

Segment trees

Giulio Ermanno Pibiri
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18/10/2017

Problems we would like to solve efficiently

A	3	1	-2	4	6	13	2	0
	0	1	2	3	4	5	6	7

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- `sum(i)` reports the sum of the first $i+1$ integers
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$$\text{sum}(5) = 25$$

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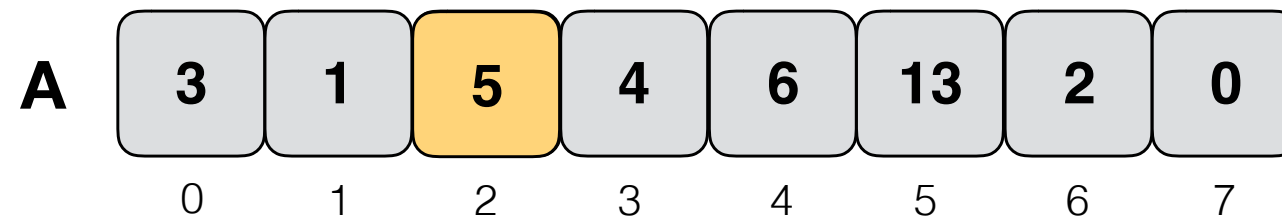
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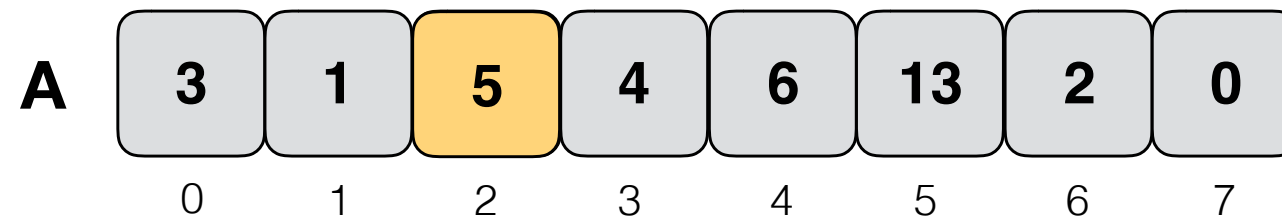
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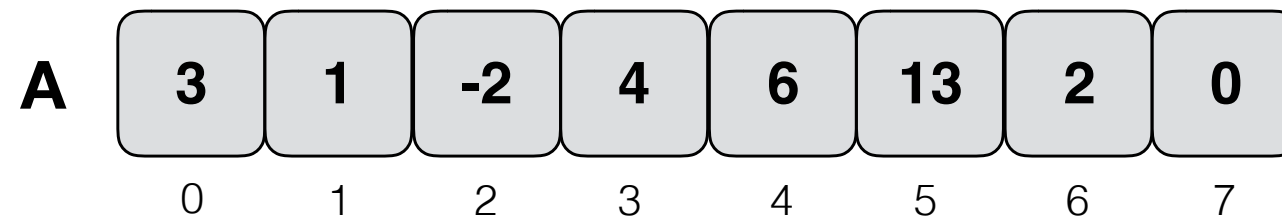
`update(2, 5)`

`sum(5) = 32`

Range MIN (MAX) queries

Report the MIN (MAX) in $A[i,j]$

Problems we would like to solve efficiently



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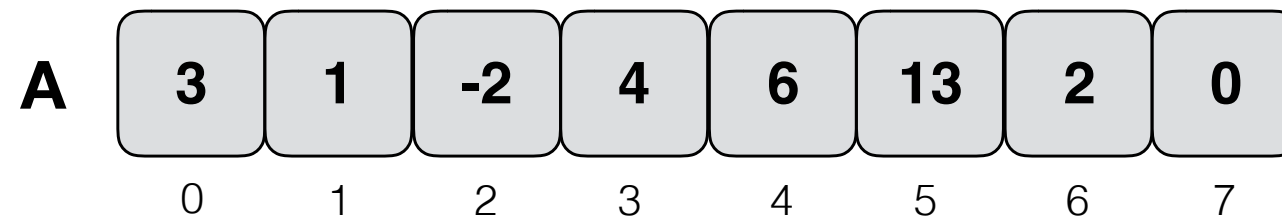
Range MIN (MAX) queries

Report the MIN (MAX) in $A[i,j]$

`min(1,3) = -2`

`max(4,7) = 0`

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$$\text{update}(2, 5)$$

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Range MIN (MAX) queries

Report the MIN (MAX) in $A[i,j]$

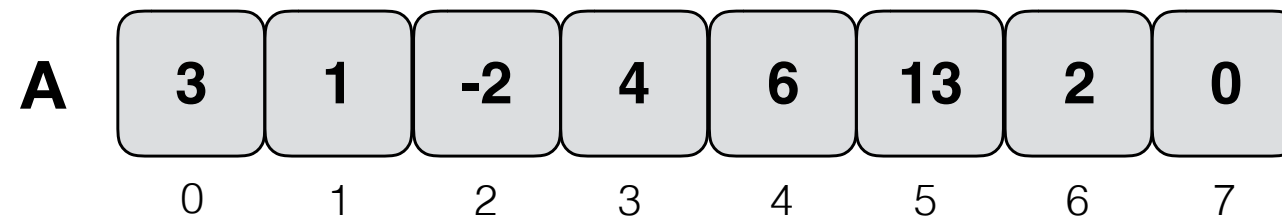
$$\min(1,3) = -2$$

$$\max(4,7) = 0$$

Range SUM queries

Report the sum of the elements in $A[i,j]$

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$$\text{sum}(3) = 6$$

$$\text{sum}(5) = 25$$

$$\text{update}(2, 5)$$

$$\text{sum}(5) = 32$$

Range MIN (MAX) queries

Report the MIN (MAX) in $A[i,j]$

$$\min(1,3) = -2$$

$$\max(4,7) = 0$$

Range SUM queries

Report the sum of the elements in $A[i,j]$

$$\text{sum}(1,3) = 3$$

$$\text{sum}(4,7) = 21$$

Any solutions?

A	3	1	-2	4	6	13	2	0
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Any solutions?

A

3	1	-2	4	6	13	2	0
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1. Do nothing
2. Pre-calculate all queries

Any solutions?

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3	1	-2	4	6	13	2	0
0	1	2	3	4	5	6	7

1. Do nothing
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(Static) Prefix sums

1.
update: $O(1)$
sum: $O(n)$
Space: no auxiliary space
2.
update: $O(n)$
sum: $O(1)$
Space: no auxiliary space

3	4	2	6	12	25	27	27
0	1	2	3	4	5	6	7

Any solutions?

A

3	1	-2	4	6	13	2	0
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0	1	2	3	4	5	6	7

1. Do nothing
2. Pre-calculate all queries

Range MIN (MAX) and SUM queries

1.
Query time: $O(n)$
Space: no auxiliary space
2.
Query time: $O(1)$
Space: $O(n^2)$
Building time: $O(n^2)$

0	1	2	3	4	5	6	7	
3	1	-2	-2	-2	-2	-2	-2	0
	1	-2	-2	-2	-2	-2	-2	1
		-2	-2	-2	-2	-2	-2	2
			4	4	4	2	0	3
				6	6	2	0	4
					13	2	0	5
						2	0	6
							0	7

An efficient solution

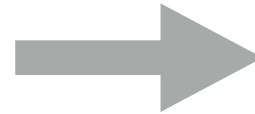
Remember

An efficient solution is the one that gives
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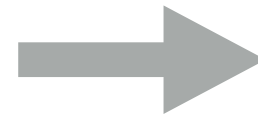
Idea

Impose a complete (static)
binary tree over the array:
a **segment tree**.

An efficient solution

Remember

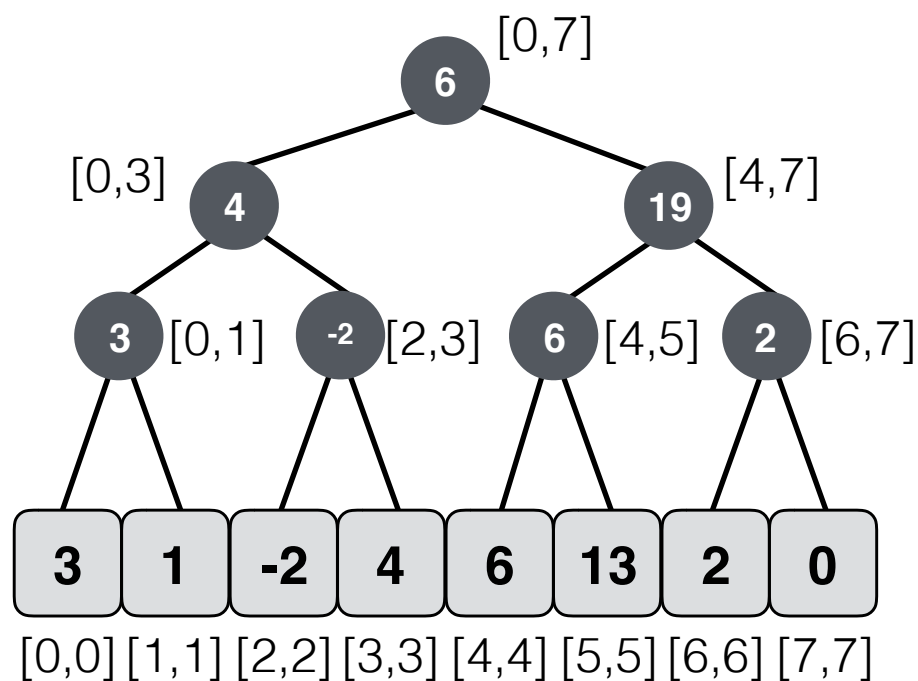
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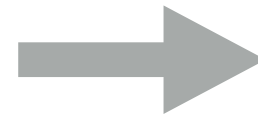
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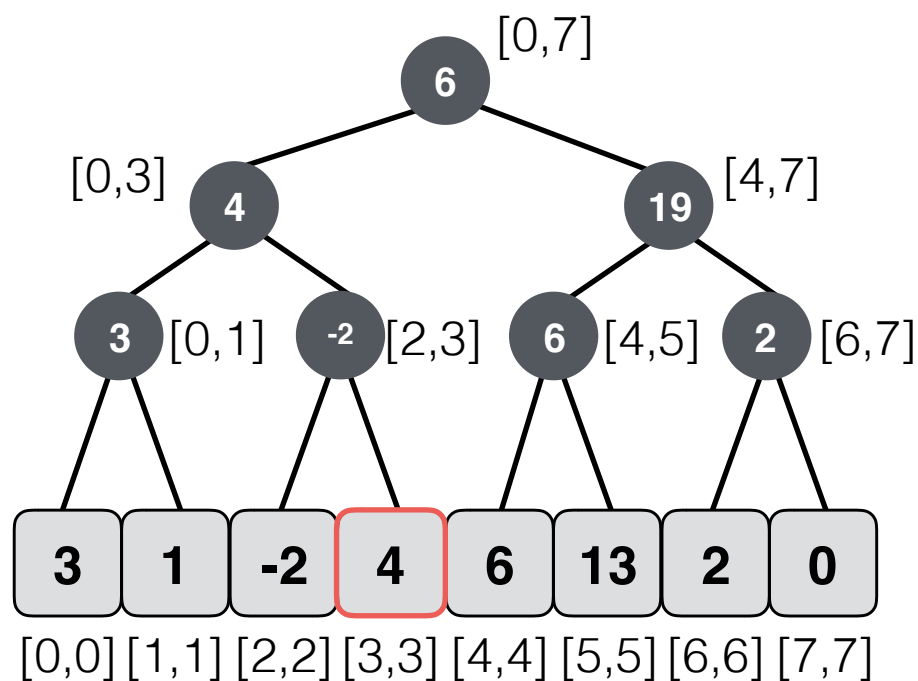
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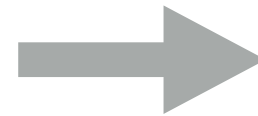


$\text{sum}(3) = (4) +$

An efficient solution

Remember

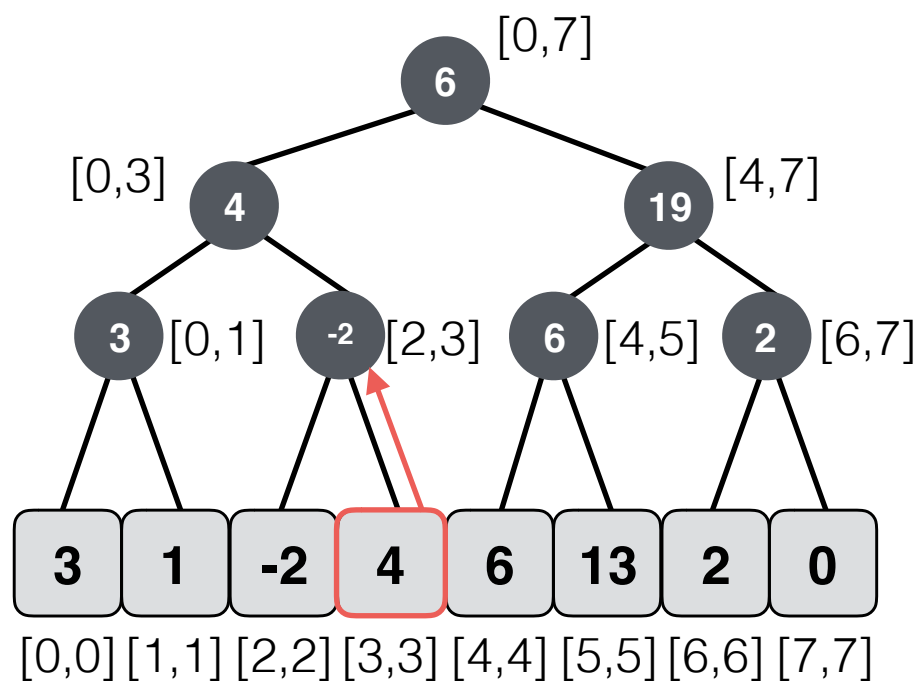
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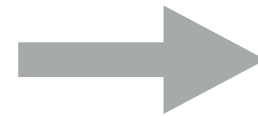


$$\text{sum}(3) = (4) + (-2) +$$

An efficient solution

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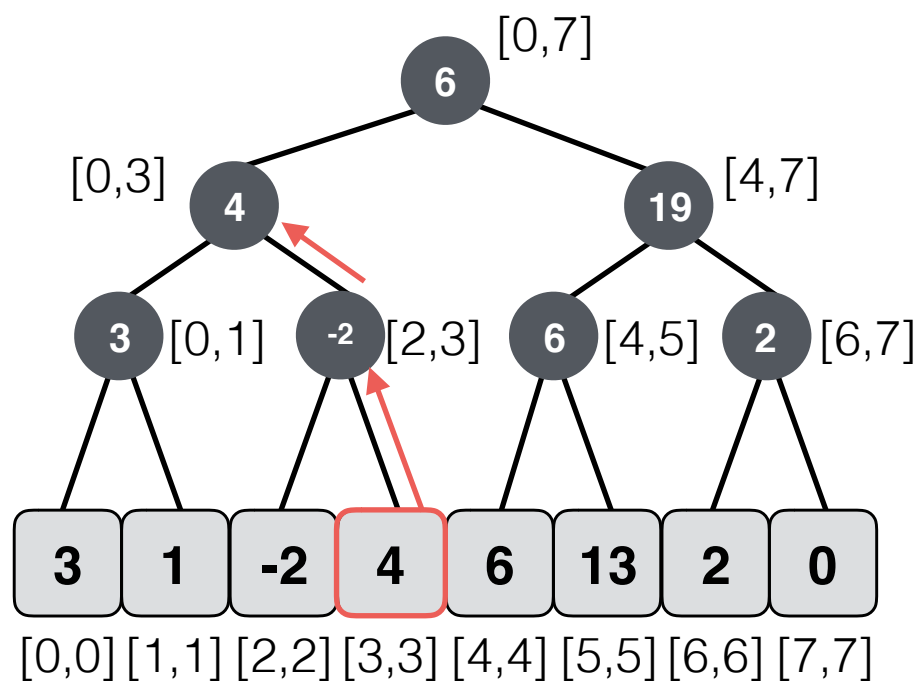
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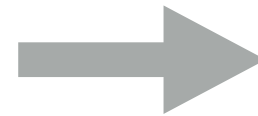


$$\text{sum}(3) = (4) + (-2) + (4) = 6$$

An efficient solution

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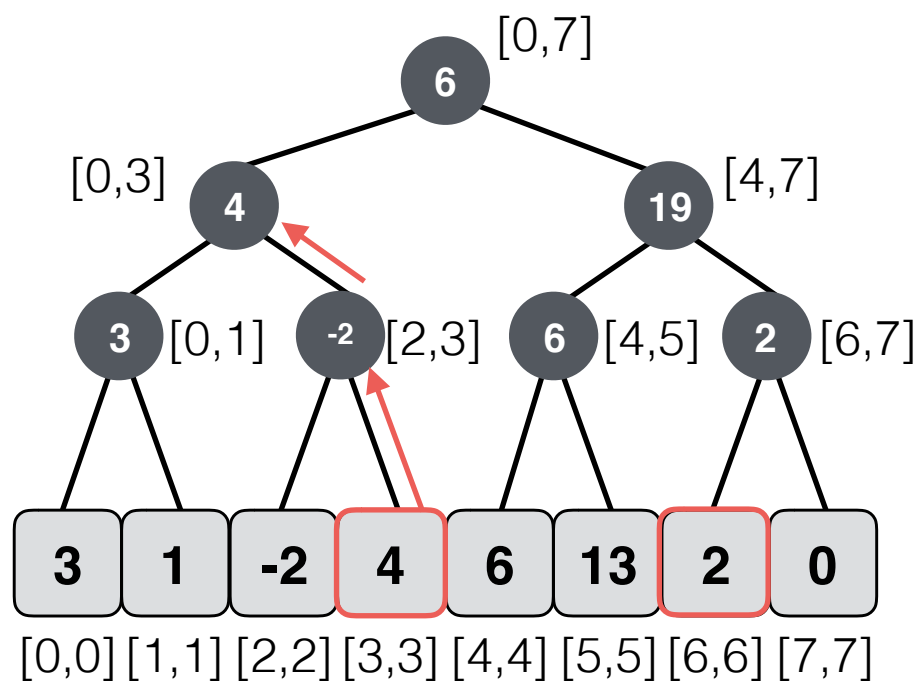
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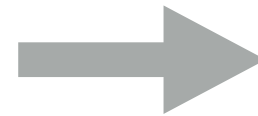
$$\text{sum}(3) = (4) + (-2) + (4) = 6$$

$$\text{sum}(6) = (2) +$$

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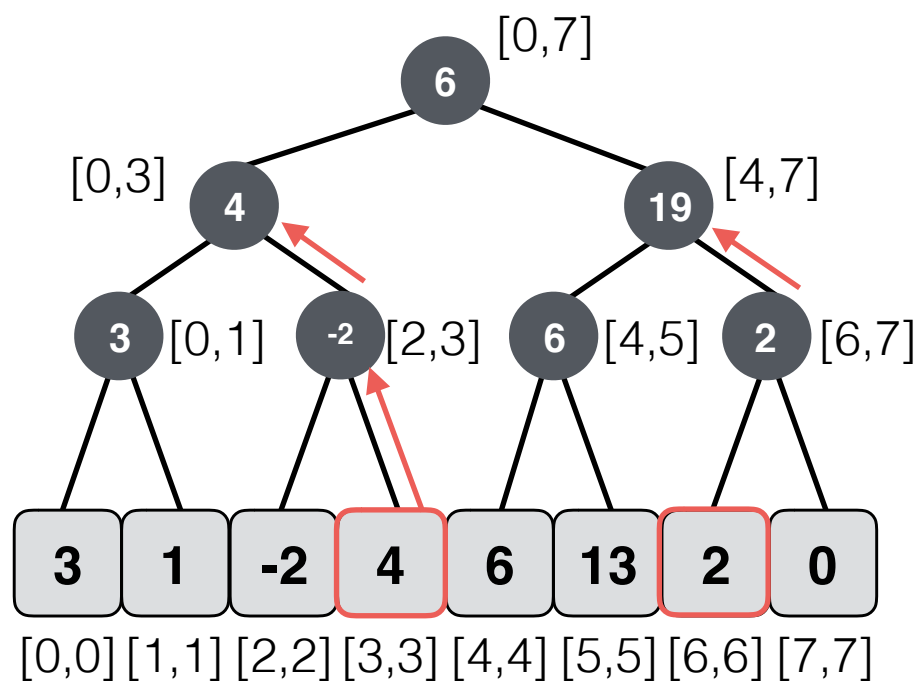
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$$\text{sum}(3) = (4) + (-2) + (4) = 6$$

$$\text{sum}(6) = (2) + (19) +$$

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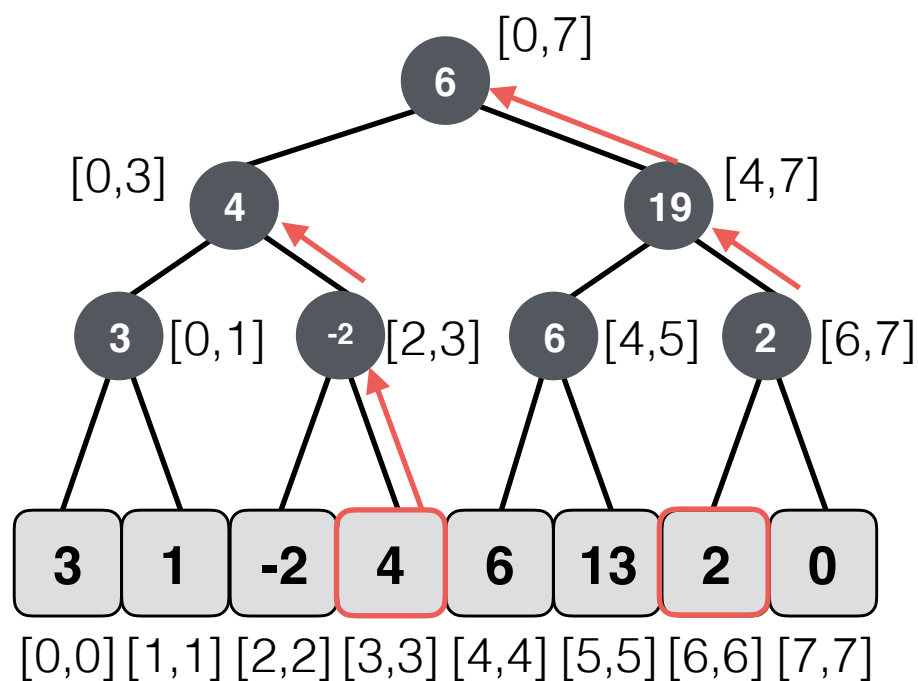
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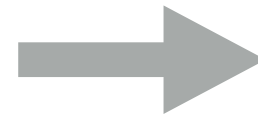
$$\text{sum}(3) = (4) + (-2) + (4) = 6$$

$$\text{sum}(6) = (2) + (19) + (6) = 27$$

An efficient solution

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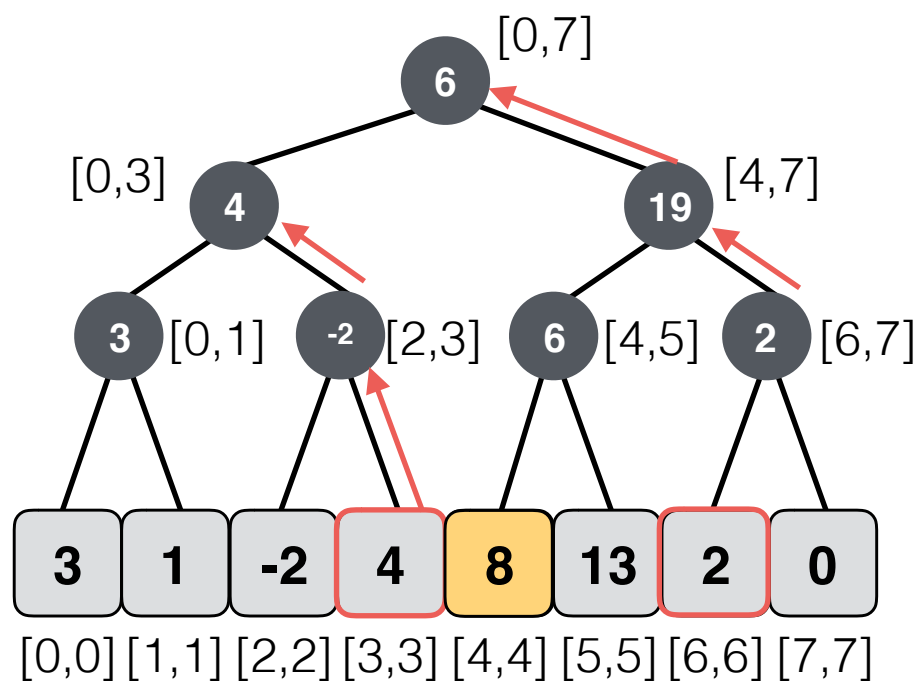
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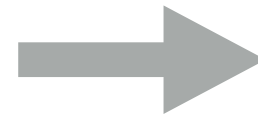
$$\text{sum}(6) = (2) + (19) + (6) = 27$$

update(4, 8)

An efficient solution

Remember

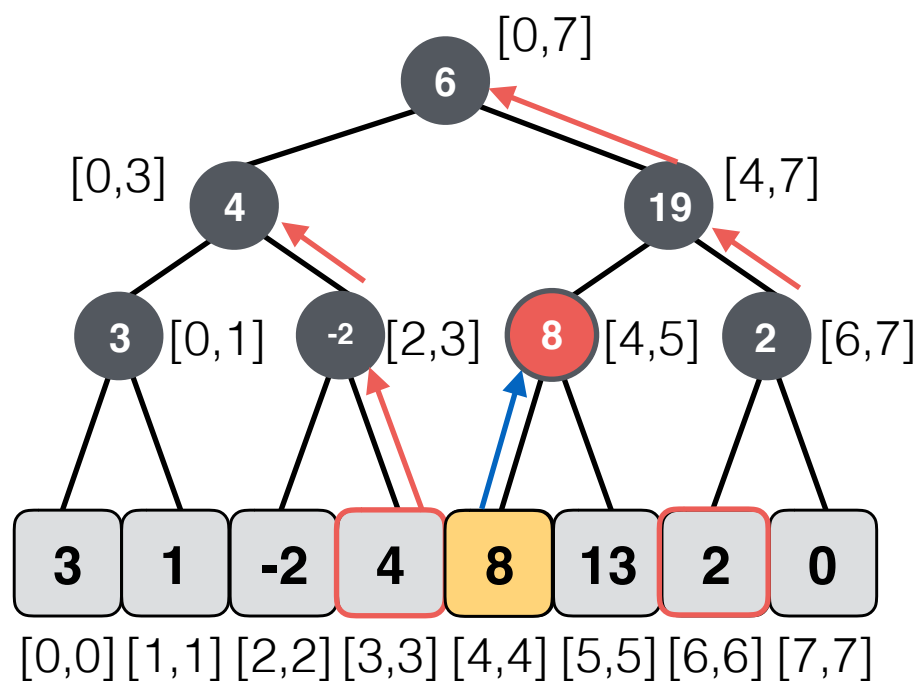
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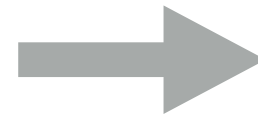
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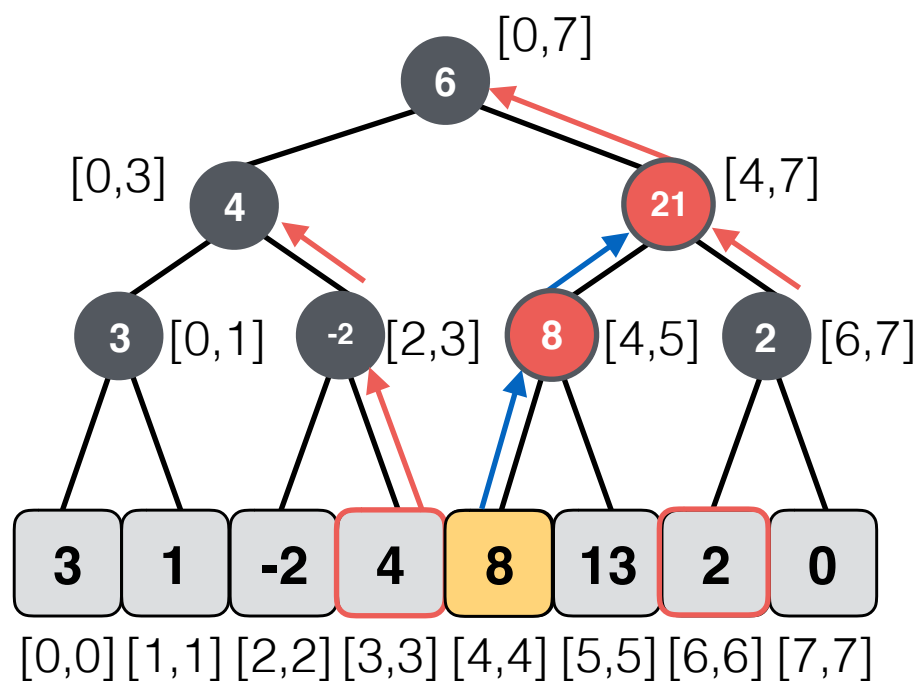
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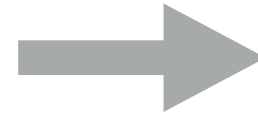
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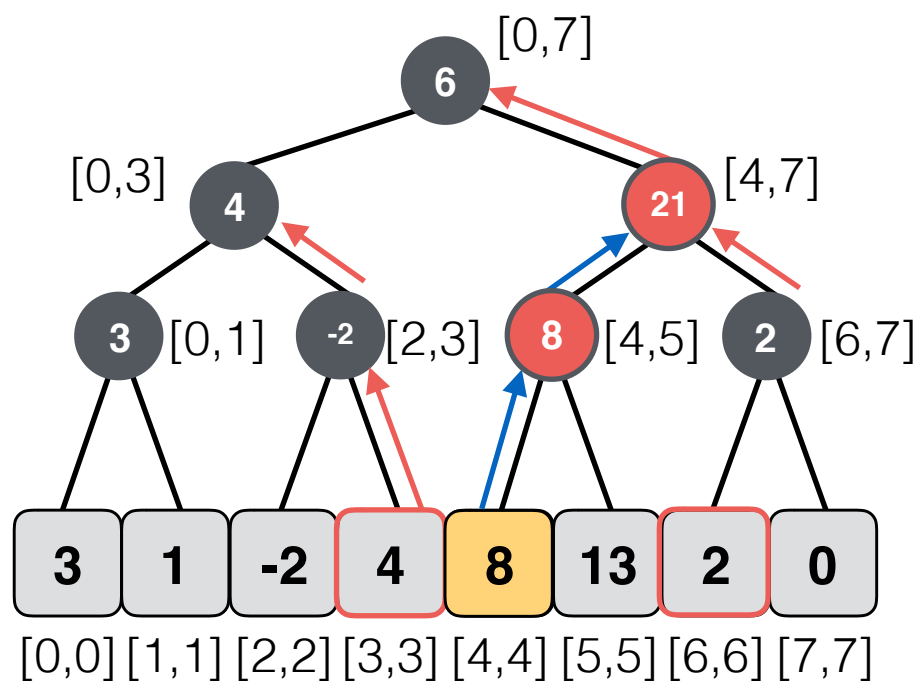
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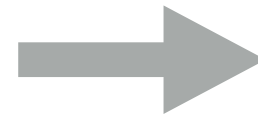
update(4, 8)

sum and update in $O(\log n)$

An efficient solution

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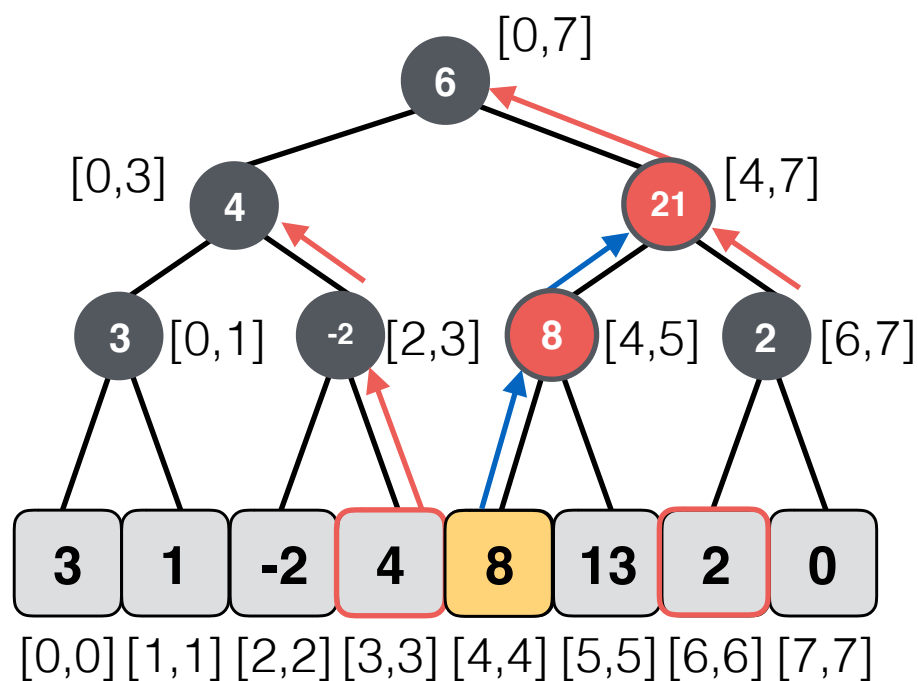
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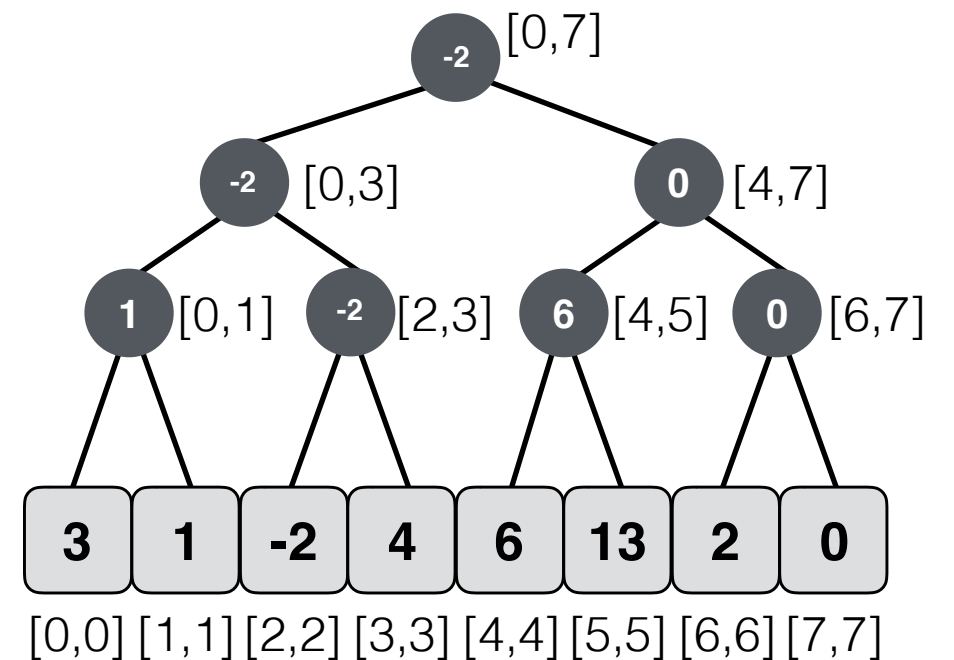
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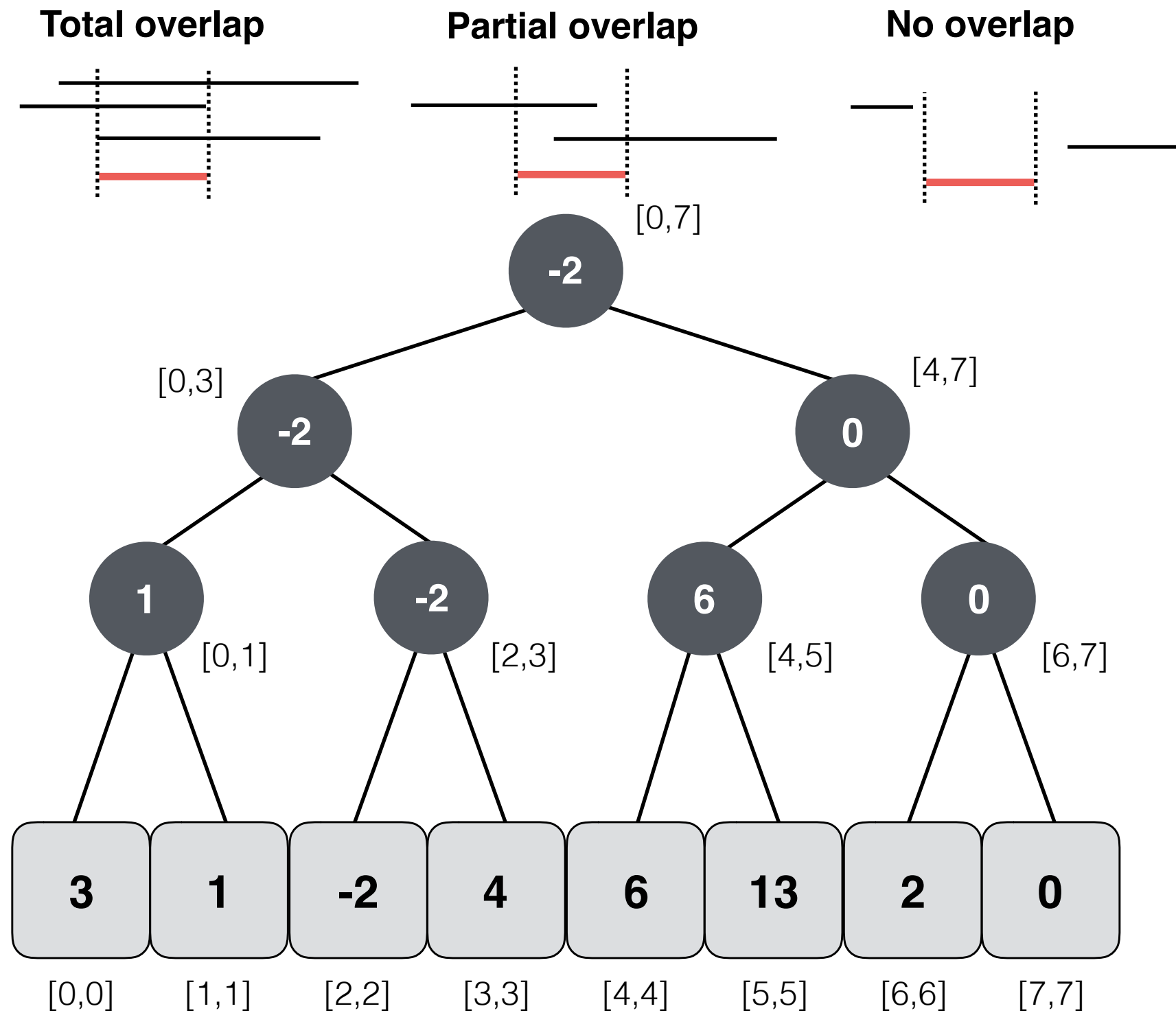
What we consider next, stay tuned!

Range MIN queries



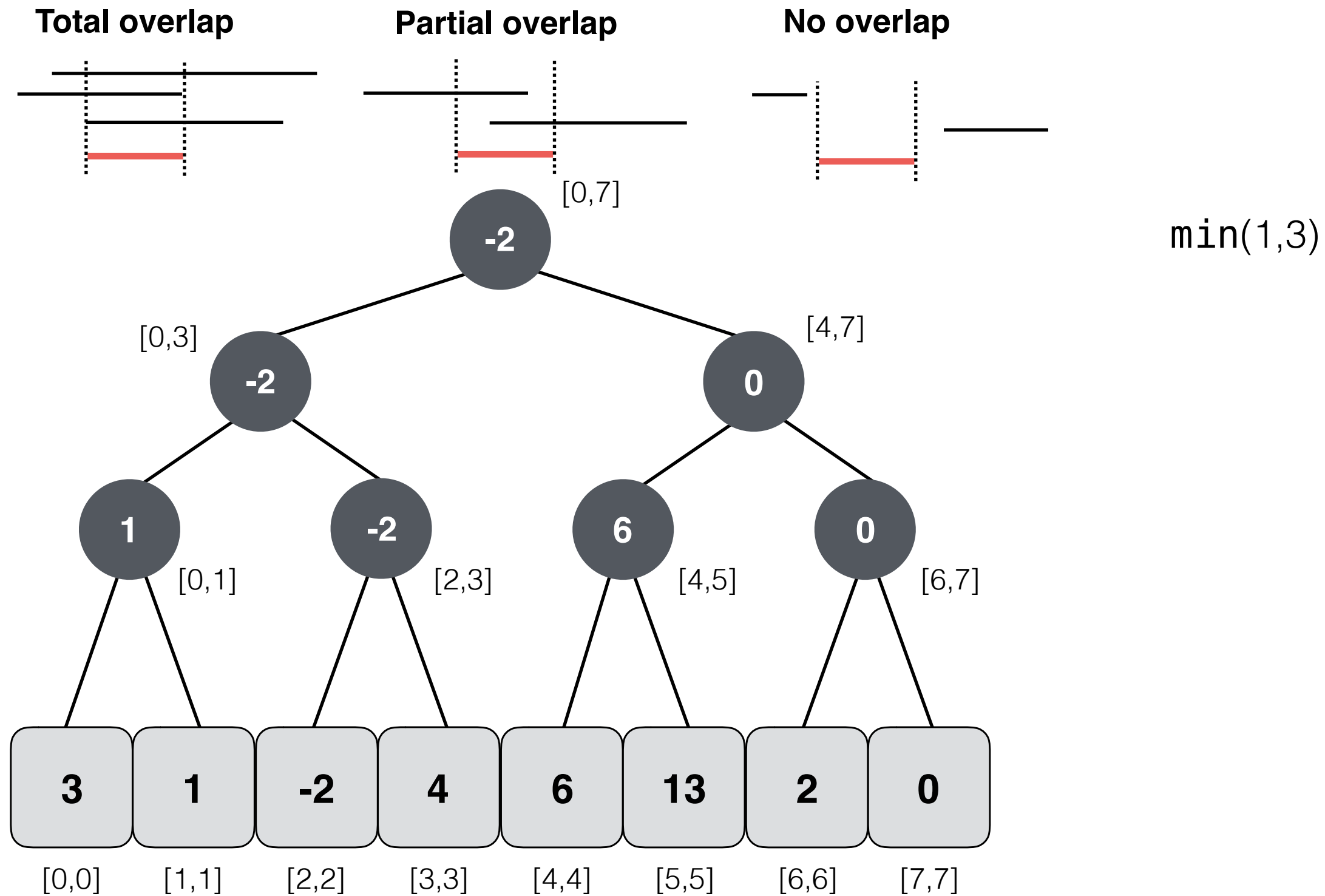
Range MIN Queries with Segment Trees

Consider a segment tree with n leaves ($2n - 1$ nodes in total).



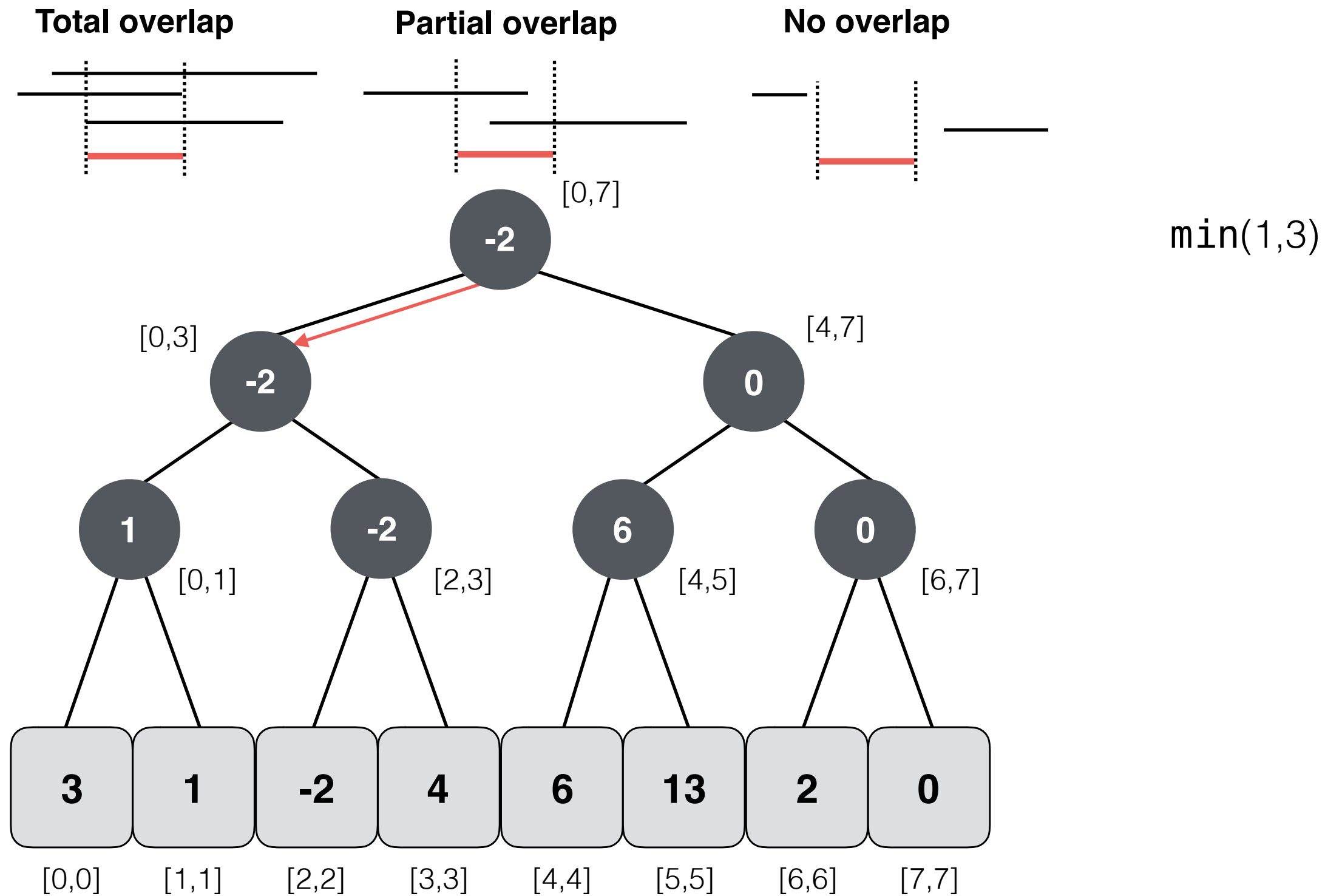
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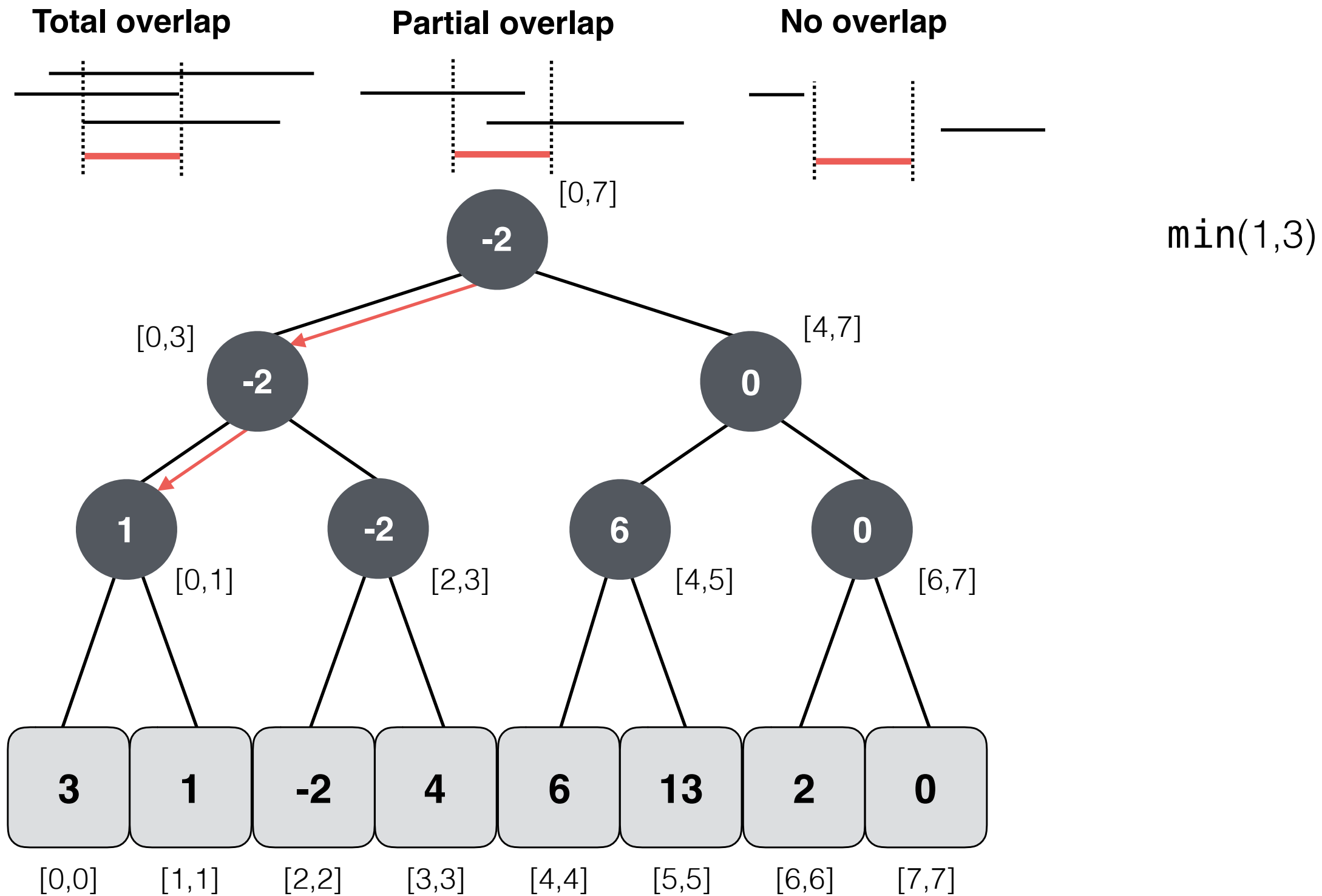
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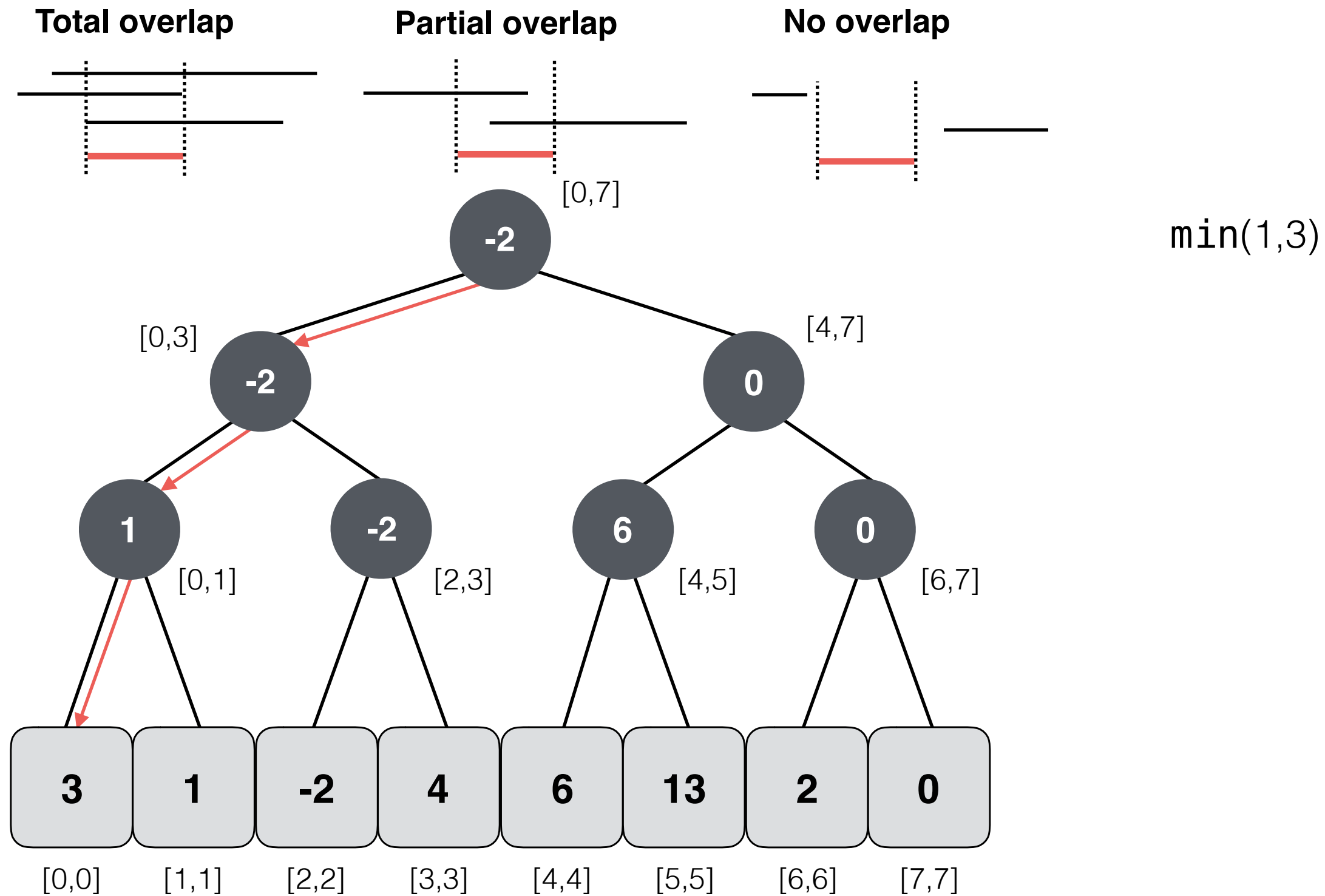
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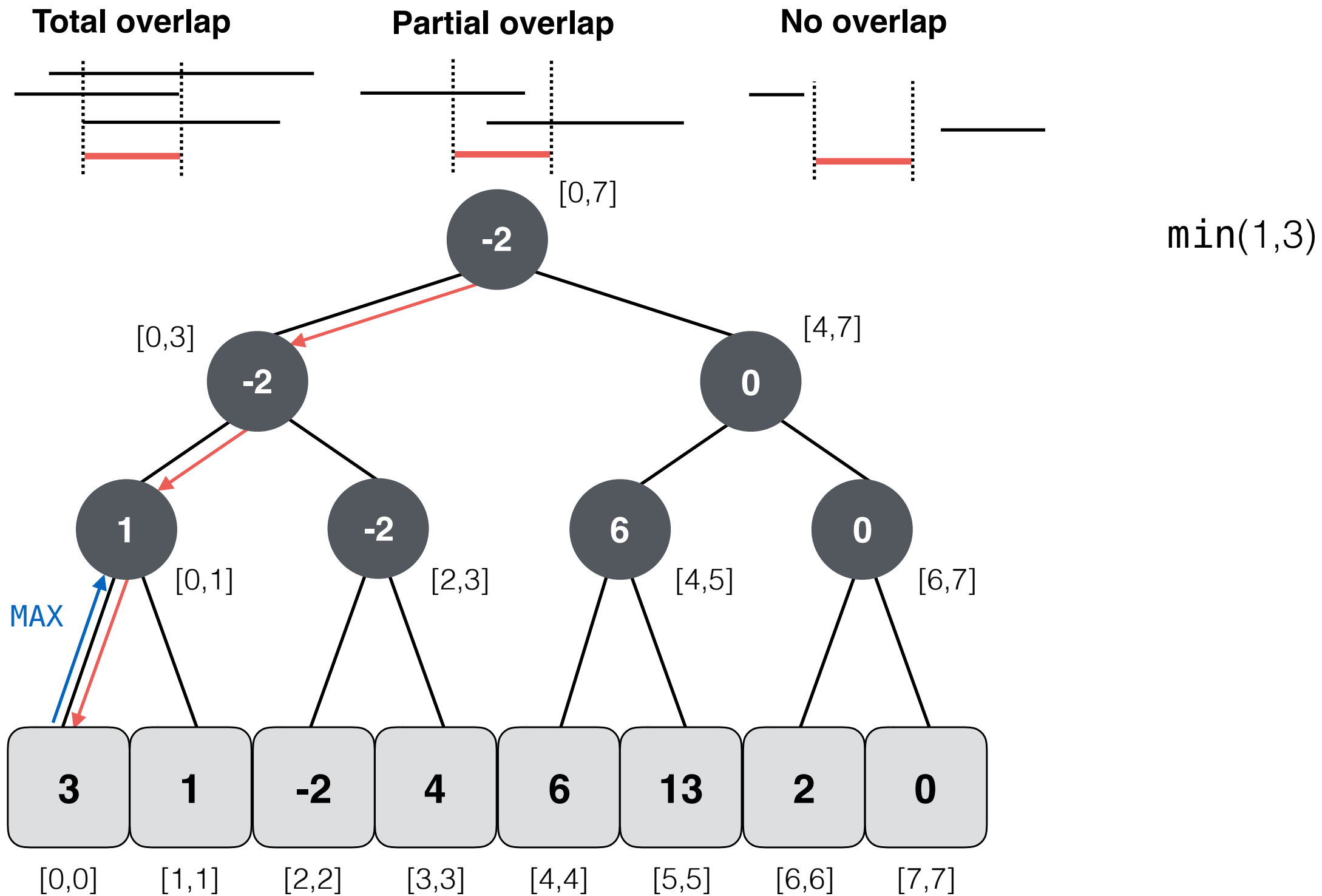
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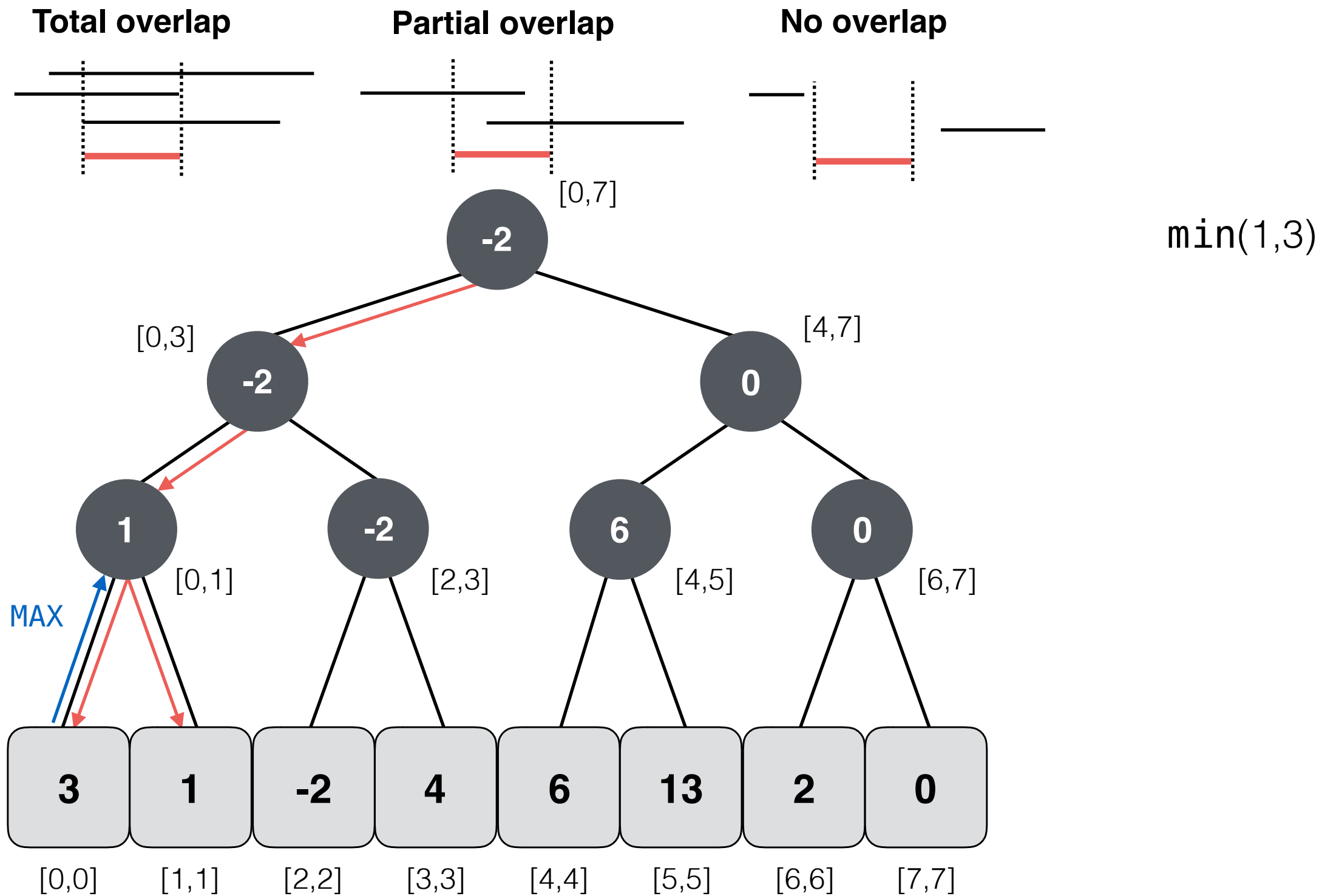
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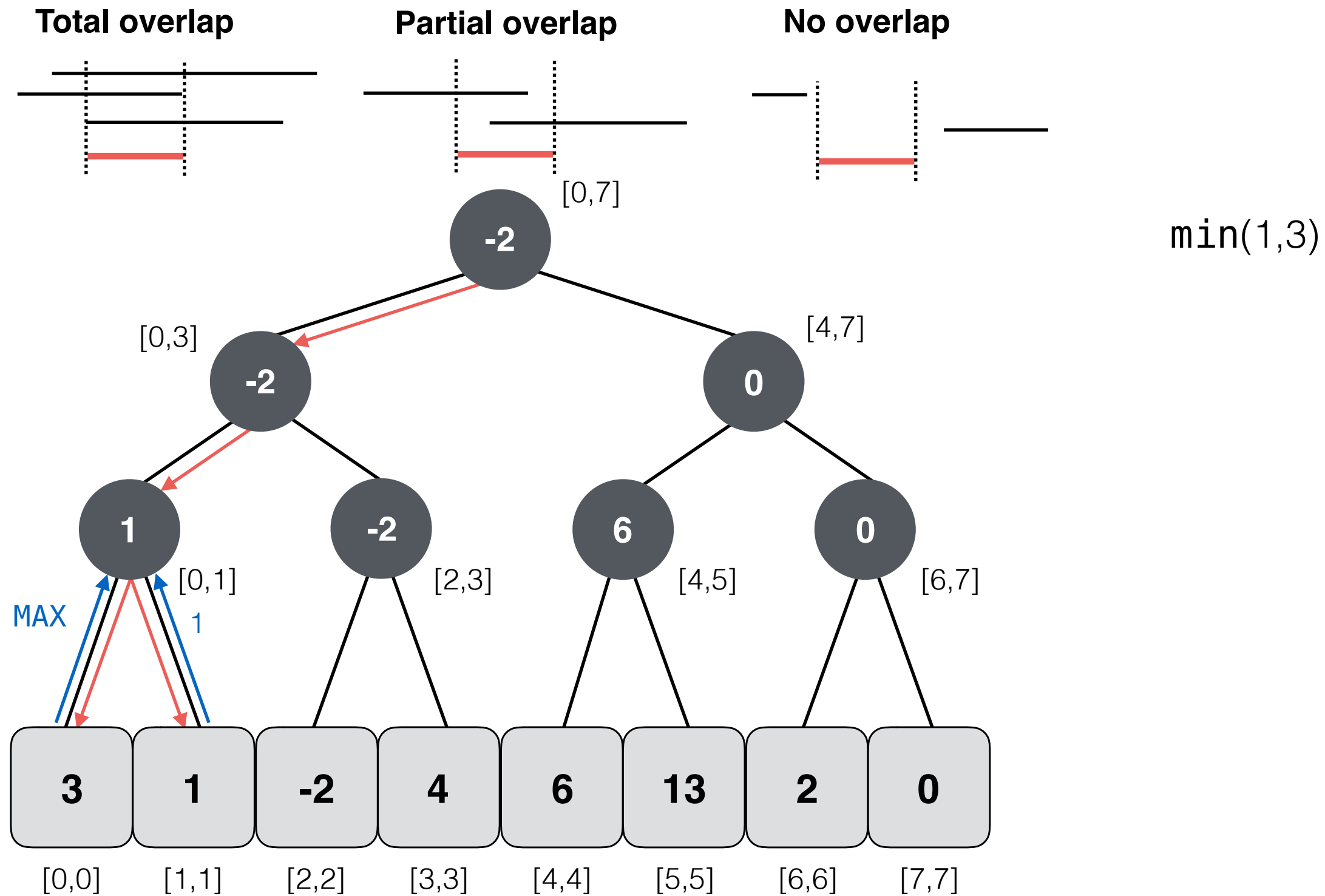
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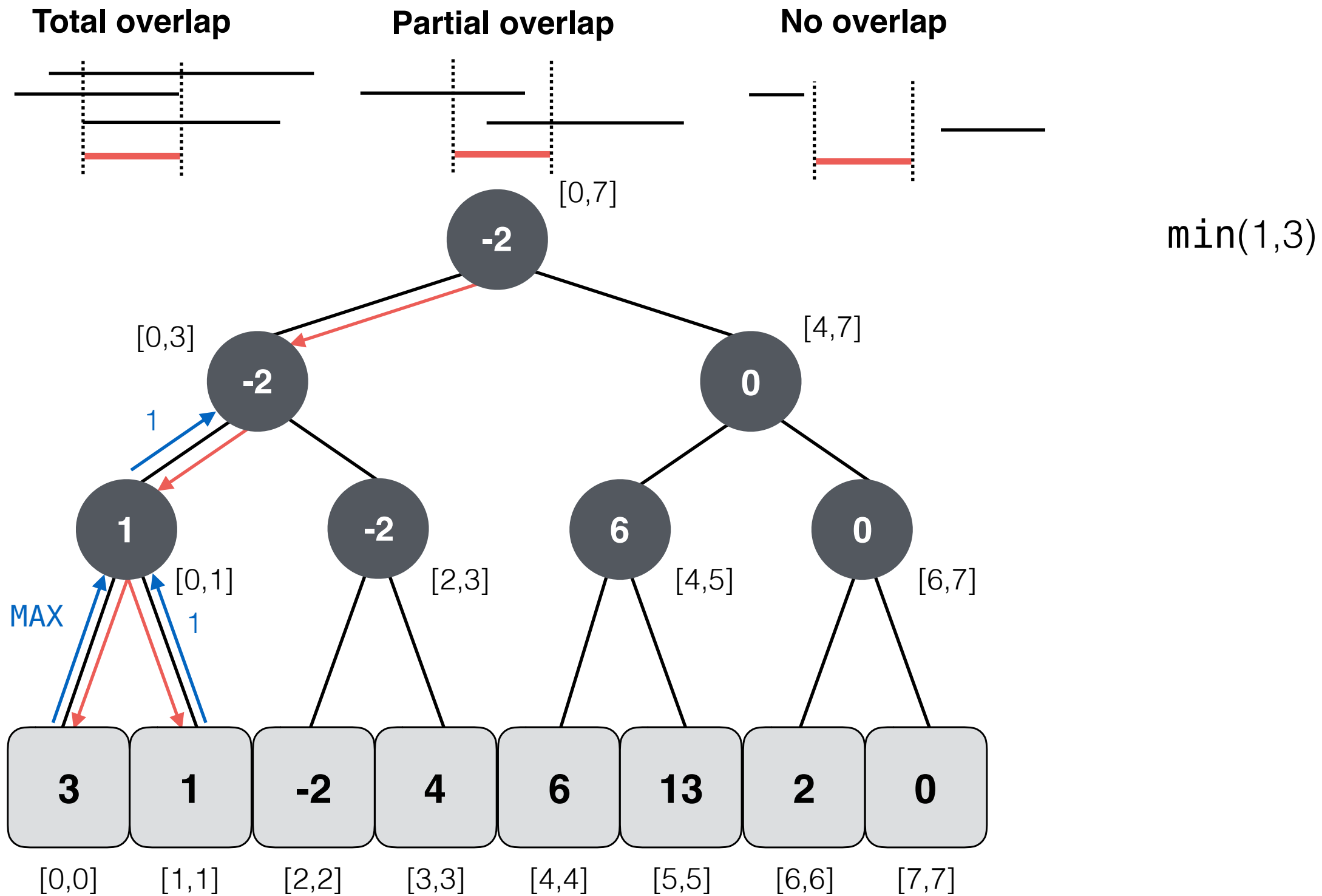
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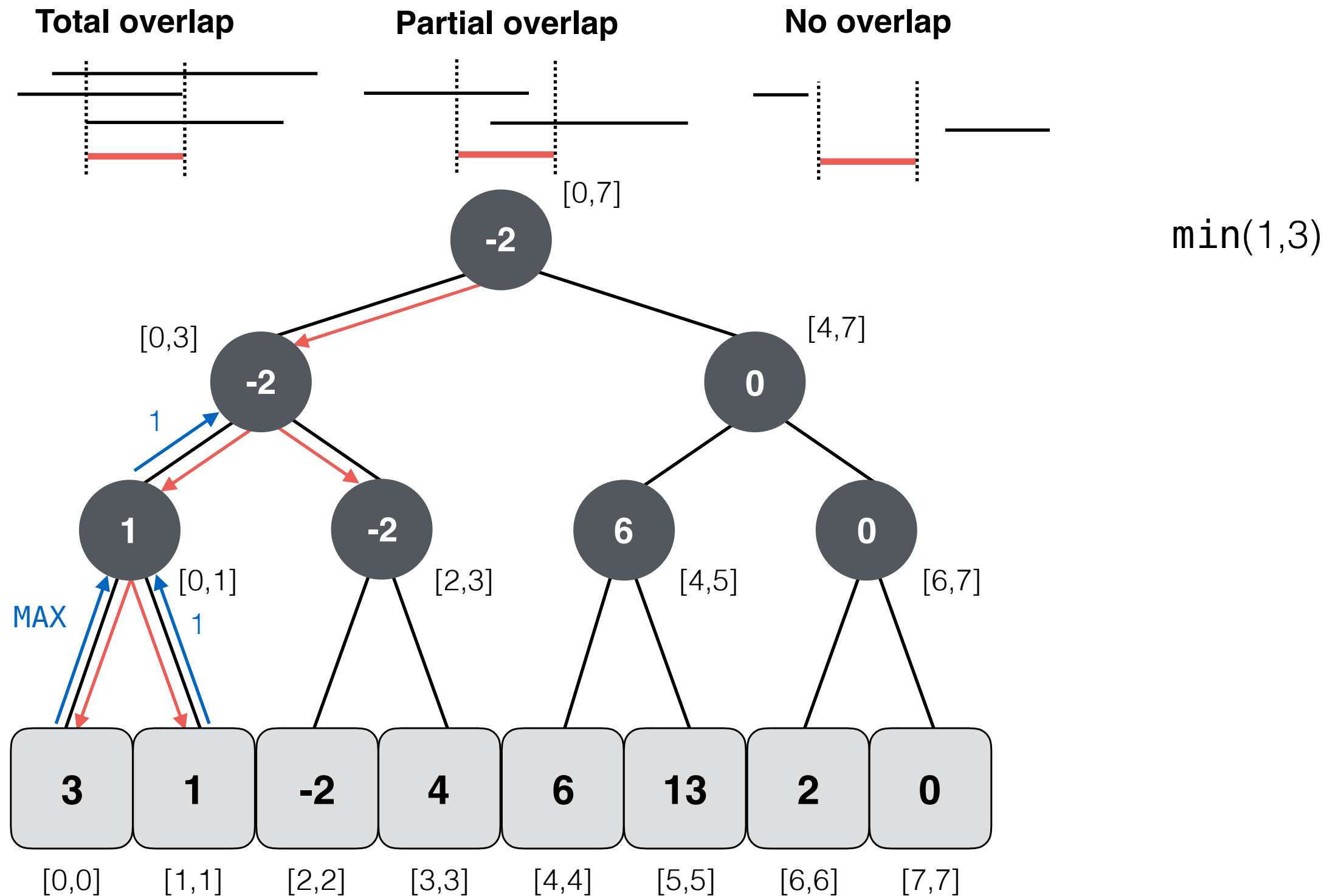
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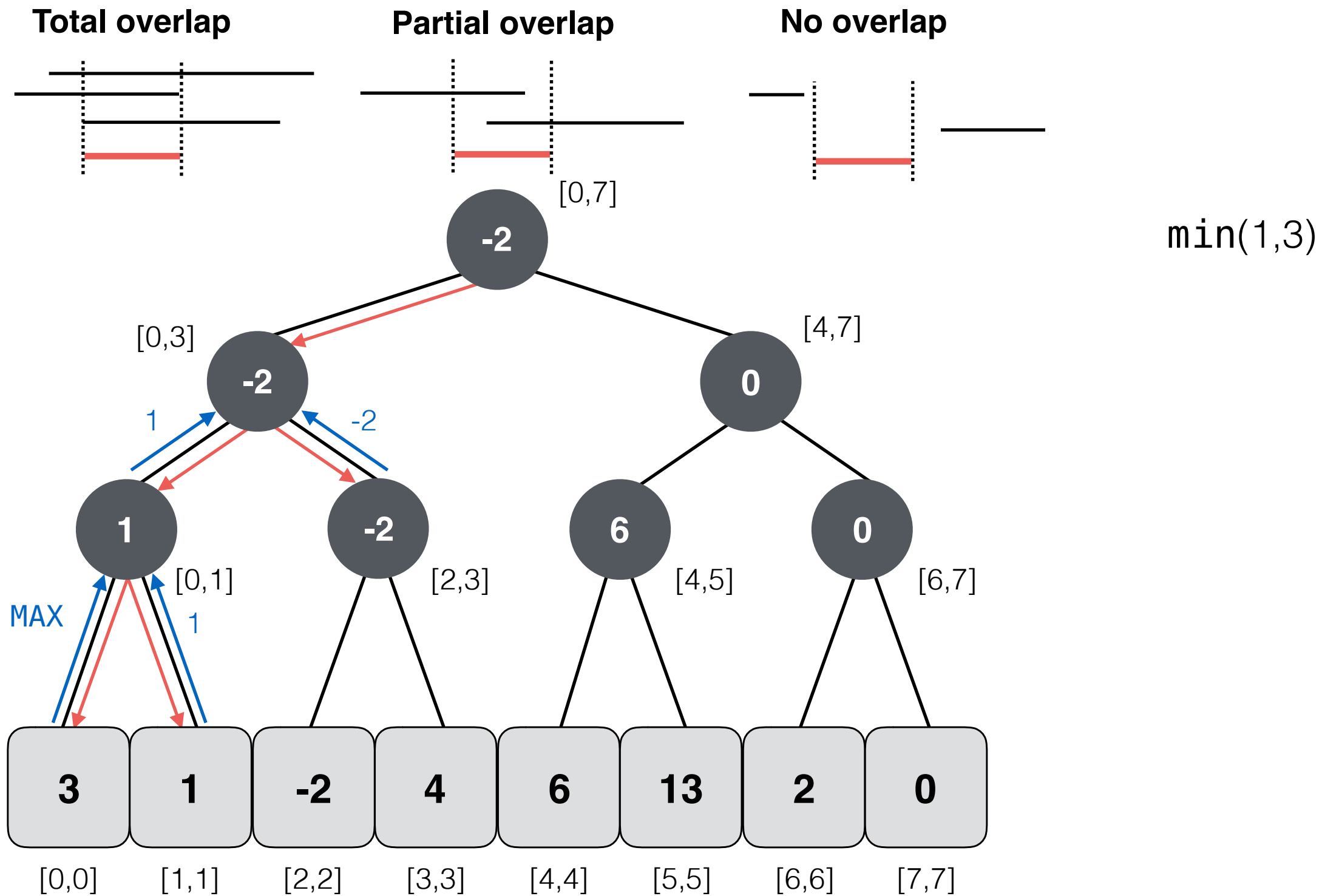
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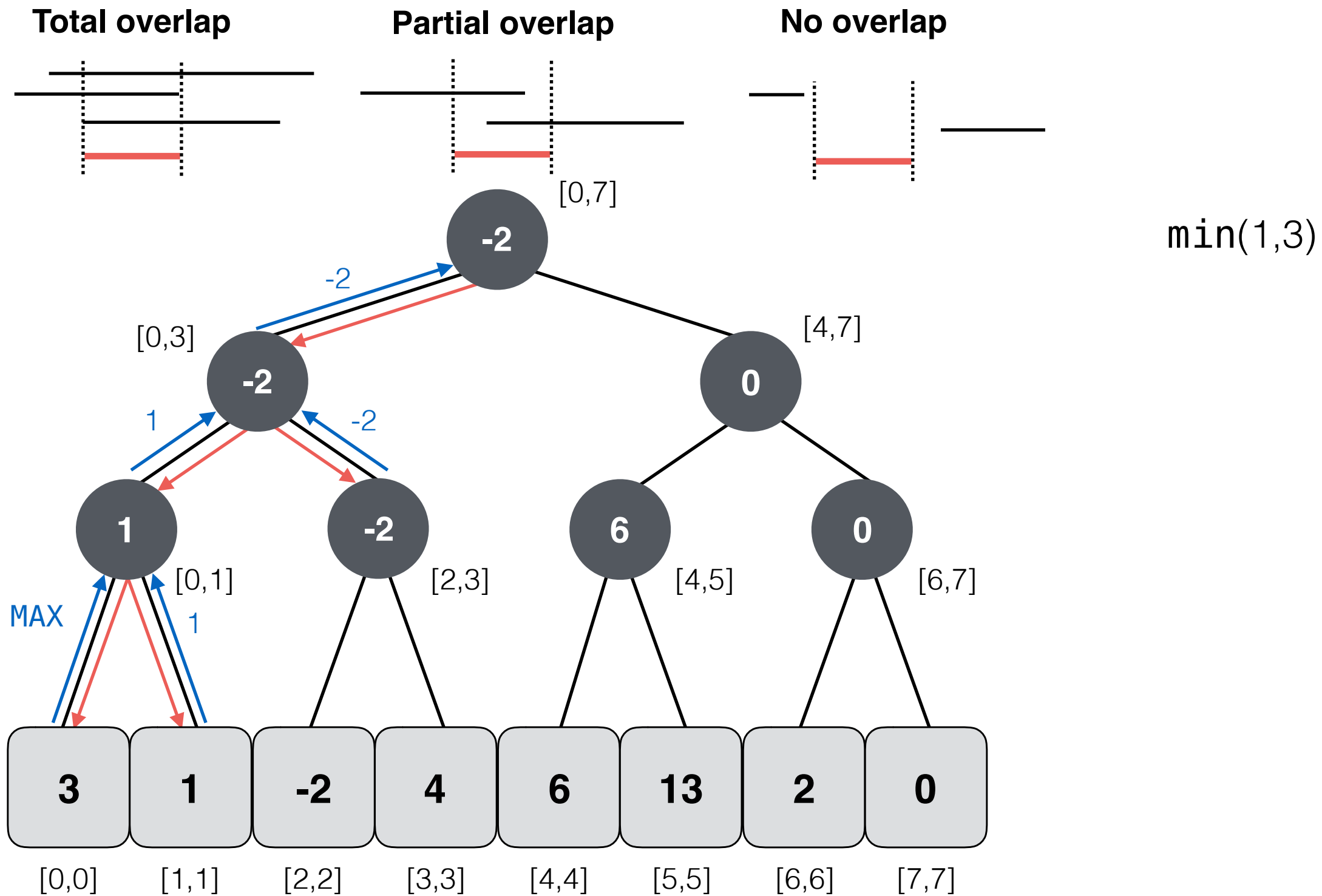
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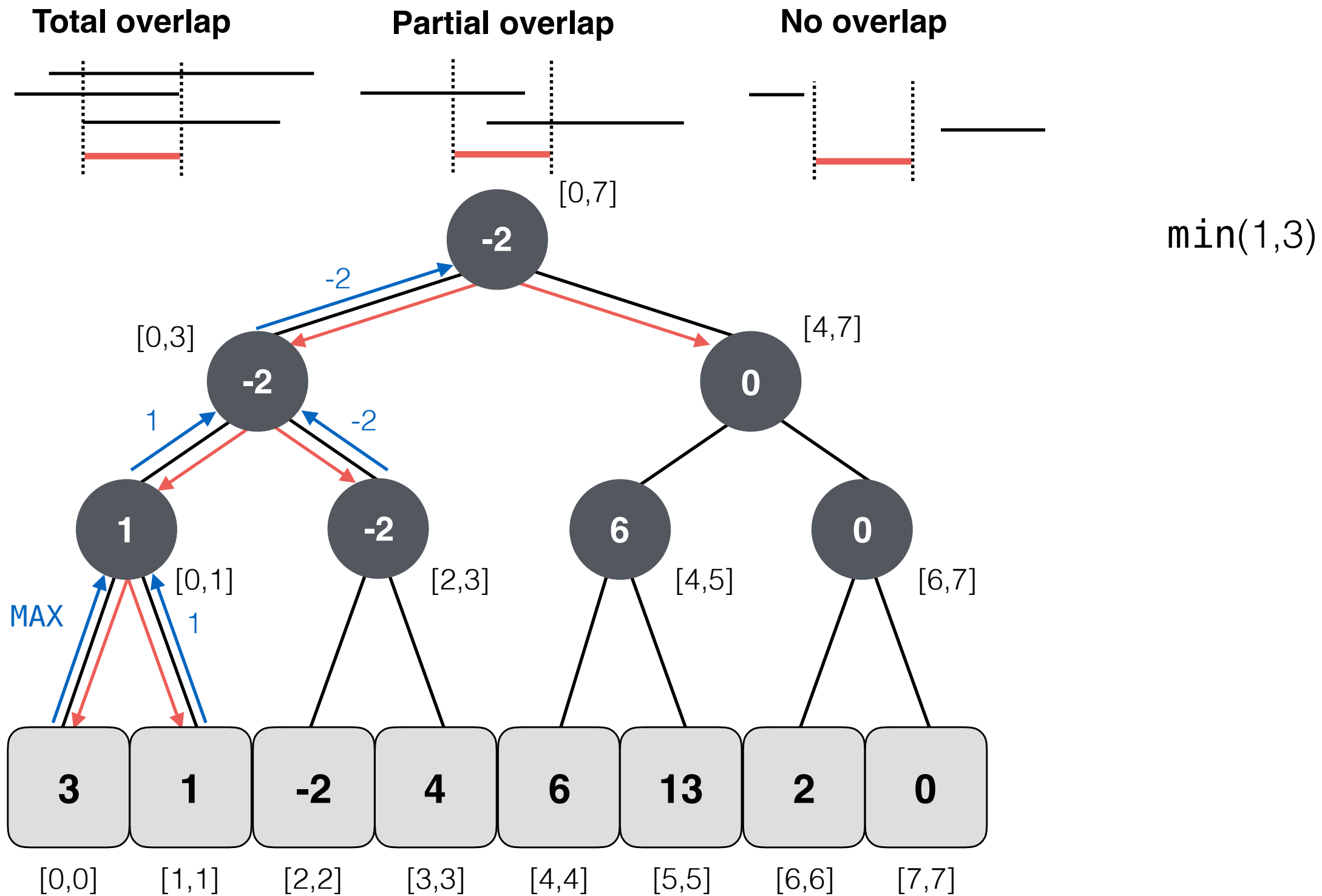
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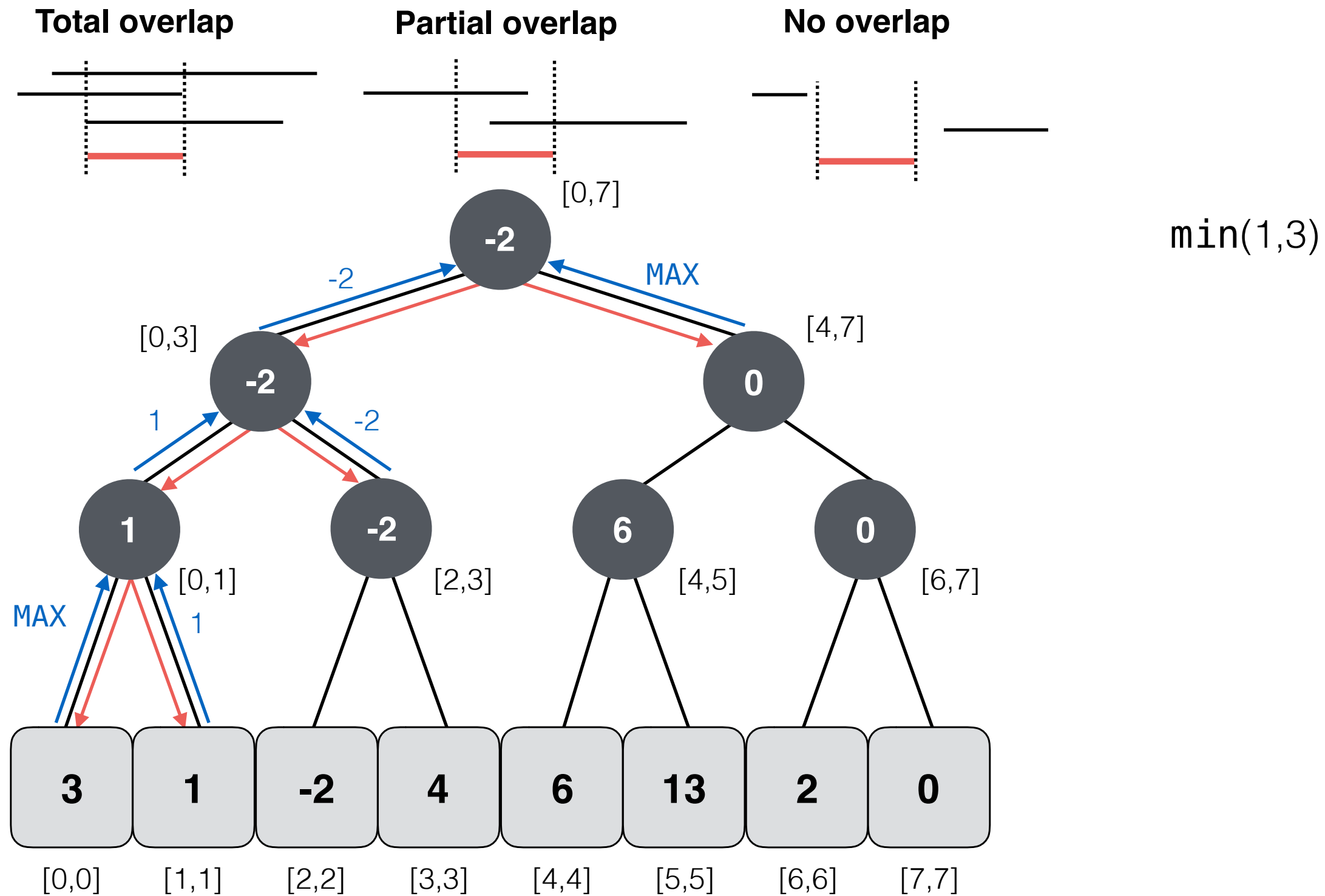
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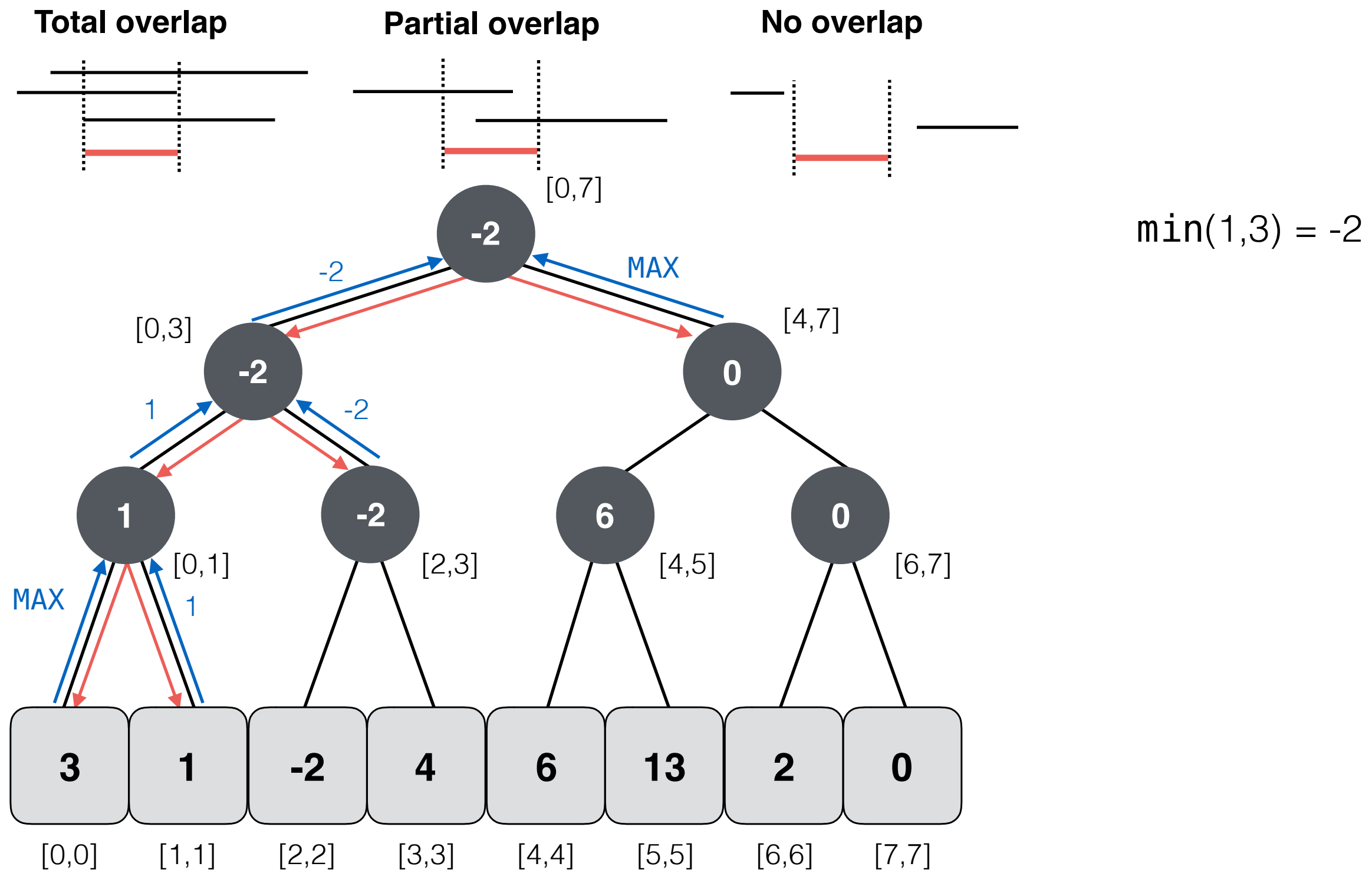
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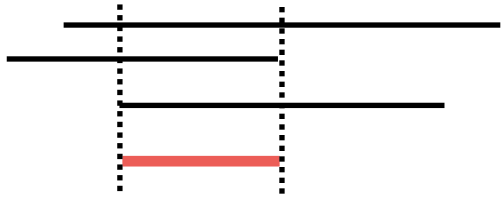
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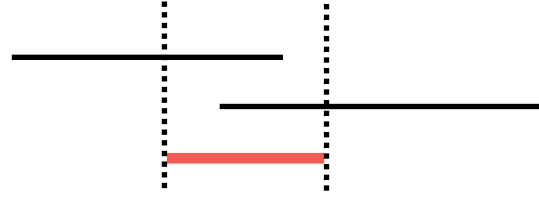
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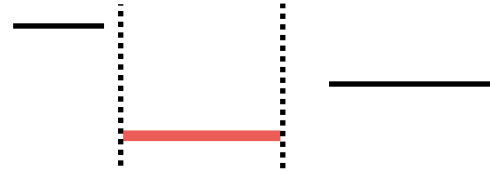
Total overlap



