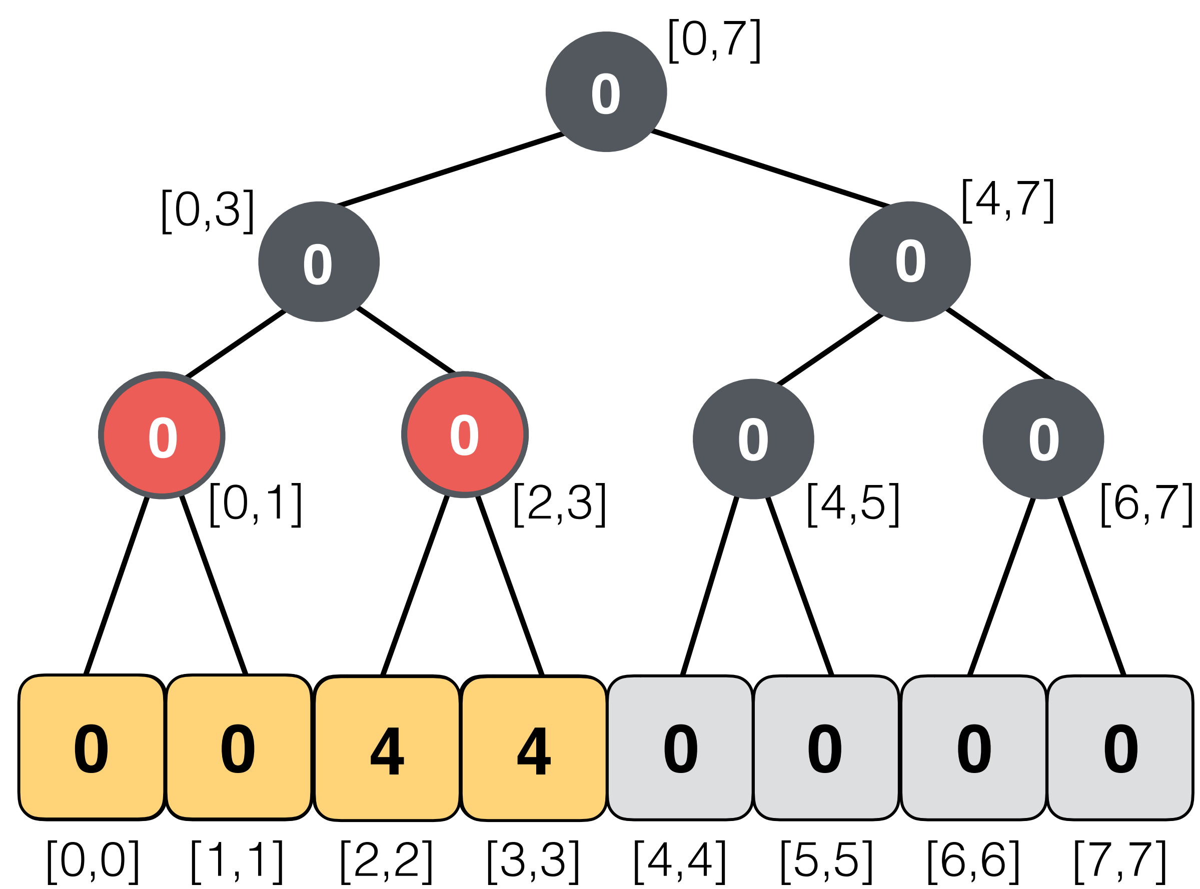

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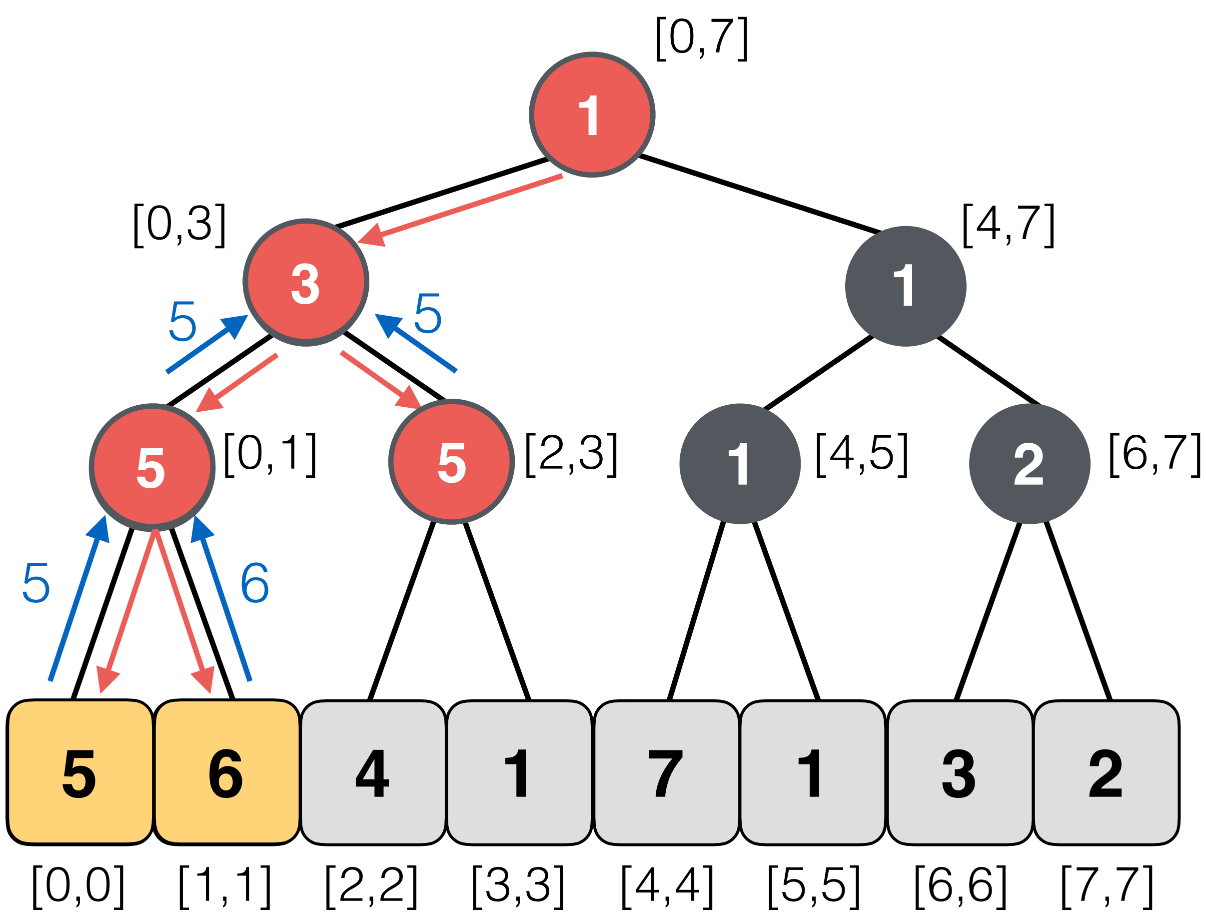

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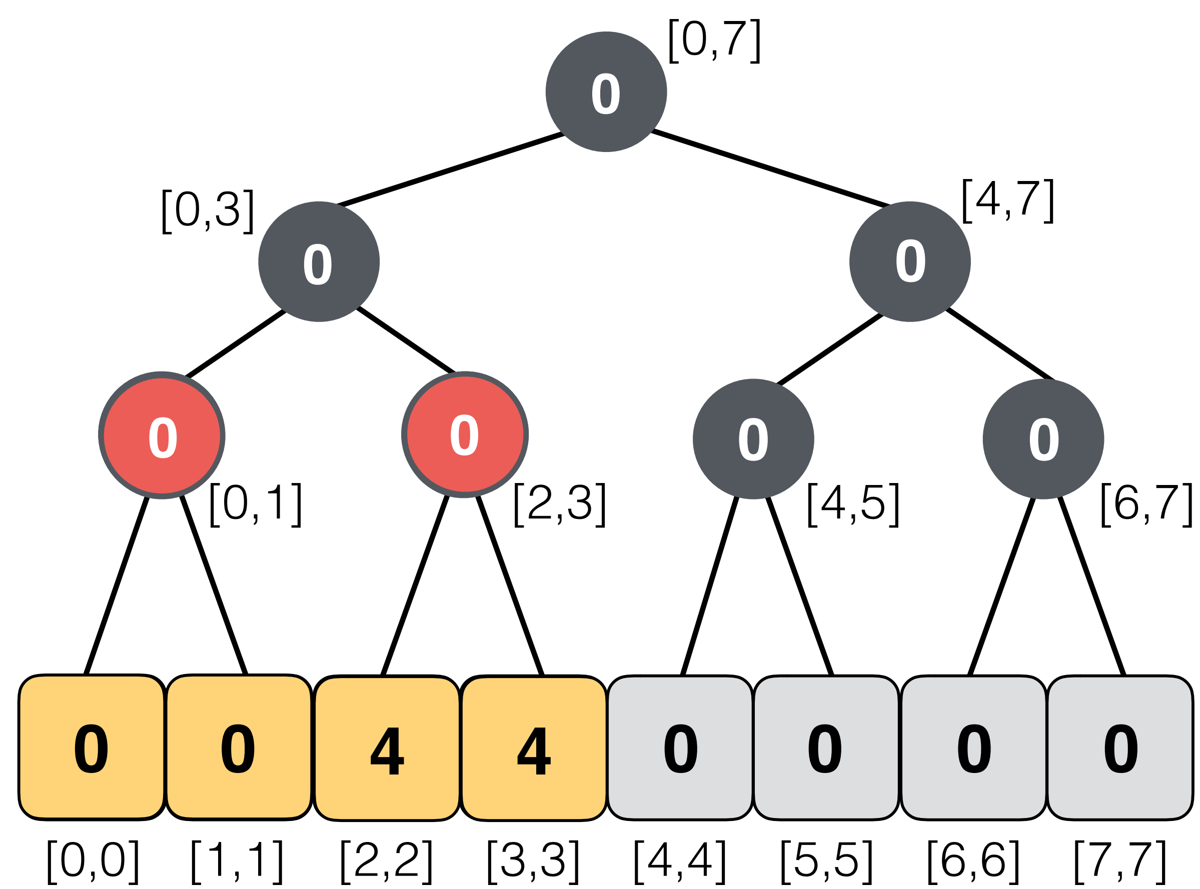

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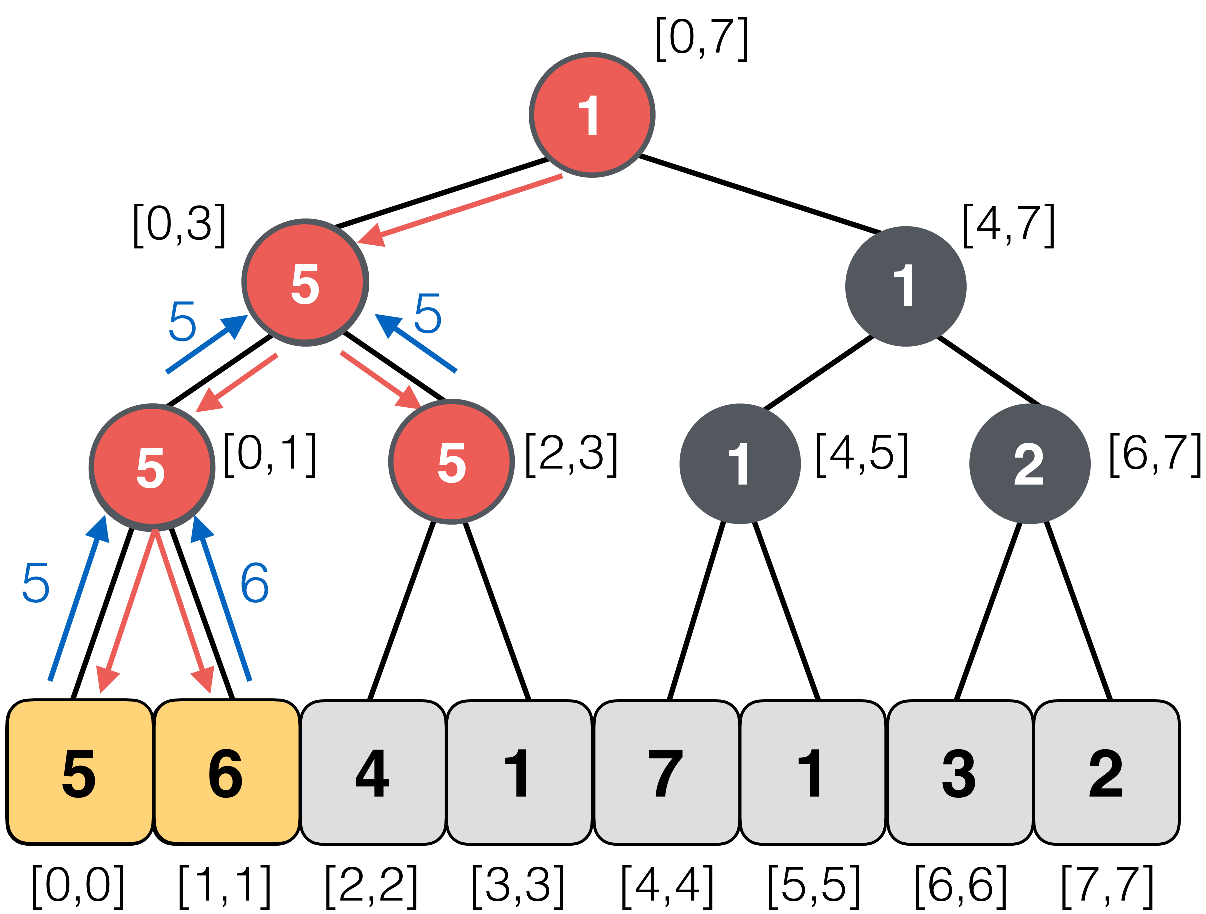

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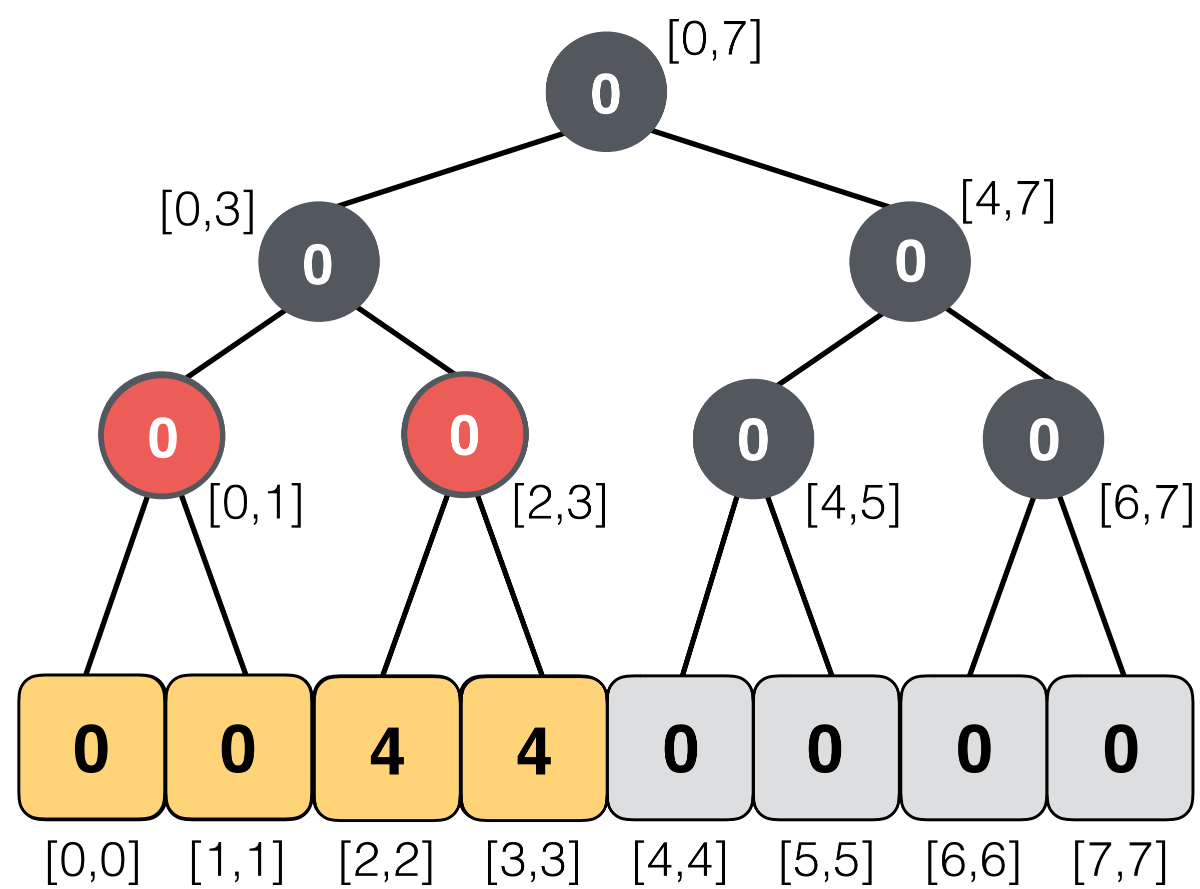

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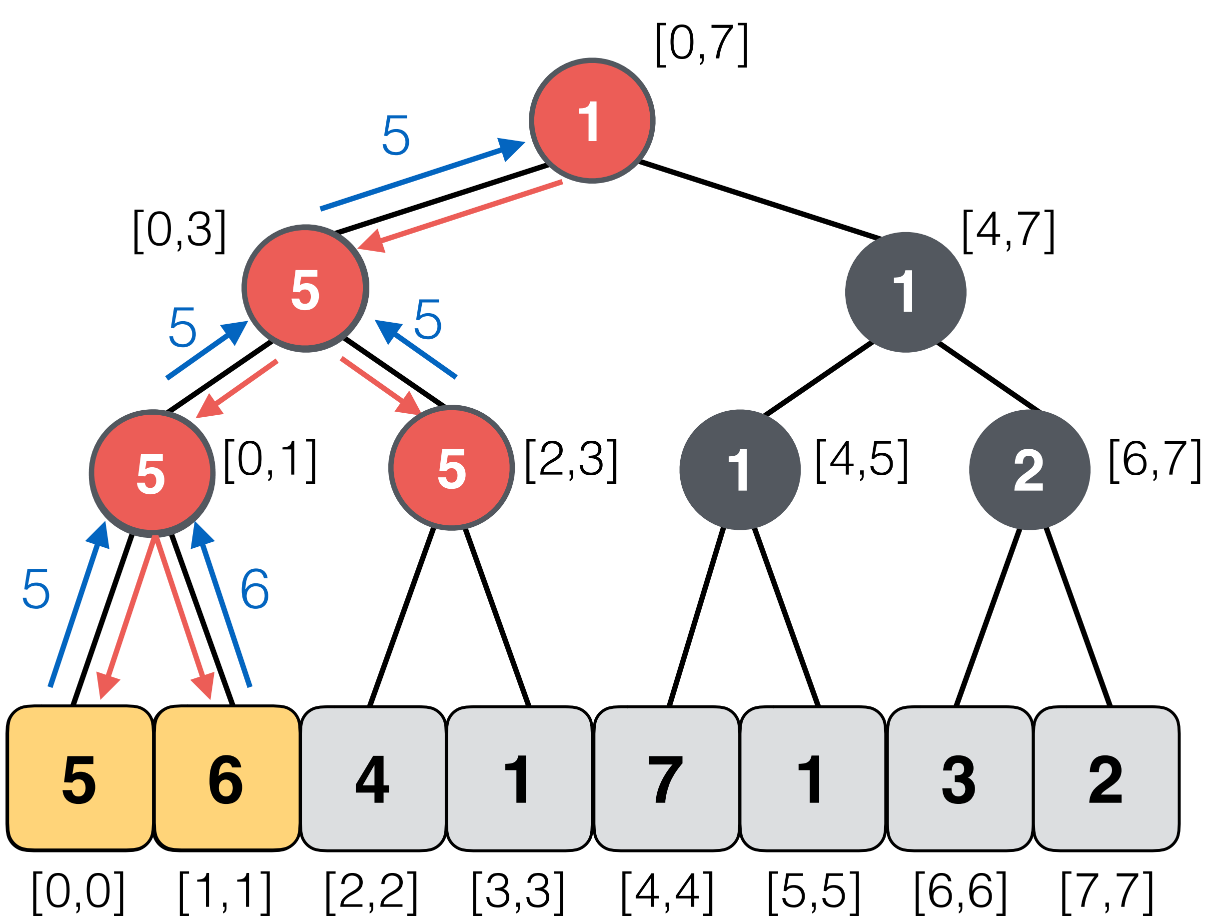

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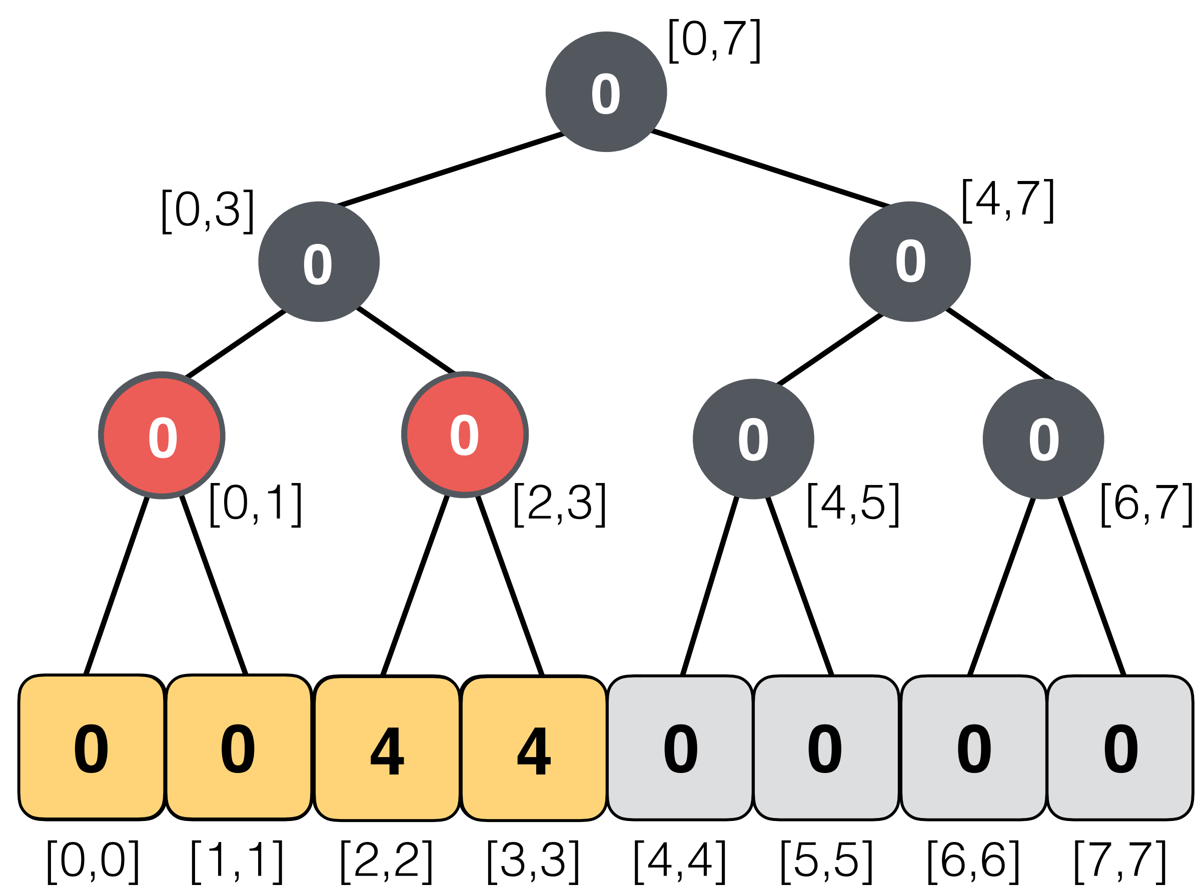

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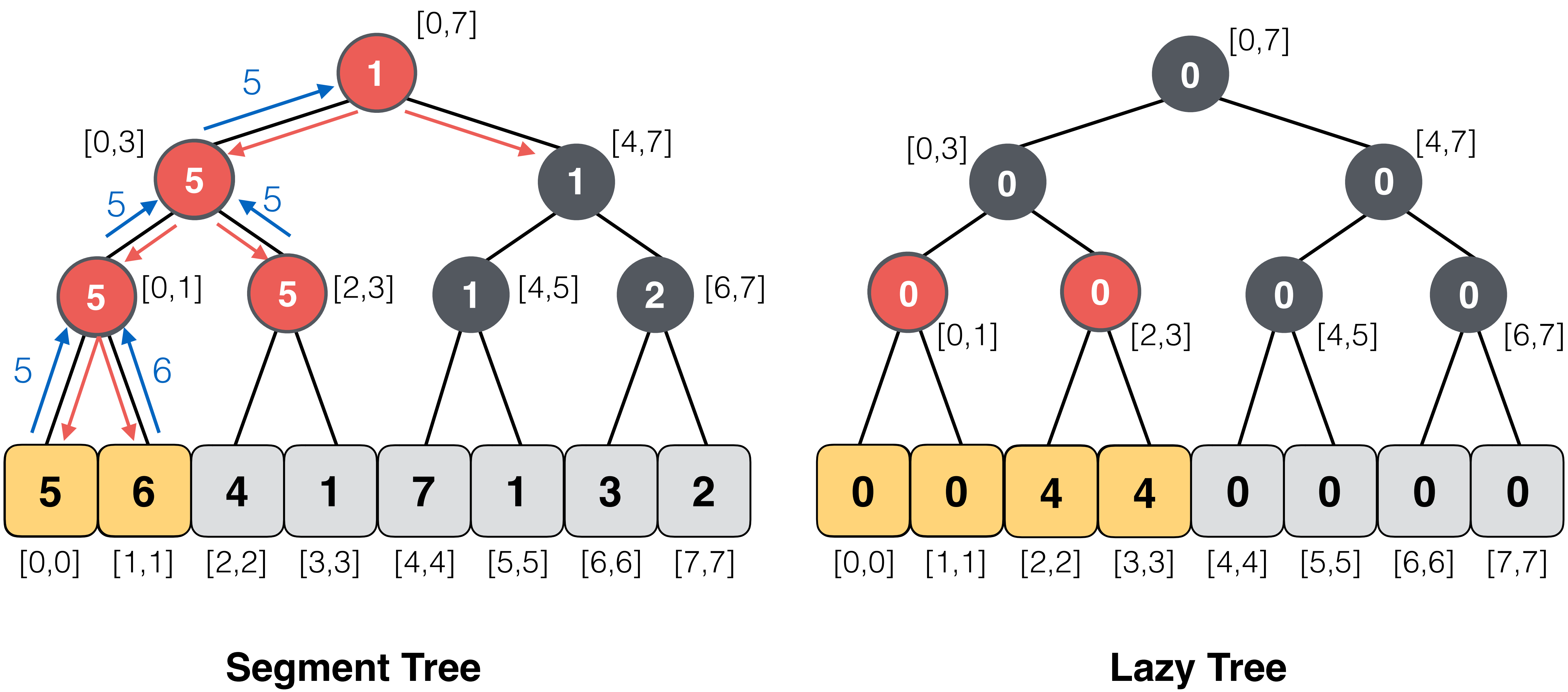