Partial overlap



No overlap

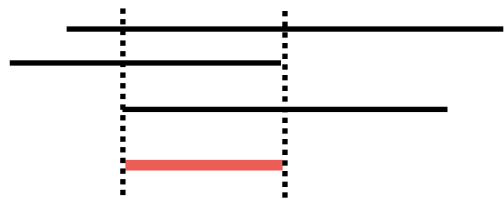


$$\min(1, 3) = -2$$

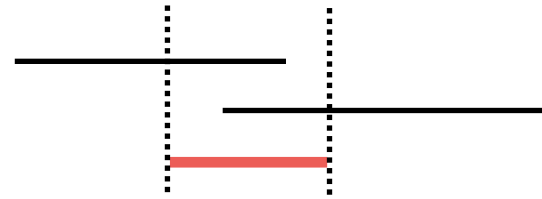
Range MIN Queries with Segment Trees

Consider a segment tree with n leaves ($2n - 1$ nodes in total).

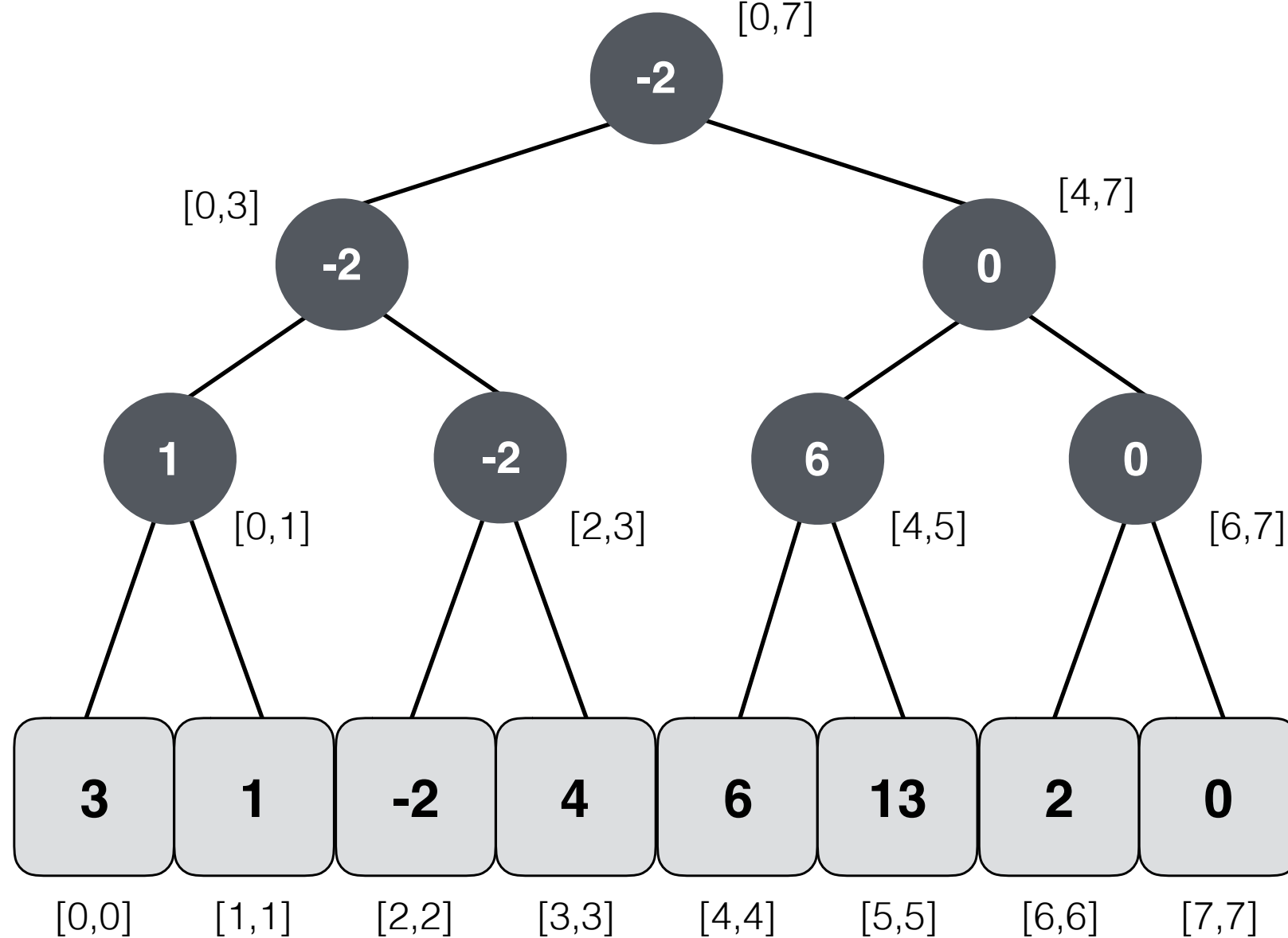
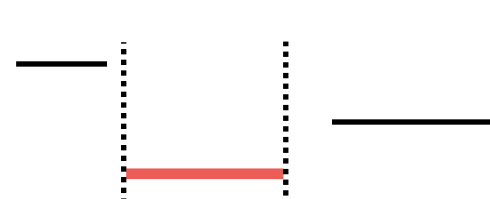
Total overlap



Partial overlap



No overlap

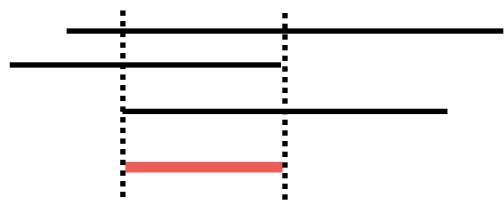


$$\min(1,3) = -2$$

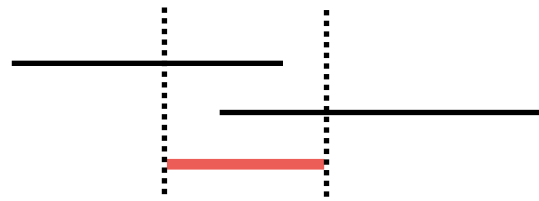
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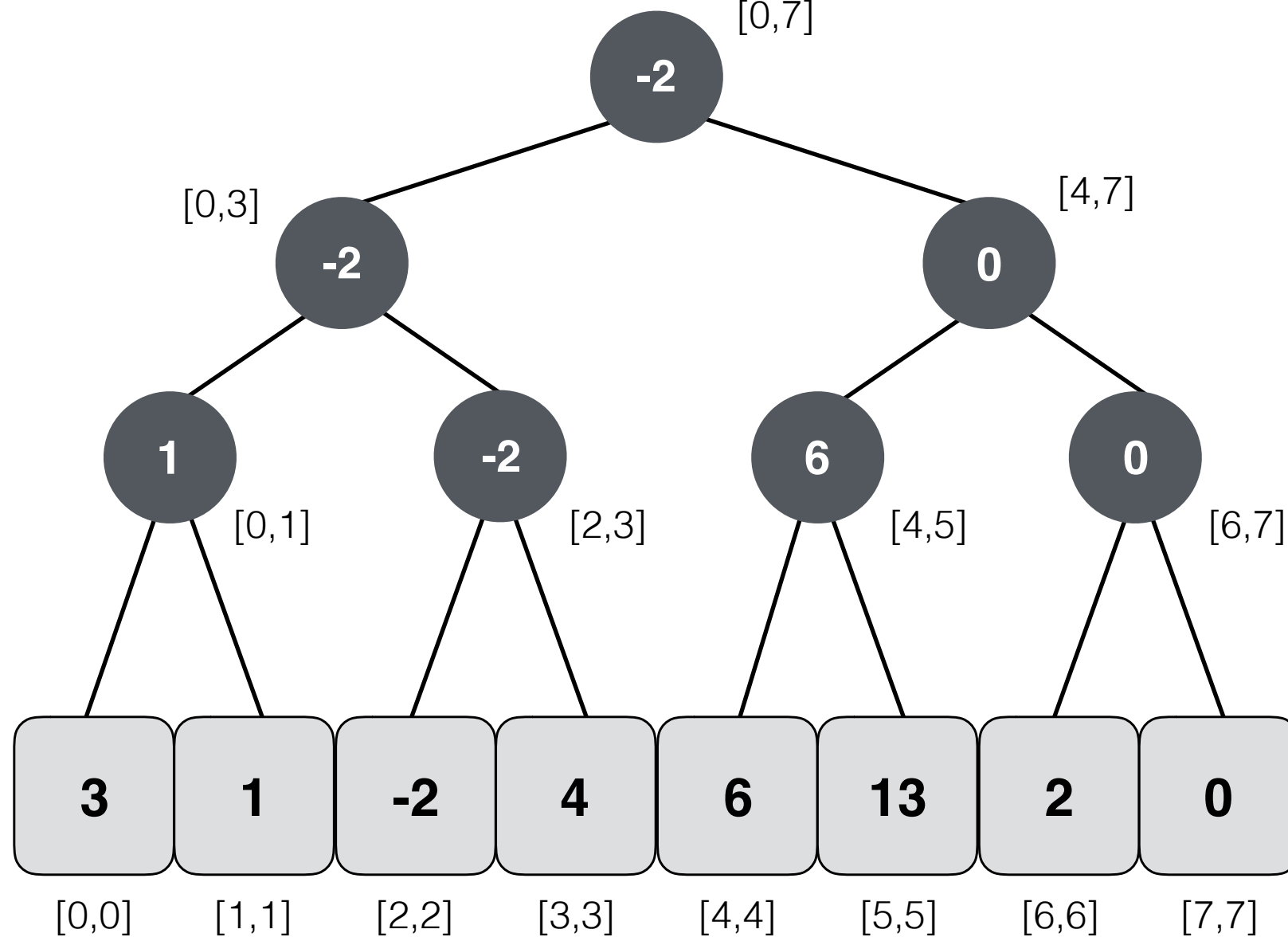
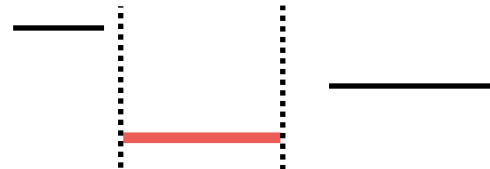
Total overlap



Partial overlap



No overlap



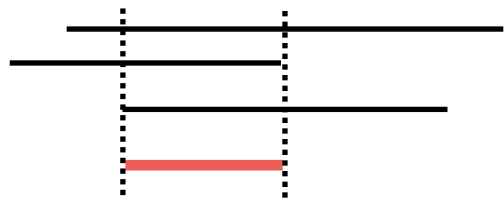
$\min(1,3) = -2$

$\min(3,6)$

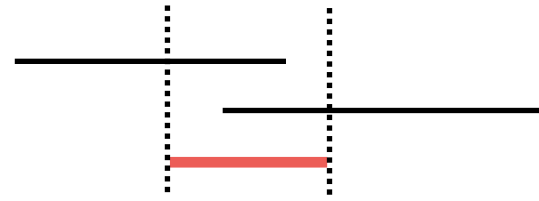
Range MIN Queries with Segment Trees

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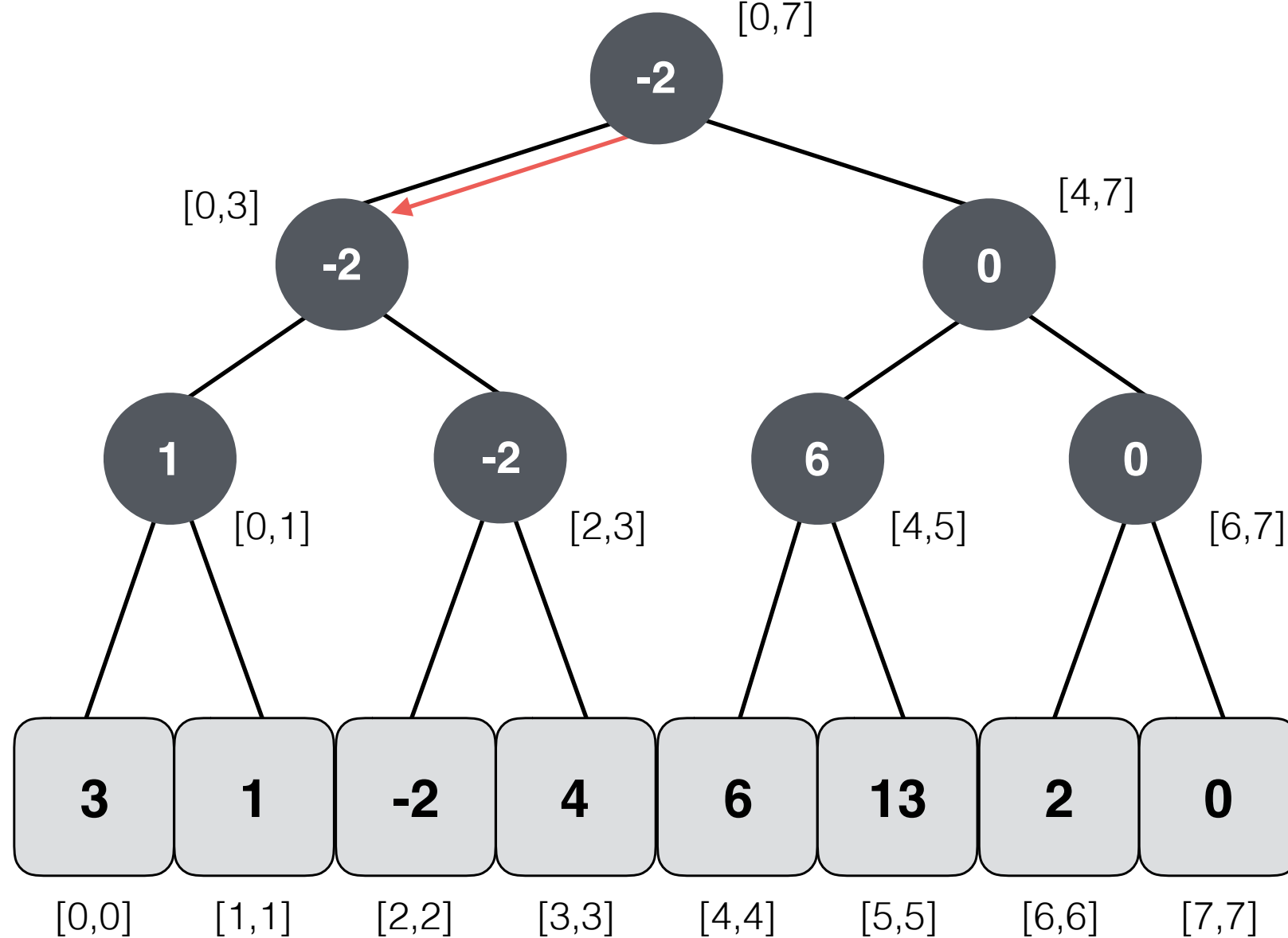
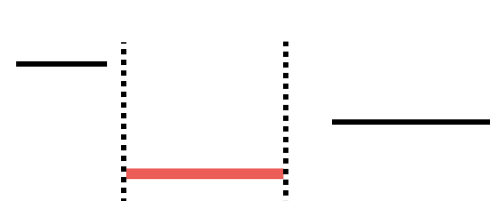
Total overlap



Partial overlap



No overlap



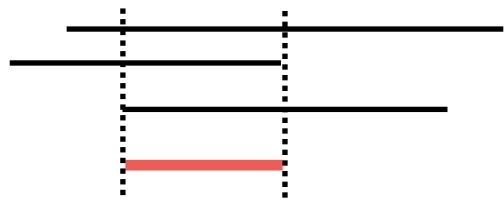
$\min(1,3) = -2$

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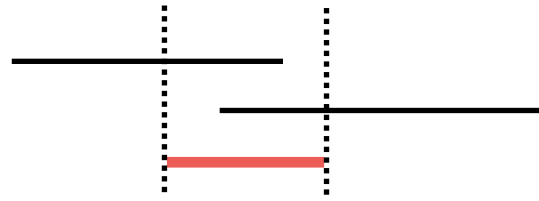
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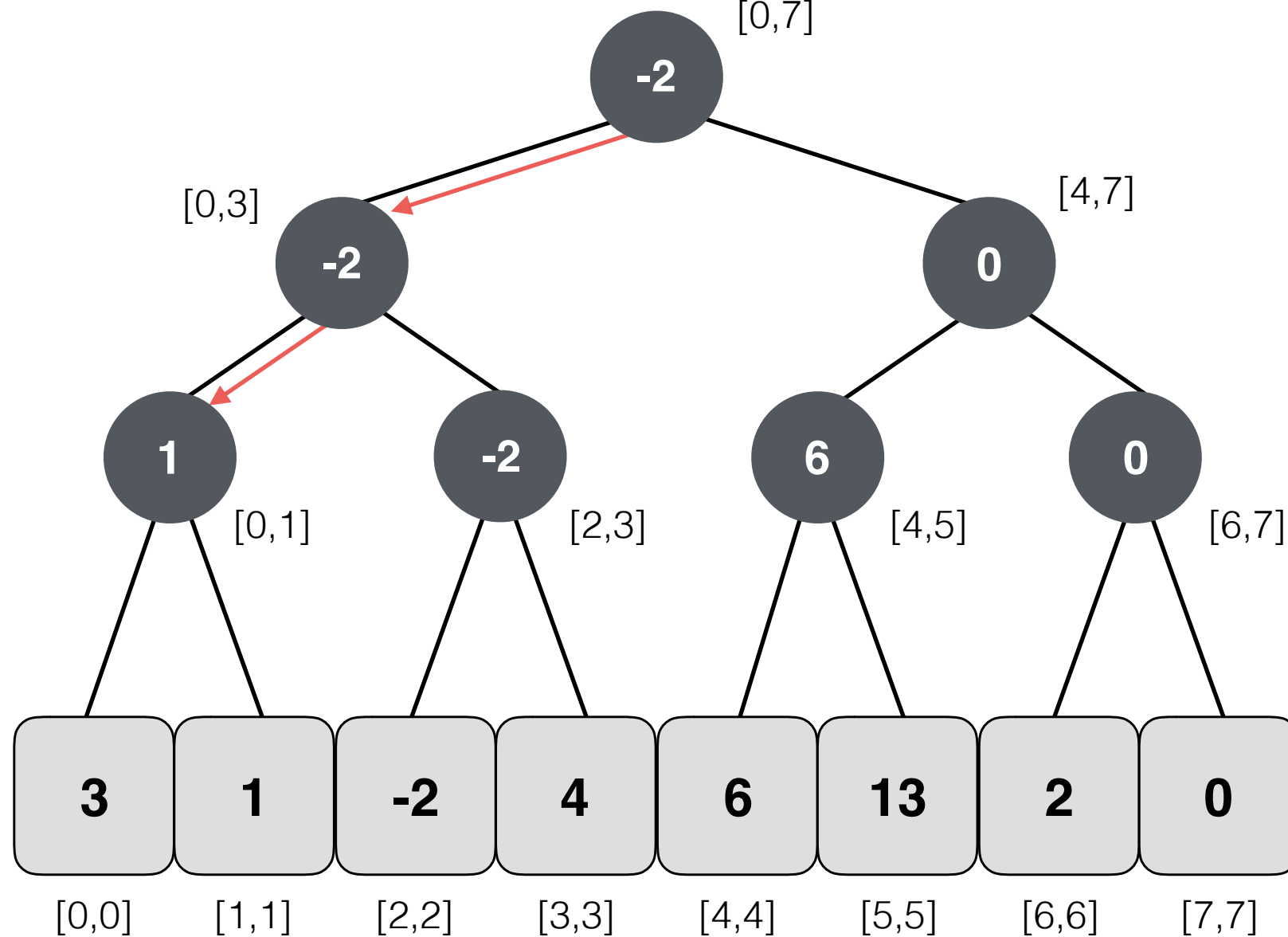
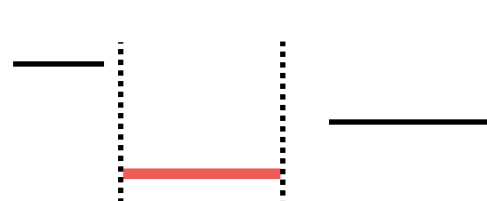
Total overlap



Partial overlap



No overlap



$\min(1,3) = -2$

$\min(3,6)$

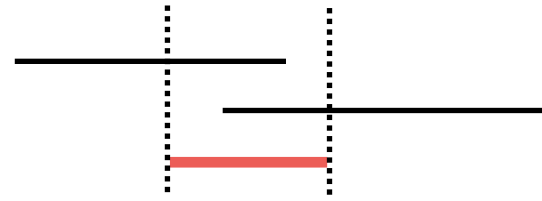
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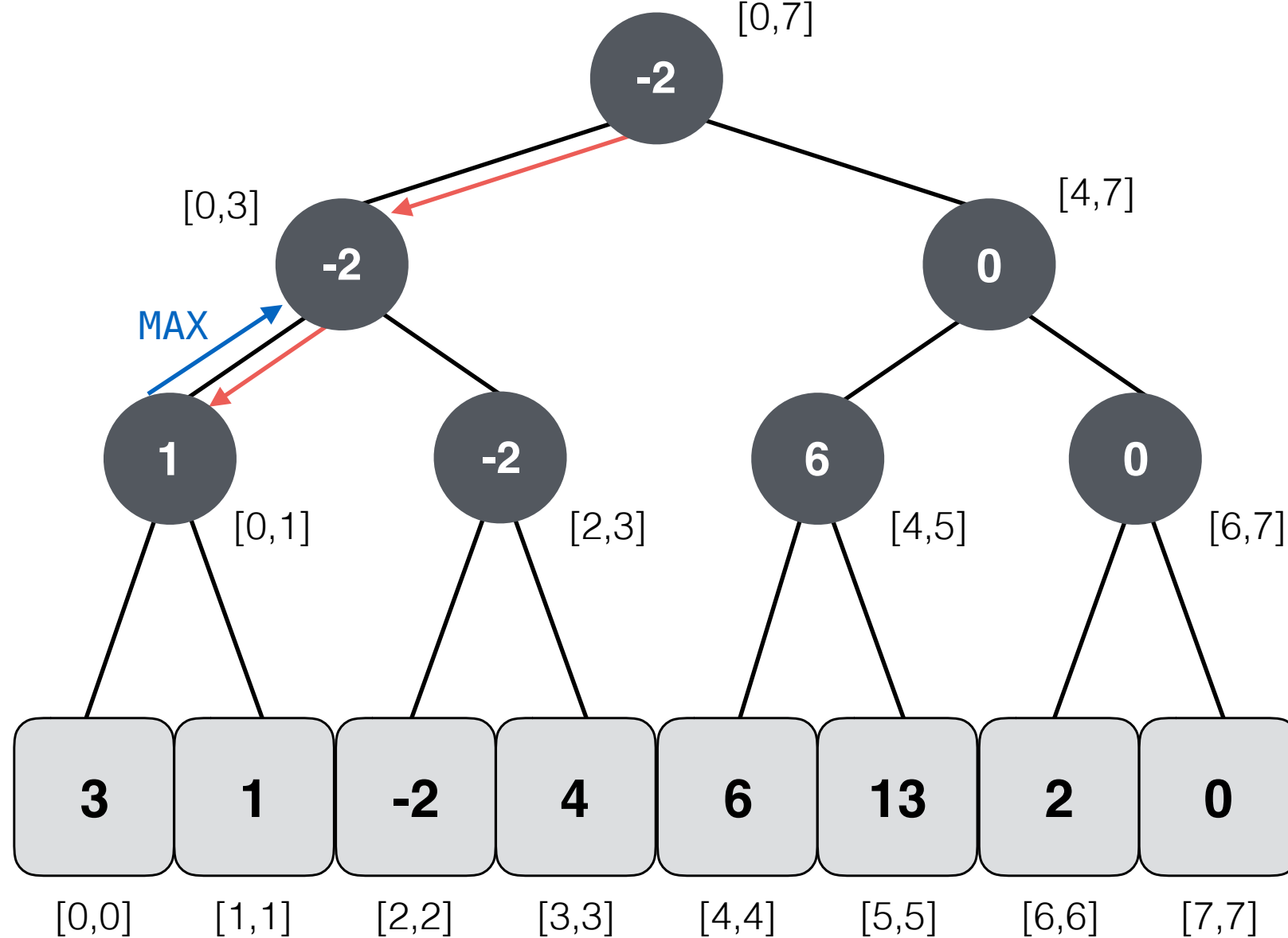
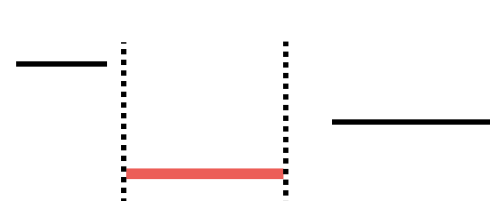
Total overlap



Partial overlap



No overlap



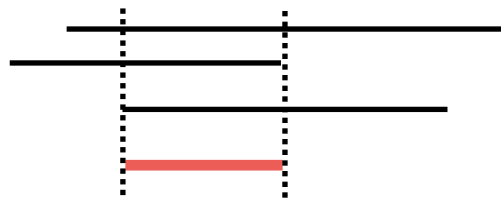
$\min(1,3) = -2$

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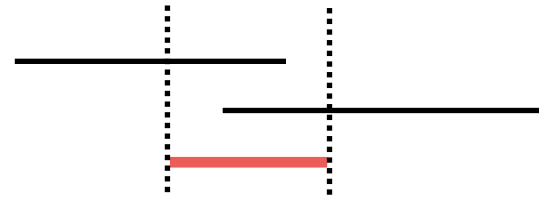
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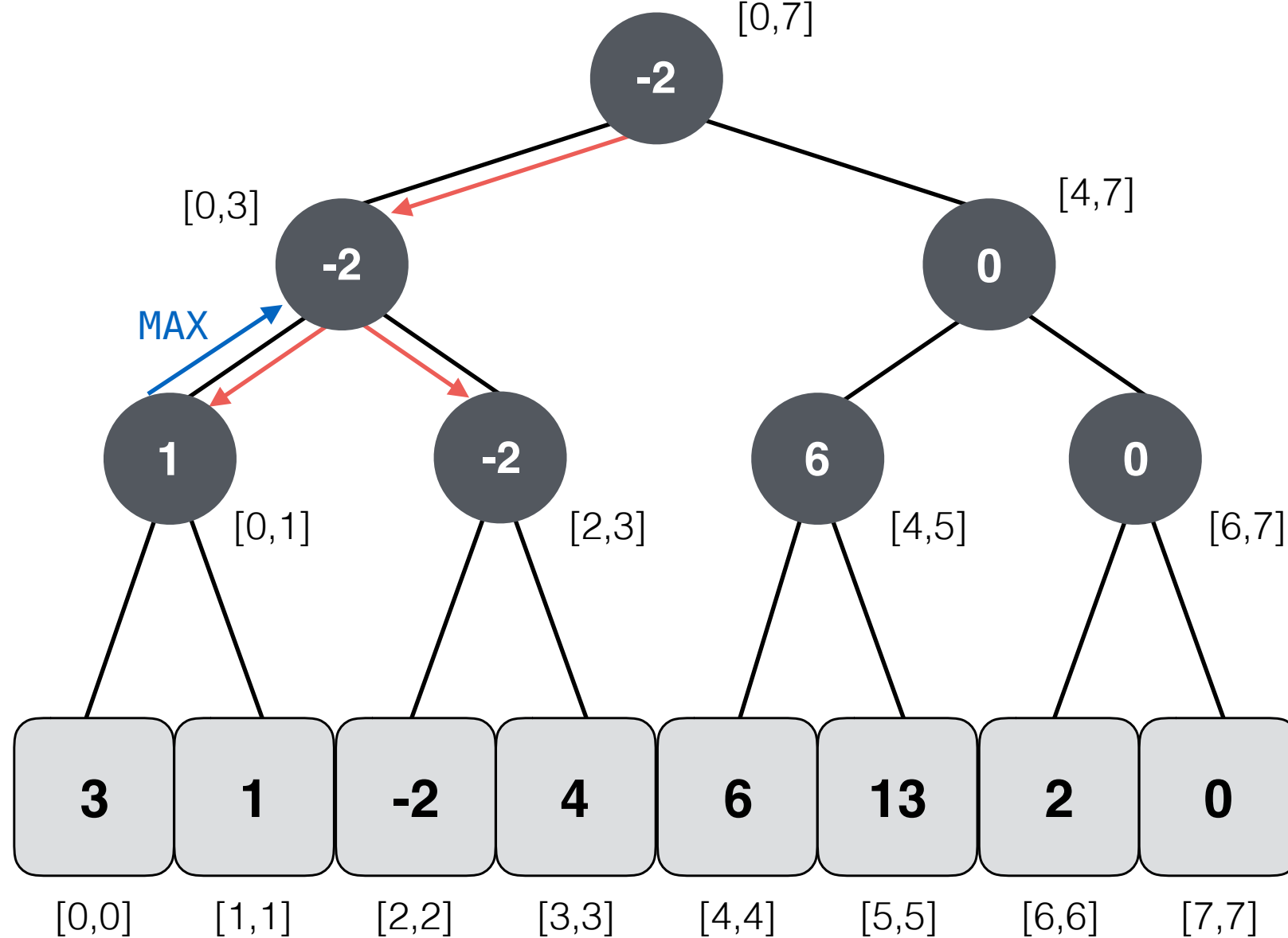
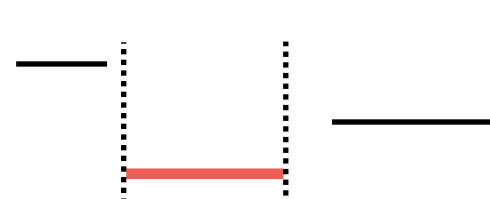
Total overlap



Partial overlap



No overlap



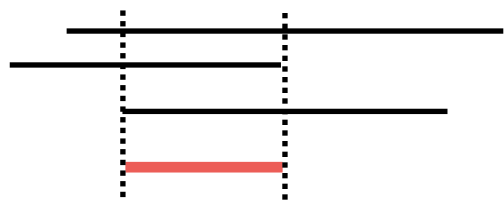
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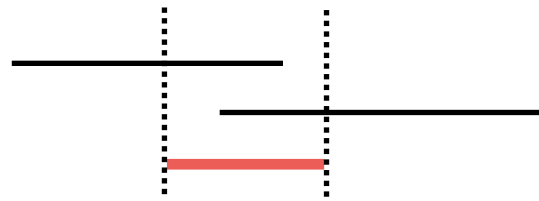
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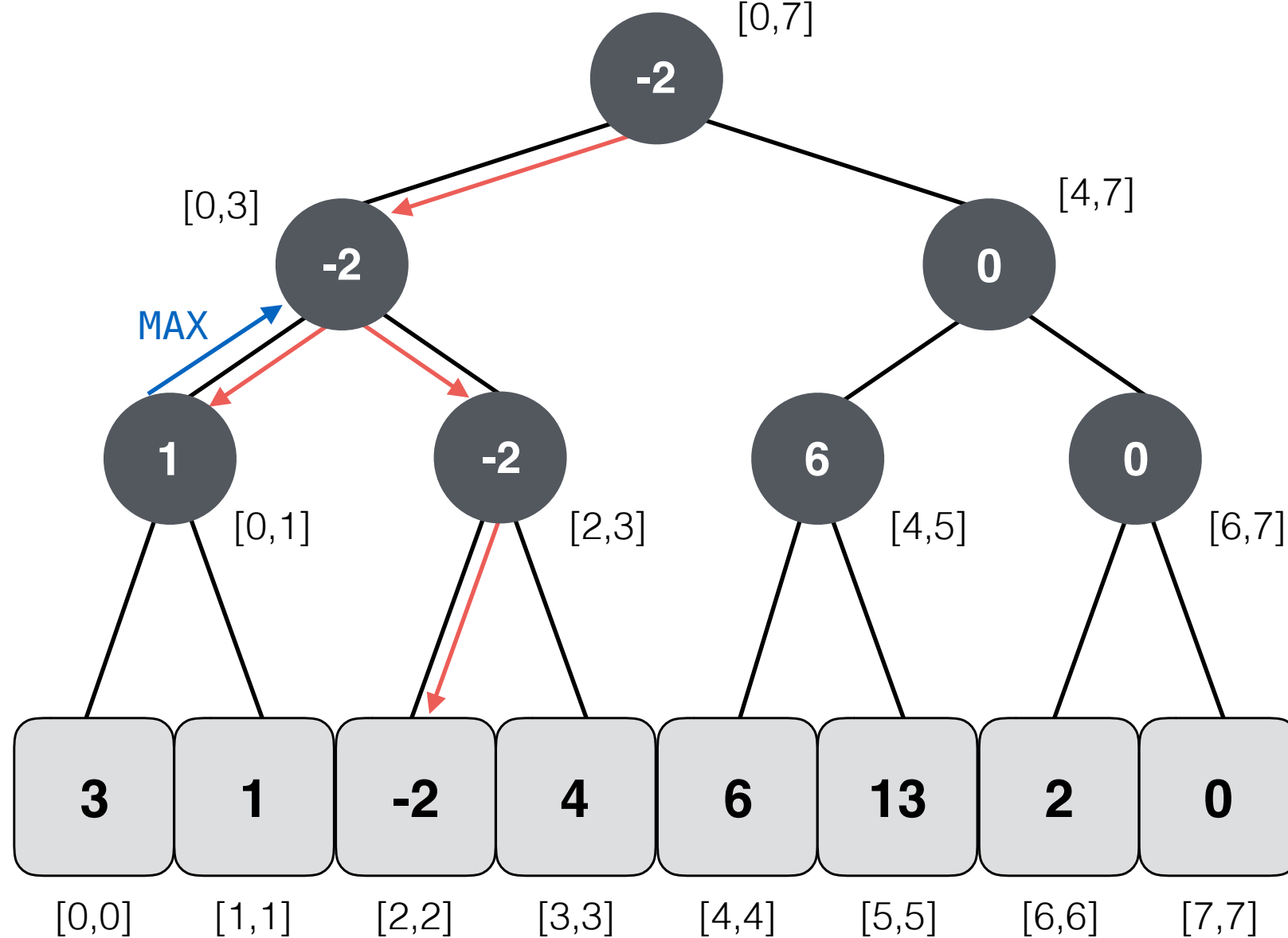
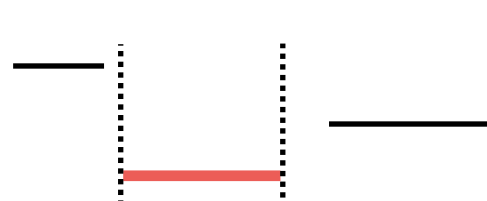
Total overlap



Partial overlap



No overlap

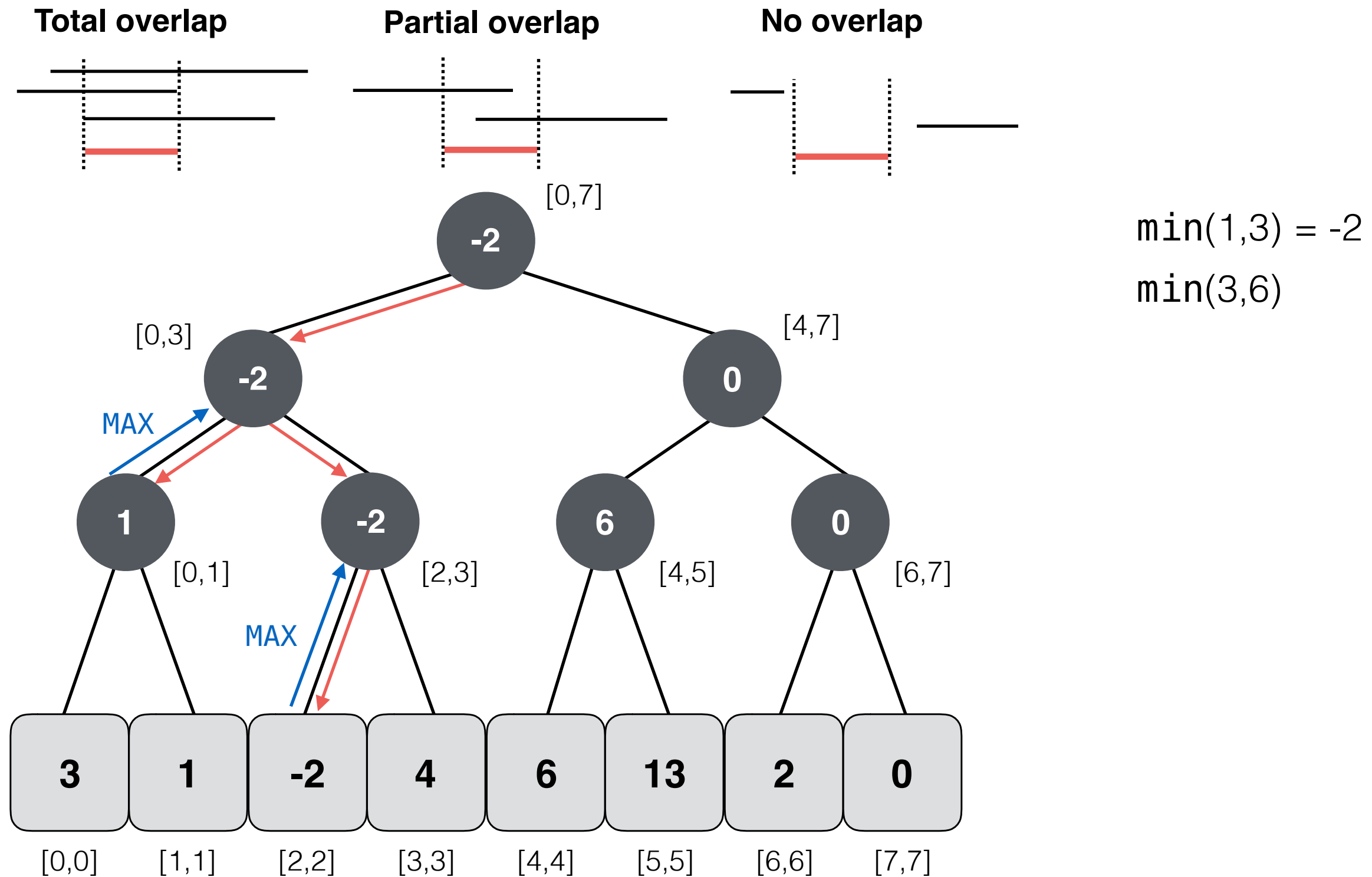


$\min(1,3) = -2$

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Range MIN Queries with Segment Trees

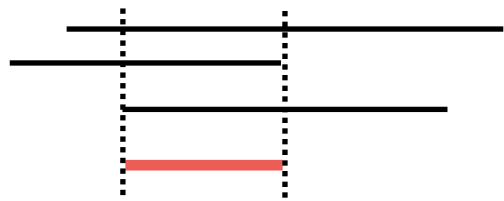
Consider a segment tree with n leaves ($2n - 1$ nodes in total).



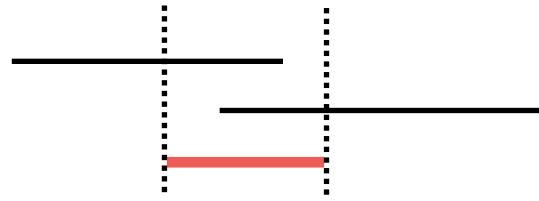
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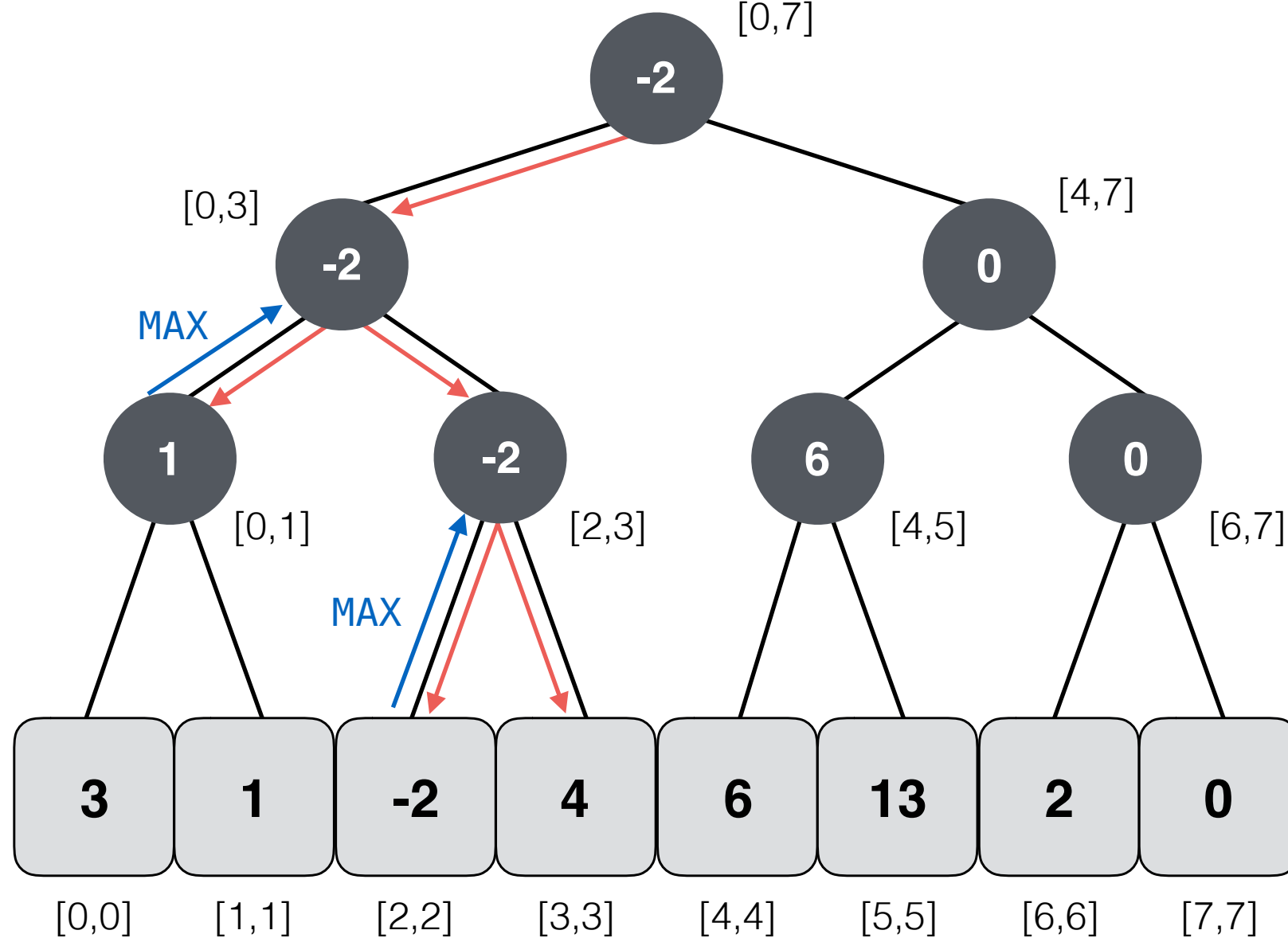
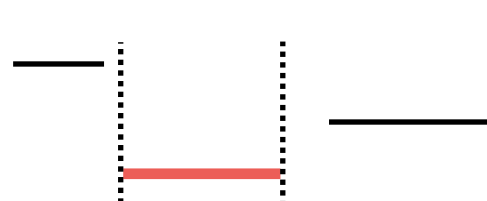
Total overlap



Partial overlap



No overlap

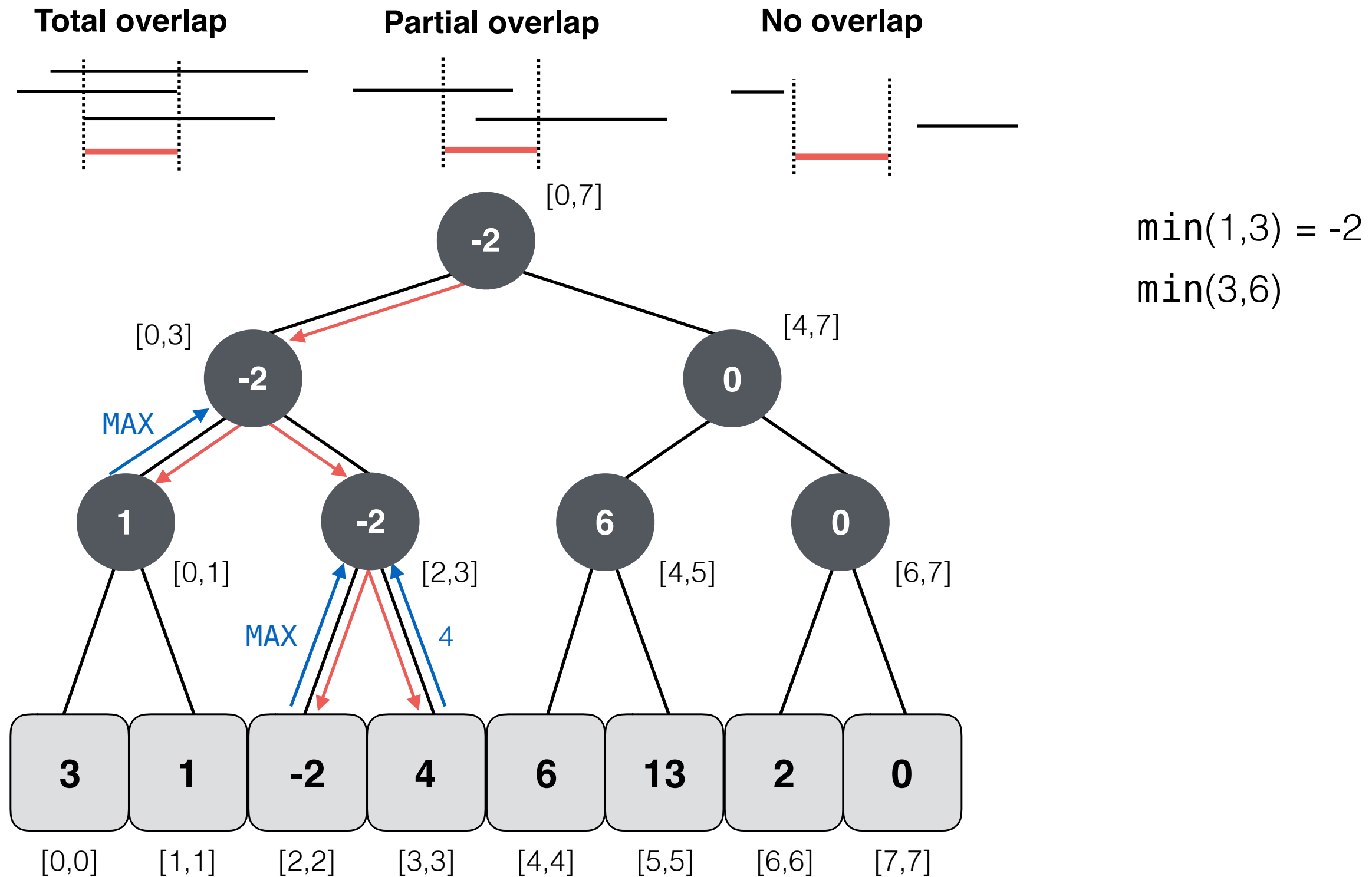


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Range MIN Queries with Segment Trees

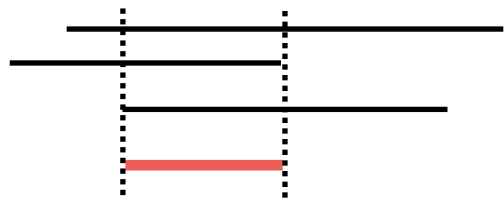
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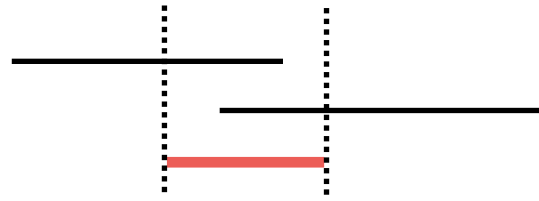
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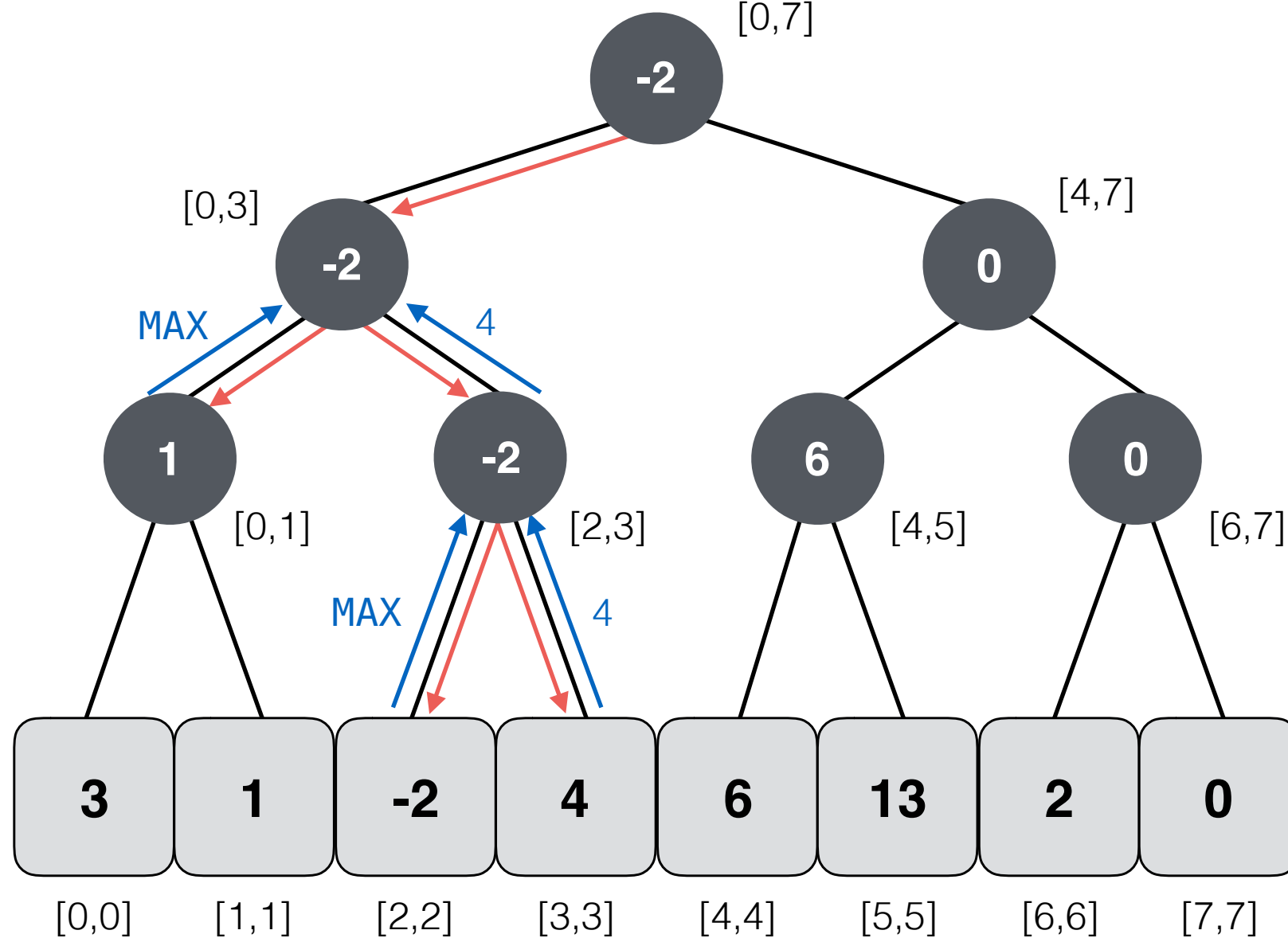
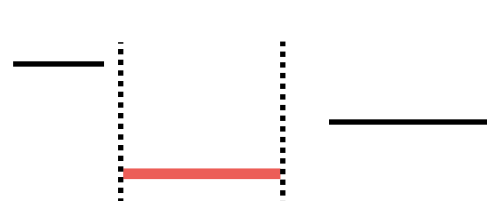
Total overlap



Partial overlap



No overlap



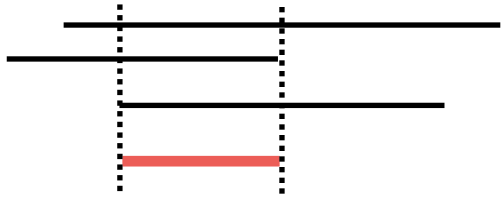
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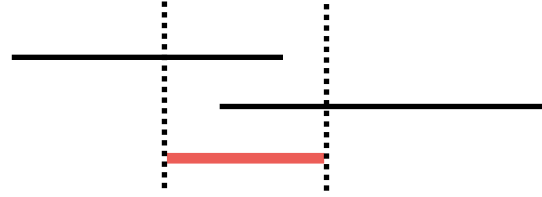
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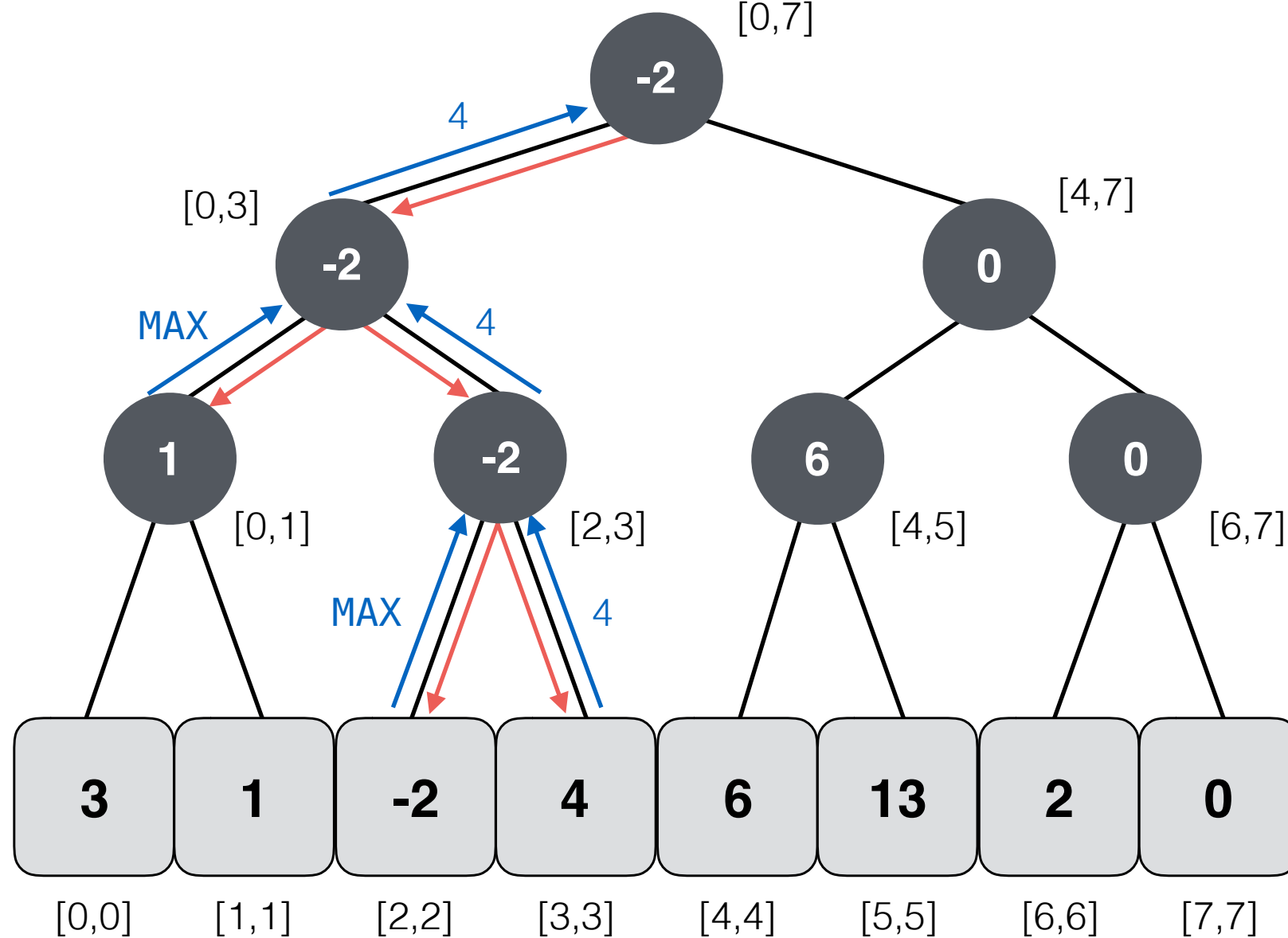
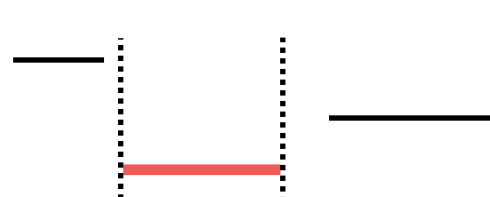
Total overlap



Partial overlap



No overlap



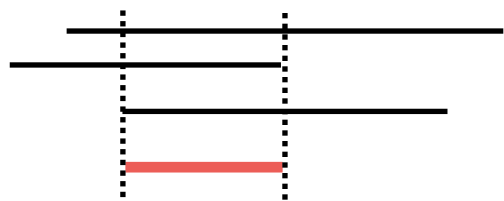
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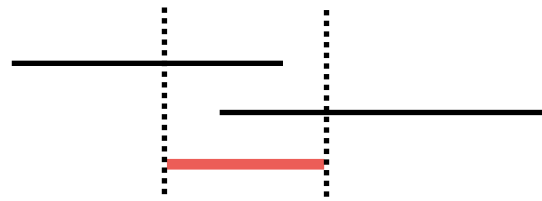
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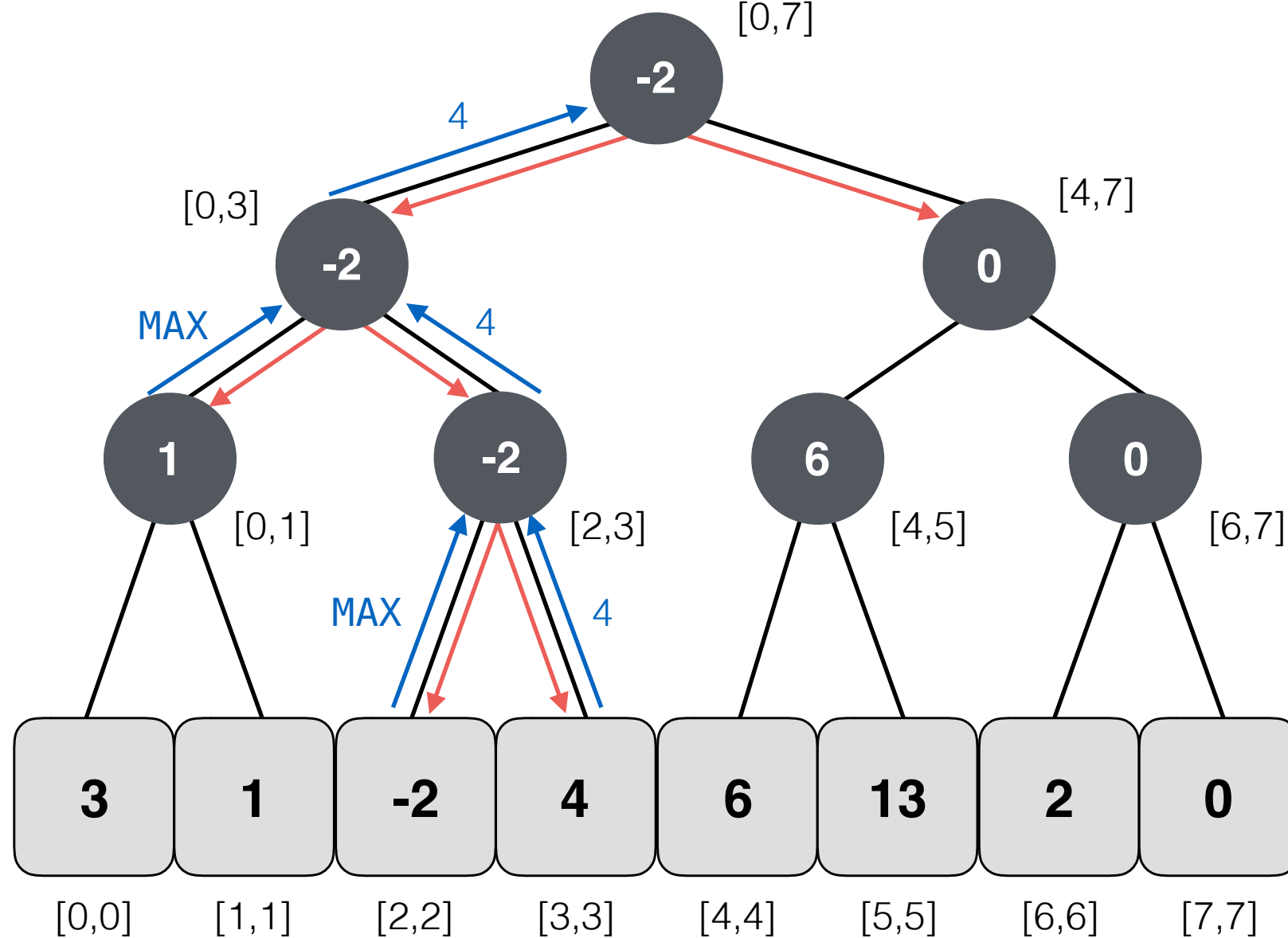
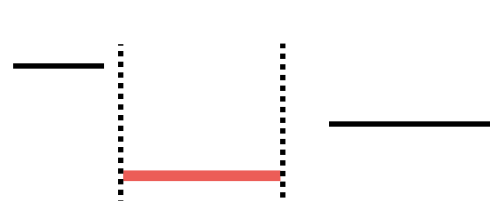
Total overlap



Partial overlap



No overlap



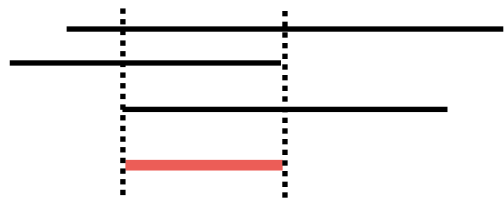
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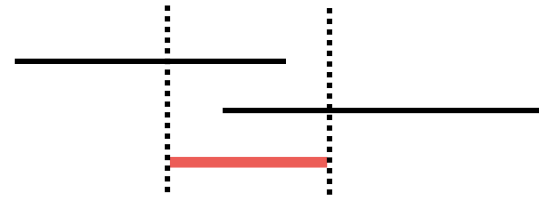
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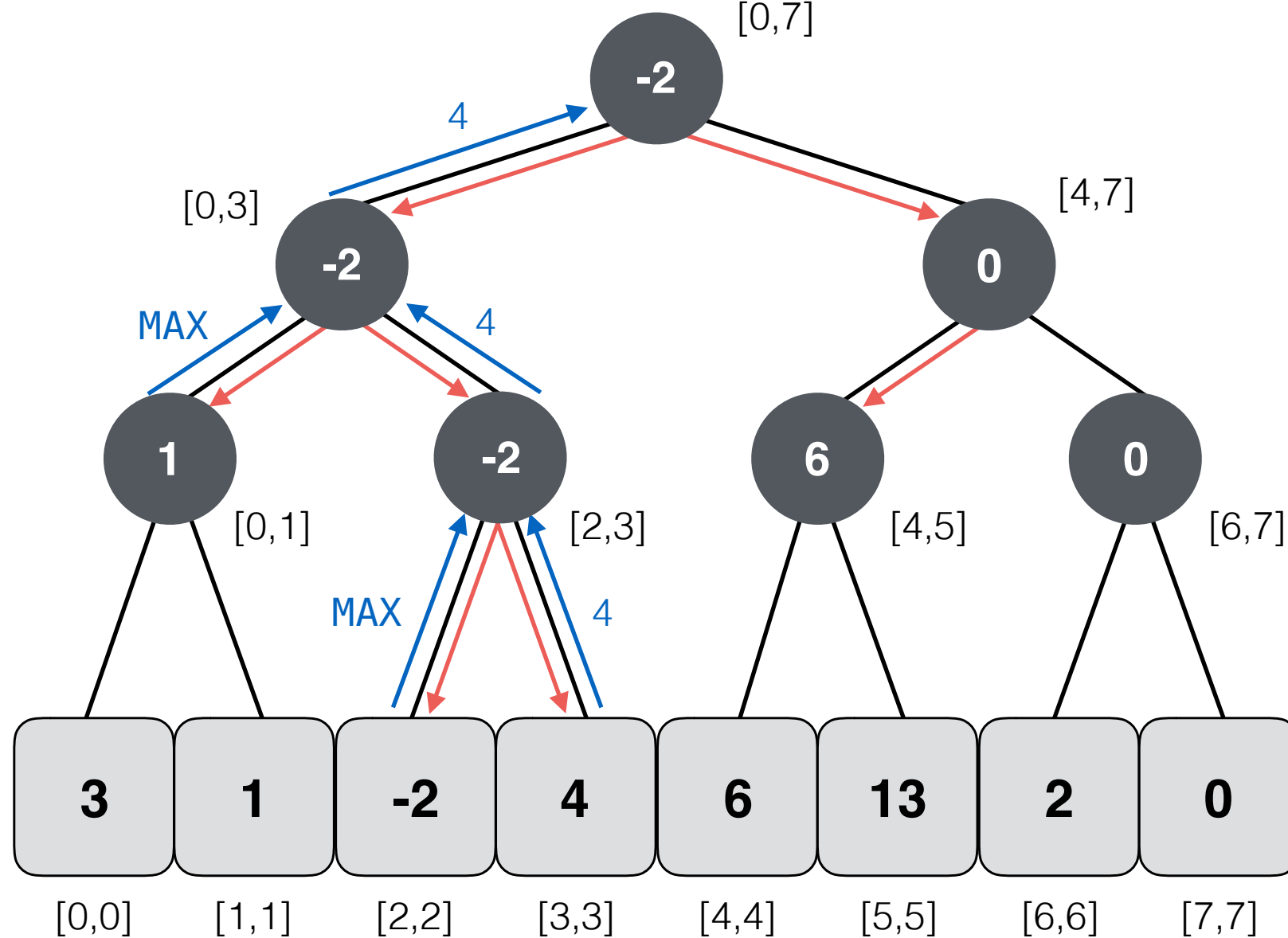
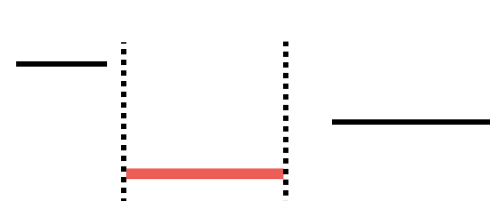
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Partial overlap



No overlap



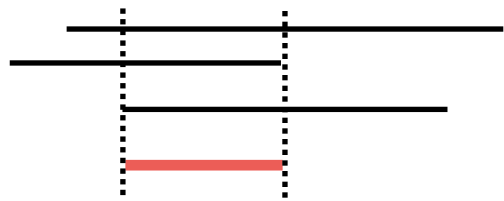
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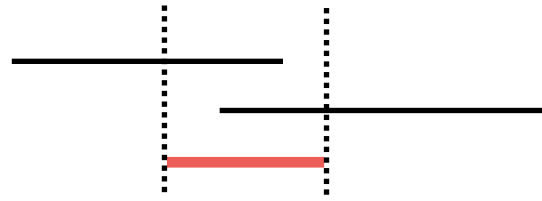
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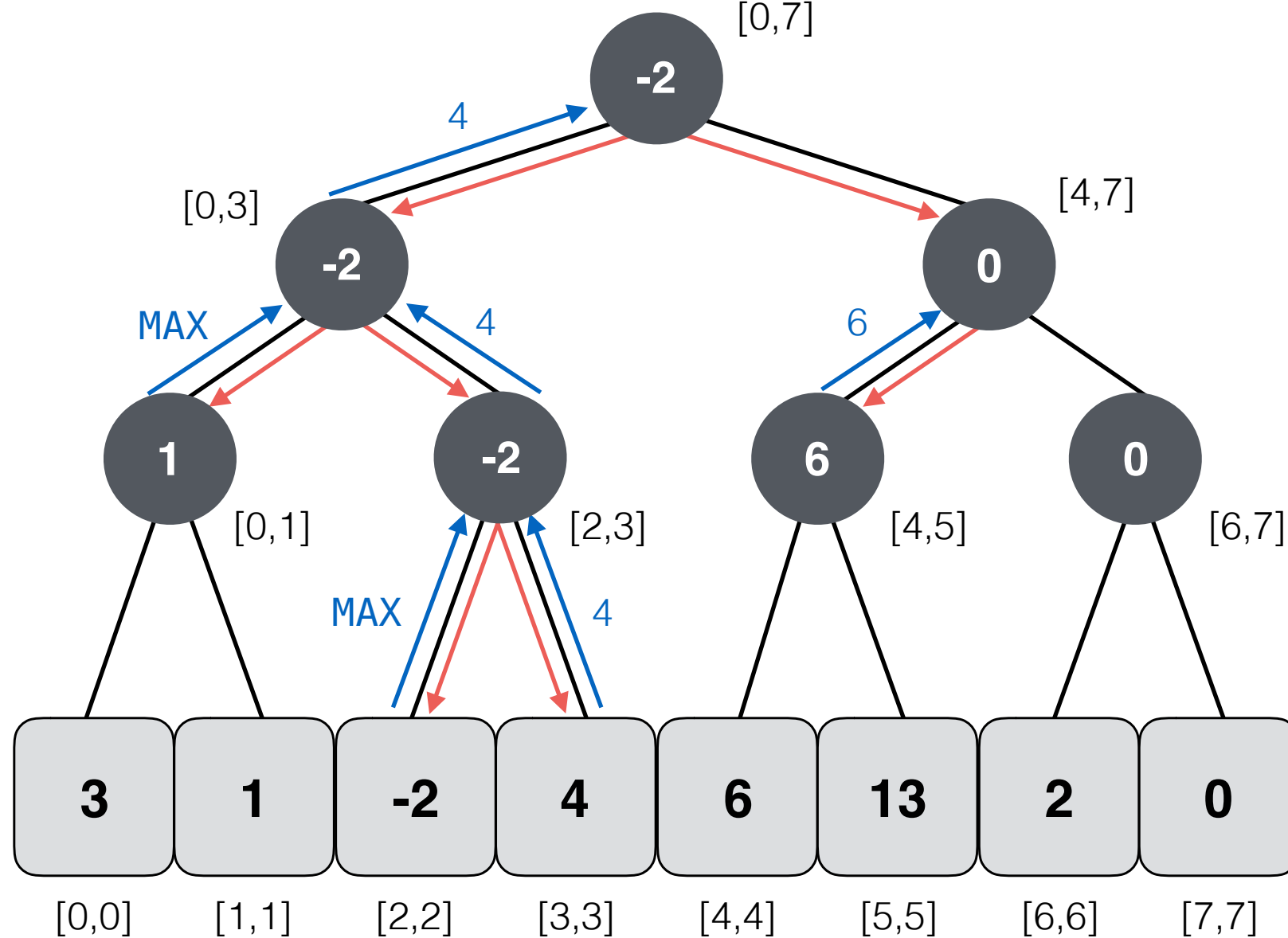
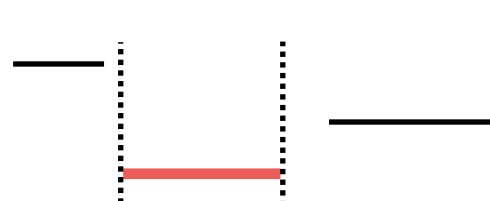
Total overlap



Partial overlap



No overlap



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$\min(3,6)$

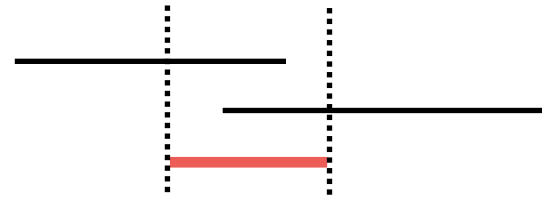
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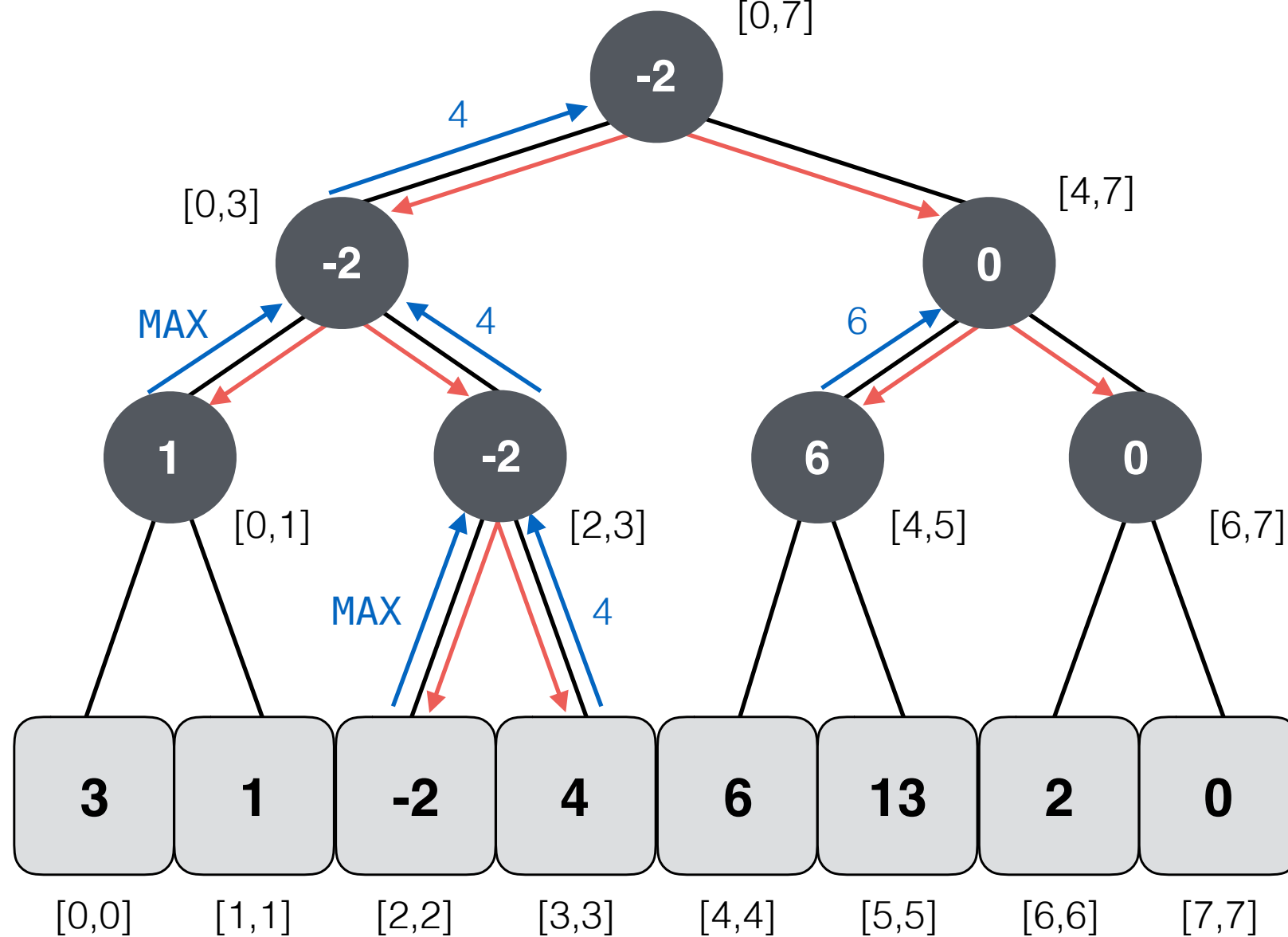
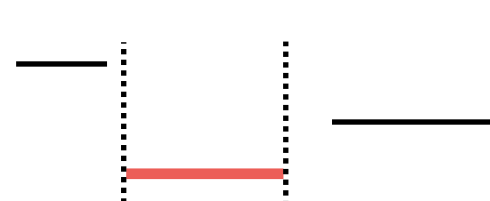
Total overlap



Partial overlap



No overlap

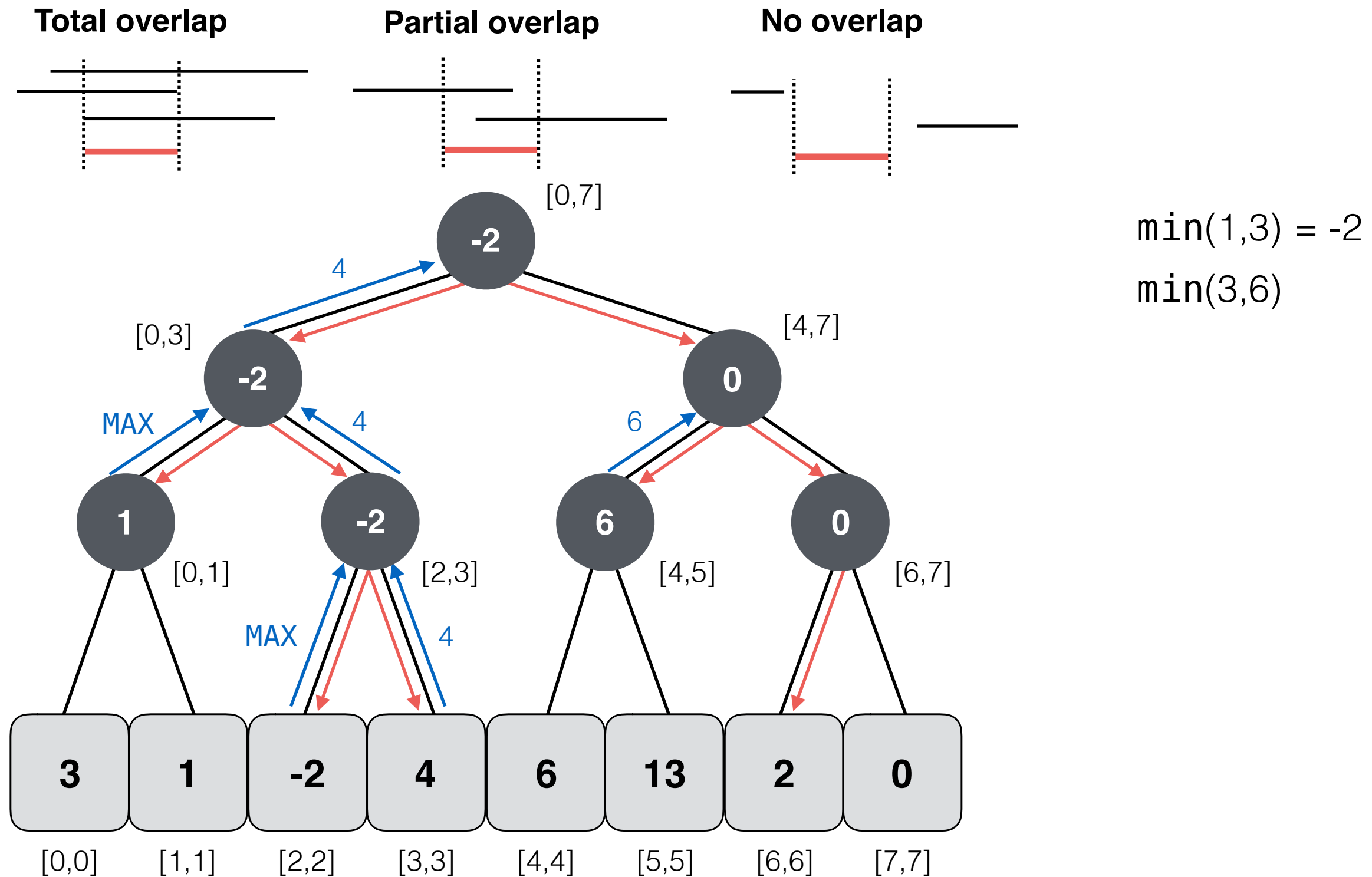


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Range MIN Queries with Segment Trees

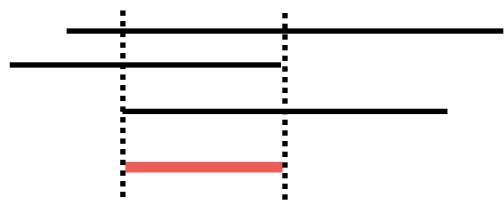
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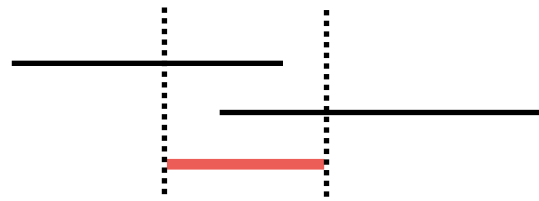
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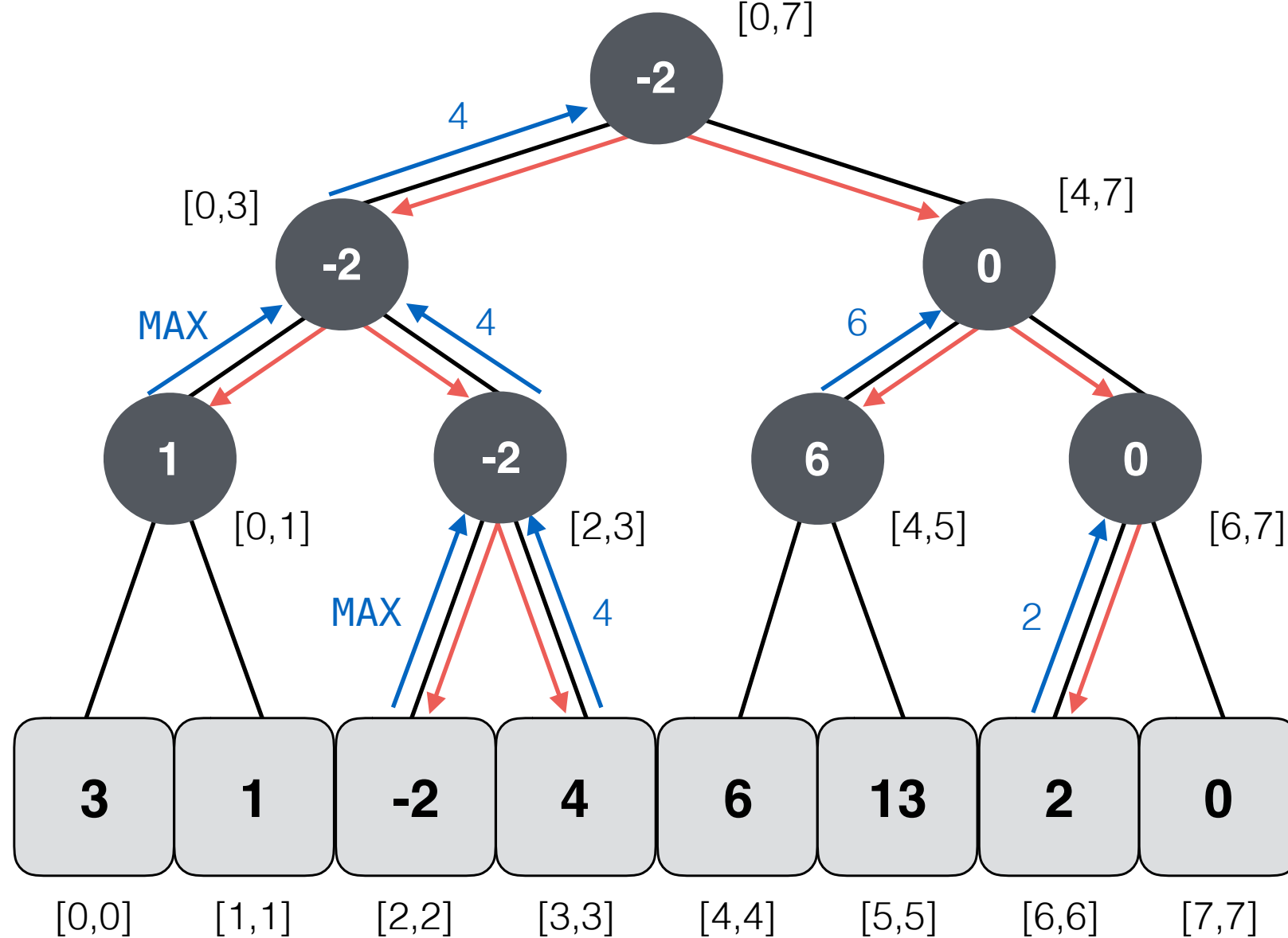
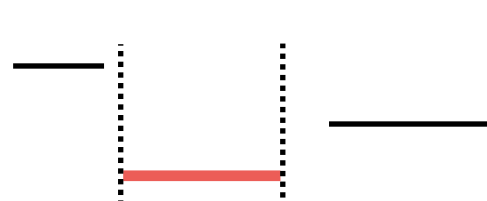
Total overlap



Partial overlap



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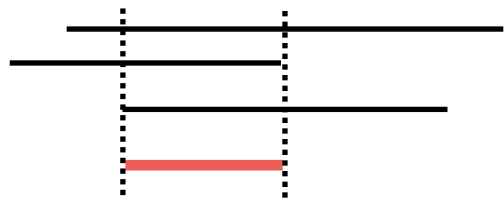
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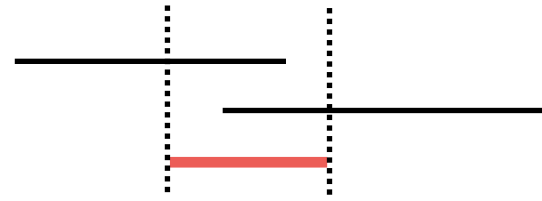
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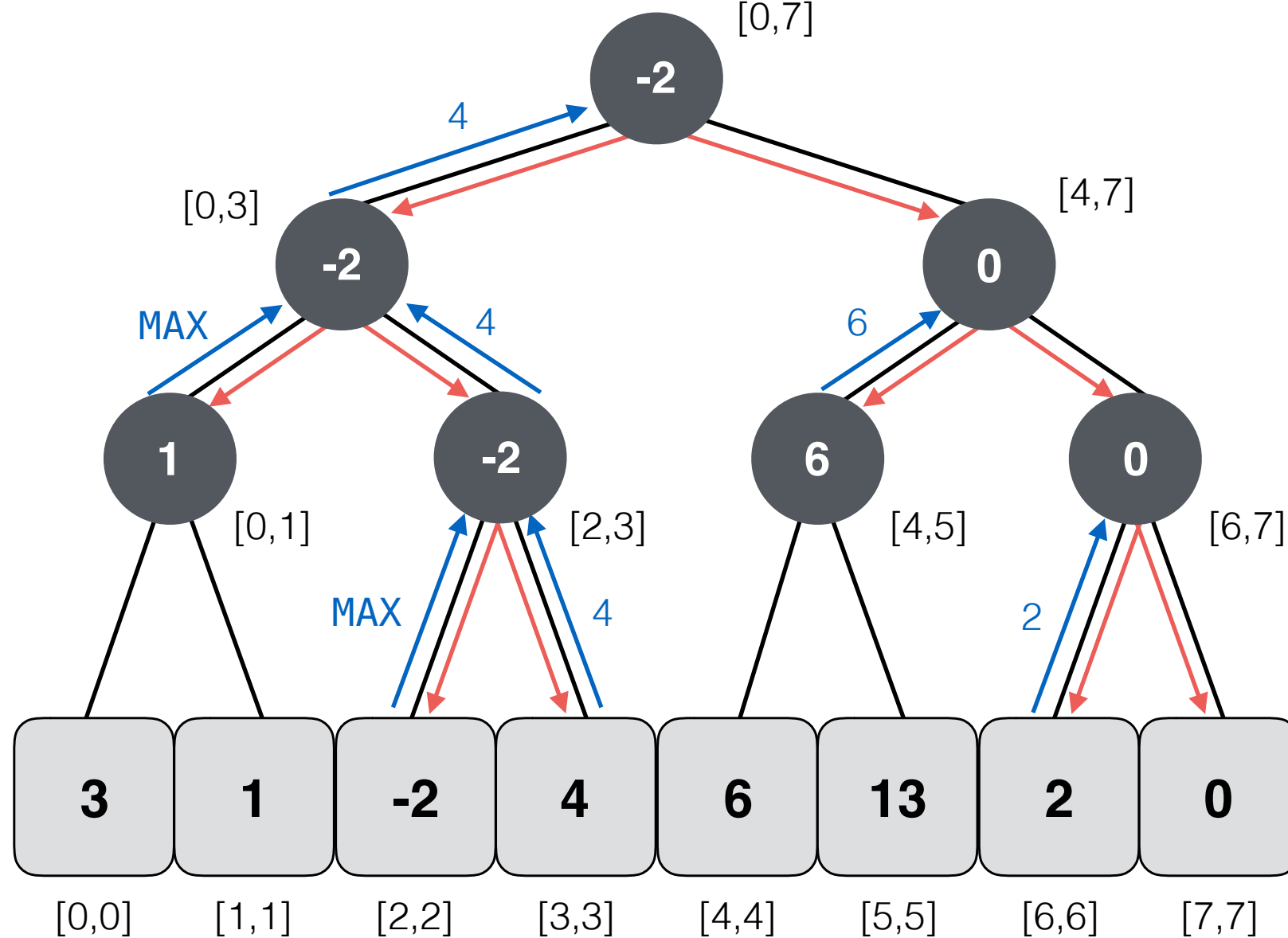
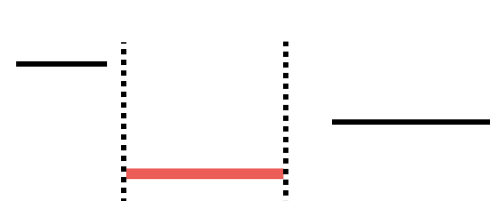
Total overlap



Partial overlap



No overlap

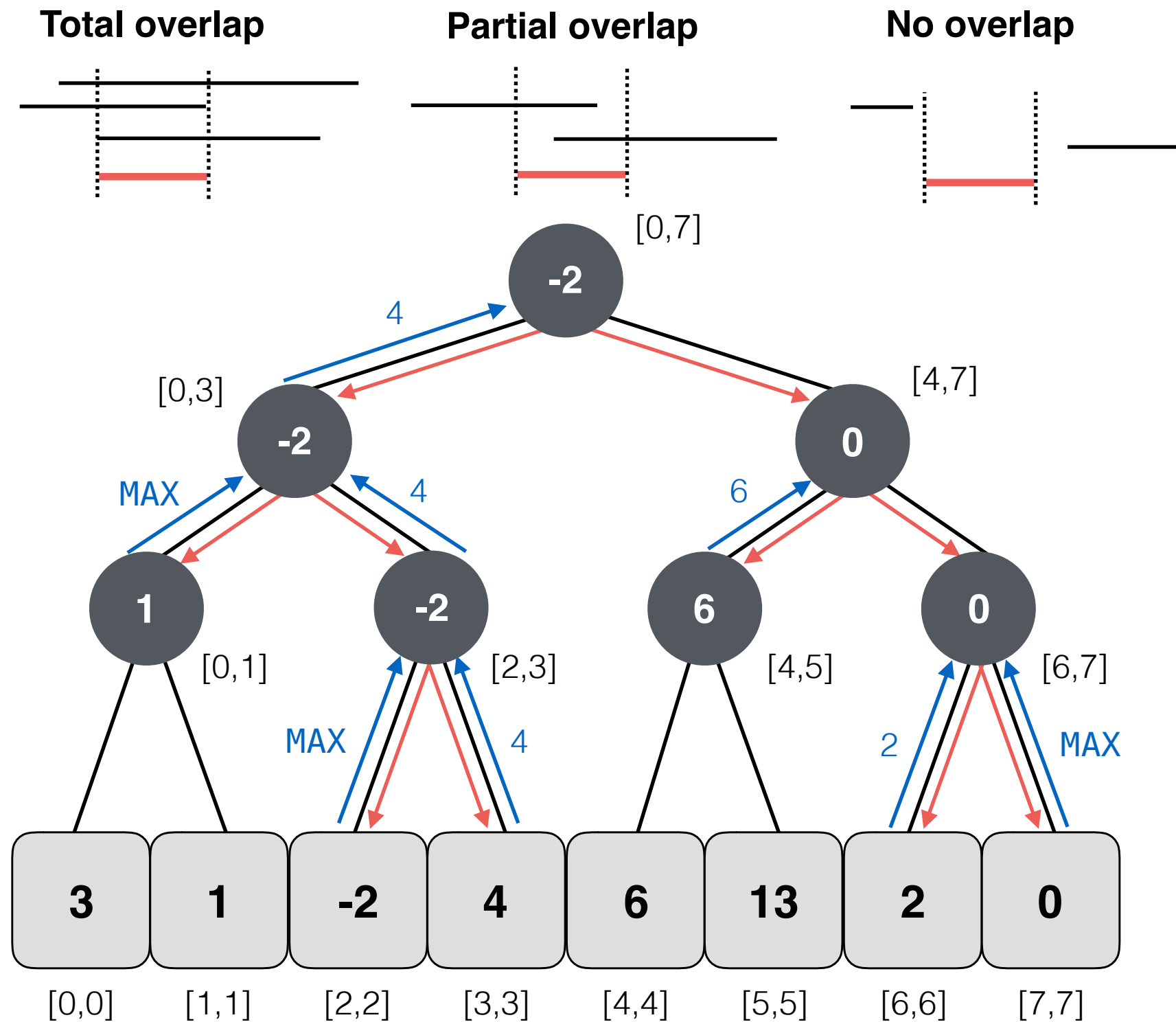


$$\min(1,3) = -2$$

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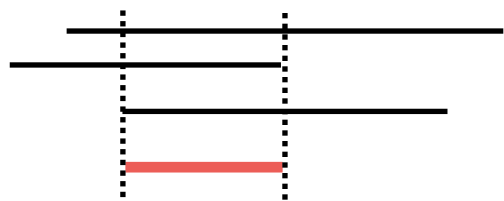
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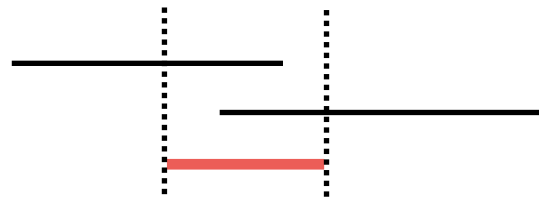
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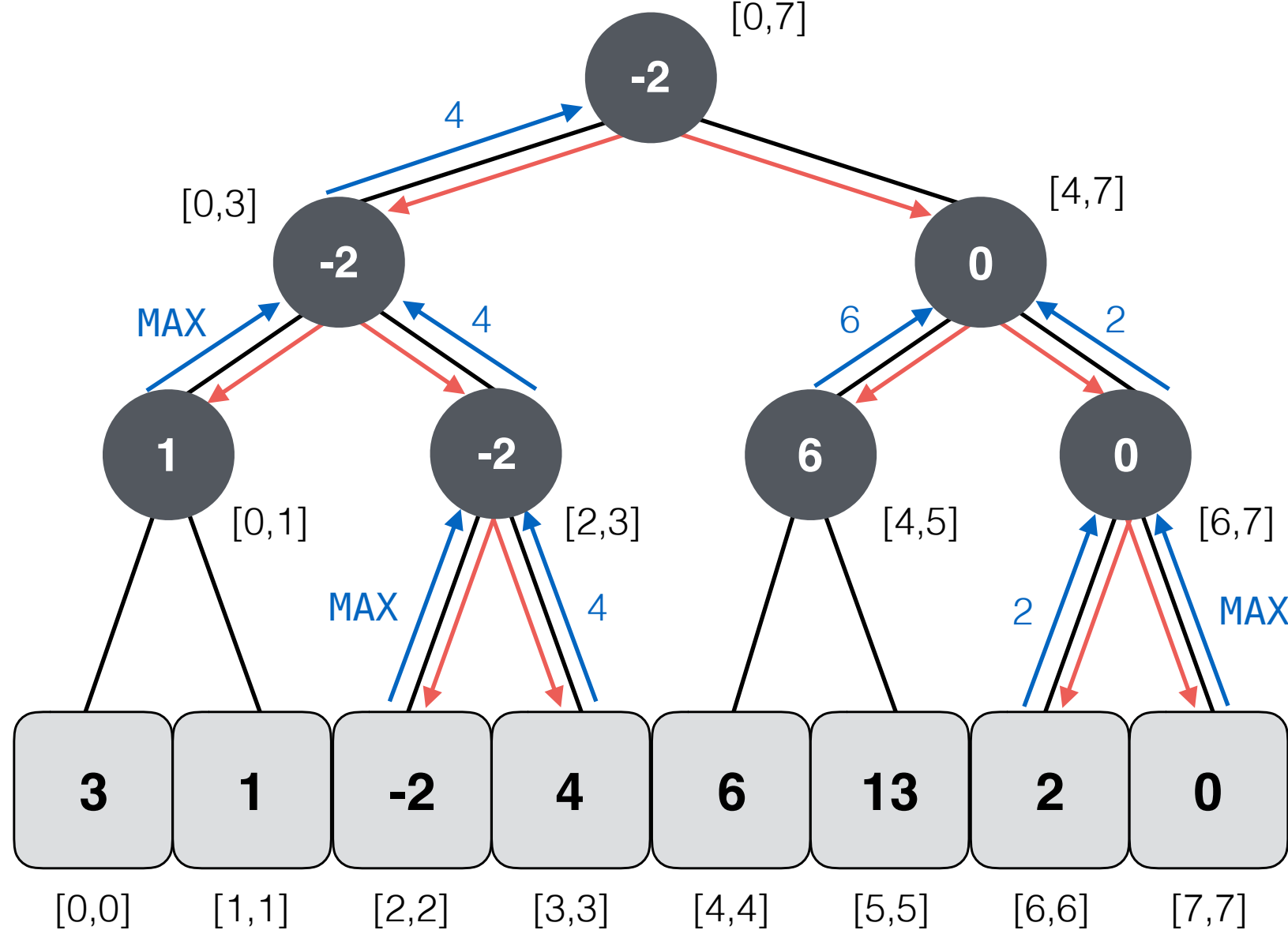
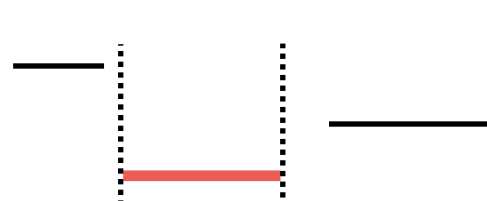
Total overlap



Partial overlap



No overlap

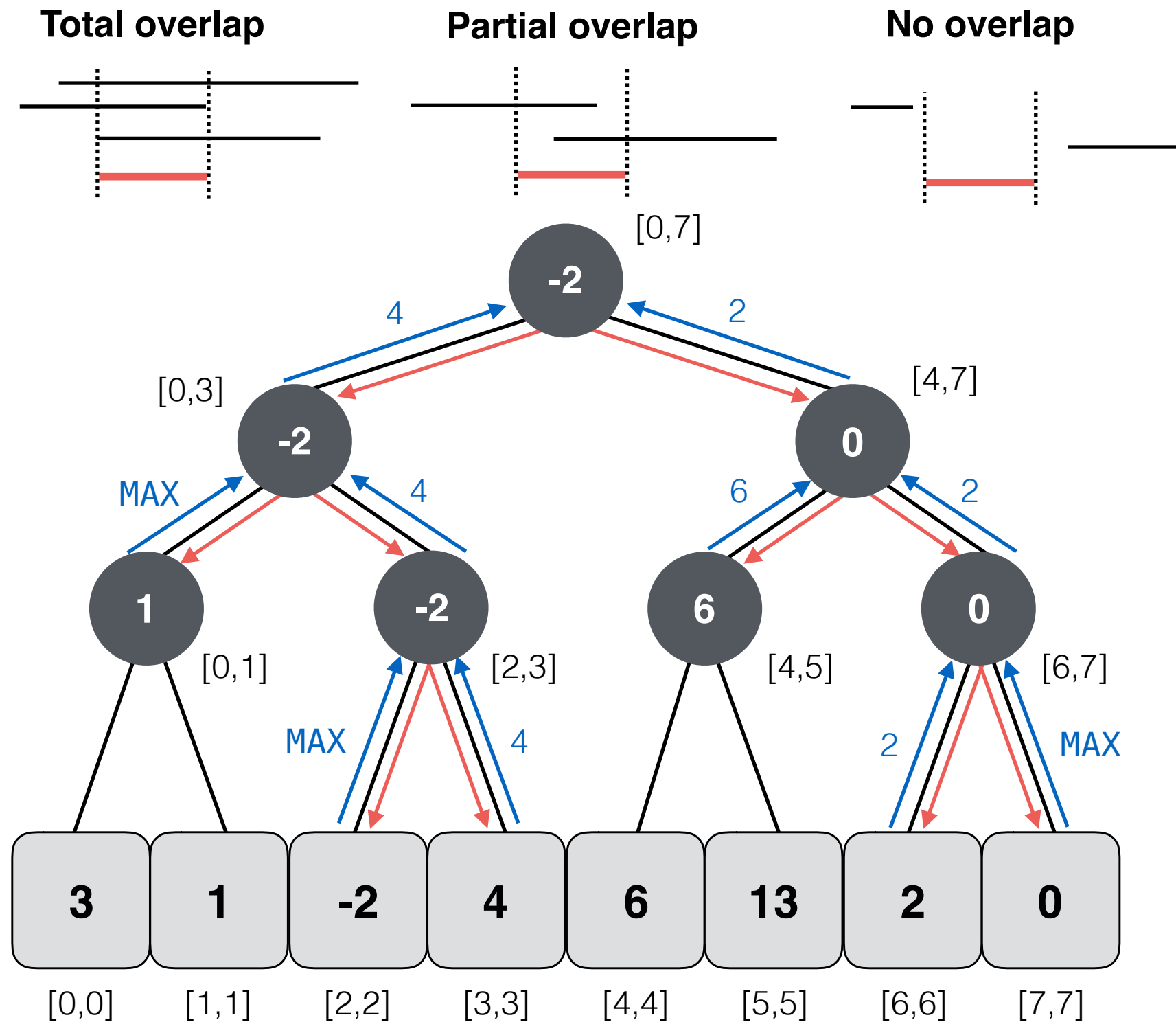


$\min(1,3) = -2$

$\min(3,6)$

Range MIN Queries with Segment Trees

Consider a segment tree with n leaves ($2n - 1$ nodes in total).

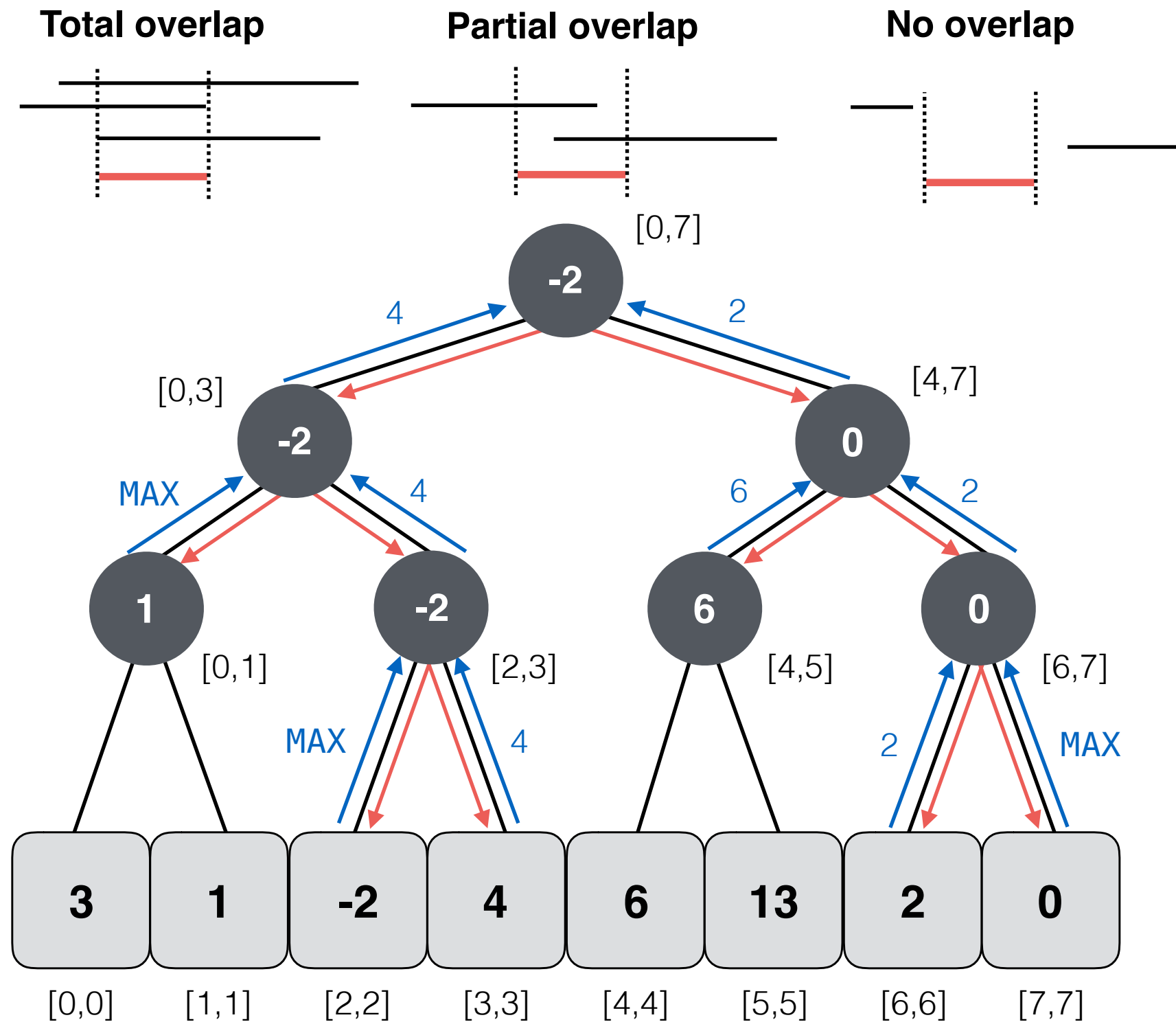


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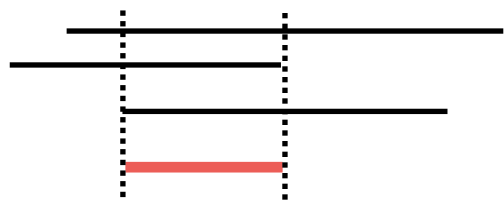
$$\text{min}(1,3) = -2$$

$$\text{min}(3,6) = 2$$

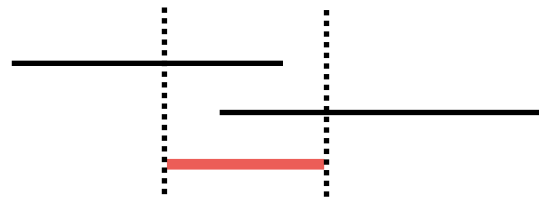
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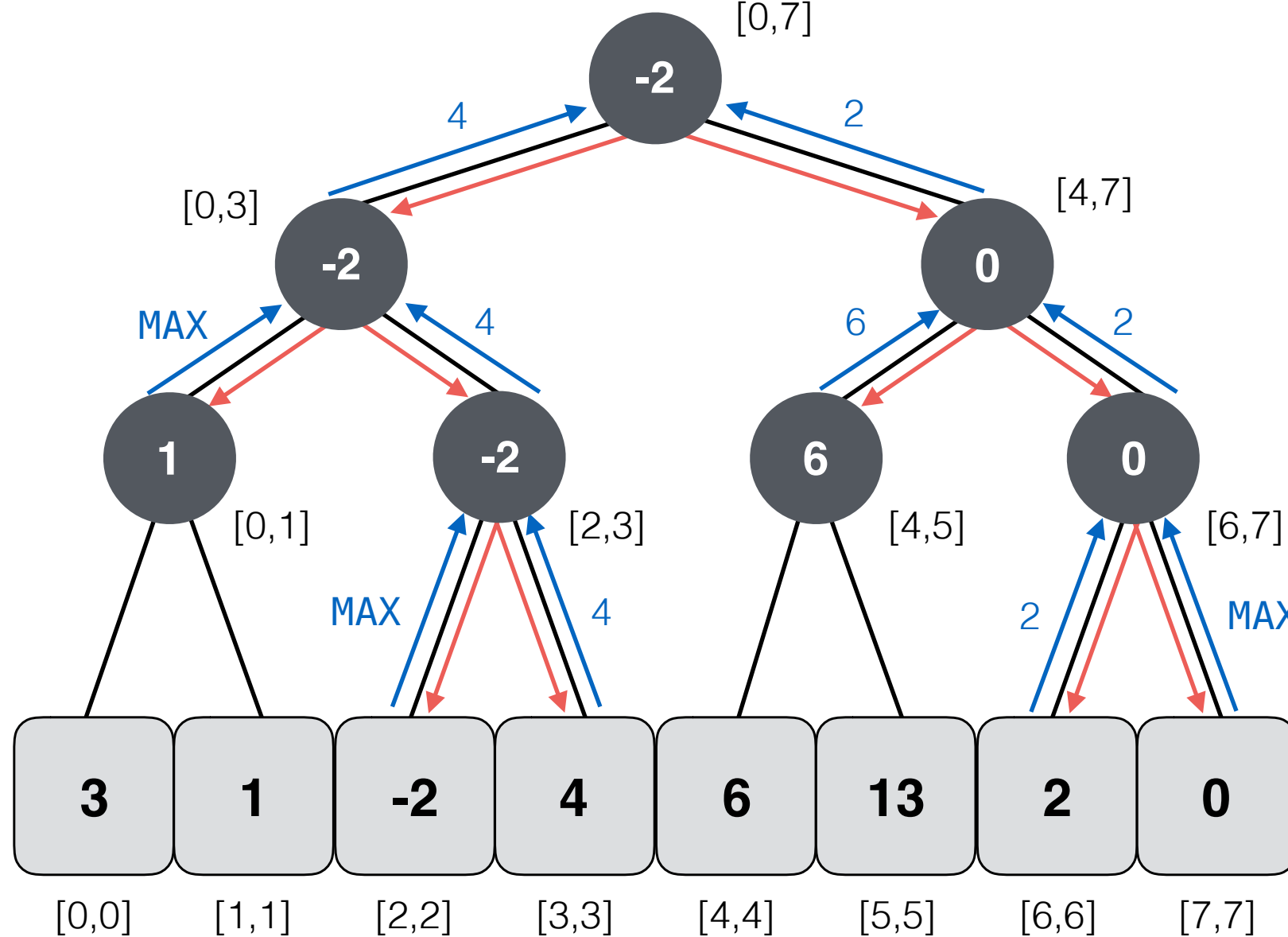
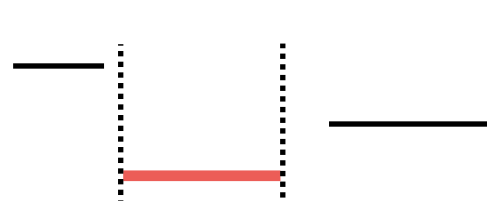
Total overlap



Partial overlap



No overlap



$$\min(1,3) = -2$$

$$\min(3,6) = 2$$

Query time: $O(\log n)$

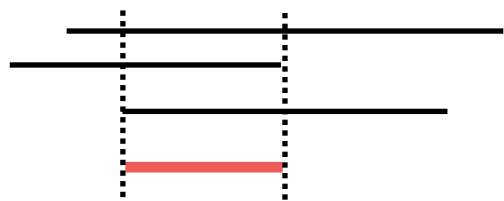
Space: $O(n)$

Building time: $O(n)$

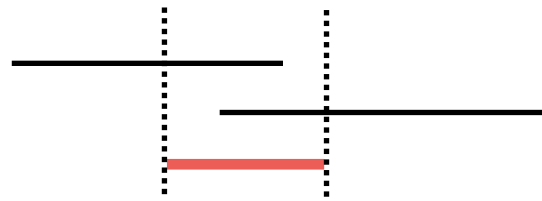
Range MIN Queries with Segment Trees

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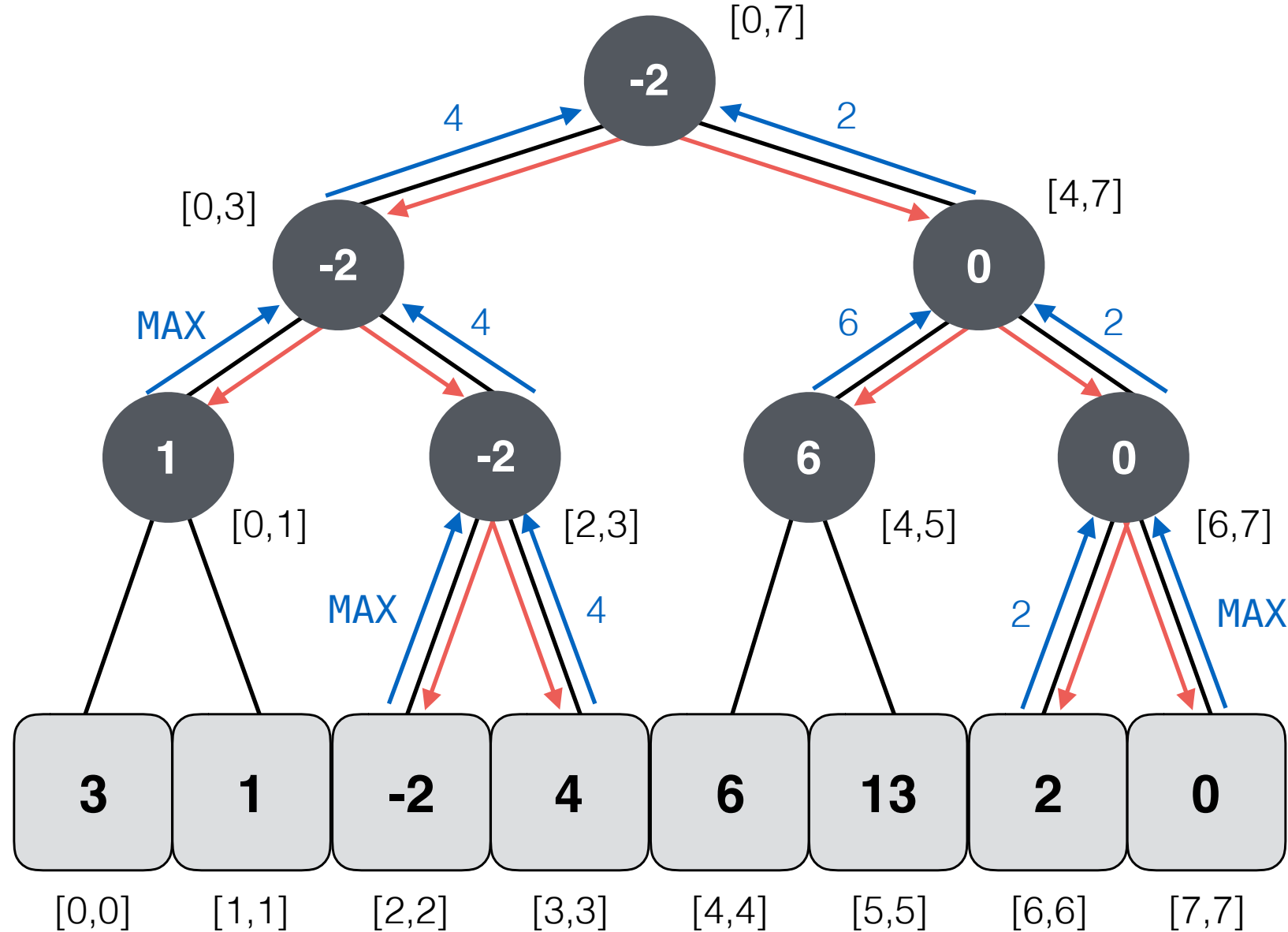
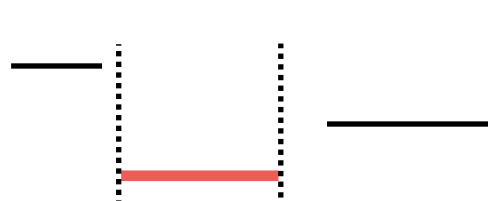
Total overlap



Partial overlap



No overlap



$$\min(1, 3) = -2$$

$$\min(3, 6) = 2$$

Query time: $O(\log n)$

Space: $O(n)$

Building time: $O(n)$



How do we represent trees?

Pointers

```
node* root = nullptr;
std::deque<node*> q;
int n = 0;
std::cin >> n;

for (int i = 0; i < n; ++i) {
    int x = 0;
    std::cin >> x;
    node* n = new node(x);
    q.push_back(n);
}

node* last = nullptr;
if (n % 2) {
    last = q.back();
    q.pop_back();
}

auto min_parent = [&](node* left, node* right) {
    int min = std::min<int>(left->key, right->key);
    node* parent = new node(min, left, right);
    q.push_back(parent);
};

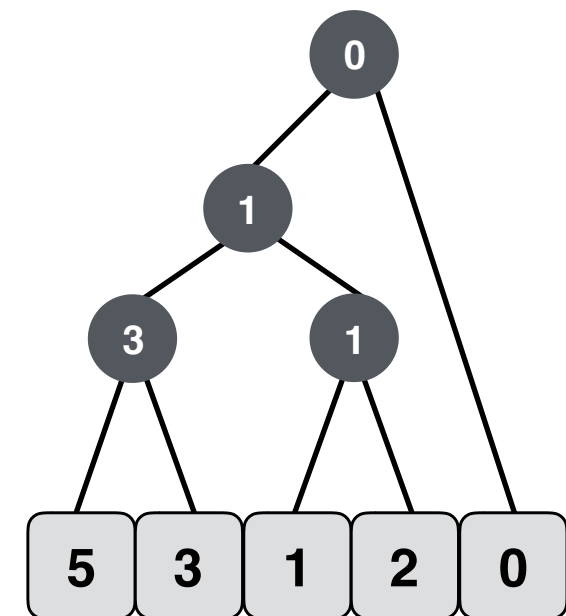
while (q.size() != 1) {
    min_parent(q[0], q[1]);
    q.pop_front();
    q.pop_front();
}

if (last != nullptr) {
    min_parent(q.front(), last);
    q.pop_front();
}

root = q.front();
```

```
struct node {
    node(int k,
        node* l = nullptr,
        node* r = nullptr)
        : key(k), left(l), right(r)
    {}

    int key;
    node* left;
    node* right;
};
```



How do we represent trees?

Arrays

```
std::vector<int> tree;

int n = 0;
std::cin >> n;
int tree_size = 2 * n - 1;
tree.resize(tree_size);

int h = ceil(log2(n));
// left-most node id following level order
int left_most_node = (int(1) << (h - 1)) - 1;
int offset = LEFT(left_most_node);

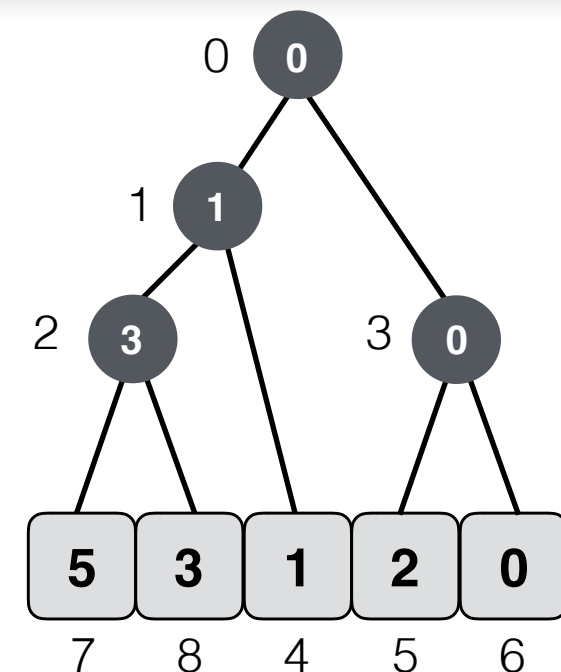
// set leaves circularly

// 1. go forward
int i = 0;
for (int j = offset; j != tree_size; ++i, ++j) {
    int x = 0;
    std::cin >> x;
    tree[j] = x;
}

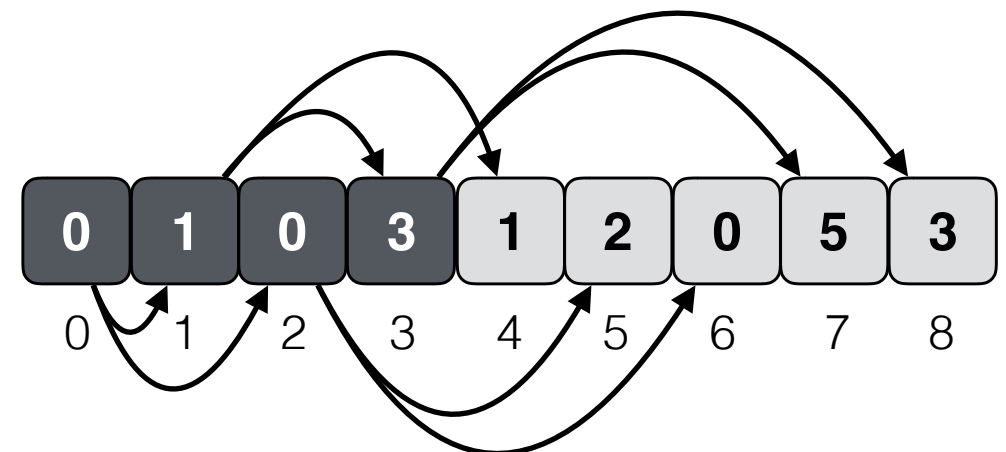
// 2. fall back
for (int j = 0; i != n; ++i, ++j) {
    int x = 0;
    std::cin >> x;
    tree[n - 1 + j] = x;
}

// set internal nodes
for (int i = tree_size - 1; i != 0; i -= 2) {
    int min = std::min<int>(tree[i], tree[i - 1]);
    tree[PARENT(i)] = min;
}
```

```
#define LEFT(i)      2 * i + 1
#define RIGHT(i)     2 * i + 2
#define PARENT(i)    (i - 1) / 2
```



Pointers are implicit!



How do we represent trees?

Arrays

```
std::vector<int> tree;

int n = 0;
std::cin >> n;
int tree_size = 2 * n - 1;
tree.resize(tree_size);

int h = ceil(log2(n));
// left-most node id following level order
int left_most_node = (int(1) << (h - 1)) - 1;
int offset = LEFT(left_most_node);

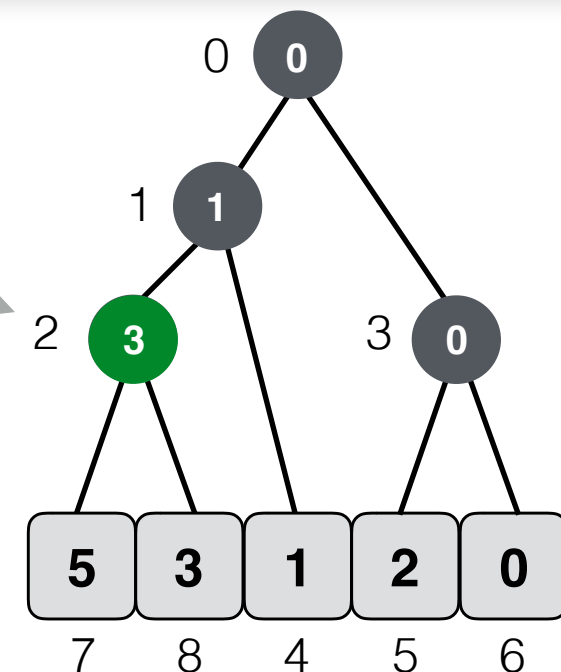
// set leaves circularly

// 1. go forward
int i = 0;
for (int j = offset; j != tree_size; ++i, ++j) {
    int x = 0;
    std::cin >> x;
    tree[j] = x;
}

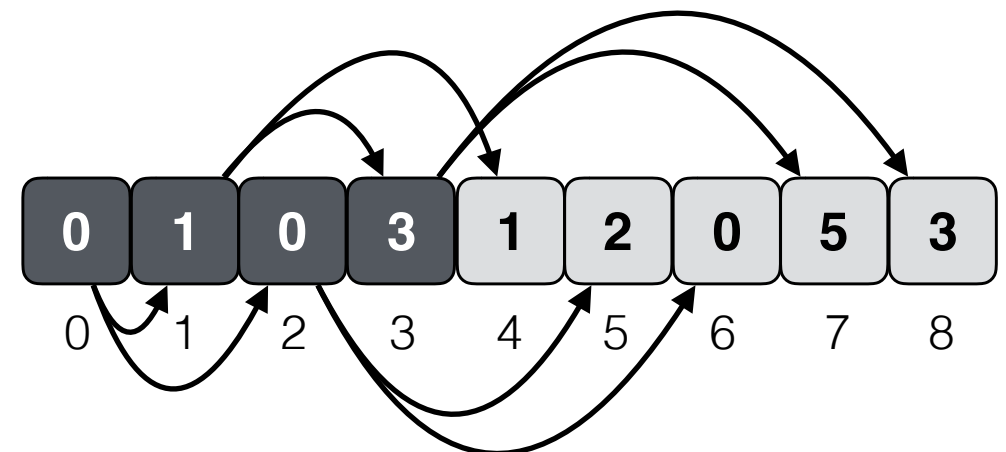
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#define LEFT(i)      2 * i + 1
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Pointers are implicit!



How do we represent trees?

Arrays

```
std::vector<int> tree;
int n = 0;
std::cin >> n;
int tree_size = 2 * n - 1;
tree.resize(tree_size);

int h = ceil(log2(n));
// left-most node id following level order
int left_most_node = (int(1) << (h - 1)) - 1;
int offset = LEFT(left_most_node);

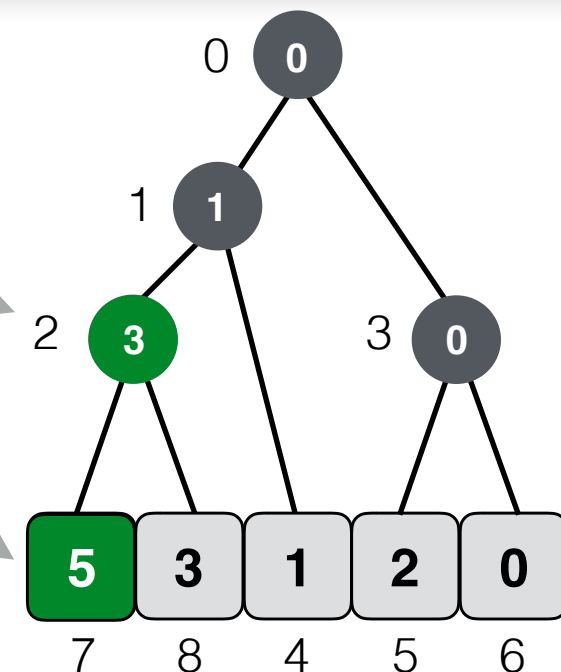
// set leaves circularly

// 1. go forward
int i = 0;
for (int j = offset; j != tree_size; ++i, ++j) {
    int x = 0;
    std::cin >> x;
    tree[j] = x;
}

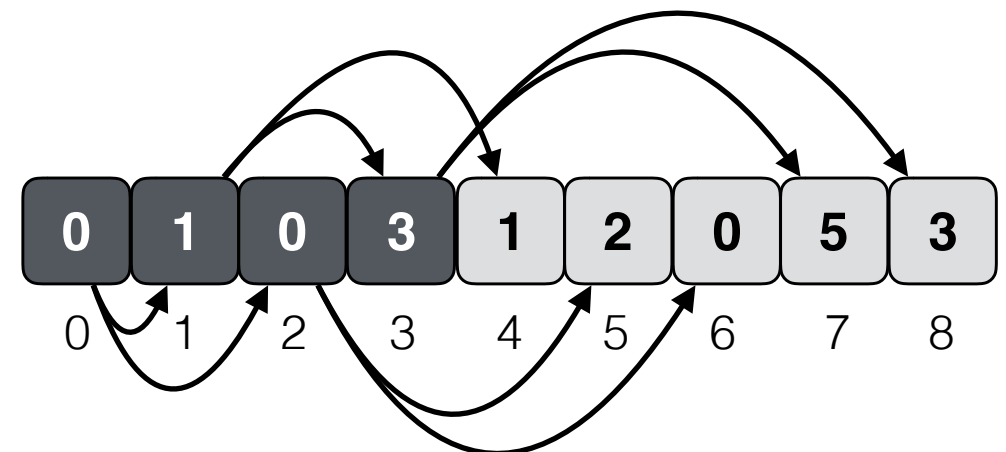
// 2. fall back
for (int j = 0; i != n; ++i, ++j) {
    int x = 0;
    std::cin >> x;
    tree[n - 1 + j] = x;
}

// set internal nodes
for (int i = tree_size - 1; i != 0; i -= 2) {
    int min = std::min<int>(tree[i], tree[i - 1]);
    tree[PARENT(i)] = min;
}
```

```
#define LEFT(i)    2 * i + 1
#define RIGHT(i)   2 * i + 2
#define PARENT(i) (i - 1) / 2
```



Pointers are implicit!



How do we represent trees?

Remember

Be skeptic: *measure* first and then conclude.

How do we represent trees?

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Be skeptic: *measure* first and then conclude.

Pointers VS. Arrays

How do we represent trees?

Remember

Be skeptic: *measure* first and then conclude.

Pointers VS. Arrays

Experiment over 5 million nodes

Visit the tree and compute the sum of all nodes.

```
[→ segment_trees git:(master) x g++ -std=c++11 -O3 trees_with_pointers.cpp -o trees_with_pointers
[→ segment_trees git:(master) x ./trees_with_pointers < input7
building took: 19.3895 [sec]
sum is: 3676408720
sum took: 0.118645 [sec]
[→ segment_trees git:(master) x g++ -std=c++11 -O3 trees_with_arrays.cpp -o trees_with_arrays
[→ segment_trees git:(master) x ./trees_with_arrays < input7
building took: 18.0724 [sec]
sum is: 3676408920
sum took: 0.0118351 [sec]
[→ segment_trees git:(master) x █
```

How do we represent trees?

Remember

Be skeptic: *measure* first and then conclude.

Pointers VS. Arrays

Experiment over 5 million nodes

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[→ segment_trees git:(master) x g++ -std=c++11 -O3 trees_with_pointers.cpp -o trees_with_pointers
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[→ segment_trees git:(master) x ./trees_with_arrays < input7
building took: 18.0724 [sec]
sum is: 3676408920
sum took: 0.0118351 [sec]
→ segment_trees git:(master) x
```

10X

How do we represent trees?

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Pointers VS. Arrays

Experiment over 5 million nodes

Visit the tree and compute the sum of all nodes.

```
[→ segment_trees git:(master) x g++ -std=c++11 -O3 trees_with_pointers.cpp -o trees_with_pointers
[→ segment_trees git:(master) x ./trees_with_pointers < input7
building took: 19.3895 [sec]
sum is: 3676408720
sum took: 0.118645 [sec]
[→ segment_trees git:(master) x g++ -std=c++11 -O3 trees_with_arrays.cpp -o trees_with_arrays
[→ segment_trees git:(master) x ./trees_with_arrays < input7
building took: 18.0724 [sec]
sum is: 3676408920
sum took: 0.0118351 [sec]
→ segment_trees git:(master) x 10X
```

OK, we are going to adopt the array-based representation!

Building Segment Trees recursively

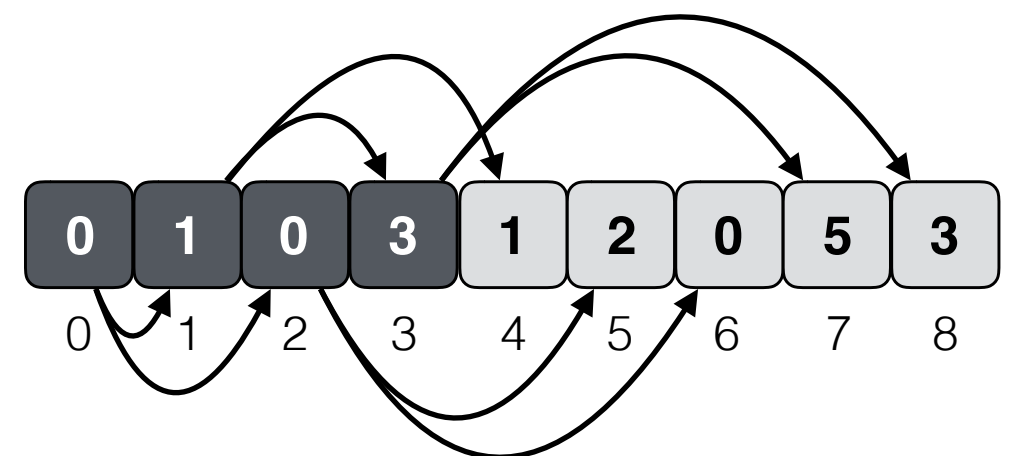
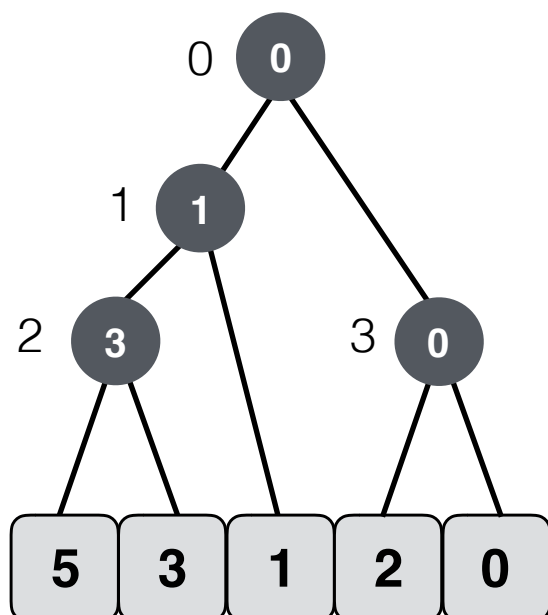
```
size_t n = leaves.size();
// round up to the next power of 2
size_t m = size_t(1) << static_cast<size_t>(ceil(log2(n)));
m_tree.resize(2 * m - 1, m_traits.invalid);

build(leaves, 0, m - 1, 0);
```

```
struct type_traits {
    IntType invalid;
    BinaryFunc funct;
};
```

```
void build(std::vector<IntType> const& leaves, size_t lo, size_t hi, size_t pos) {
    if (lo == hi) {
        m_tree[pos] = leaves[lo];
        return;
    }
    size_t mid = (lo + hi) / 2;
    build(leaves, lo, mid, LEFT(pos));
    build(leaves, mid + 1, hi, RIGHT(pos));
    m_tree[pos] = m_traits.funct(m_tree[LEFT(pos)], m_tree[RIGHT(pos)]);
}
```

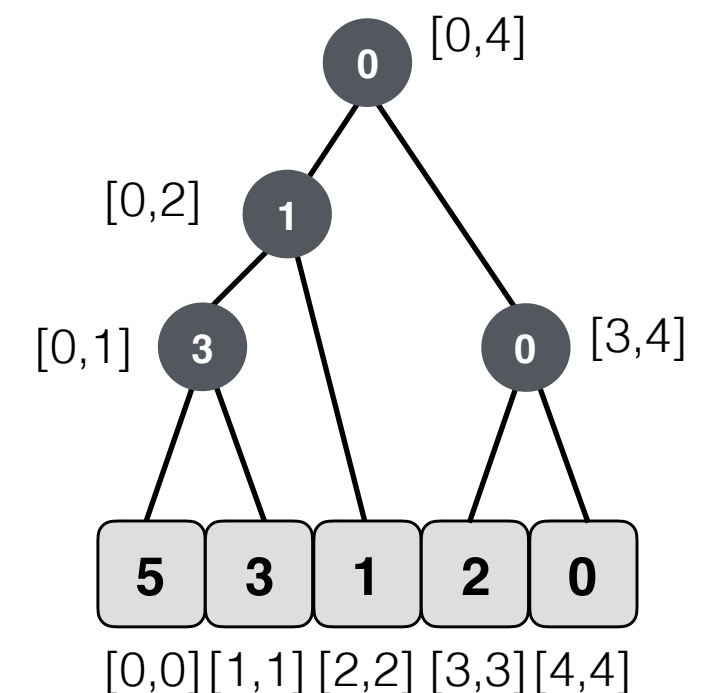
```
#define LEFT(i)    2 * i + 1
#define RIGHT(i)   2 * i + 2
#define PARENT(i) (i - 1) / 2
```



Range (MIN) Queries with Segment Trees

```
IntType rmq(range const& query, range node_segment, size_t pos) {  
    if (query.lo <= node_segment.lo  
        and query.hi >= node_segment.hi) { // total overlap  
        return m_tree[pos];  
    }  
    if (query.lo > node_segment.hi  
        or query.hi < node_segment.lo) { // no overlap  
        return m_traits.invalid;  
    }  
  
    // partial overlap  
    size_t mid = (node_segment.lo + node_segment.hi) / 2;  
    return m_traits.funct(  
        rmq(query, {node_segment.lo, mid}, LEFT(pos)),  
        rmq(query, {mid + 1, node_segment.hi}, RIGHT(pos))  
    );  
}
```

```
struct range {  
    range(size_t l, size_t h)  
        : lo(l), hi(h)  
    {}  
    size_t lo, hi;  
};
```



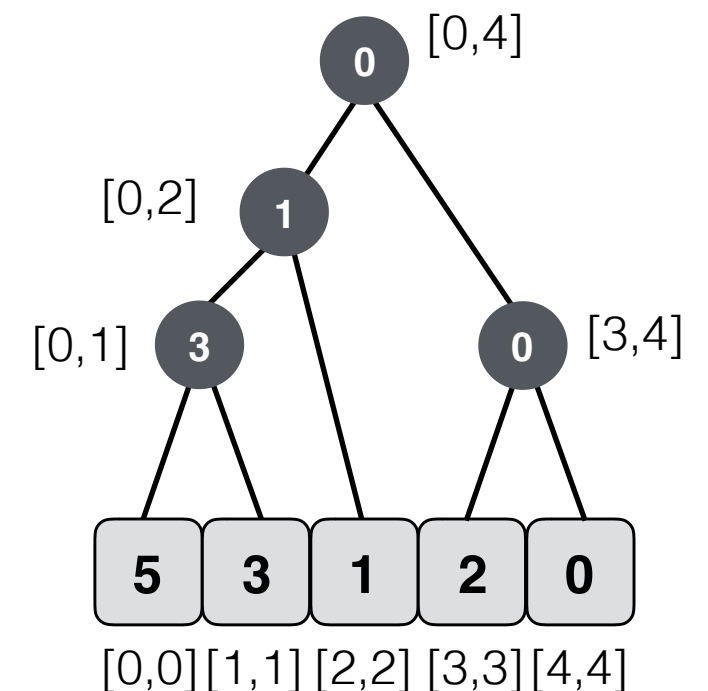
Range (MIN) Queries with Segment Trees

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    }  
    if (query.lo > node_segment.hi  
        or query.hi < node_segment.lo) { // no overlap  
        return m_traits.invalid;  
    }  
  
    // partial overlap  
    size_t mid = (node_segment.lo + node_segment.hi) / 2;  
    return m_traits.funct(  
        rmq(query, {node_segment.lo, mid}, LEFT(pos)),  
        rmq(query, {mid + 1, node_segment.hi}, RIGHT(pos))  
    );  
}
```

```
struct range {  
    range(size_t l, size_t h)  
        : lo(l), hi(h)  
    {}  
    size_t lo, hi;  
};
```



$\min(1, 3) = ?$

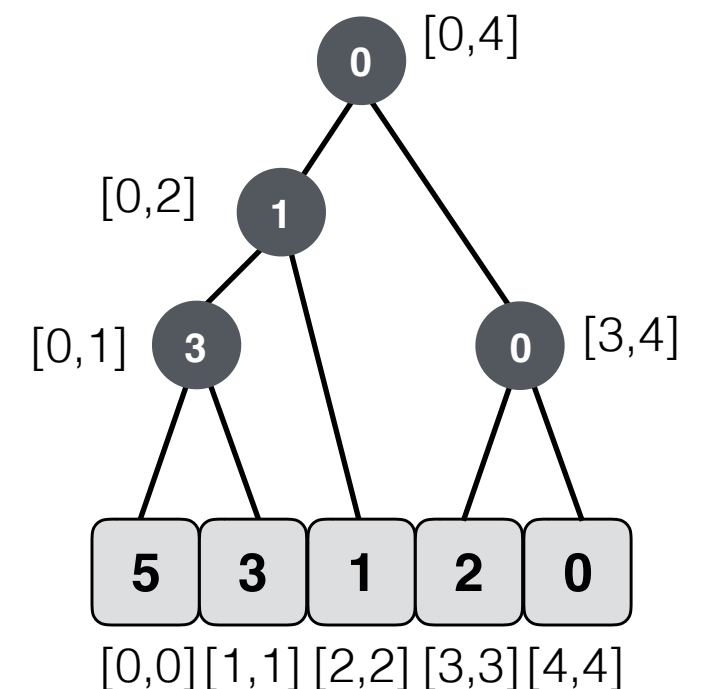


Updating Segment Trees

Let's add two new operations (updates):

- `update(i, x)` which increments the *i*-th leaf by *x*;
- `update_range(i, j, x)` which increments all leaves from *i* to *j* by *x*.

```
void update(size_t i, IntType delta, range node_segment, size_t pos) {  
    if (i > node_segment.hi  
    or  i < node_segment.lo) {  
        return;  
    }  
  
    if (node_segment.lo == node_segment.hi) { // leaf  
        m_tree[pos] += delta;  
        return;  
    }  
  
    size_t mid = (node_segment.lo + node_segment.hi) / 2;  
    update(i, delta, {node_segment.lo, mid}, LEFT(pos));  
    update(i, delta, {mid + 1, node_segment.hi}, RIGHT(pos));  
    m_tree[pos] = m_traits.funct(m_tree[LEFT(pos)], m_tree[RIGHT(pos)]);  
}
```



Updating Segment Trees

Let's add two new operations (updates):

- `update(i, x)` which increments the *i*-th leaf by *x*;
- `update_range(i, j, x)` which increments all leaves from *i* to *j* by *x*.