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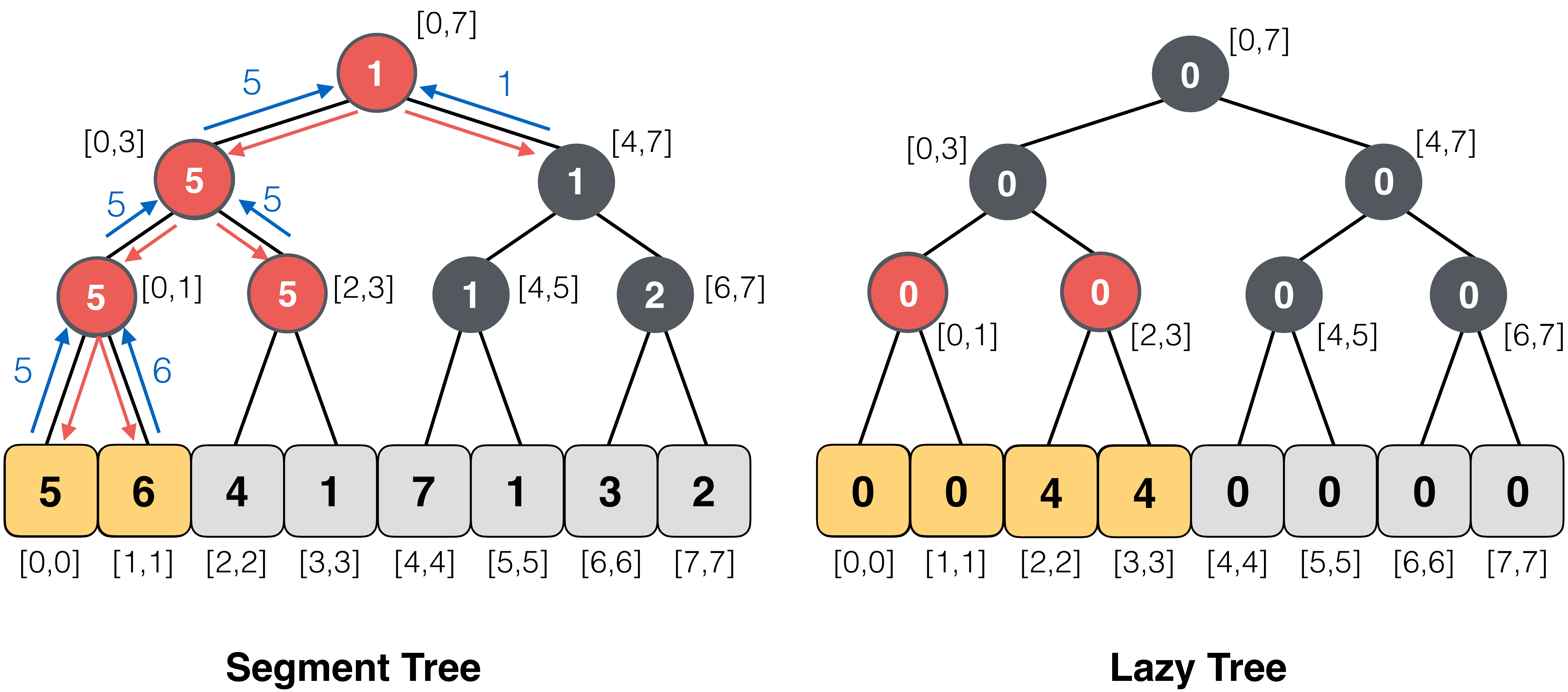

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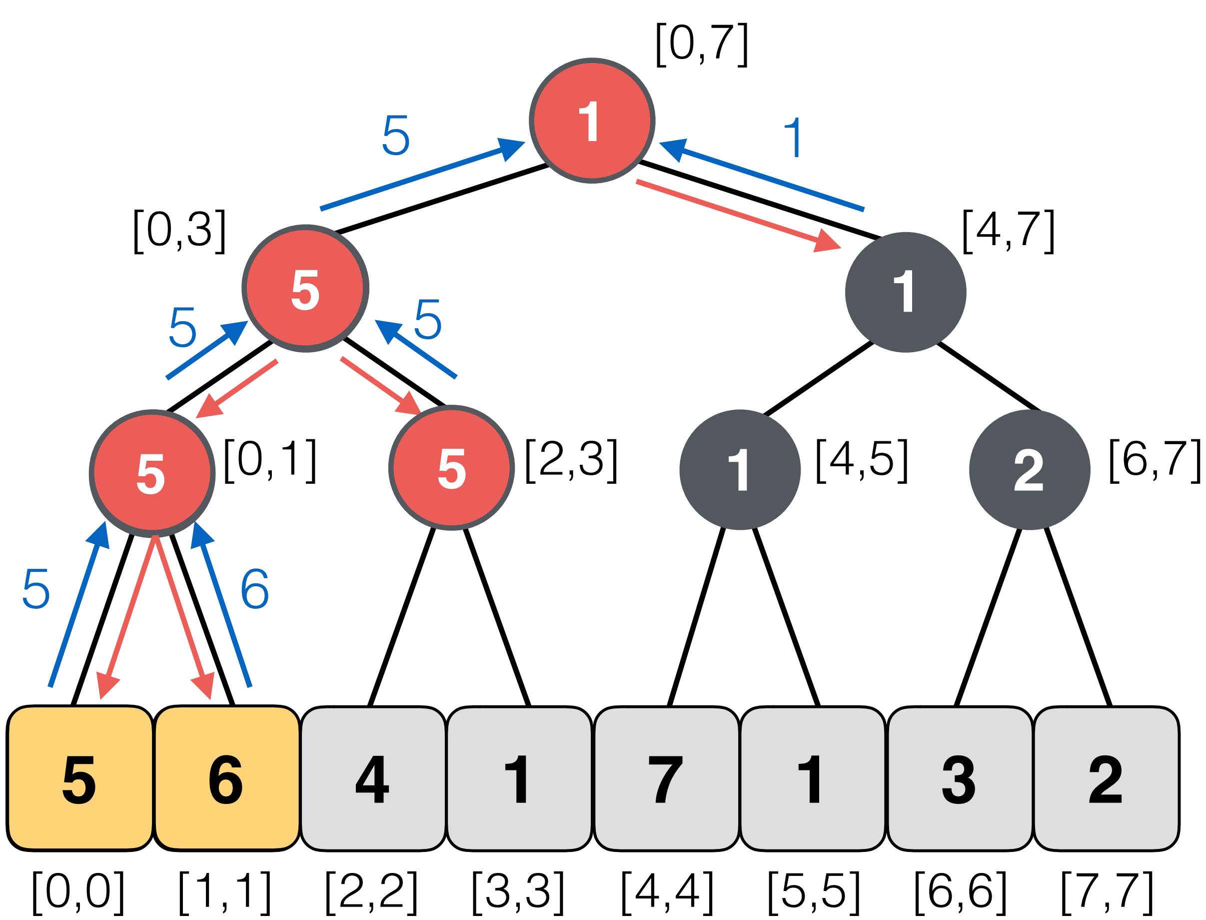

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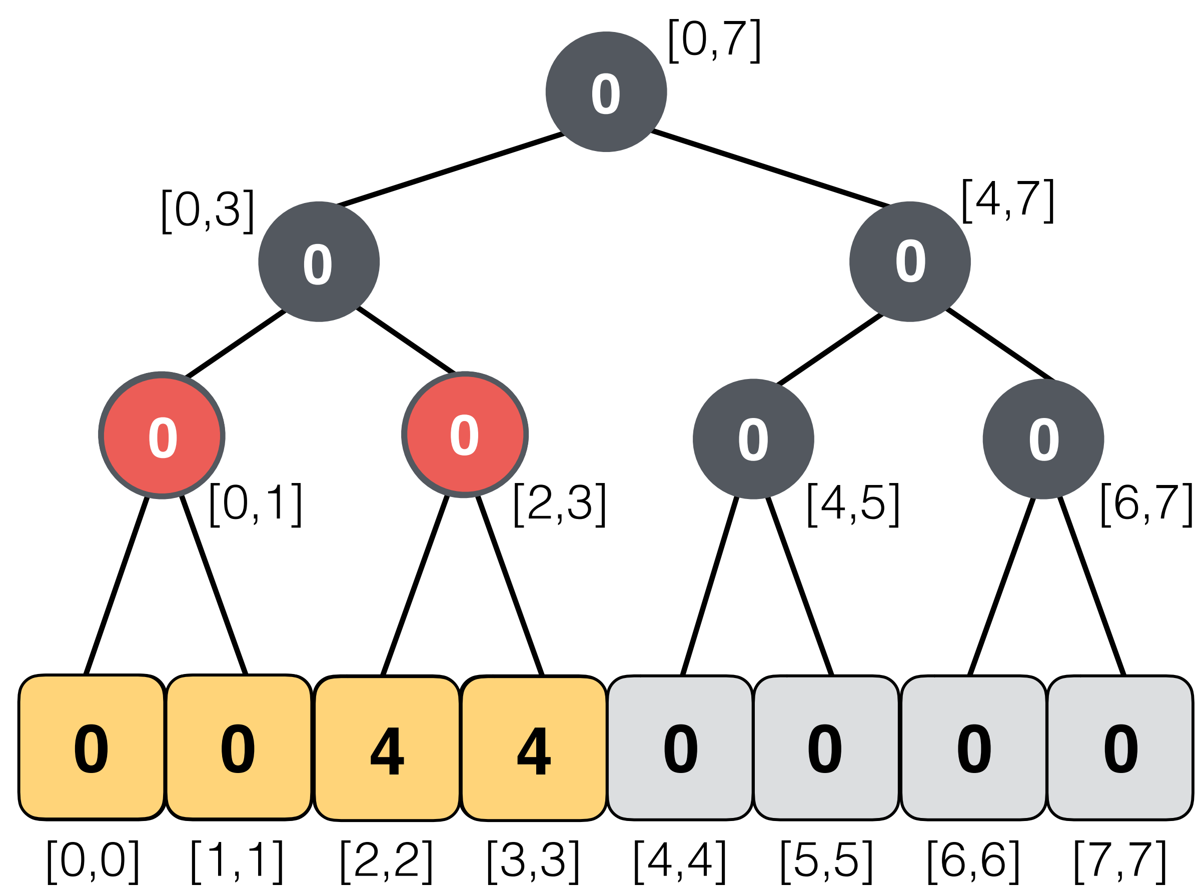

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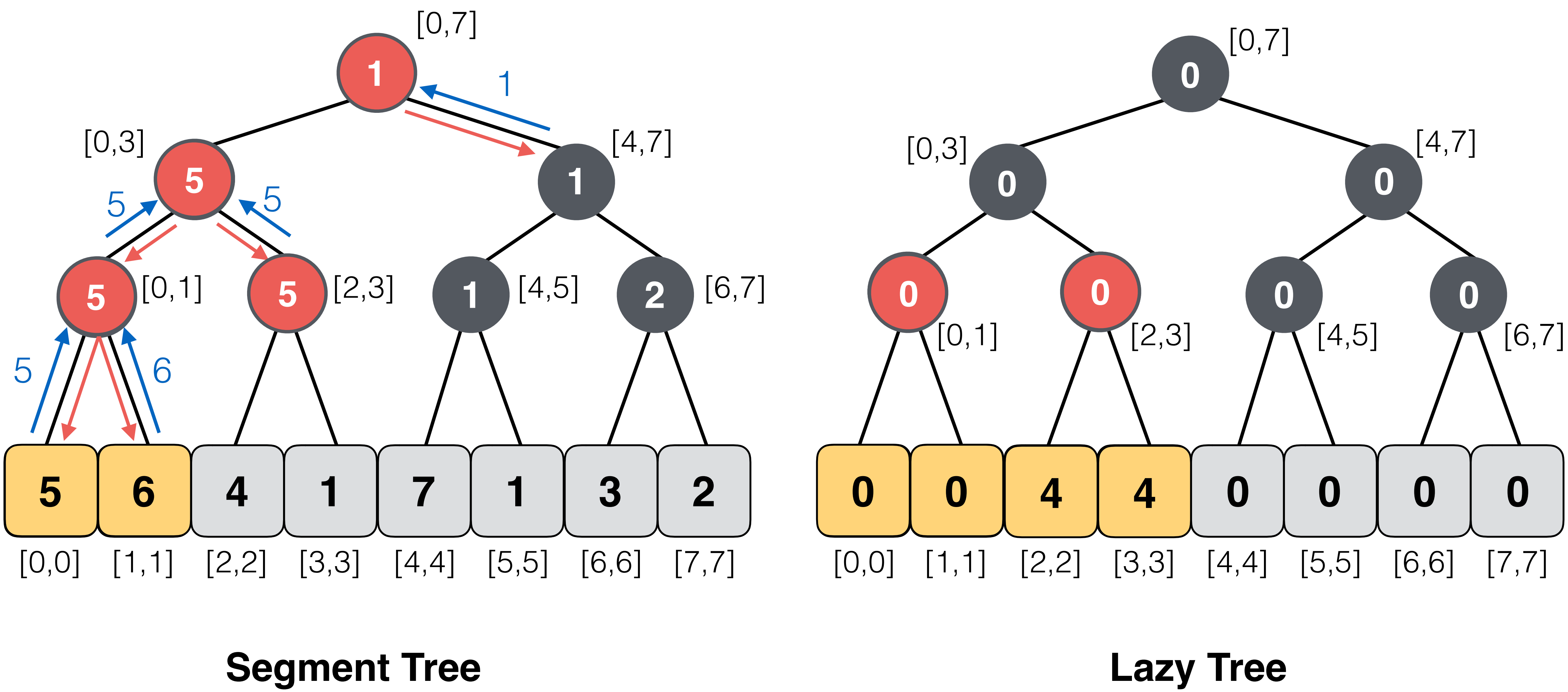

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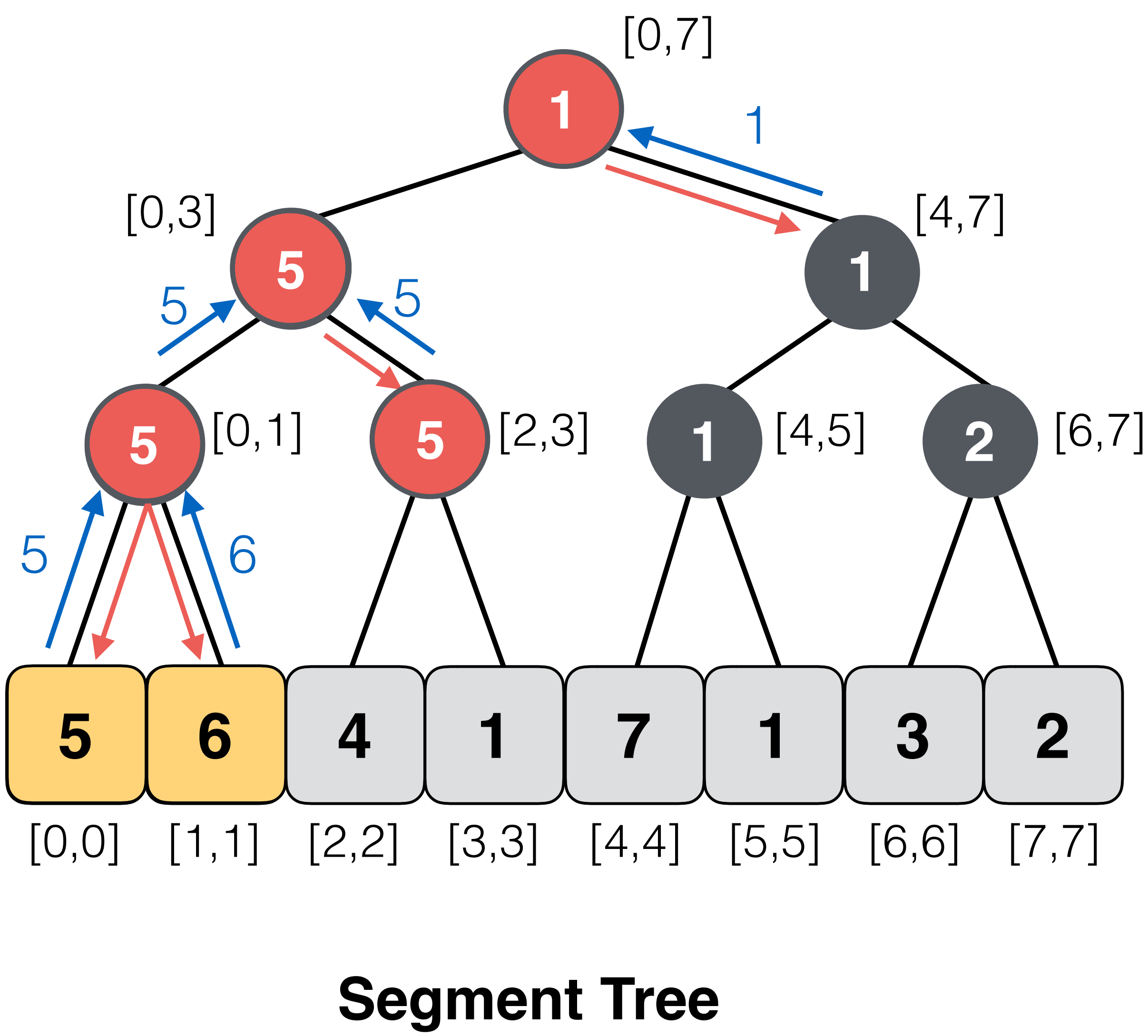

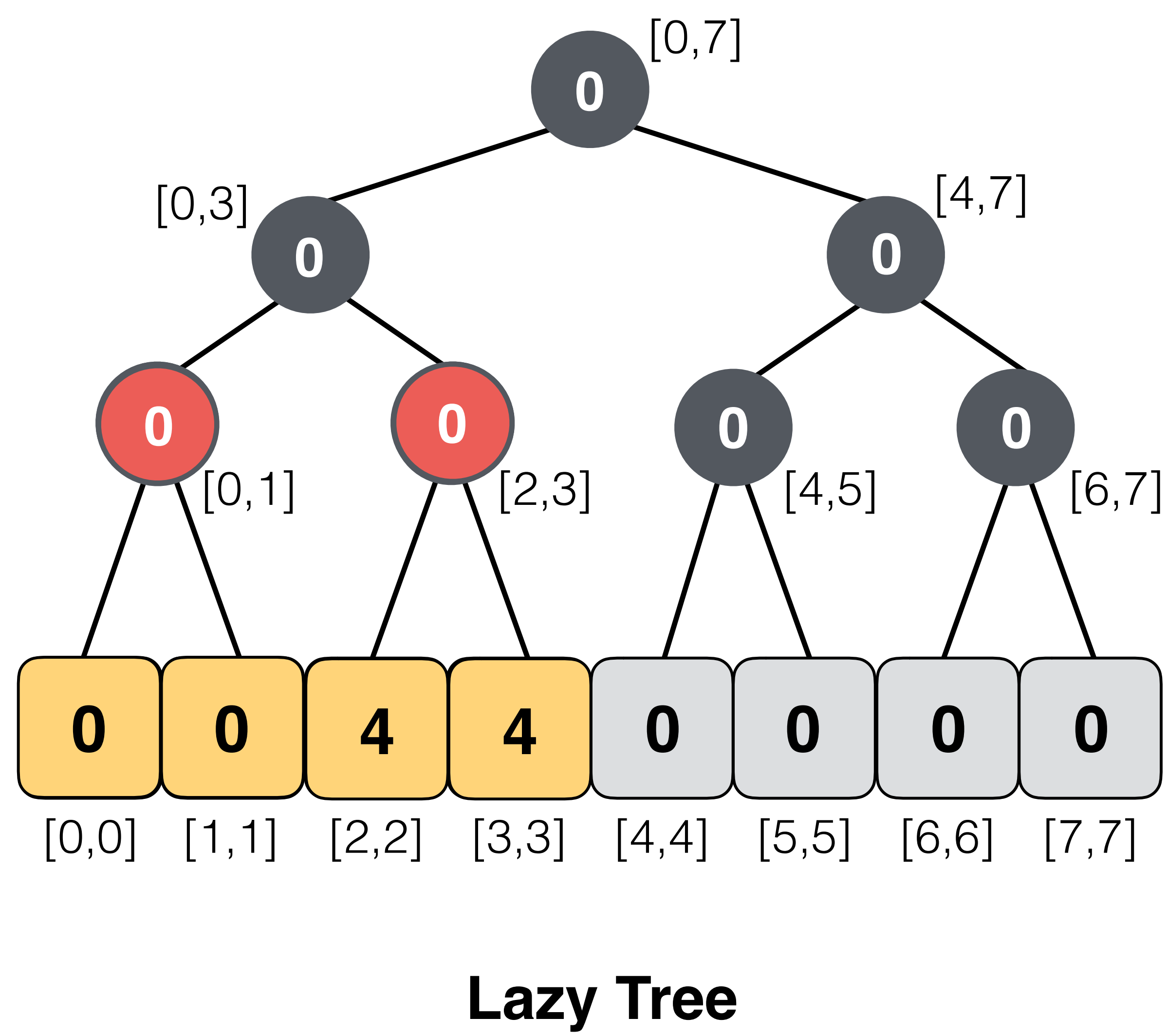

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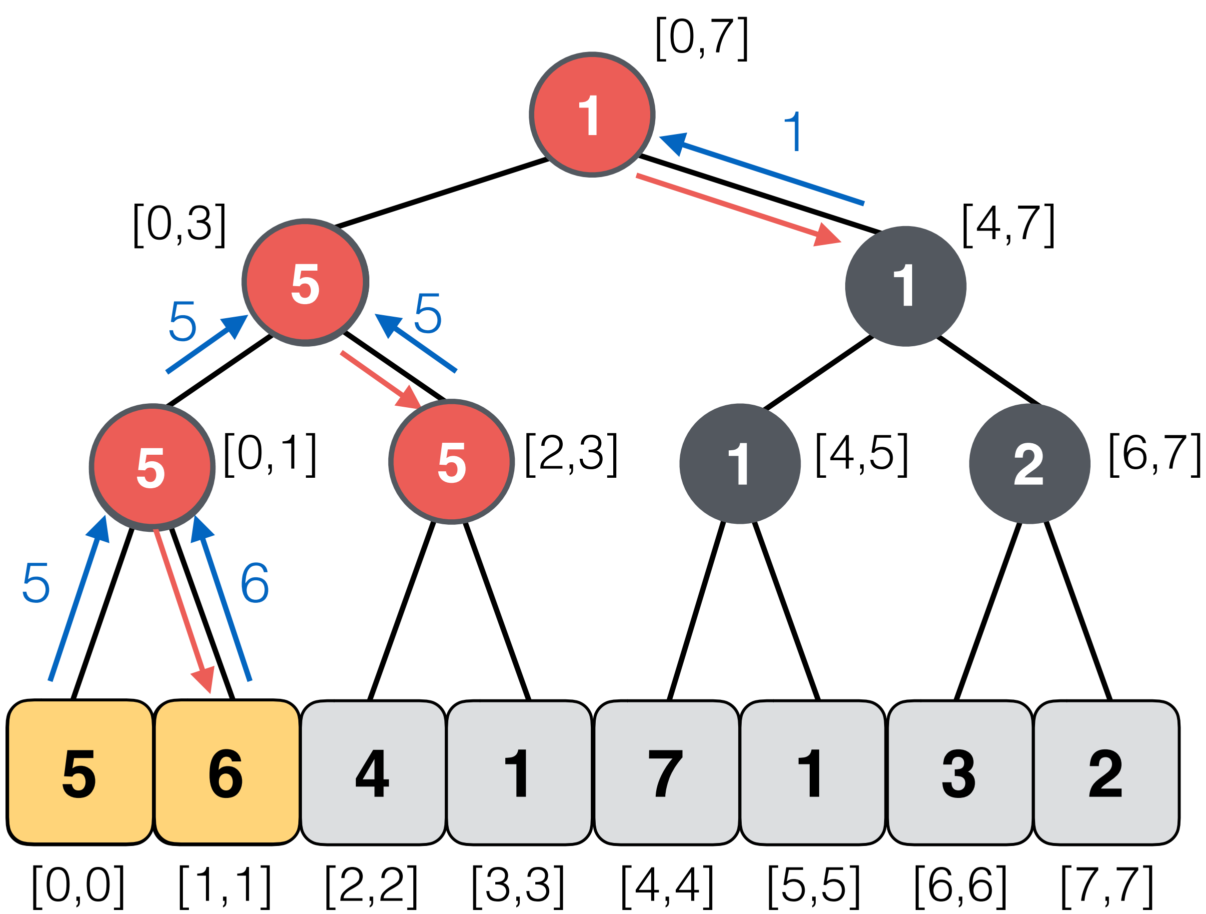

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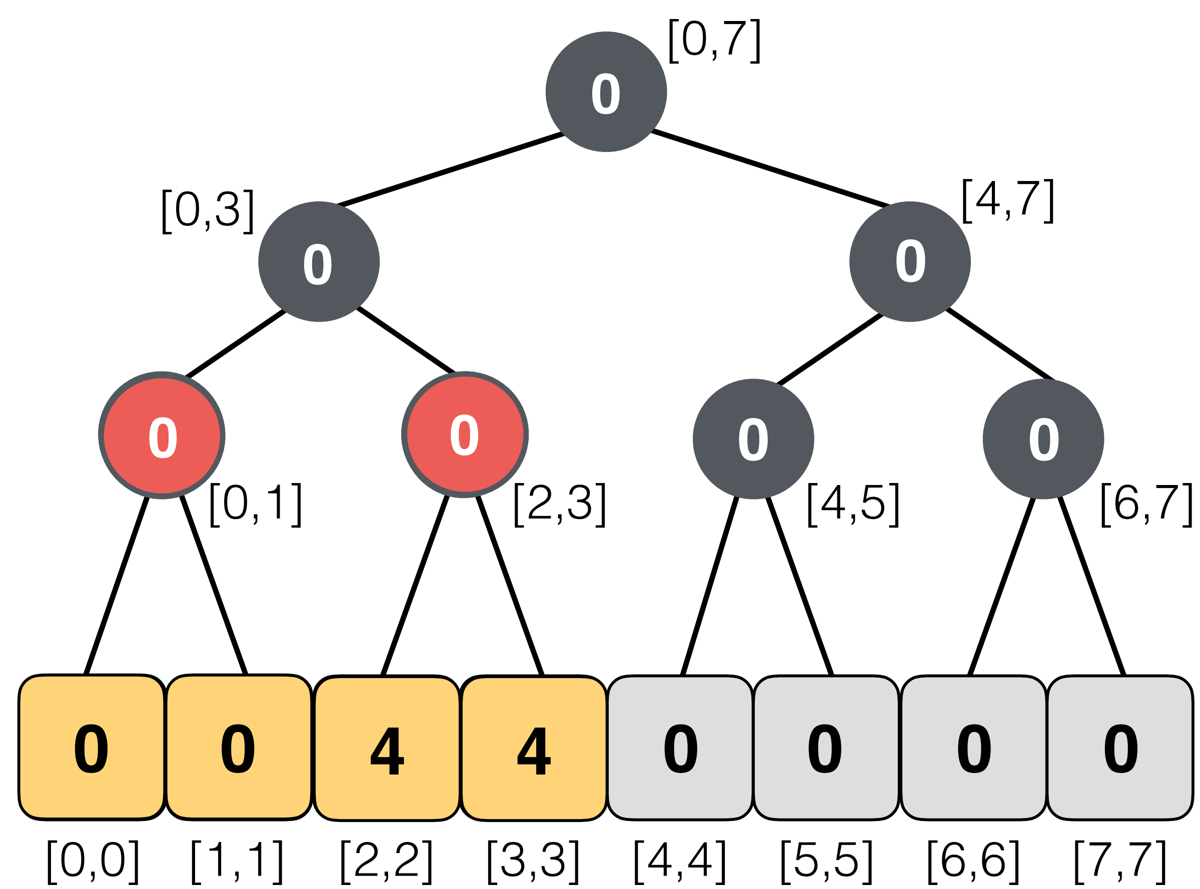