```

range const& query
void update(size_t i, IntType delta, range node_segment, size_t pos) {
if (i > node_segment.hi
or i < node_segment.lo) {
return;
}

    if (node_segment.lo == node_segment.hi) { // leaf
        m_tree[pos] += delta;
        return;
    }

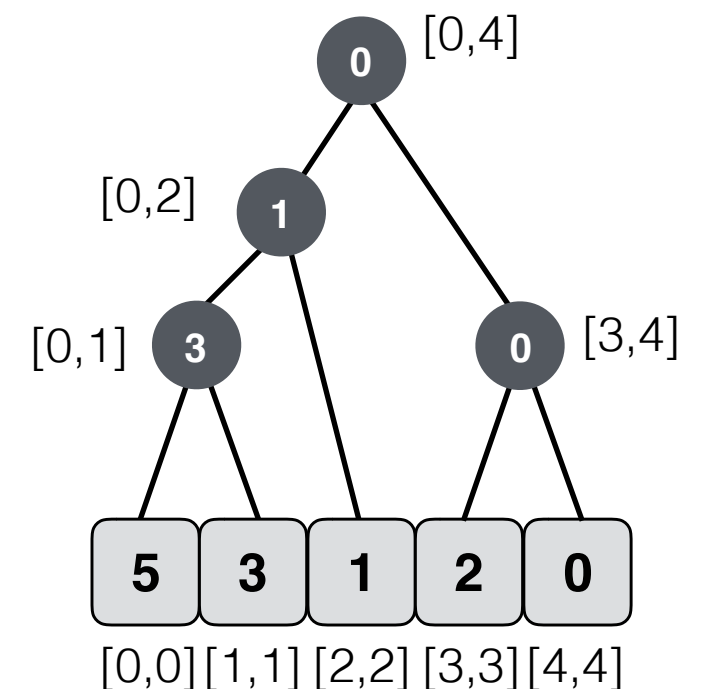
    size_t mid = (node_segment.lo + node_segment.hi) / 2;
    update(i, delta, {node_segment.lo, mid}, LEFT(pos));
    update(i, delta, {mid + 1, node_segment.hi}, RIGHT(pos));
    m_tree[pos] = m_traits.funct(m_tree[LEFT(pos)], m_tree[RIGHT(pos)]);
}

```

```

if (query.lo > node_segment.hi
or query.hi < node_segment.lo) {
    return;
}

```



Benchmarking Segment Trees



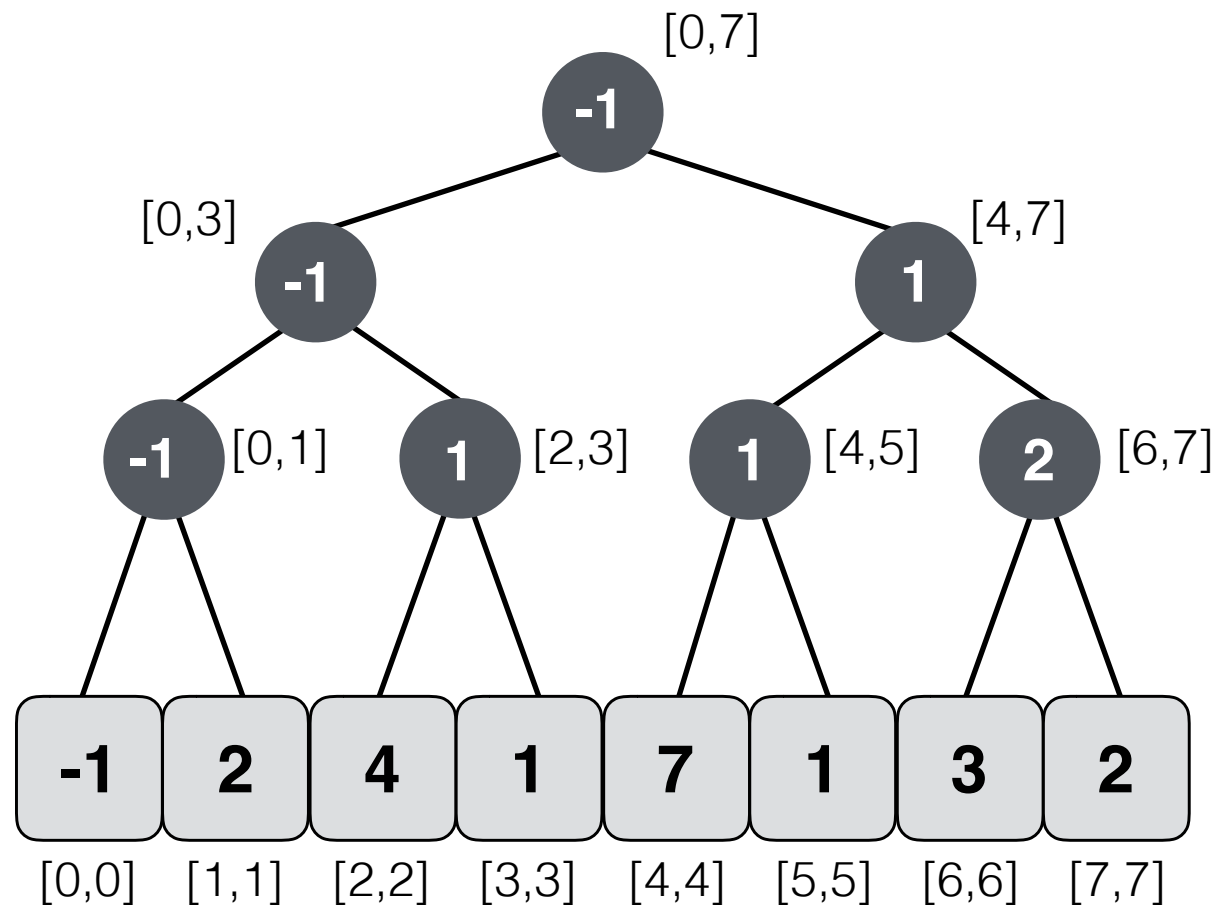
```
→ segment_trees git:(master) x python gen_data.py 5000000 100000 10000 > input13
→ segment_trees git:(master) x ./rmq_segment_tree < input13
parsing the input took: 18.5193 [sec]
building tree with 5000000 leaves
building took: 0.314939 [sec]
executing 100000 range queries
average query time: 1.74382 [musec]
executing 10000 updates
average update time: 0.561733 [musec]
executing 10000 range updates
average range update time: 2.55461 [musec]
→ segment_trees git:(master) x █
```

Lazy Propagation in Segment Trees

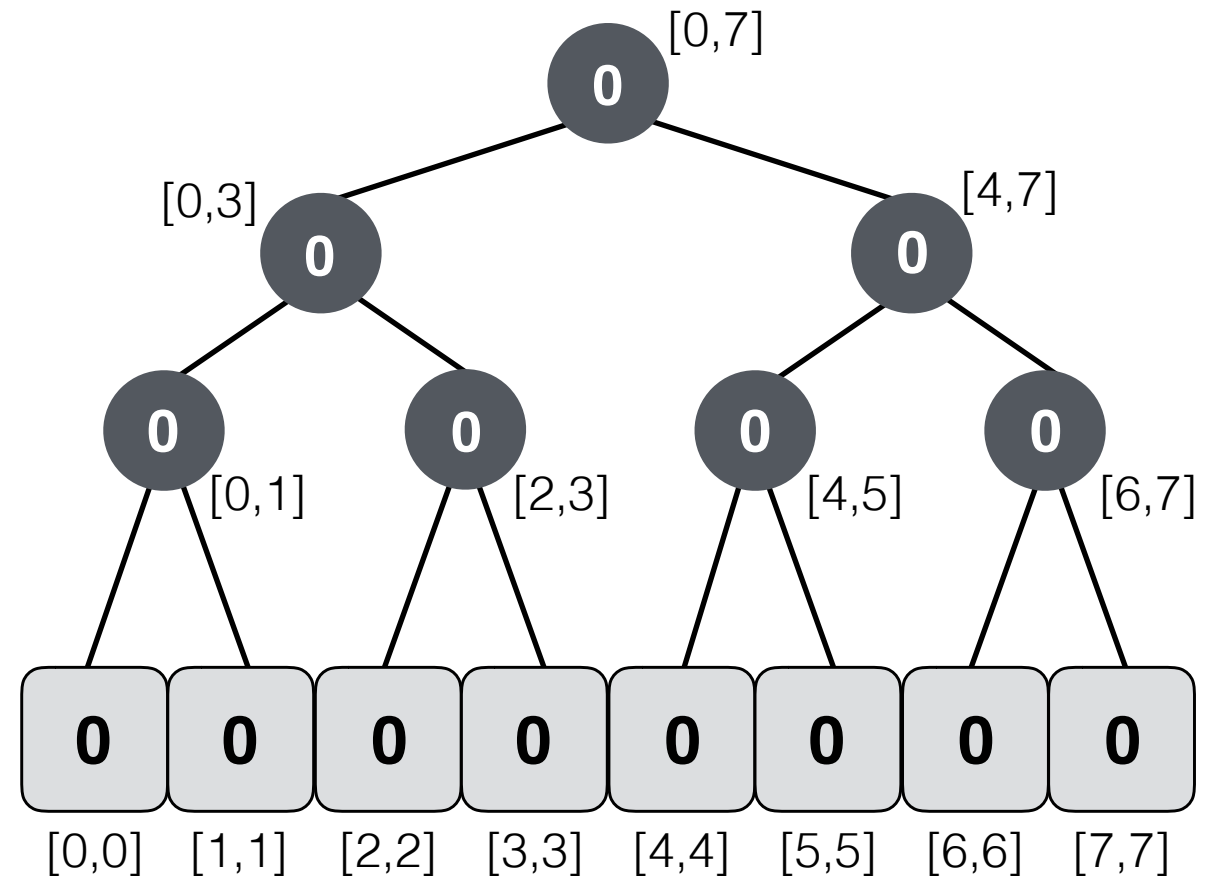
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

Lazy Propagation in Segment Trees

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Segment Tree

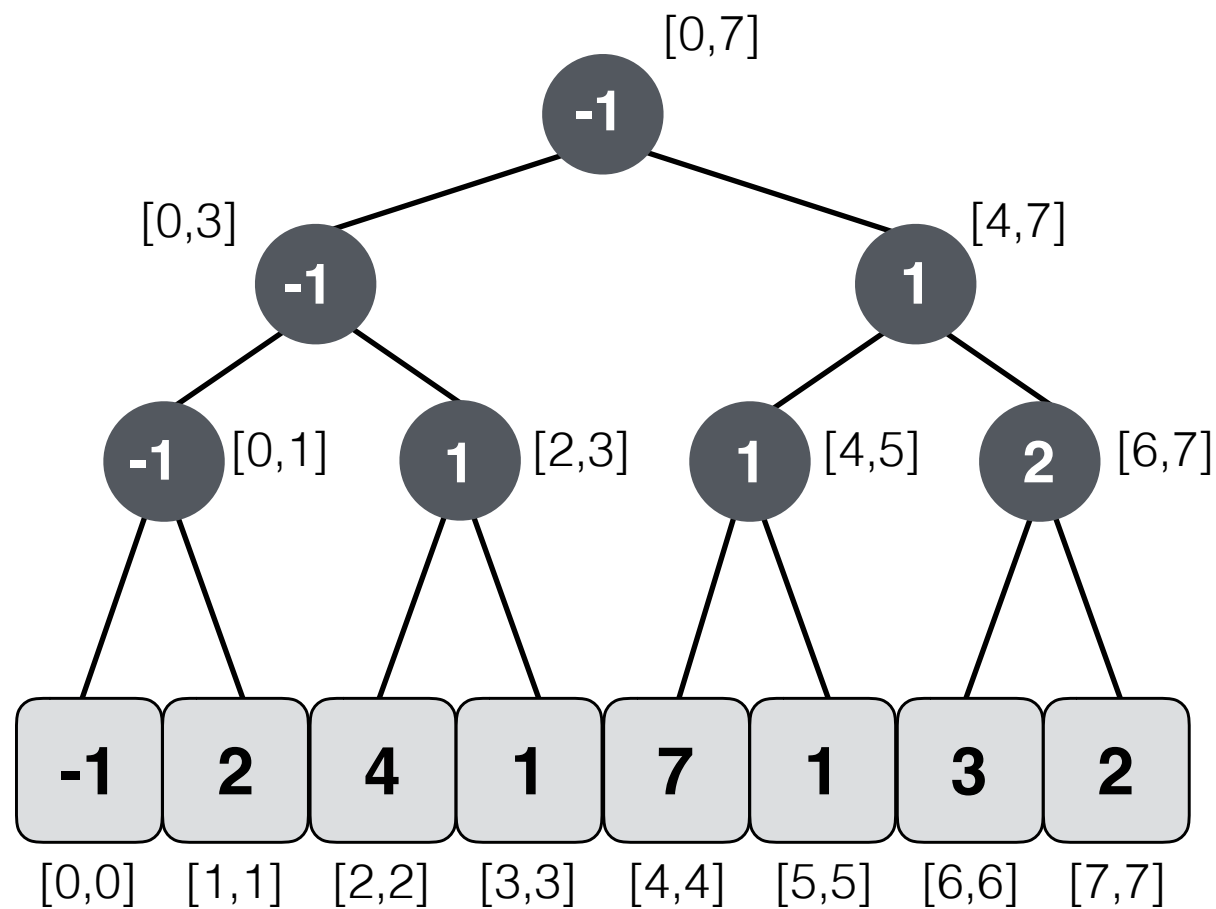


Lazy Tree

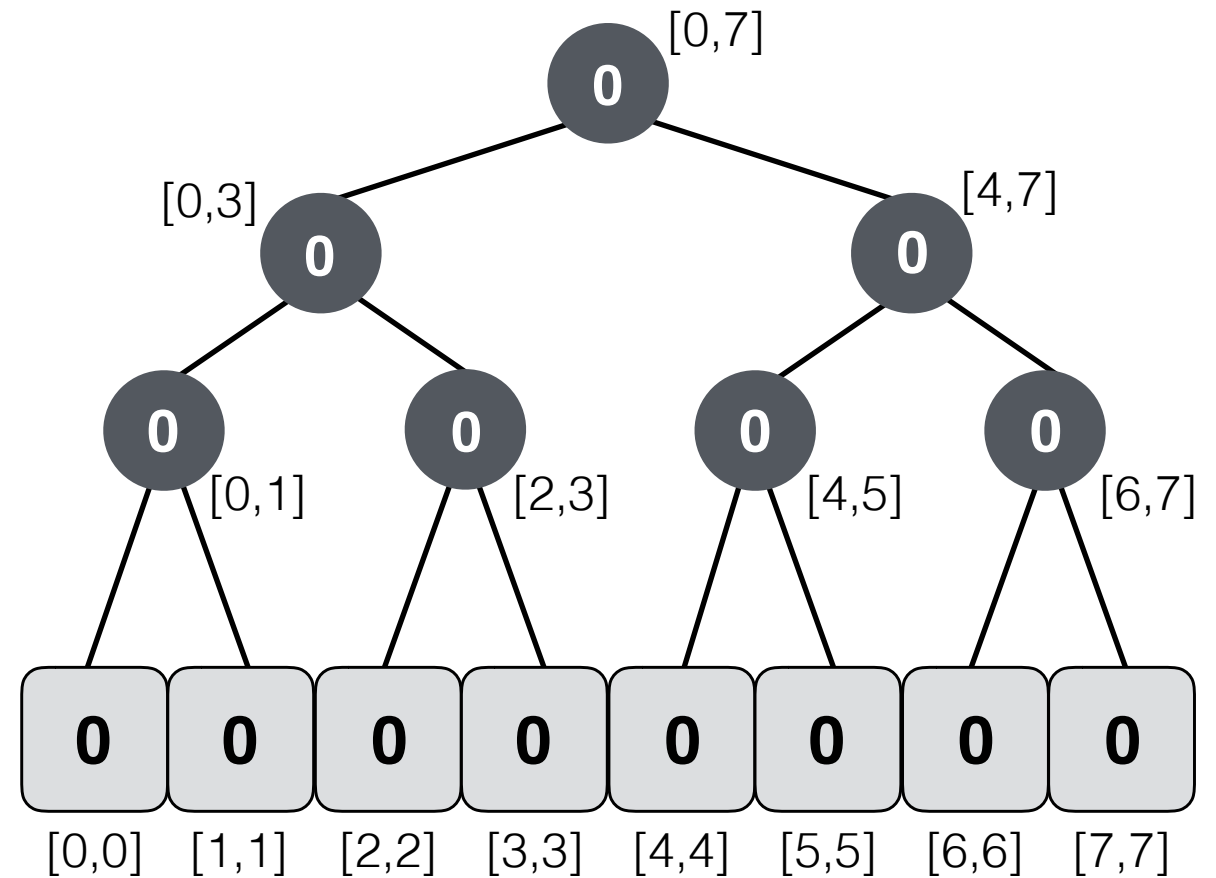
Lazy Propagation in Segment Trees

Avoid going down to the leaves and then up updating the internal nodes.
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update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

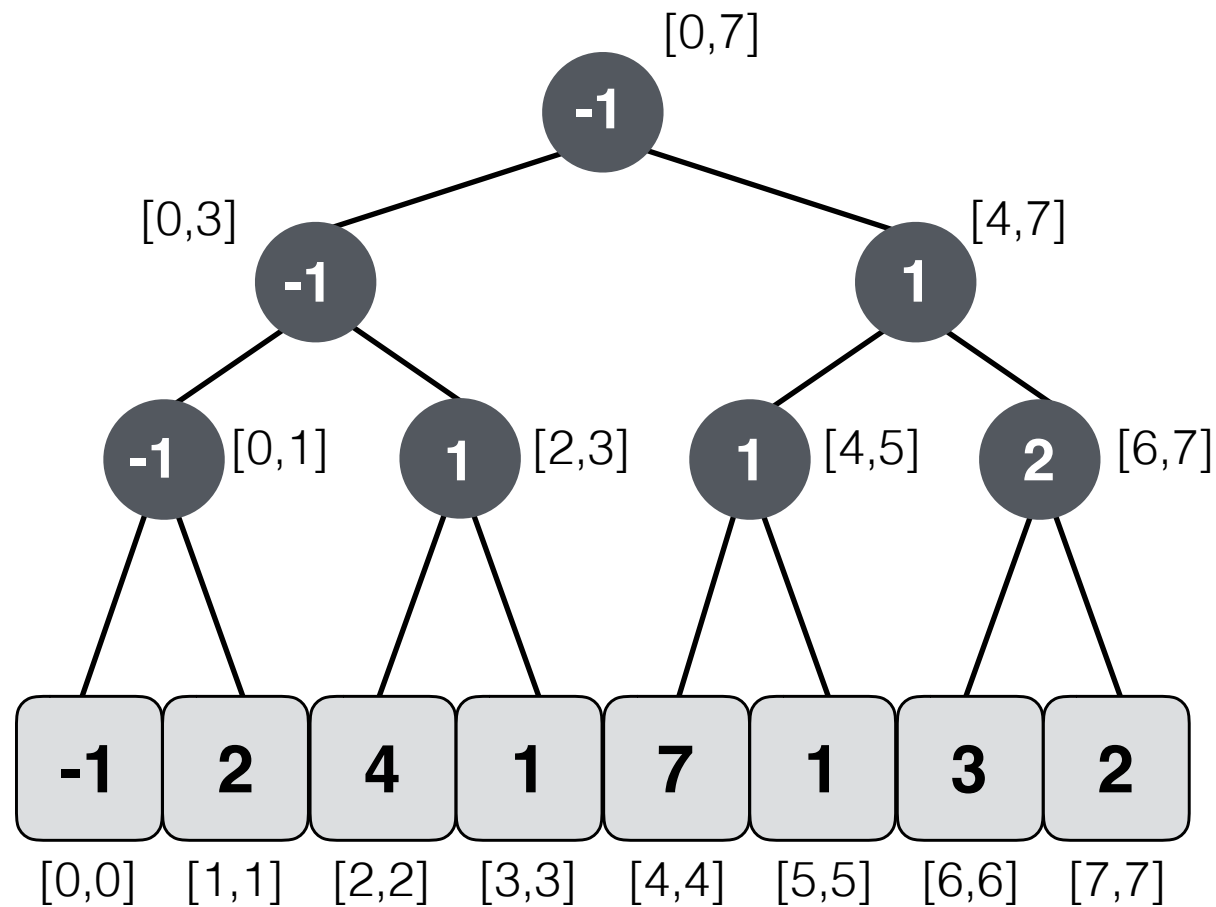


Lazy Tree

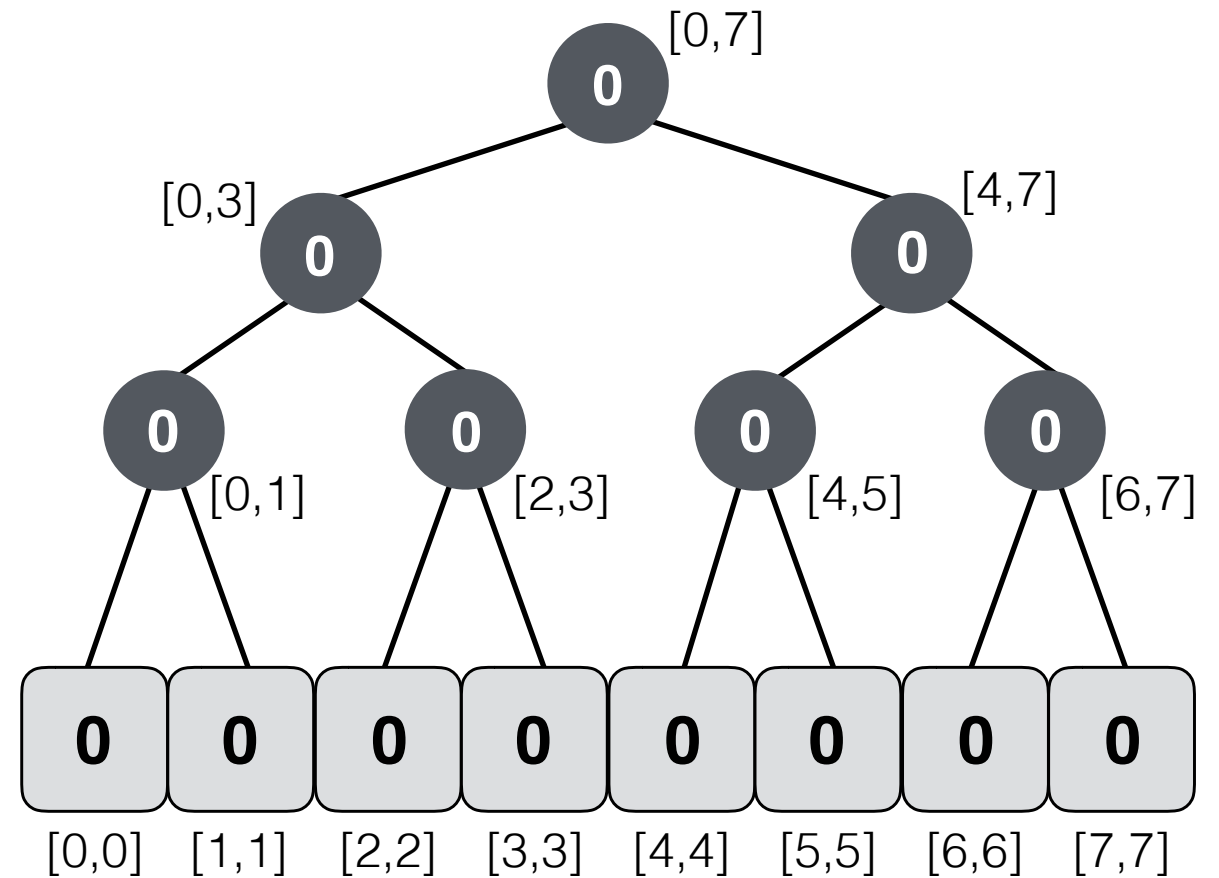
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Segment Tree

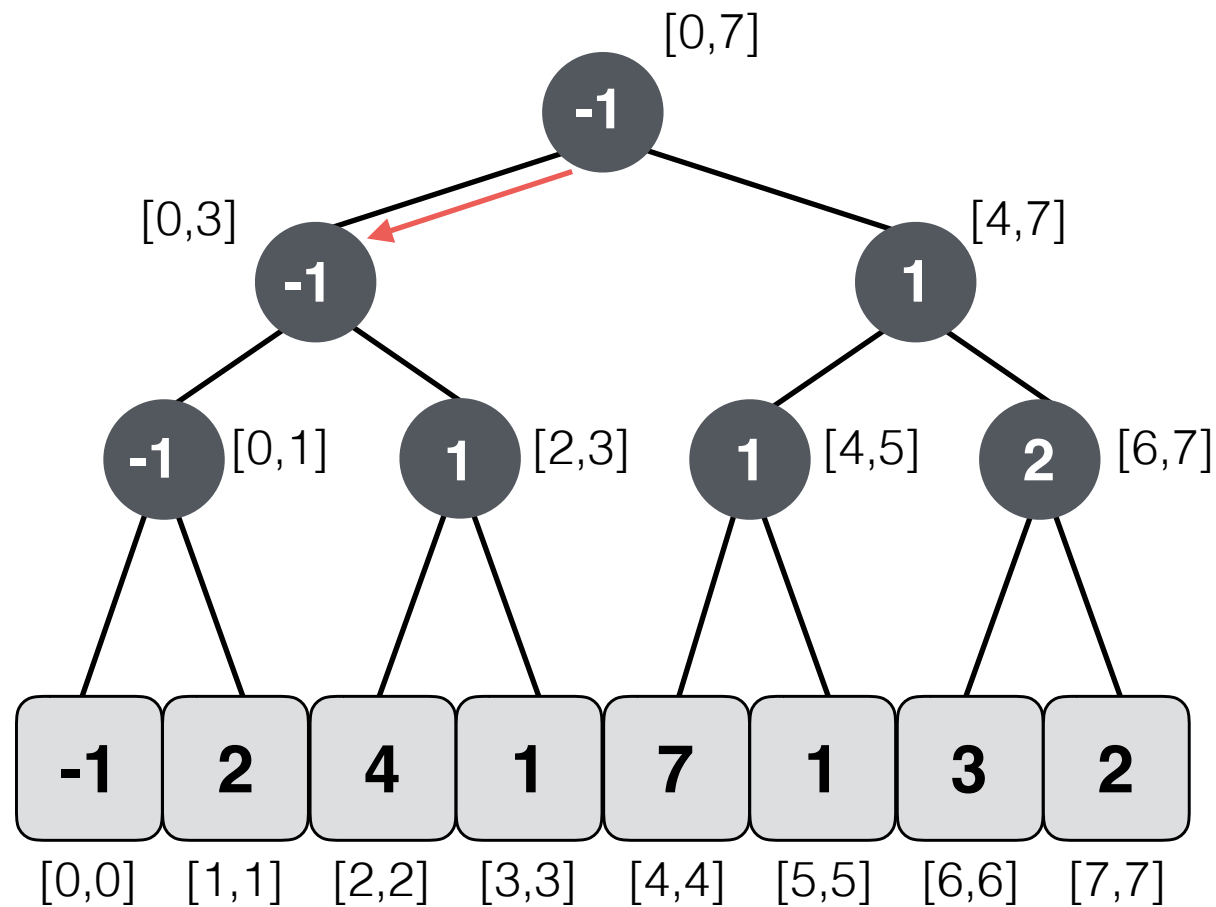


Lazy Tree

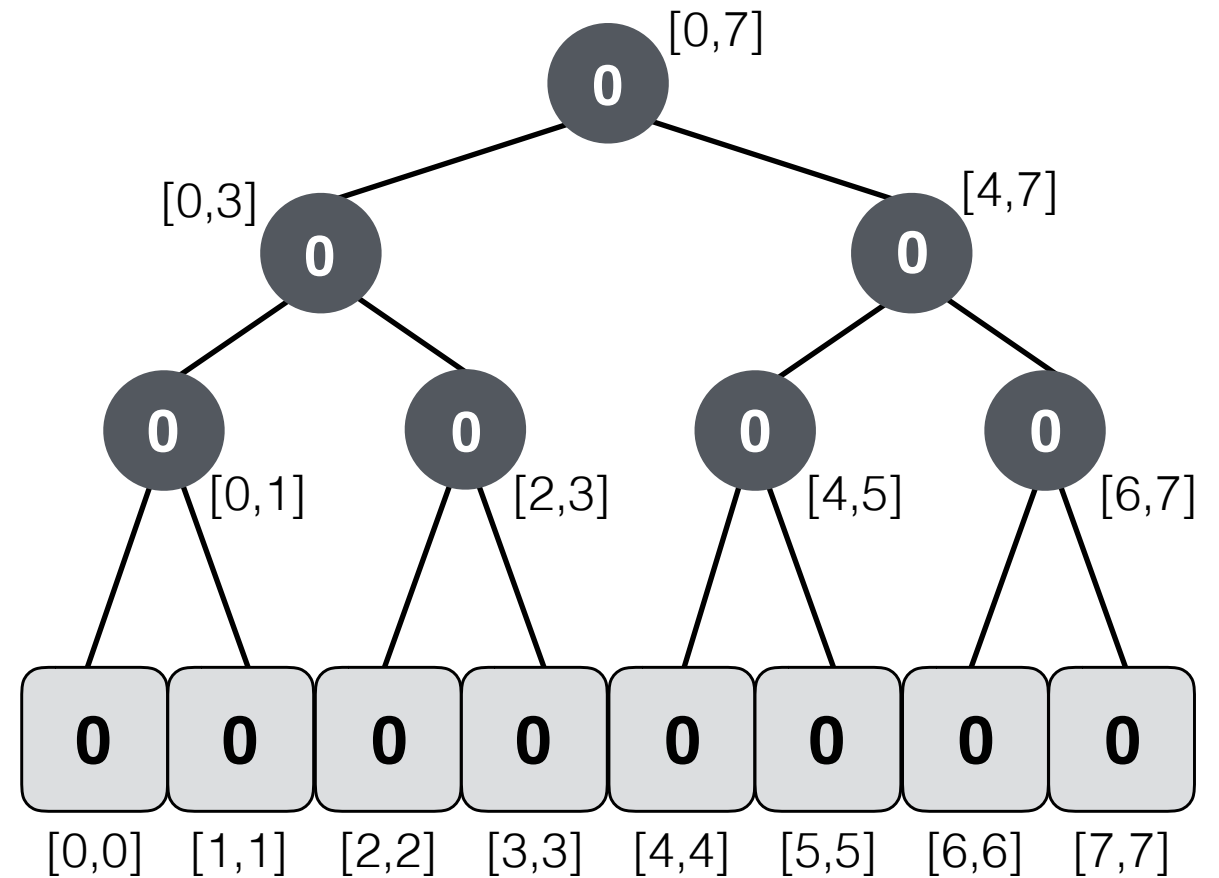
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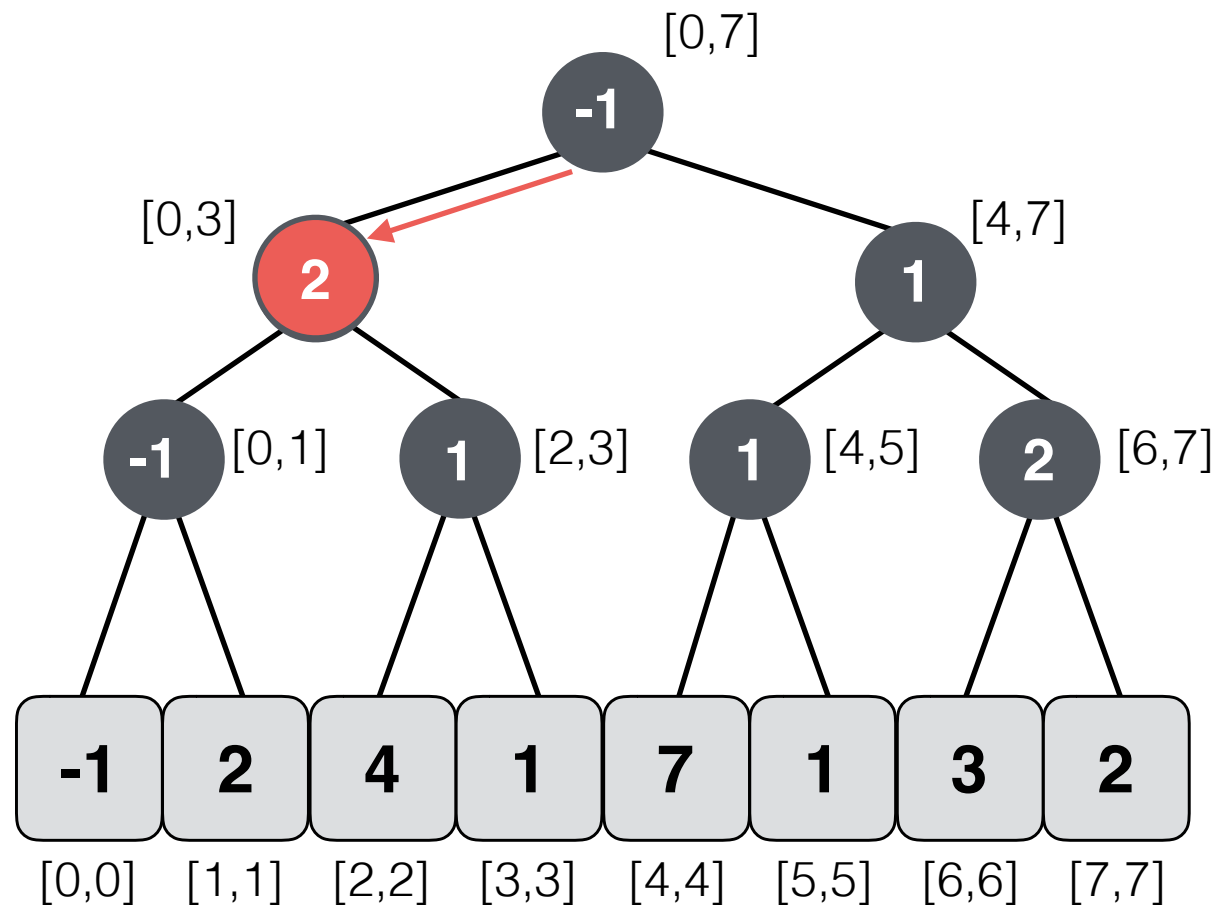


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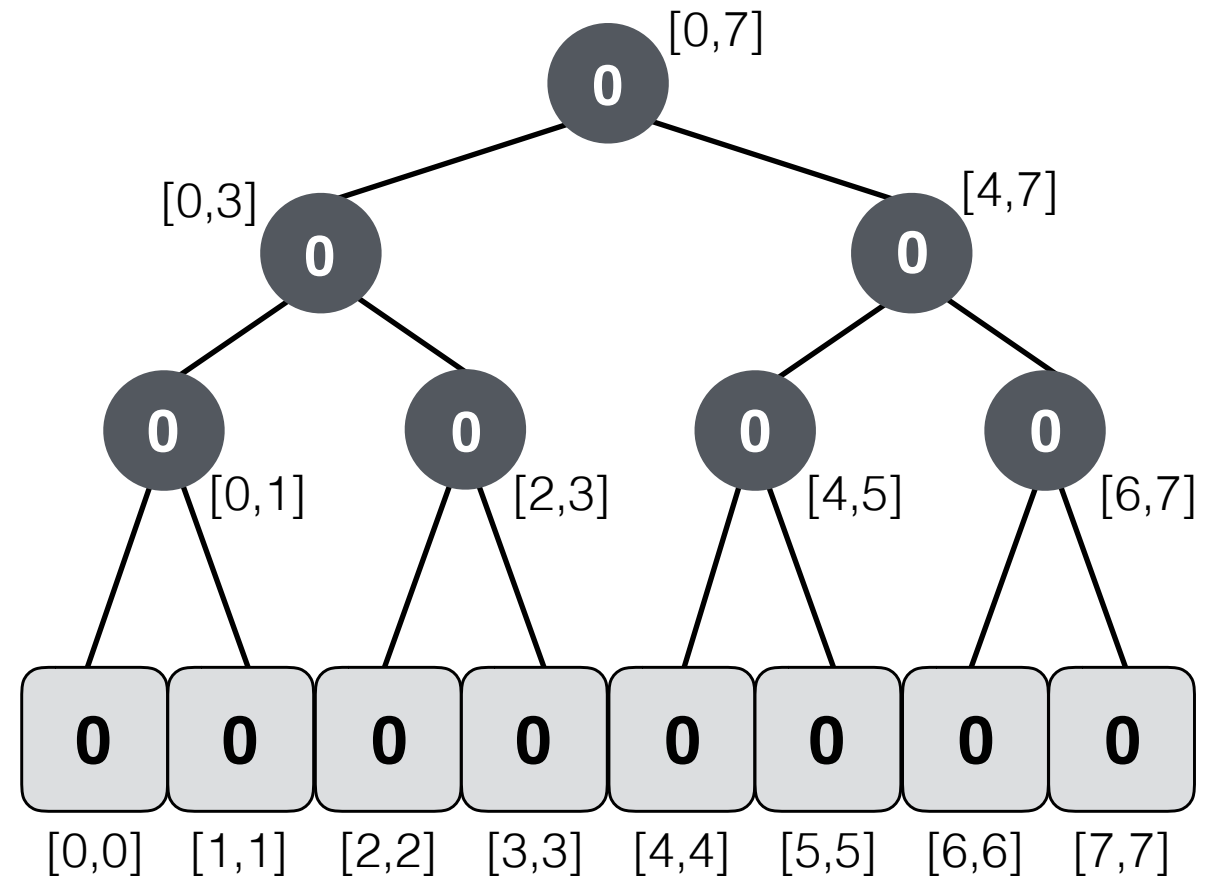
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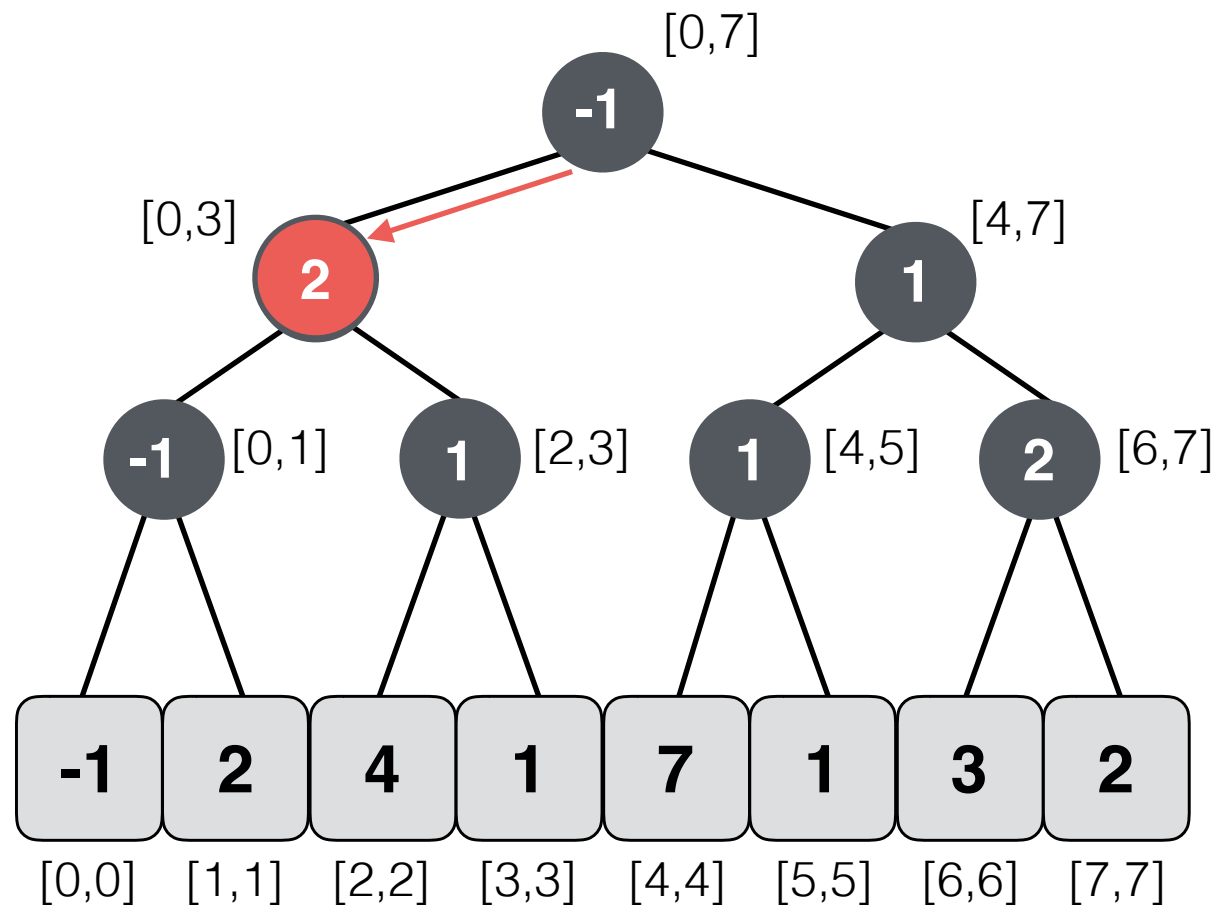


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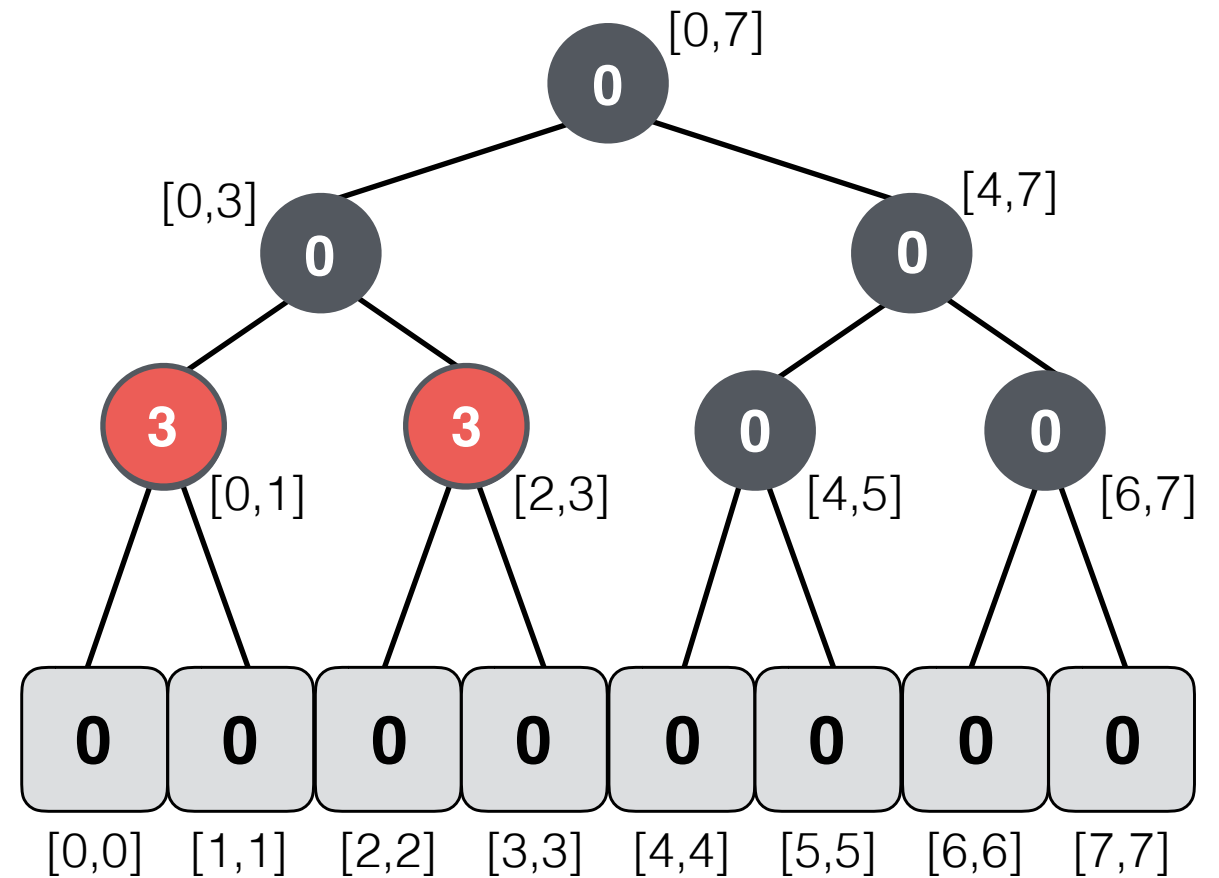
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Segment Tree

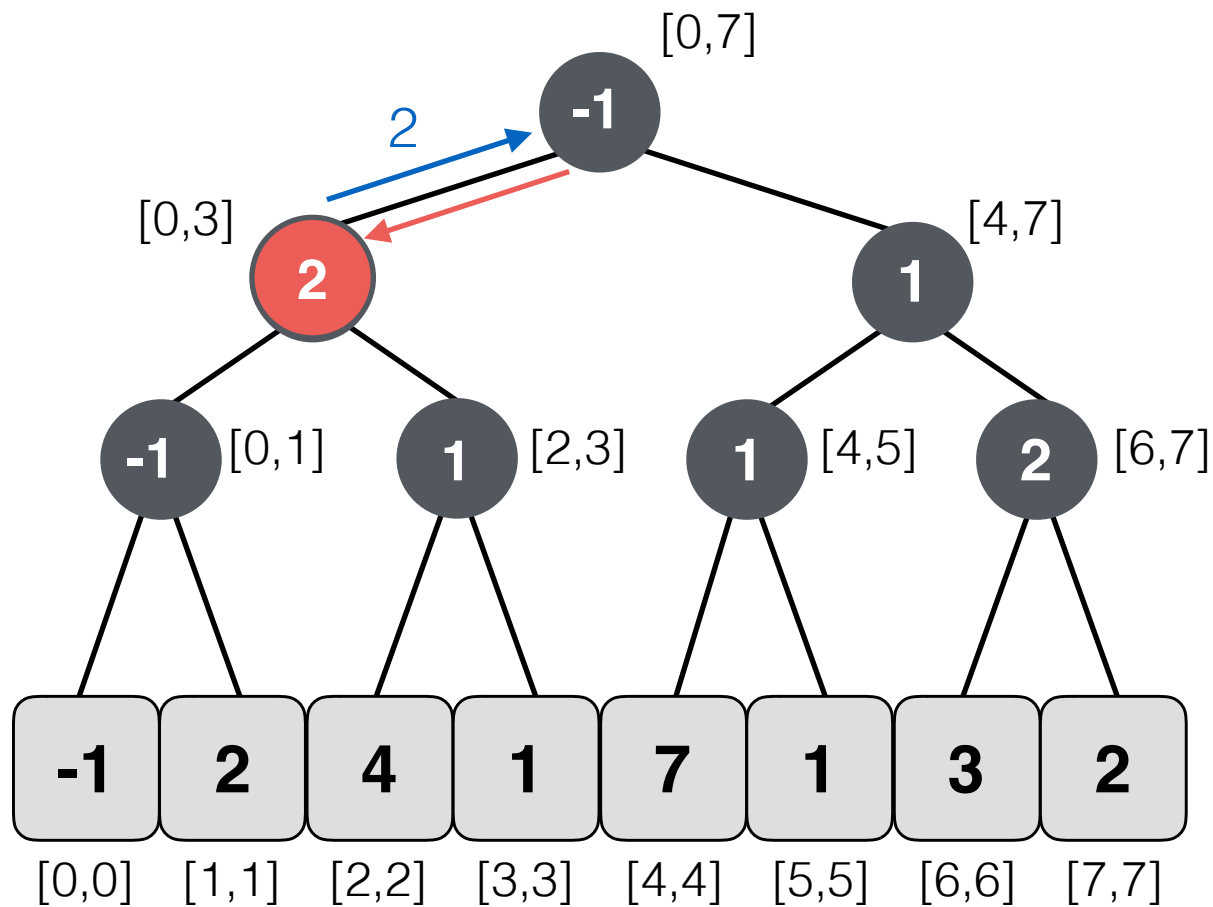


Lazy Tree

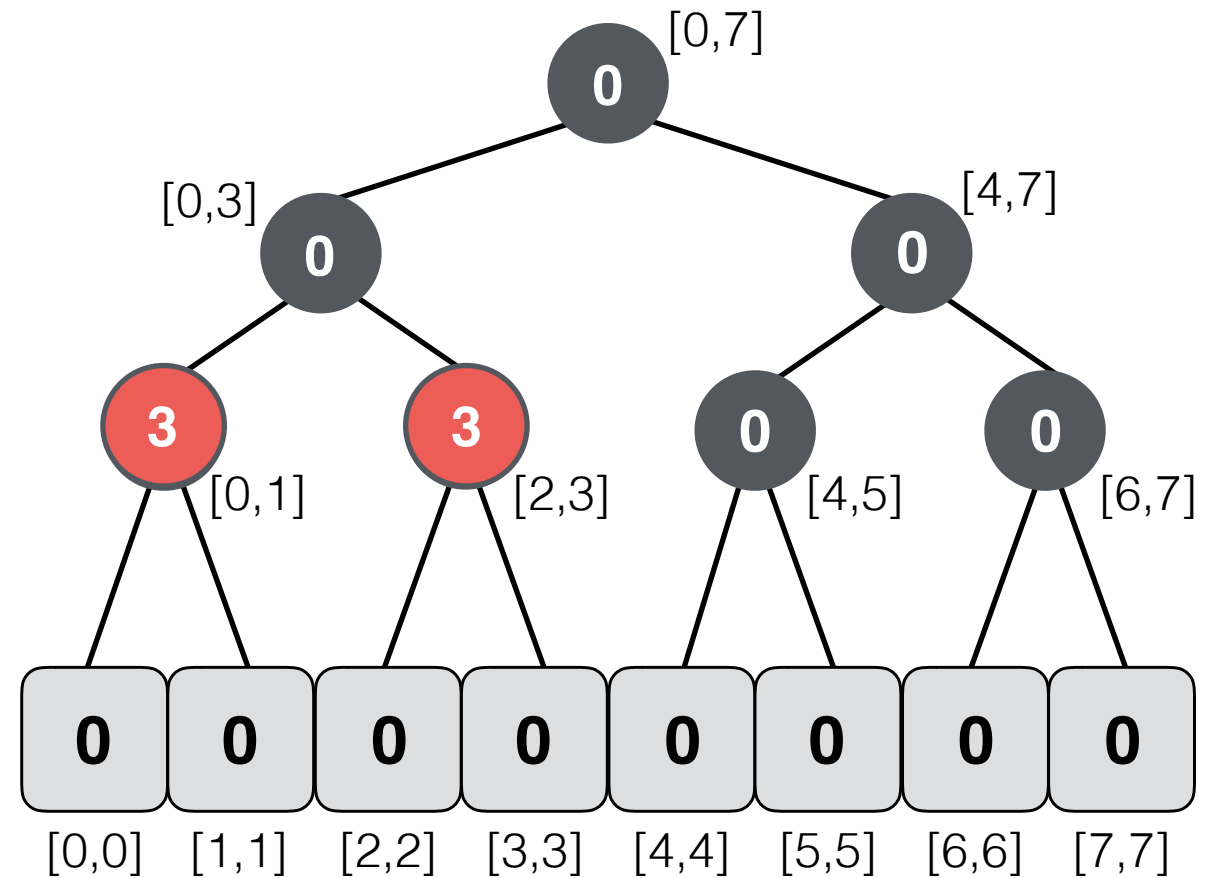
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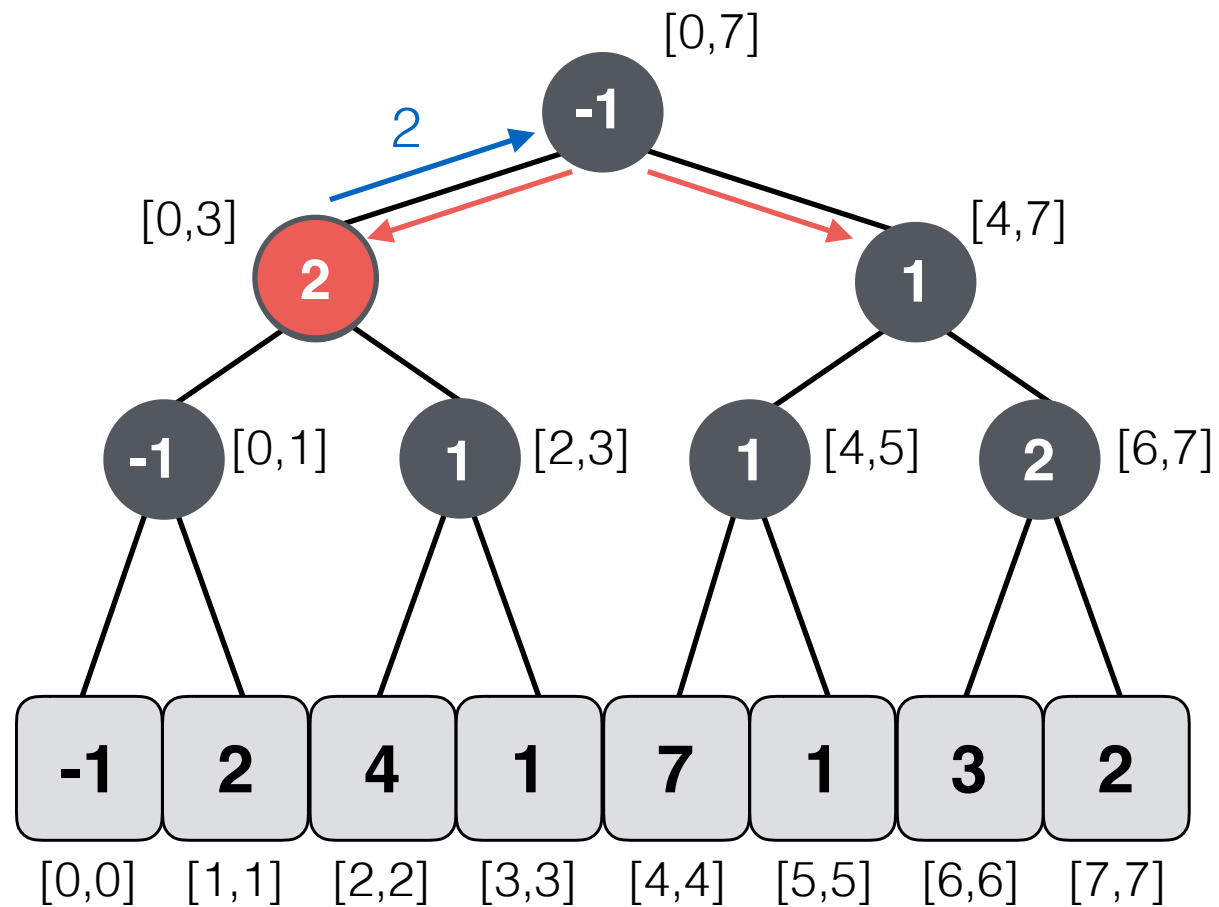


Lazy Tree

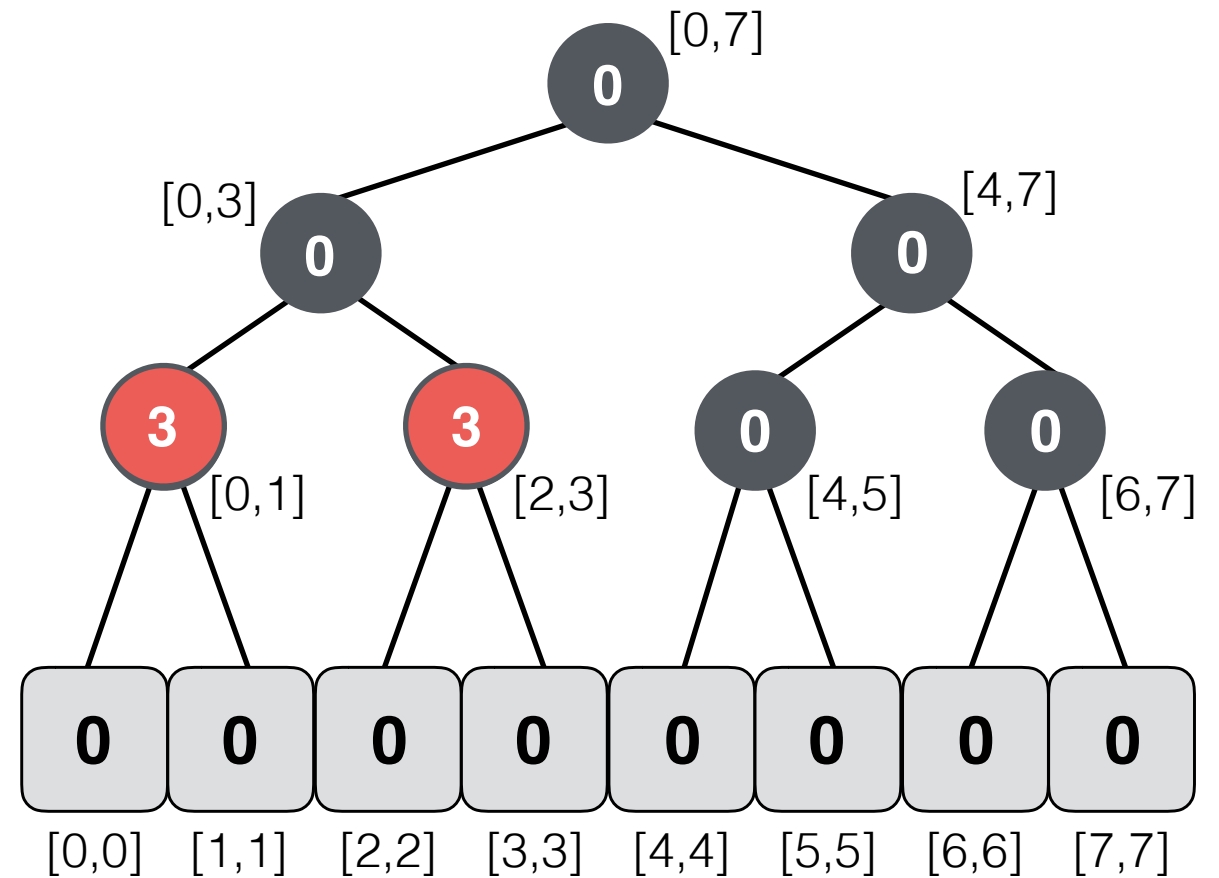
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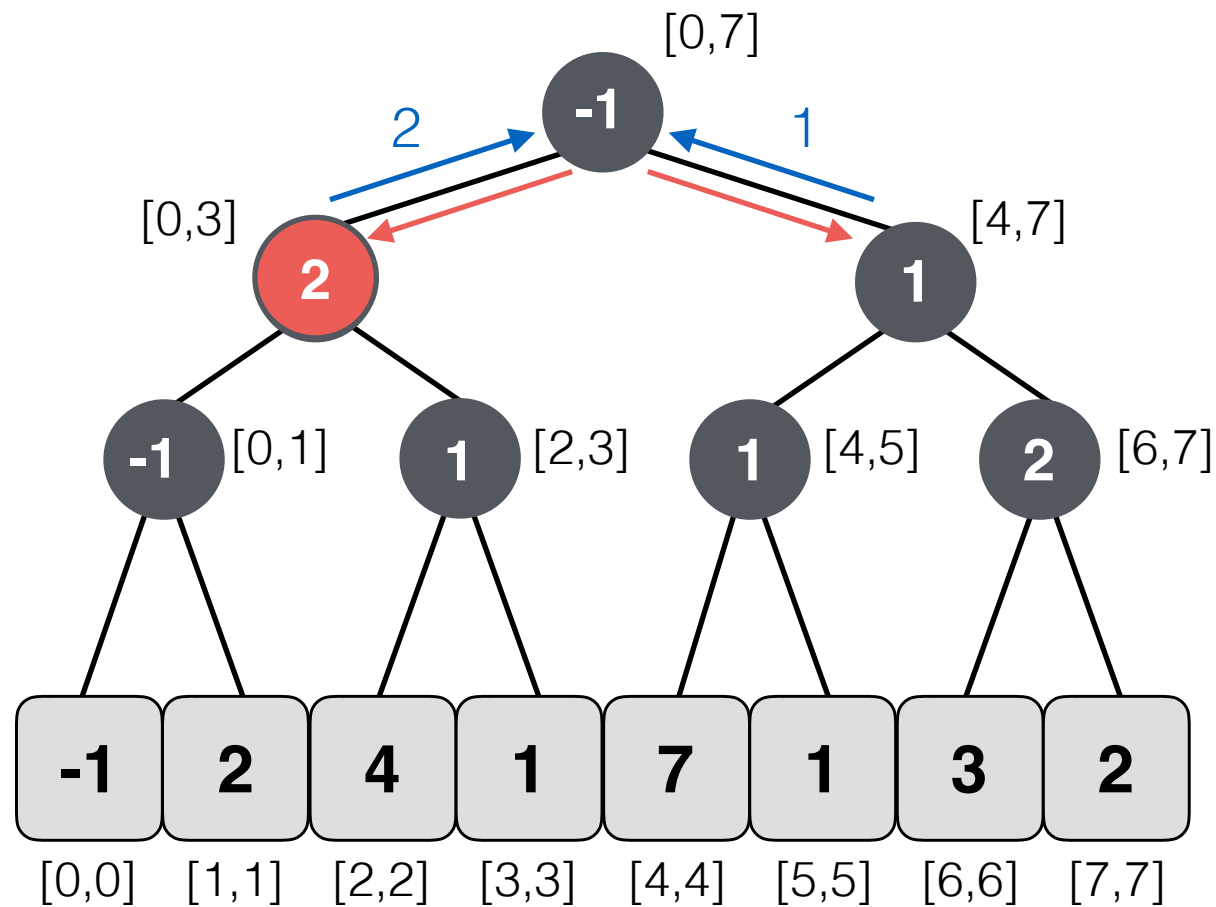


Lazy Tree

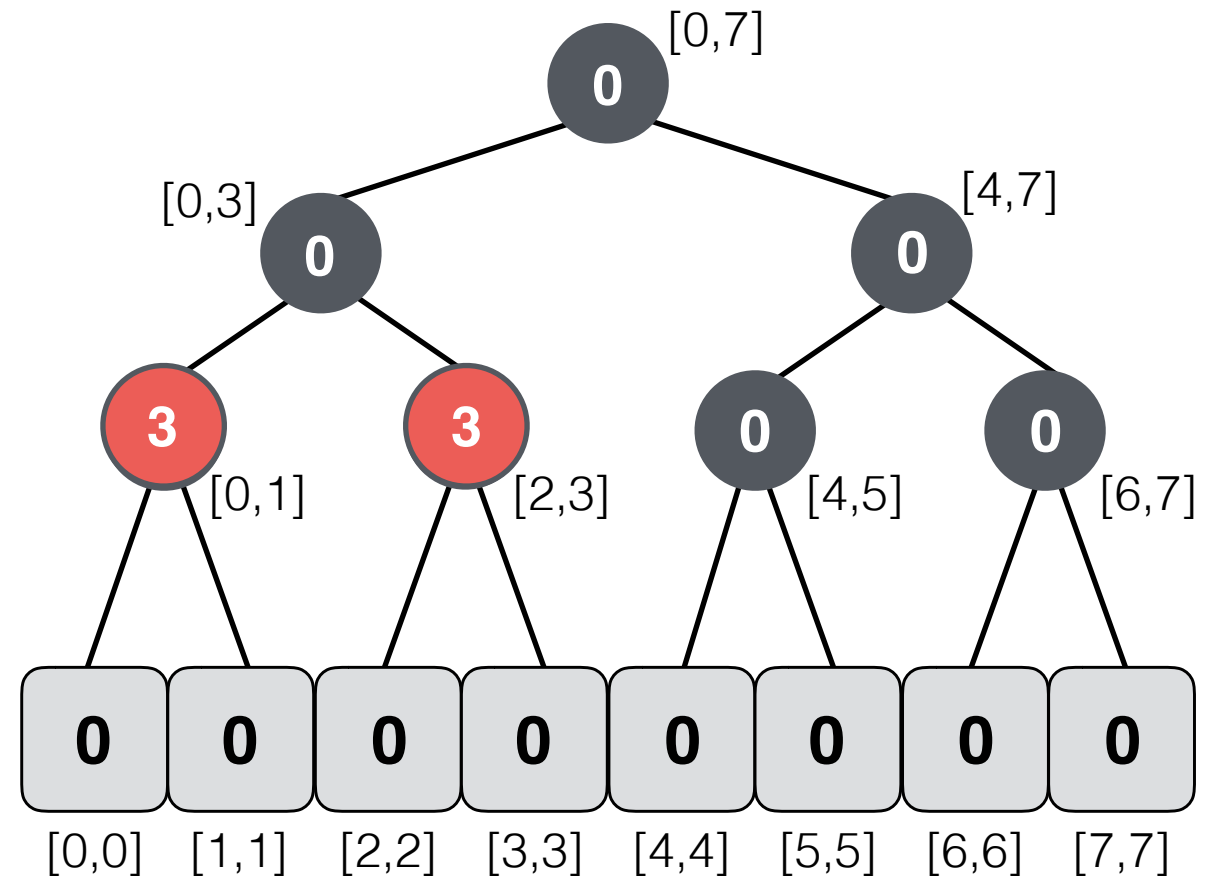
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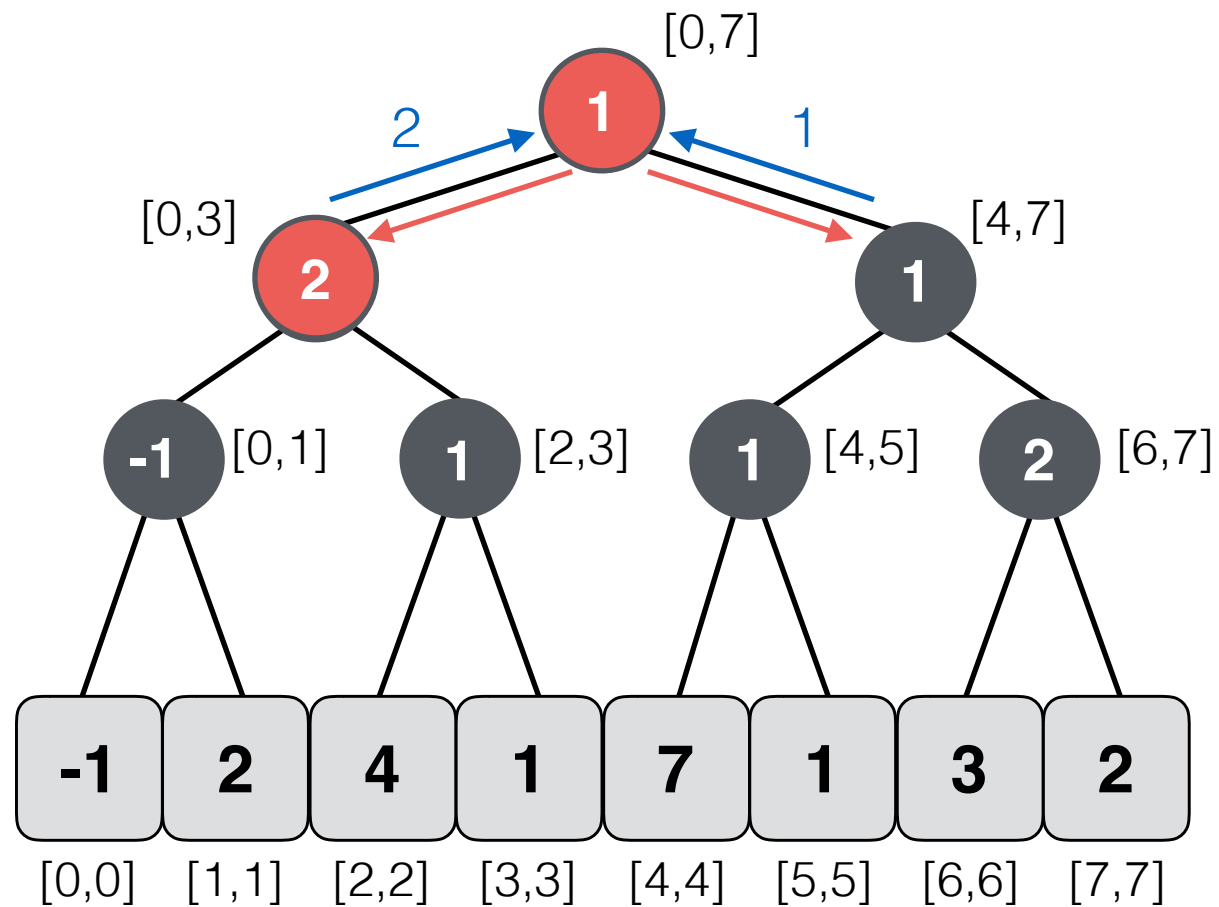


Lazy Tree

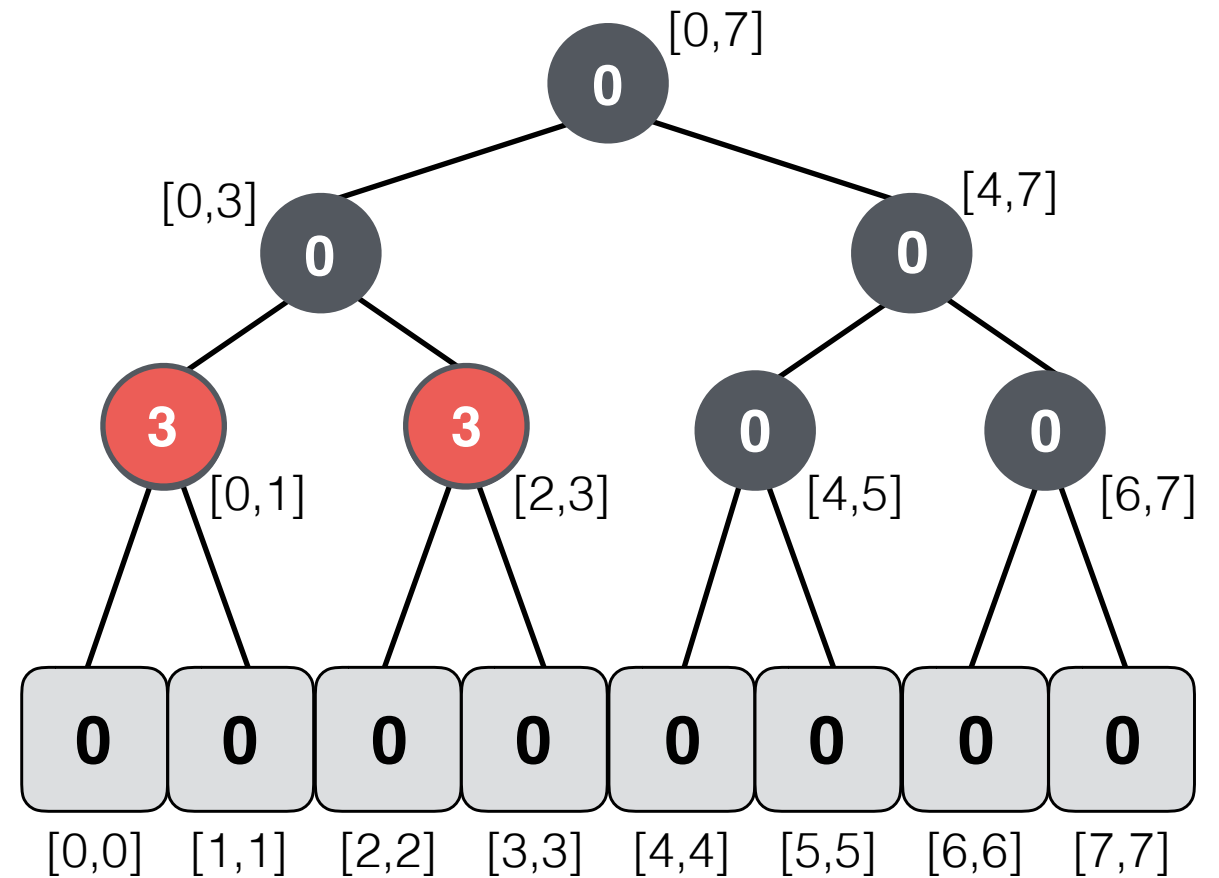
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Segment Tree

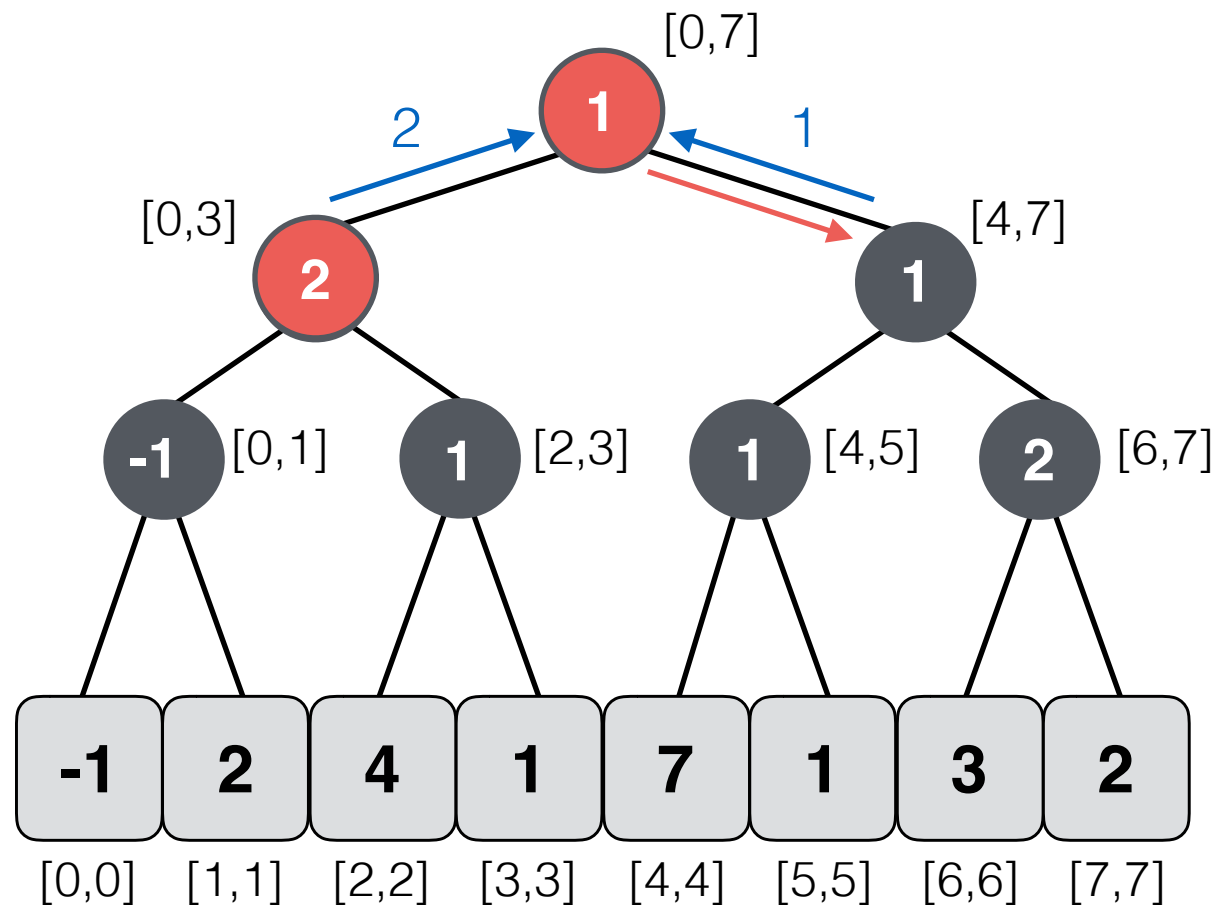


Lazy Tree

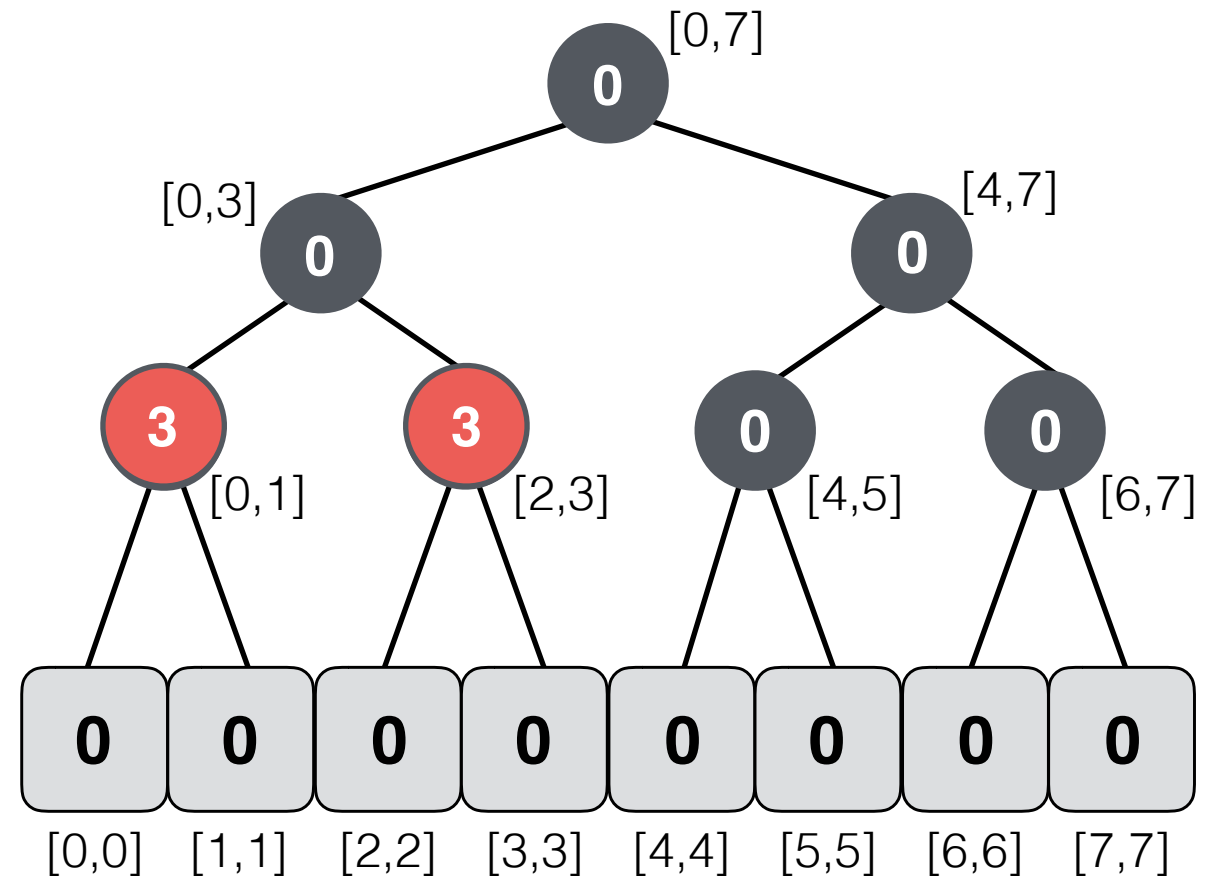
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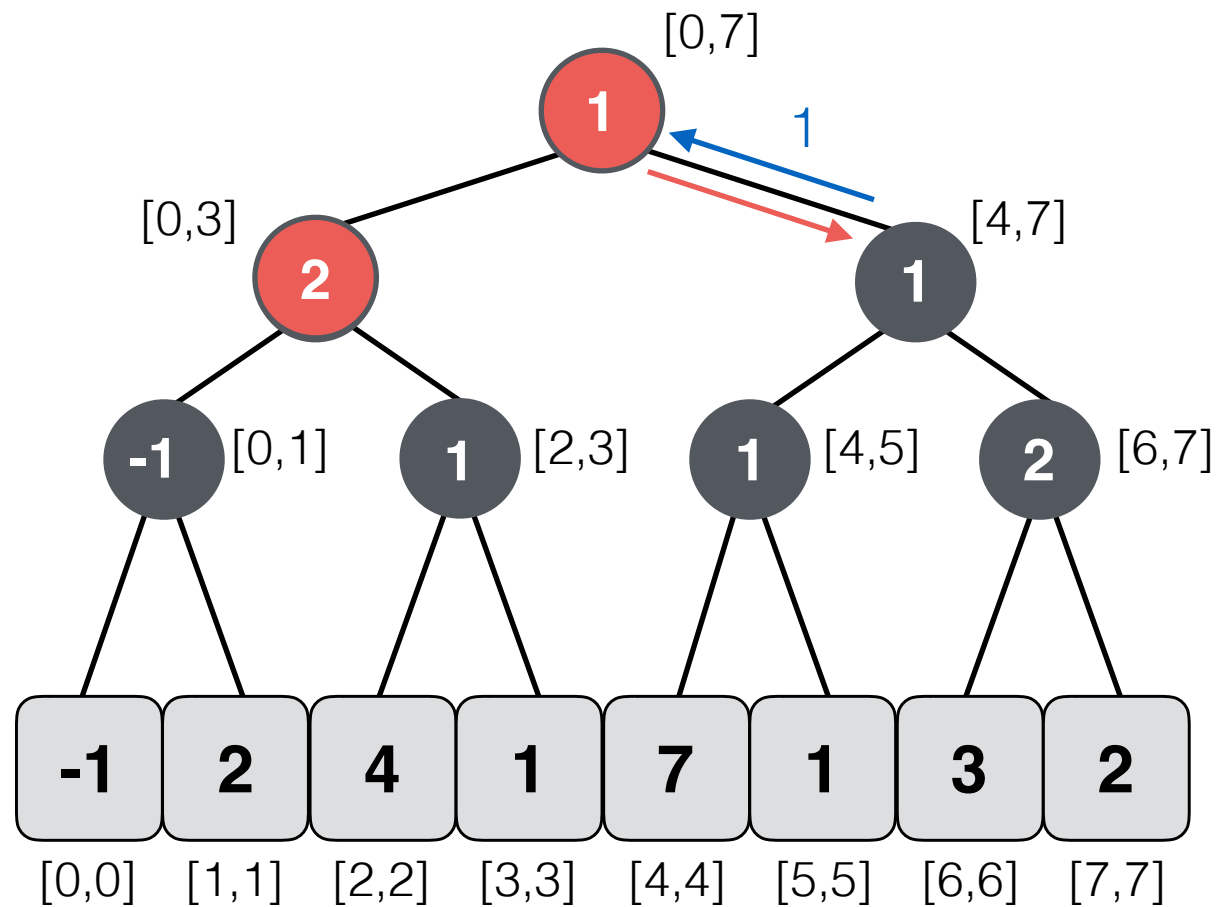


Lazy Tree

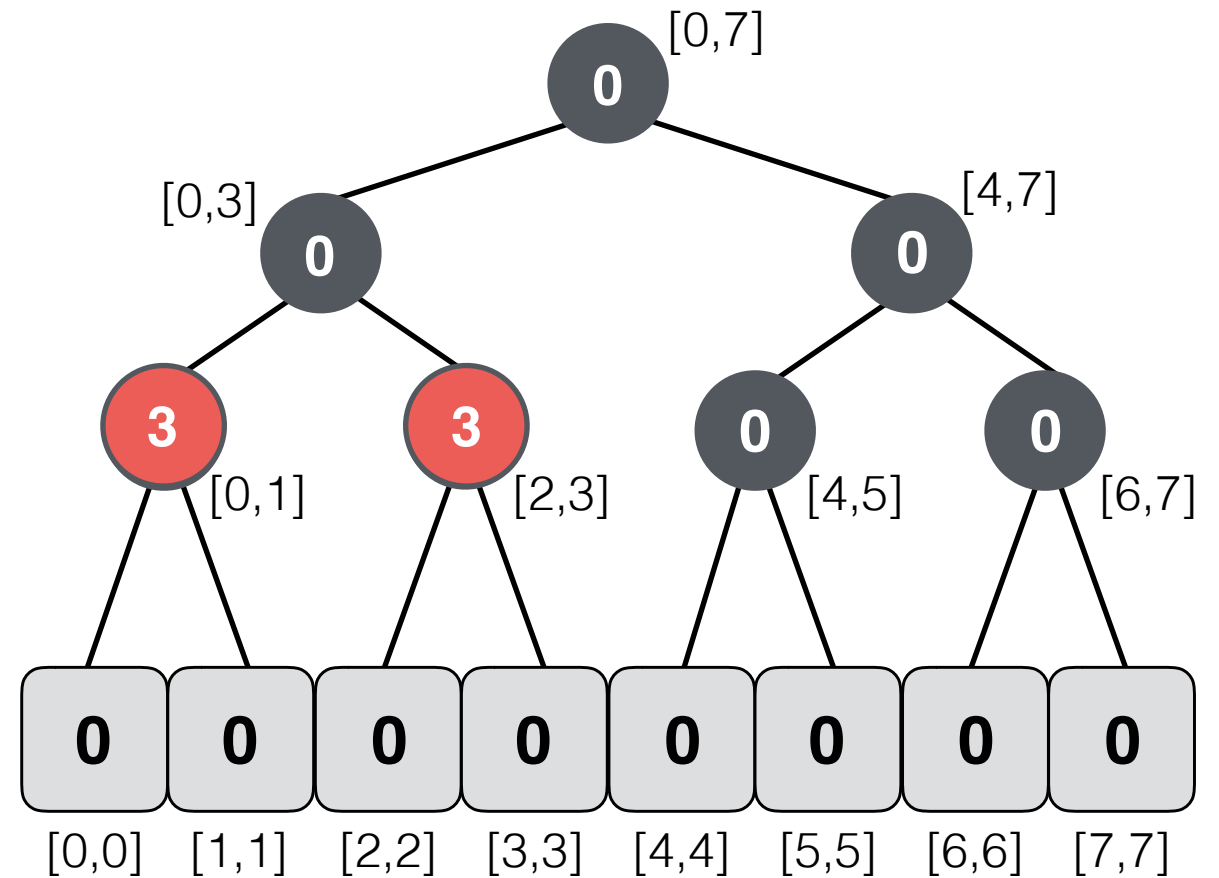
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Only update when needed.

→ update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

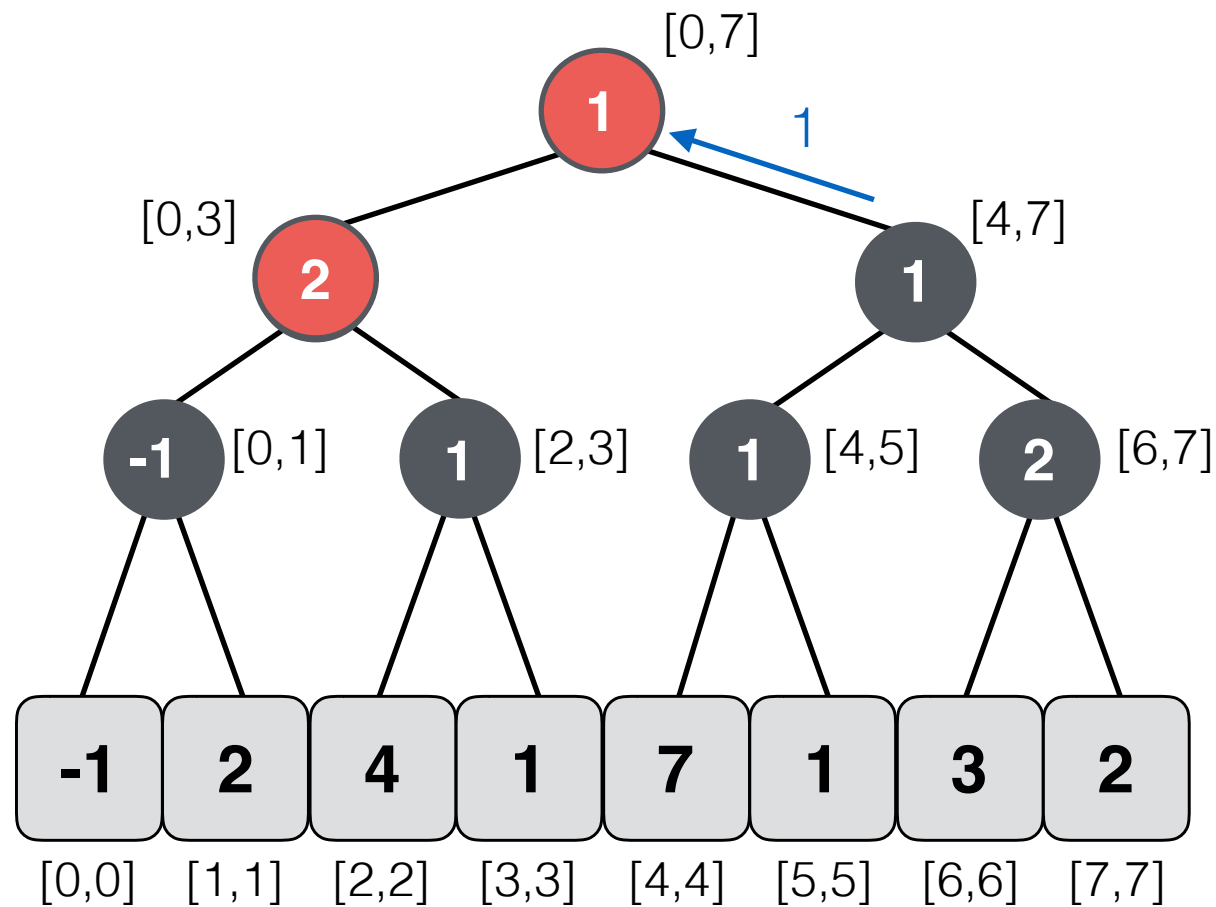


Lazy Tree

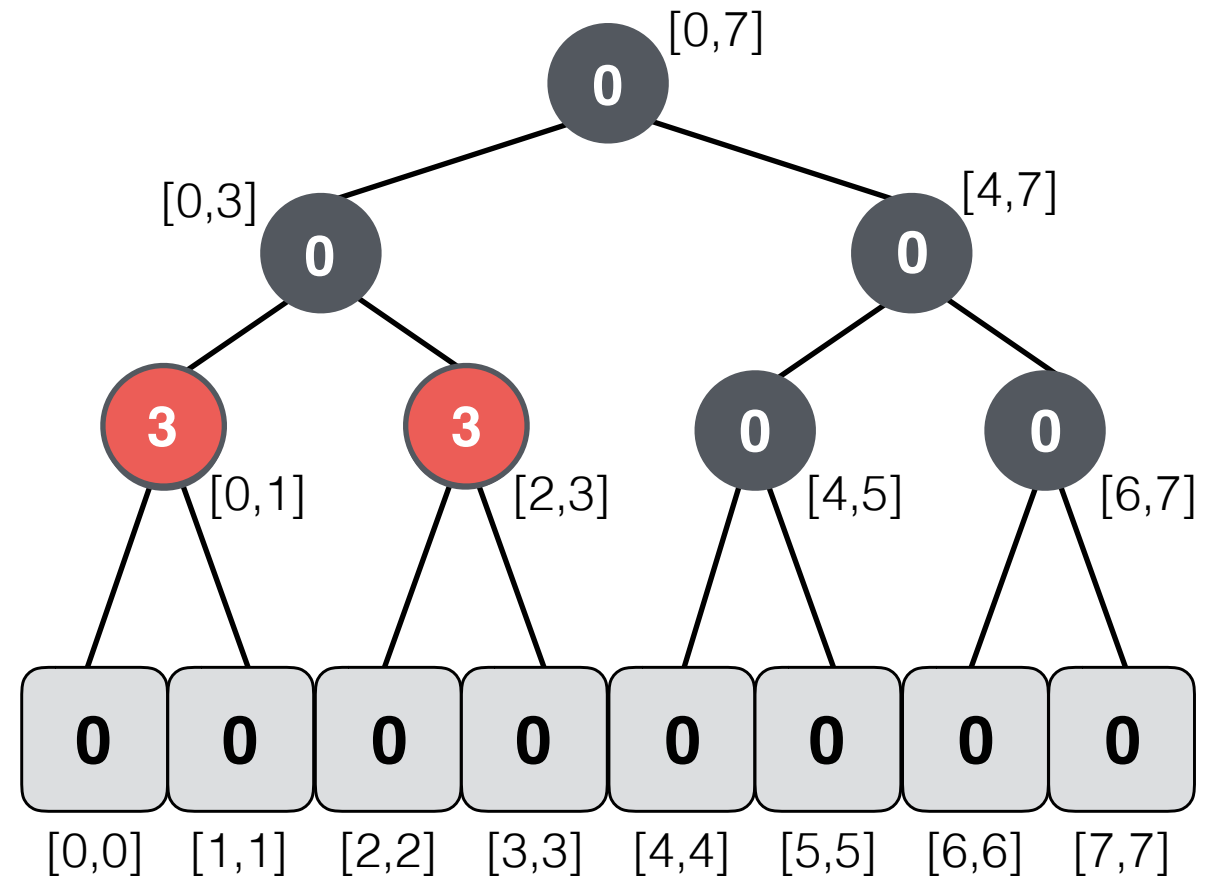
Lazy Propagation in Segment Trees

Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

→ update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

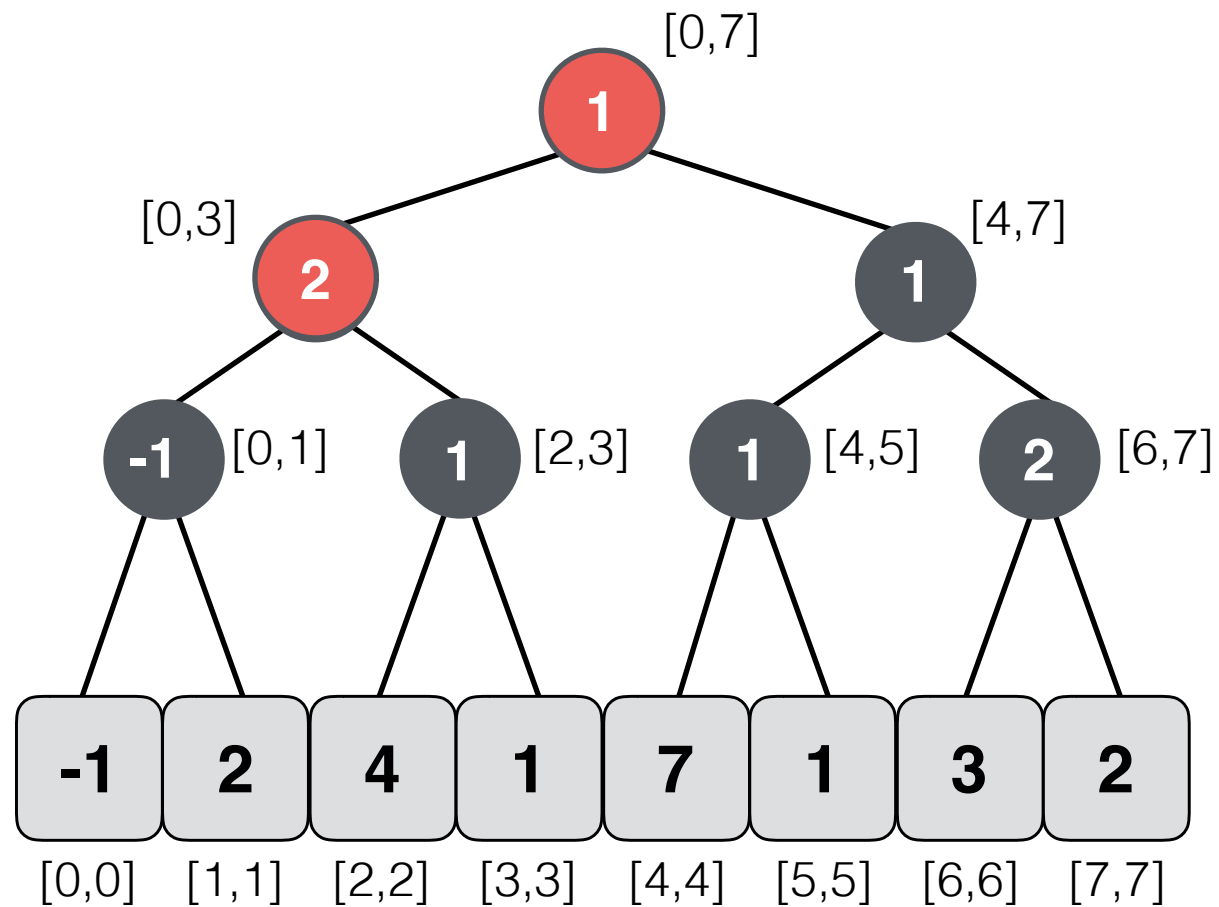


Lazy Tree

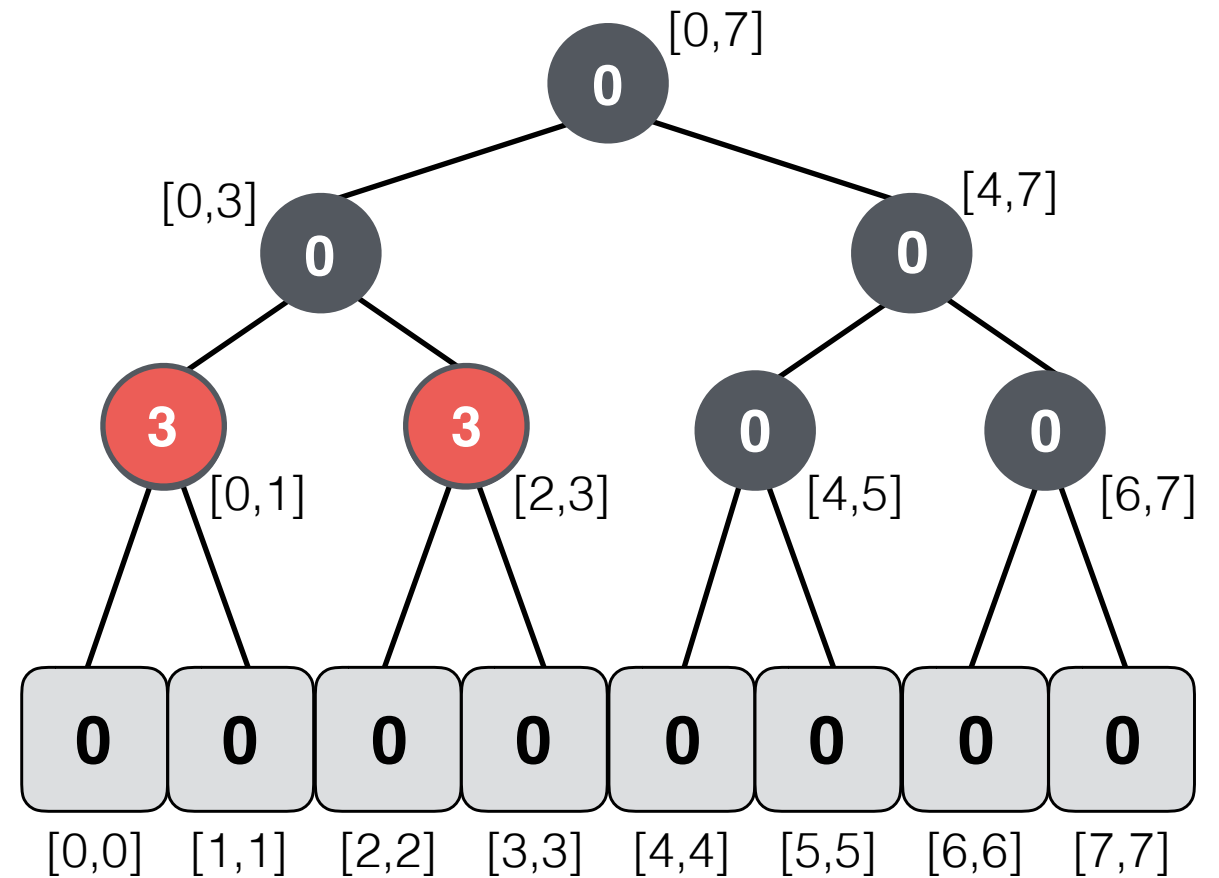
Lazy Propagation in Segment Trees

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Only update when needed.

→ update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

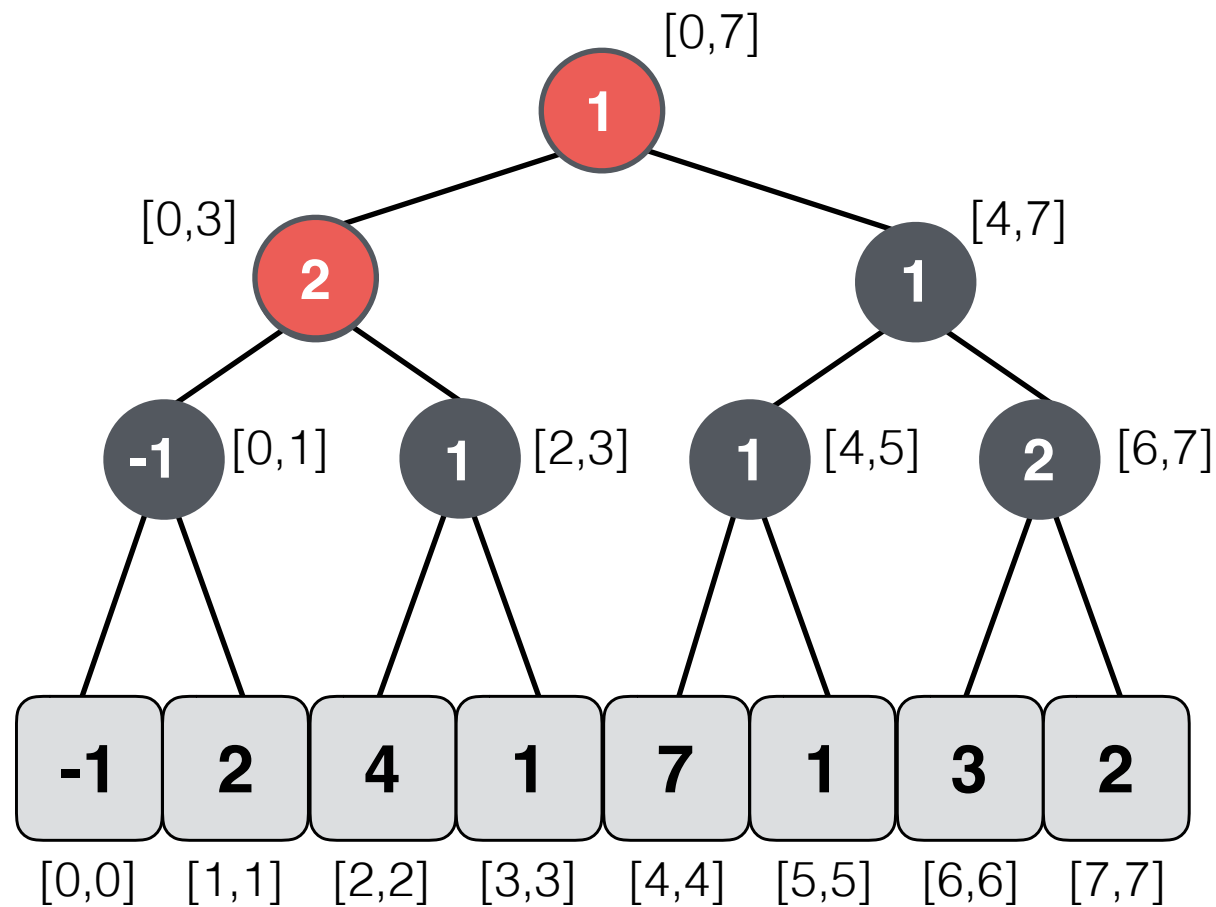