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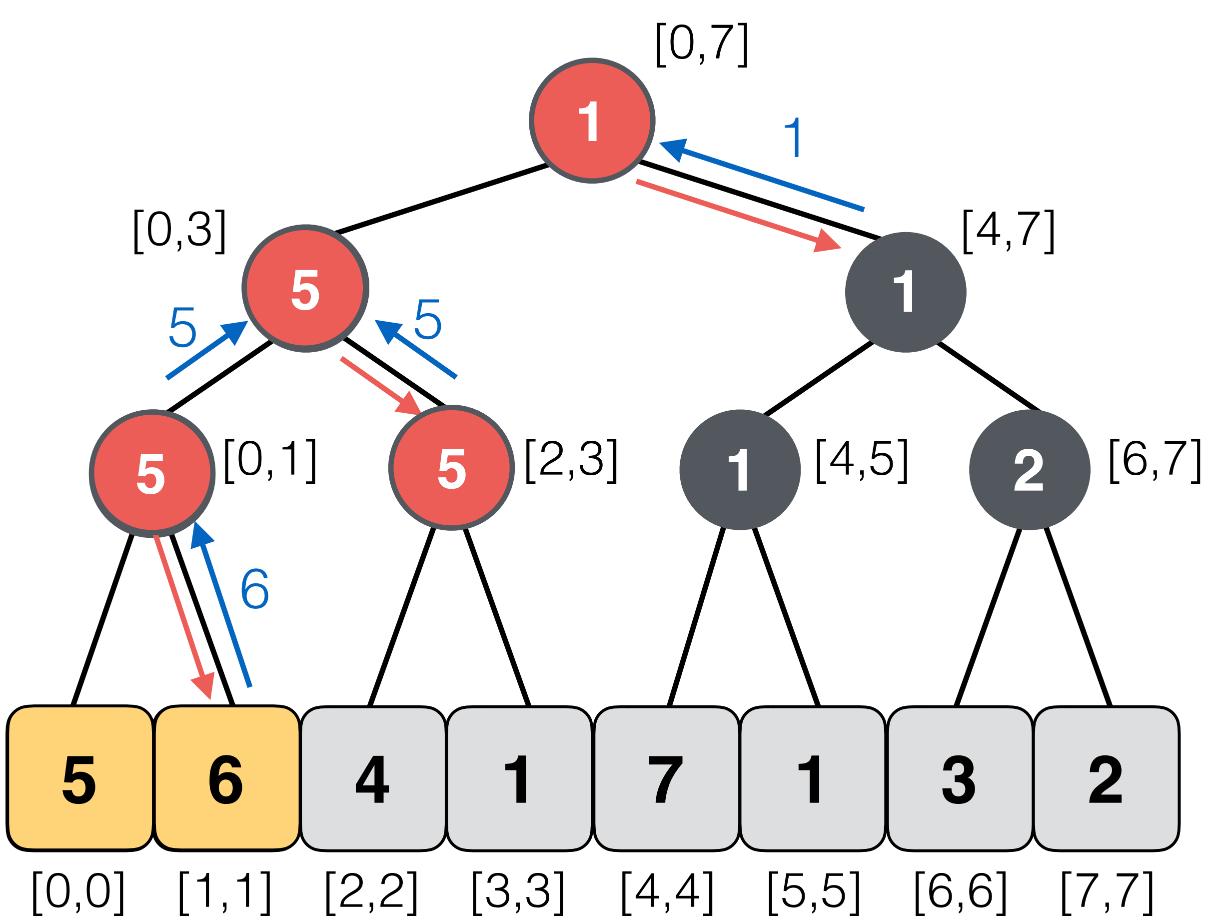

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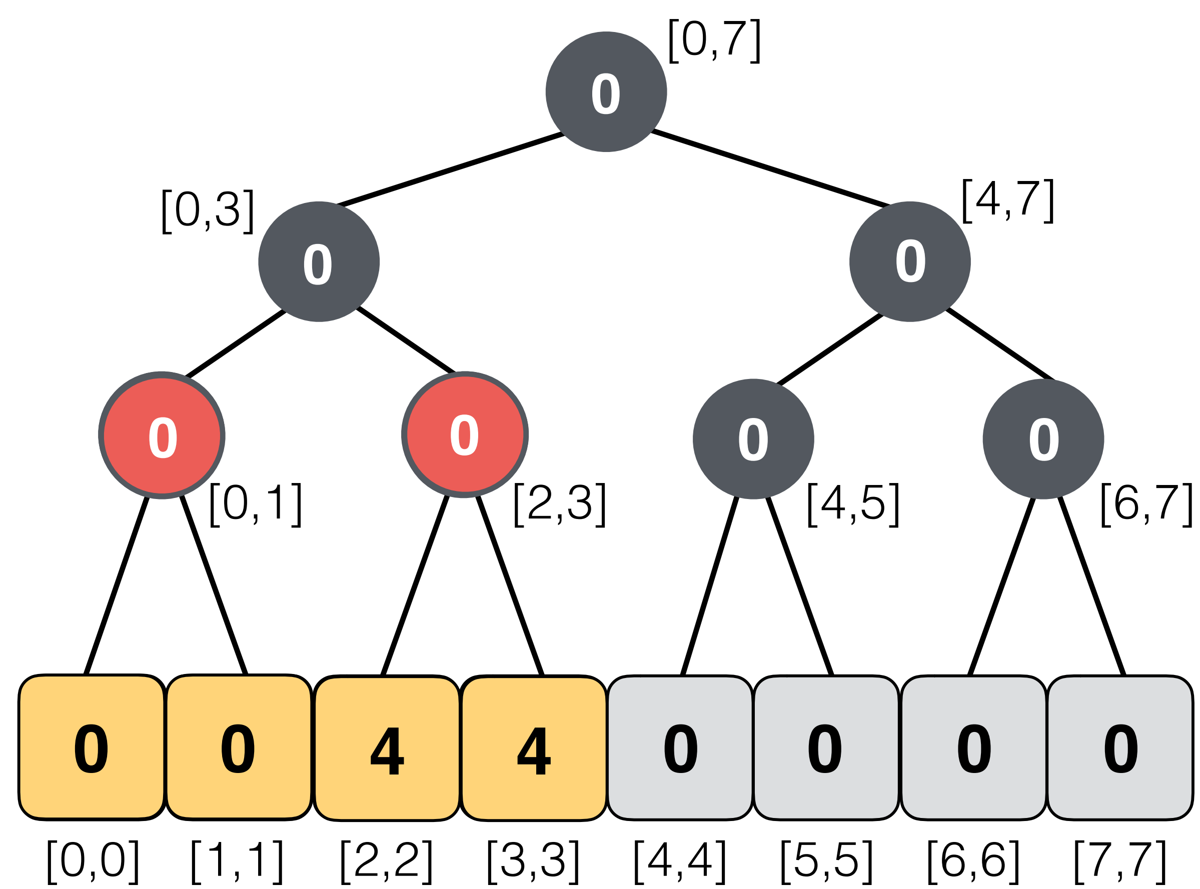

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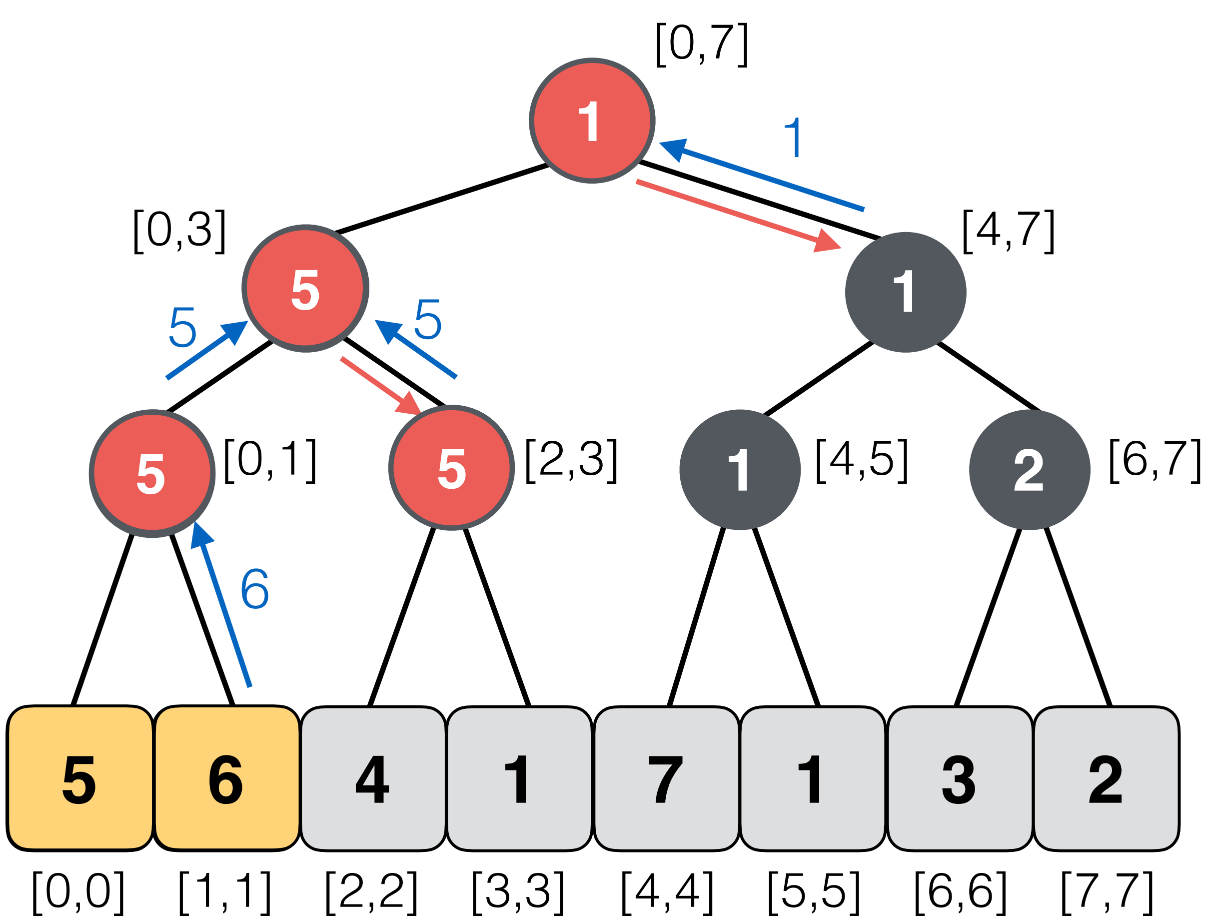

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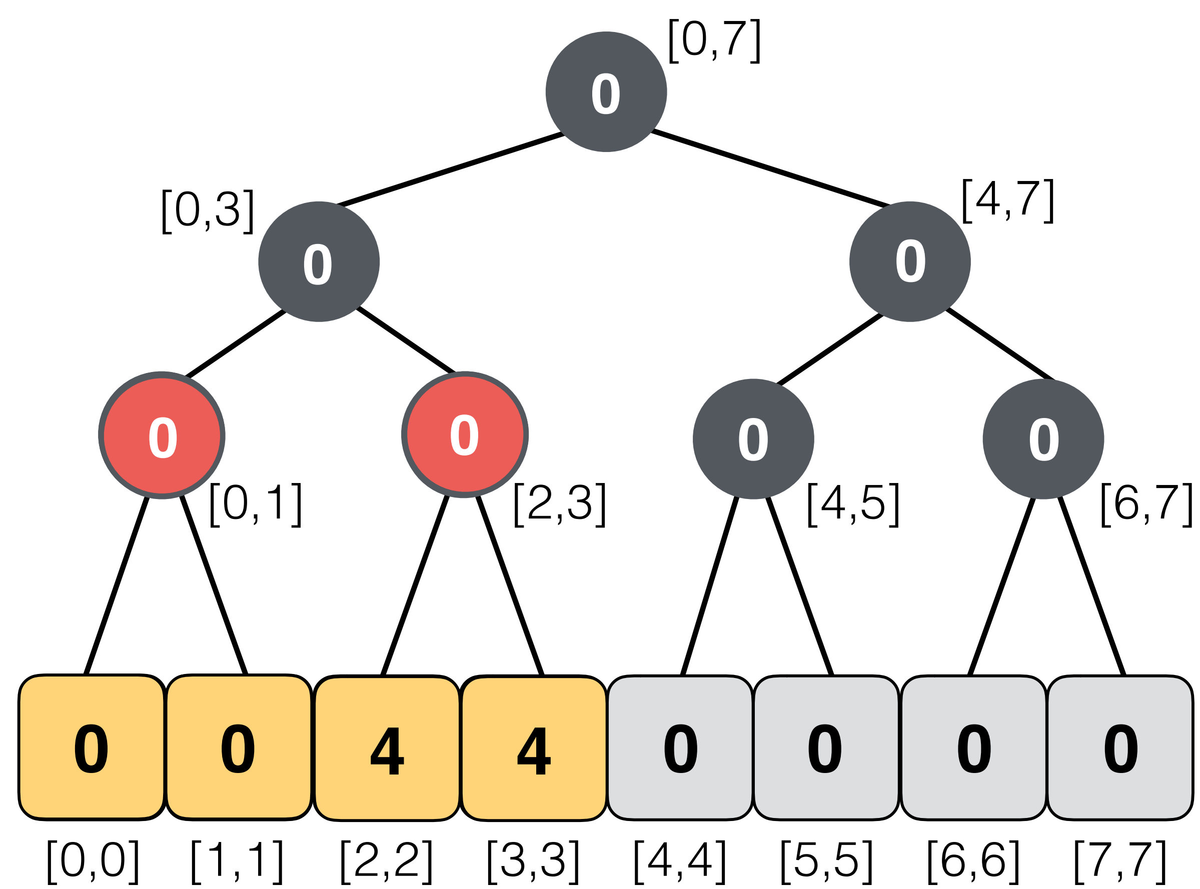

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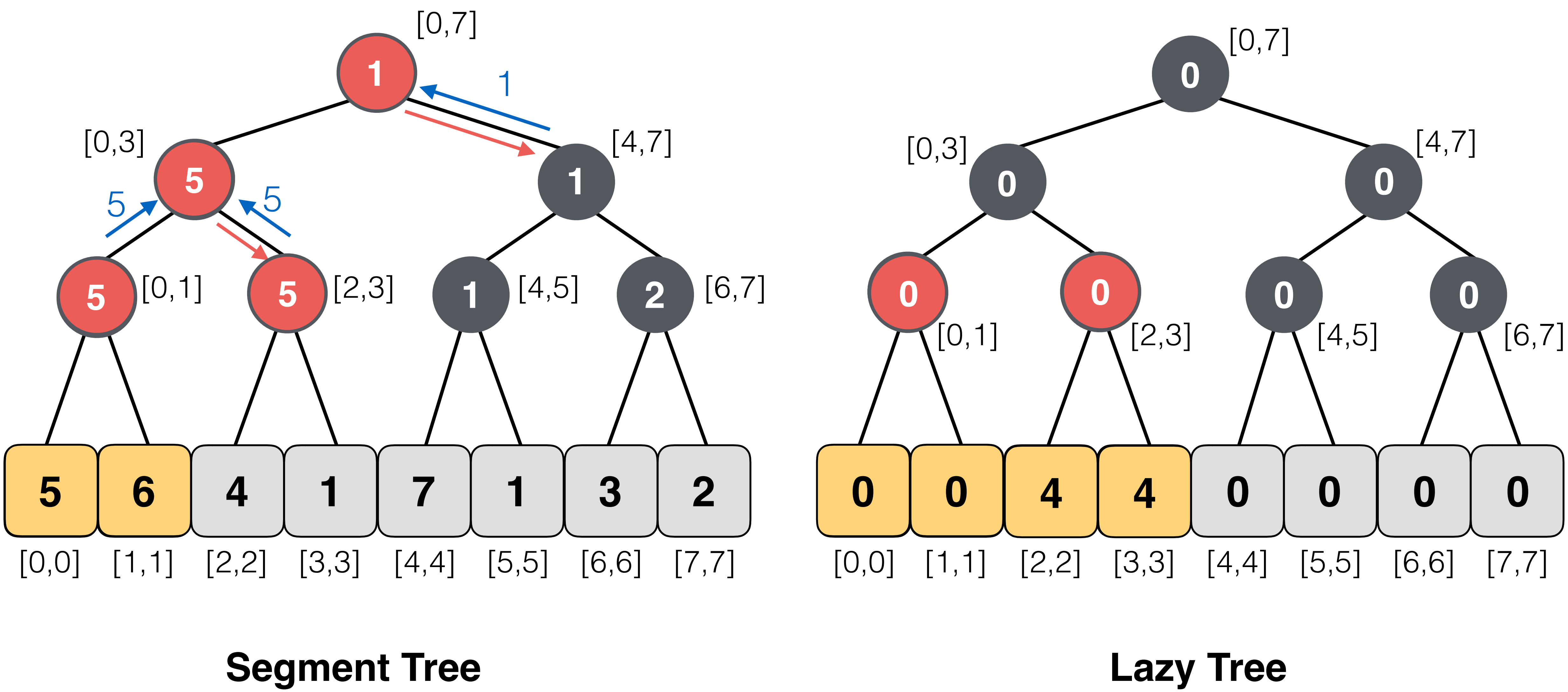

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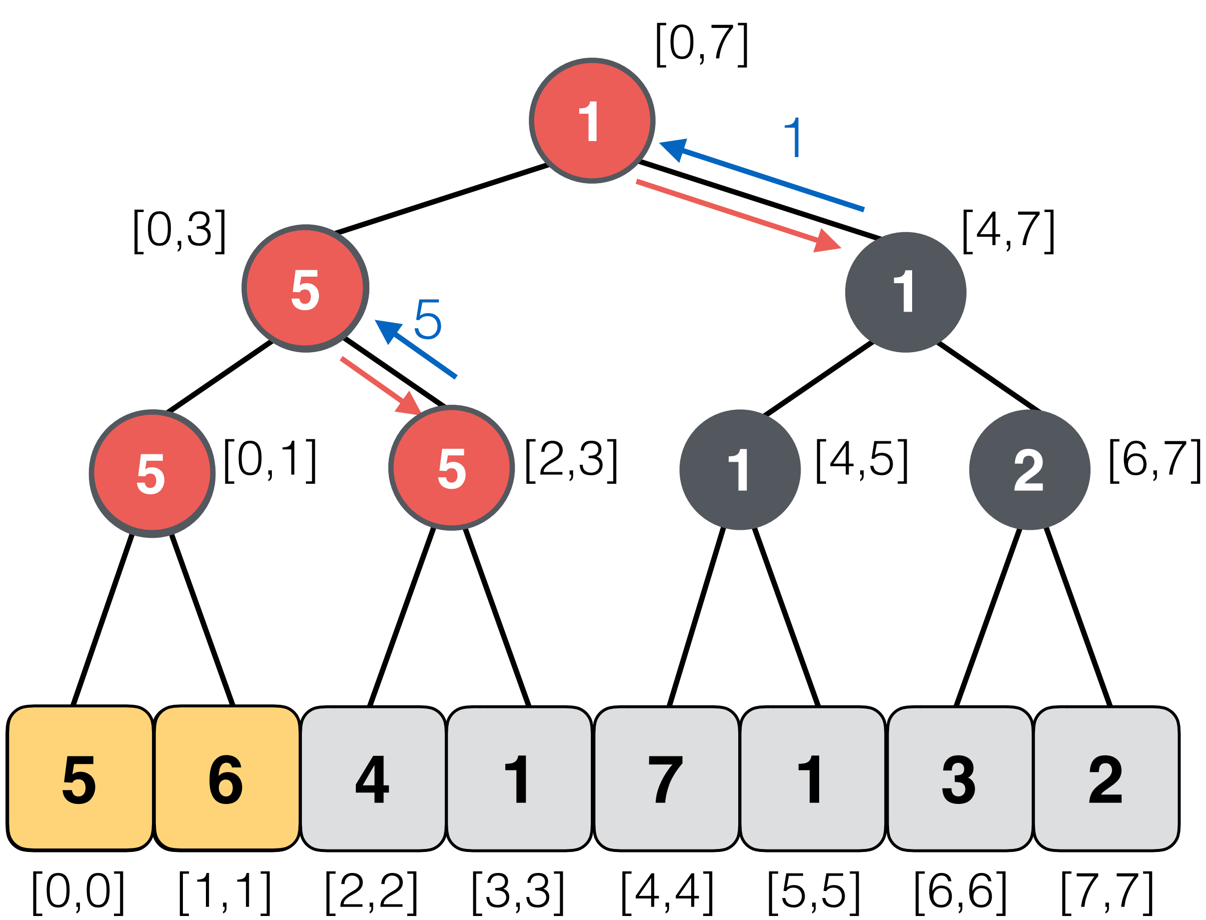

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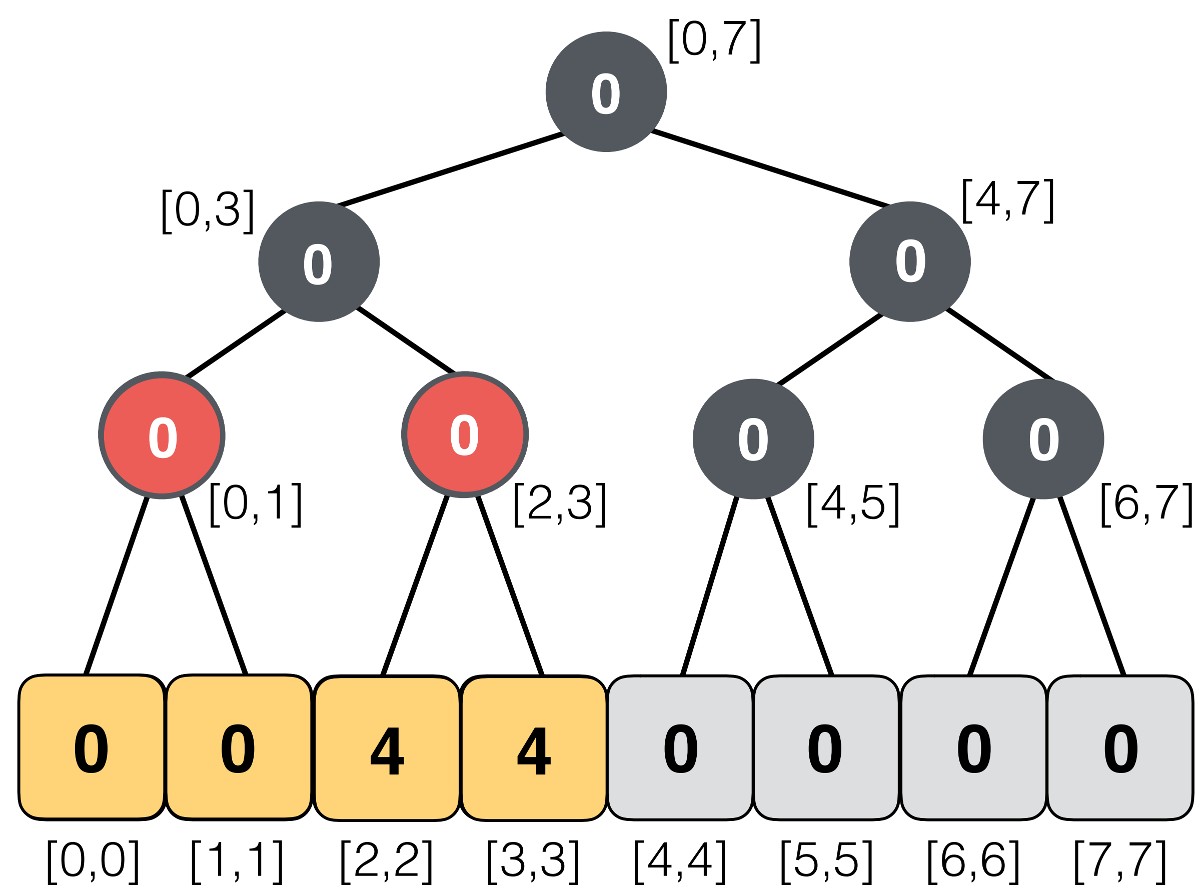

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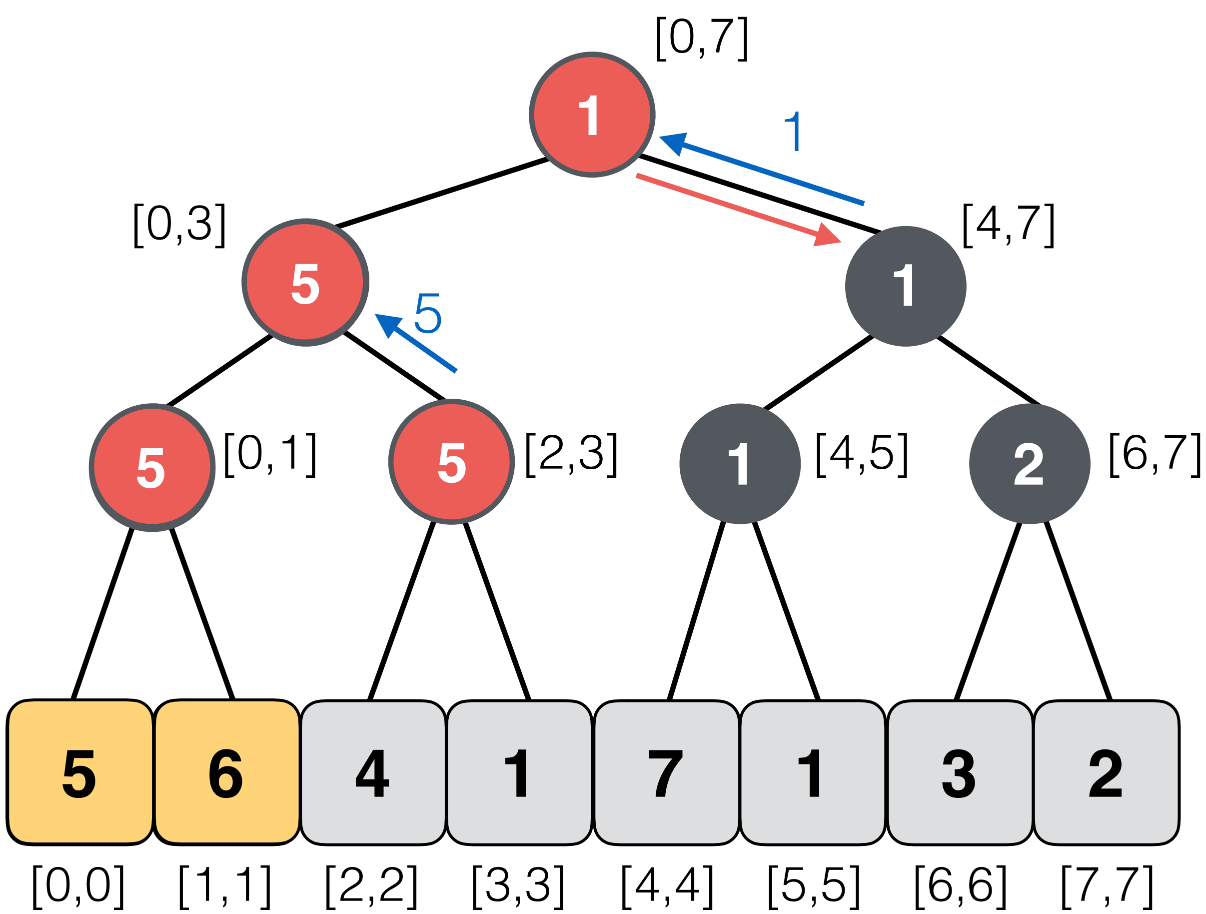