Lazy Tree

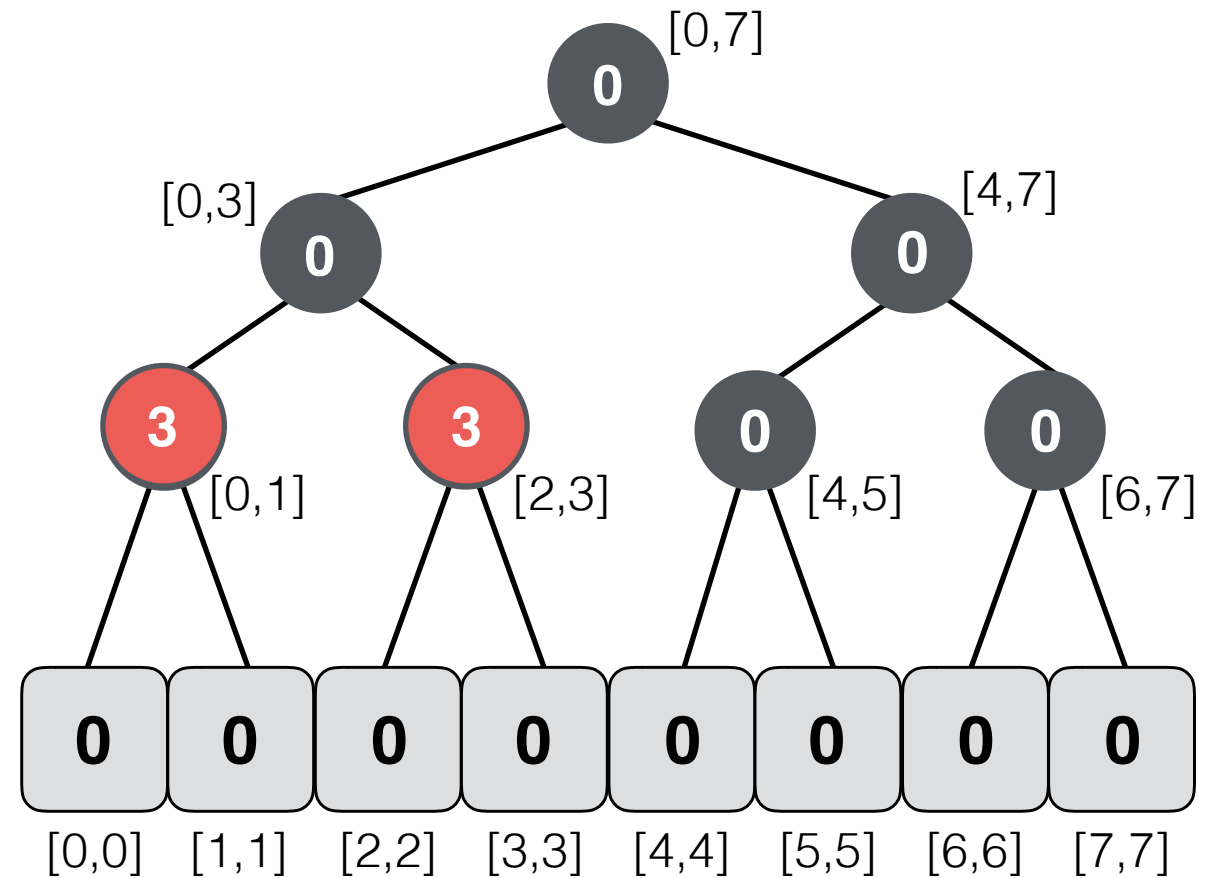
Lazy Propagation in Segment Trees

Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

→ `update_range(0,3,3)`
`update_range(0,3,1)`
`update_range(0,0,2)`
`rmq(3,5) = ?`



Segment Tree

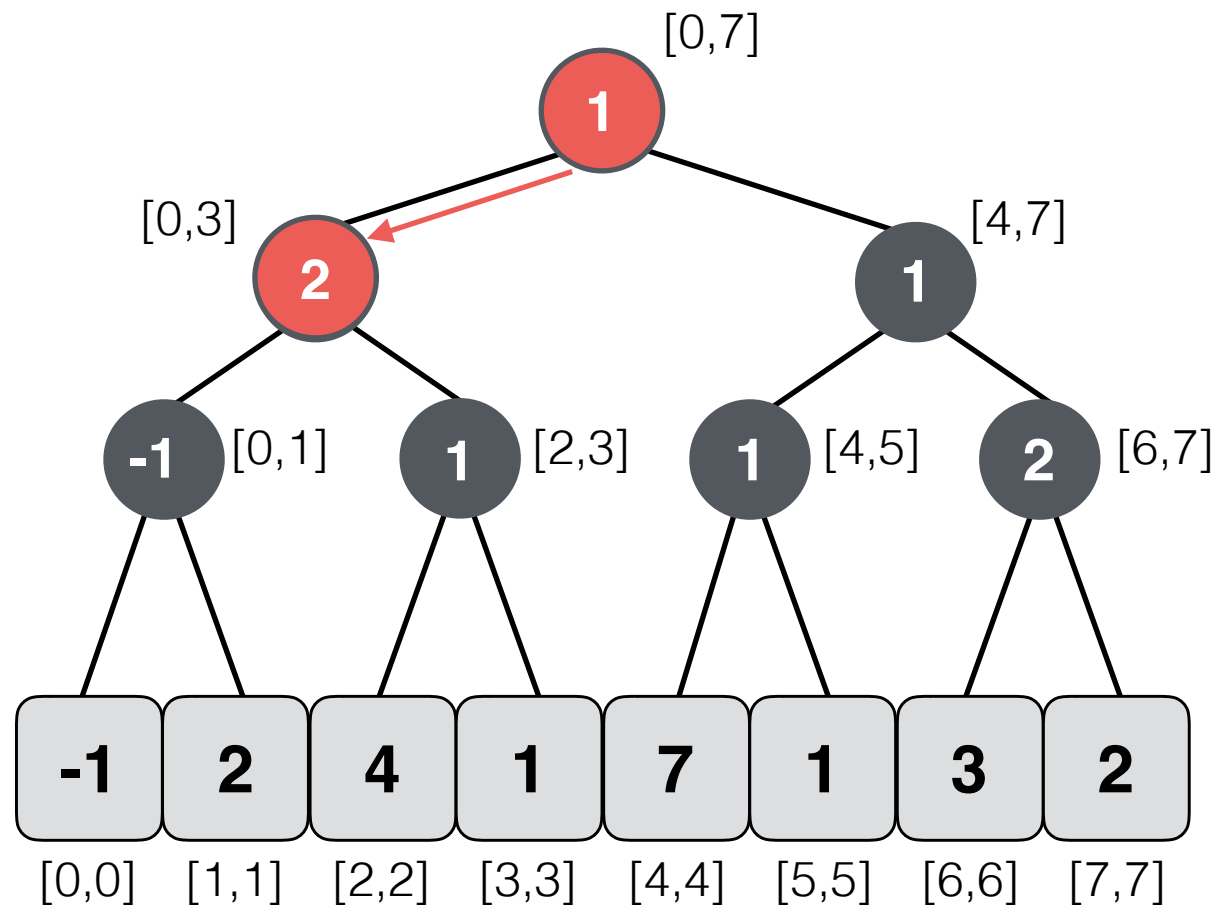


Lazy Tree

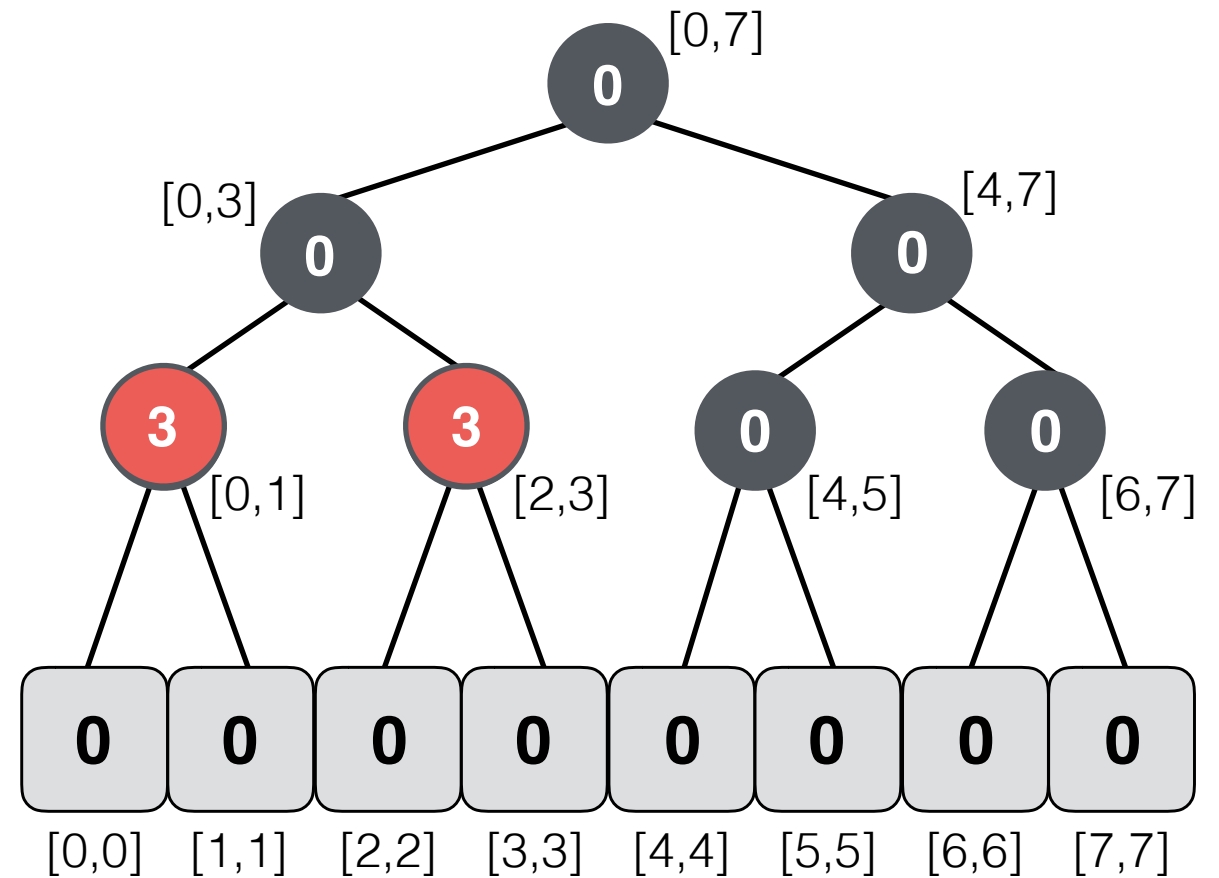
Lazy Propagation in Segment Trees

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→ `update_range(0,3,3)`
`update_range(0,3,1)`
`update_range(0,0,2)`
`rmq(3,5) = ?`



Segment Tree

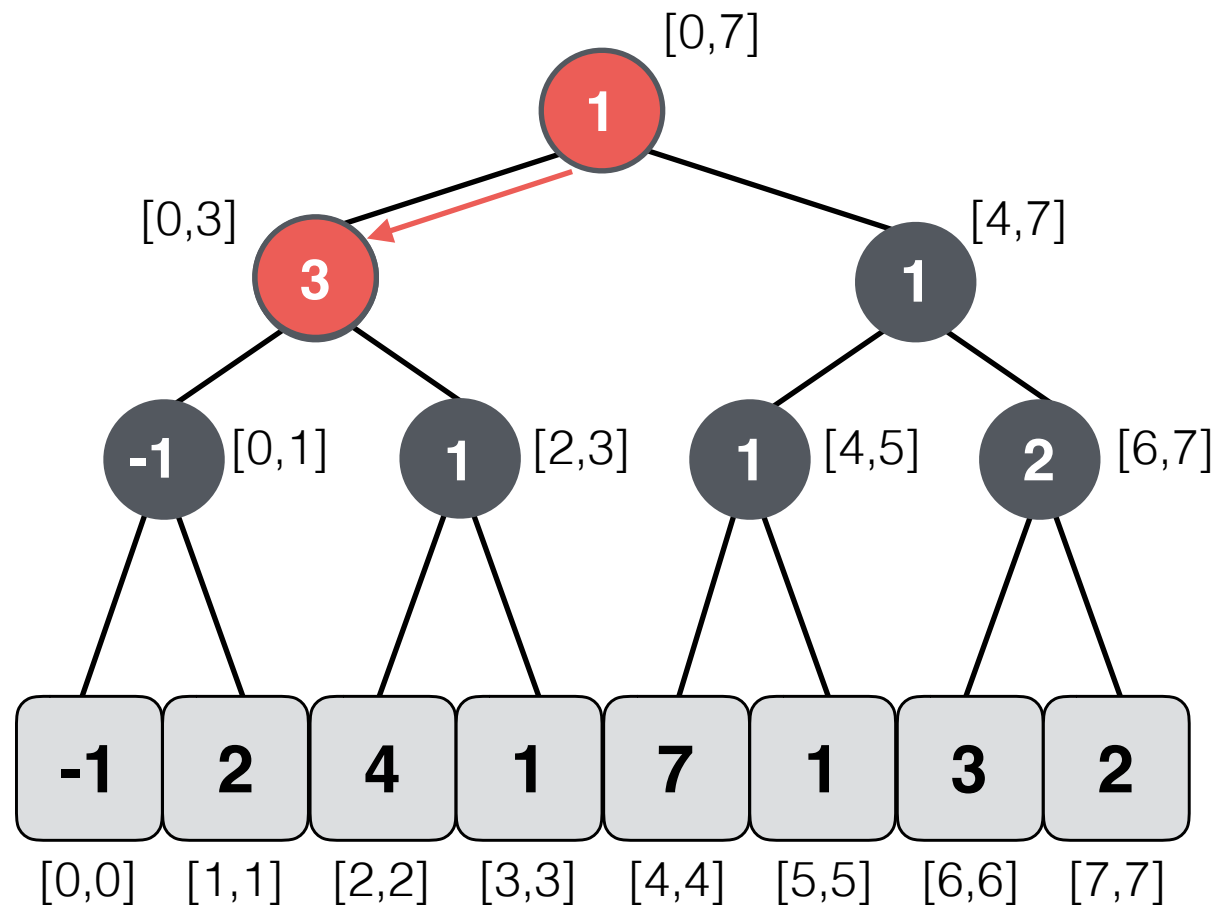


Lazy Tree

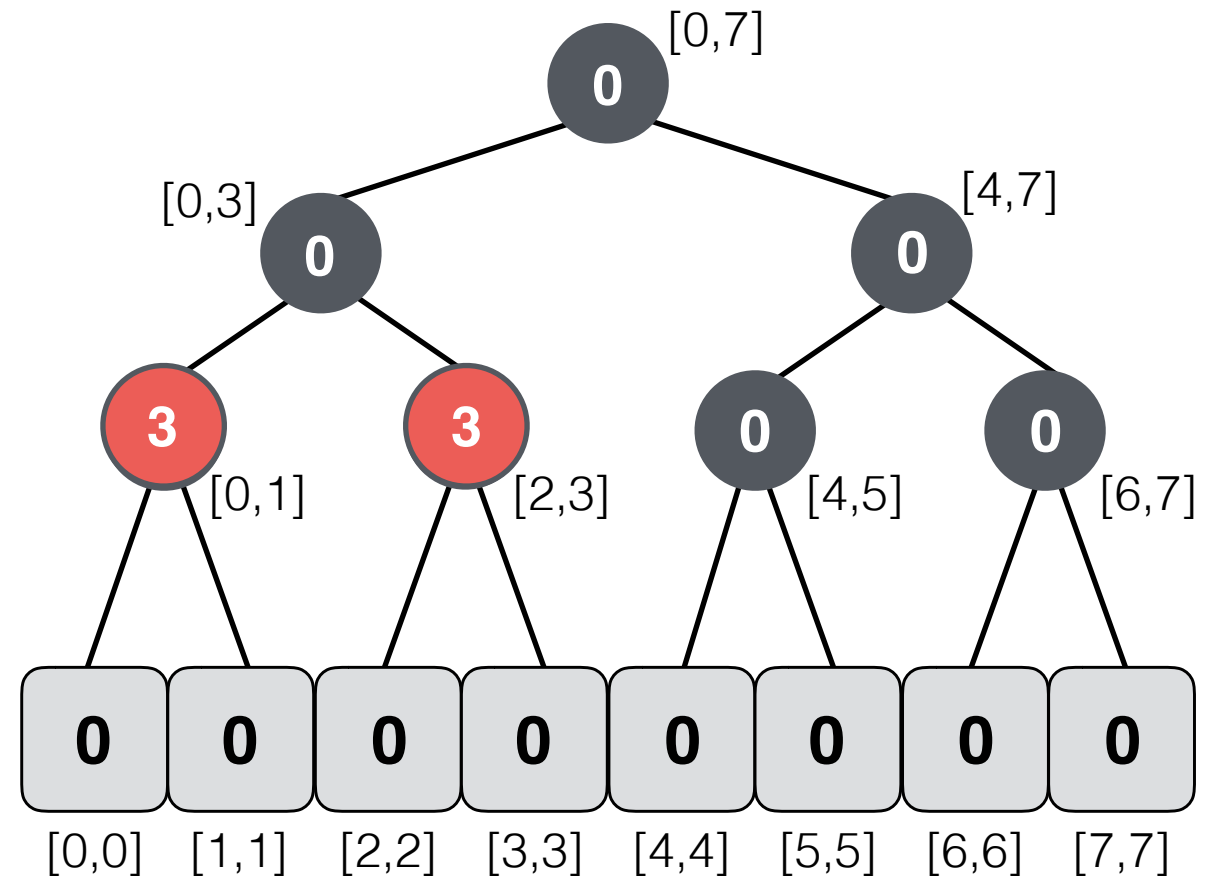
Lazy Propagation in Segment Trees

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→ `update_range(0,3,3)`
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`update_range(0,0,2)`
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Segment Tree

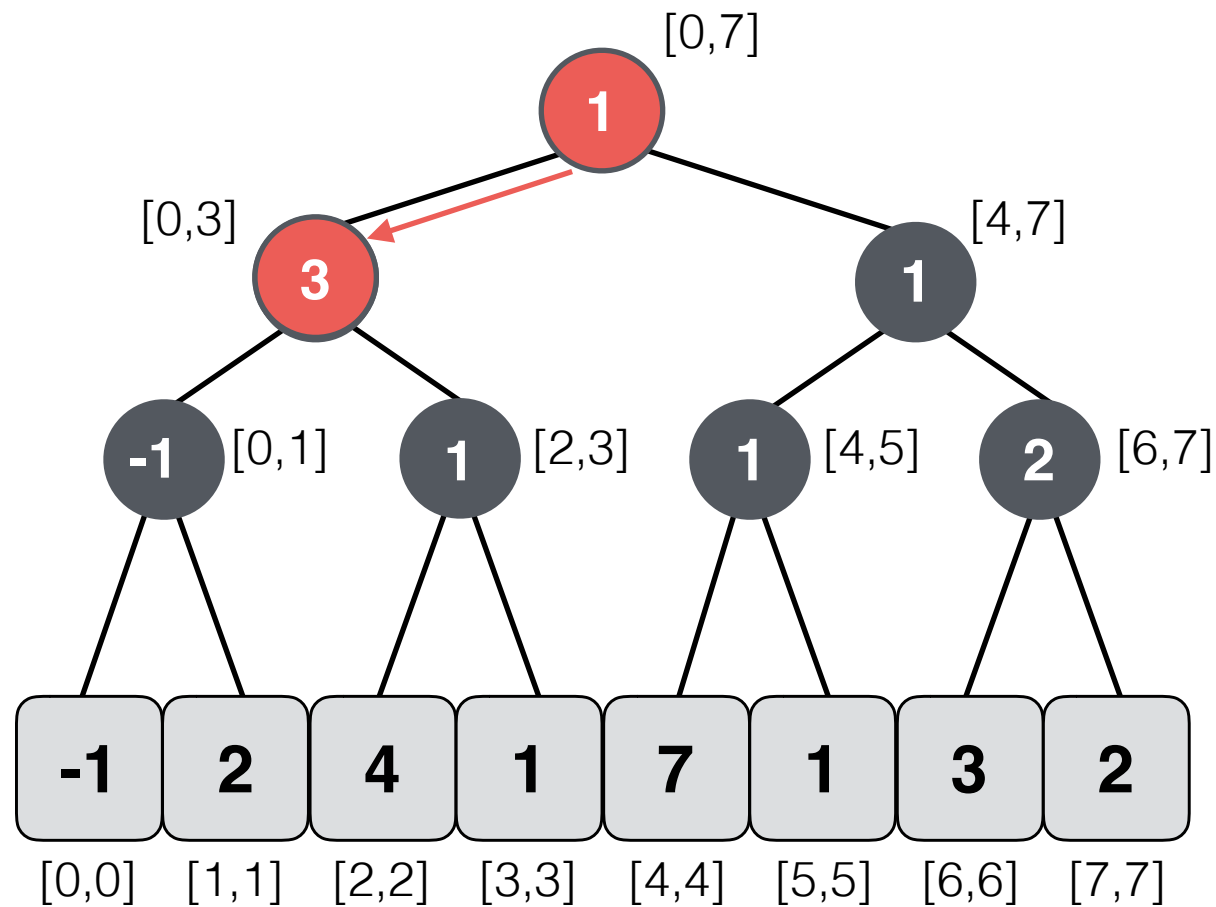


Lazy Tree

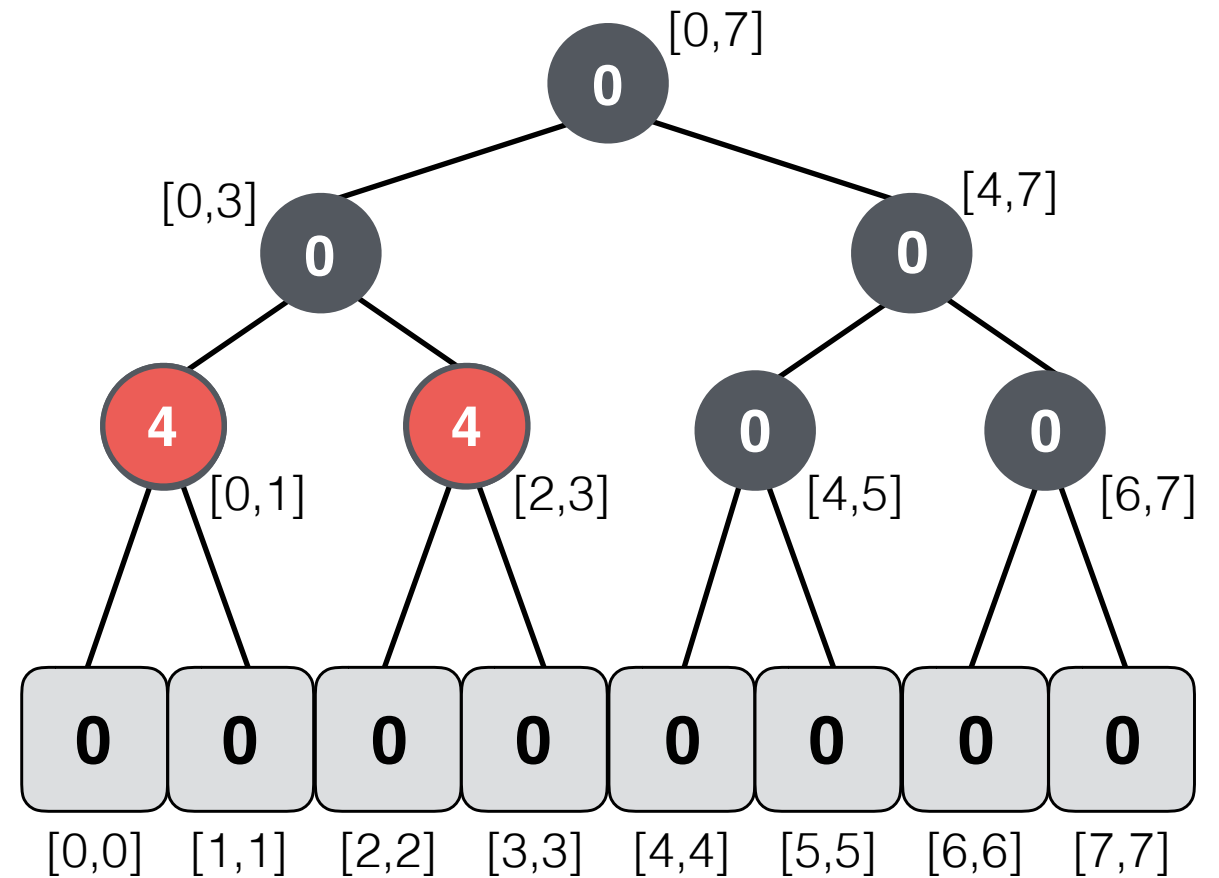
Lazy Propagation in Segment Trees

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Only update when needed.

→ `update_range(0,3,3)`
`update_range(0,3,1)`
`update_range(0,0,2)`
`rmq(3,5) = ?`



Segment Tree

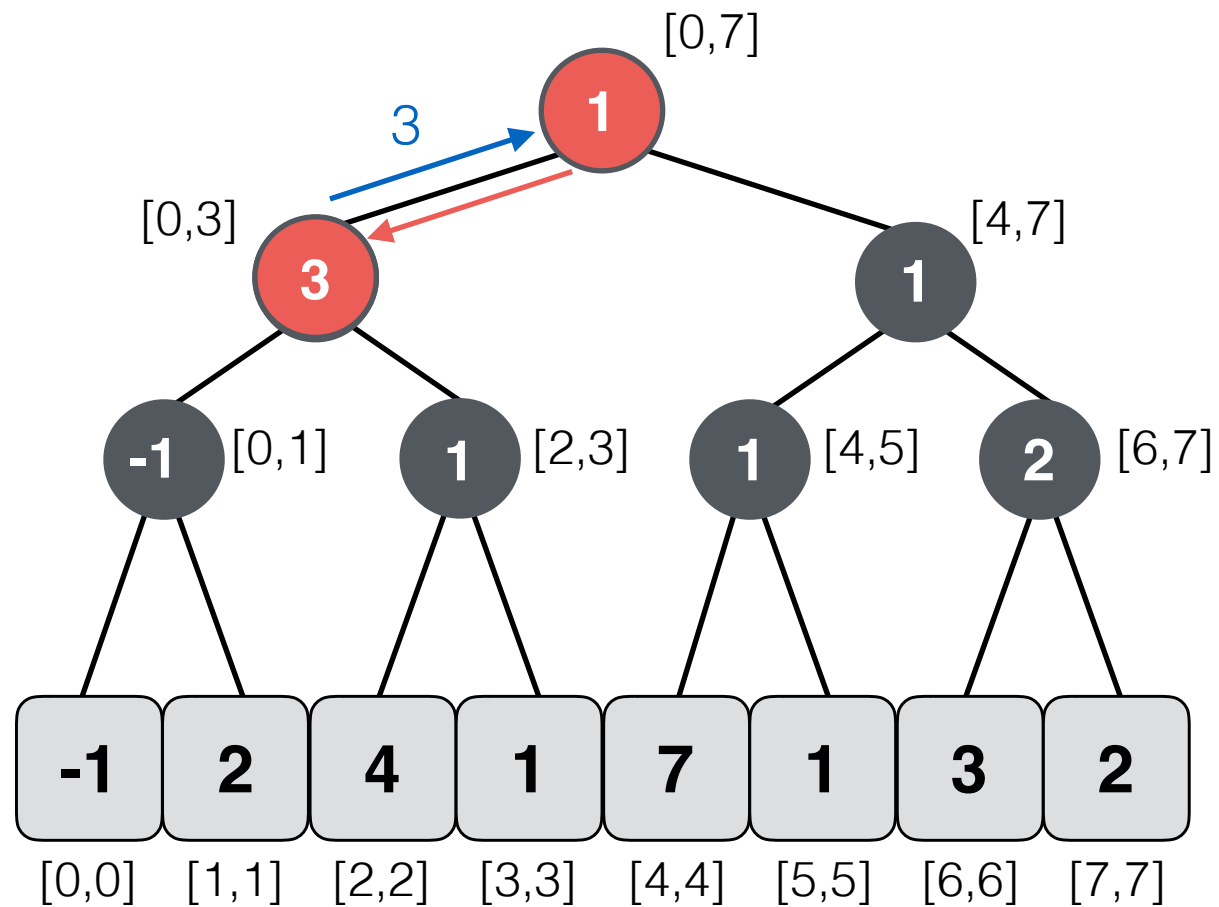


Lazy Tree

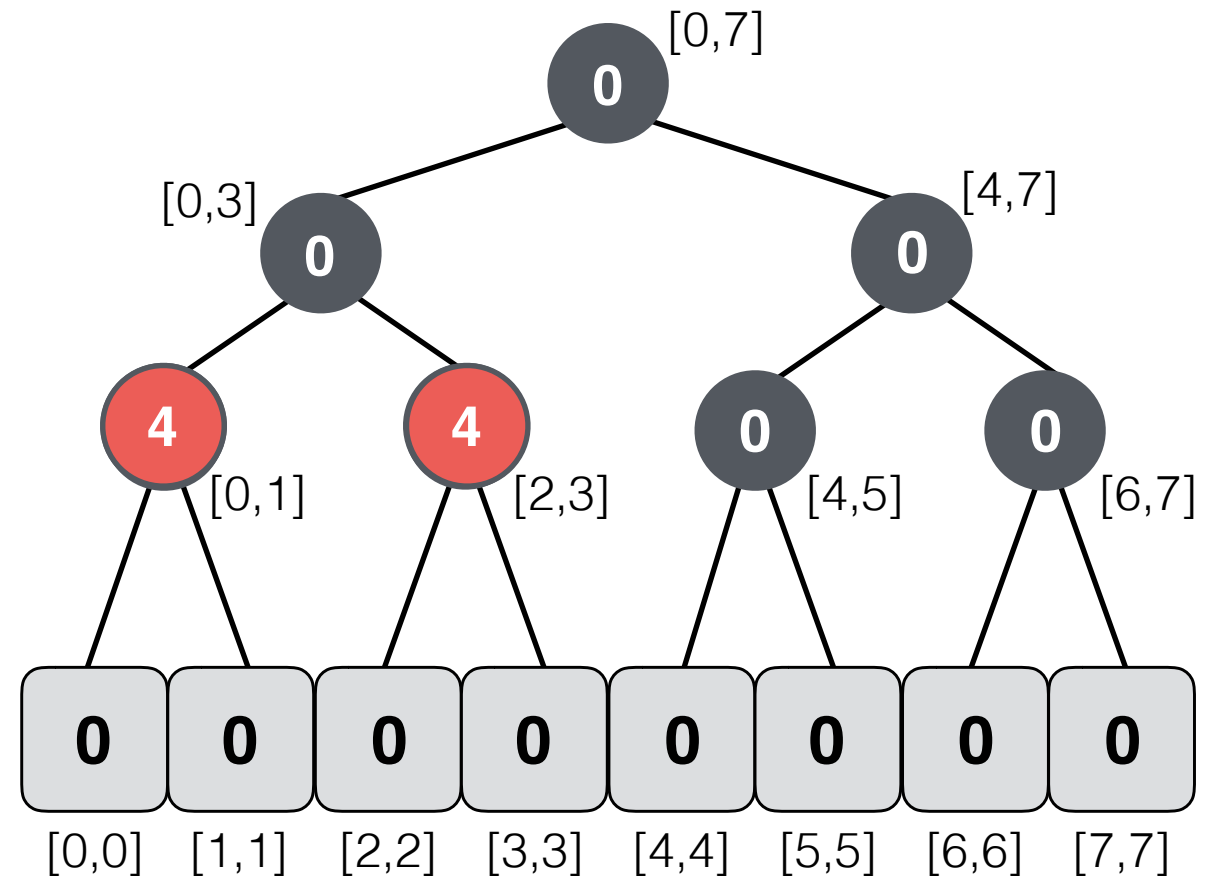
Lazy Propagation in Segment Trees

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`update_range(0,0,2)`
`rmq(3,5) = ?`



Segment Tree

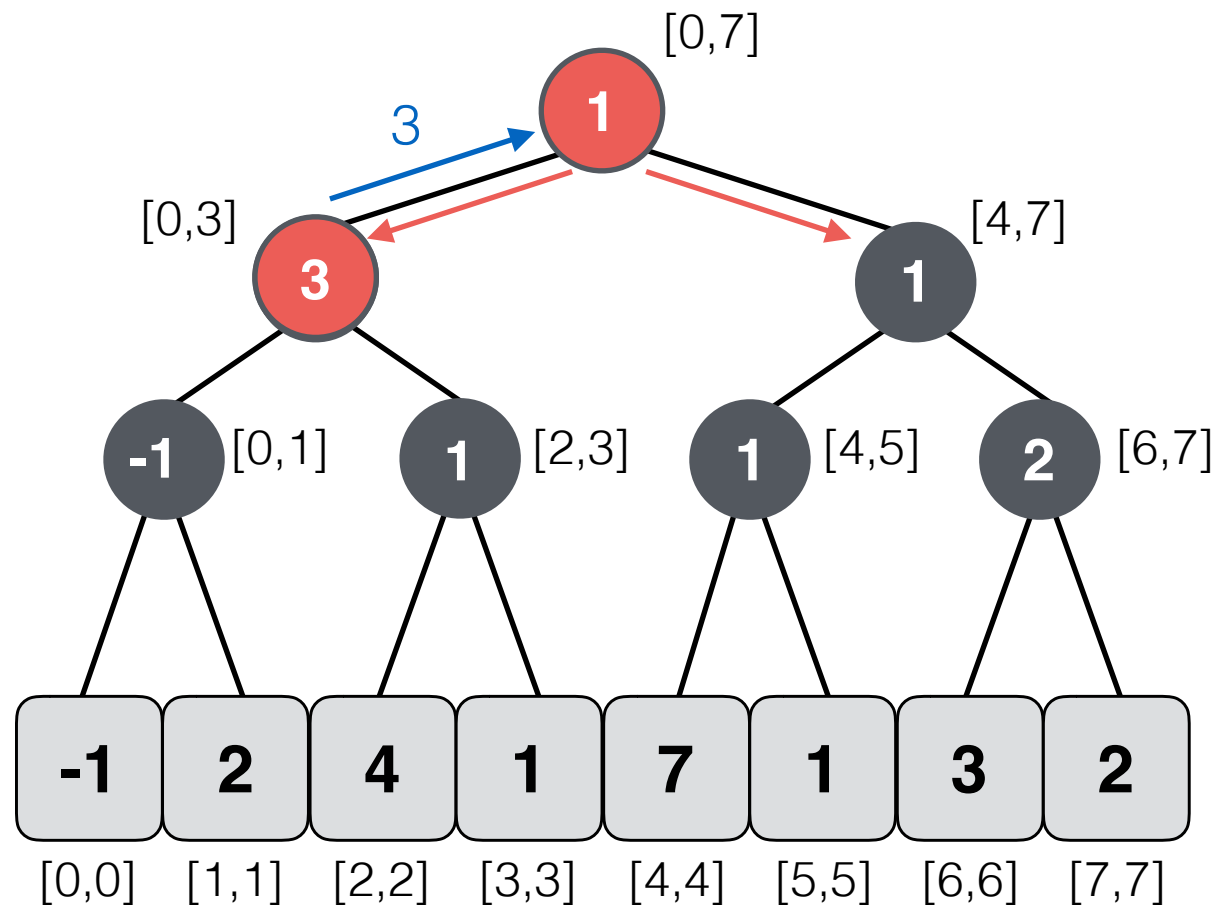


Lazy Tree

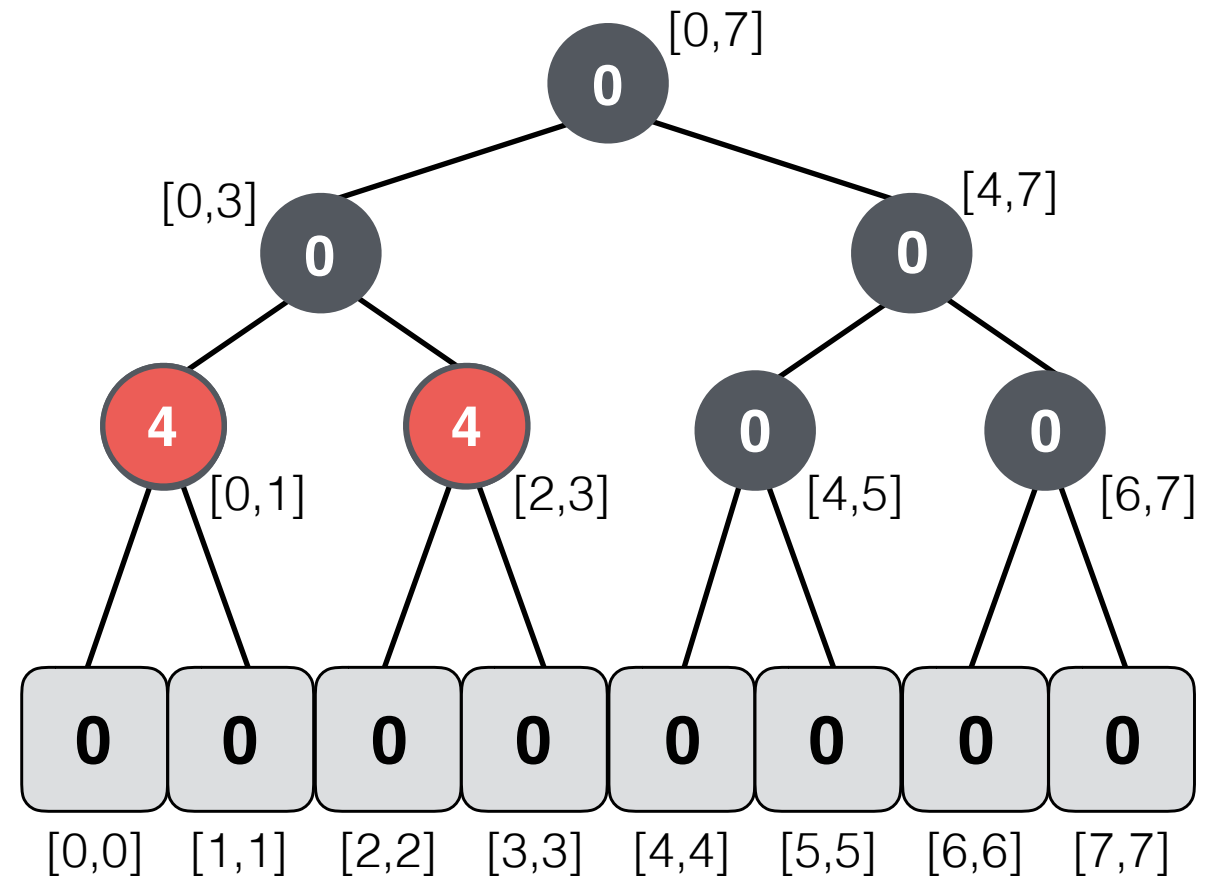
Lazy Propagation in Segment Trees

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`update_range(0,3,1)`
`update_range(0,0,2)`
`rmq(3,5) = ?`



Segment Tree

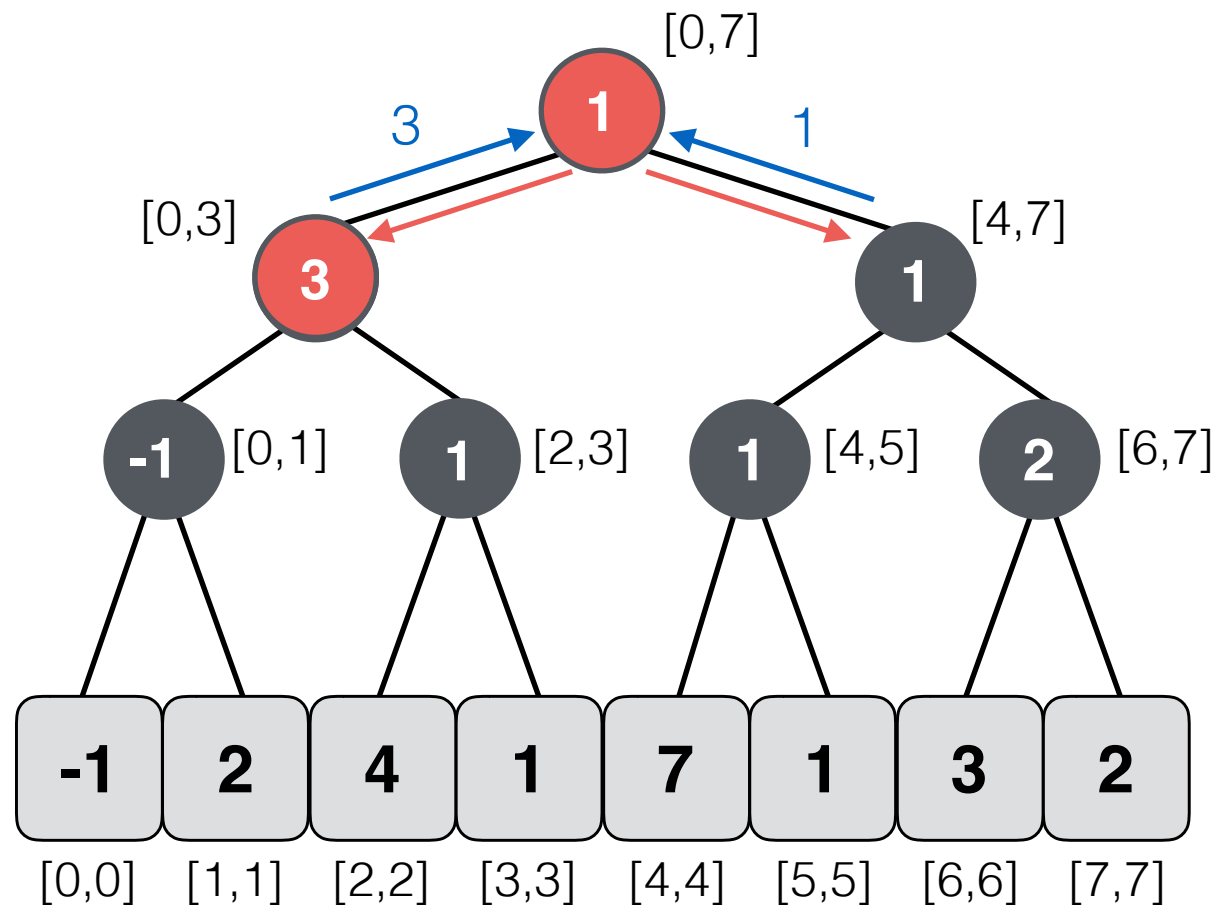


Lazy Tree

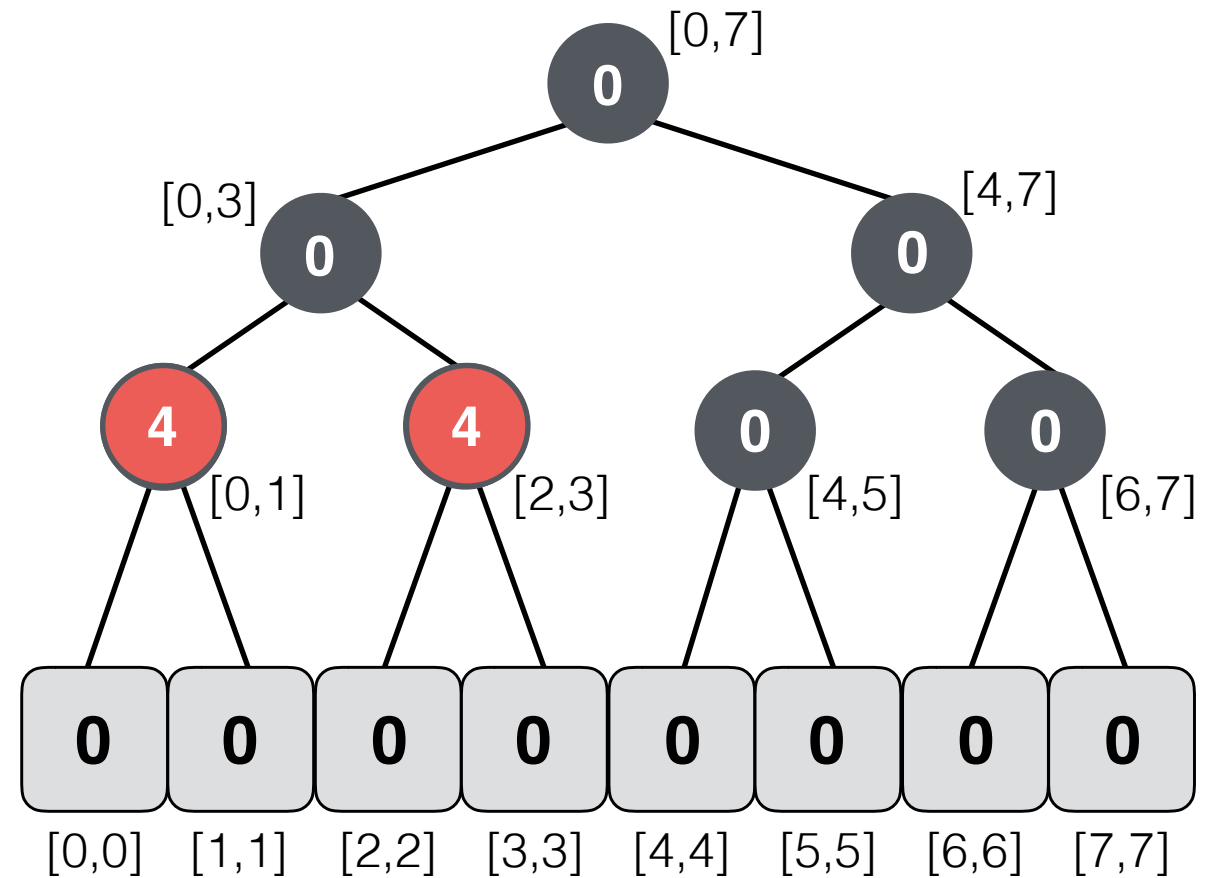
Lazy Propagation in Segment Trees

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`update_range(0,3,1)`
`update_range(0,0,2)`
`rmq(3,5) = ?`



Segment Tree

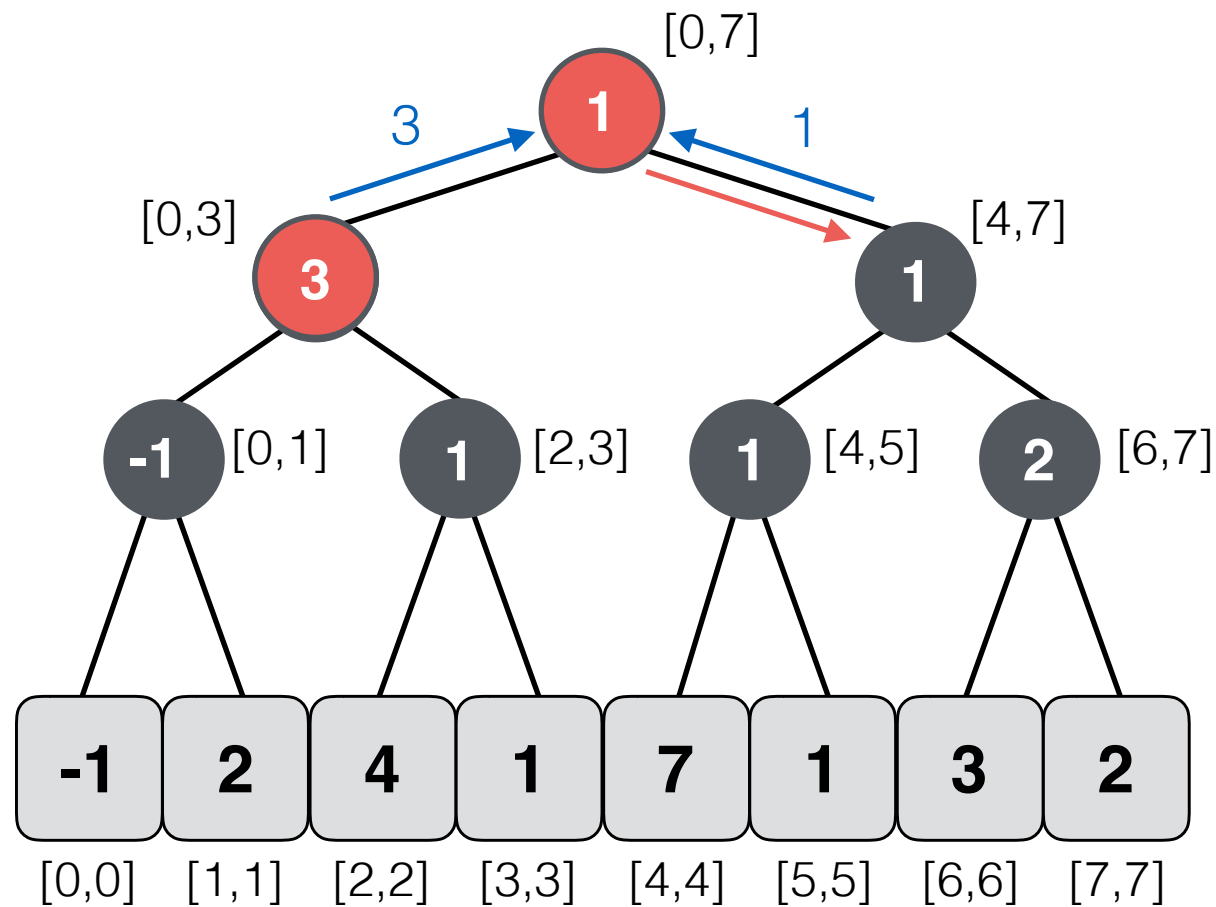


Lazy Tree

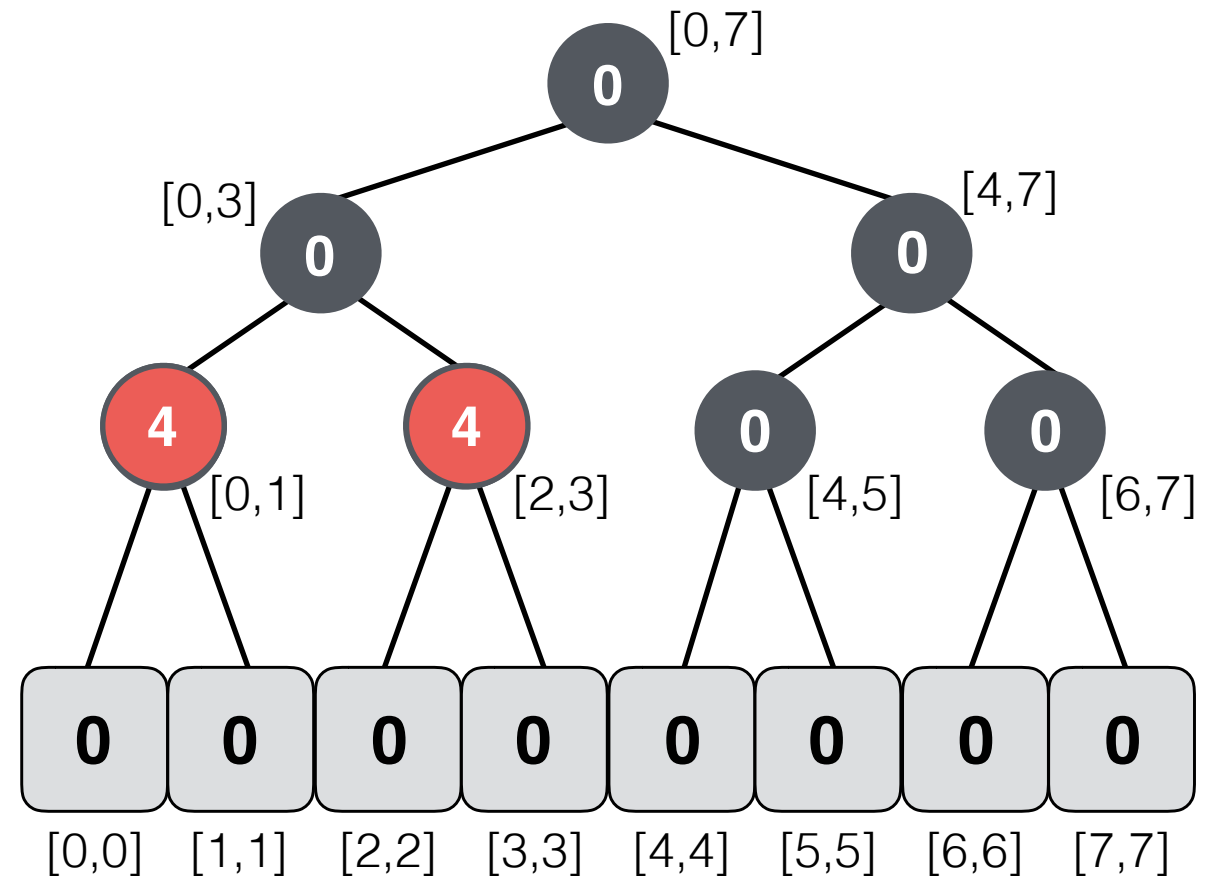
Lazy Propagation in Segment Trees

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`update_range(0,0,2)`
`rmq(3,5) = ?`



Segment Tree

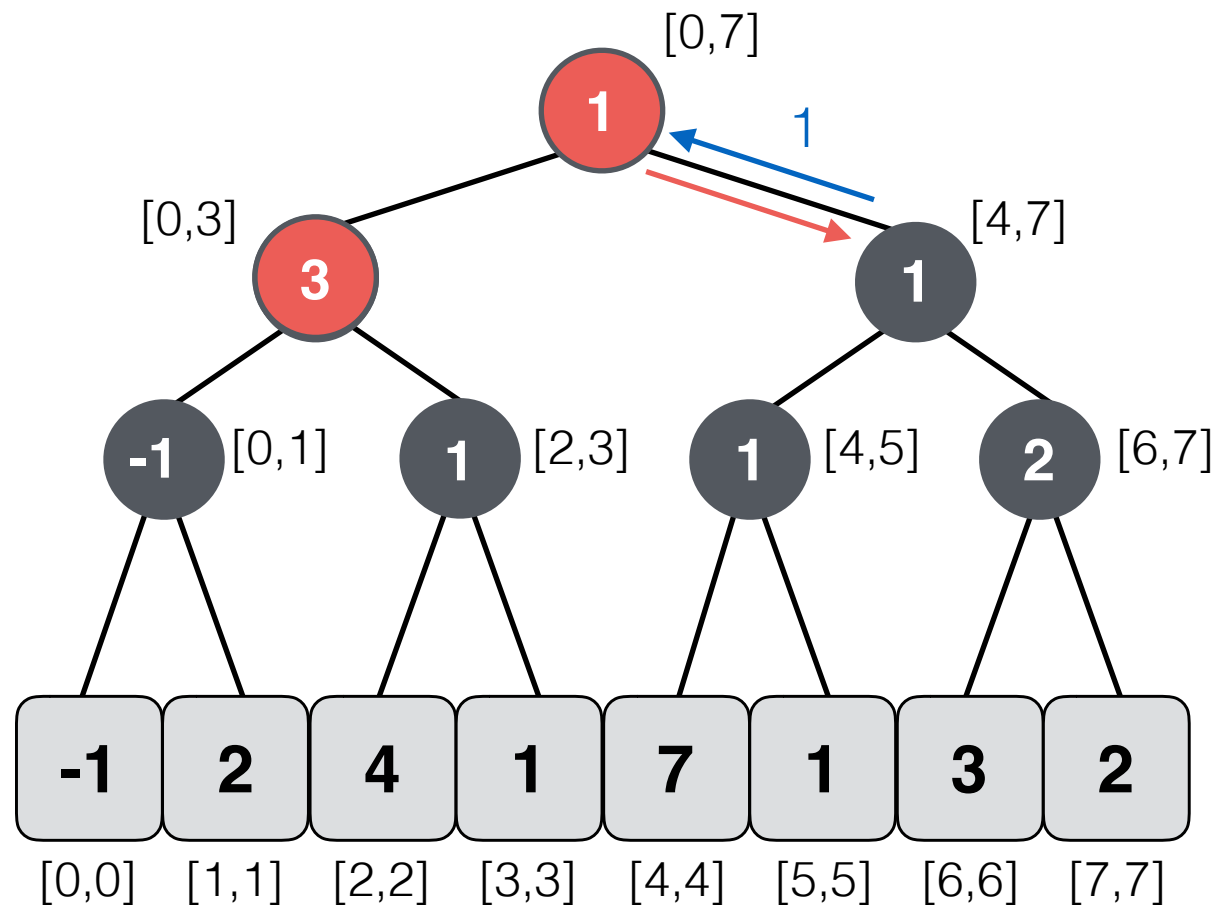


Lazy Tree

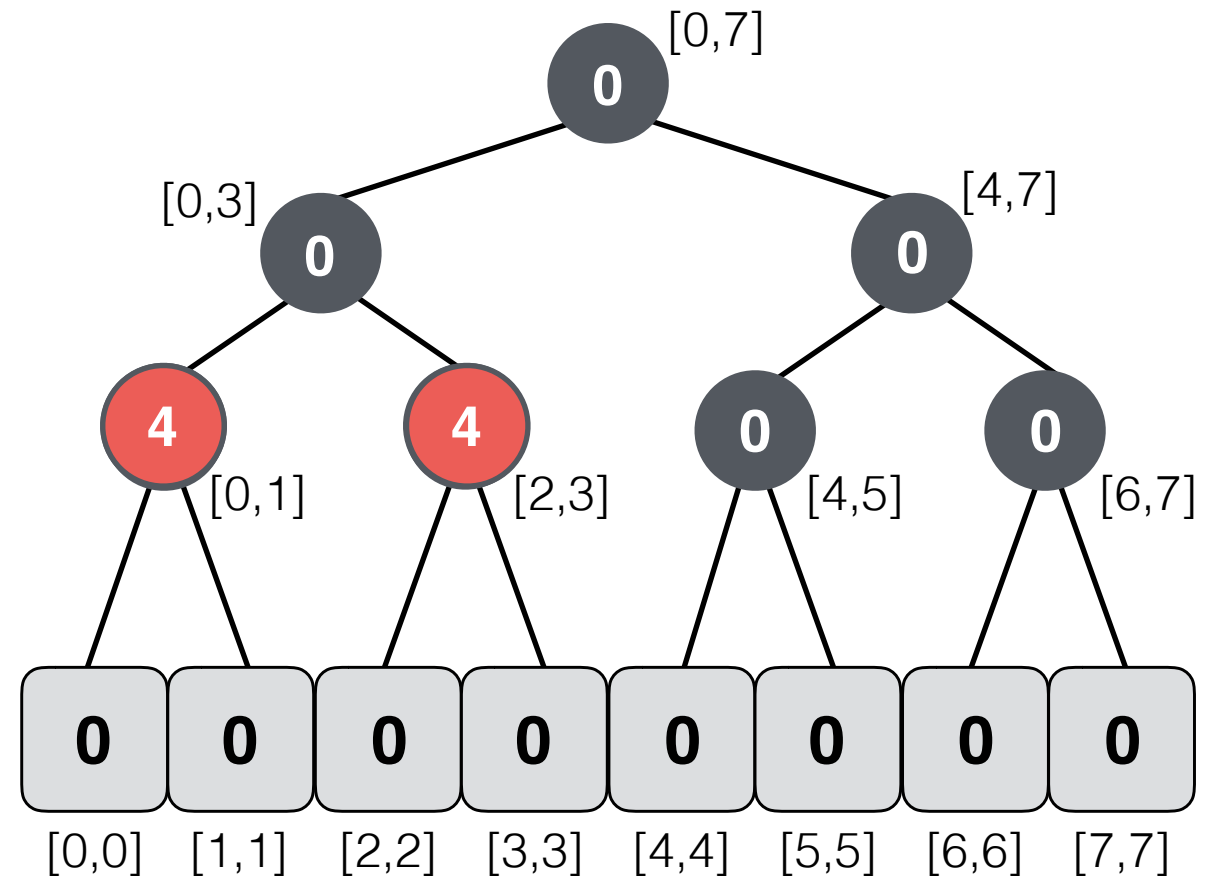
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Segment Tree

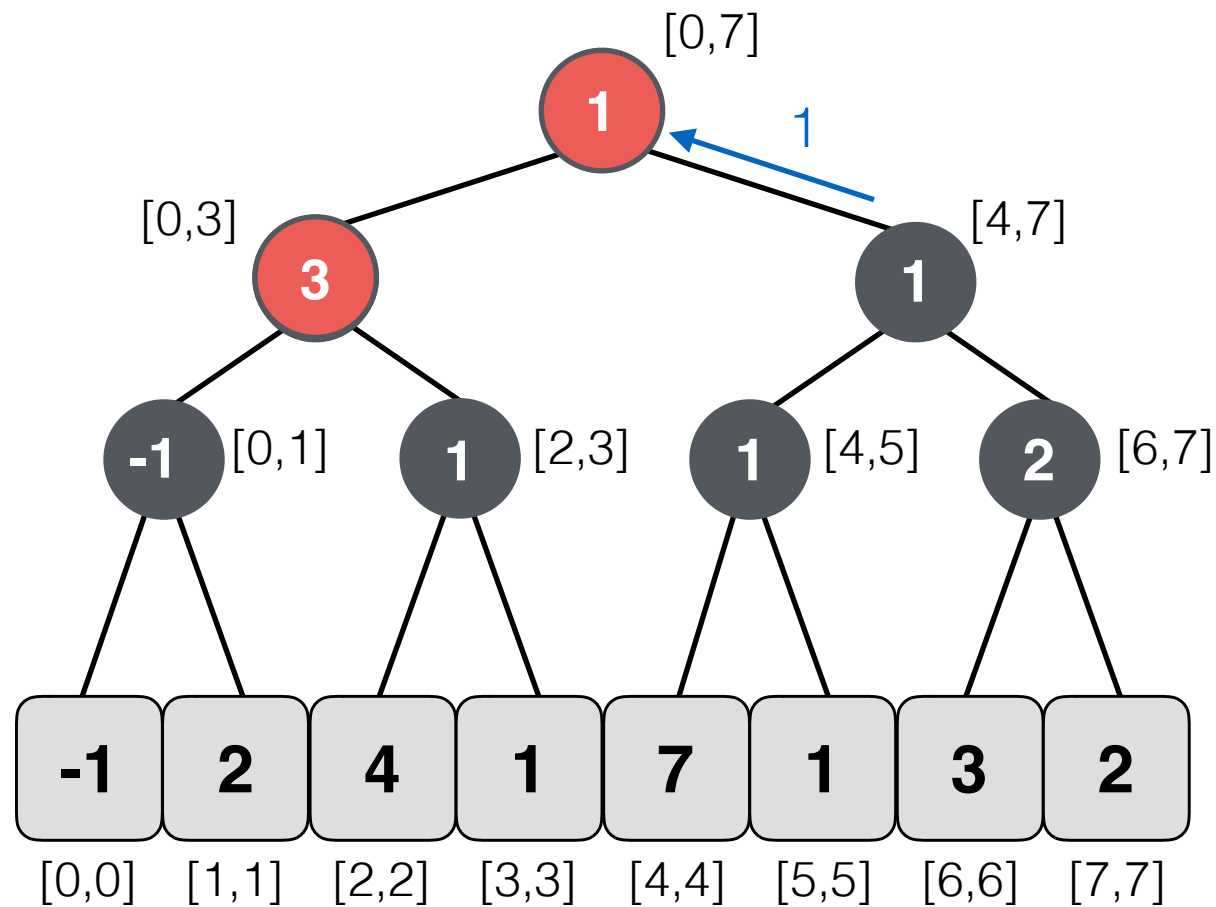


Lazy Tree

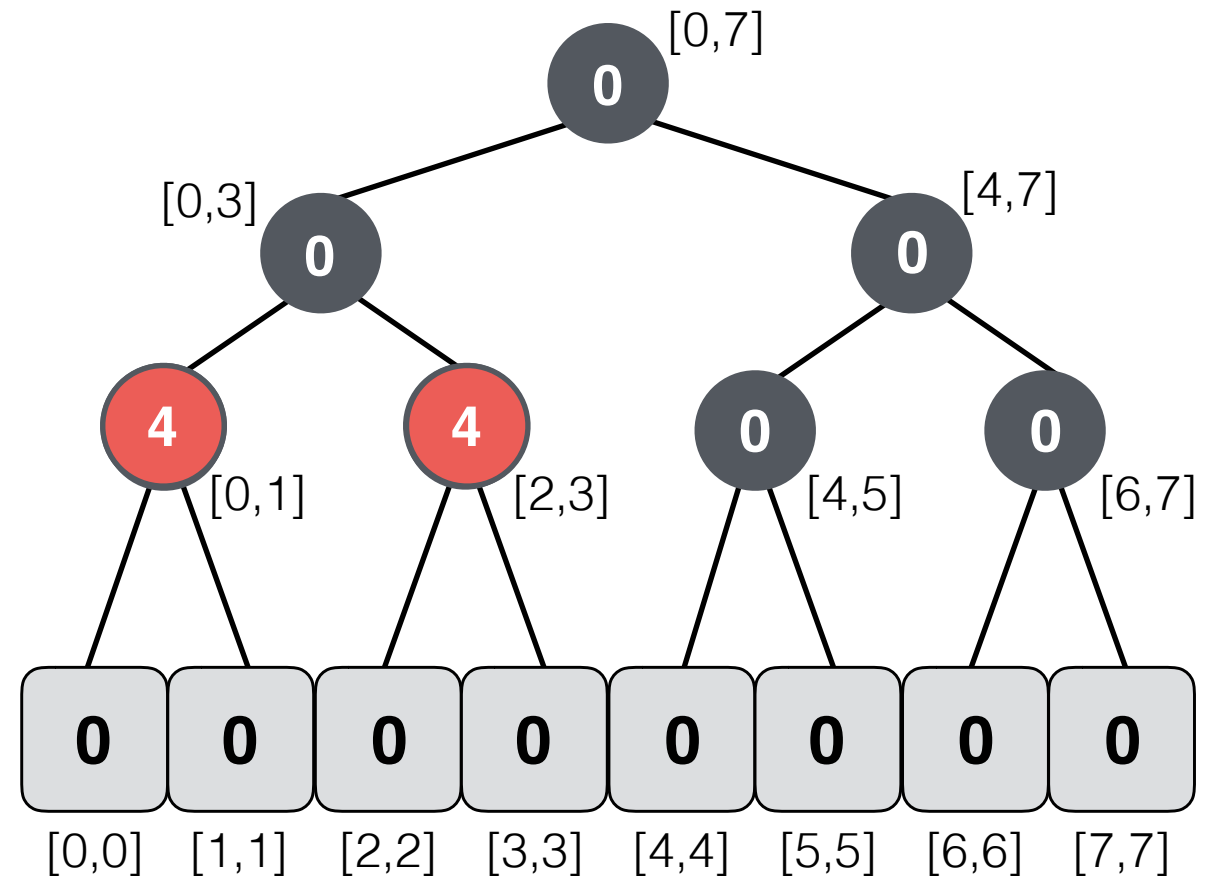
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`update_range(0,0,2)`
`rmq(3,5) = ?`



Segment Tree

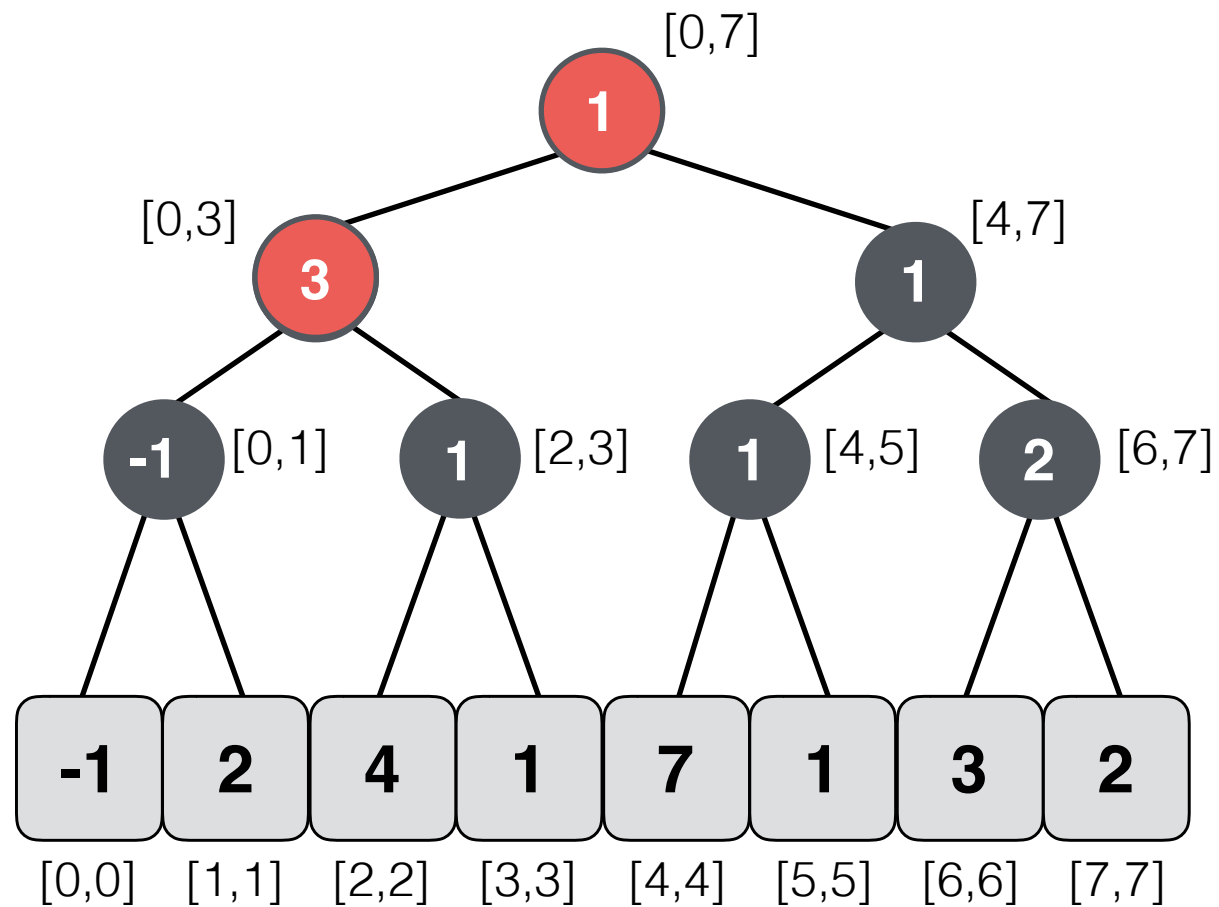


Lazy Tree

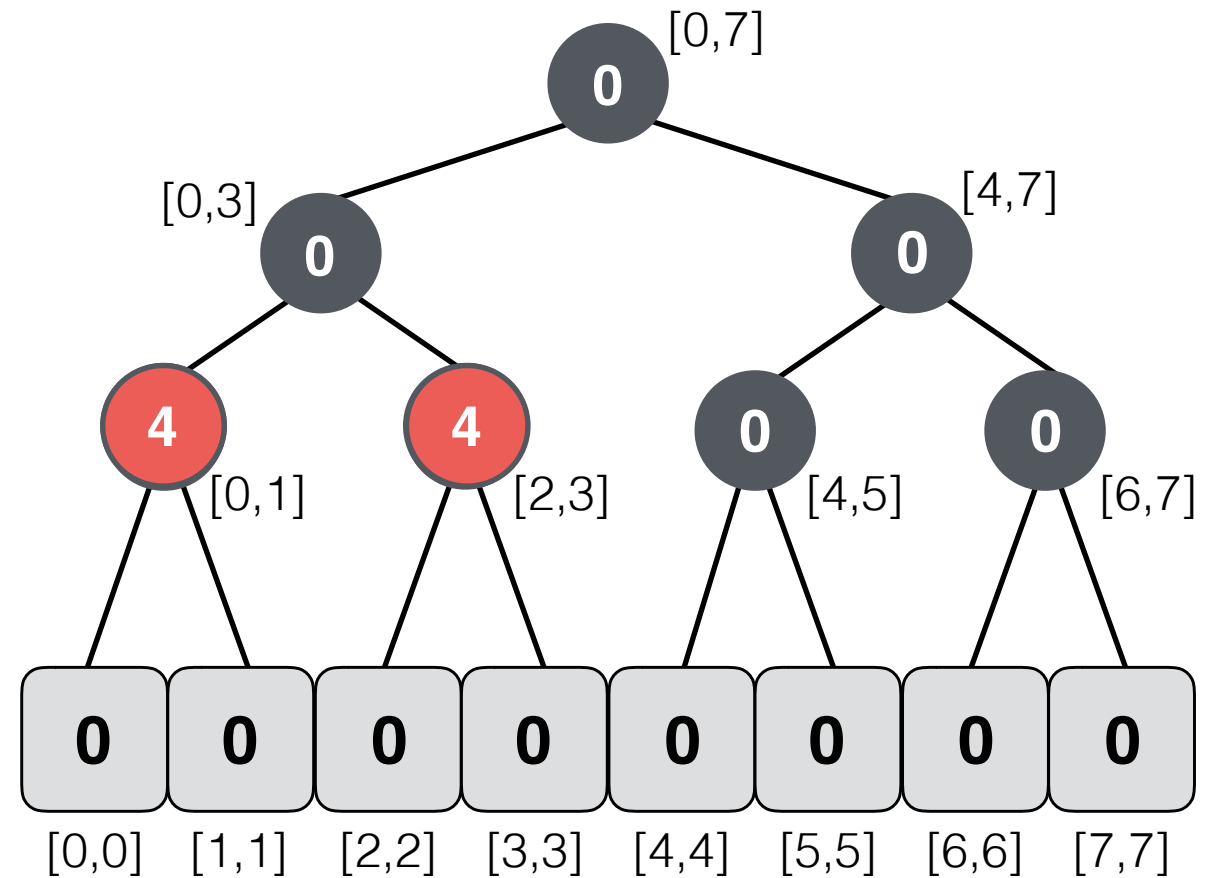
Lazy Propagation in Segment Trees

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`update_range(0,3,1)`
`update_range(0,0,2)`
`rmq(3,5) = ?`



Segment Tree

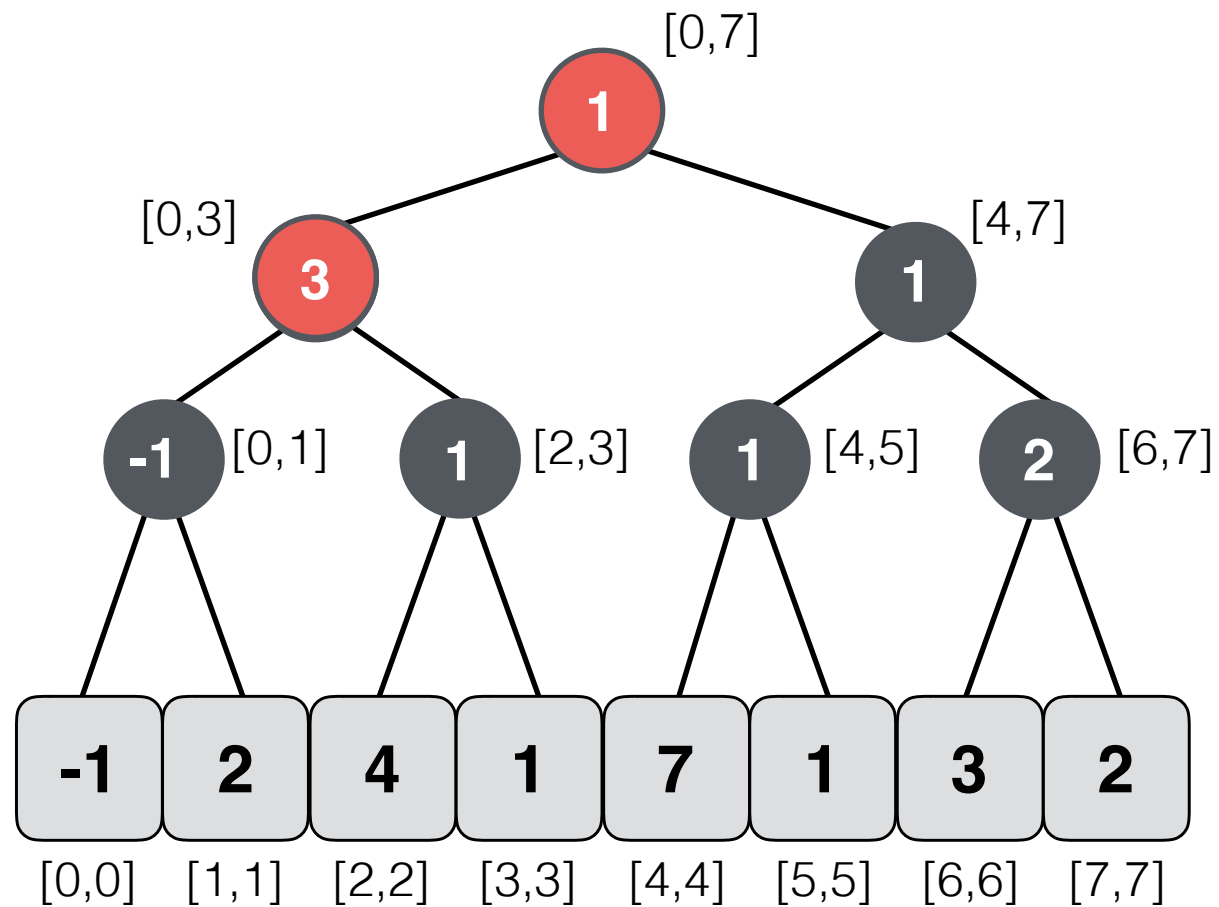


Lazy Tree

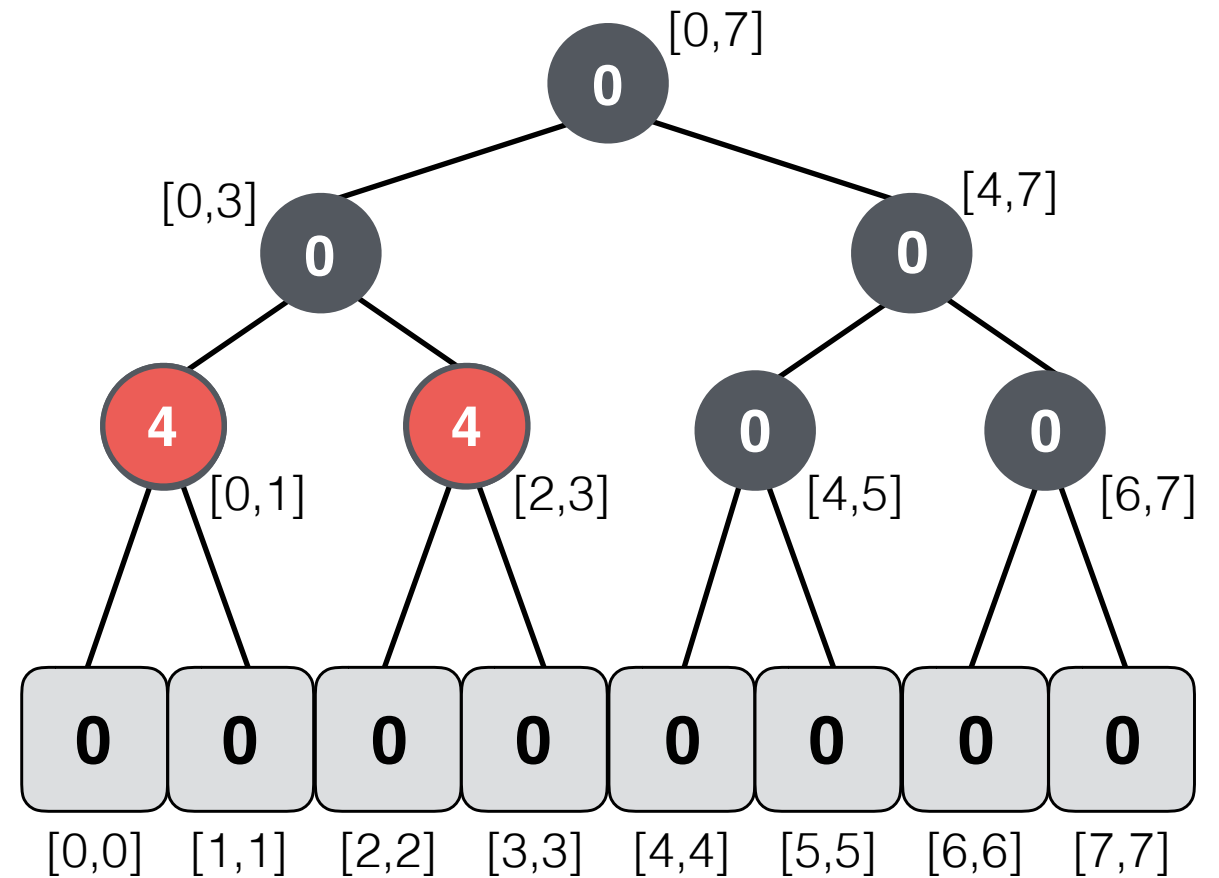
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update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

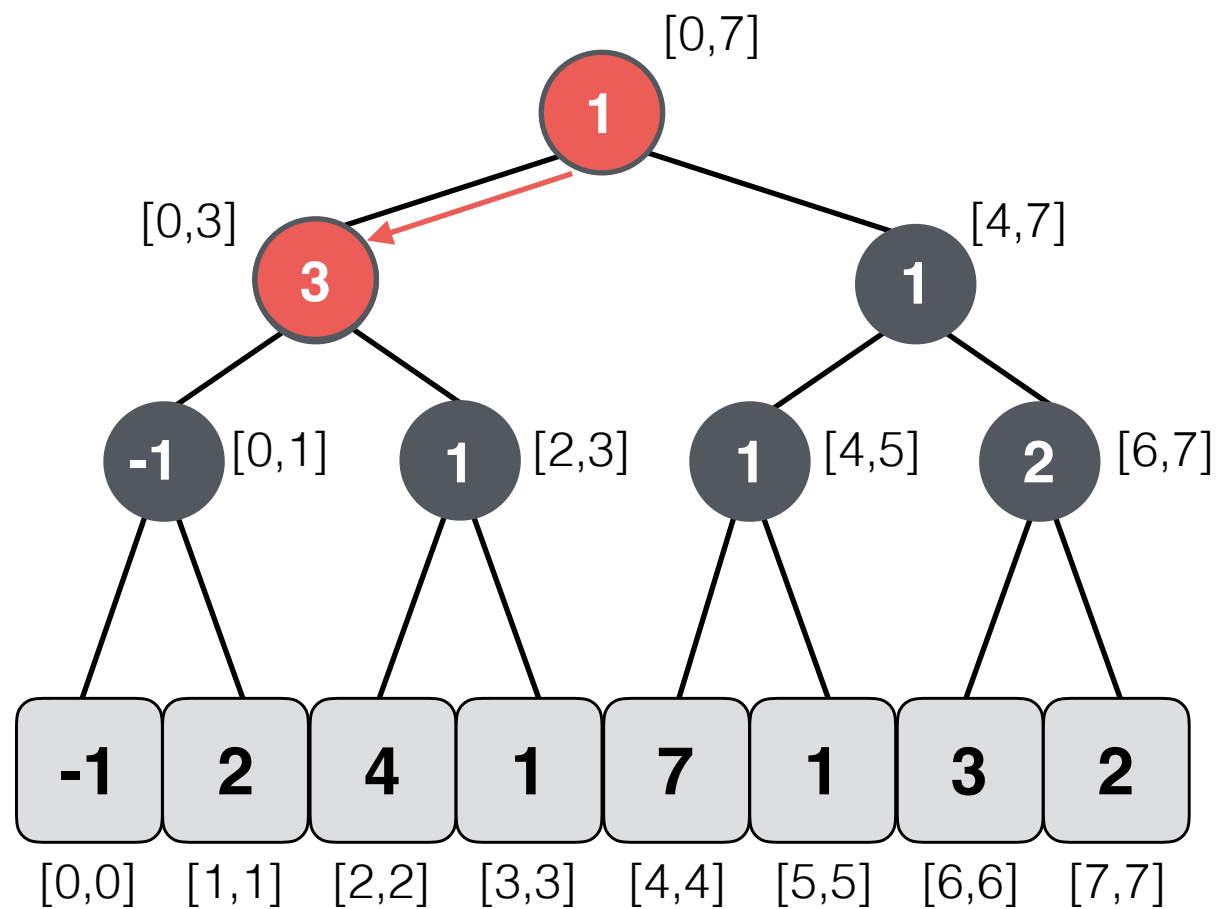


Lazy Tree

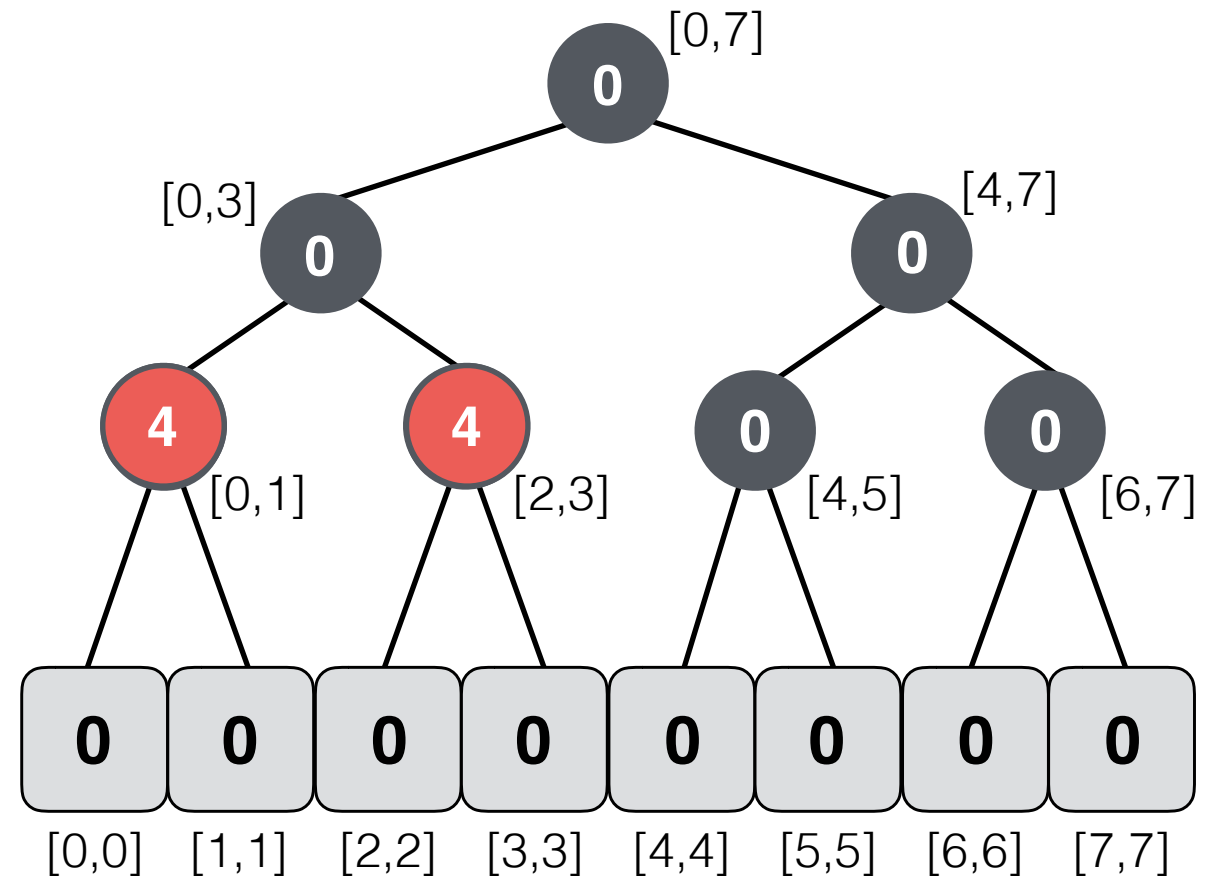
Lazy Propagation in Segment Trees

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update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

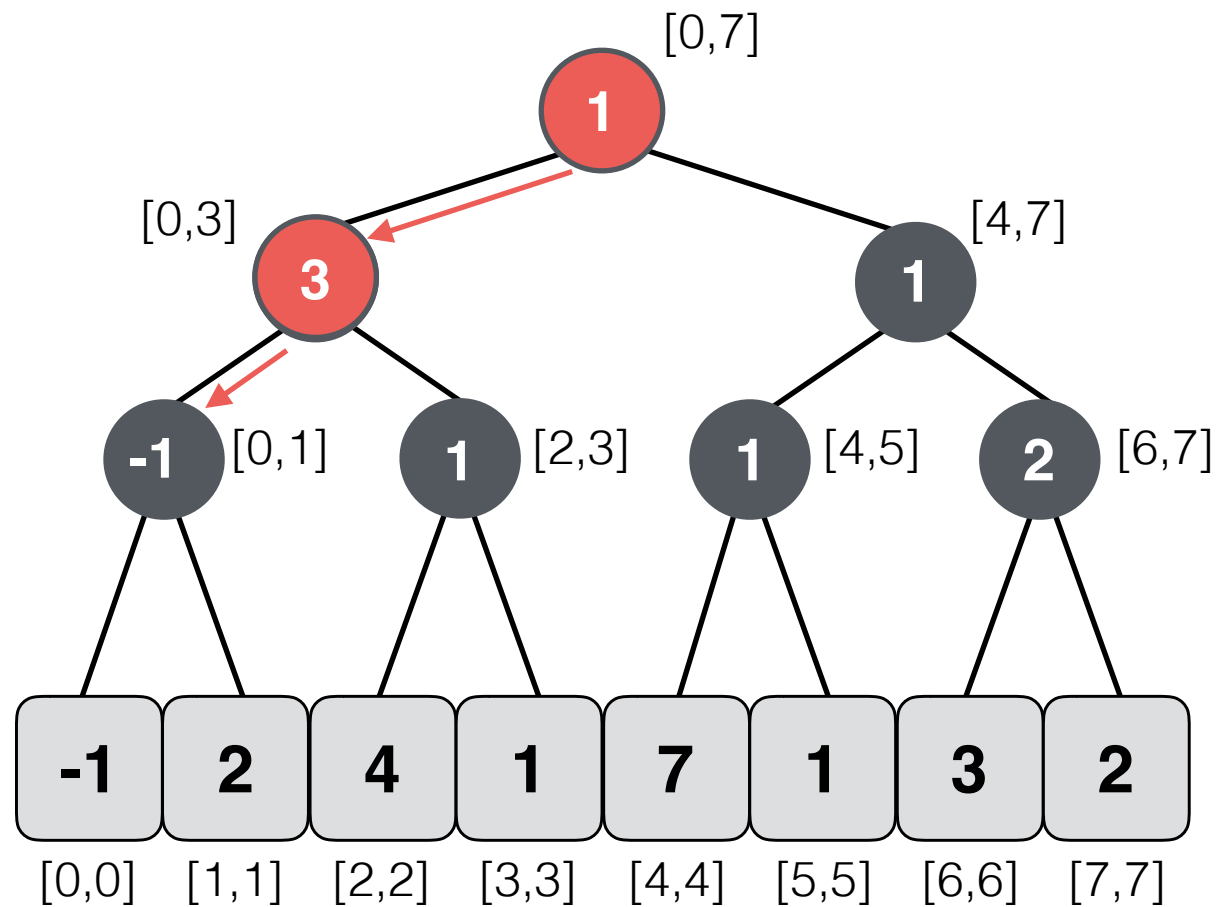


Lazy Tree

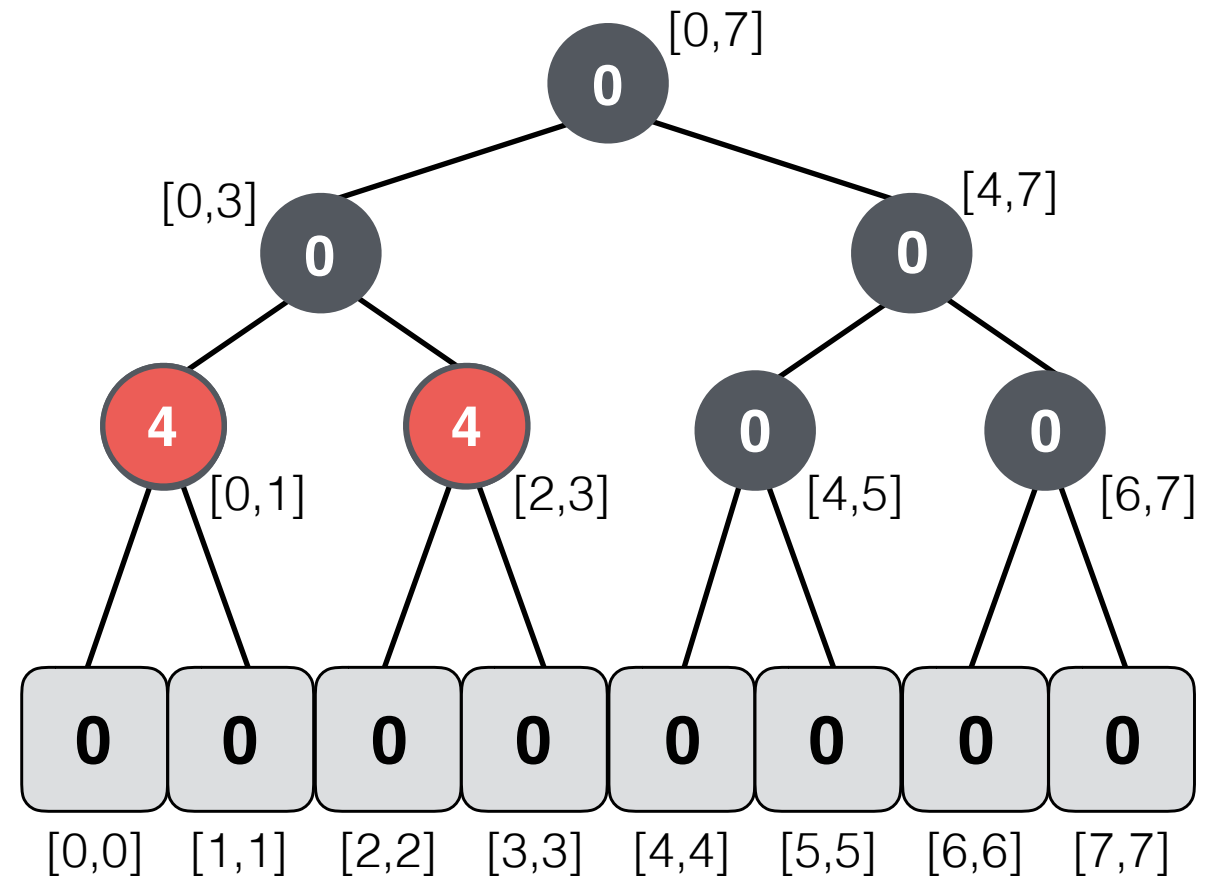
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→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