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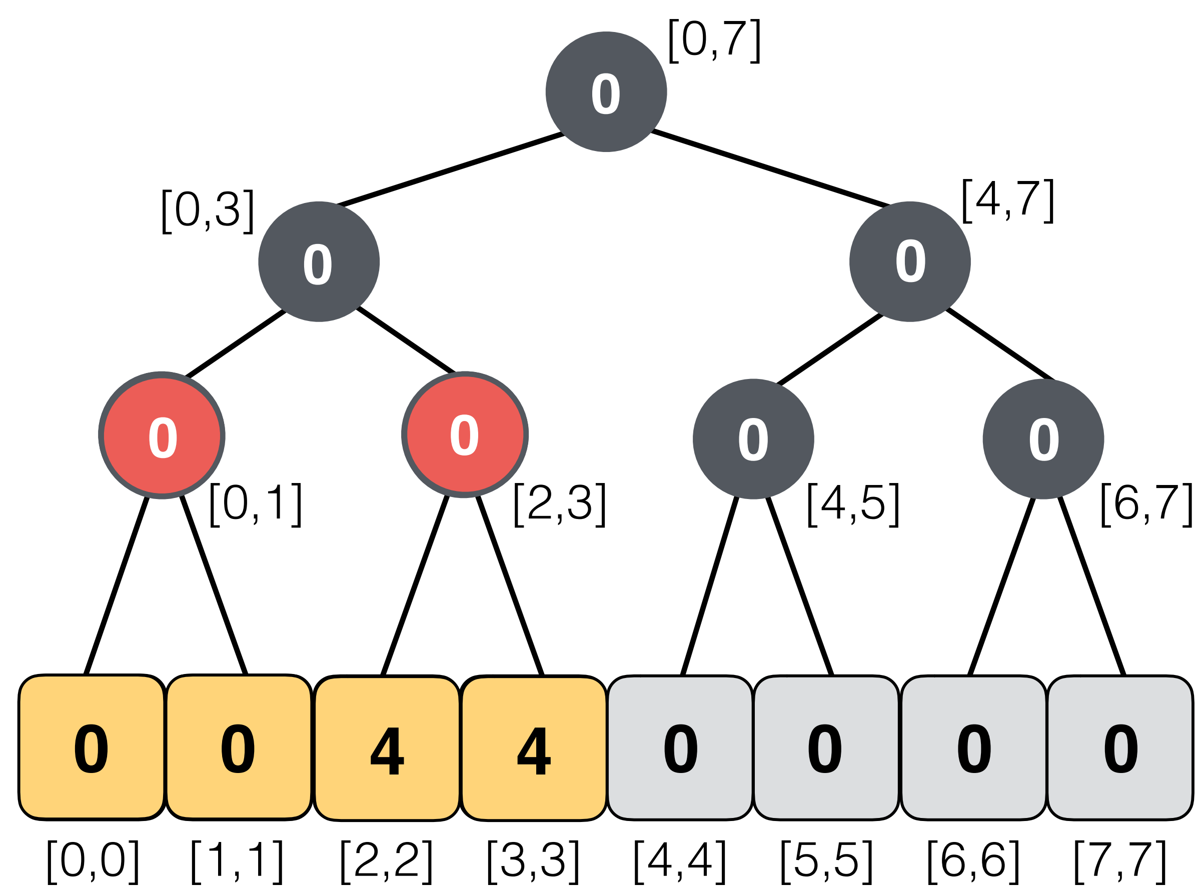

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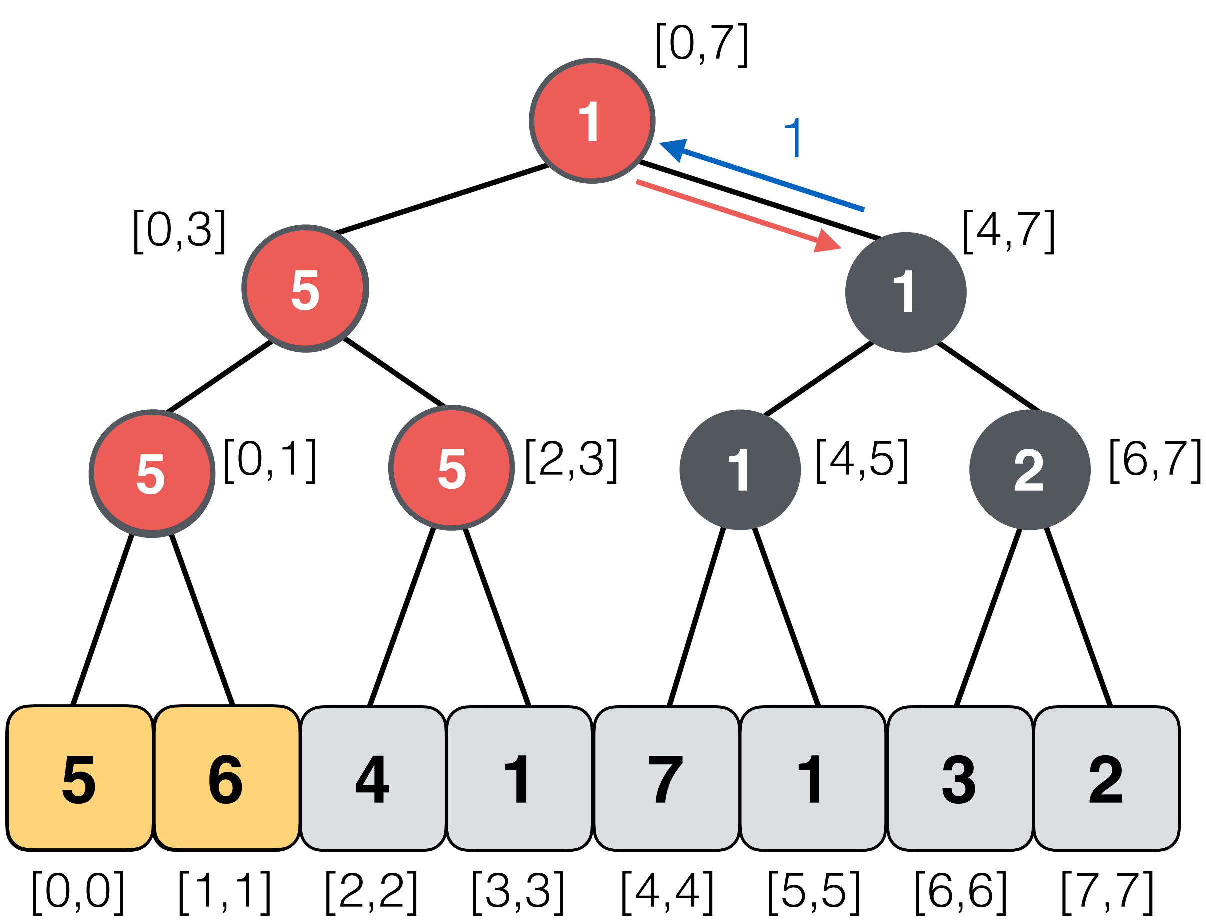

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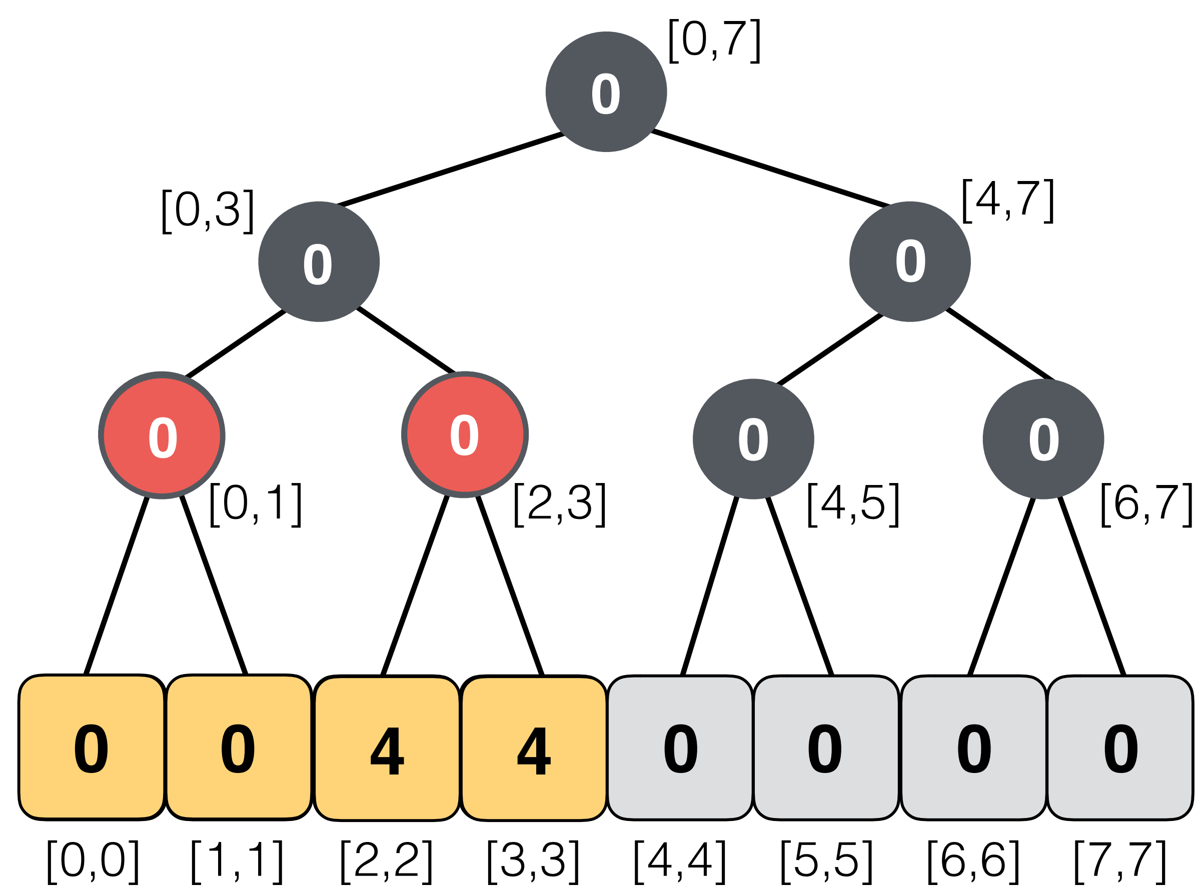

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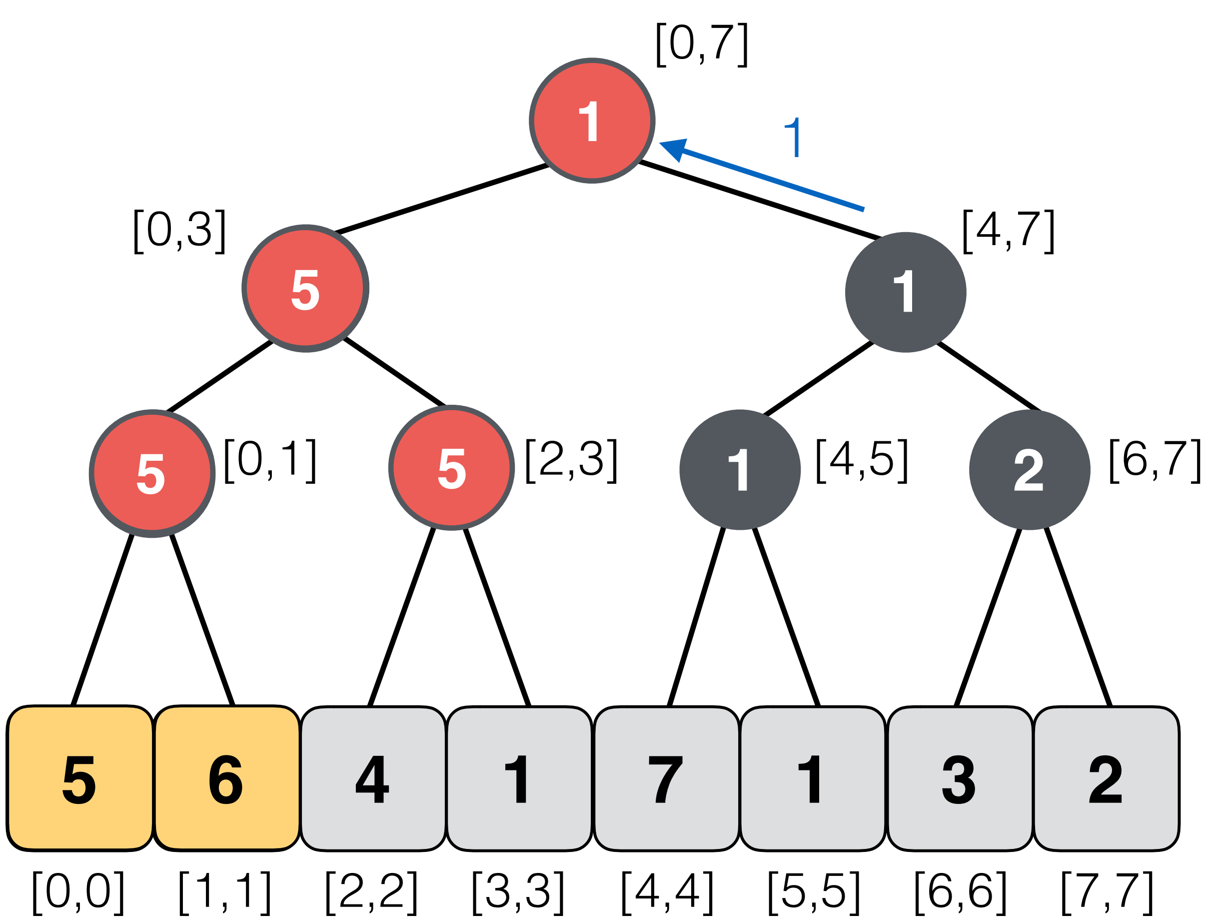

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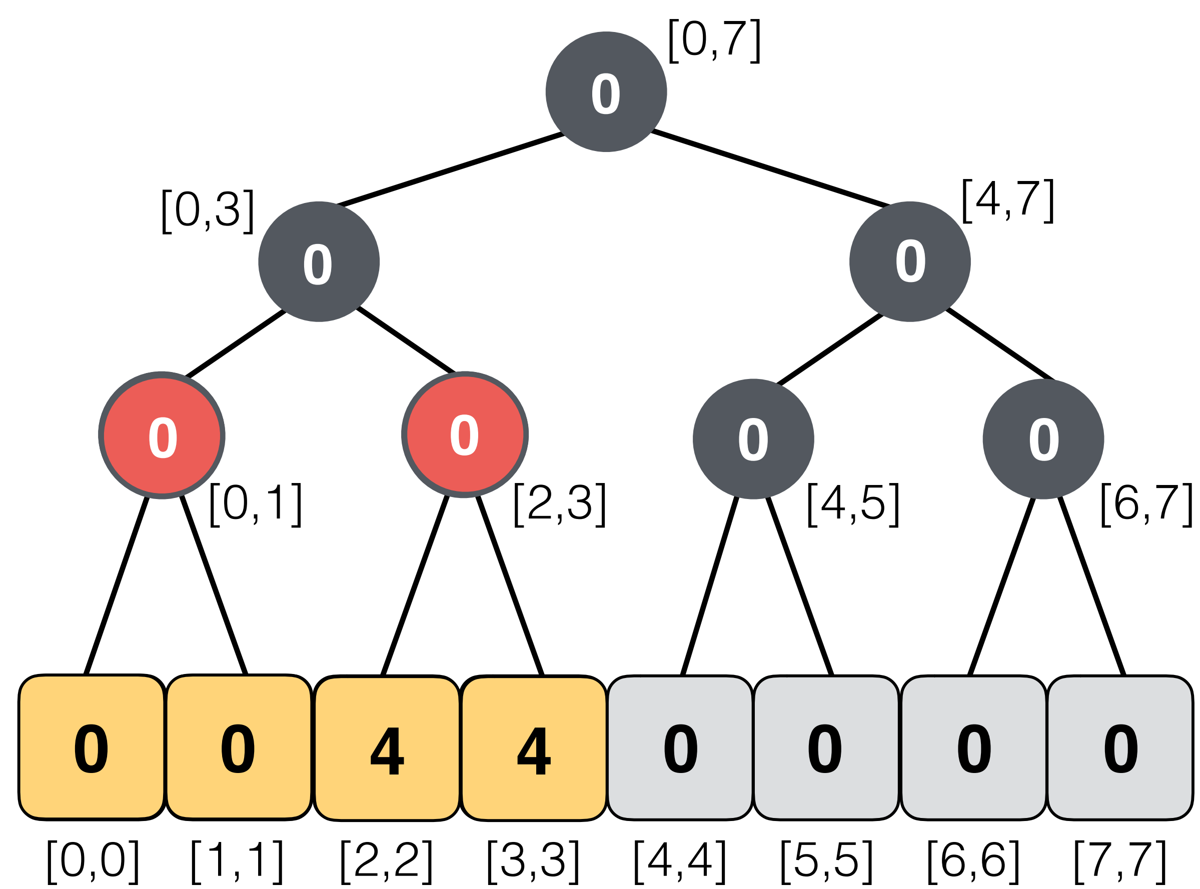

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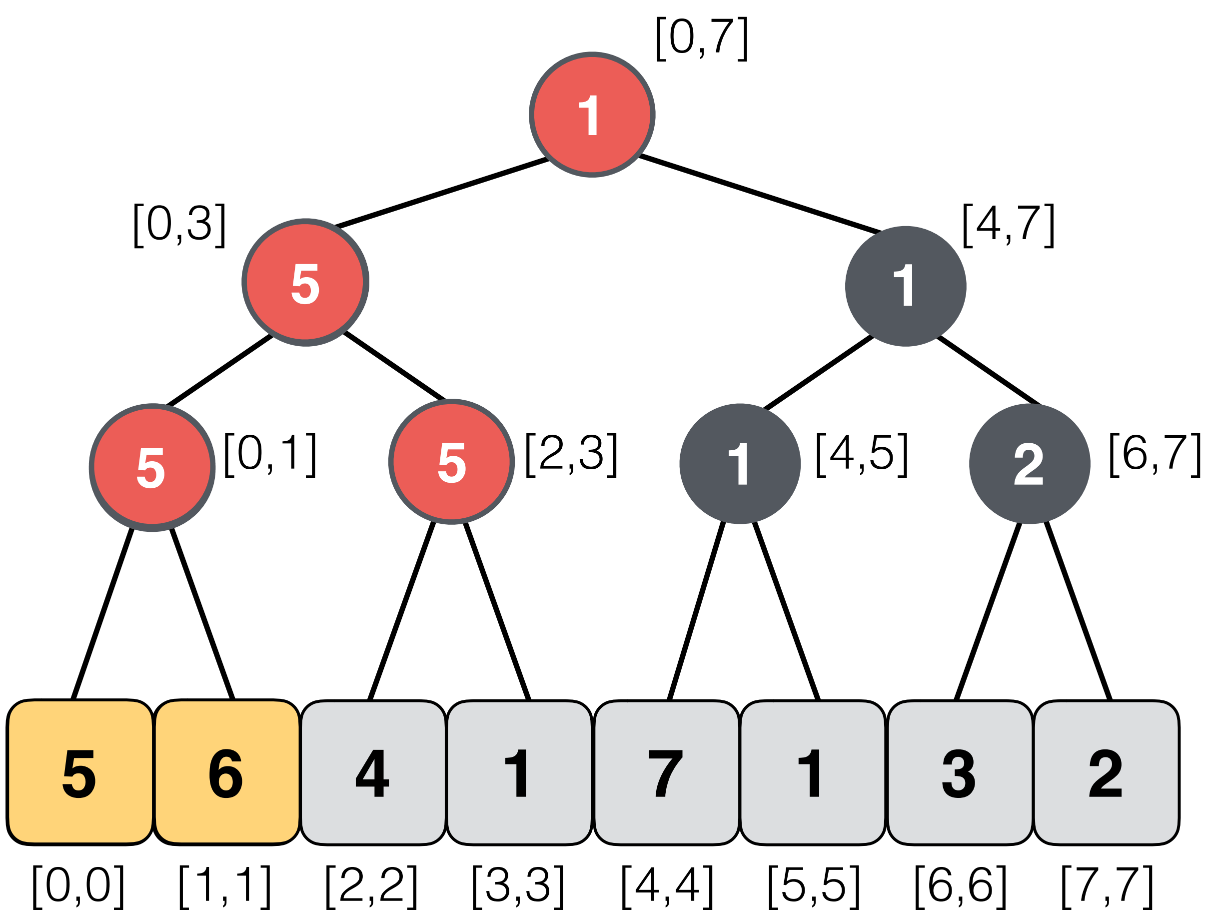

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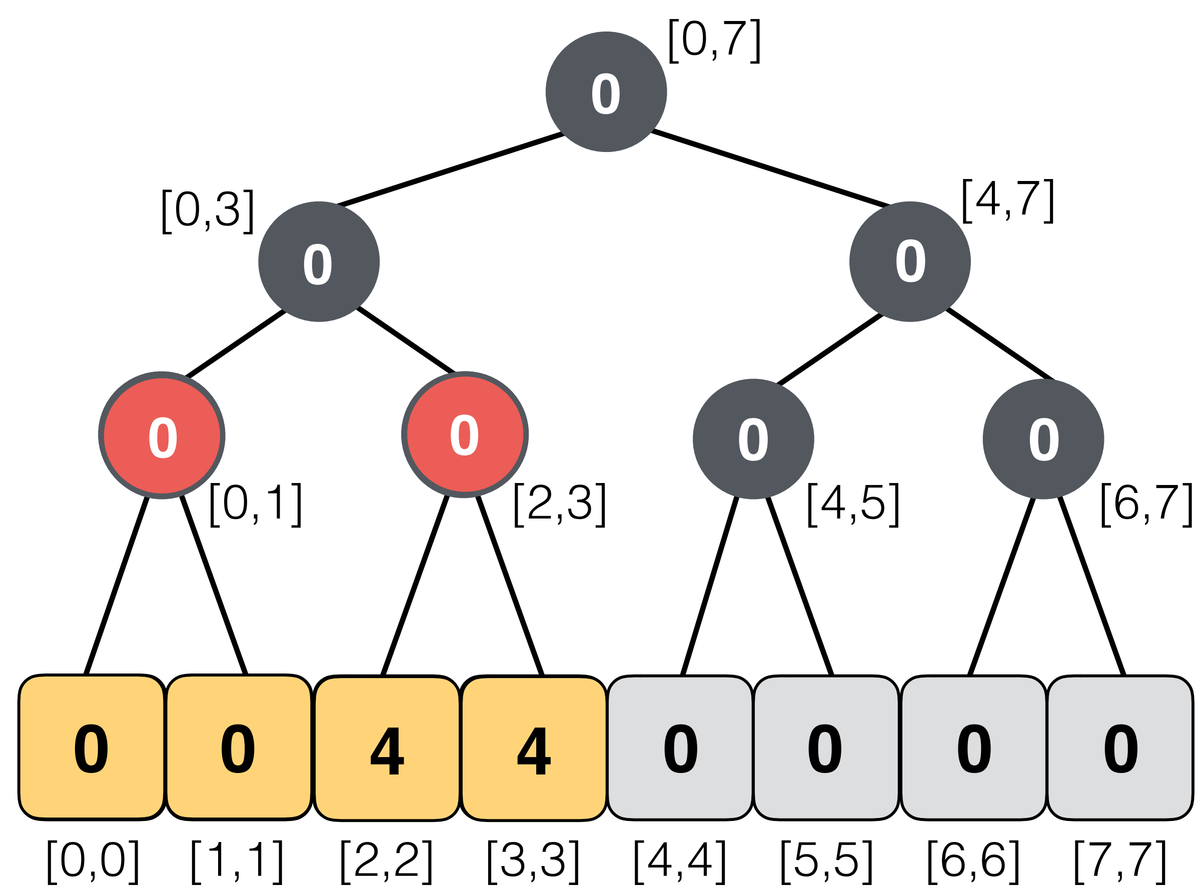

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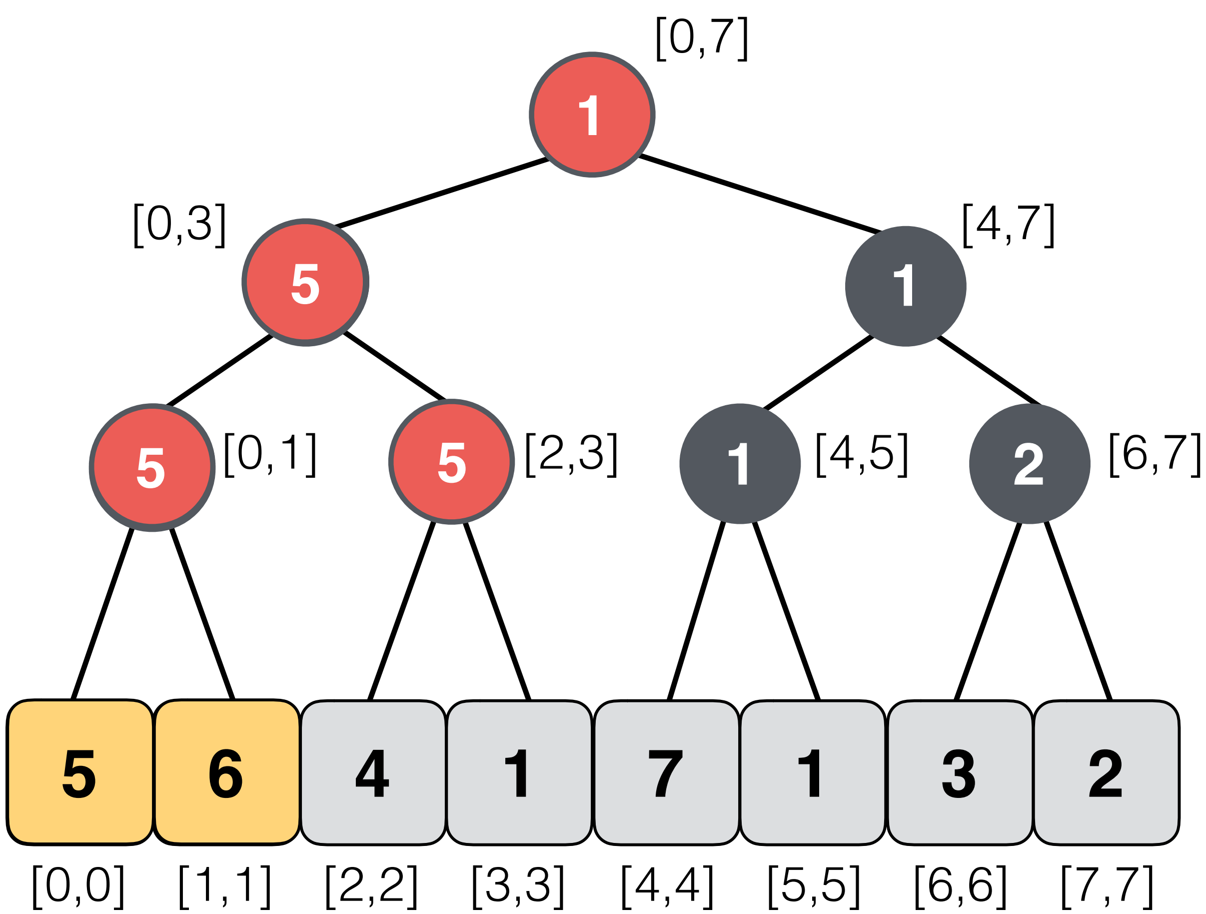

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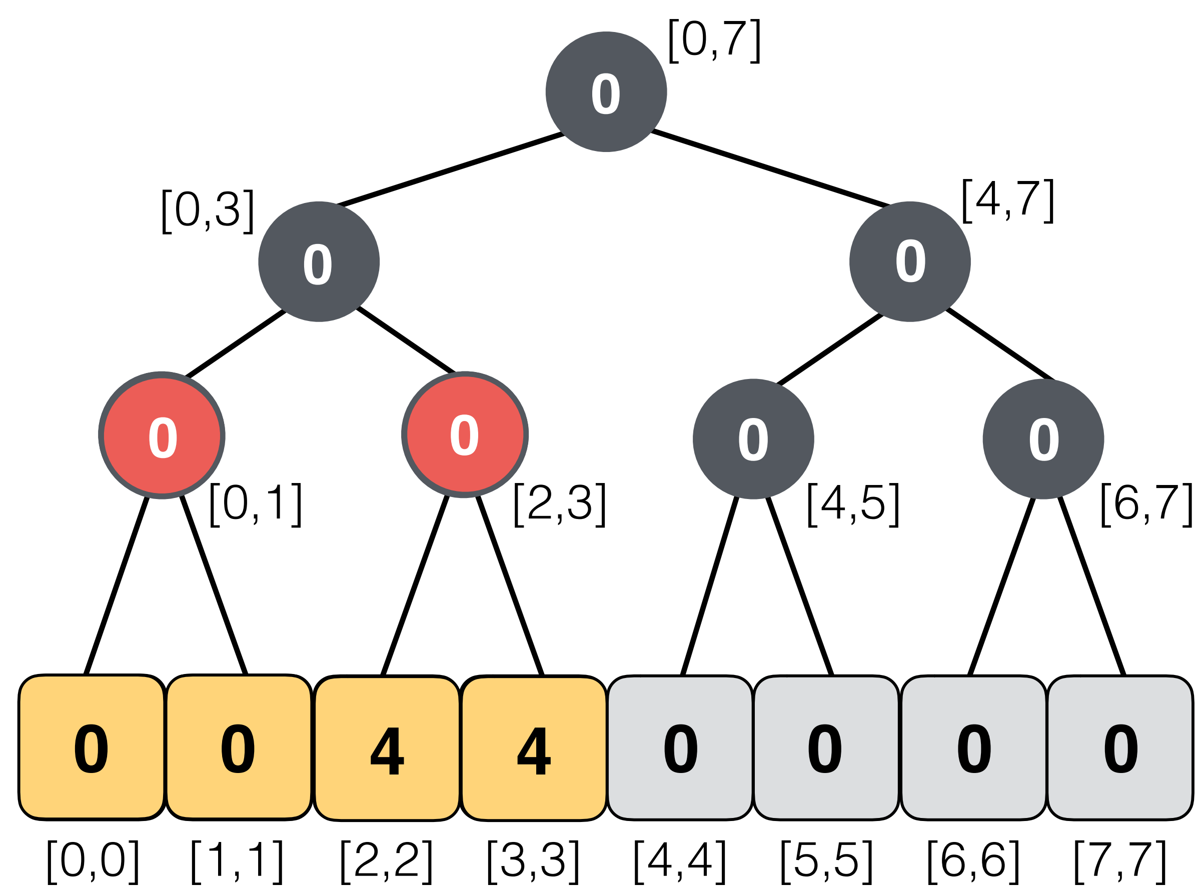

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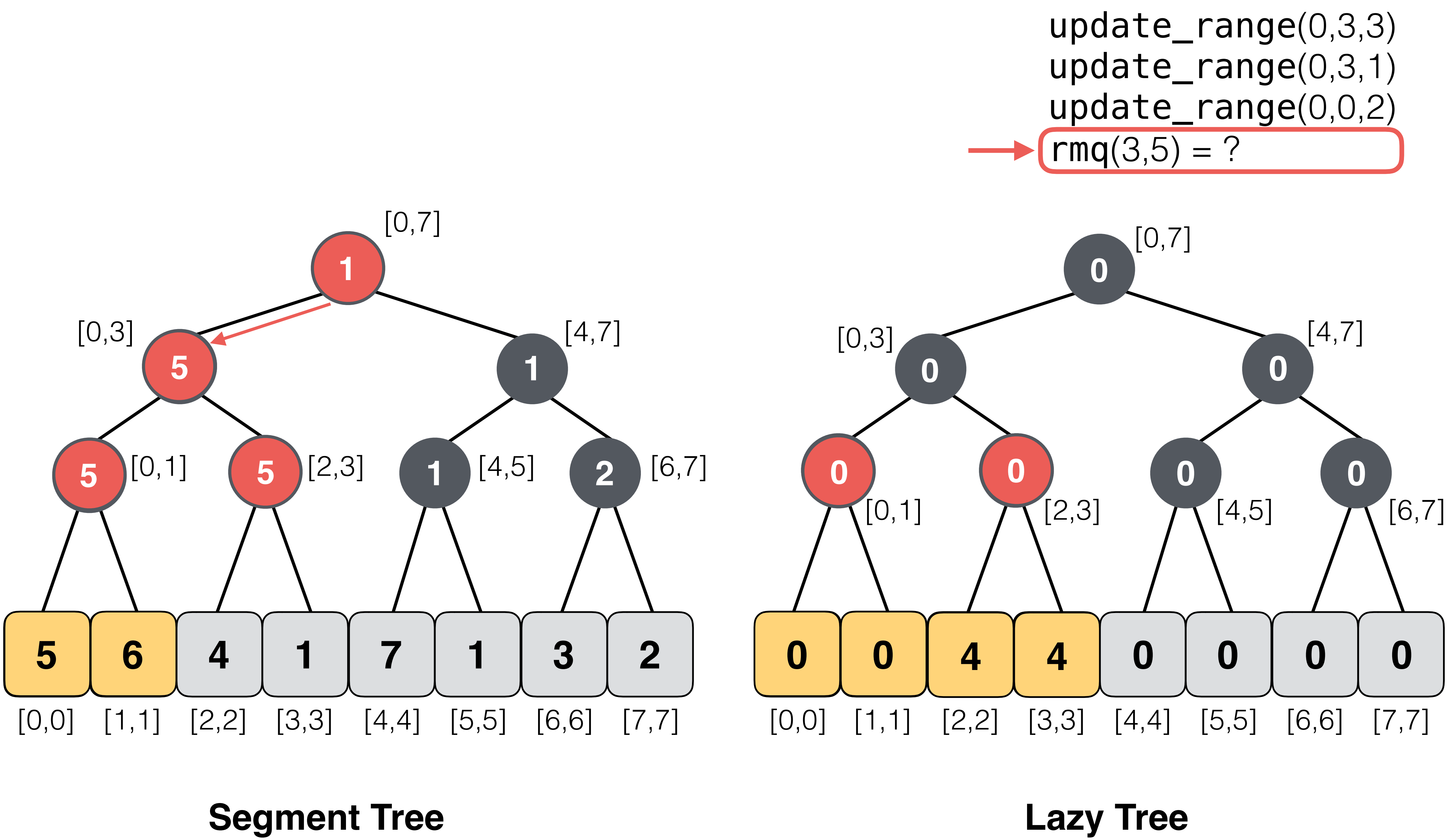

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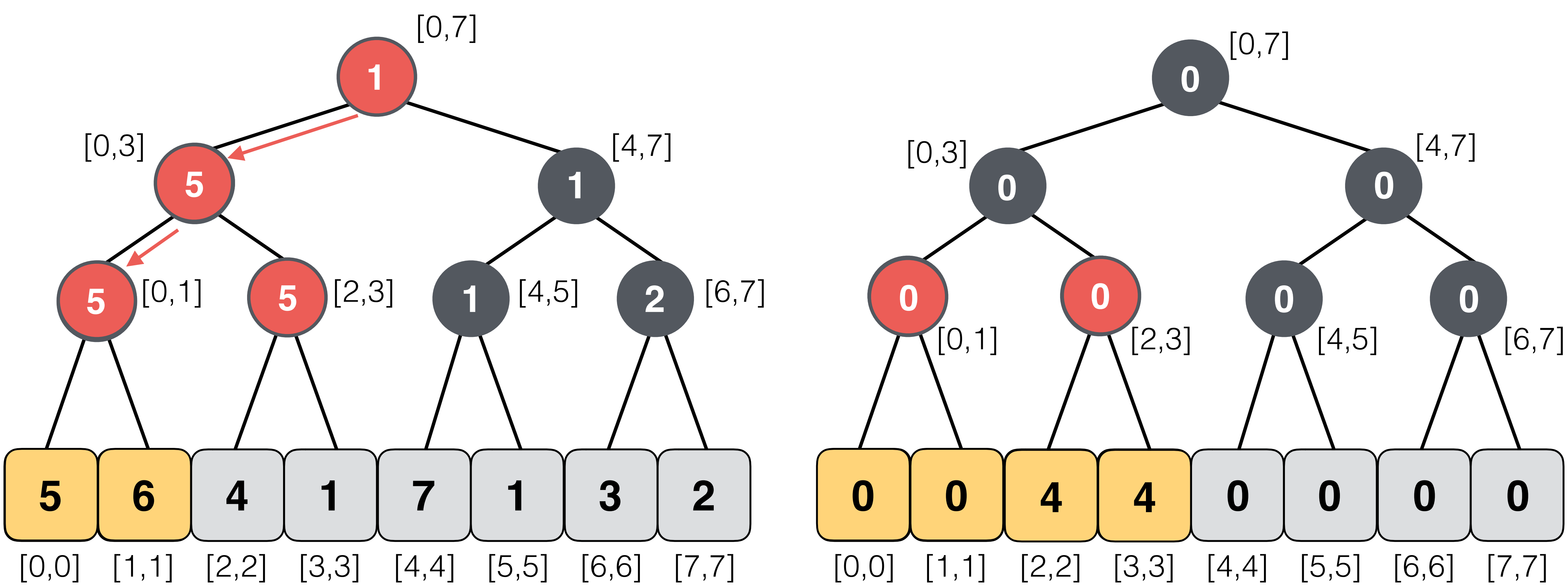

Segment Tree **Lazy Tree**

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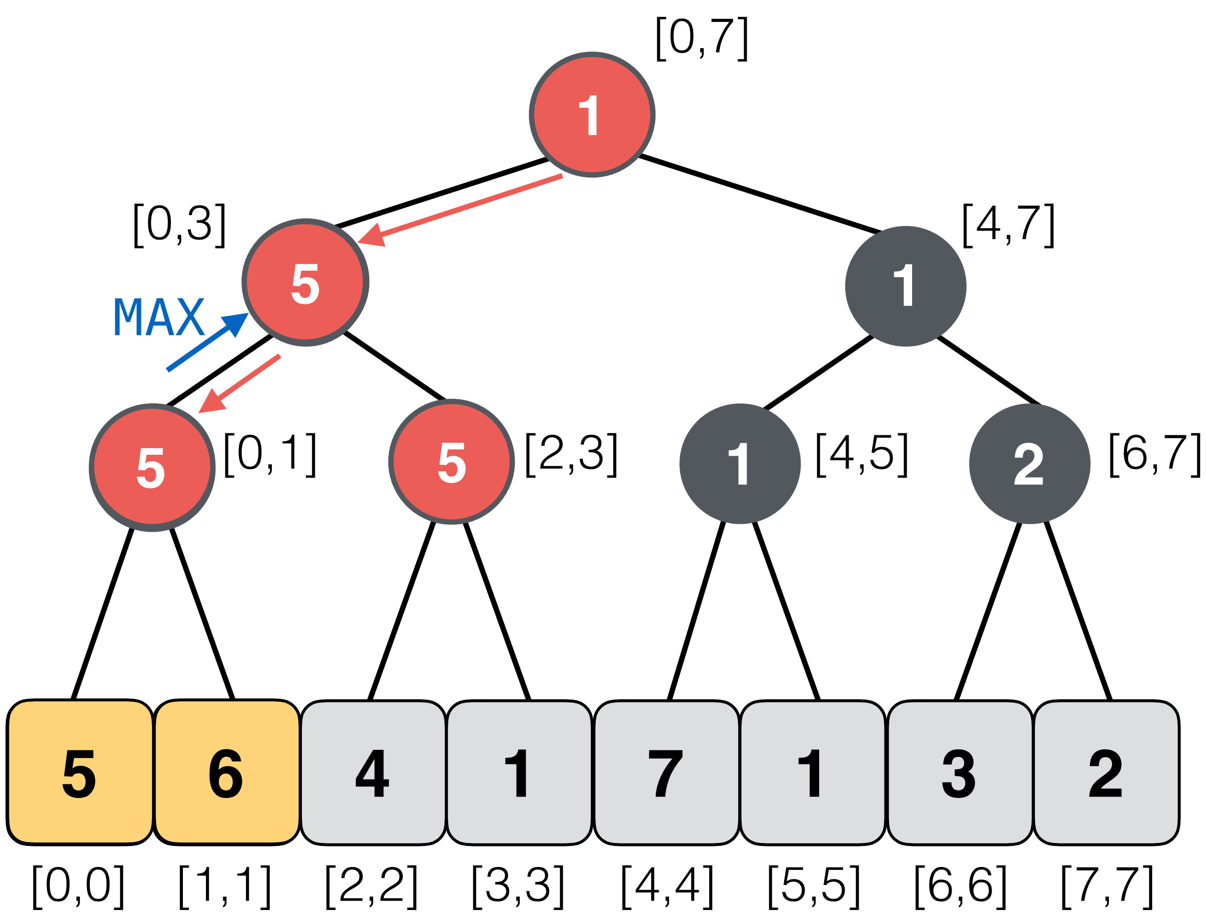

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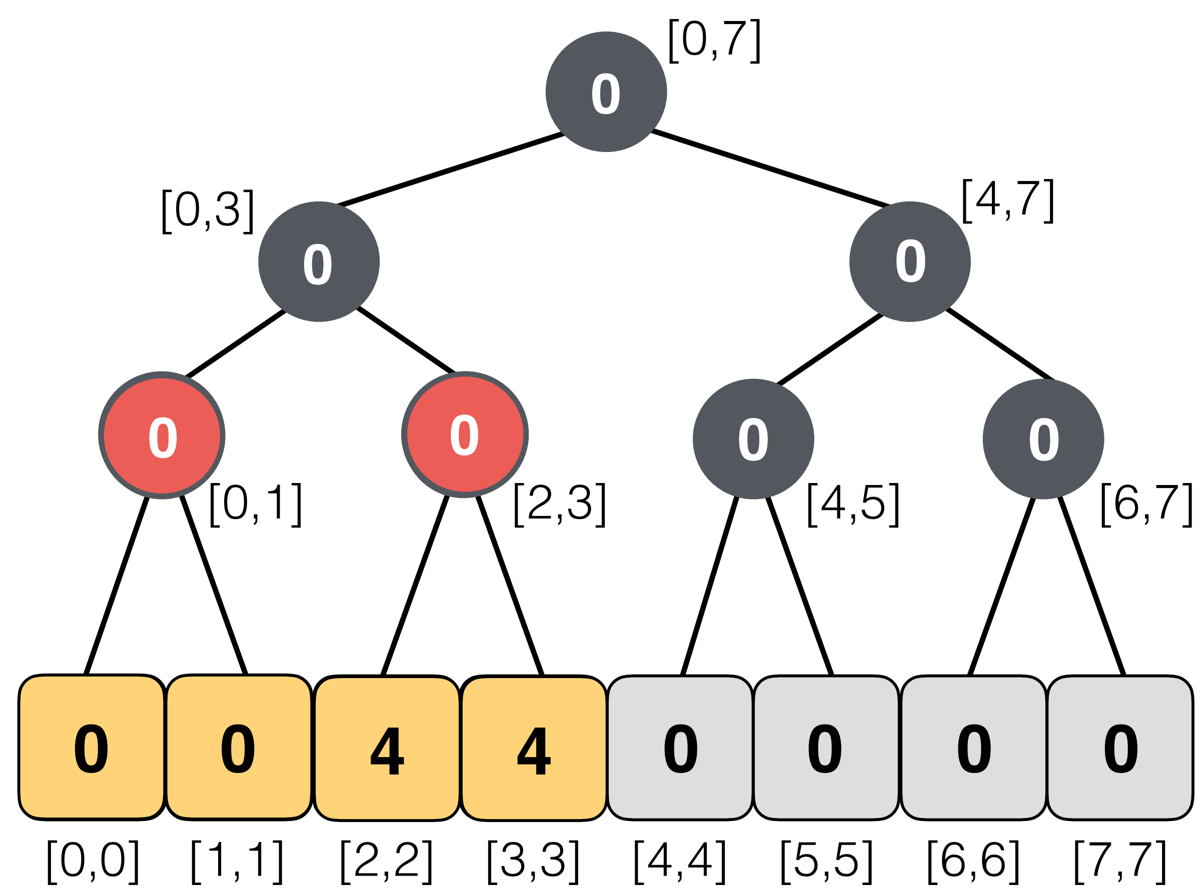

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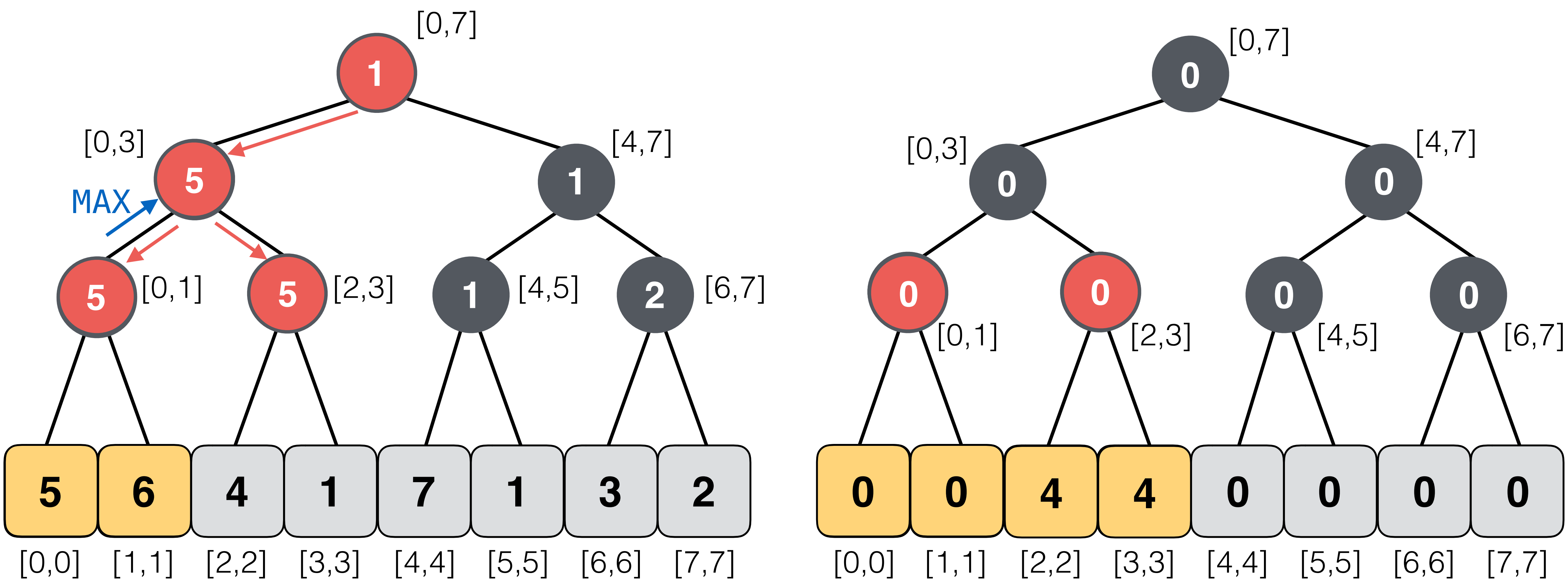

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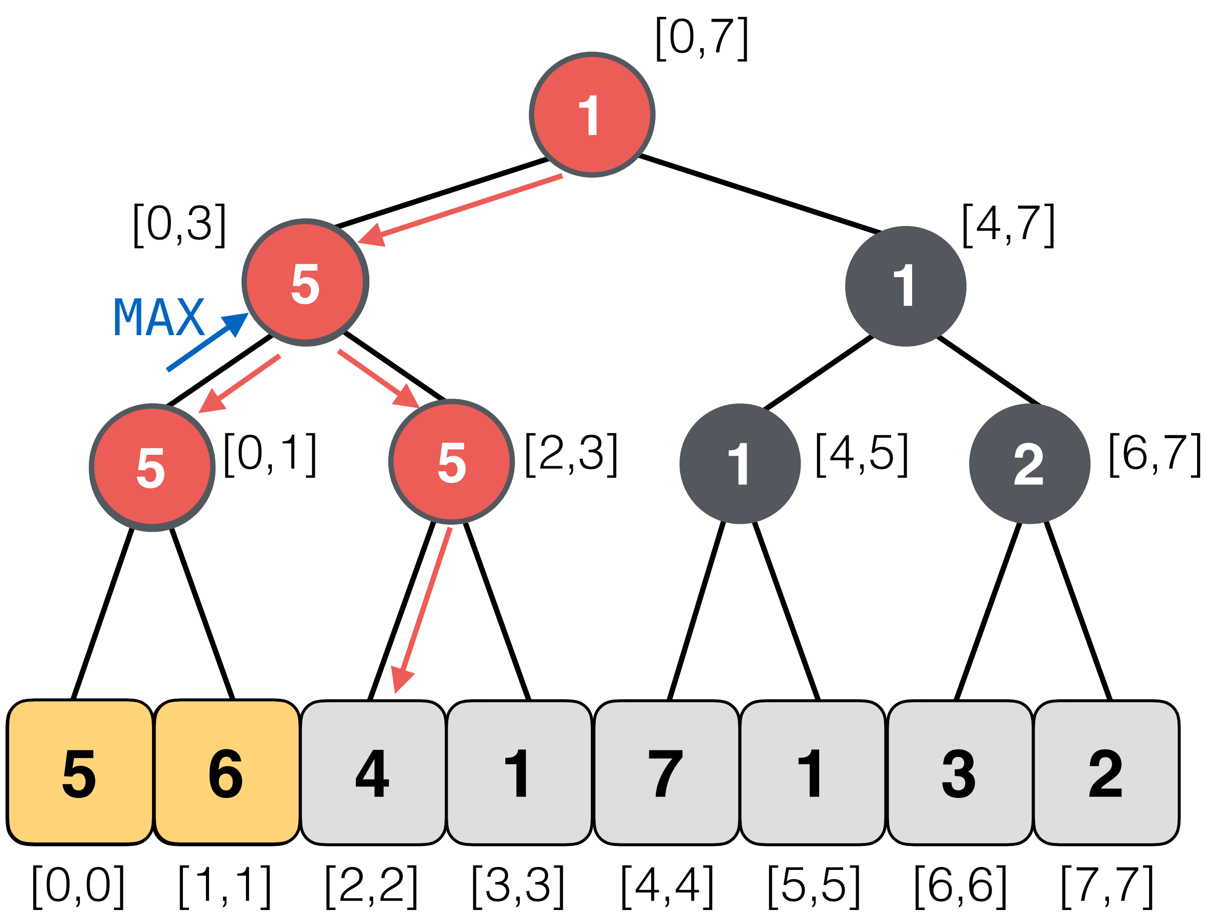

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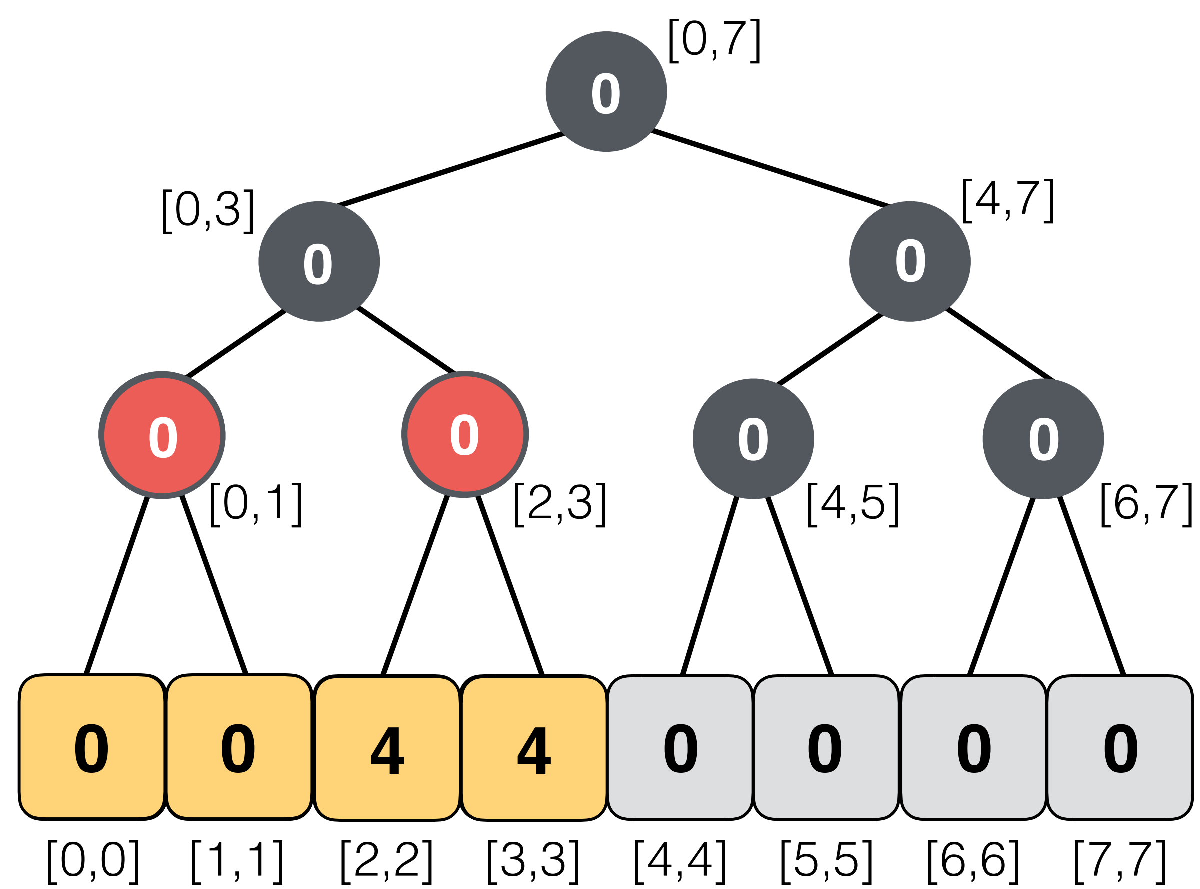