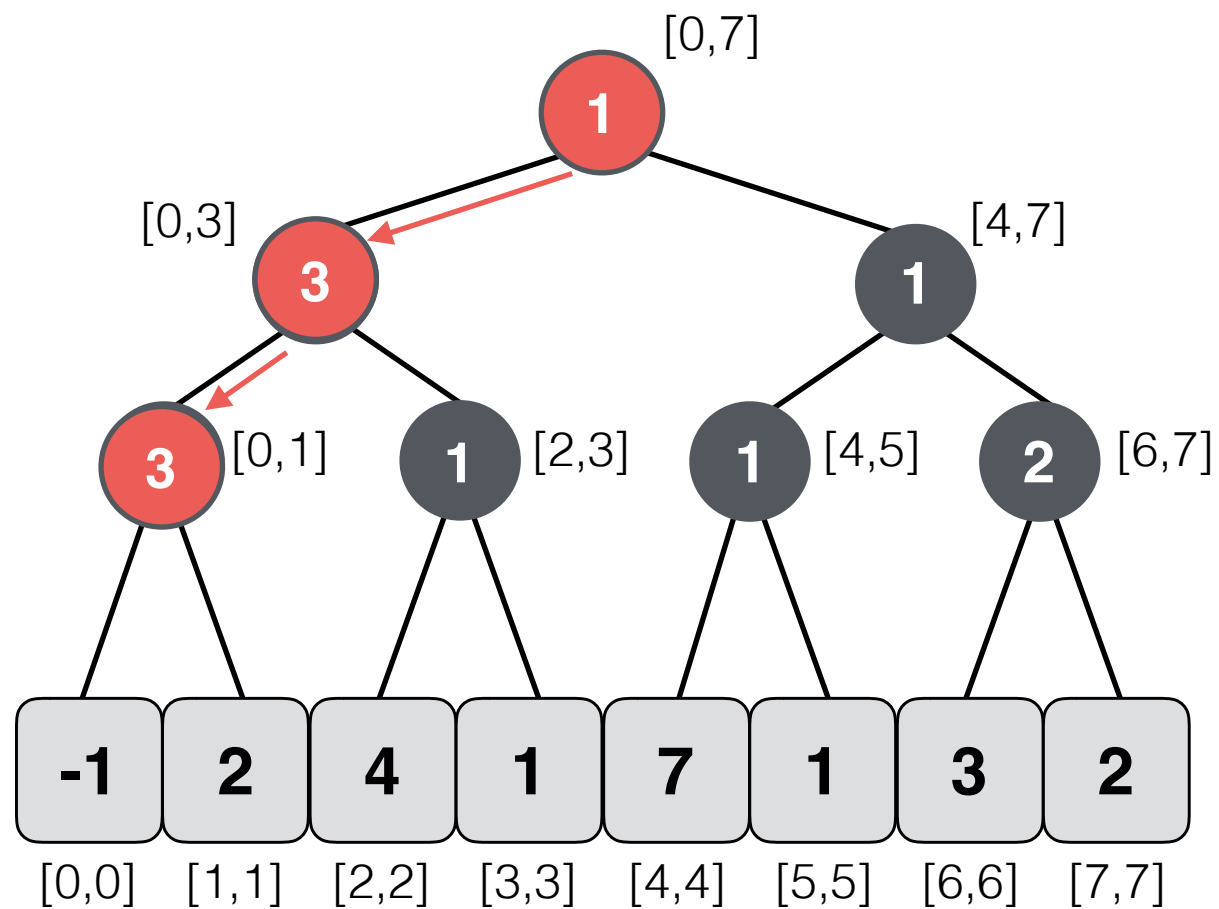


Lazy Tree

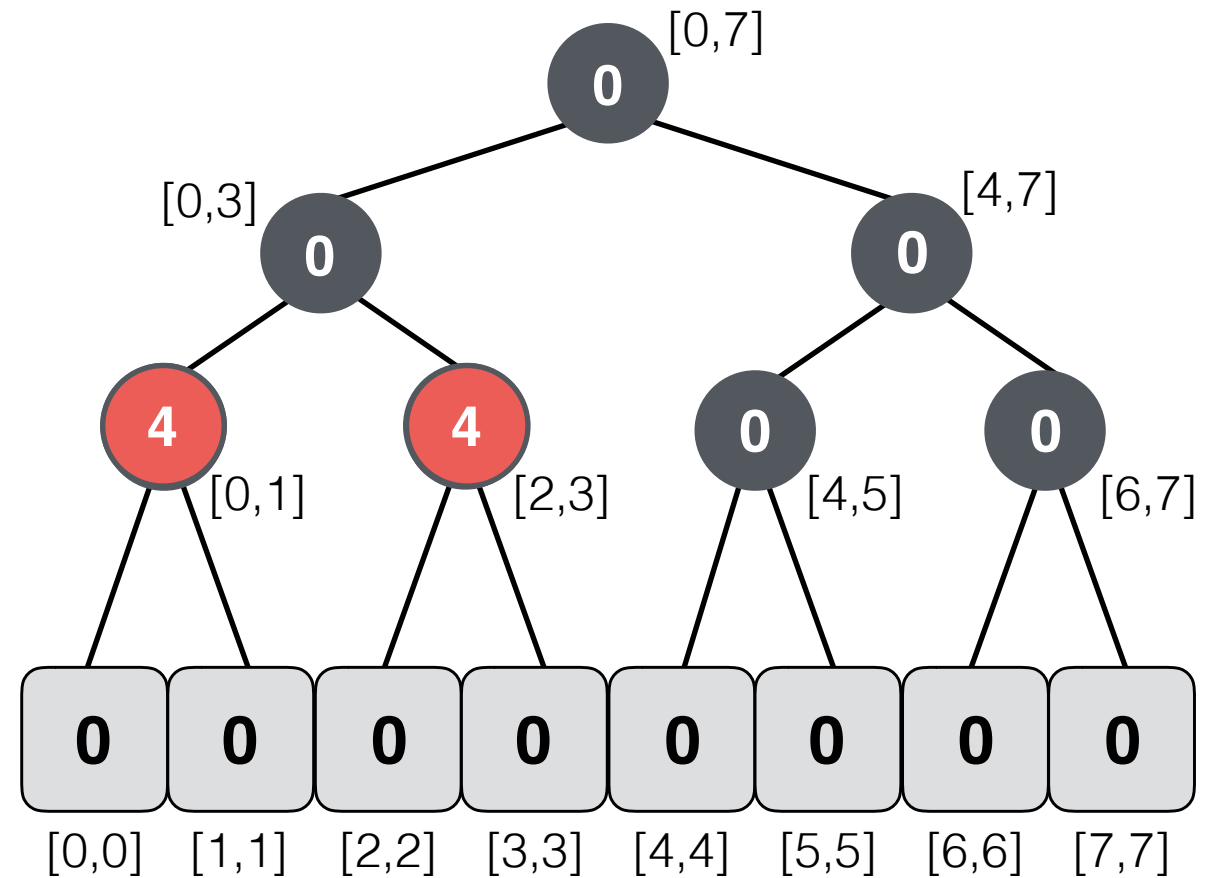
Lazy Propagation in Segment Trees

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update_range(0,3,3)
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→ update_range(0,0,2)
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Segment Tree

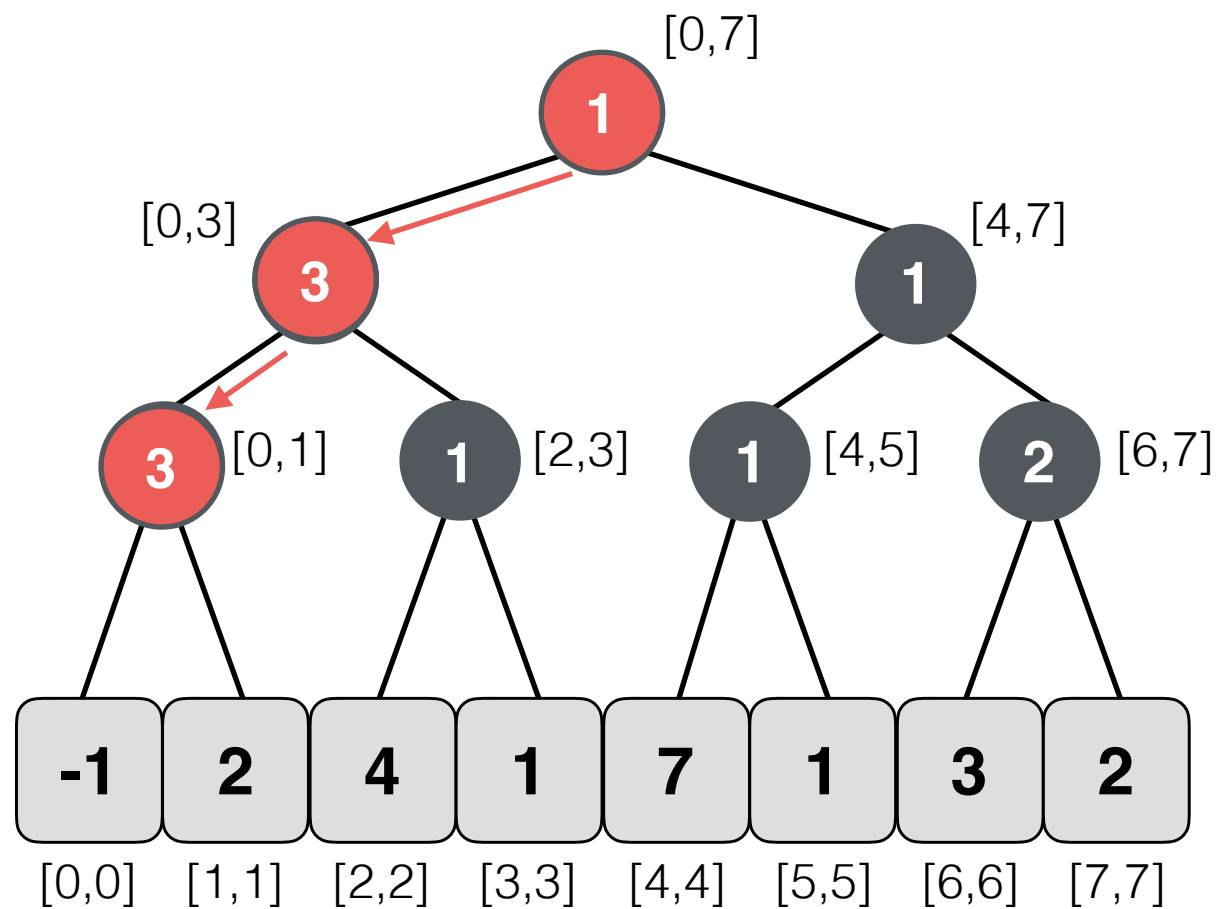


Lazy Tree

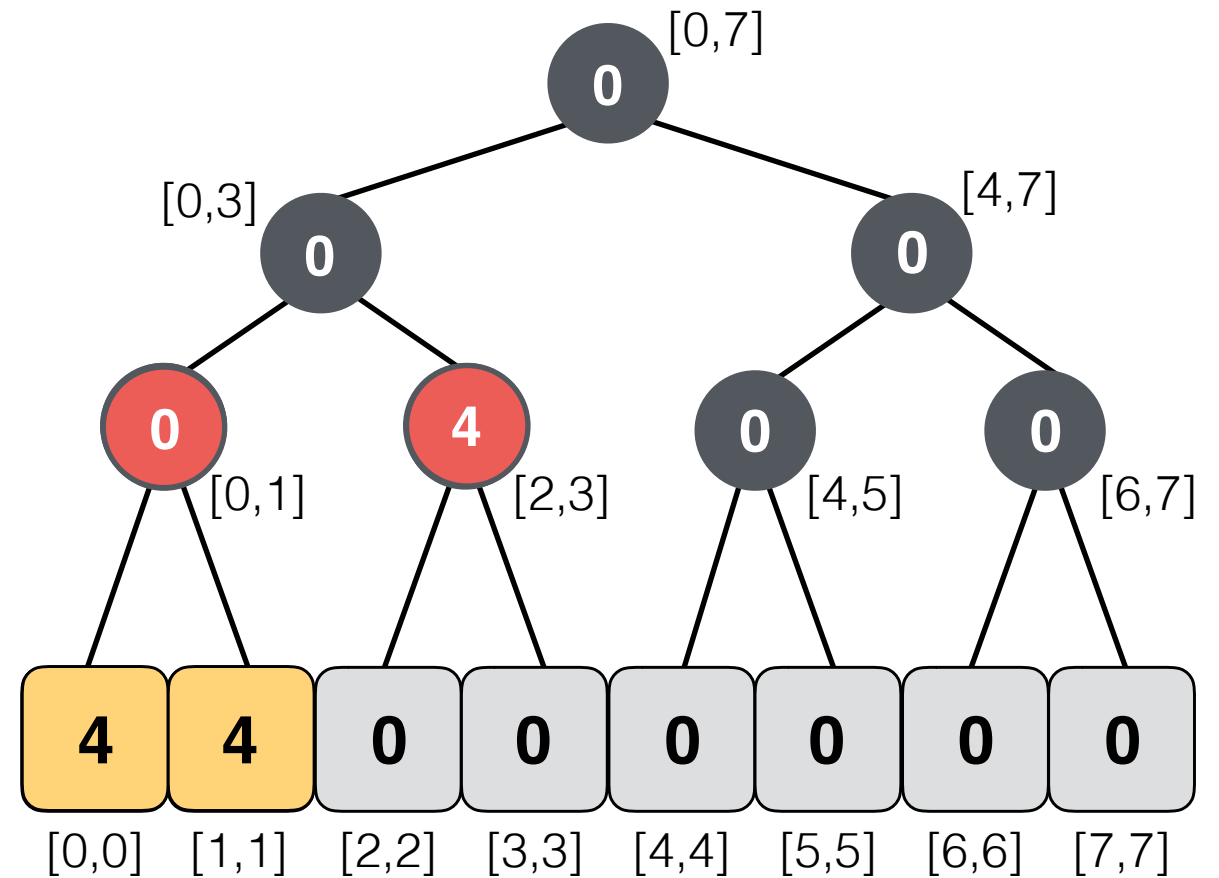
Lazy Propagation in Segment Trees

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update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

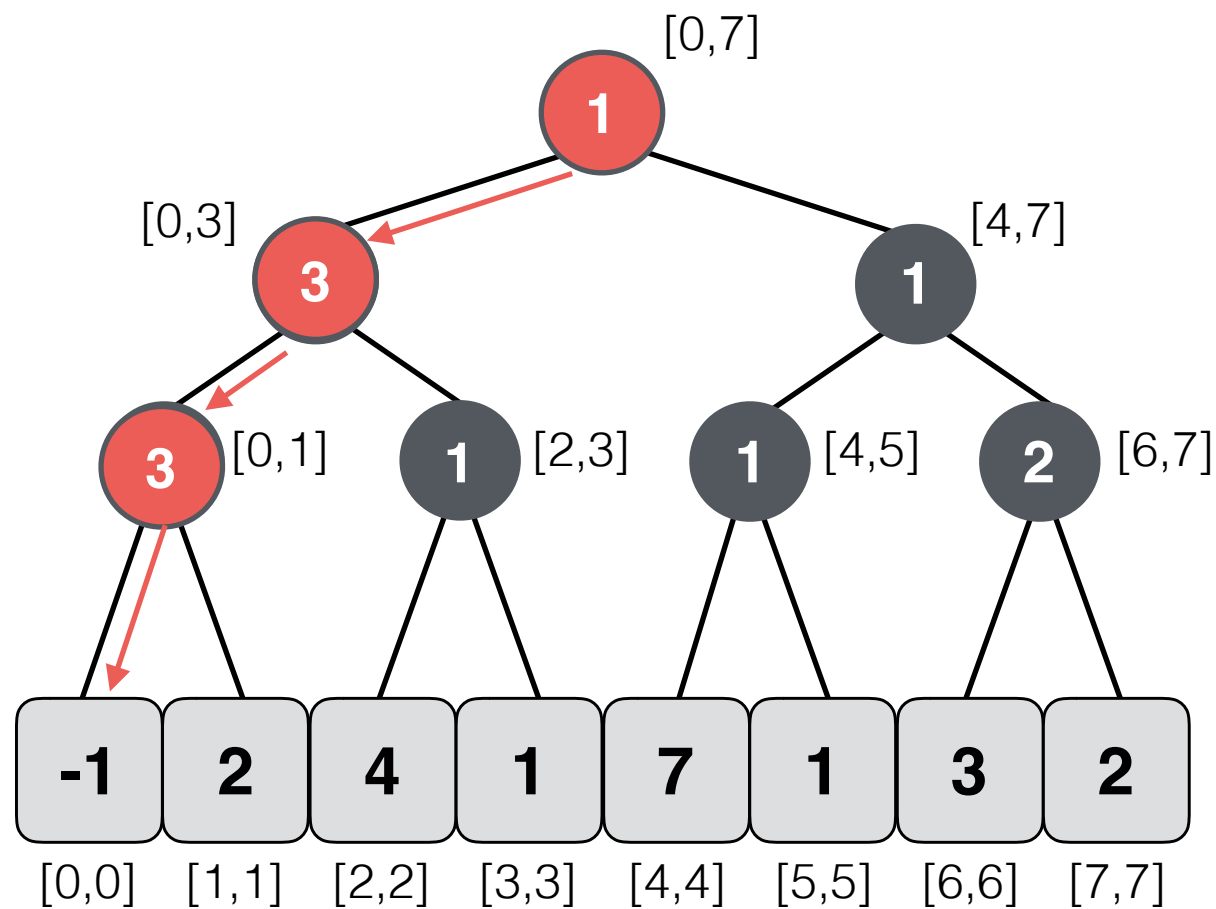


Lazy Tree

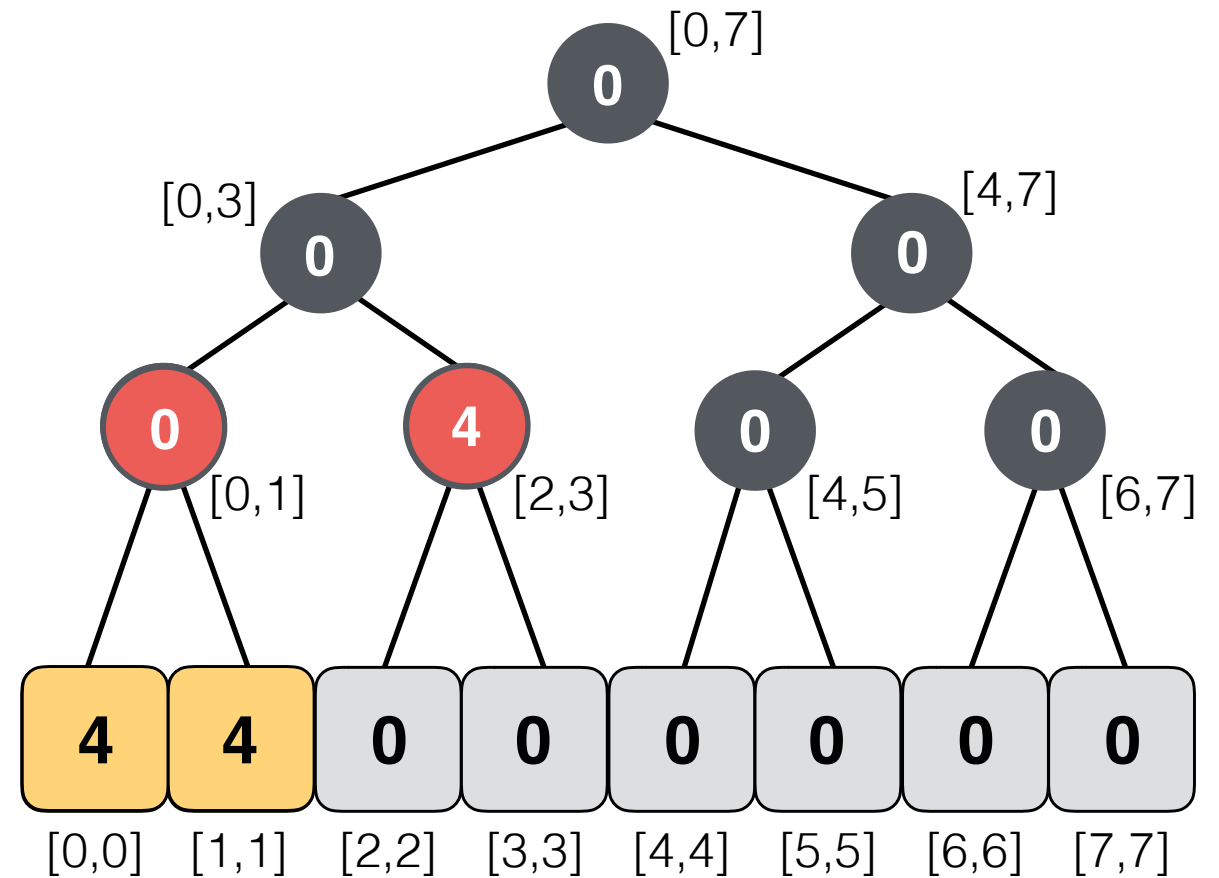
Lazy Propagation in Segment Trees

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→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

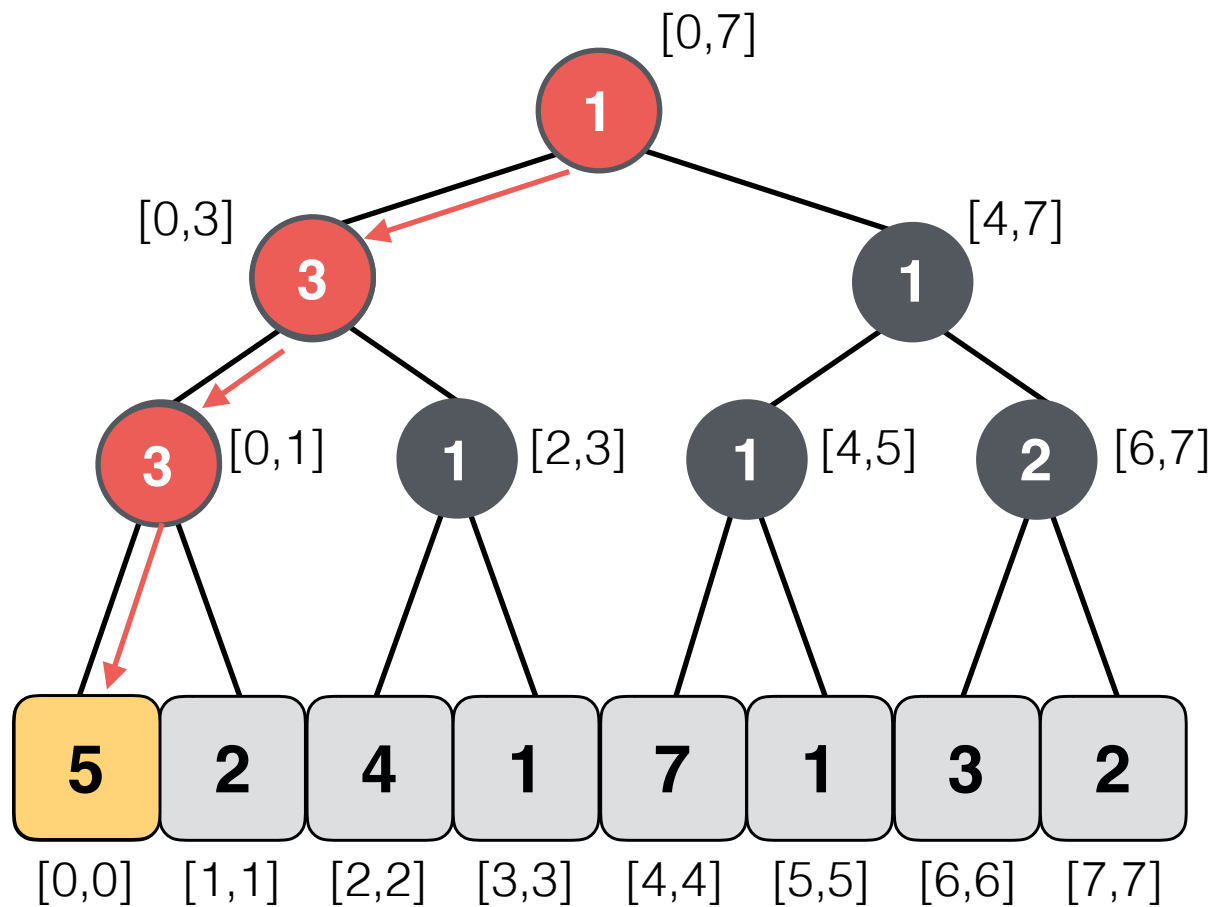


Lazy Tree

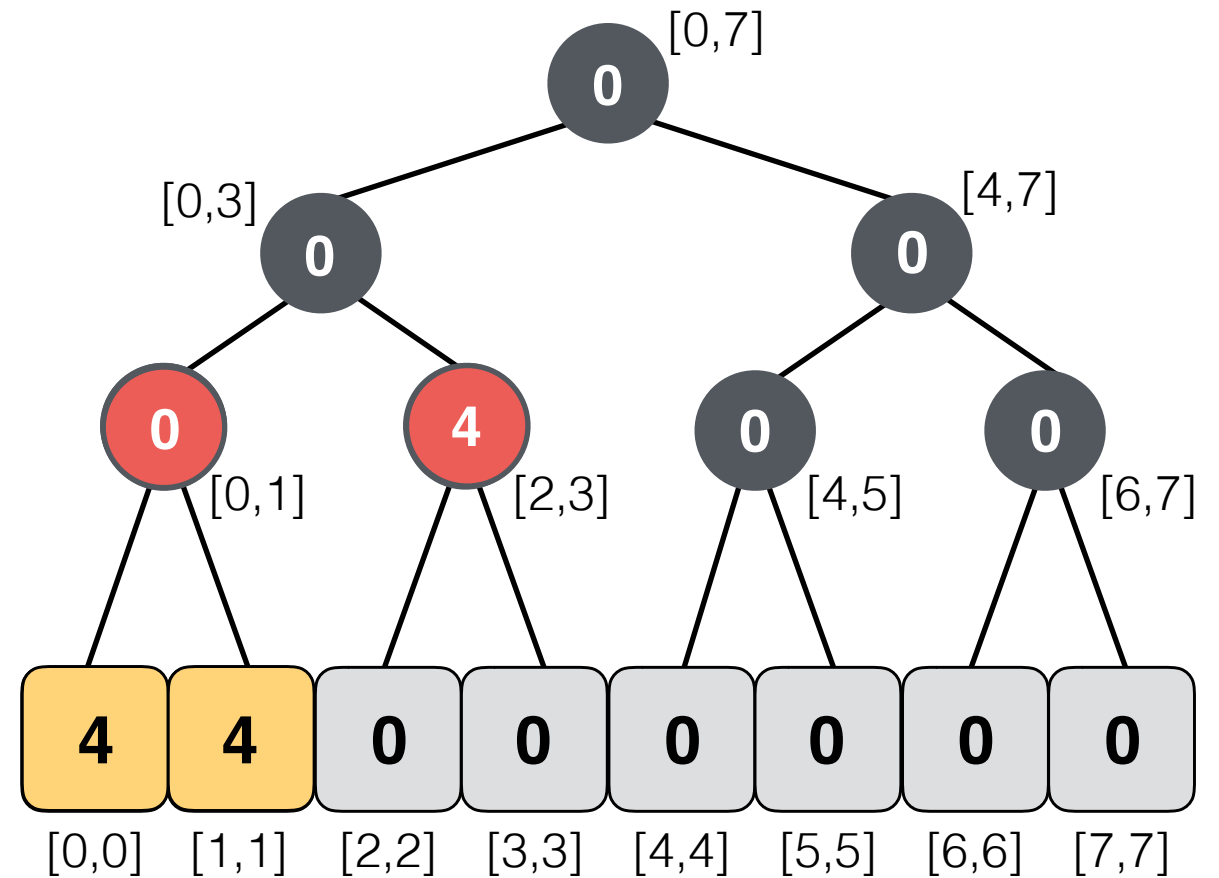
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Segment Tree

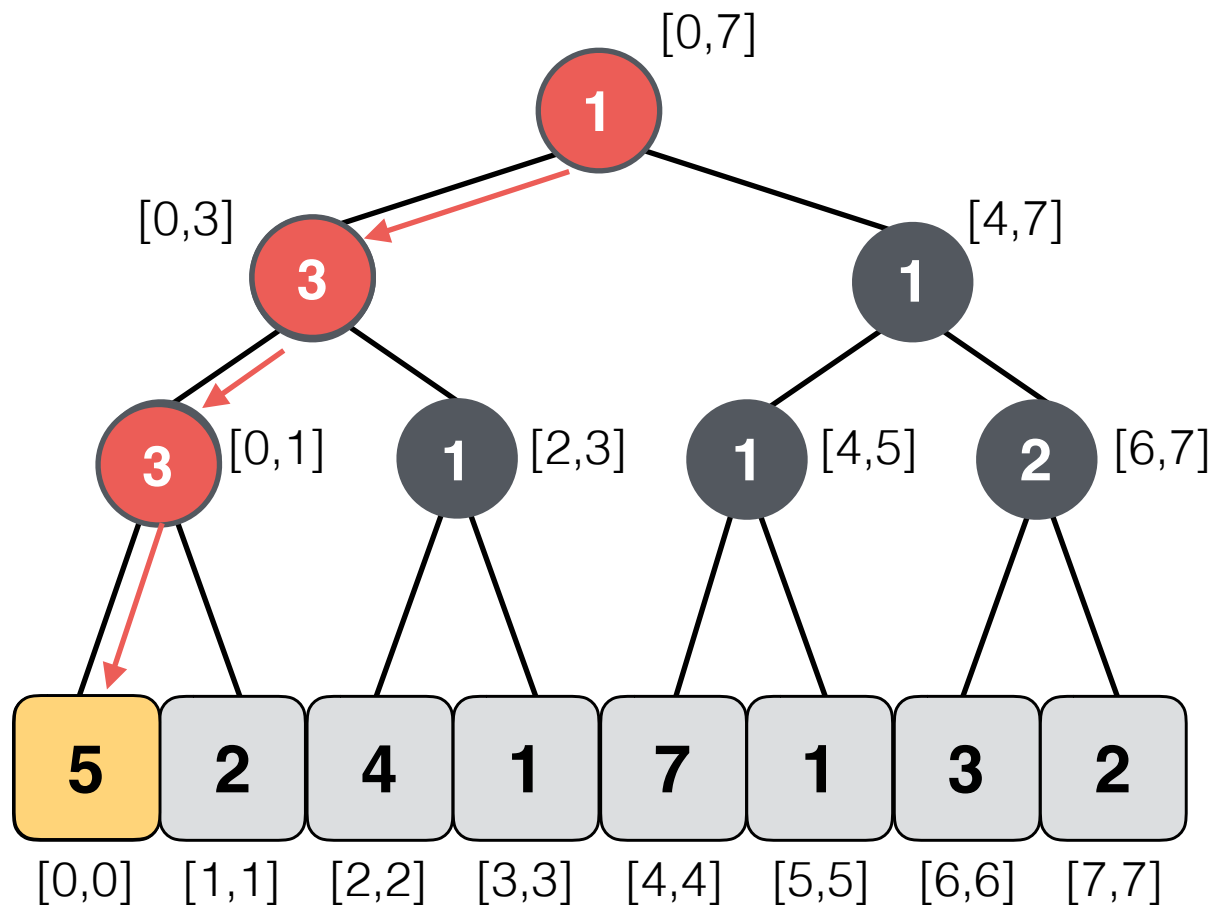


Lazy Tree

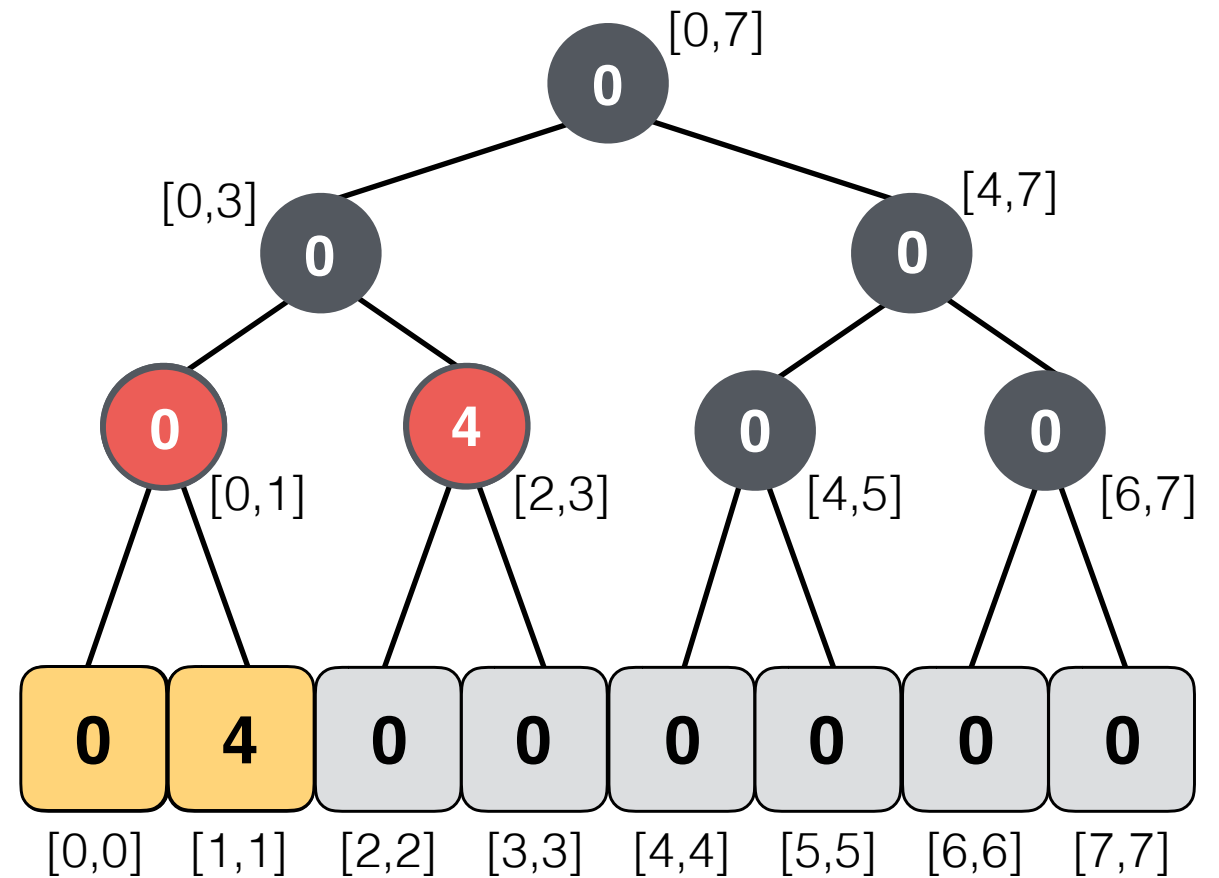
Lazy Propagation in Segment Trees

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update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

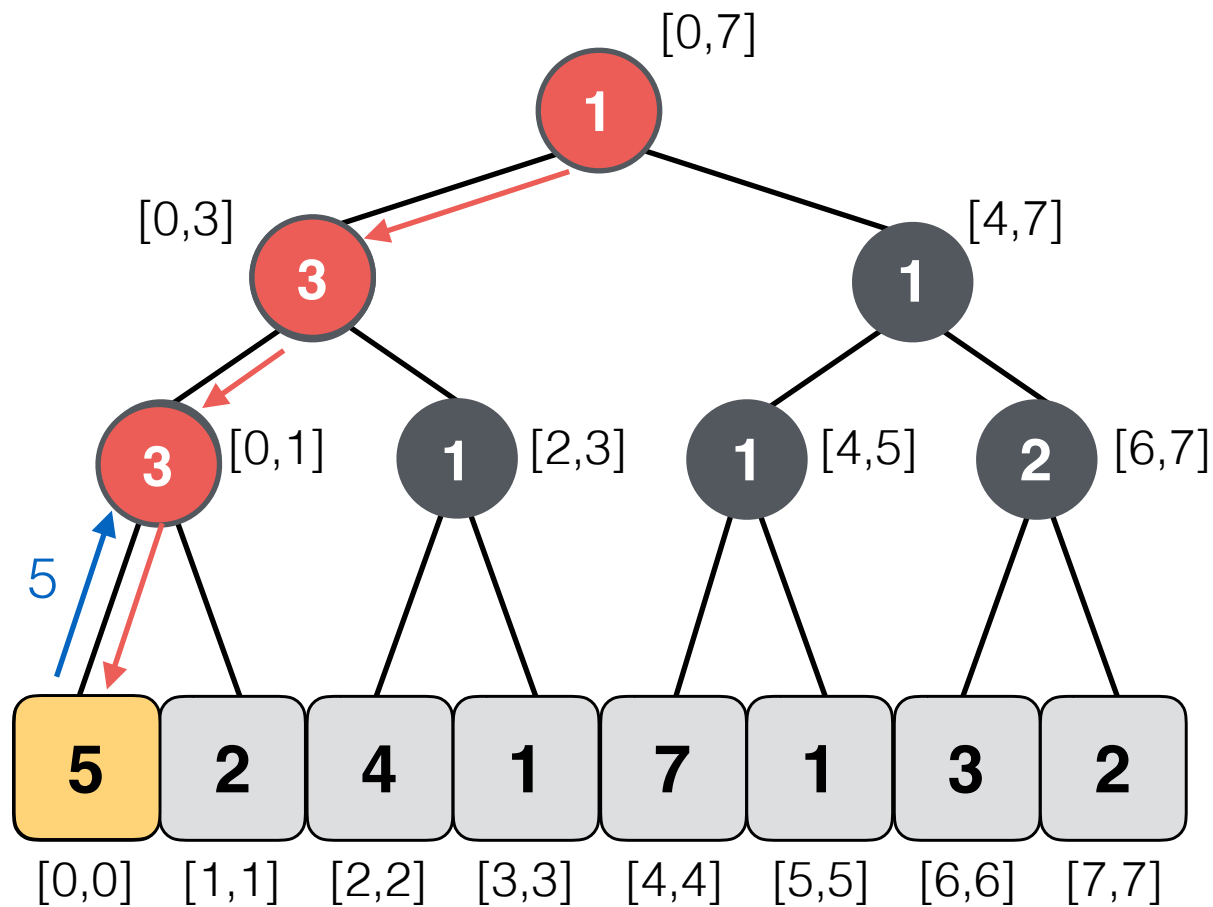


Lazy Tree

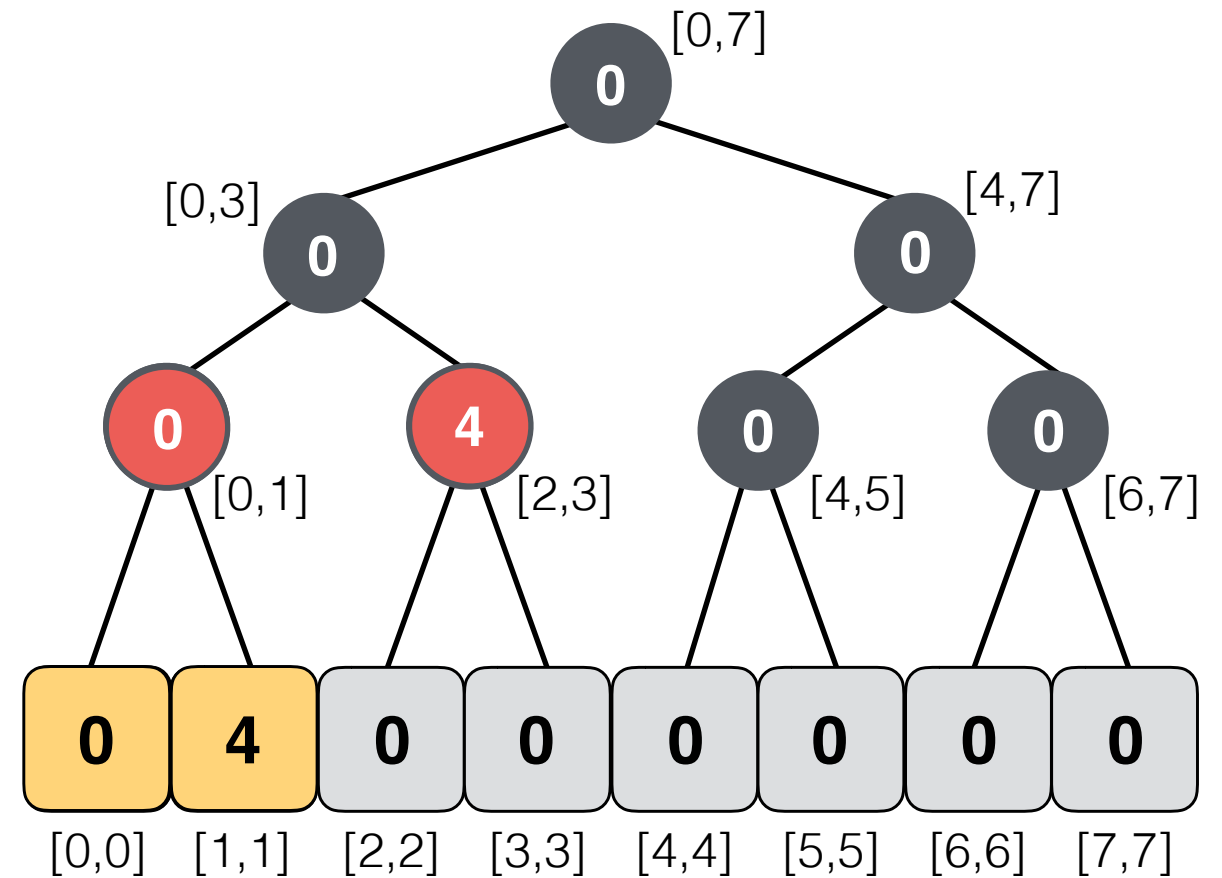
Lazy Propagation in Segment Trees

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Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

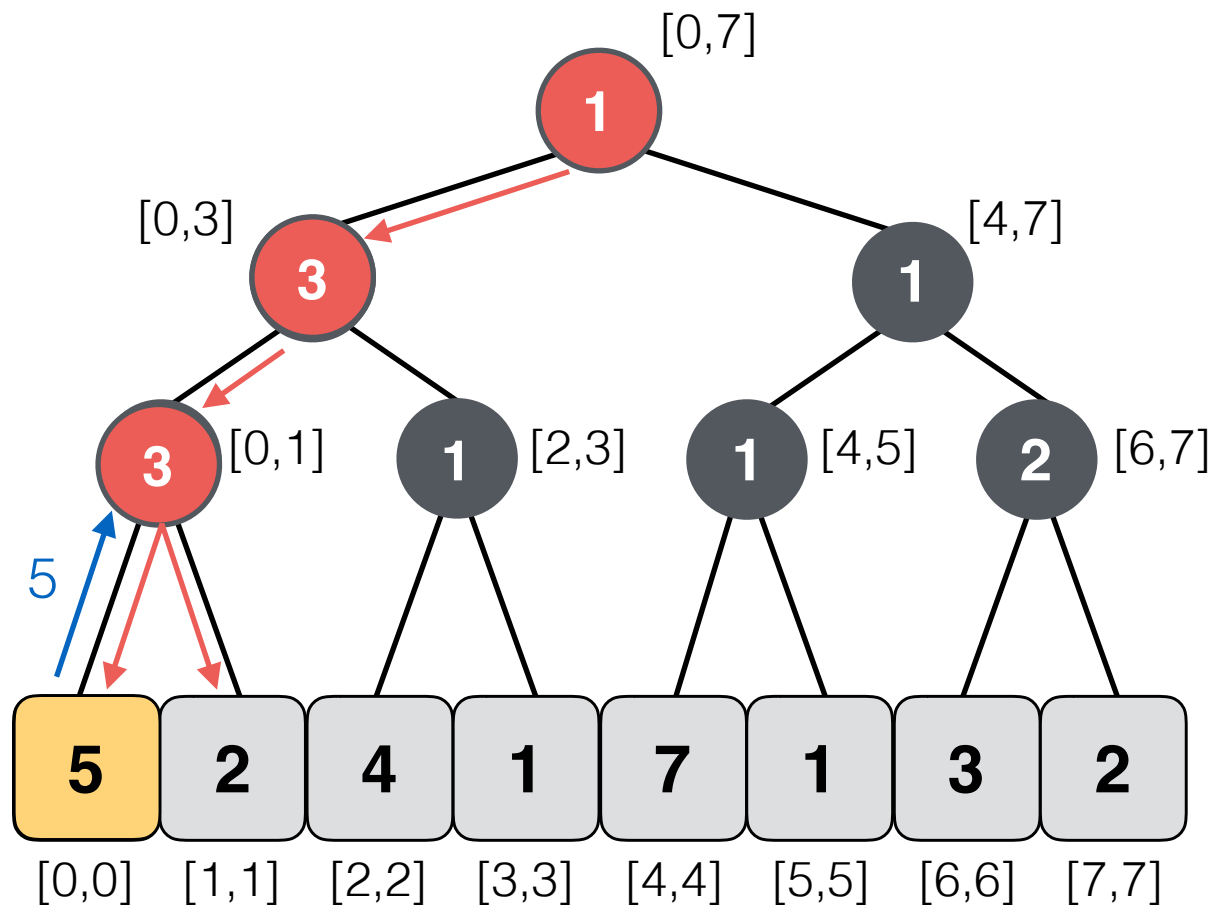


Lazy Tree

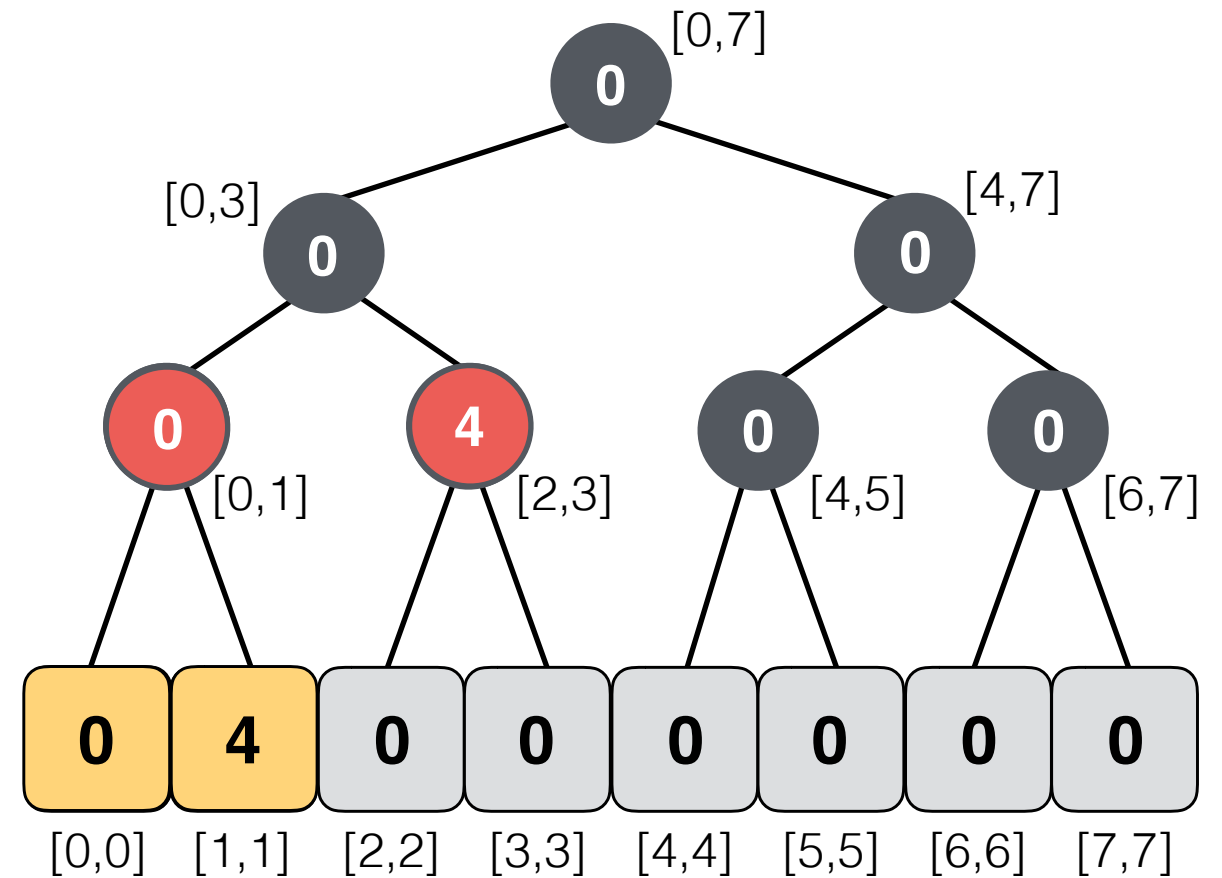
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Segment Tree

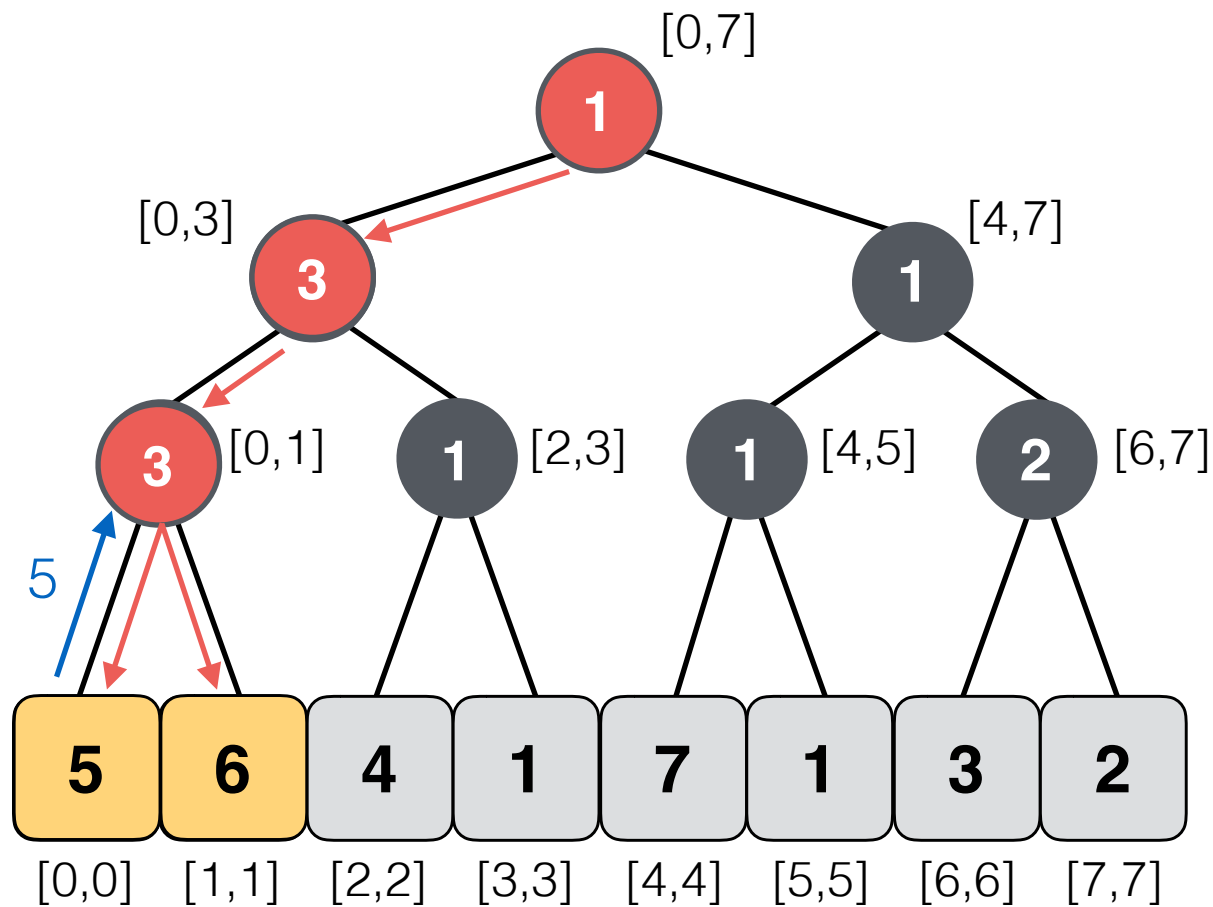


Lazy Tree

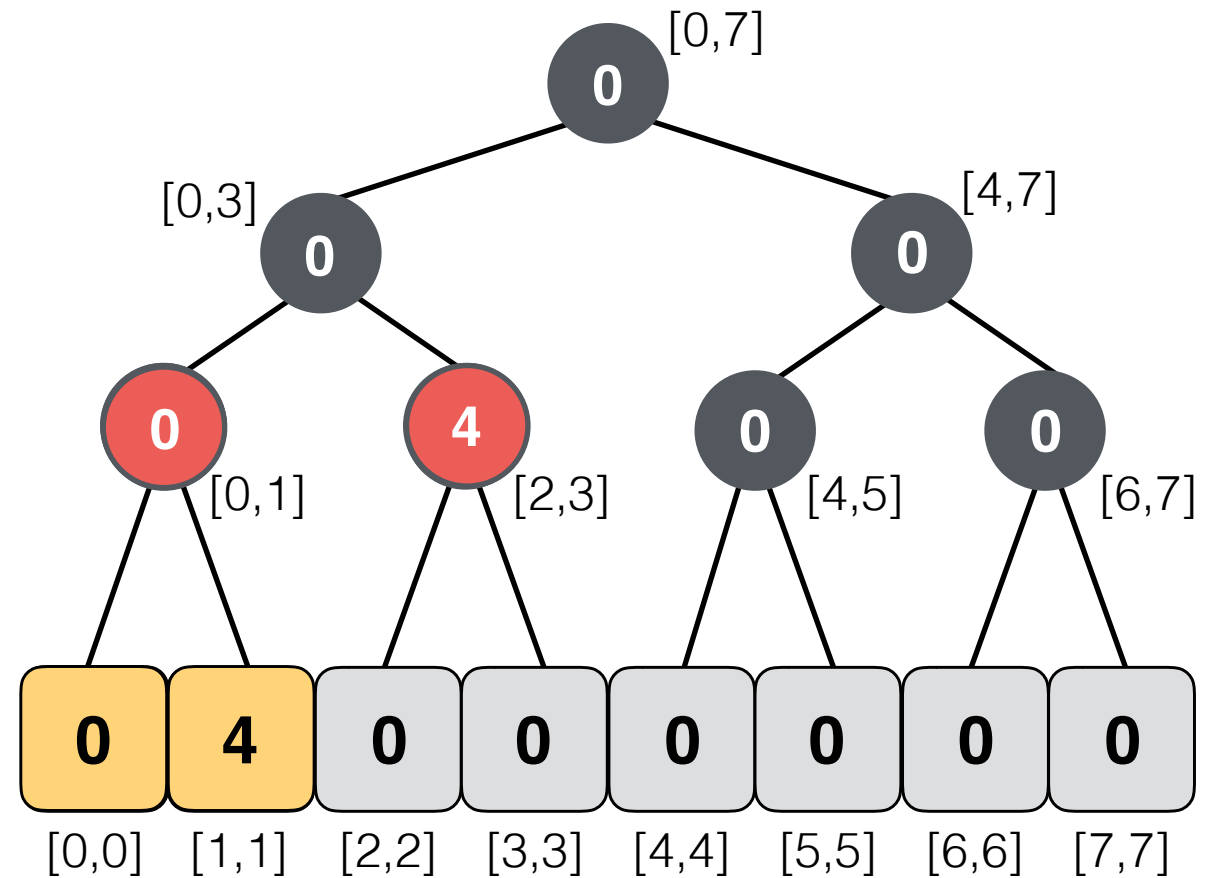
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Segment Tree

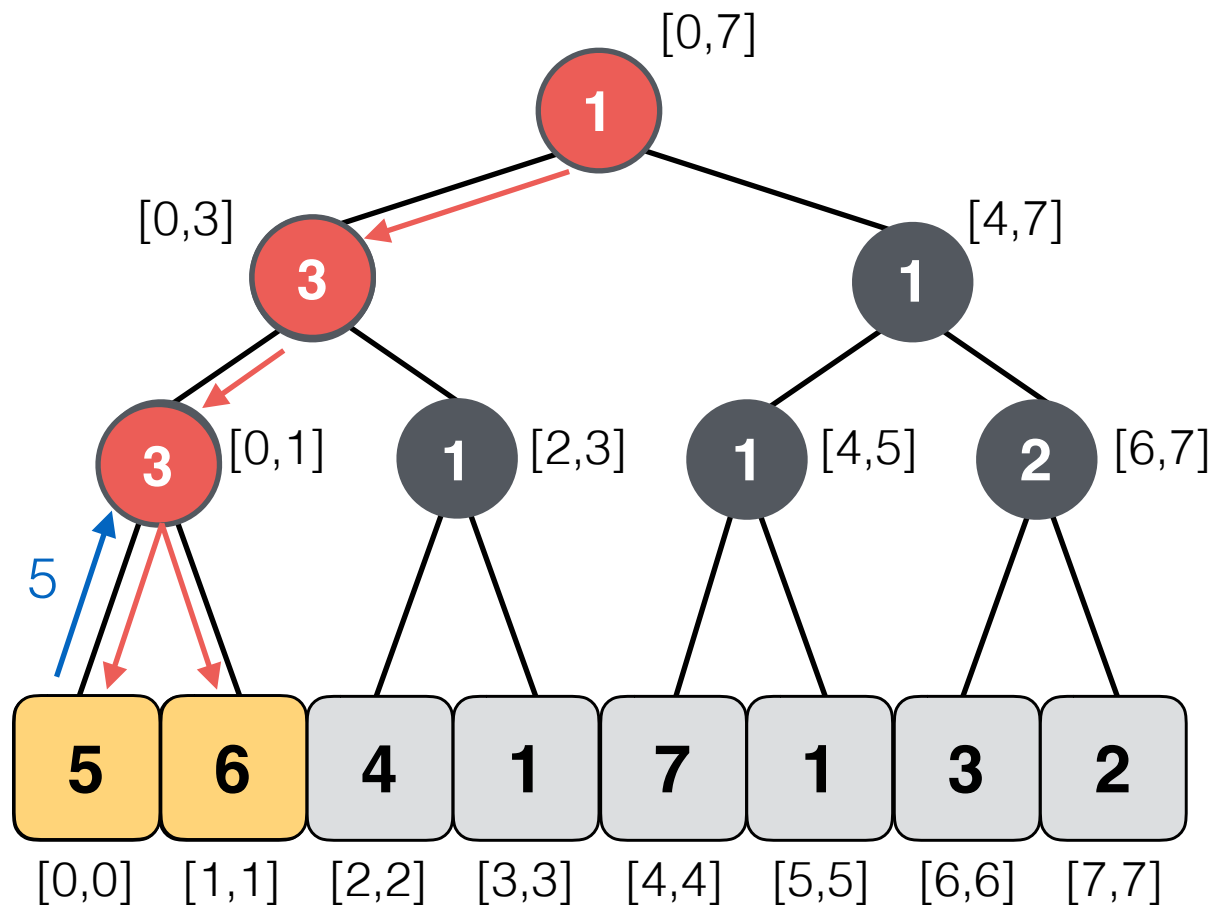


Lazy Tree

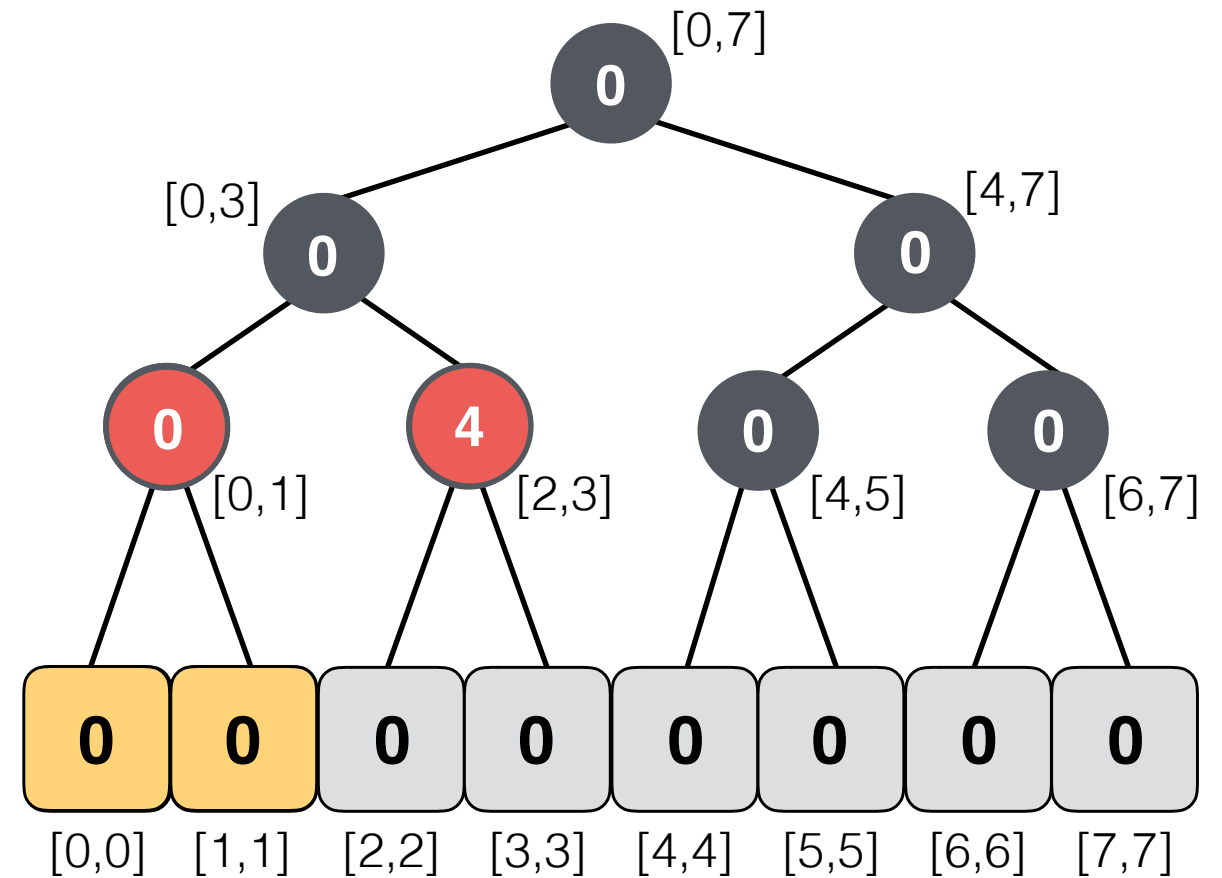
Lazy Propagation in Segment Trees

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Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

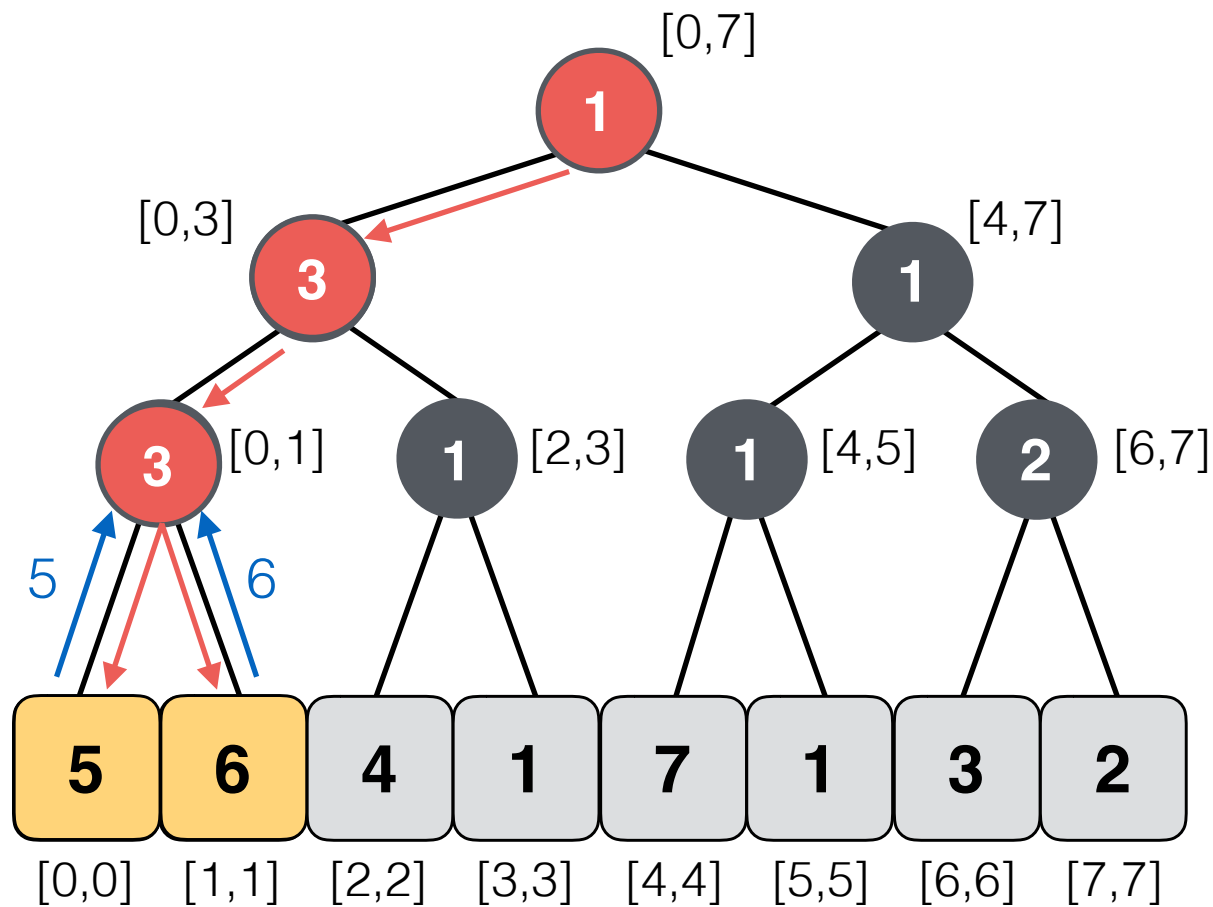


Lazy Tree

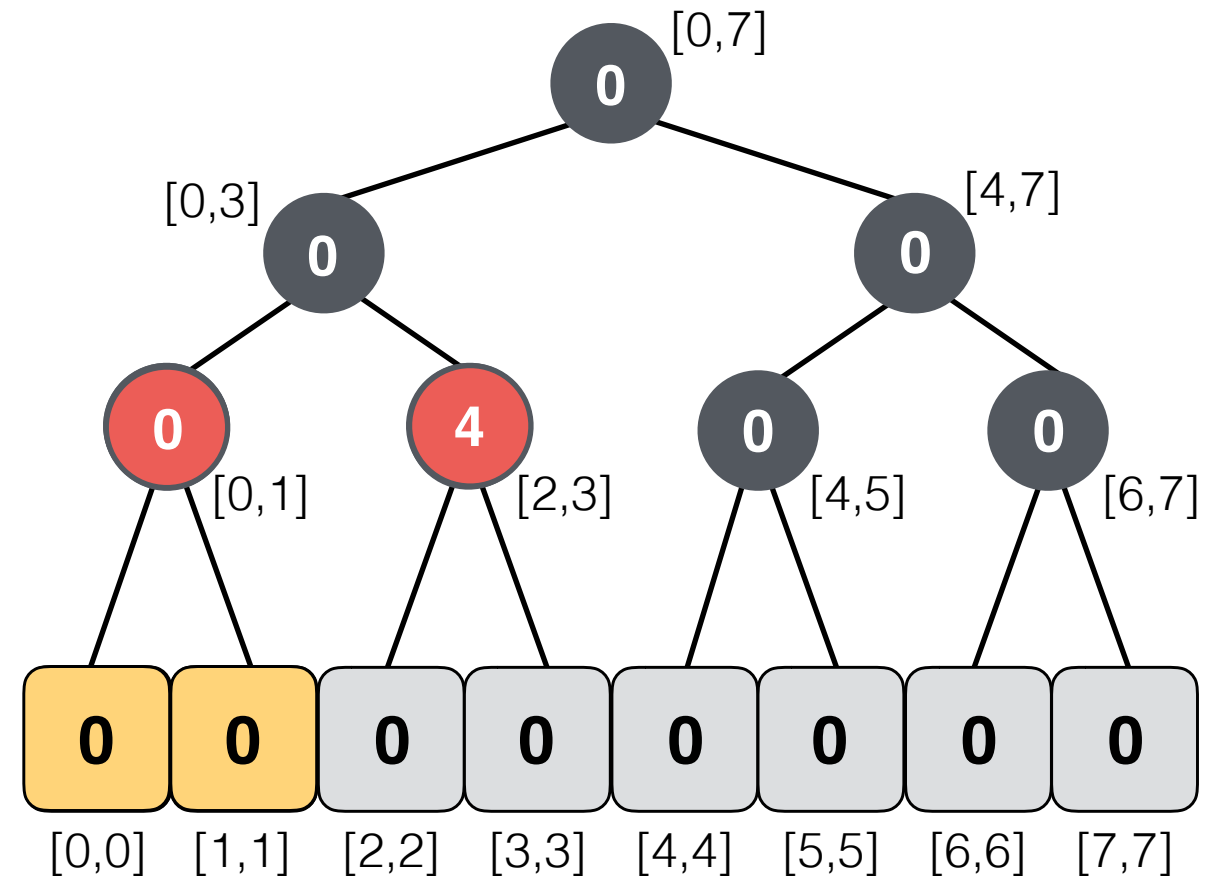
Lazy Propagation in Segment Trees

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update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

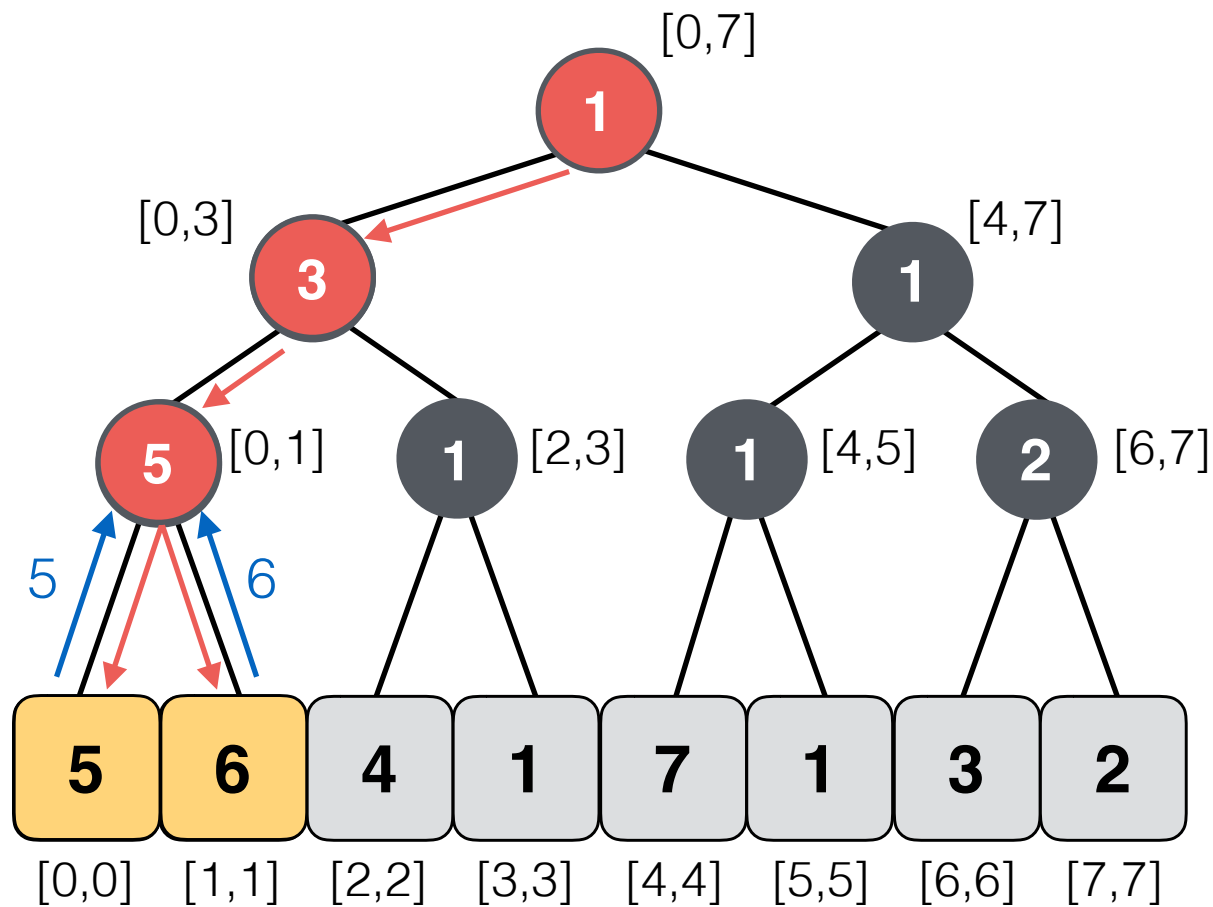


Lazy Tree

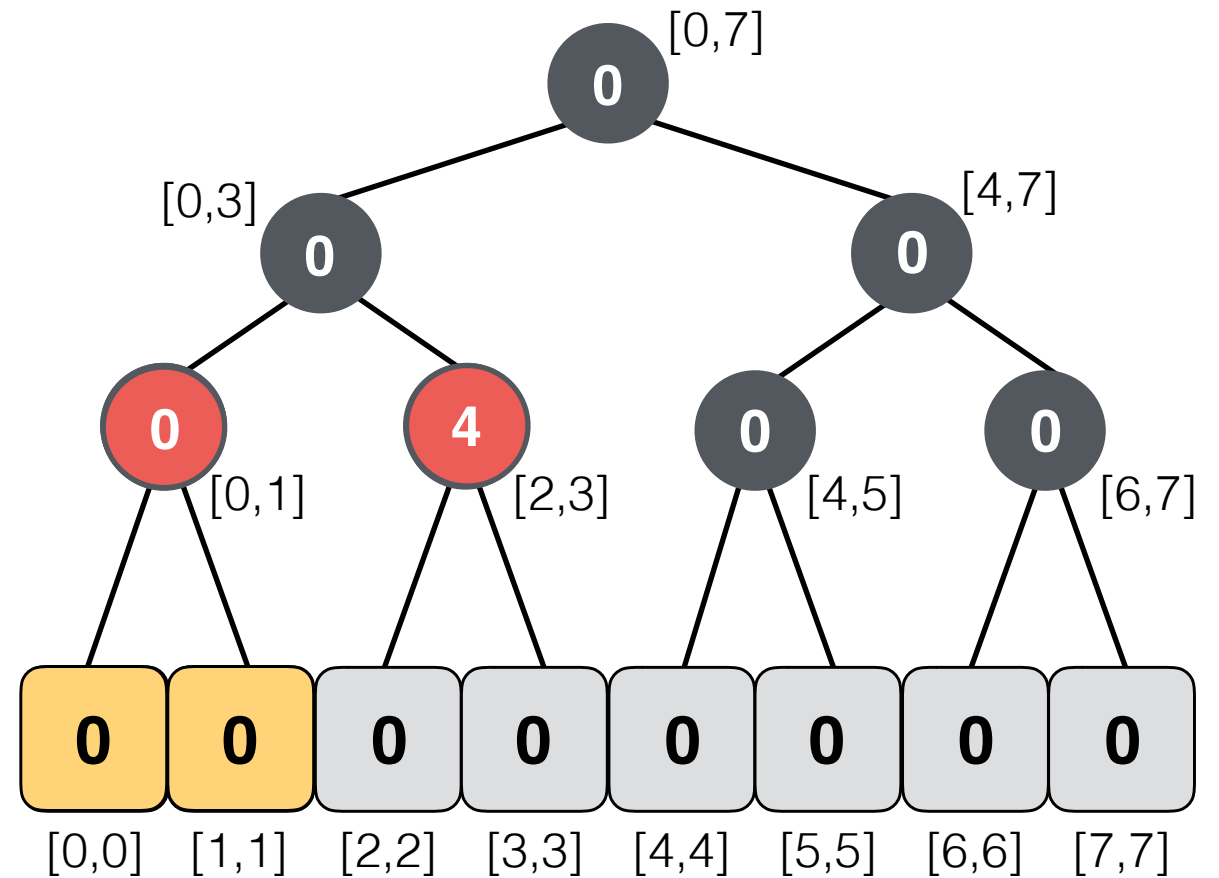
Lazy Propagation in Segment Trees

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Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

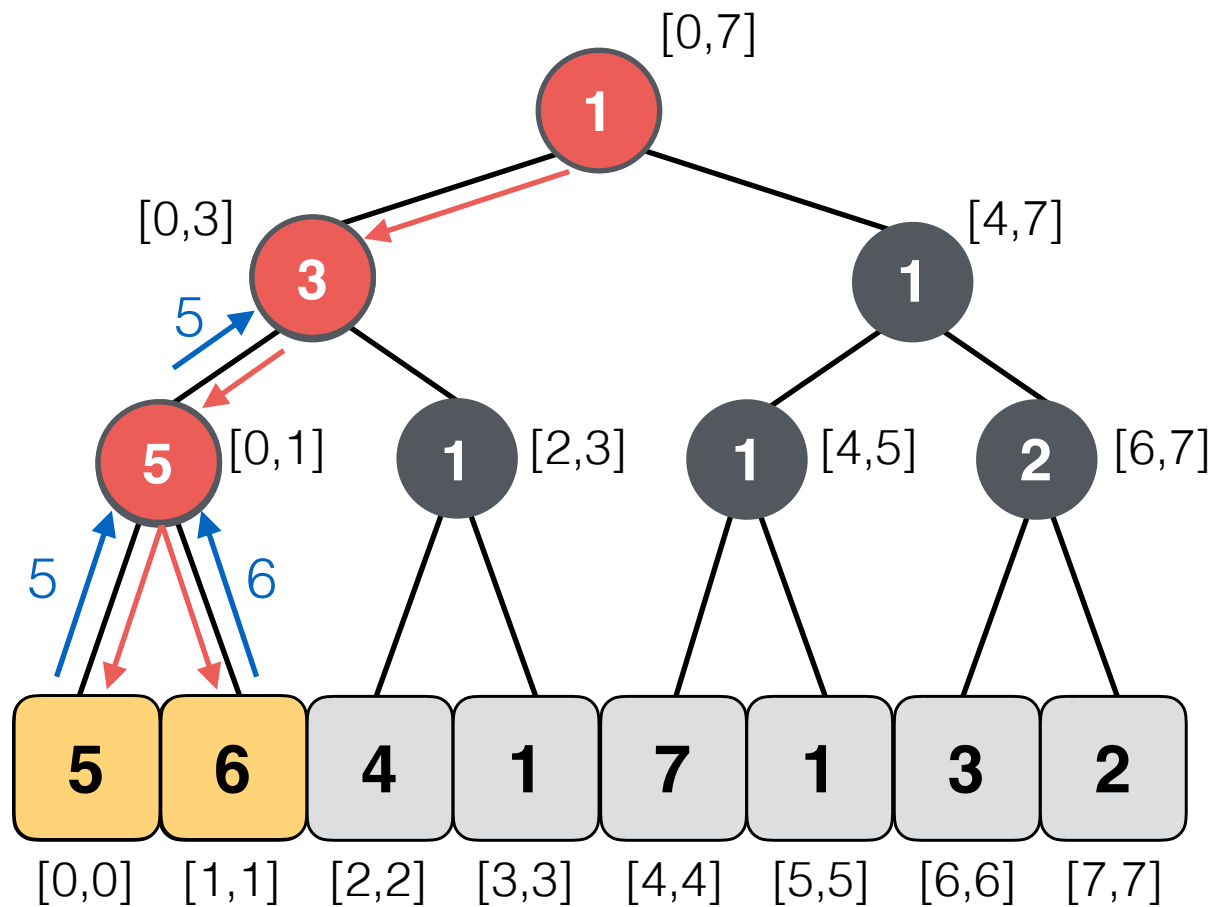


Lazy Tree

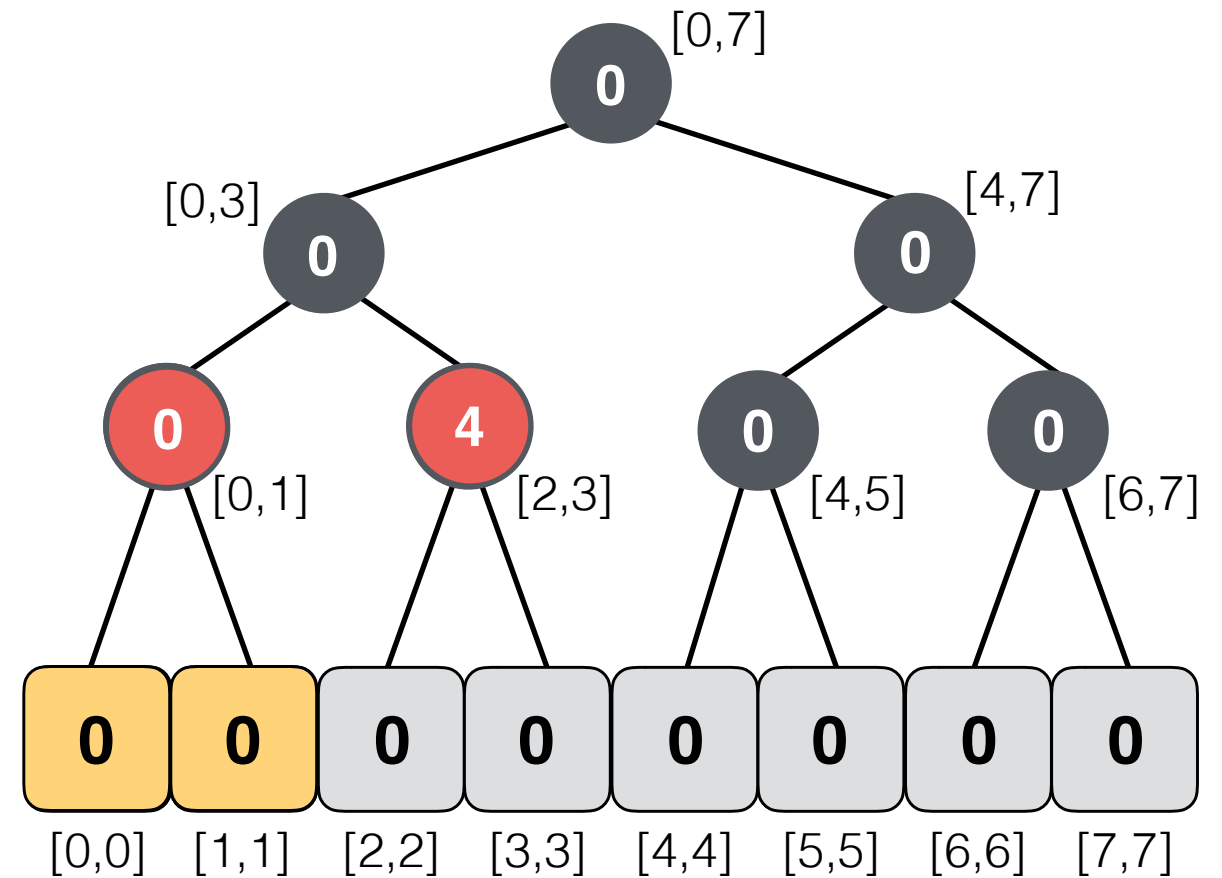
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Segment Tree

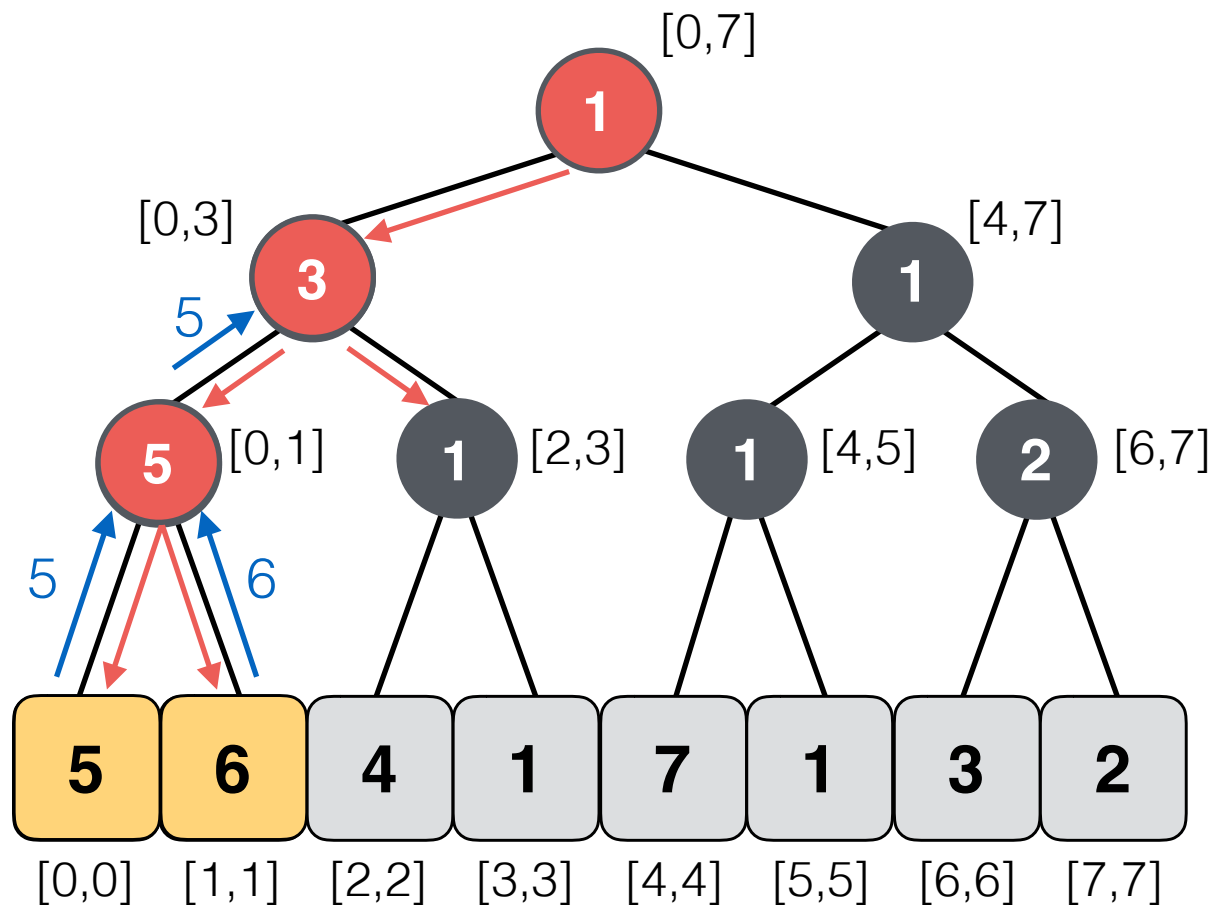


Lazy Tree

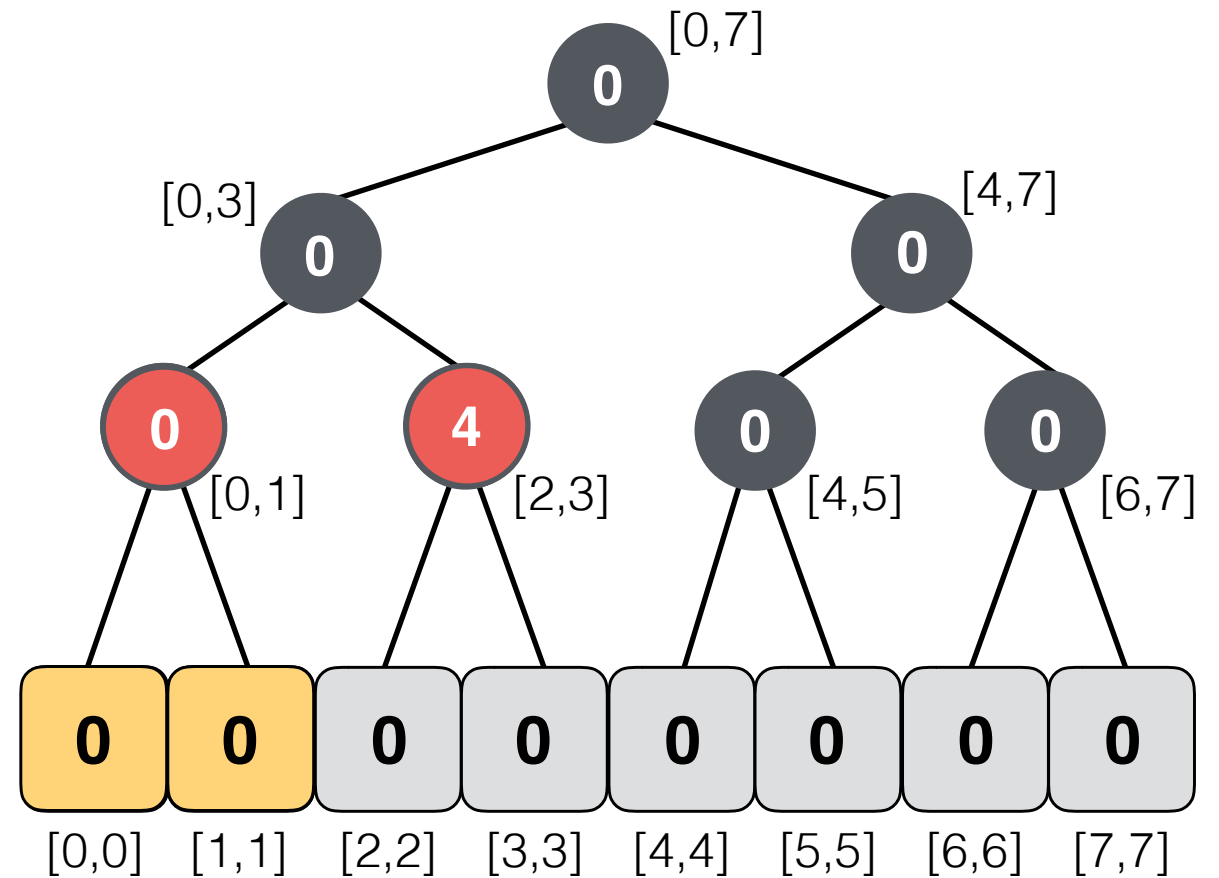
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Segment Tree

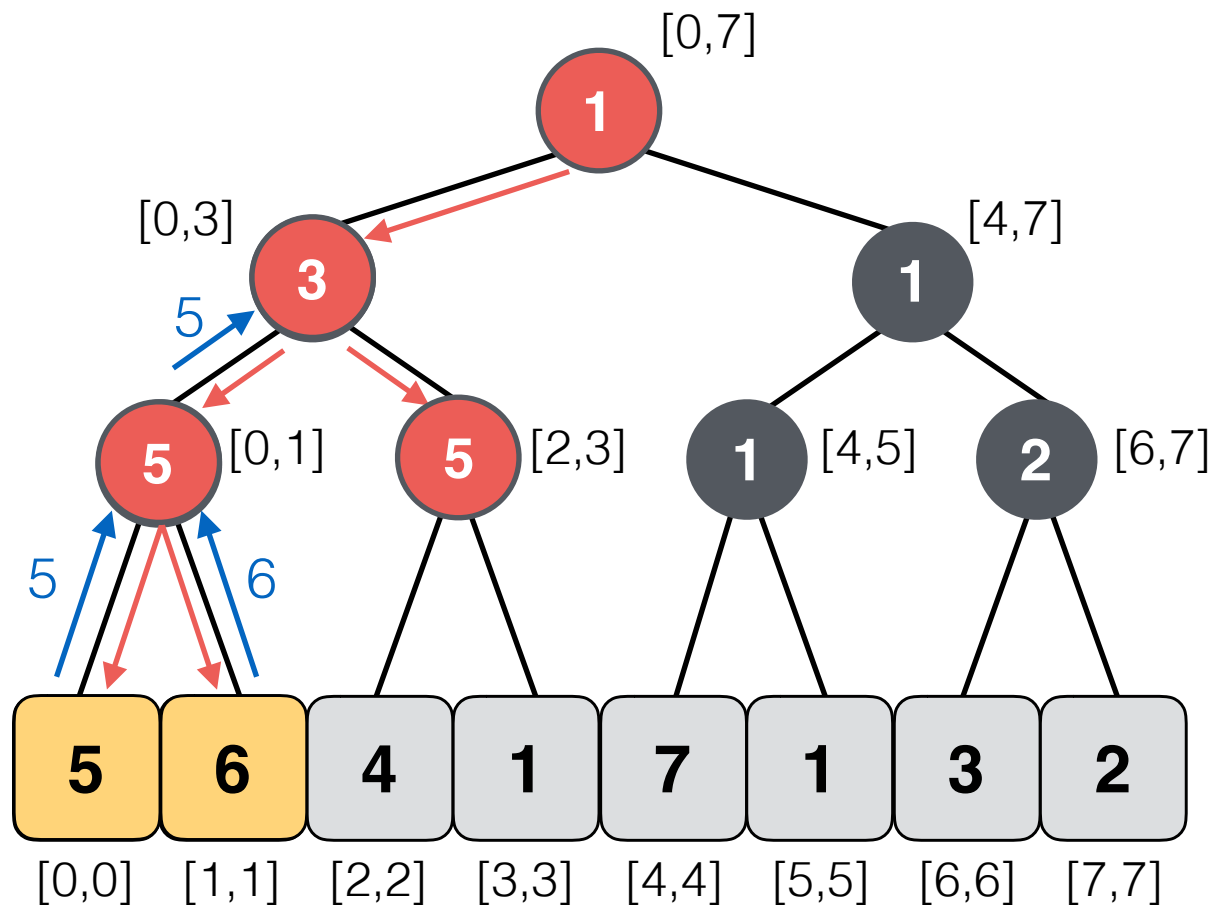


Lazy Tree

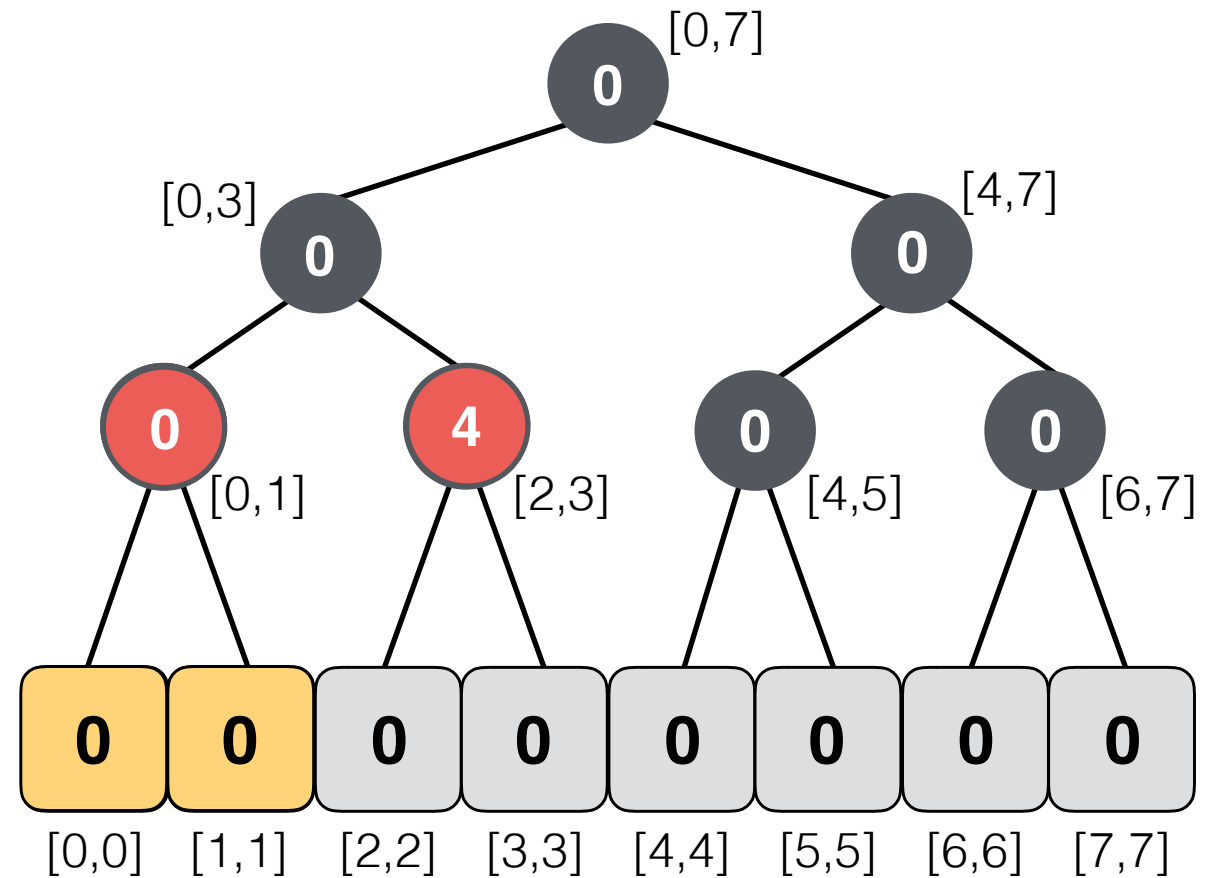
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Segment Tree

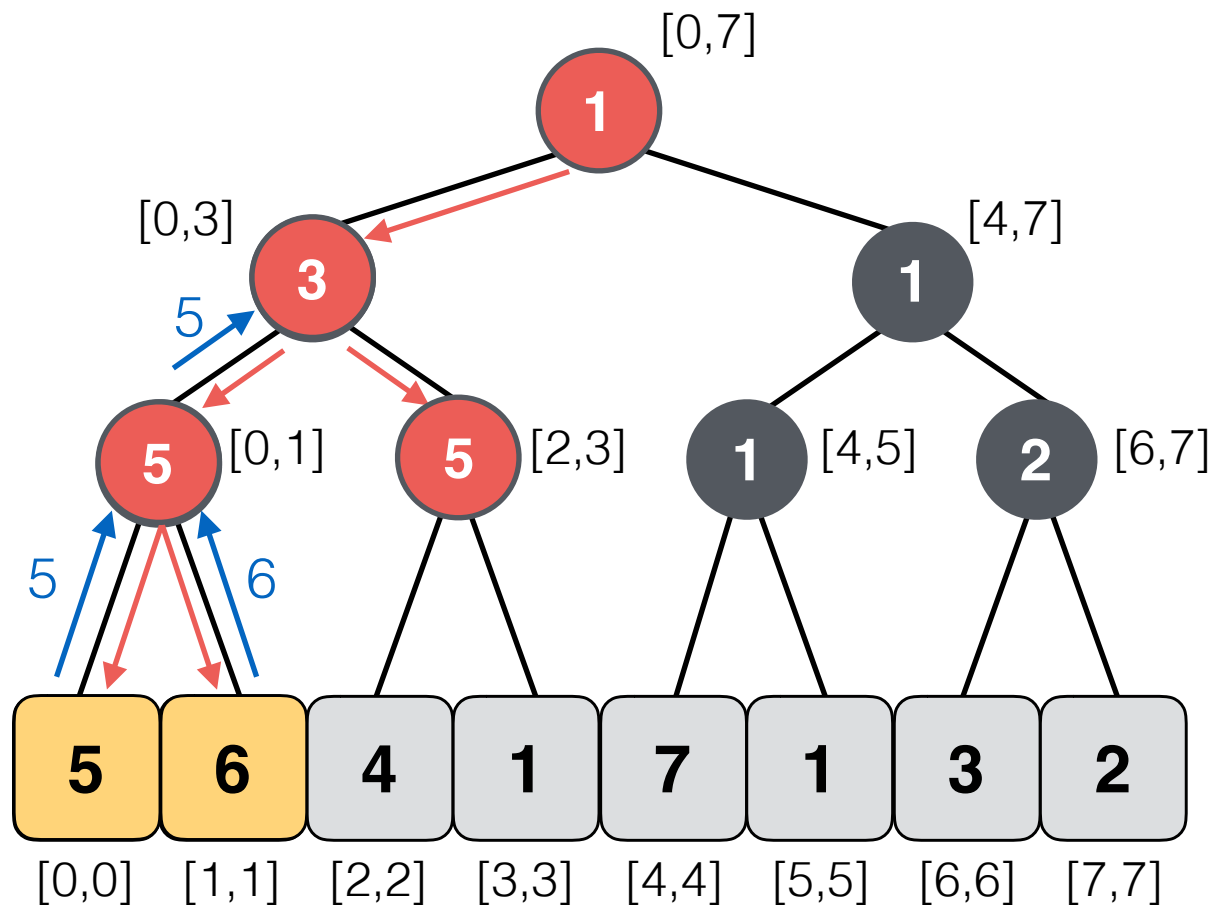


Lazy Tree

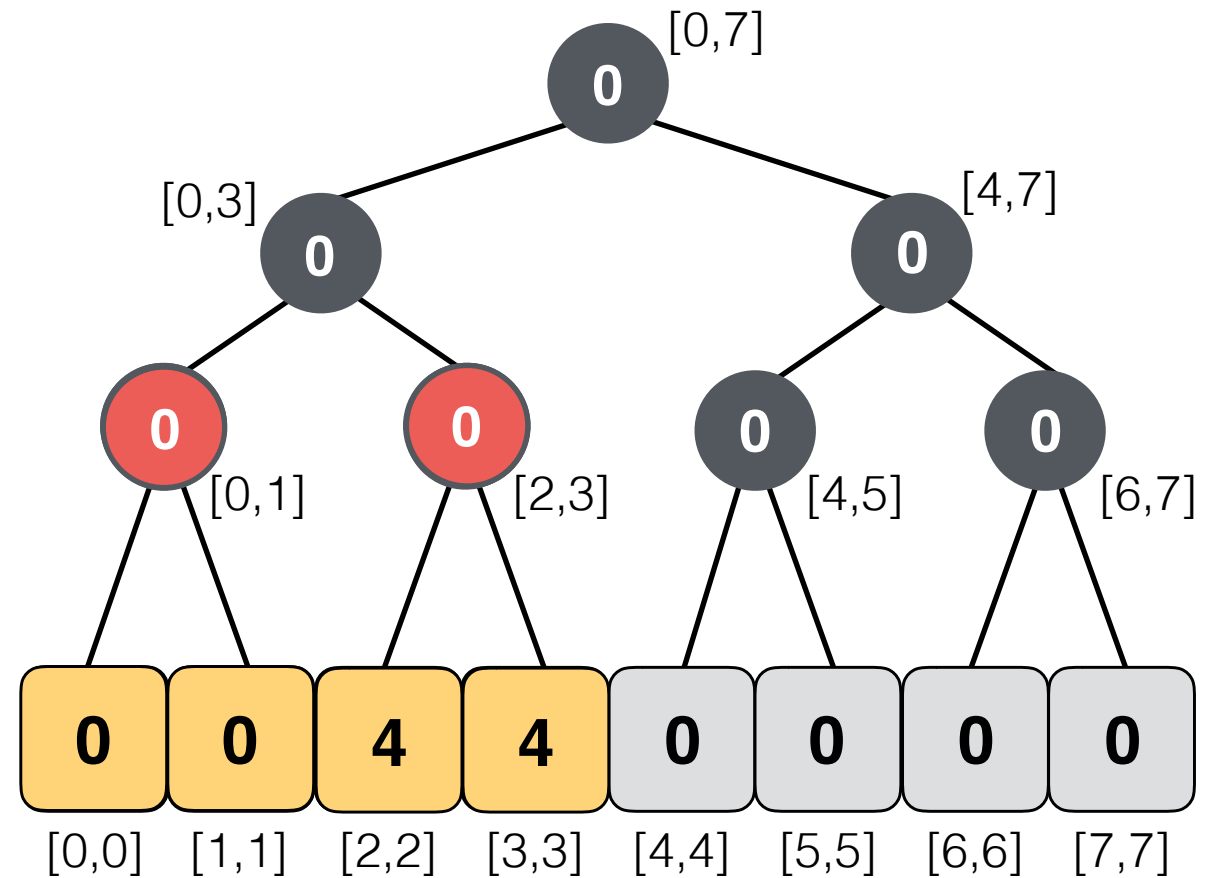
Lazy Propagation in Segment Trees

Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

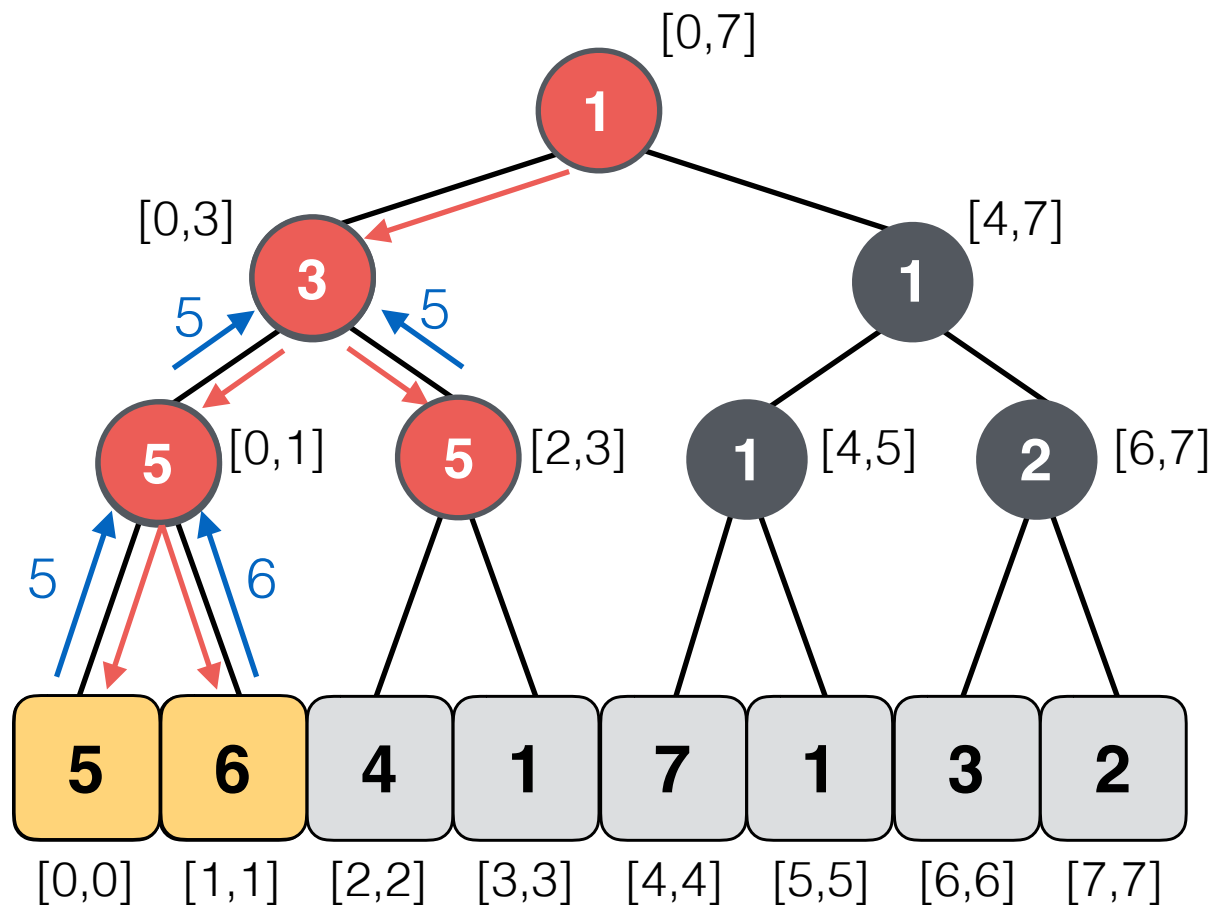


Lazy Tree

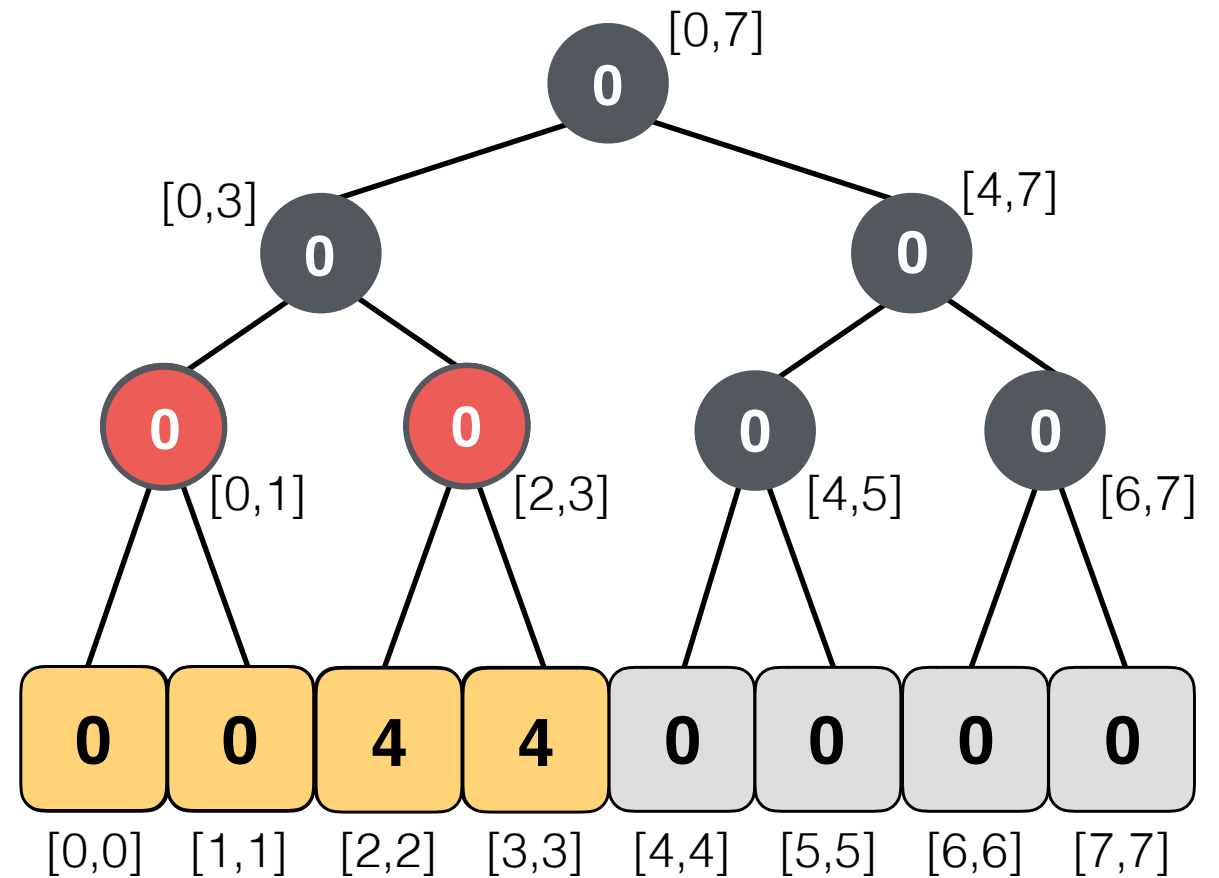
Lazy Propagation in Segment Trees

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Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

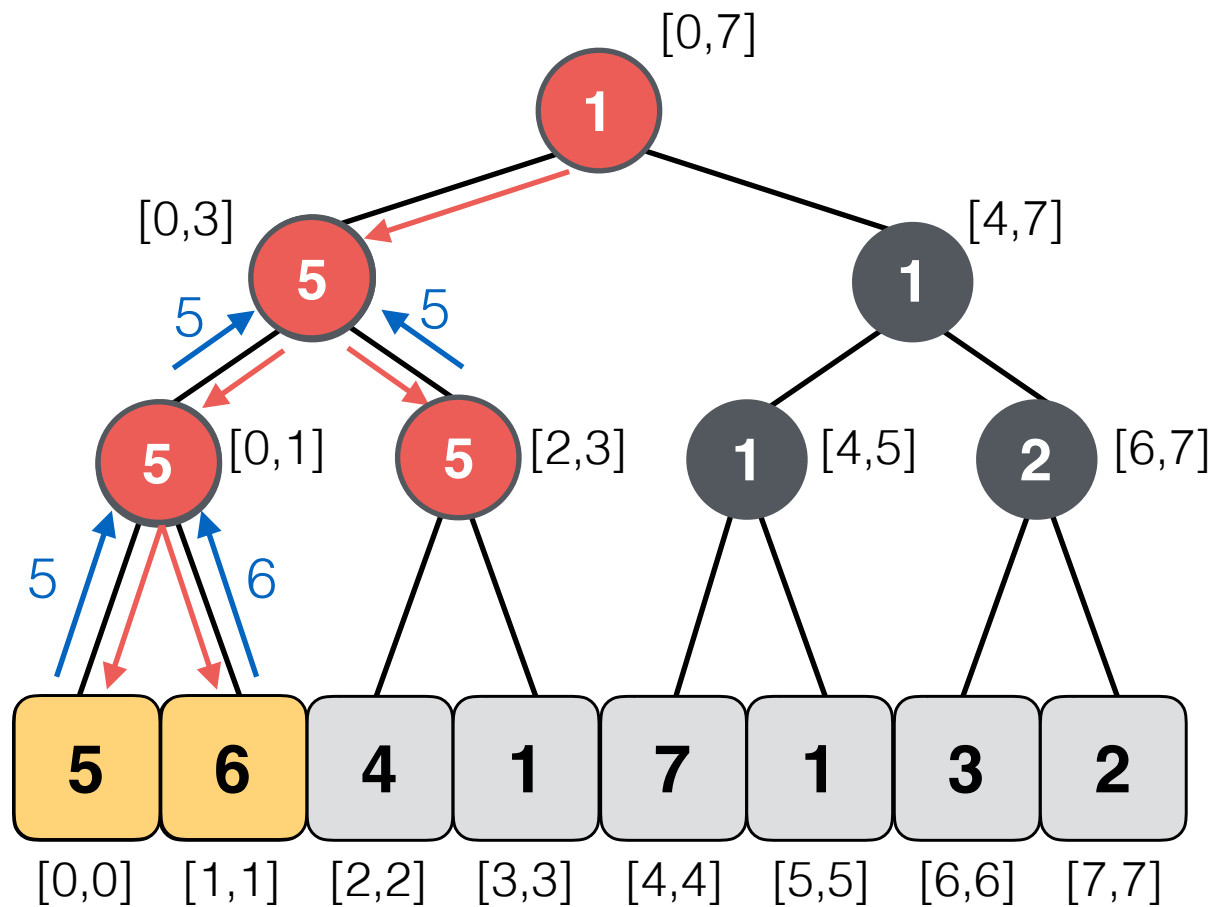