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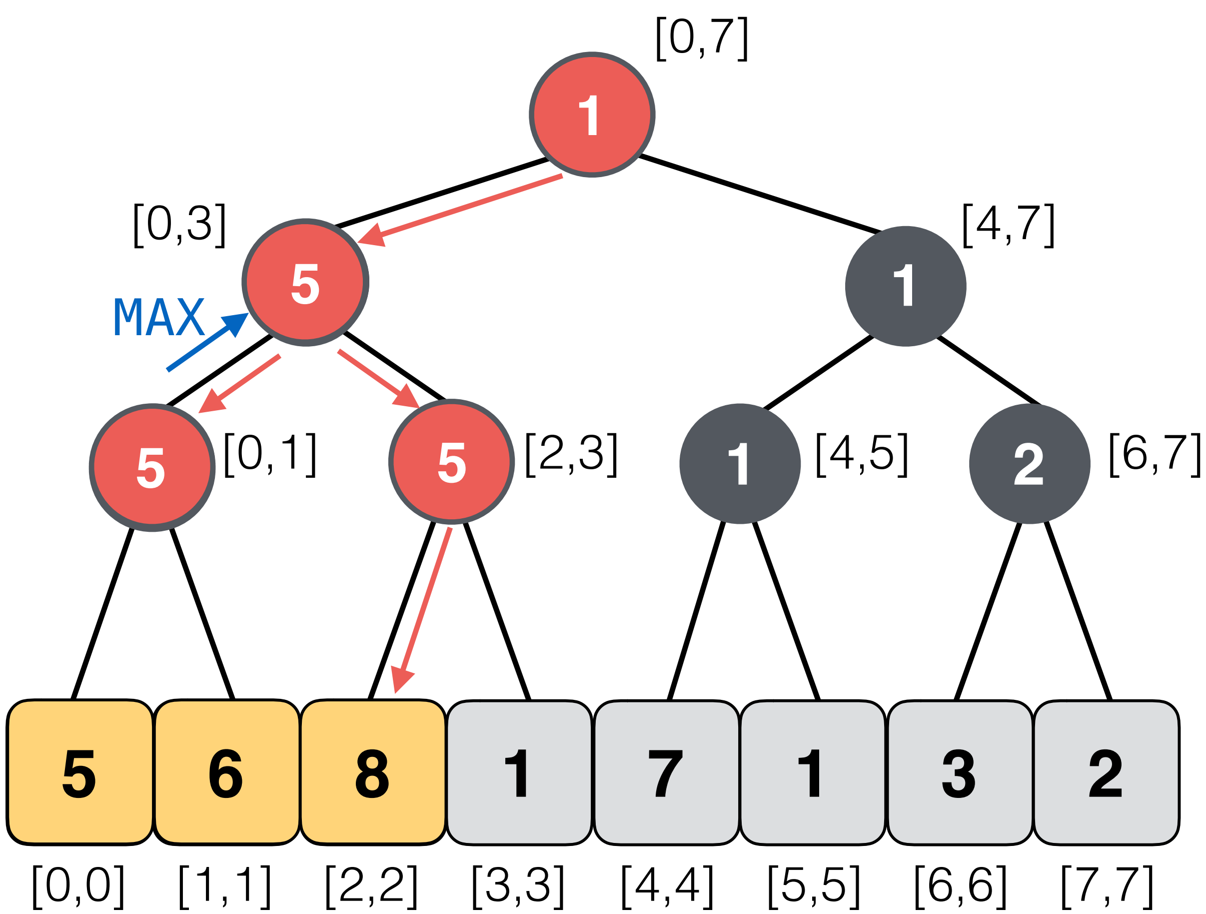

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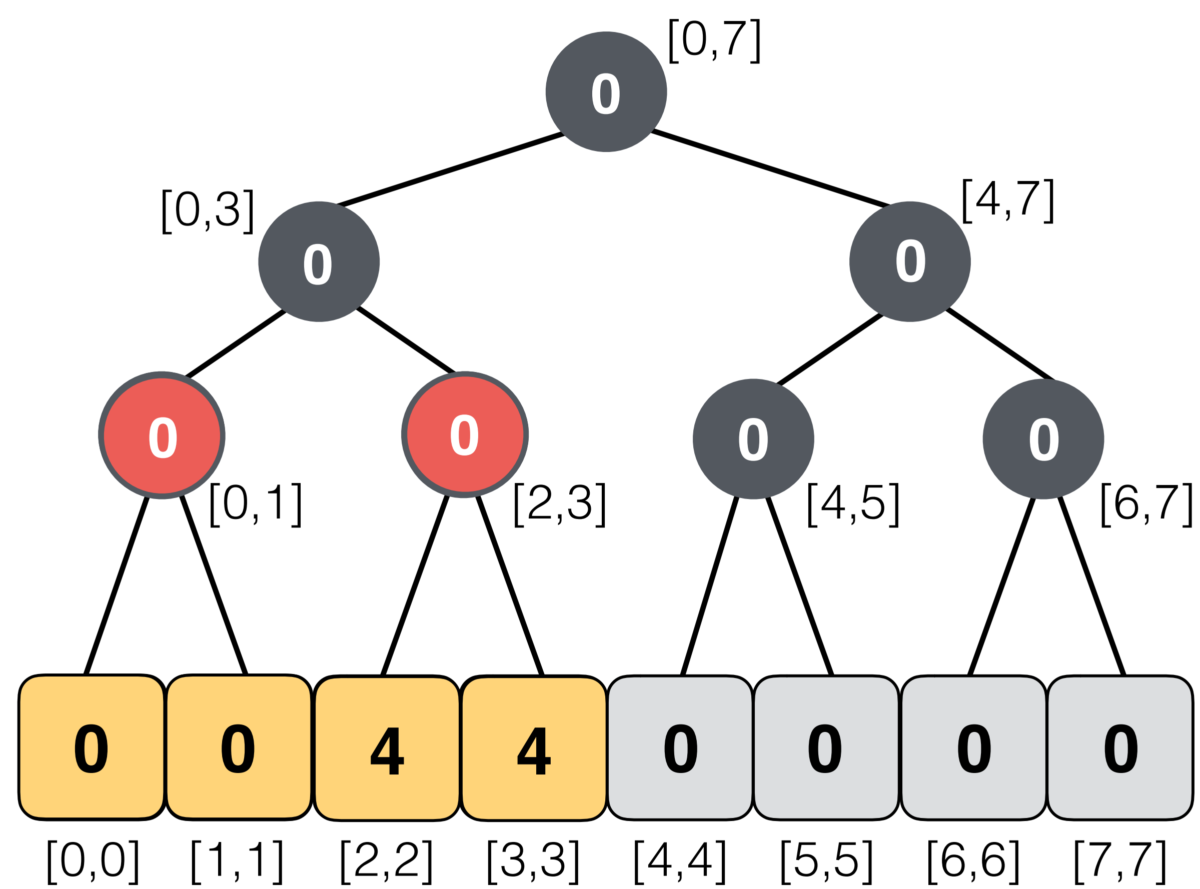

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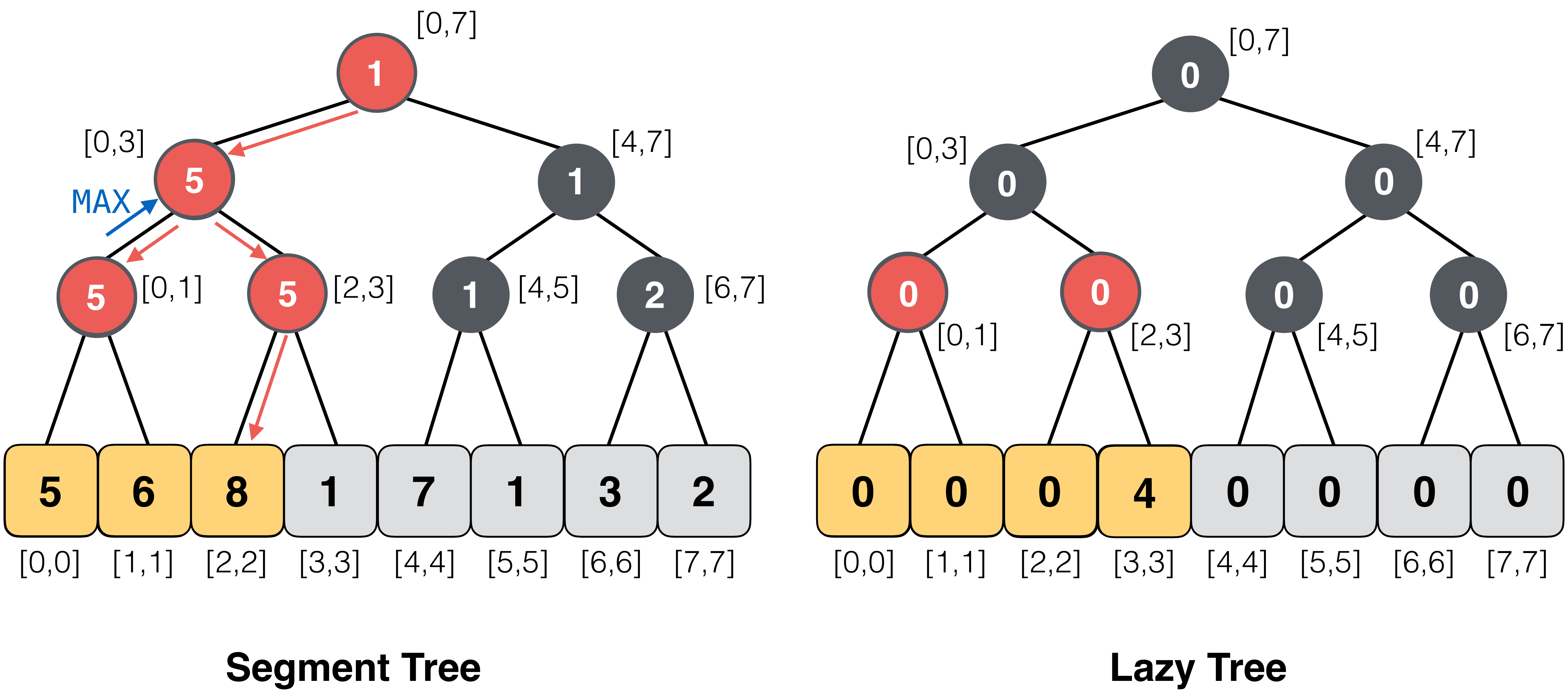

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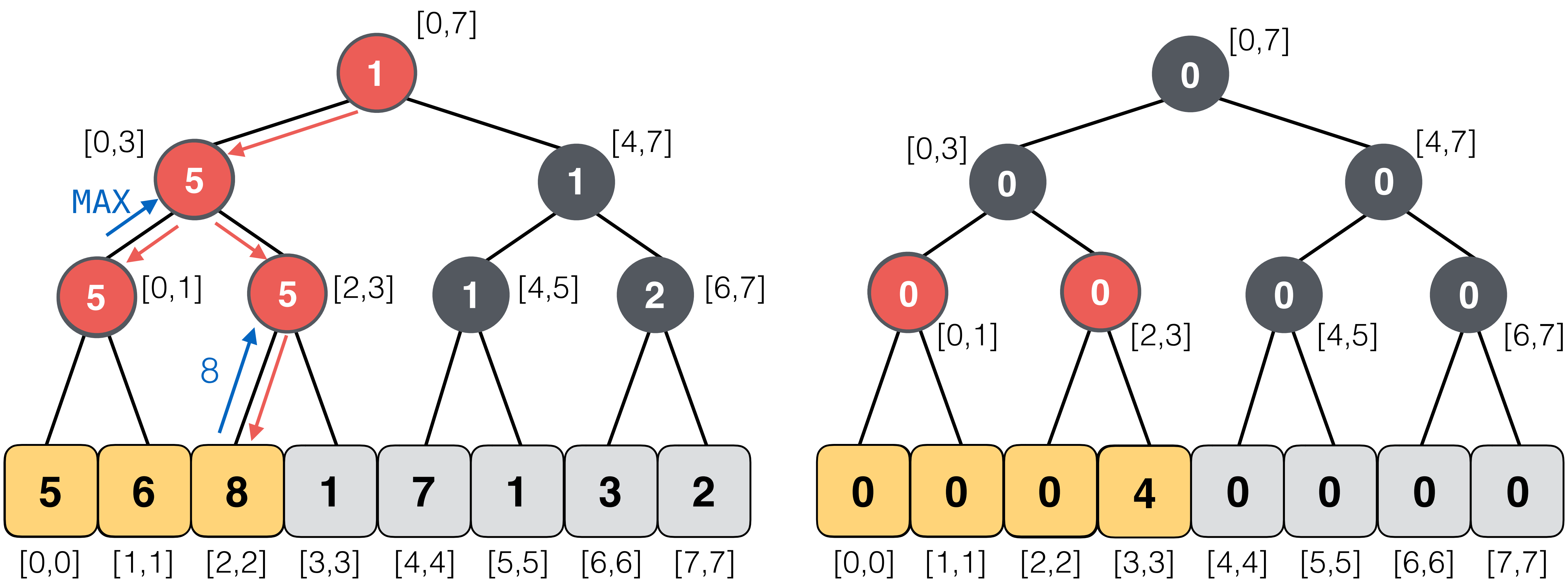

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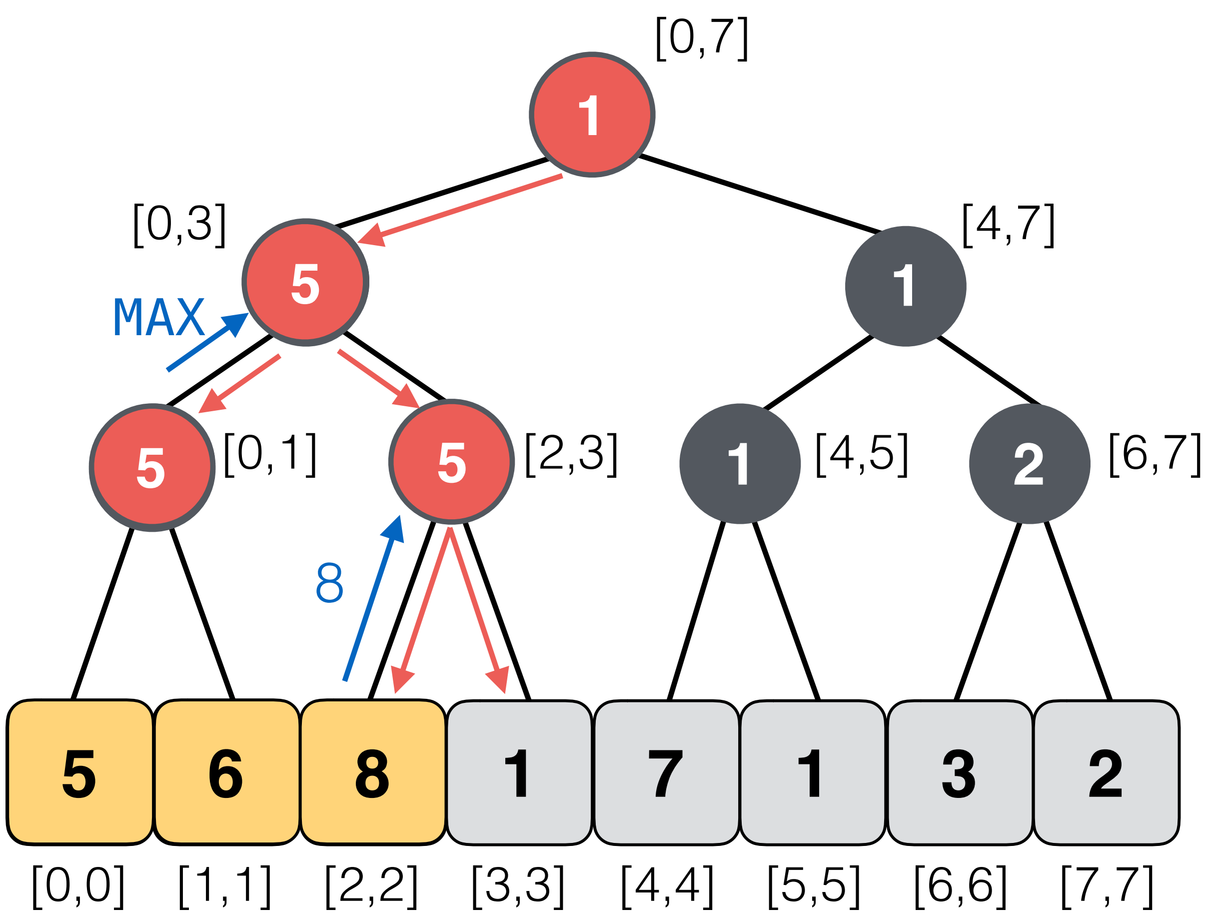

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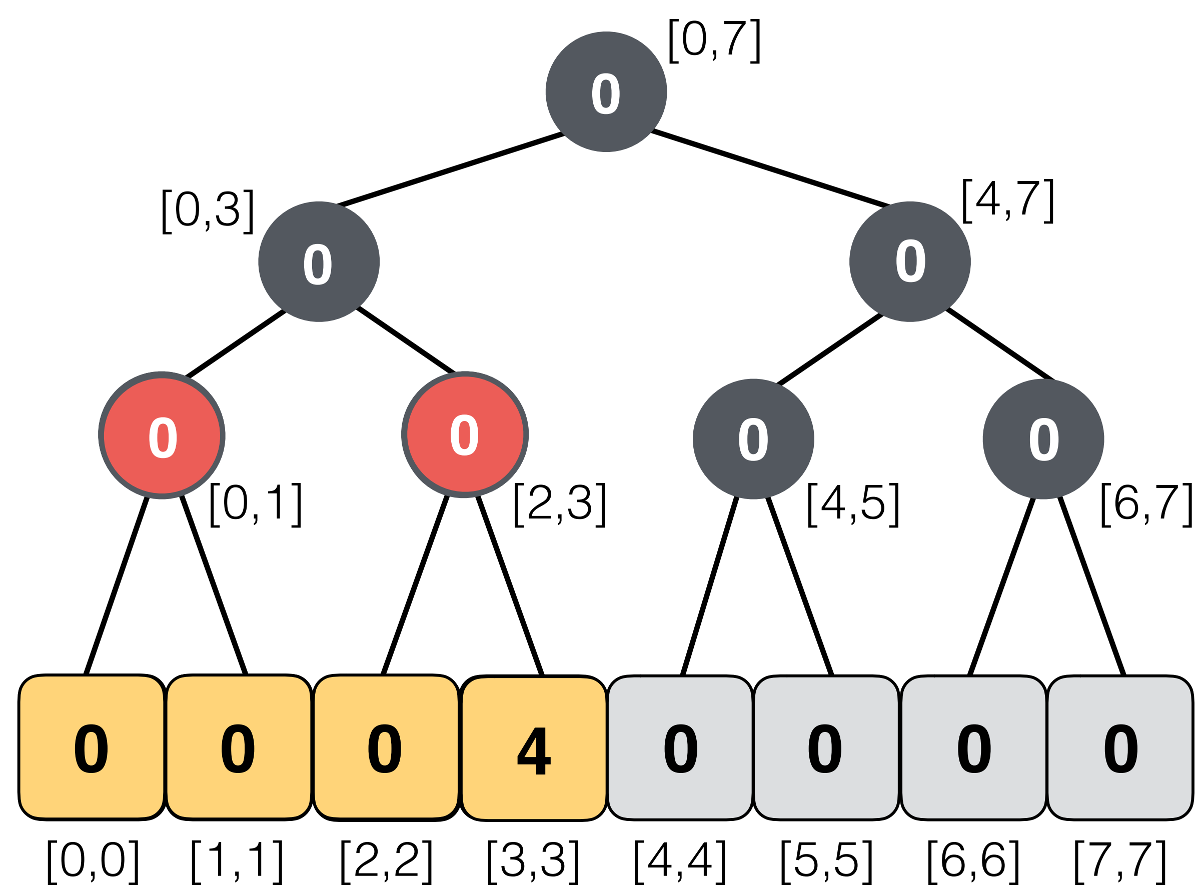

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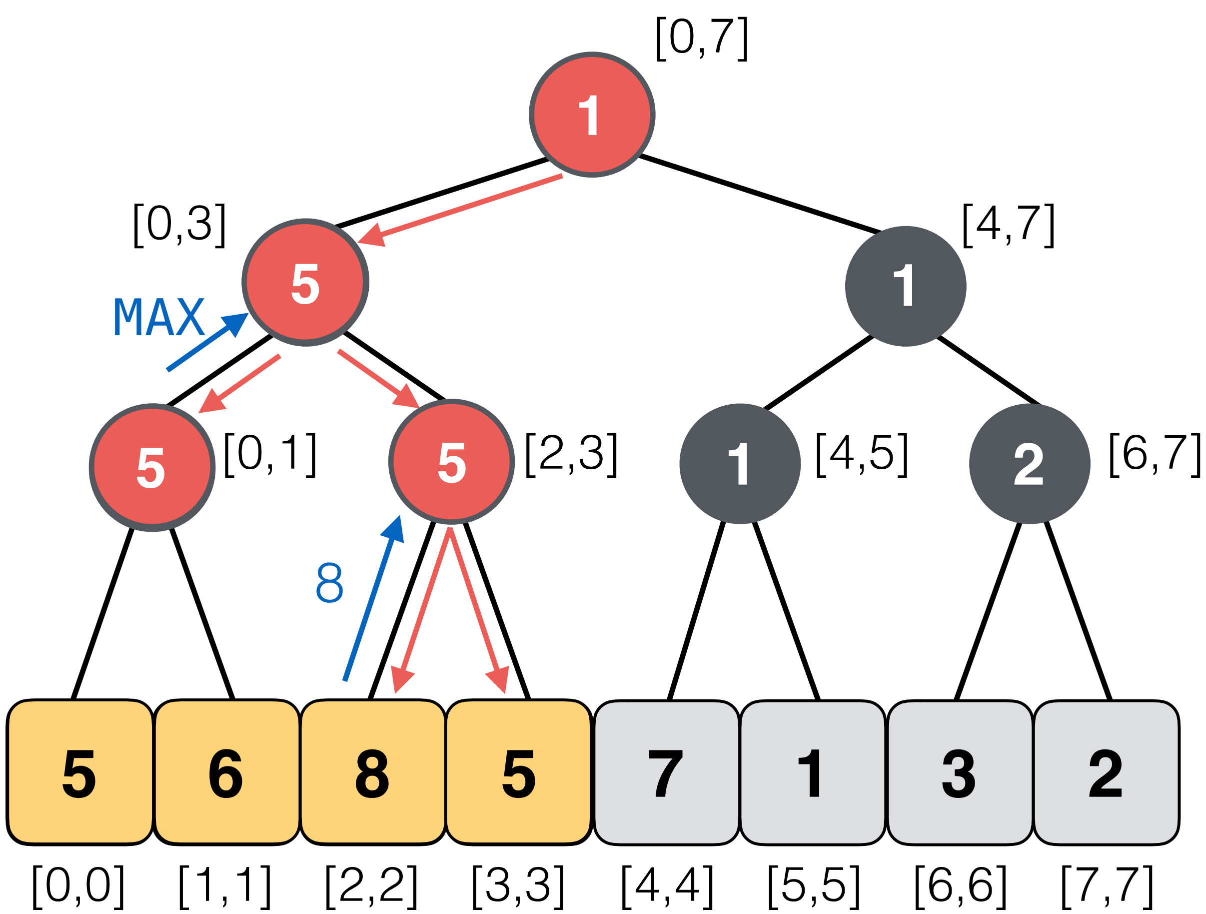

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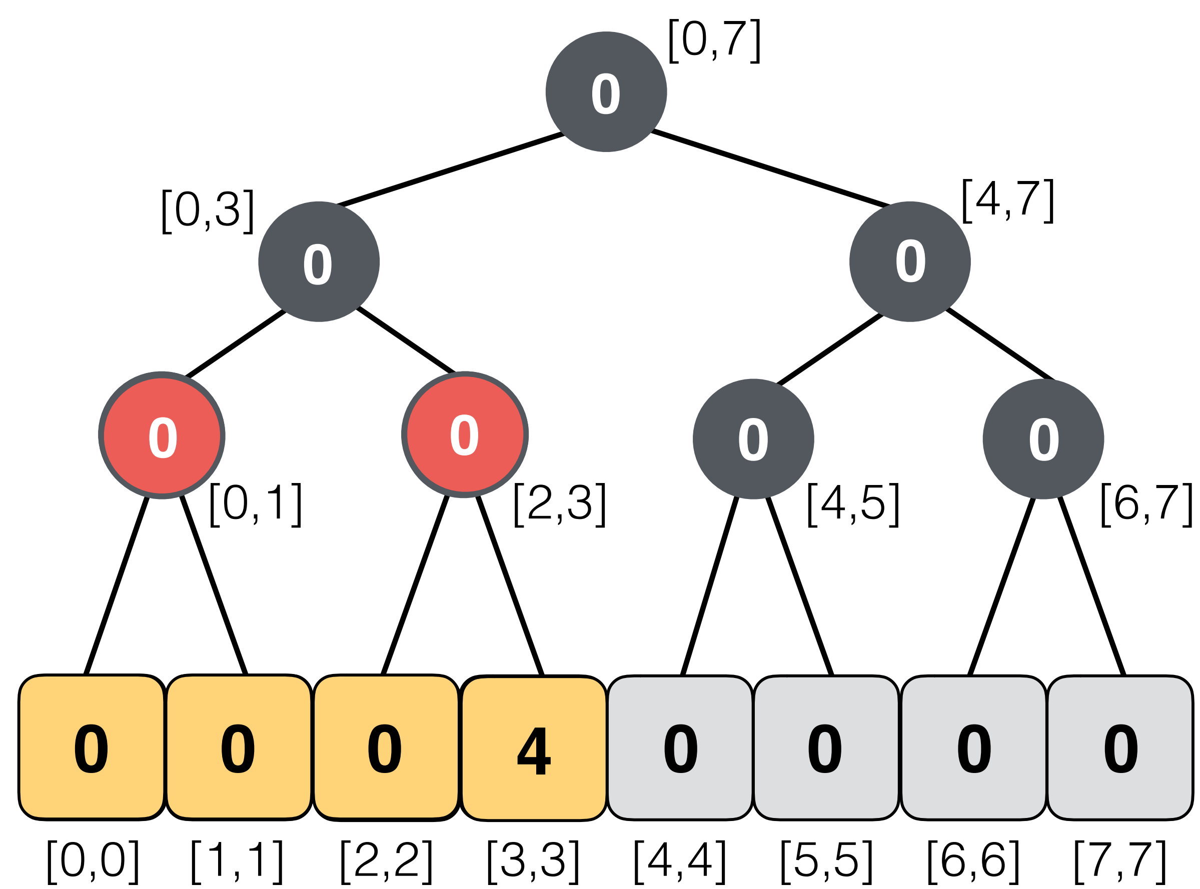

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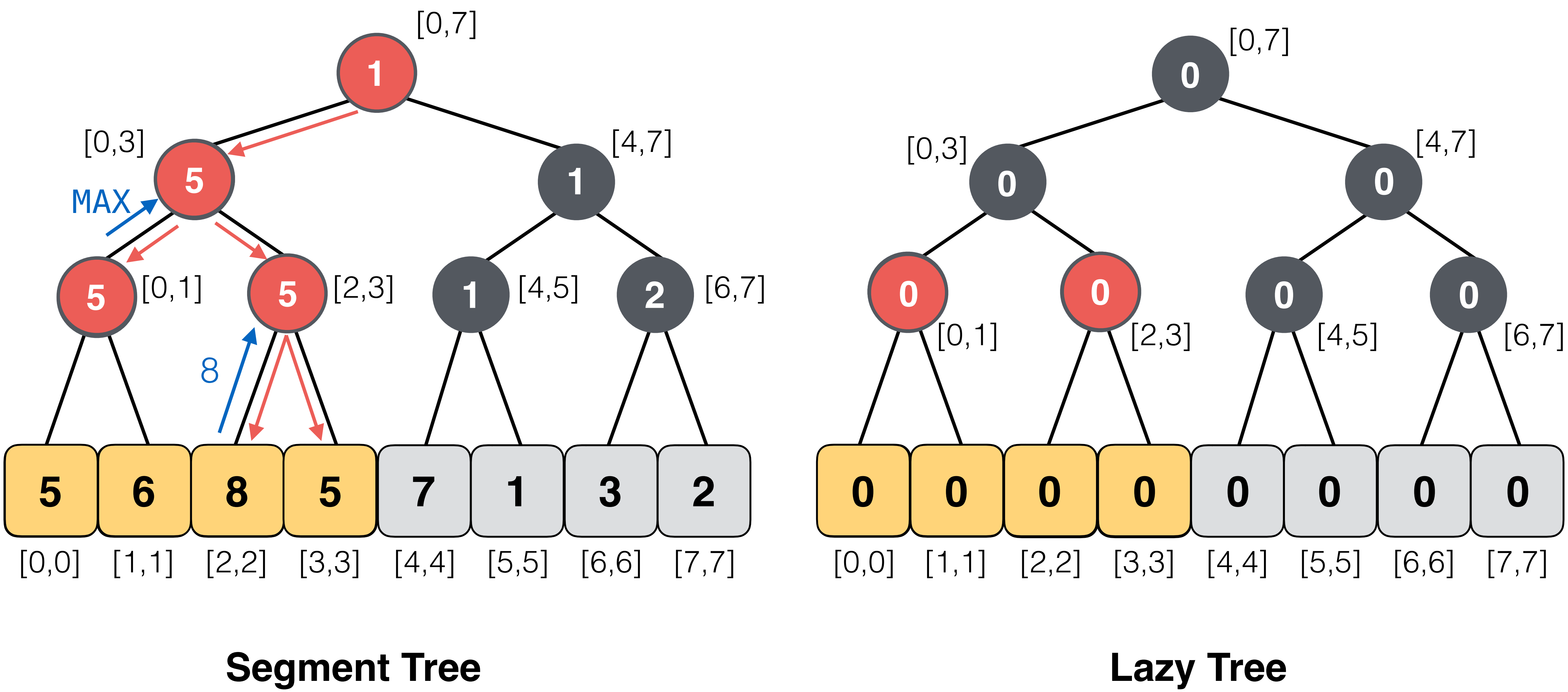

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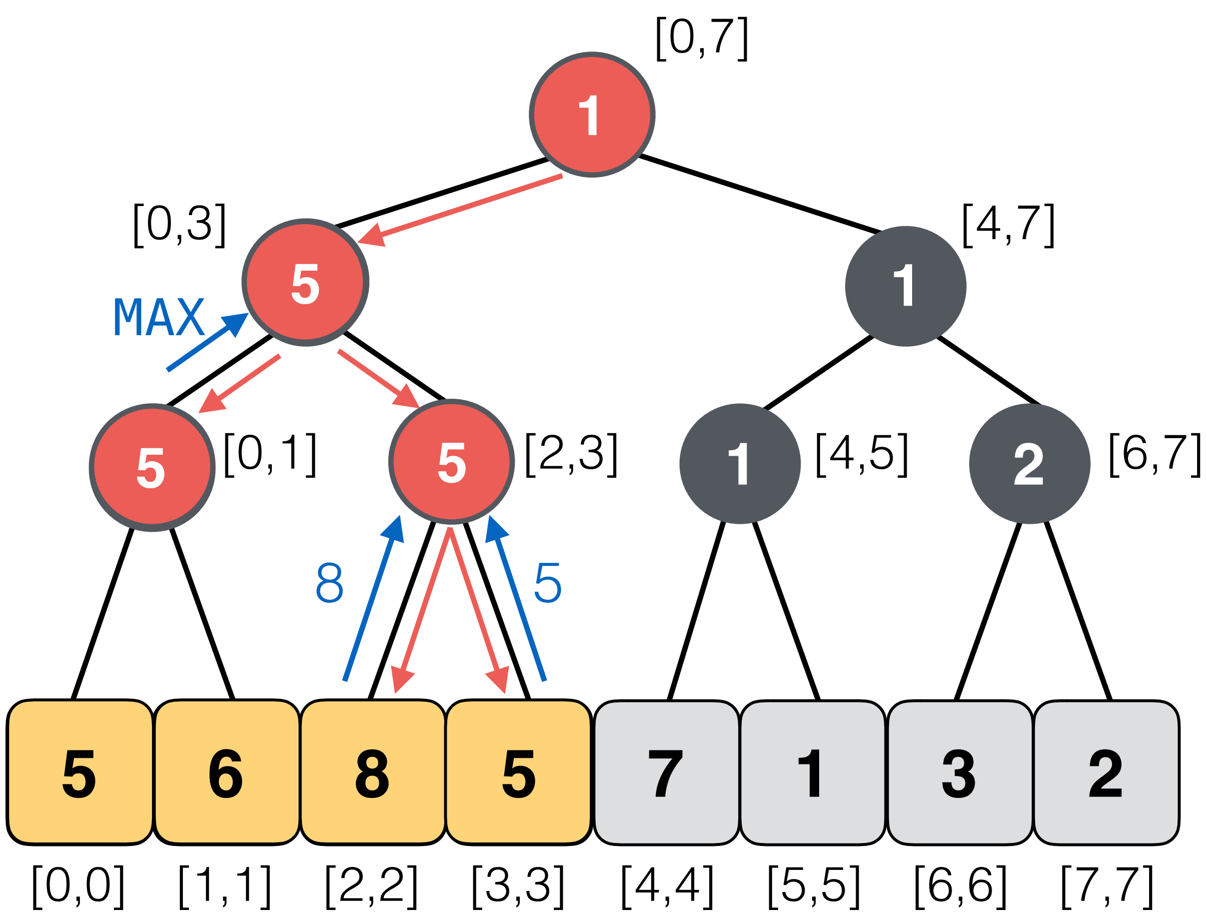

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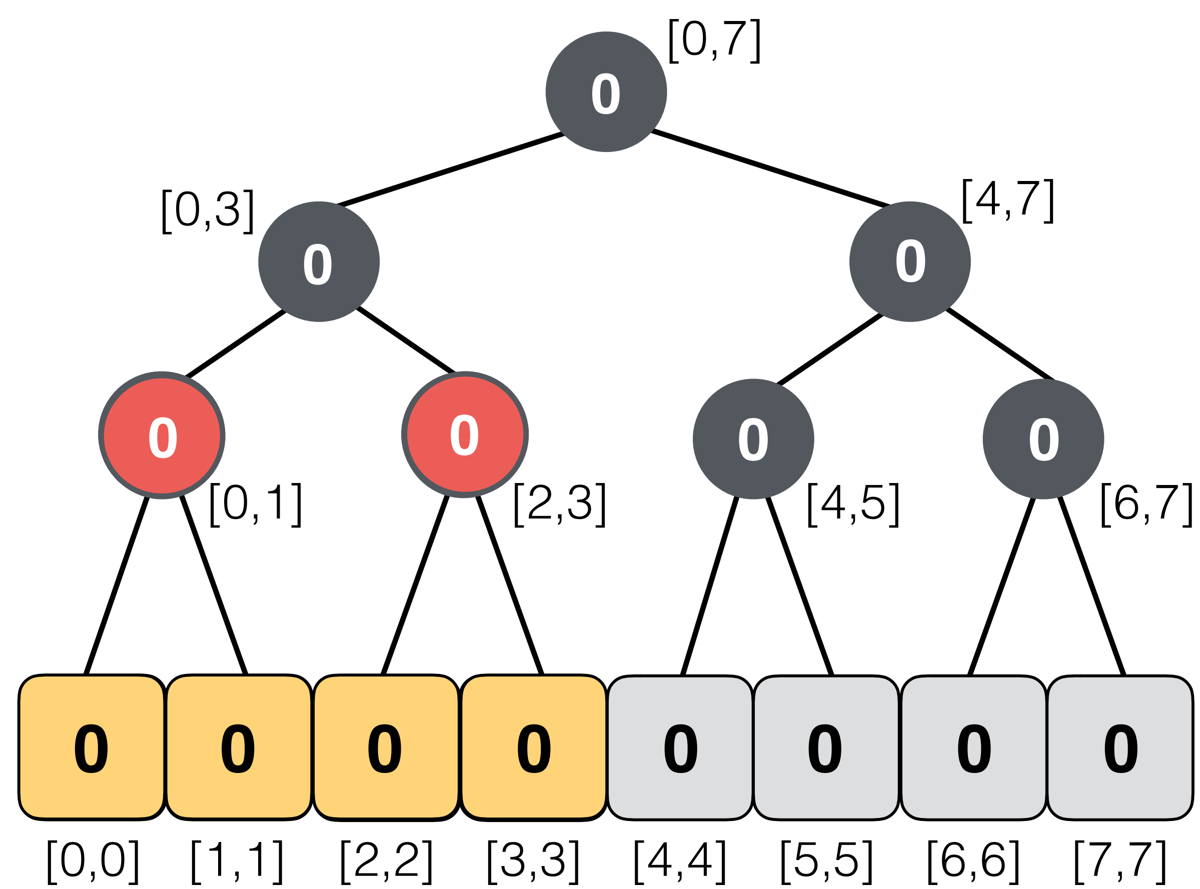

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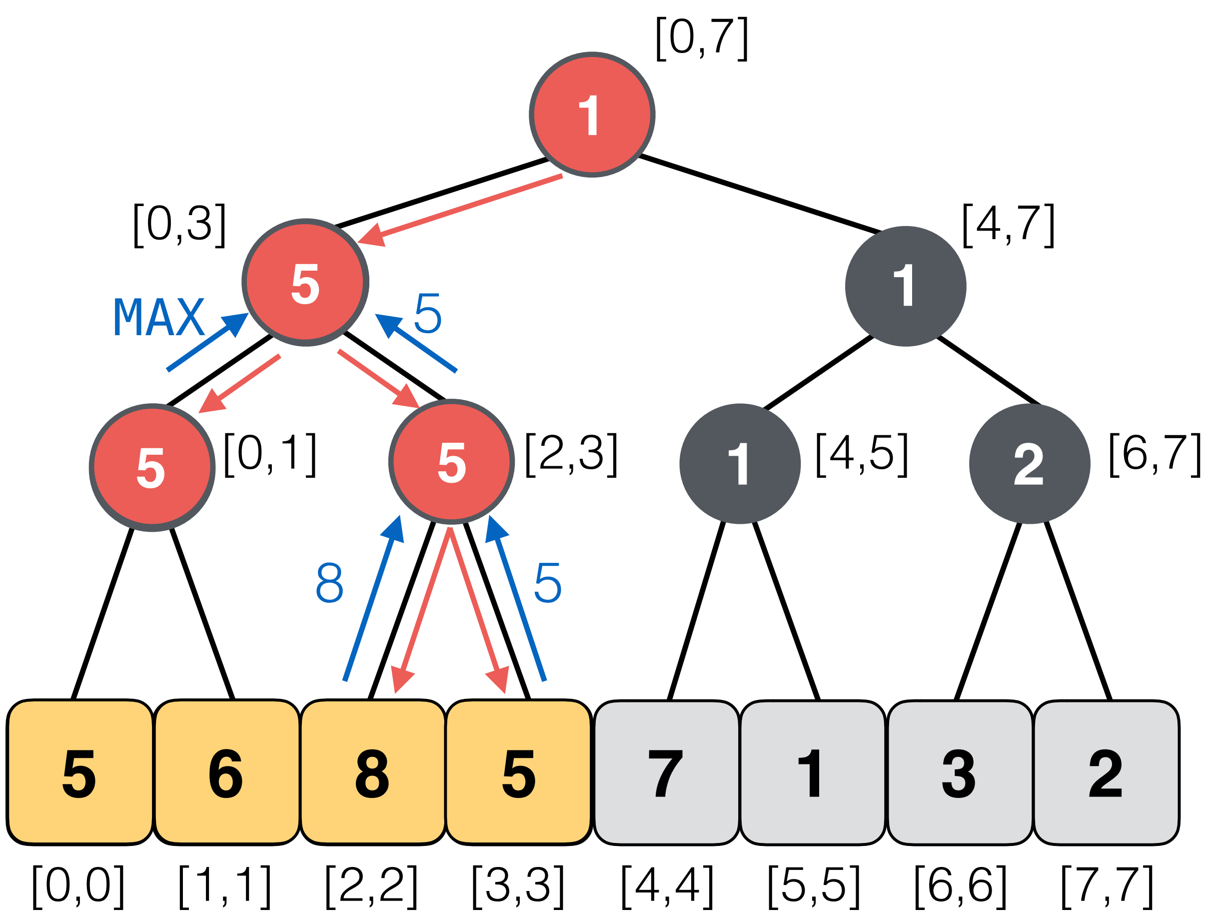

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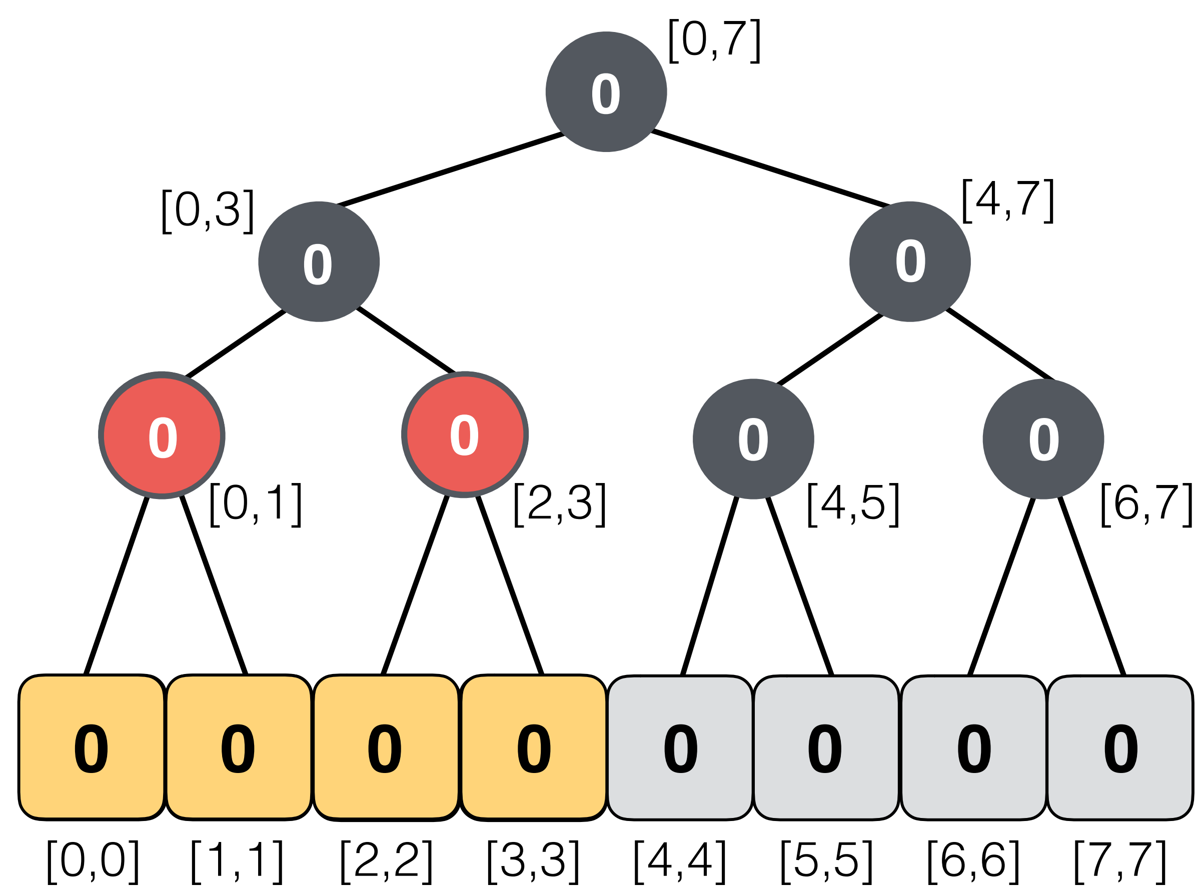

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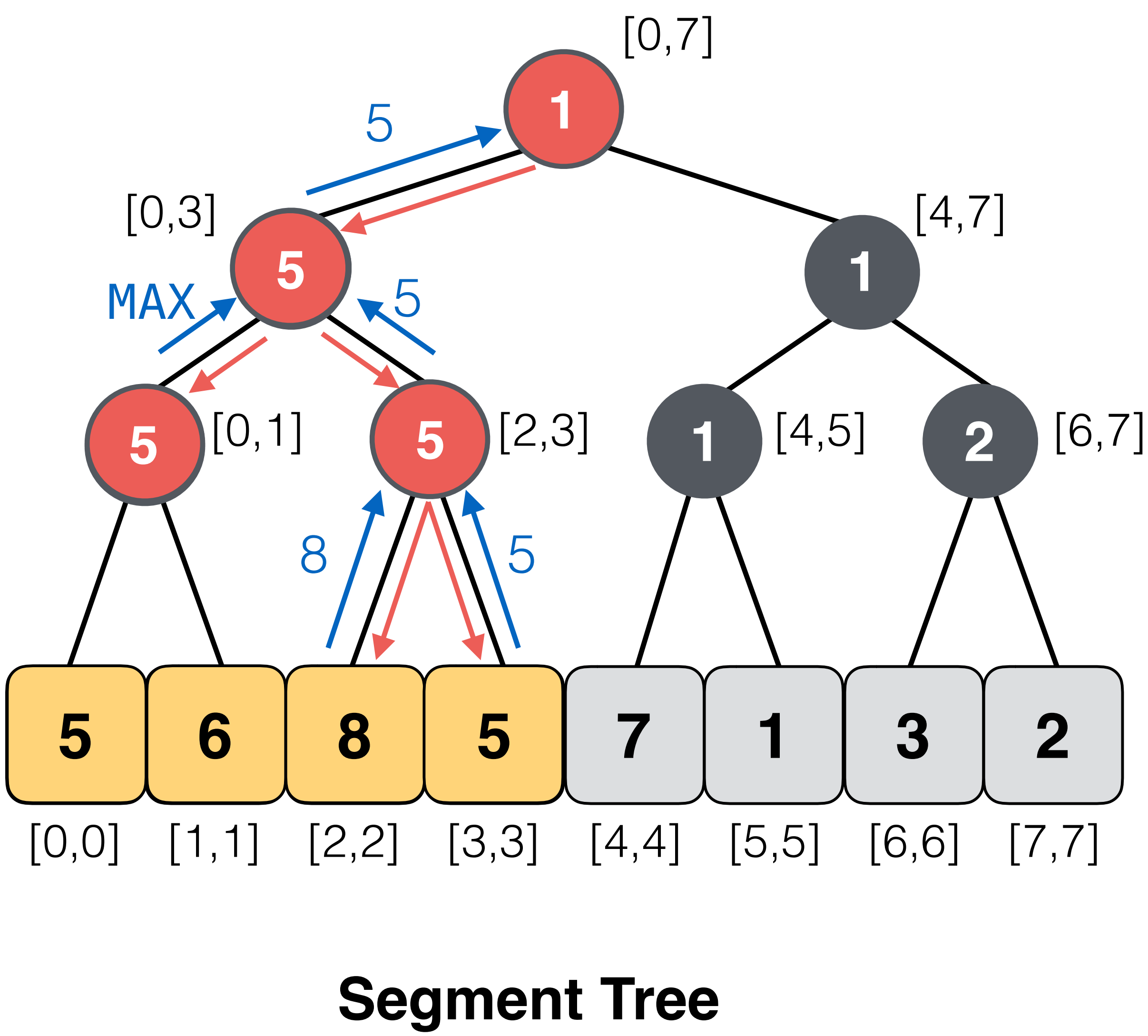

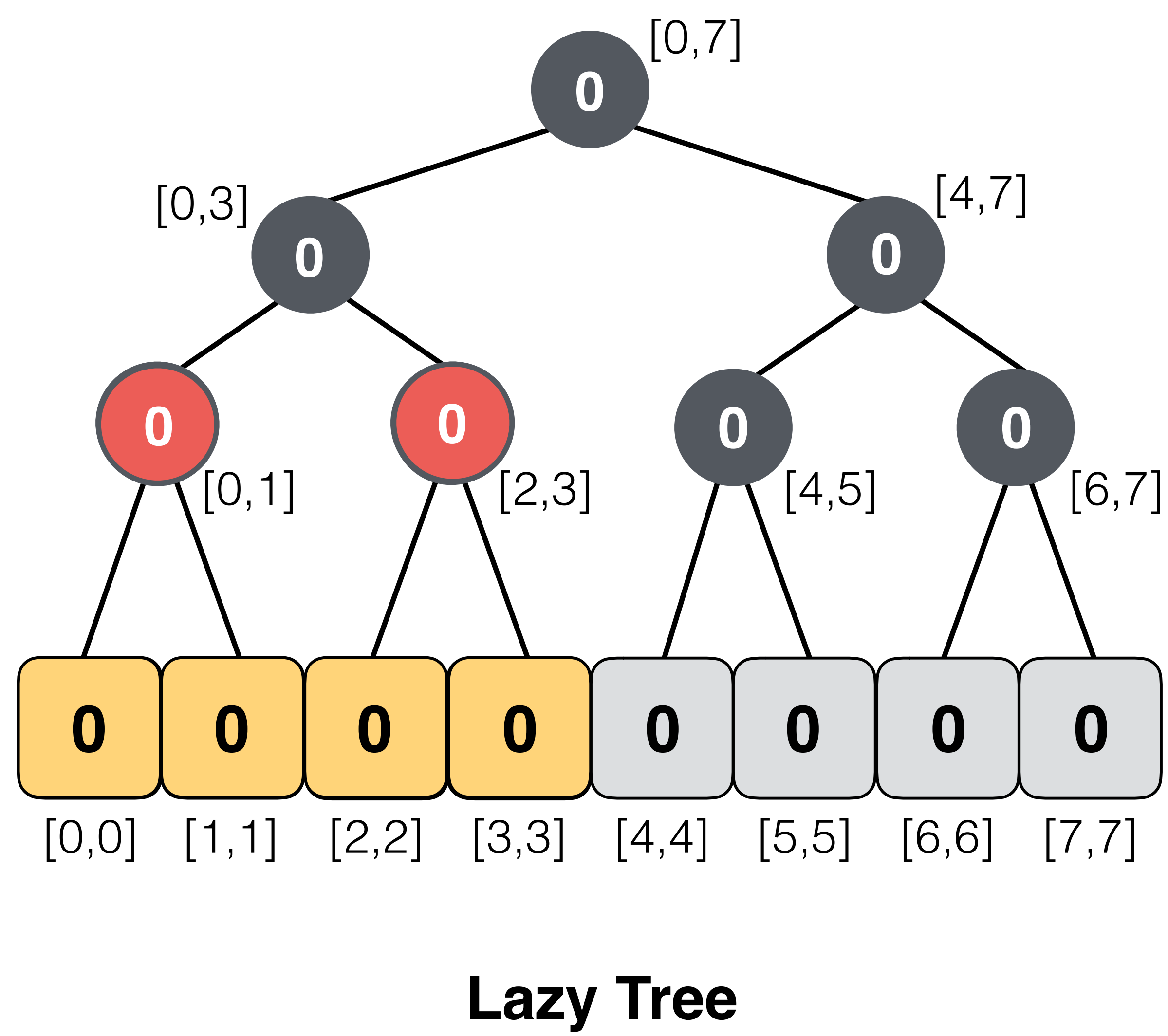

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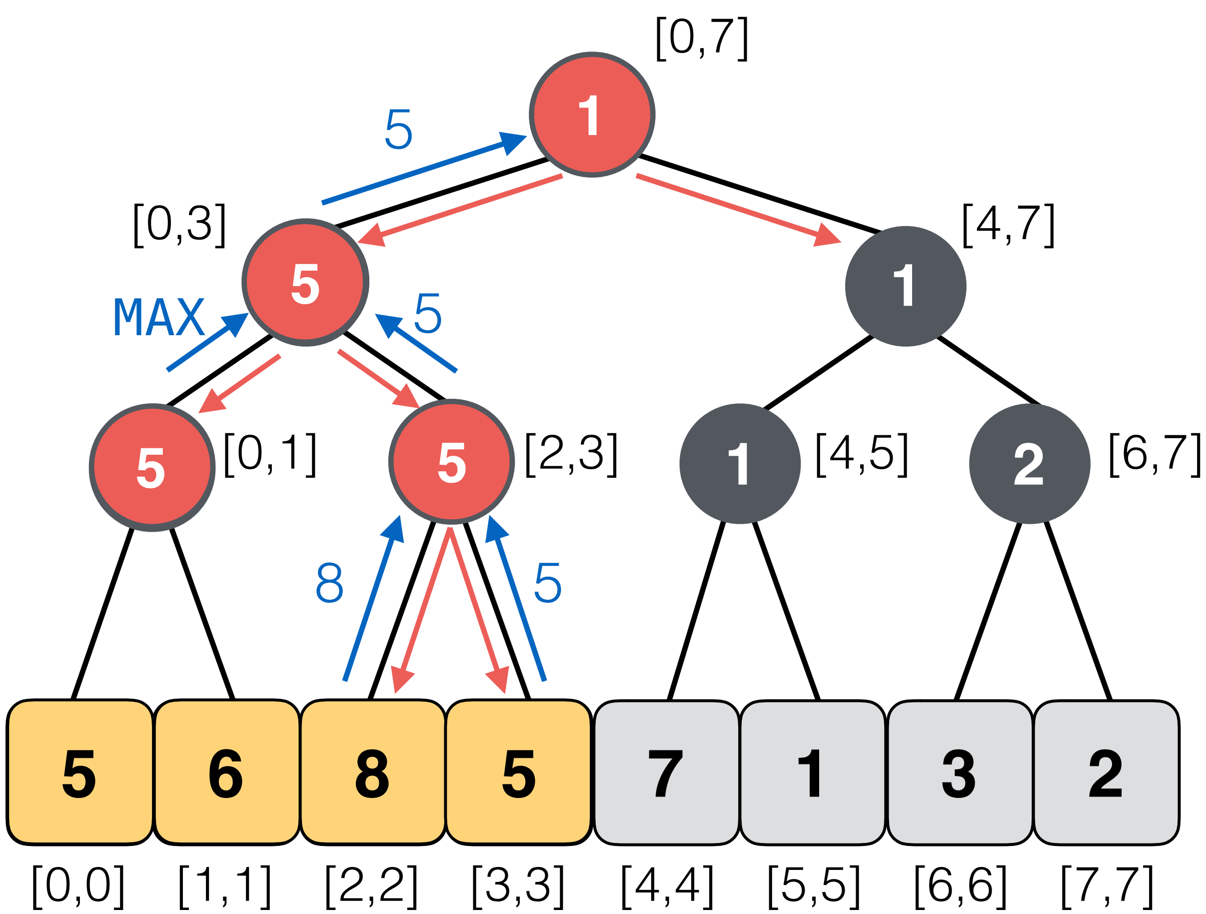

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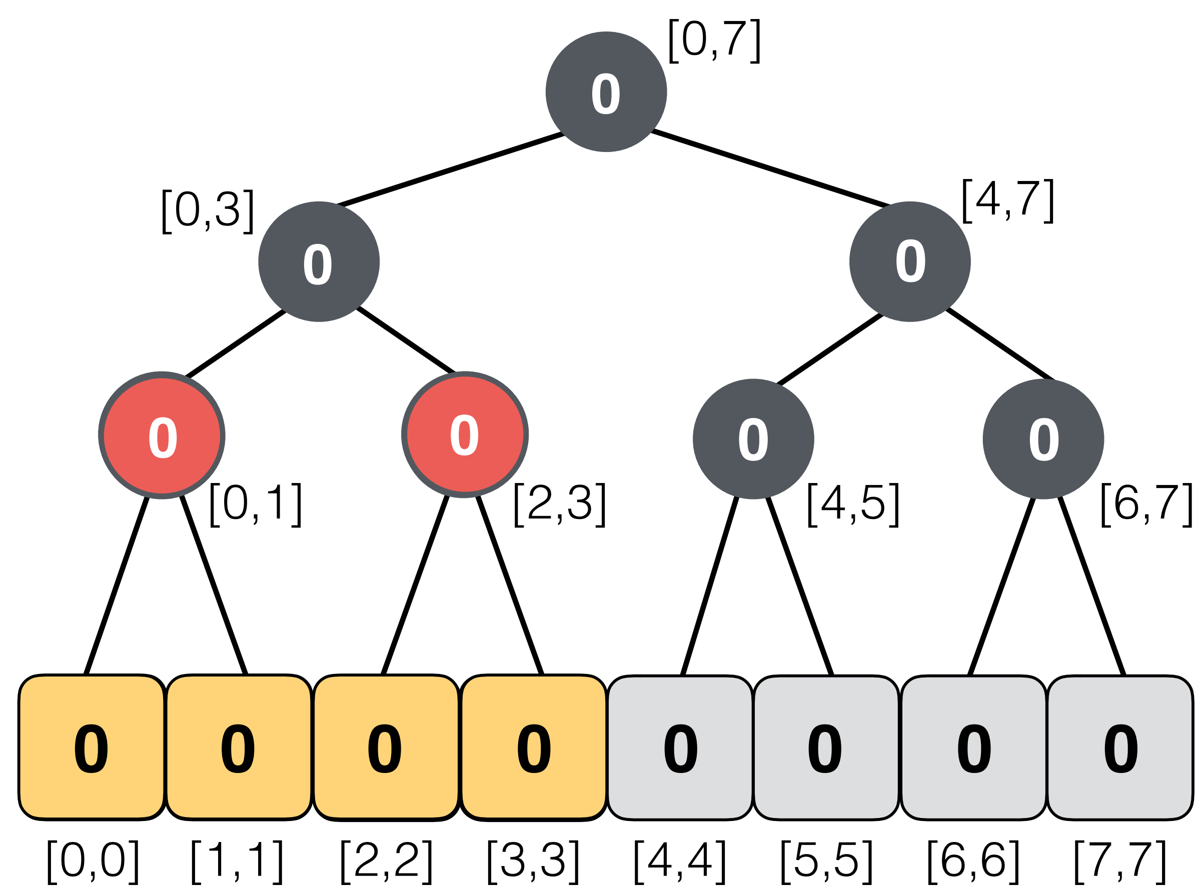

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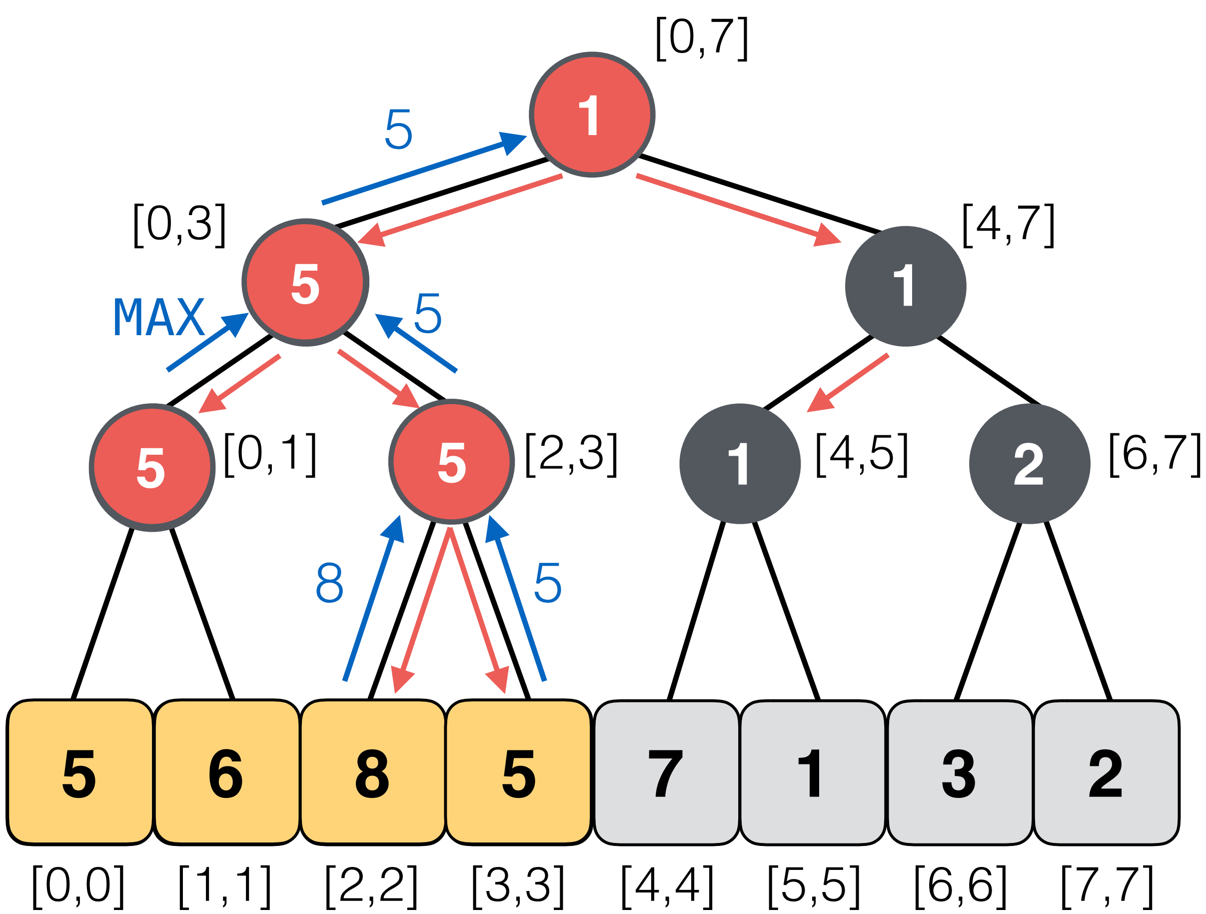

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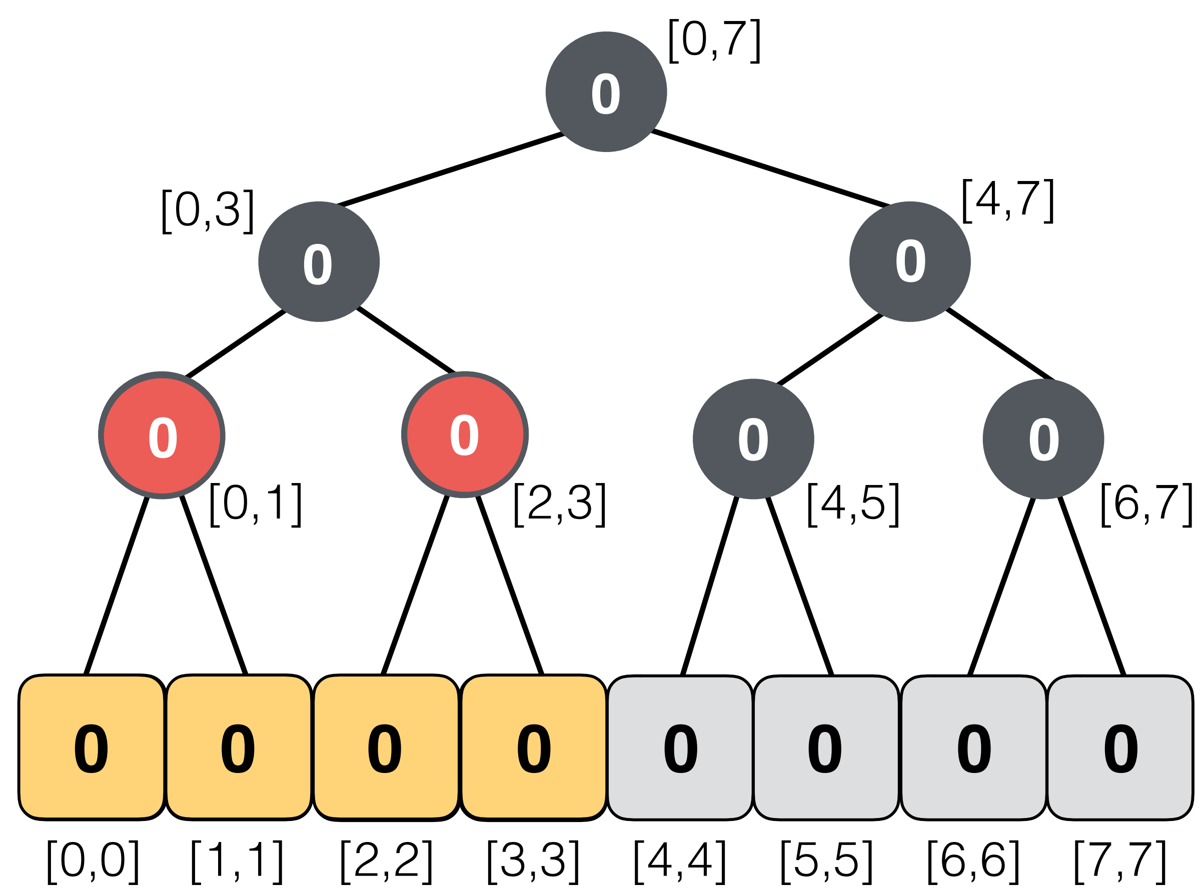

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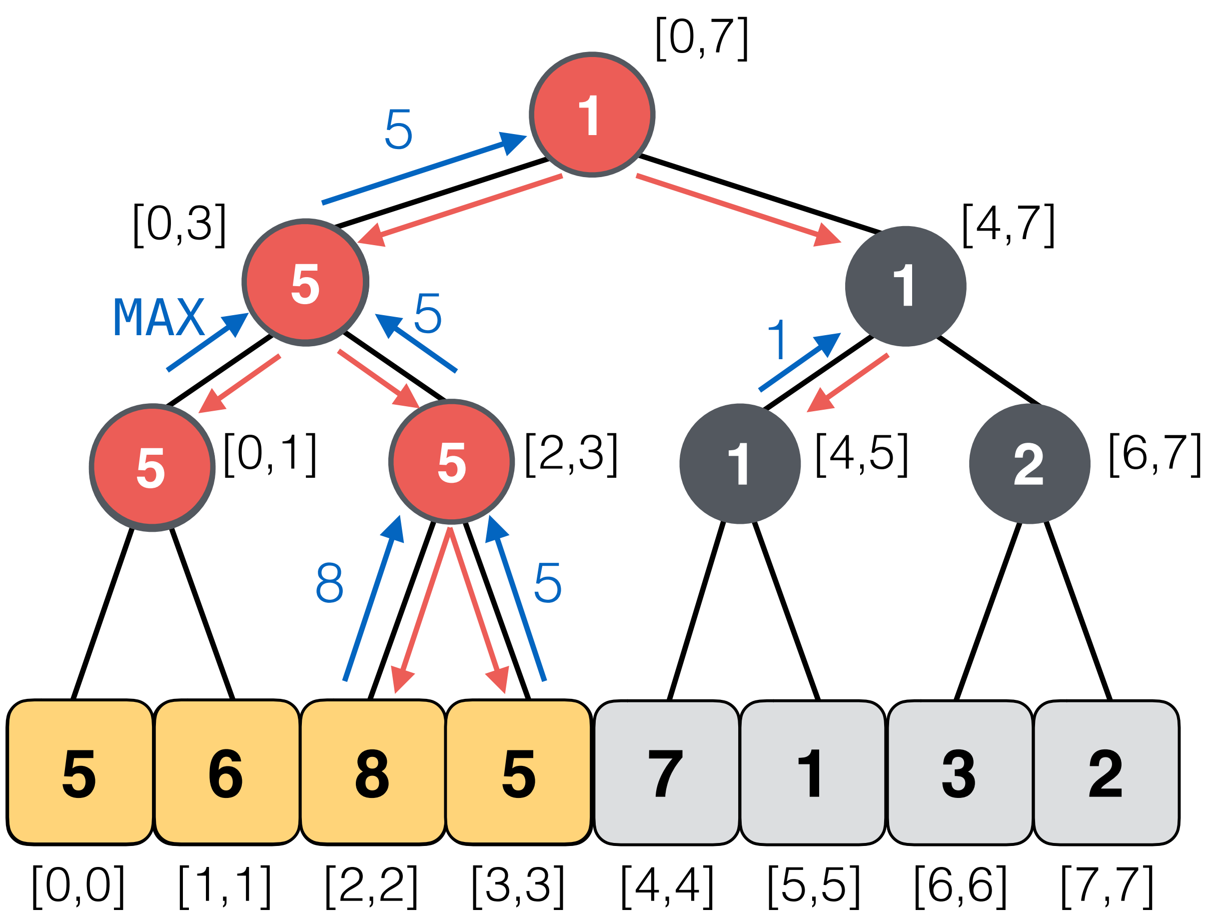

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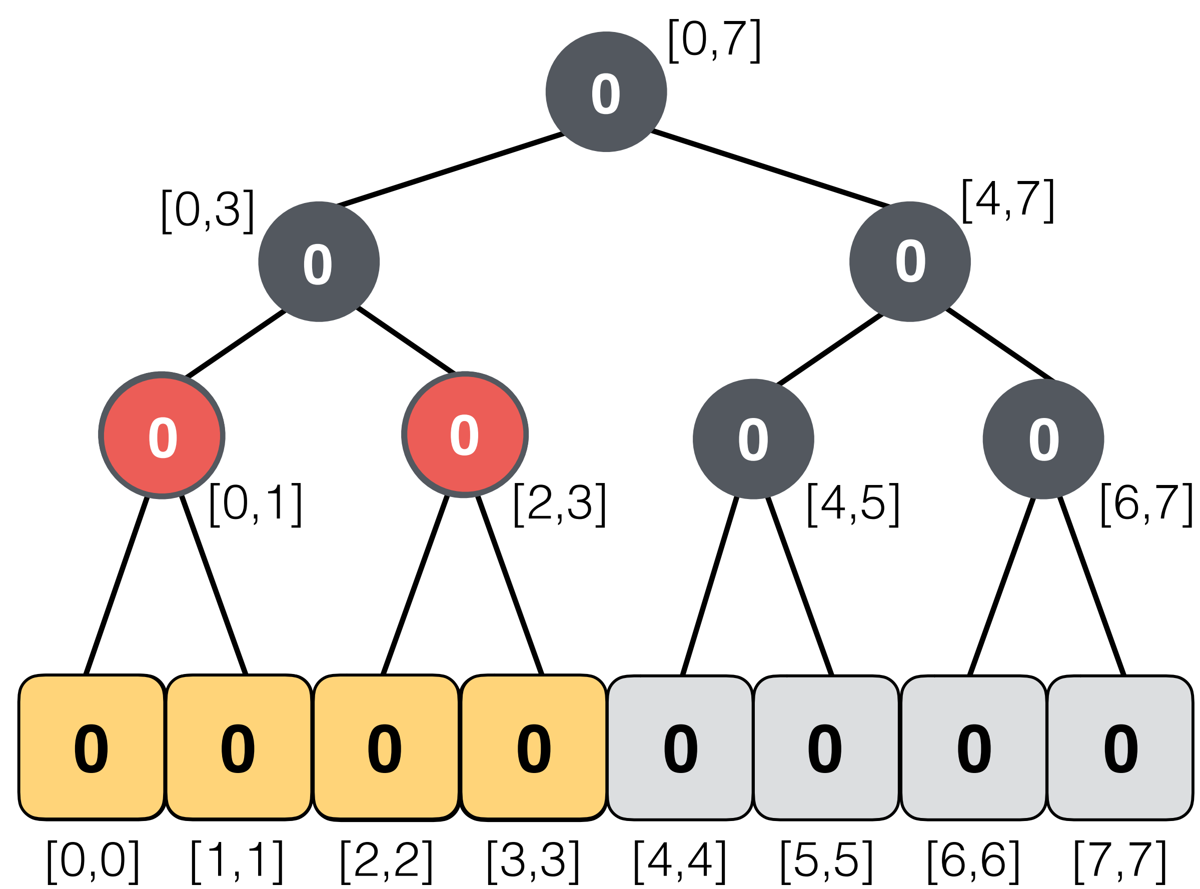

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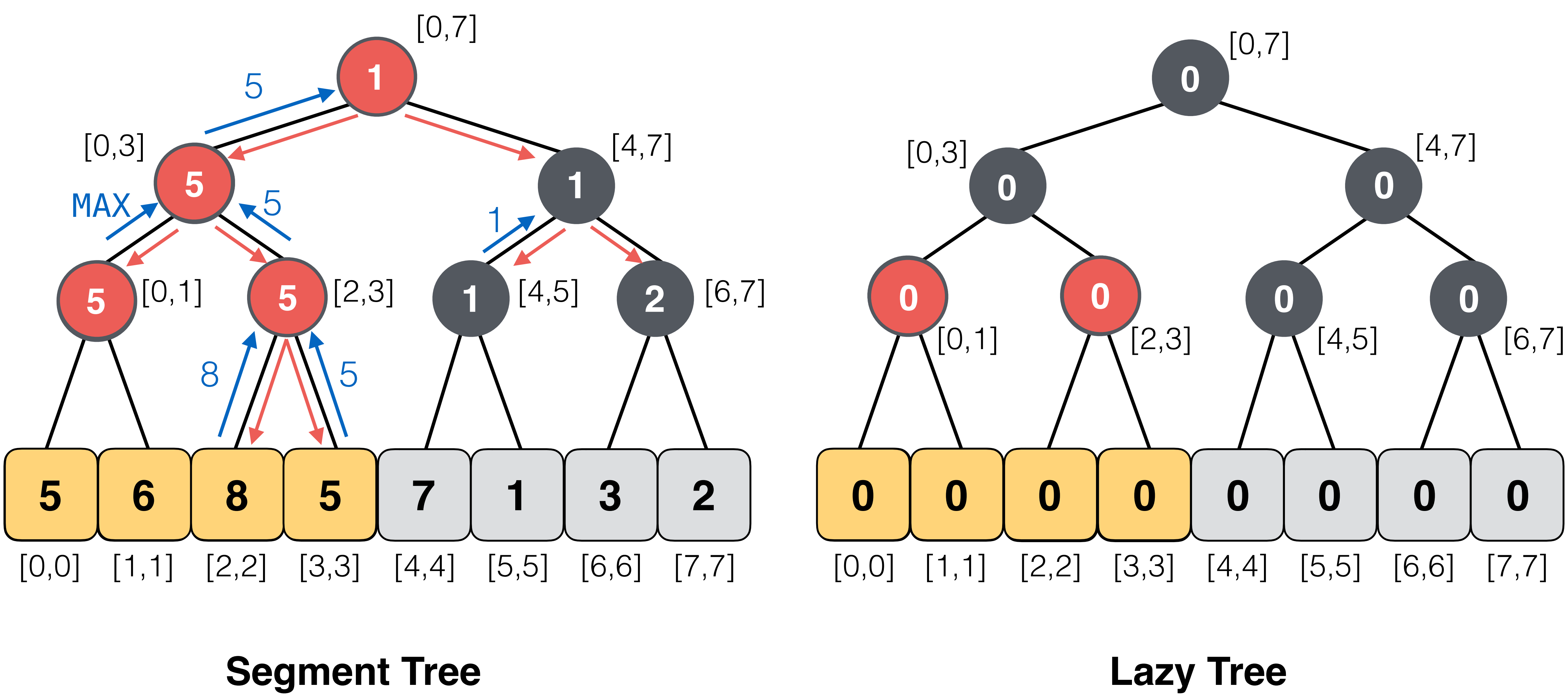

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```
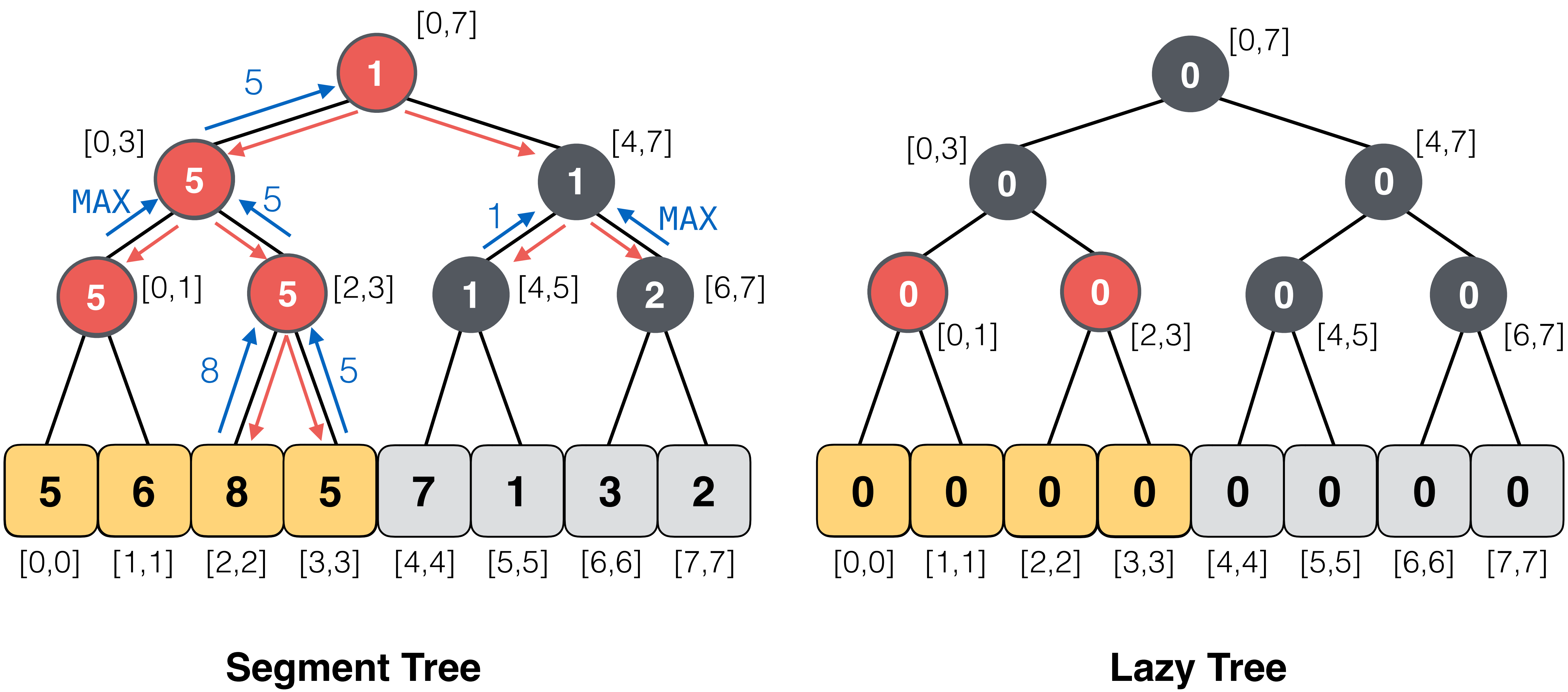
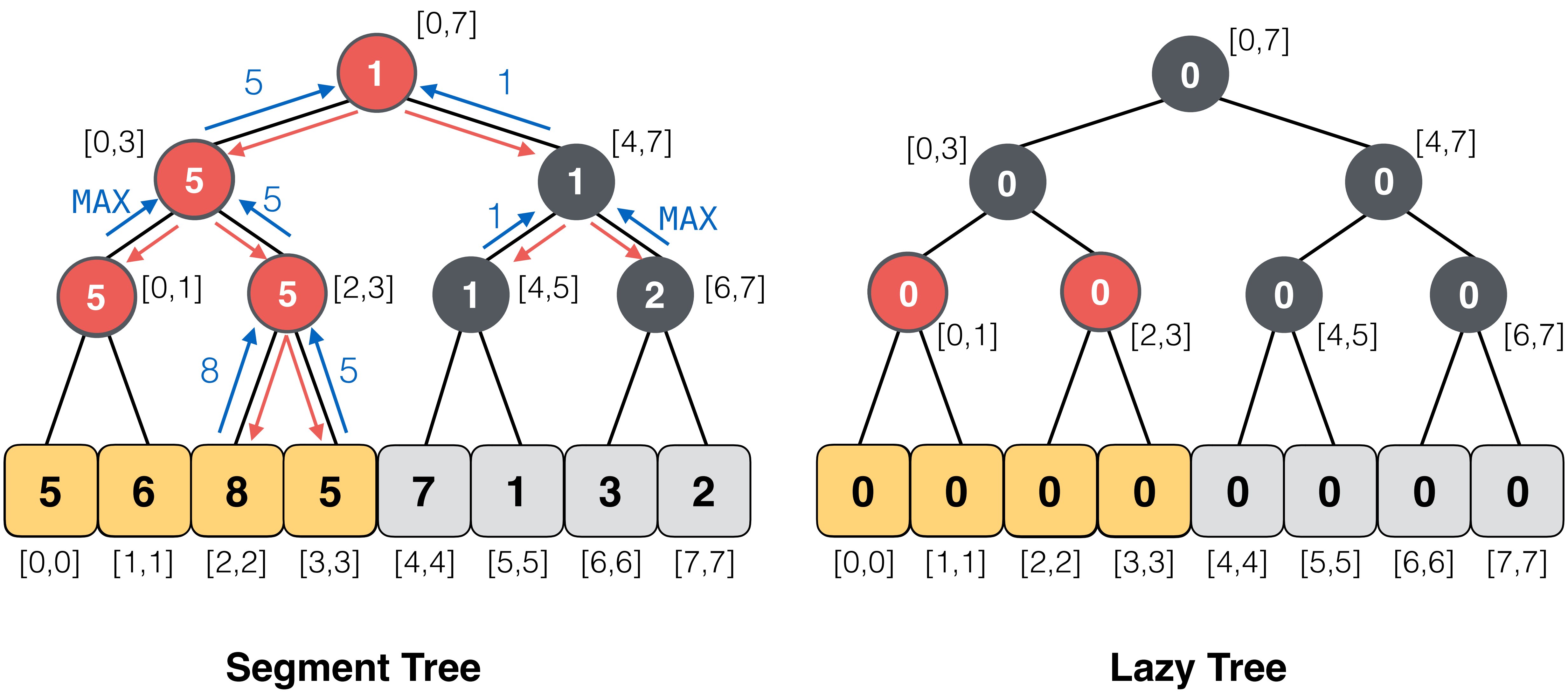



Lazy Propagation in Segment Trees

Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

`update_range`(0,3,3)
`update_range`(0,3,1)
`update_range`(0,0,2)
`rmq`(3,5) = ?



Exercises

Implement lazy propagation and test the difference in running time for a mix of updates/queries.

http://www.geeksforgeeks.org/lazy-propagation-in-segment-tree/

http://www.cdn.geeksforgeeks.org/segment-tree-set-1-sum-of-given-range/

References

Full segment tree code and benchmark at:
https://github.com/rossanoventurini/CompetitiveProgramming/tree/master/code/segment_trees

Video lectures:
https://www.youtube.com/watch?v=ZBHKZF5w4YU&list=PLrmLmBdmIlpv_jNDXtJGYTPNQ2L1gdHxu&index=22
https://www.youtube.com/watch?v=xuoQdt5pHj0&index=23&list=PLrmLmBdmIlpv_jNDXtJGYTPNQ2L1gdHxu