Lazy Tree

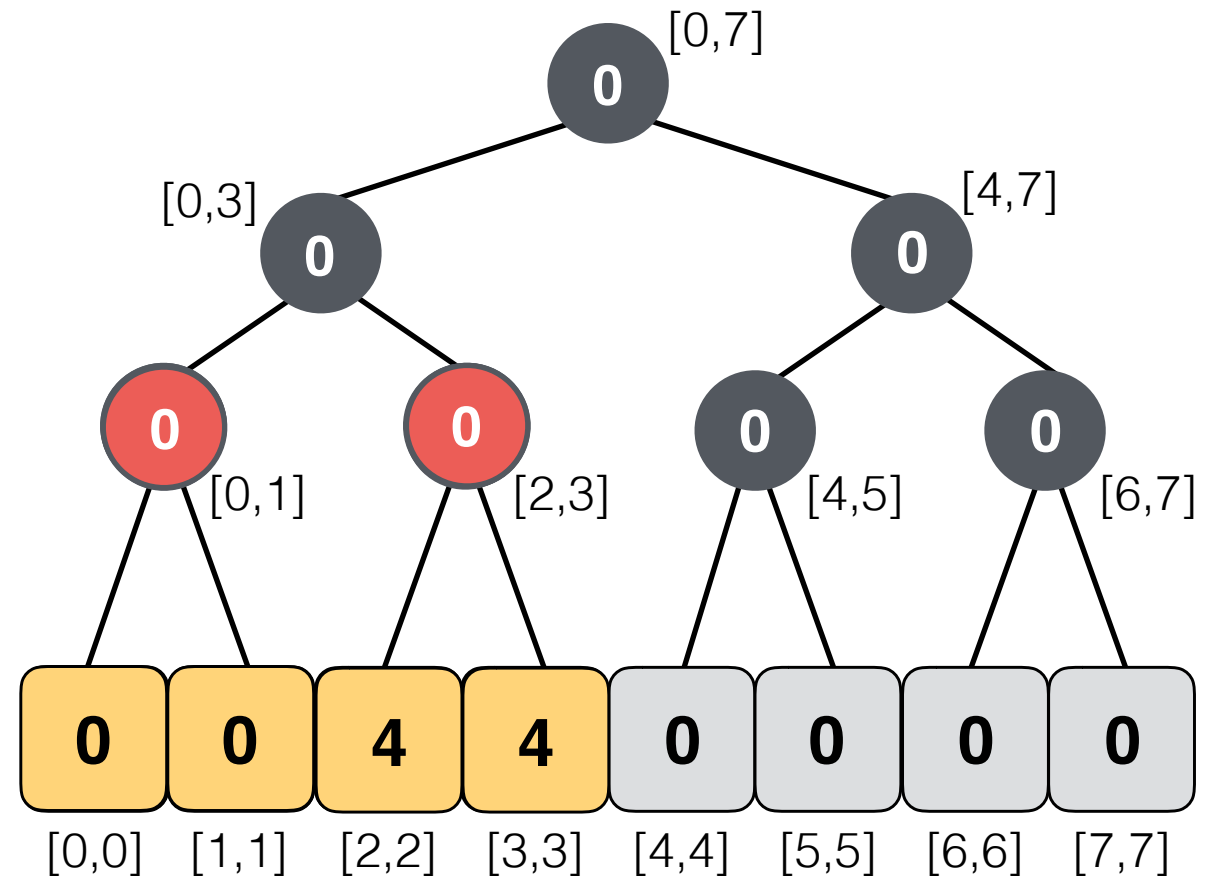
Lazy Propagation in Segment Trees

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Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

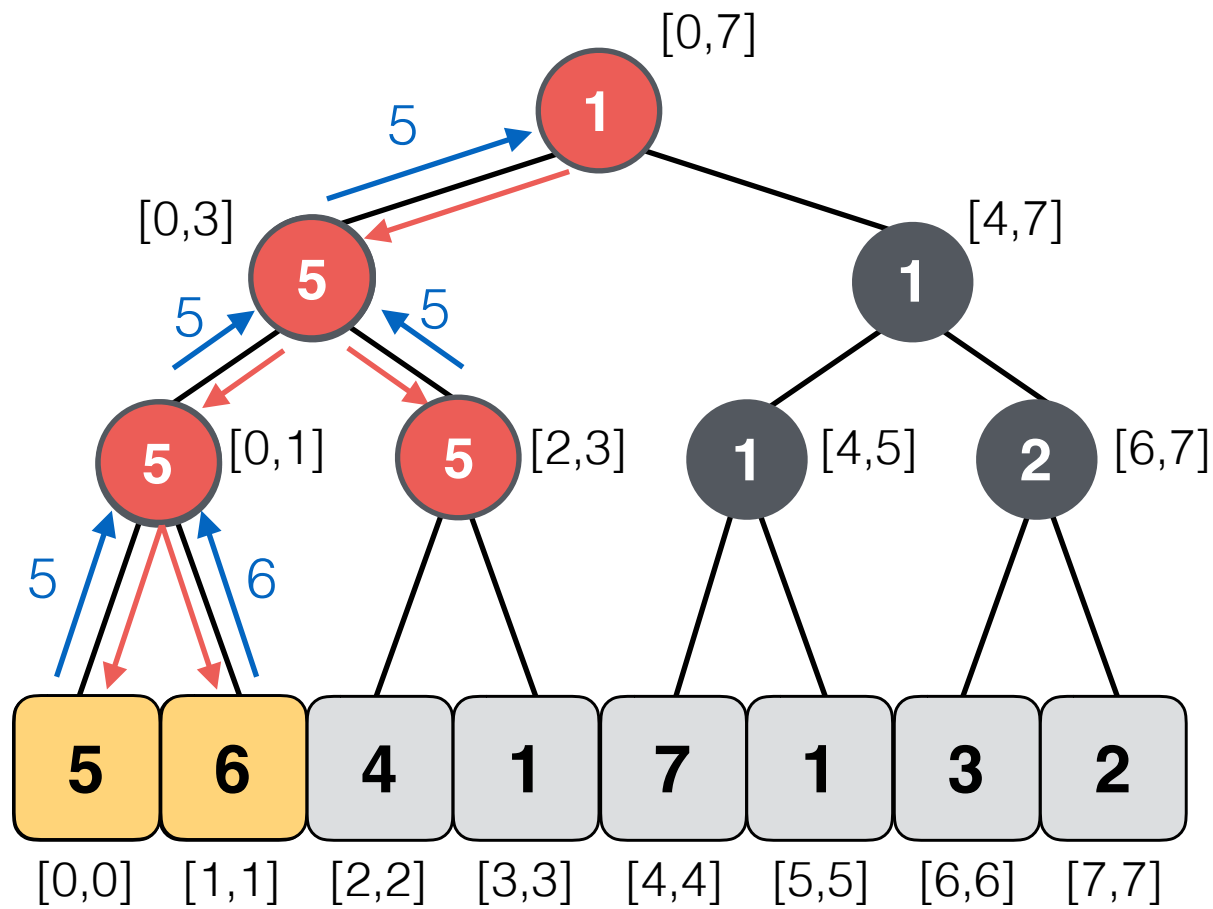


Lazy Tree

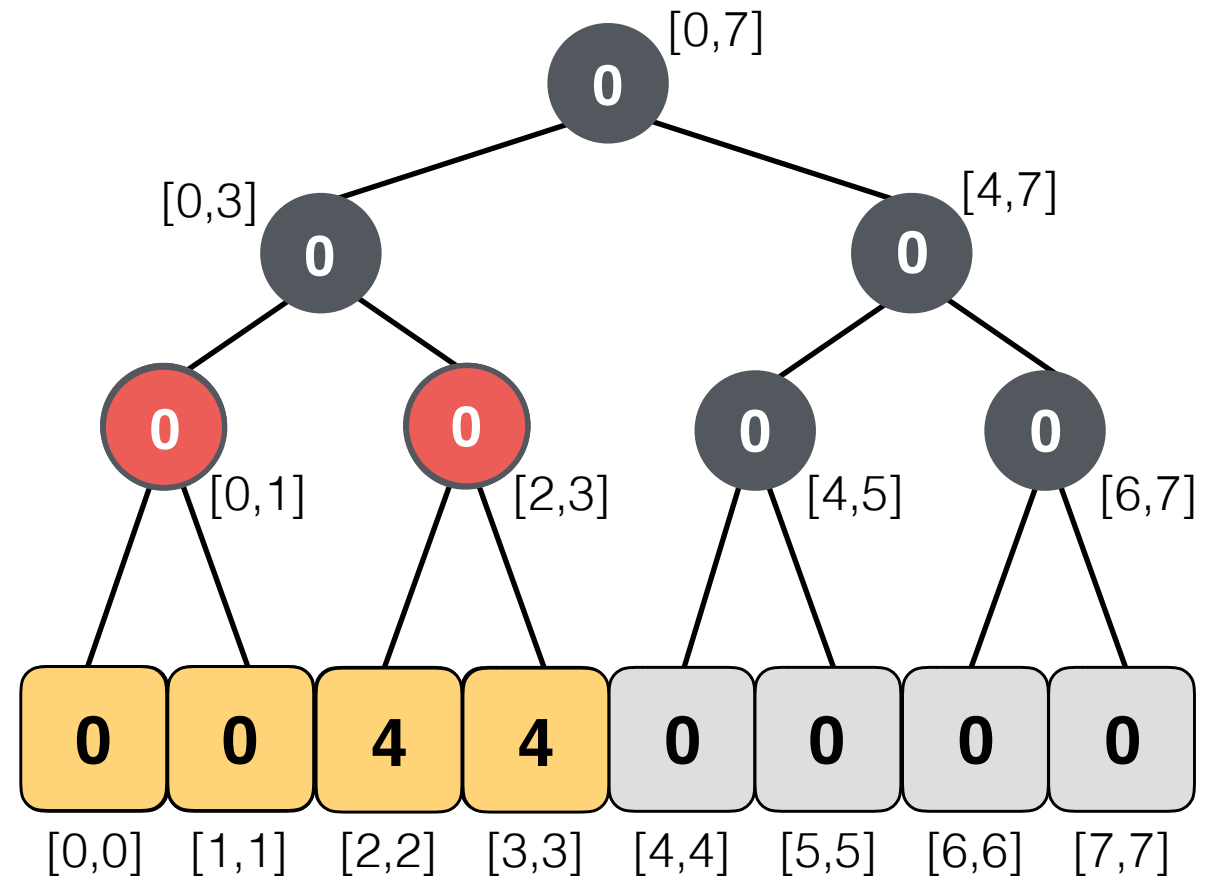
Lazy Propagation in Segment Trees

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Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

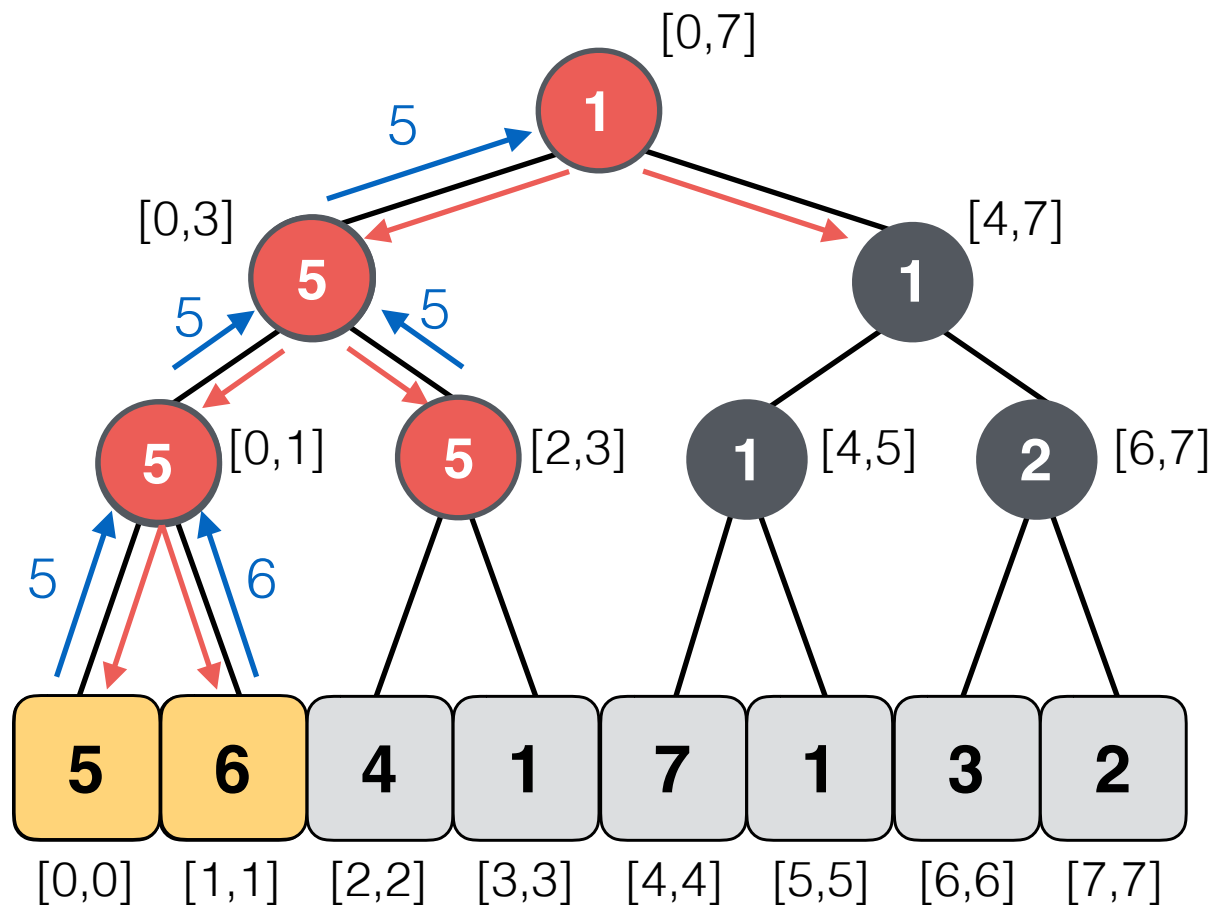


Lazy Tree

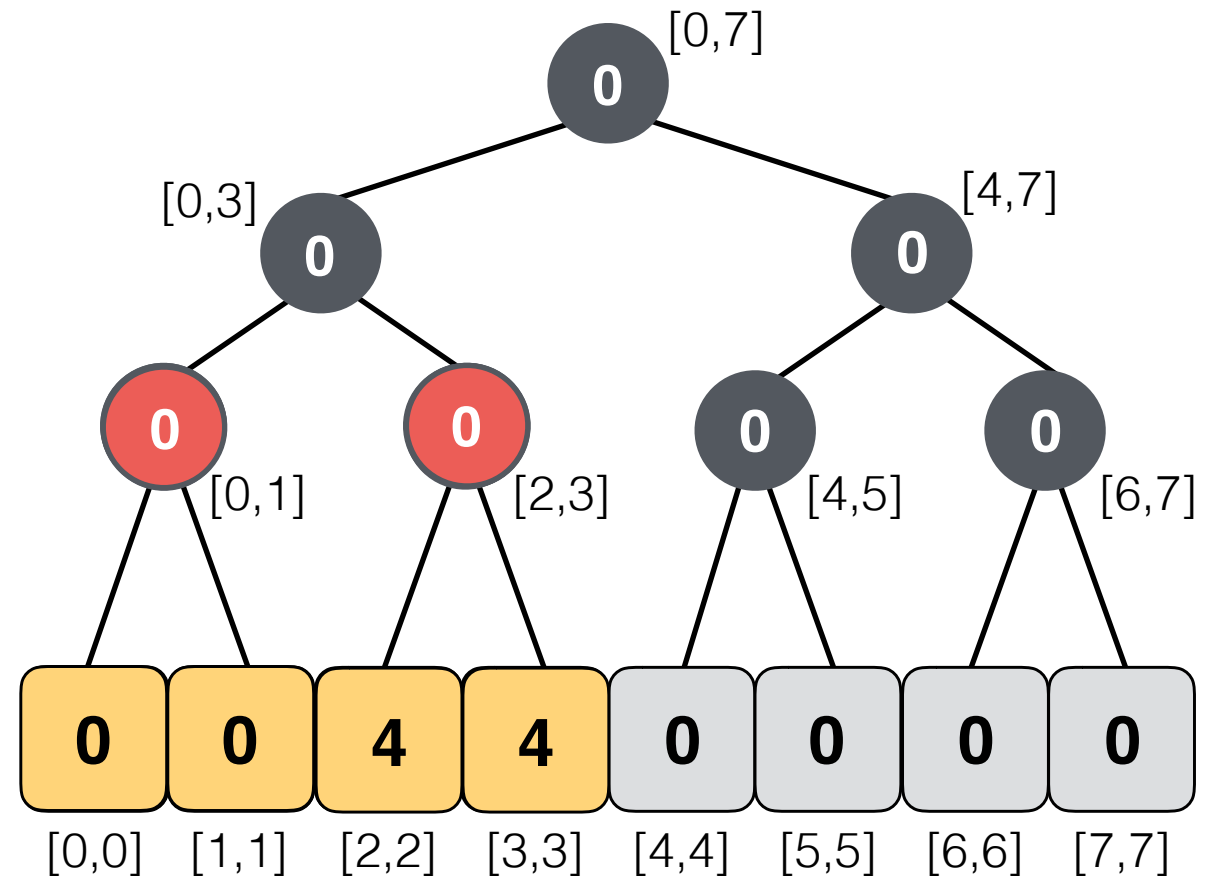
Lazy Propagation in Segment Trees

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Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

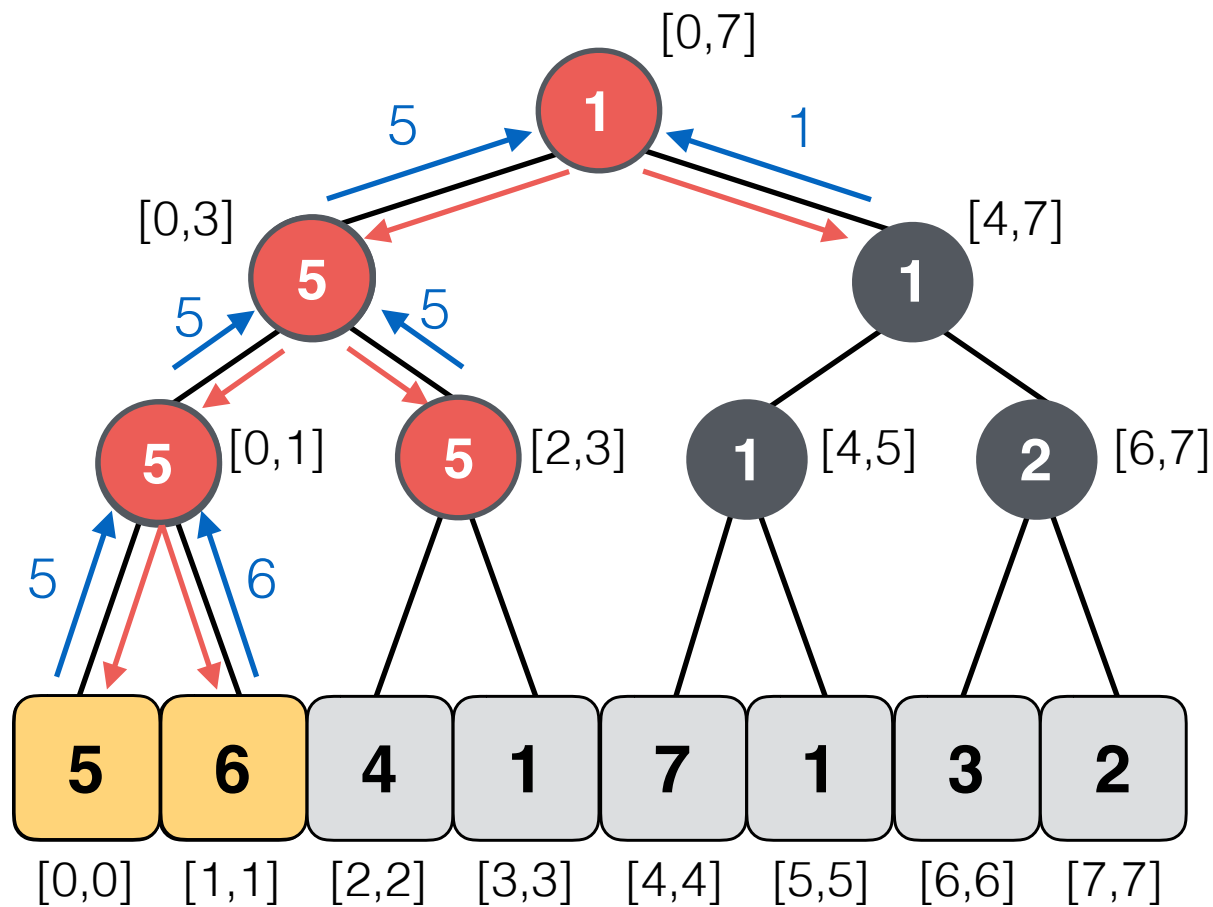


Lazy Tree

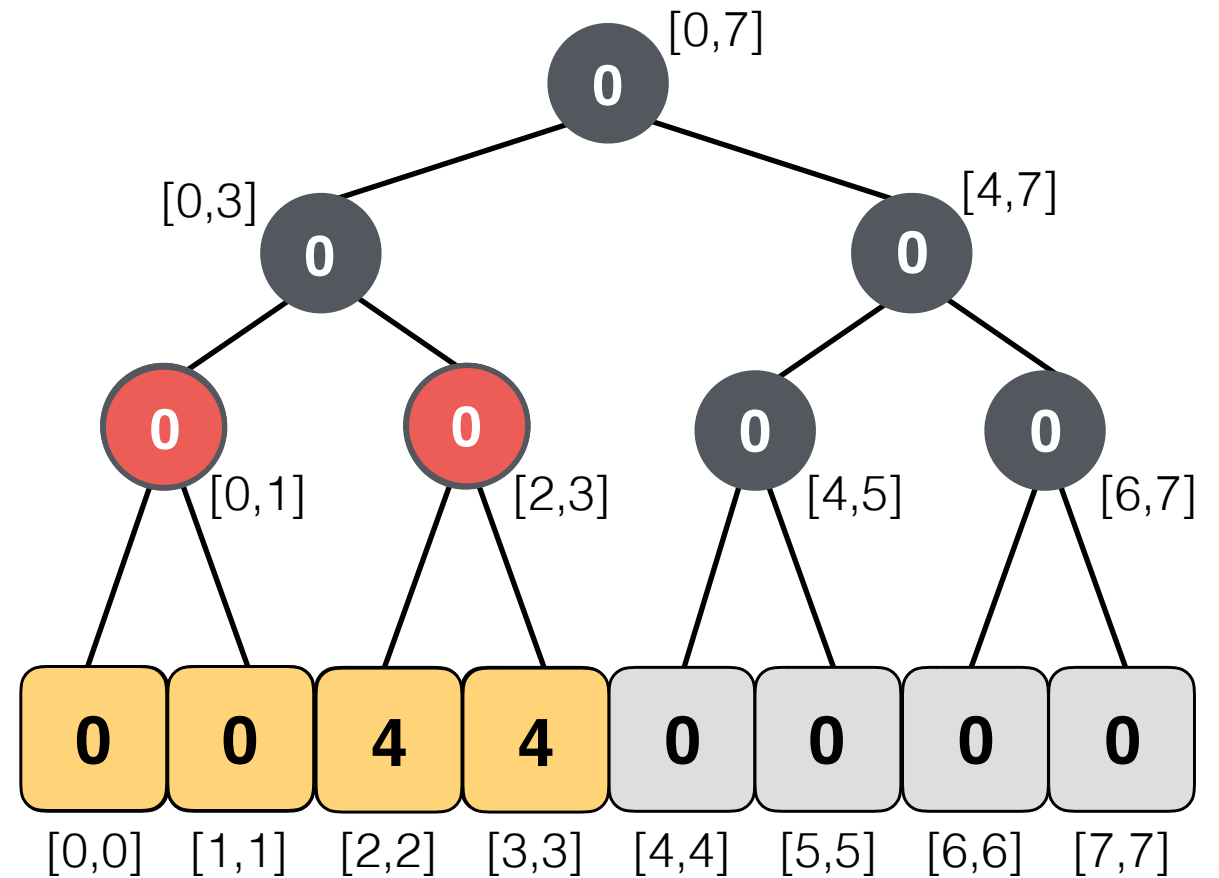
Lazy Propagation in Segment Trees

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Only update when needed.

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update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

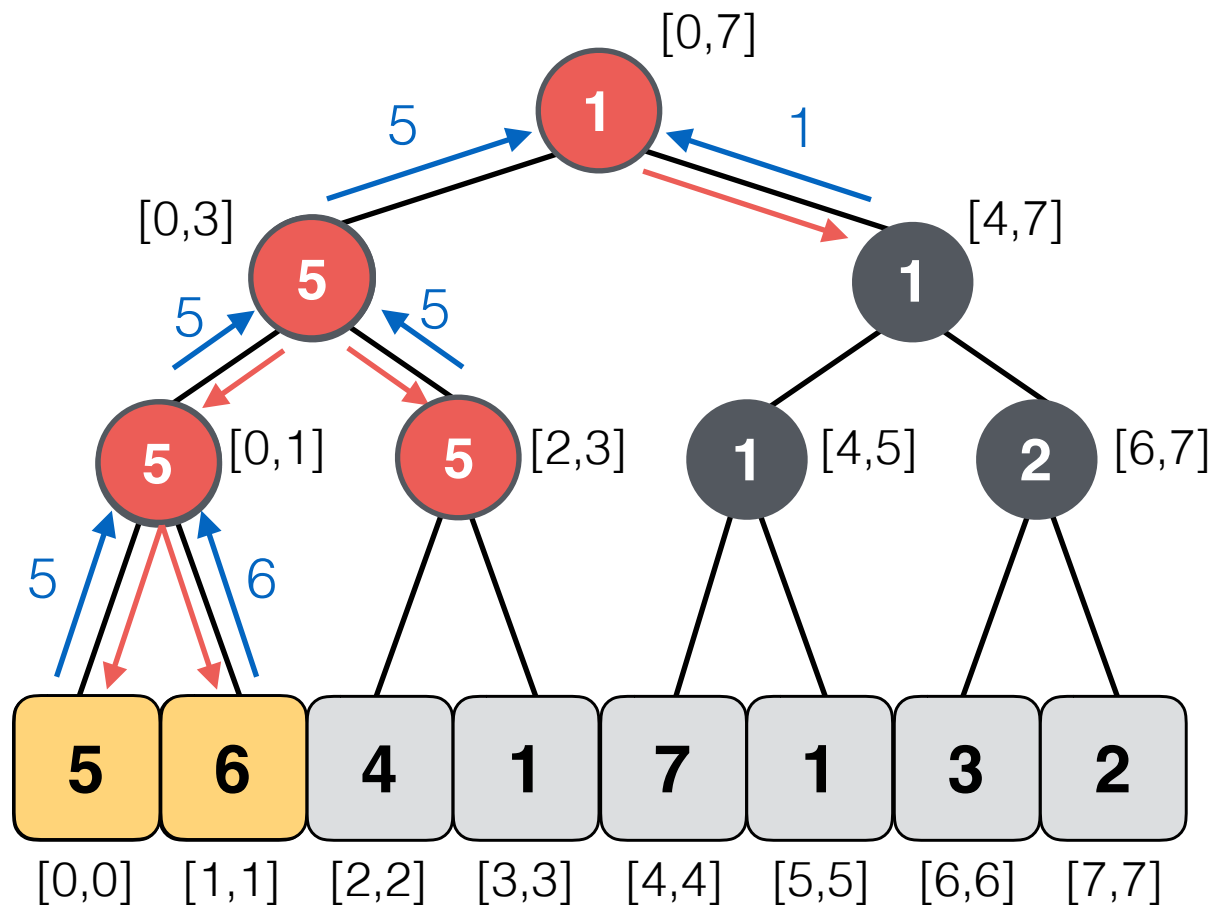


Lazy Tree

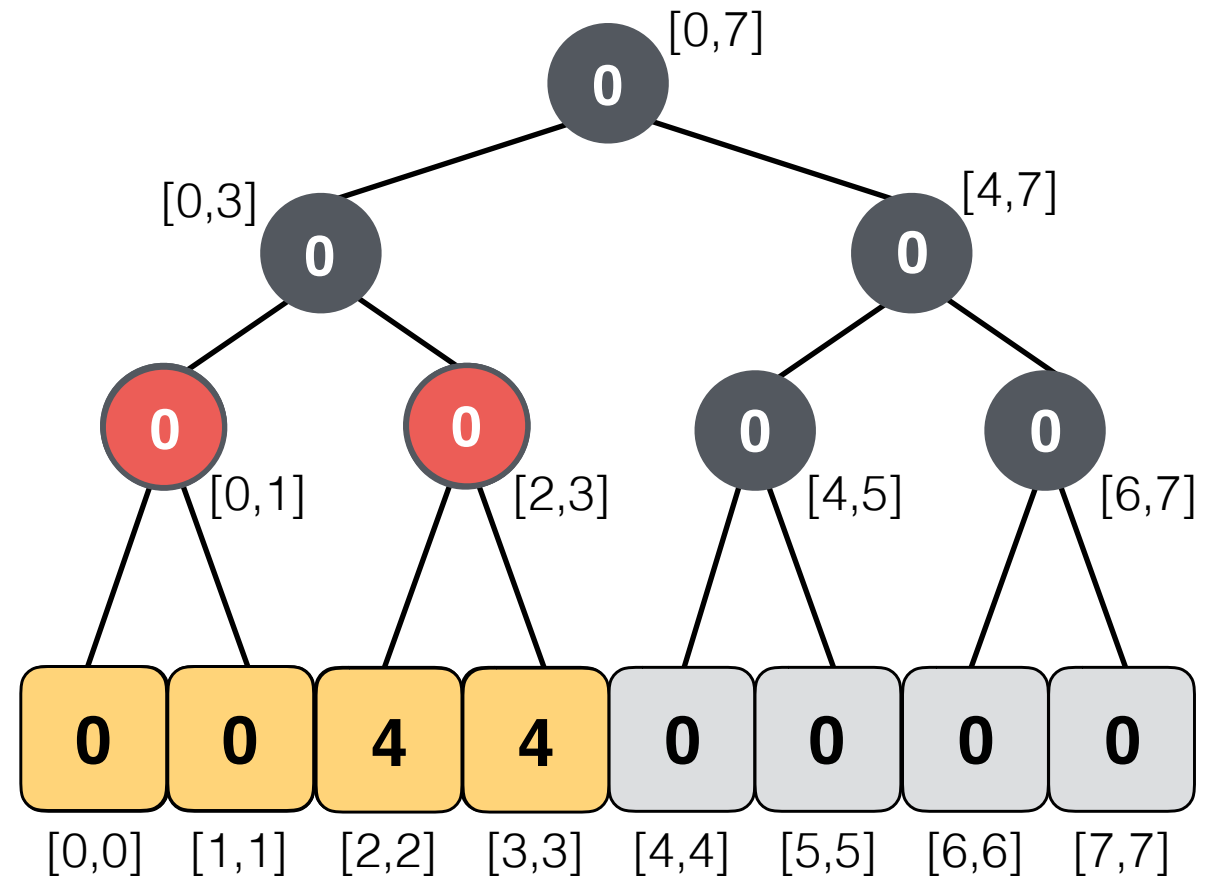
Lazy Propagation in Segment Trees

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update_range(0,3,3)
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→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

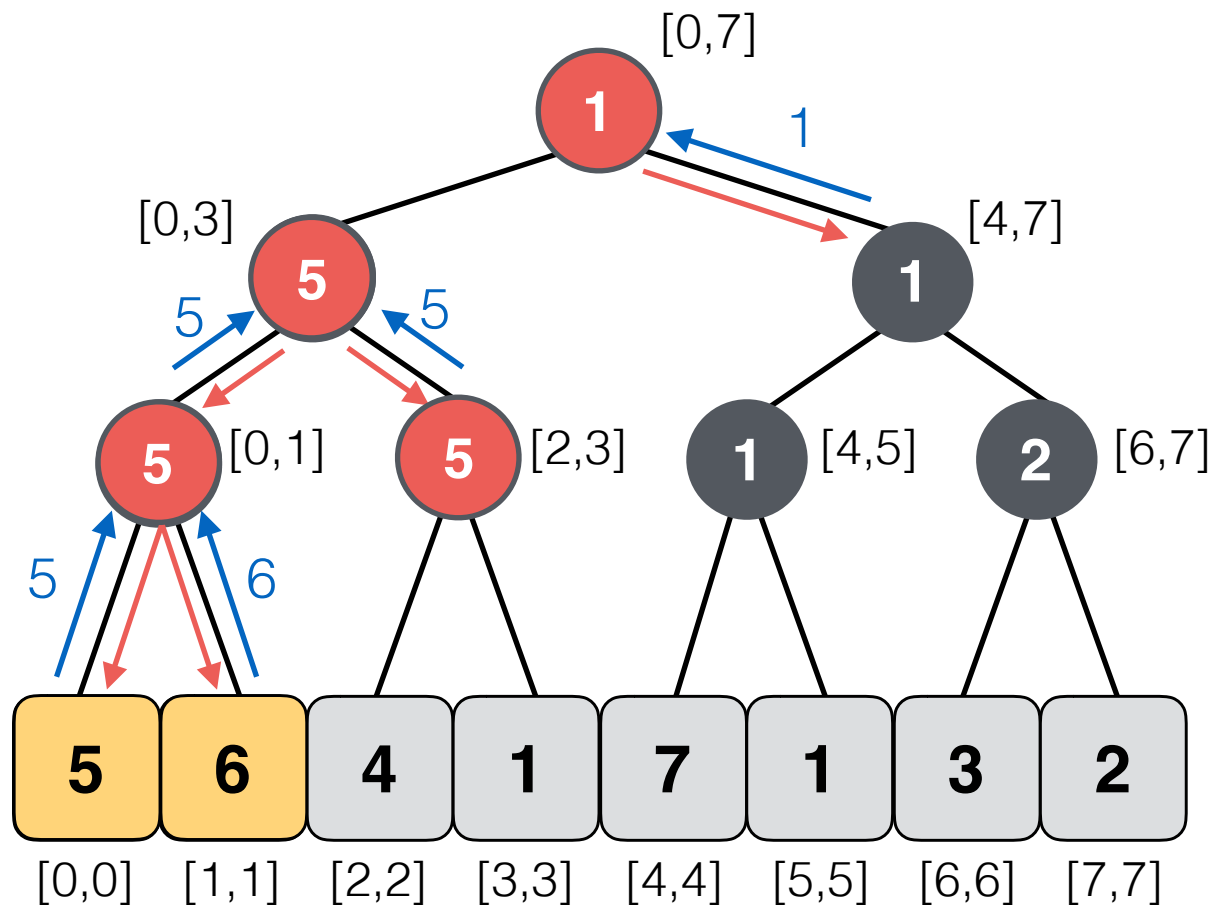


Lazy Tree

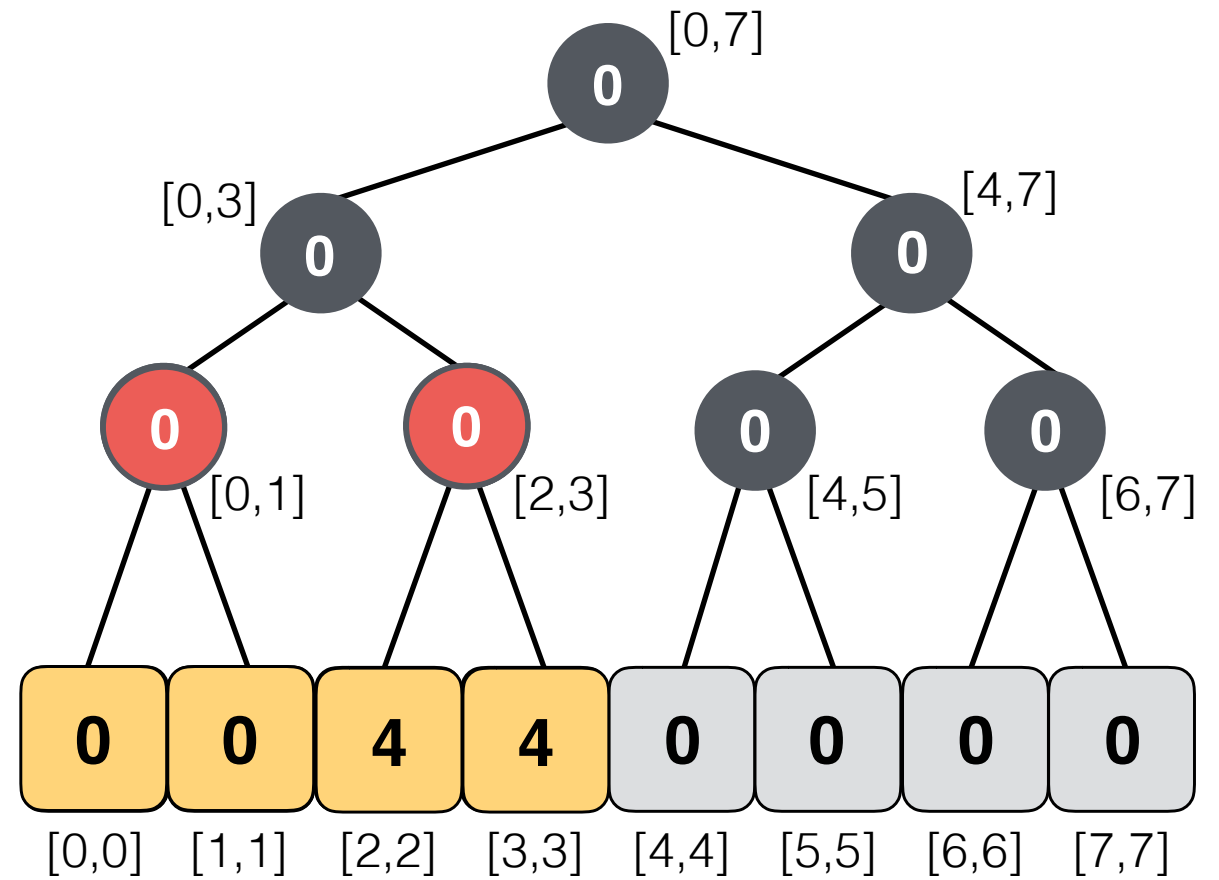
Lazy Propagation in Segment Trees

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update_range(0,3,3)
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→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

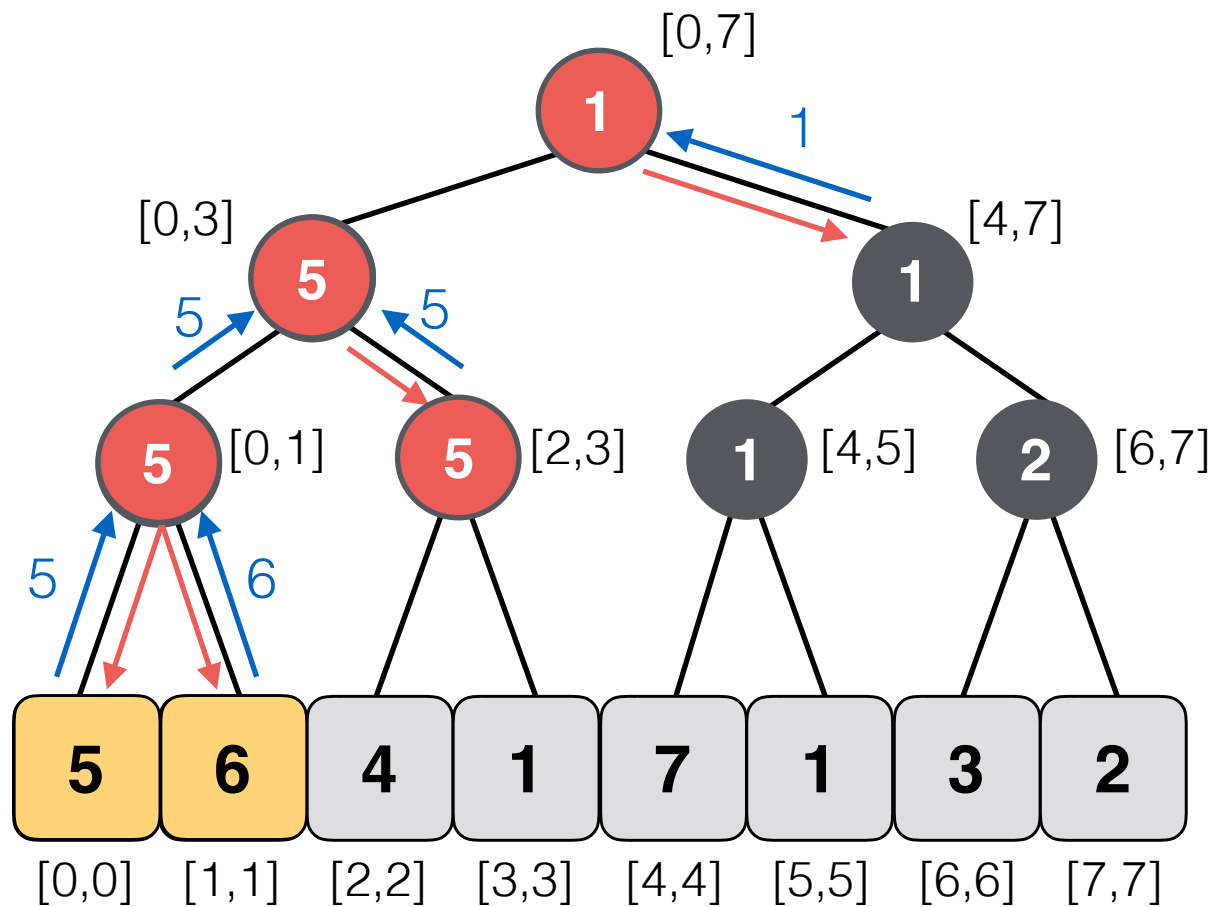


Lazy Tree

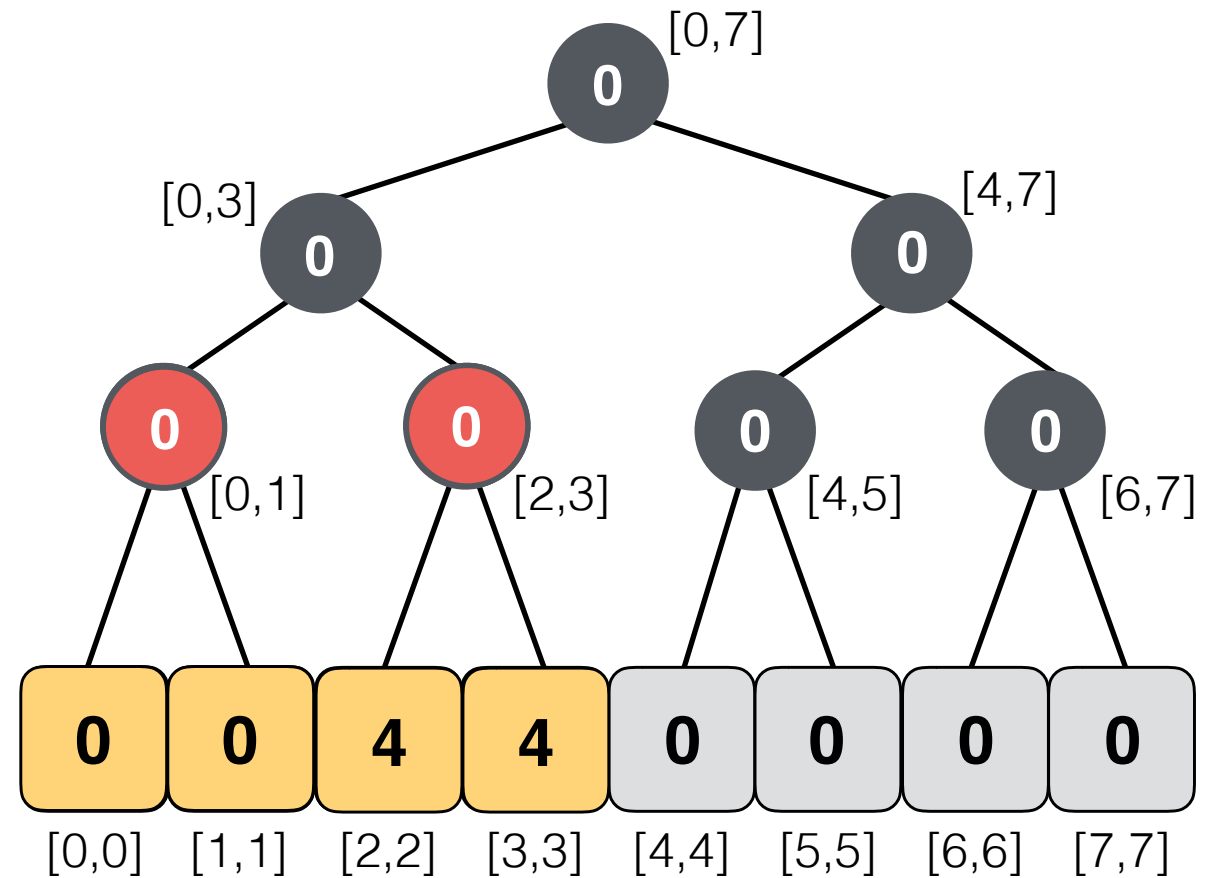
Lazy Propagation in Segment Trees

Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

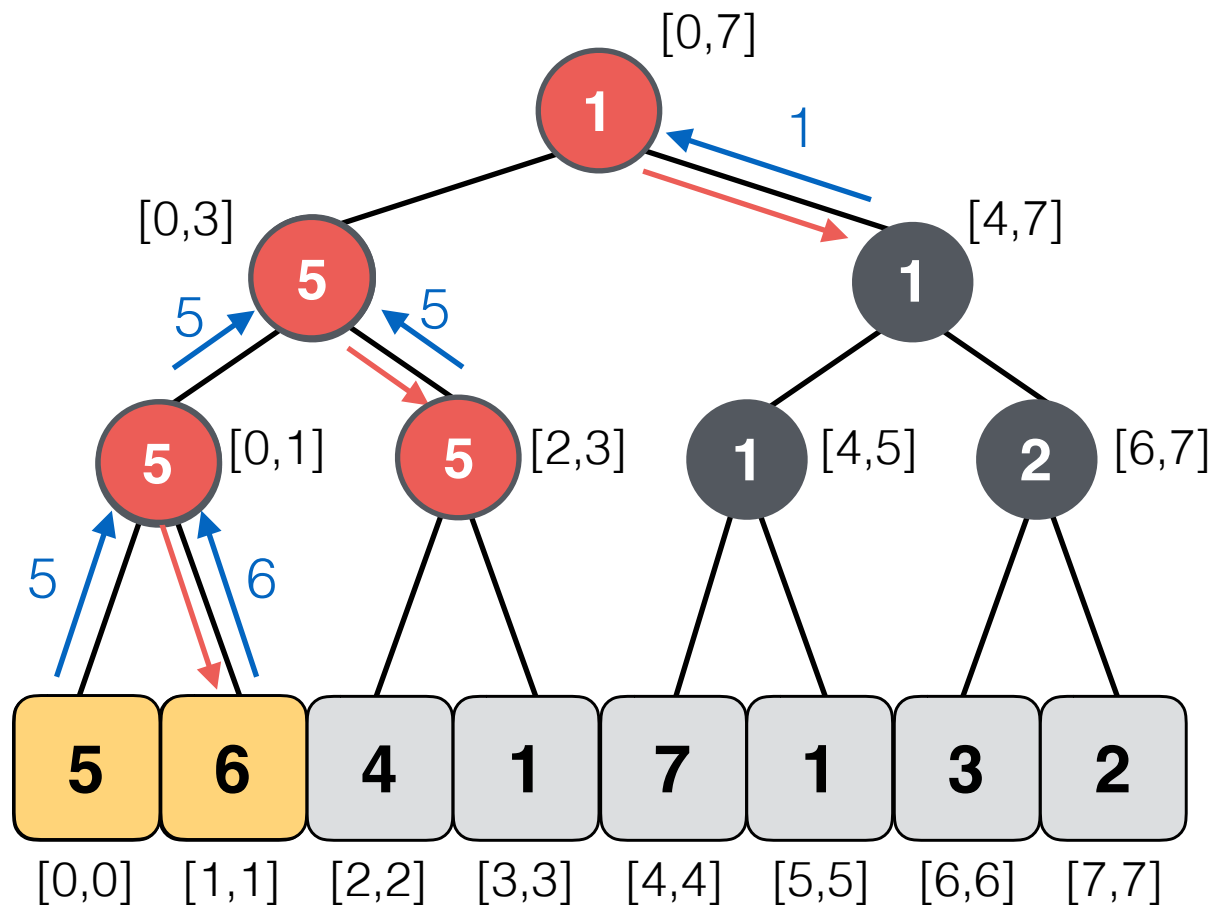


Lazy Tree

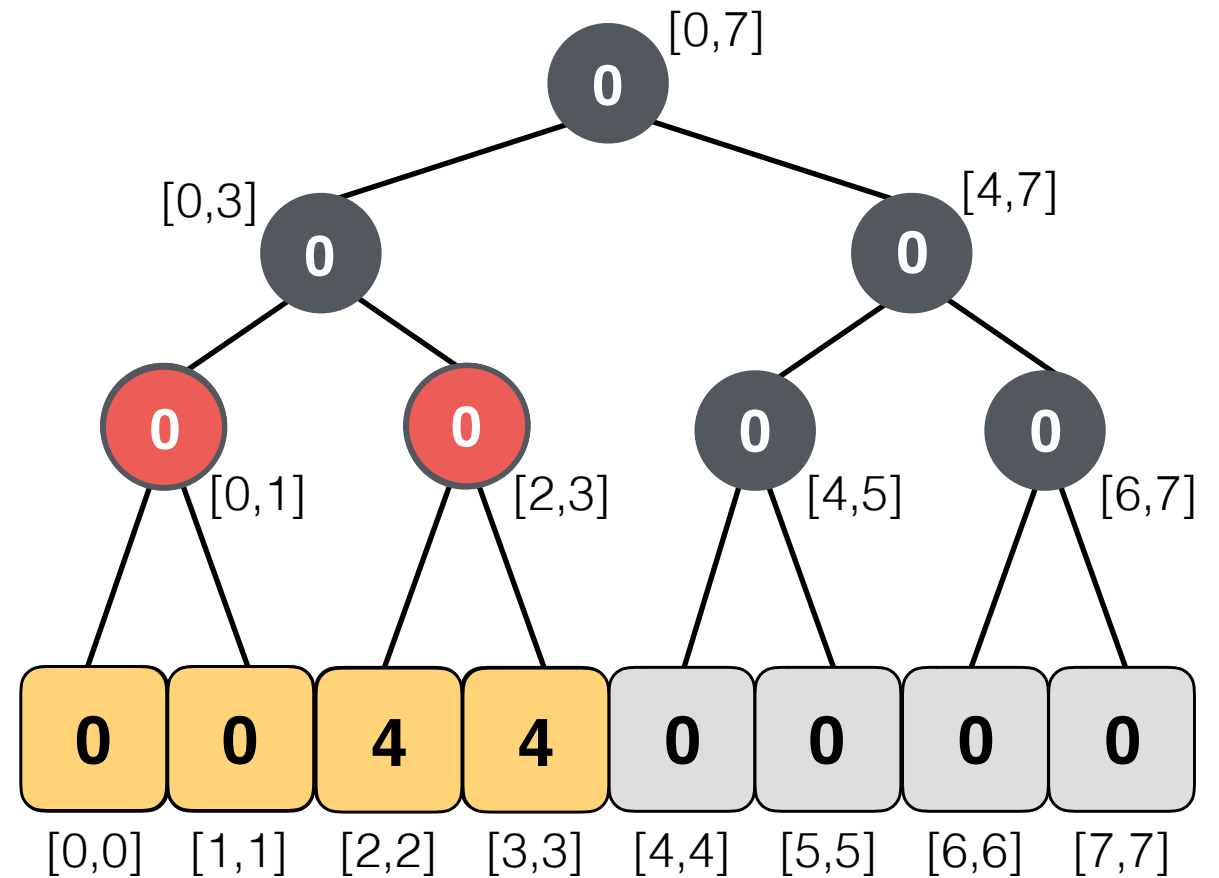
Lazy Propagation in Segment Trees

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Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

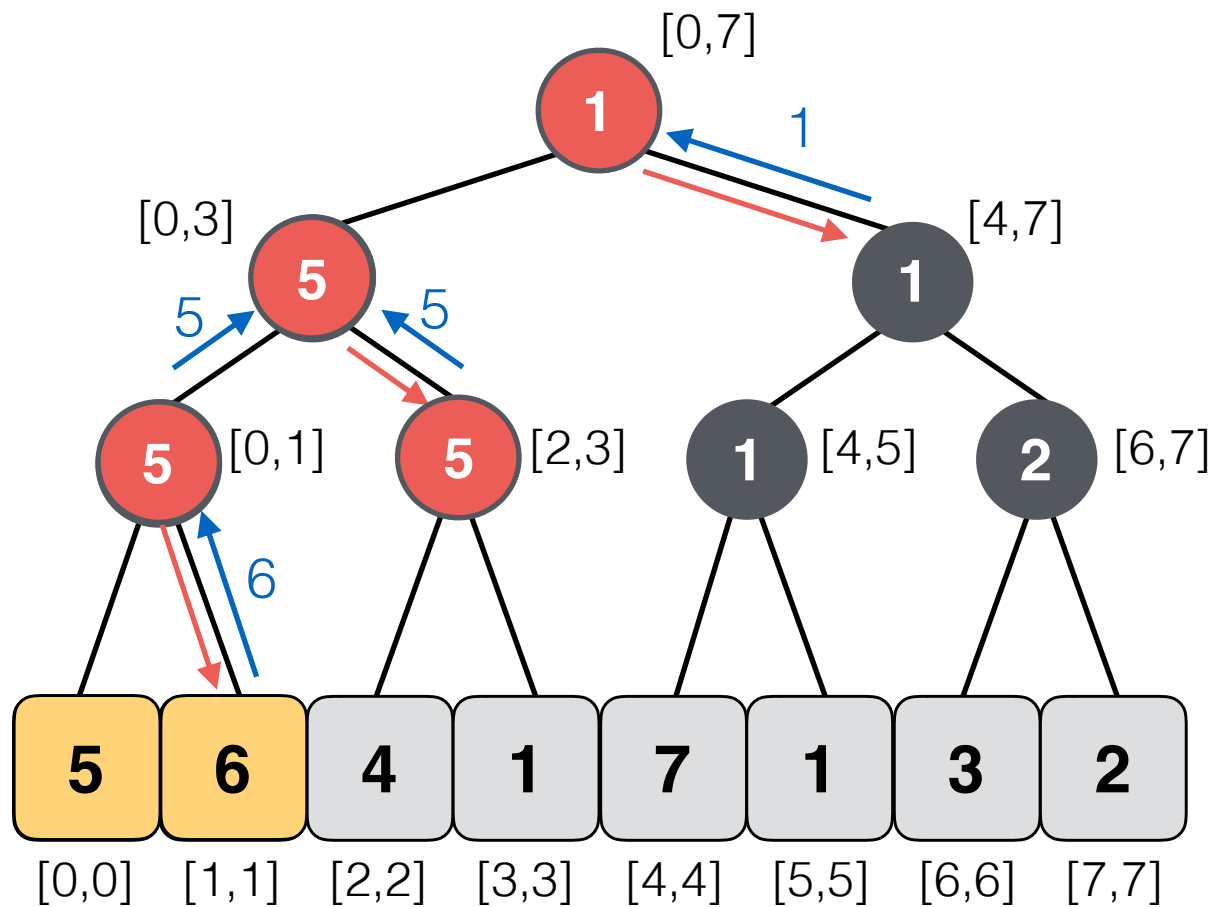


Lazy Tree

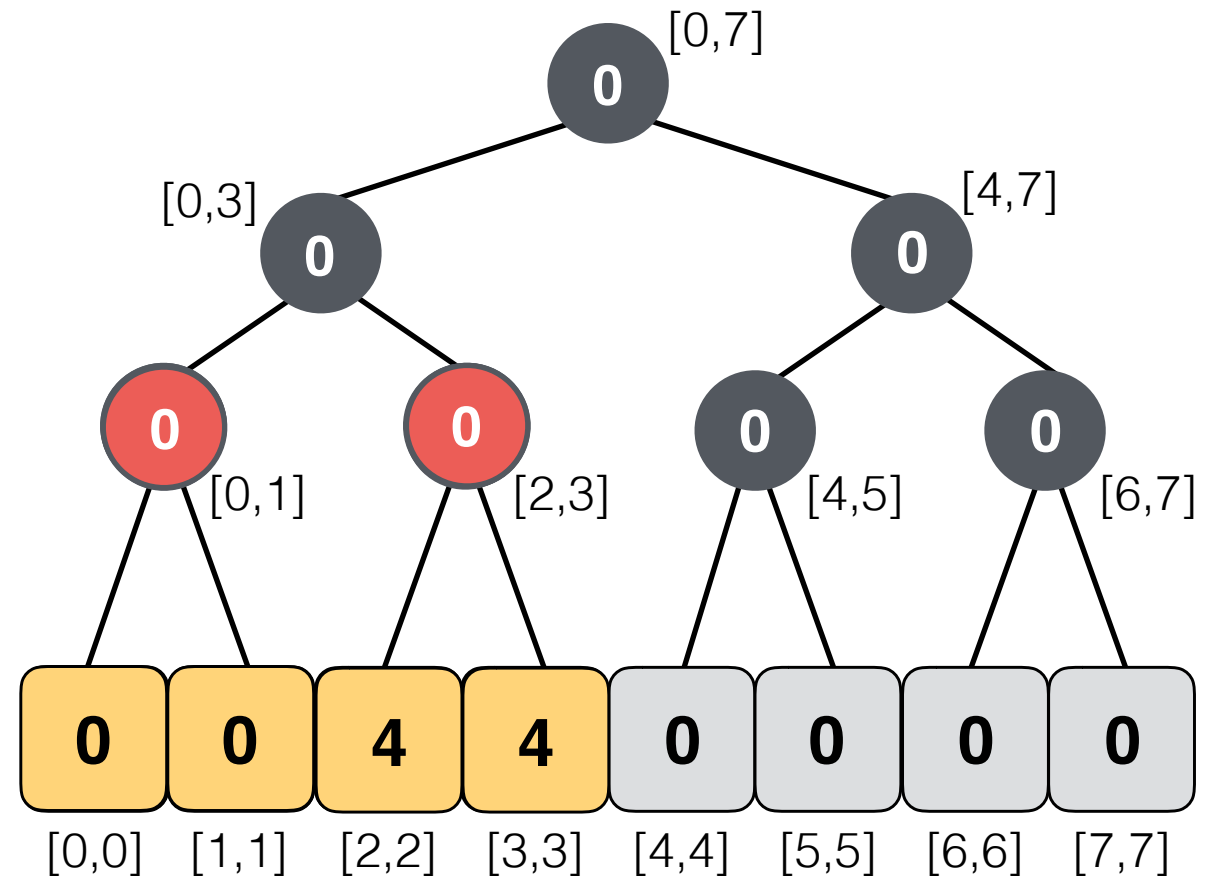
Lazy Propagation in Segment Trees

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update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

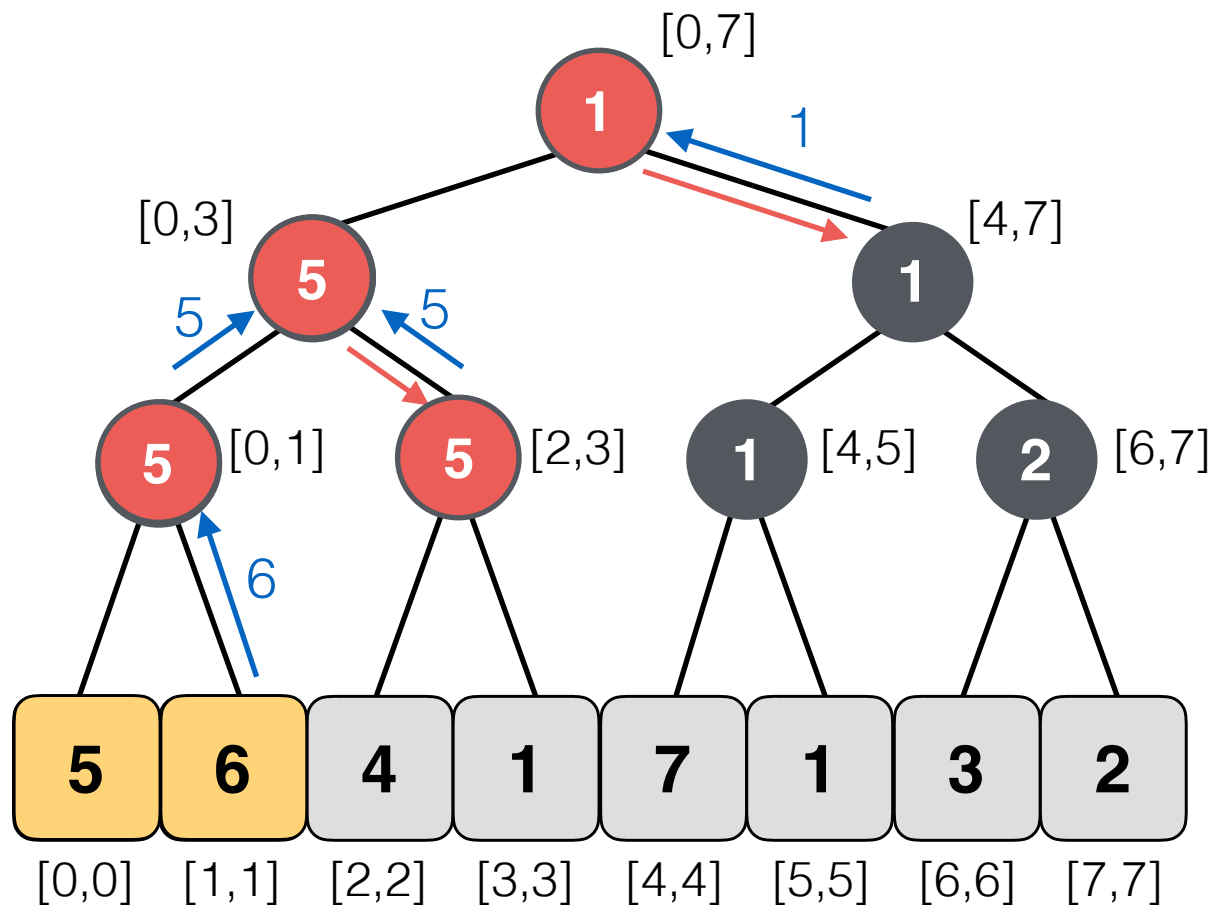


Lazy Tree

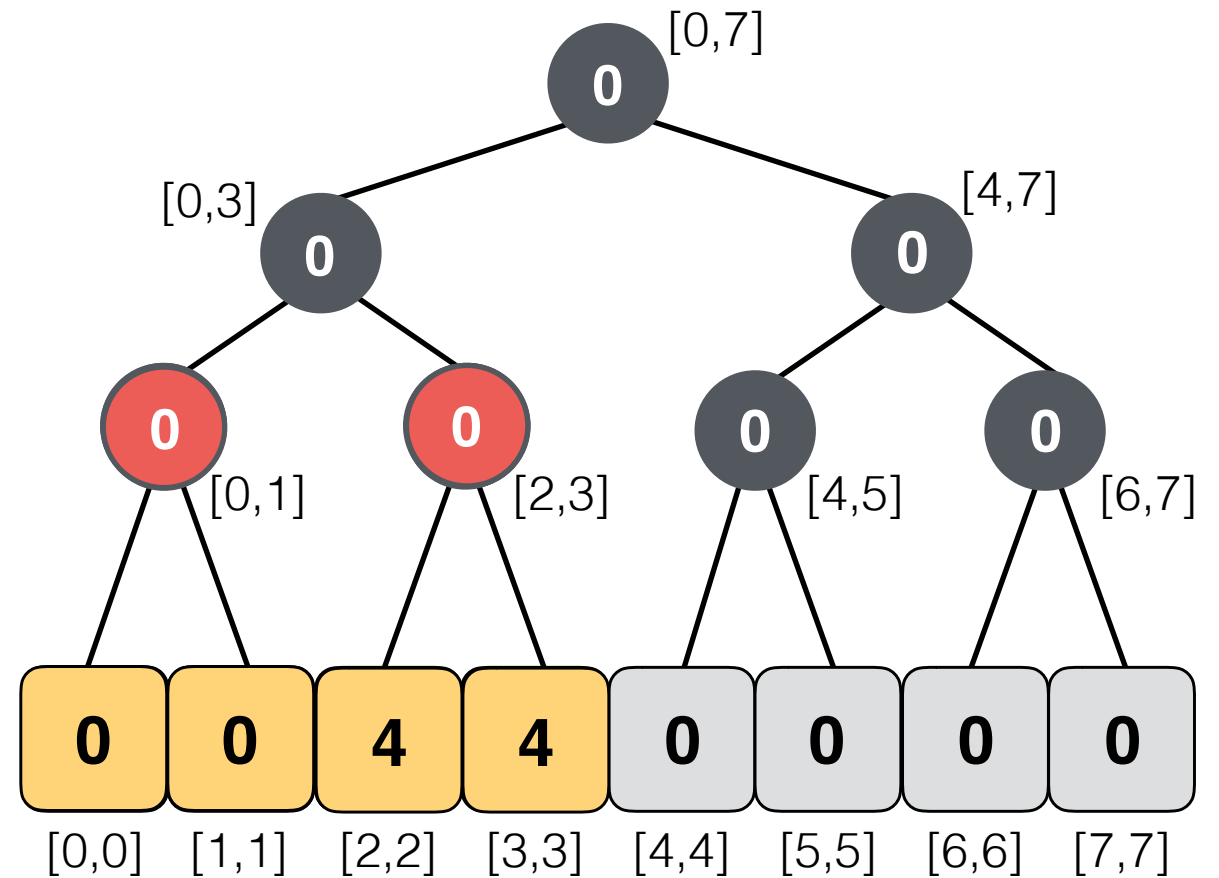
Lazy Propagation in Segment Trees

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update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

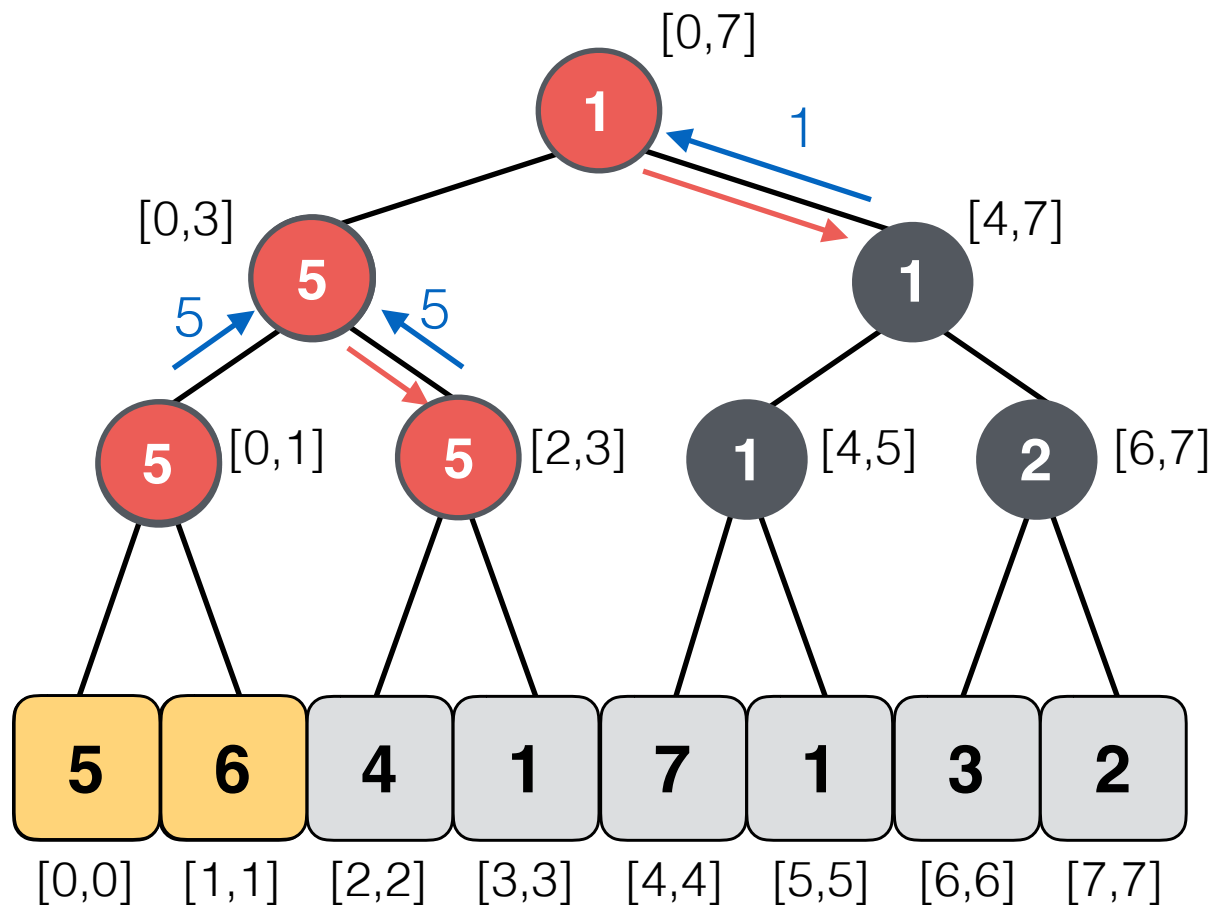


Lazy Tree

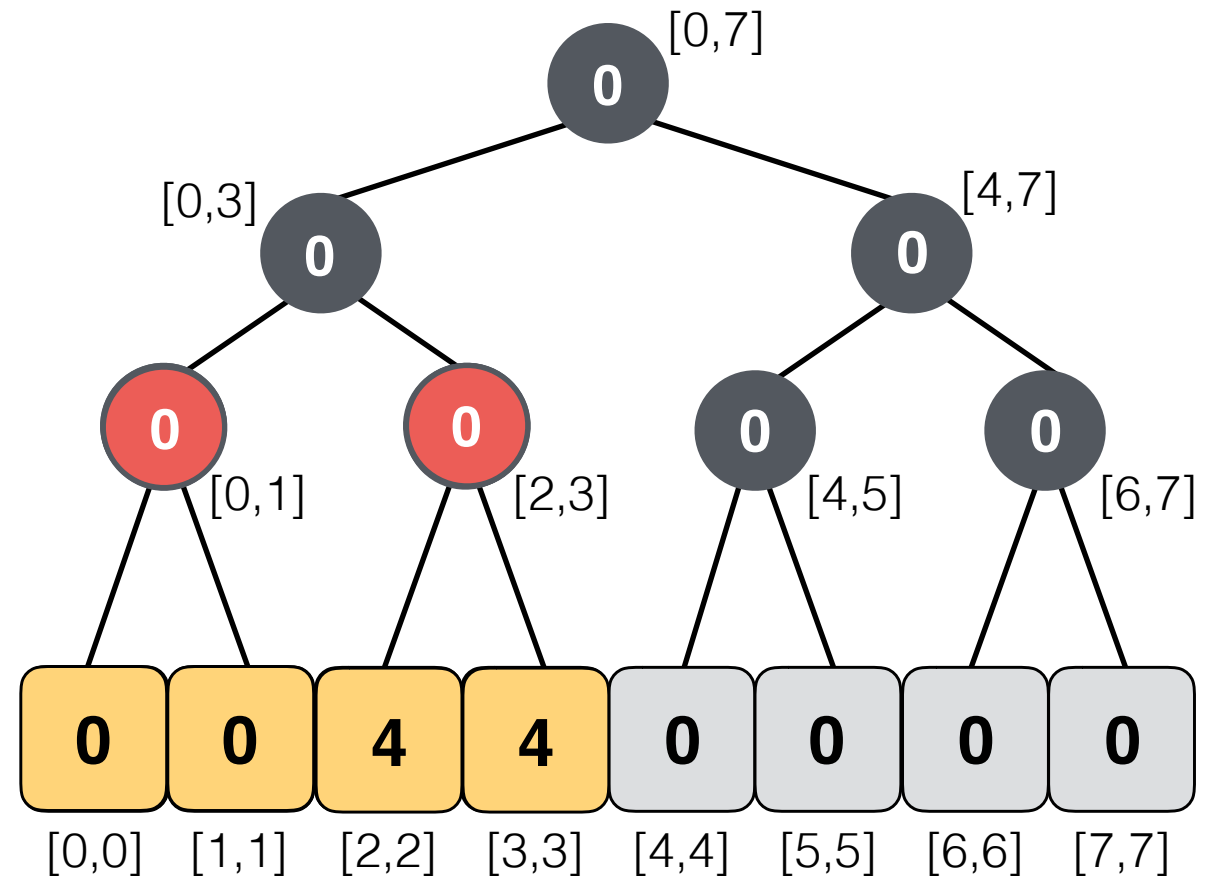
Lazy Propagation in Segment Trees

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Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

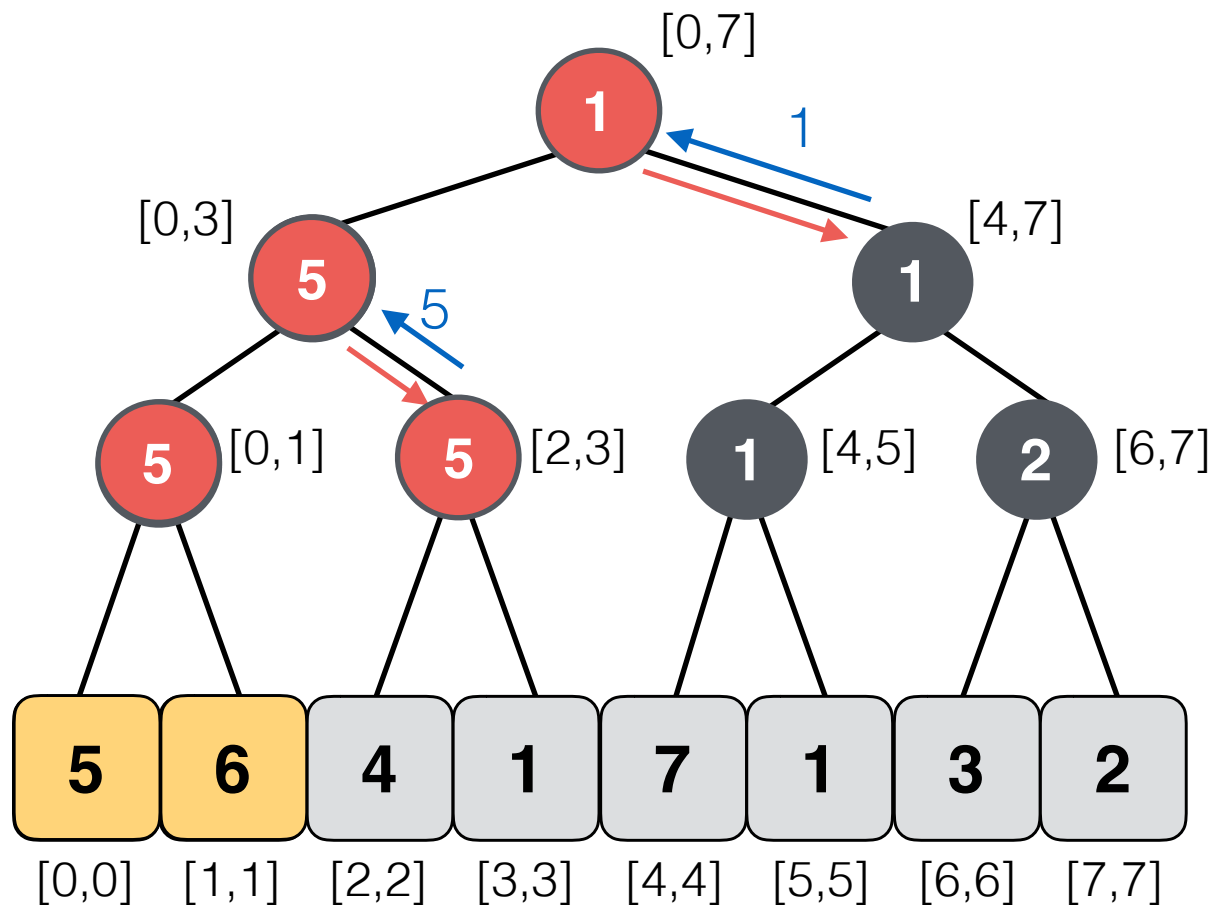


Lazy Tree

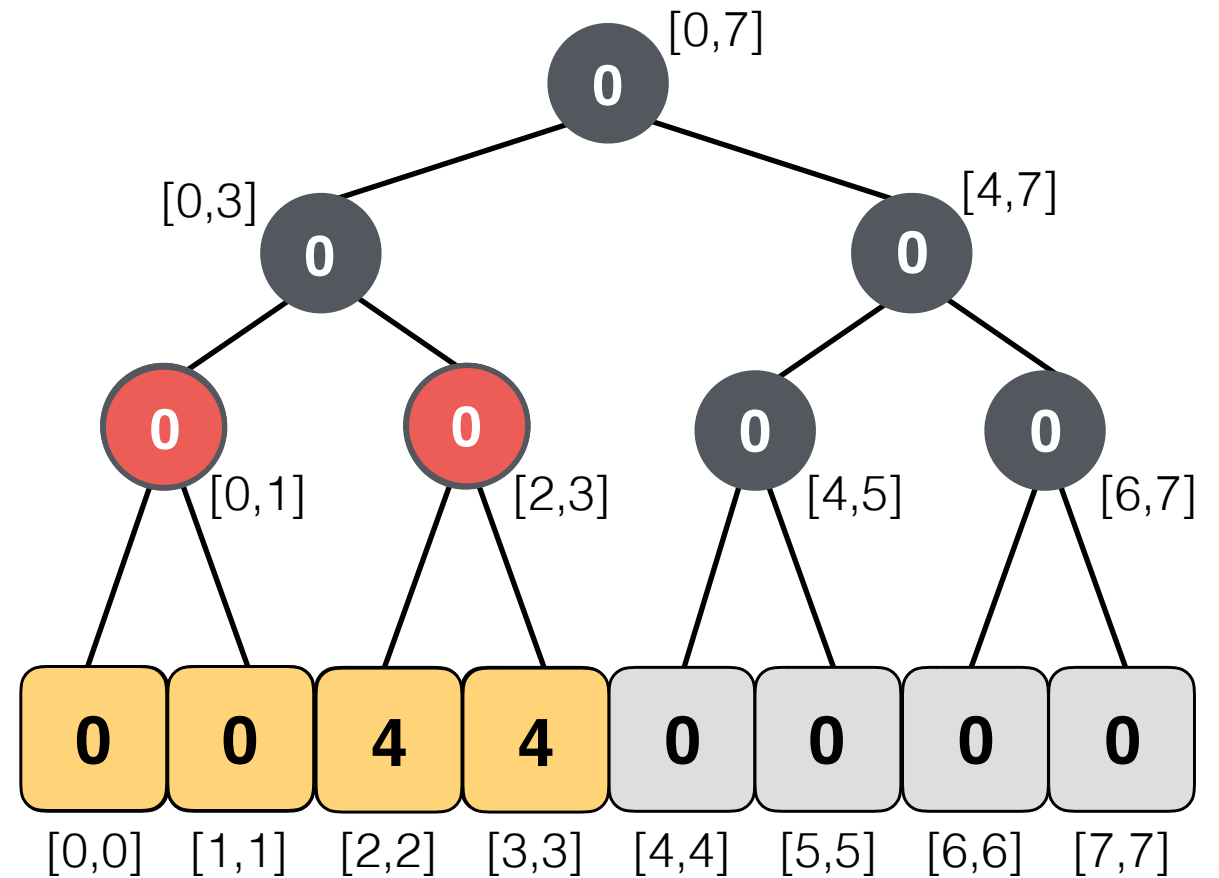
Lazy Propagation in Segment Trees

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Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

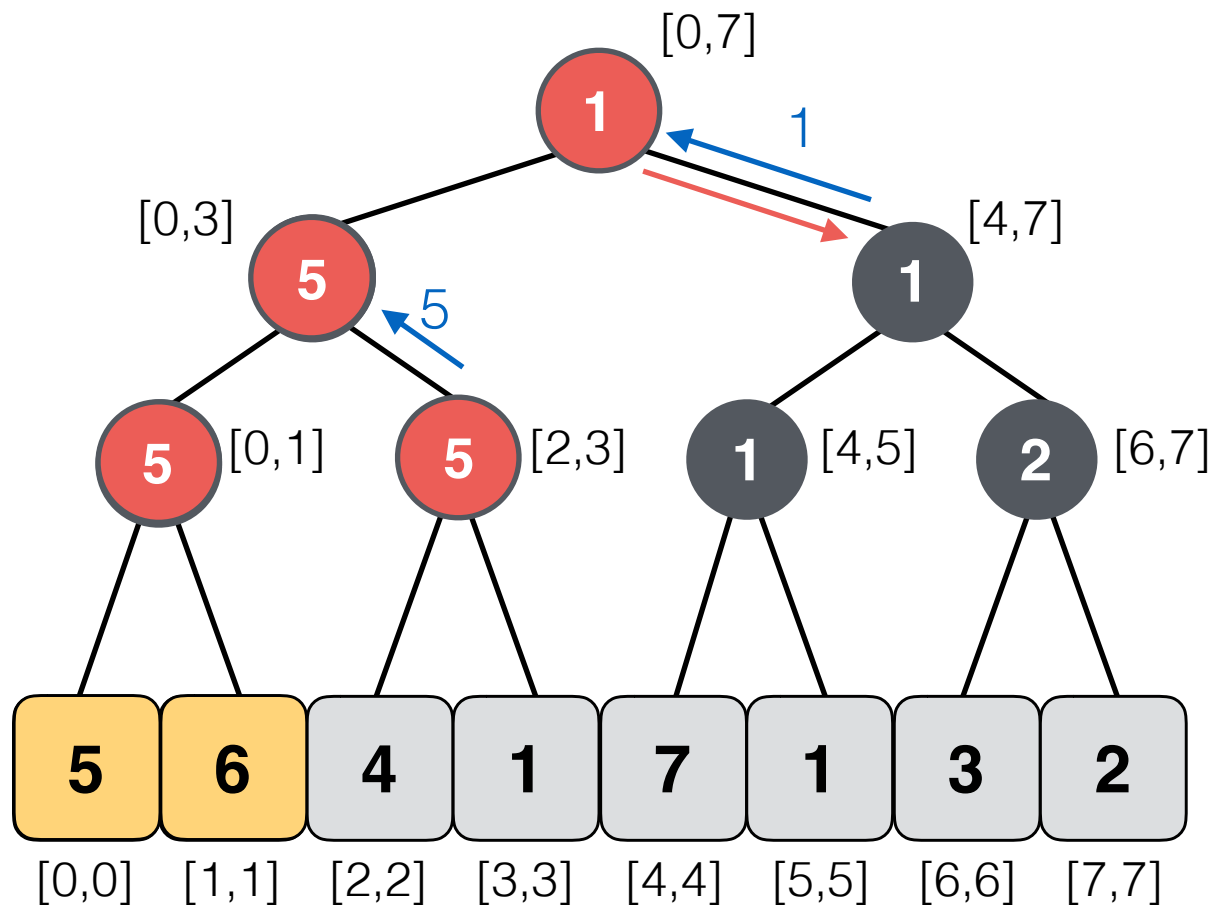


Lazy Tree

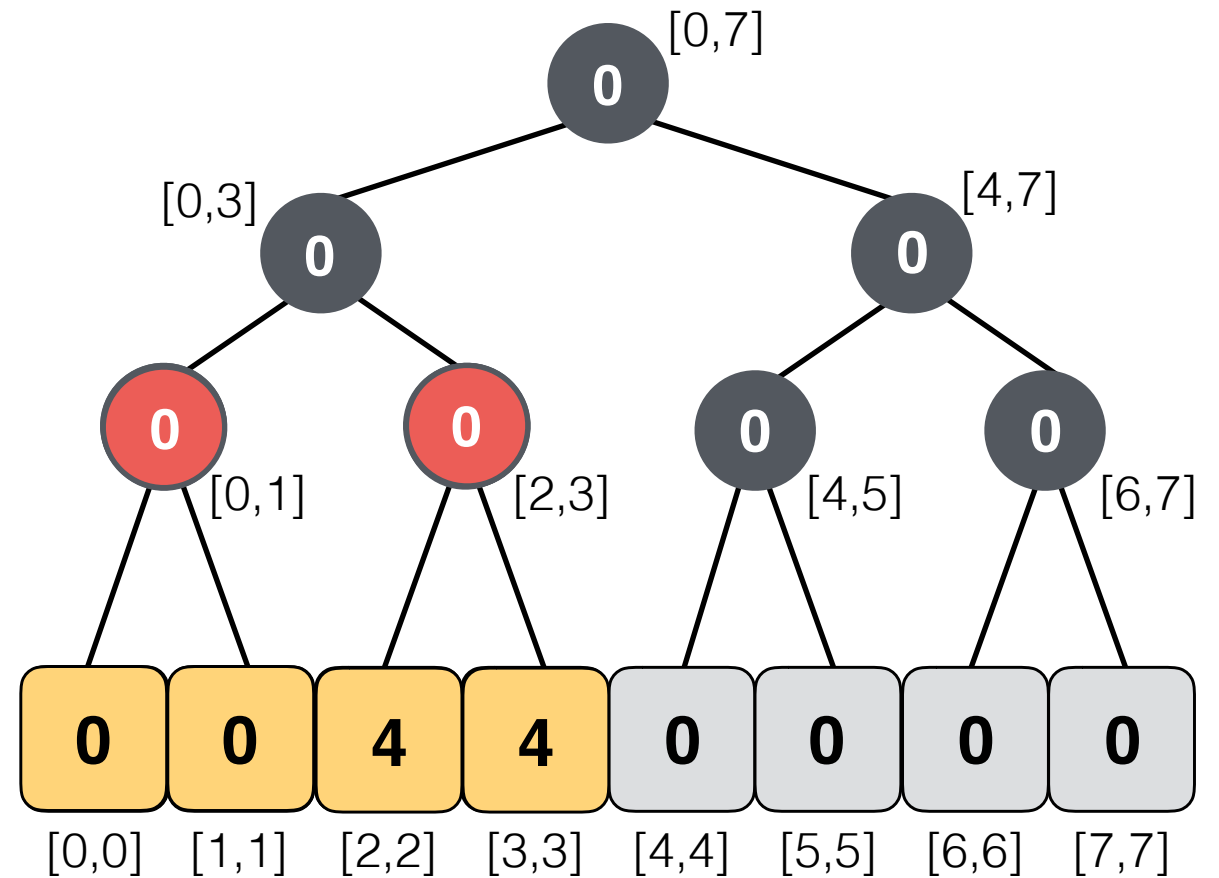
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update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

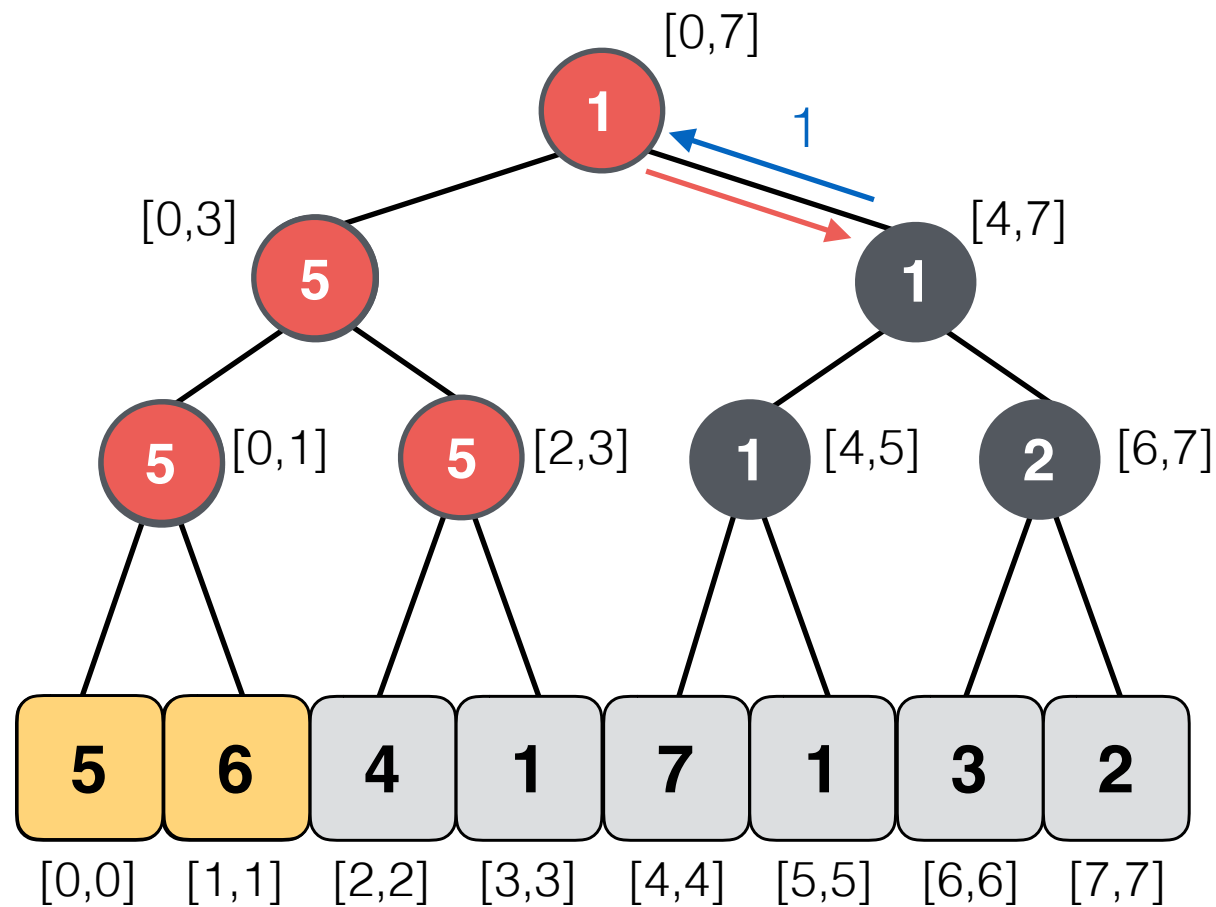


Lazy Tree

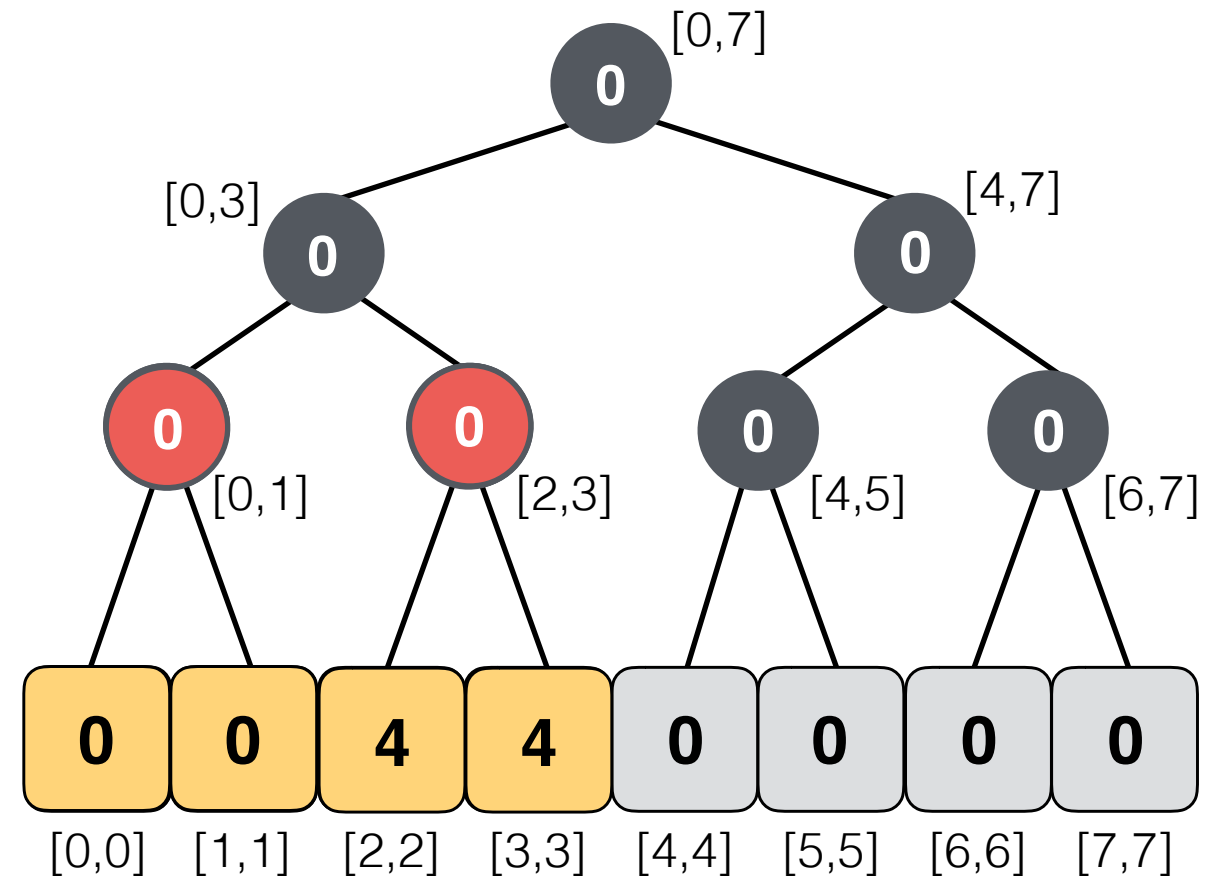
Lazy Propagation in Segment Trees

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Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

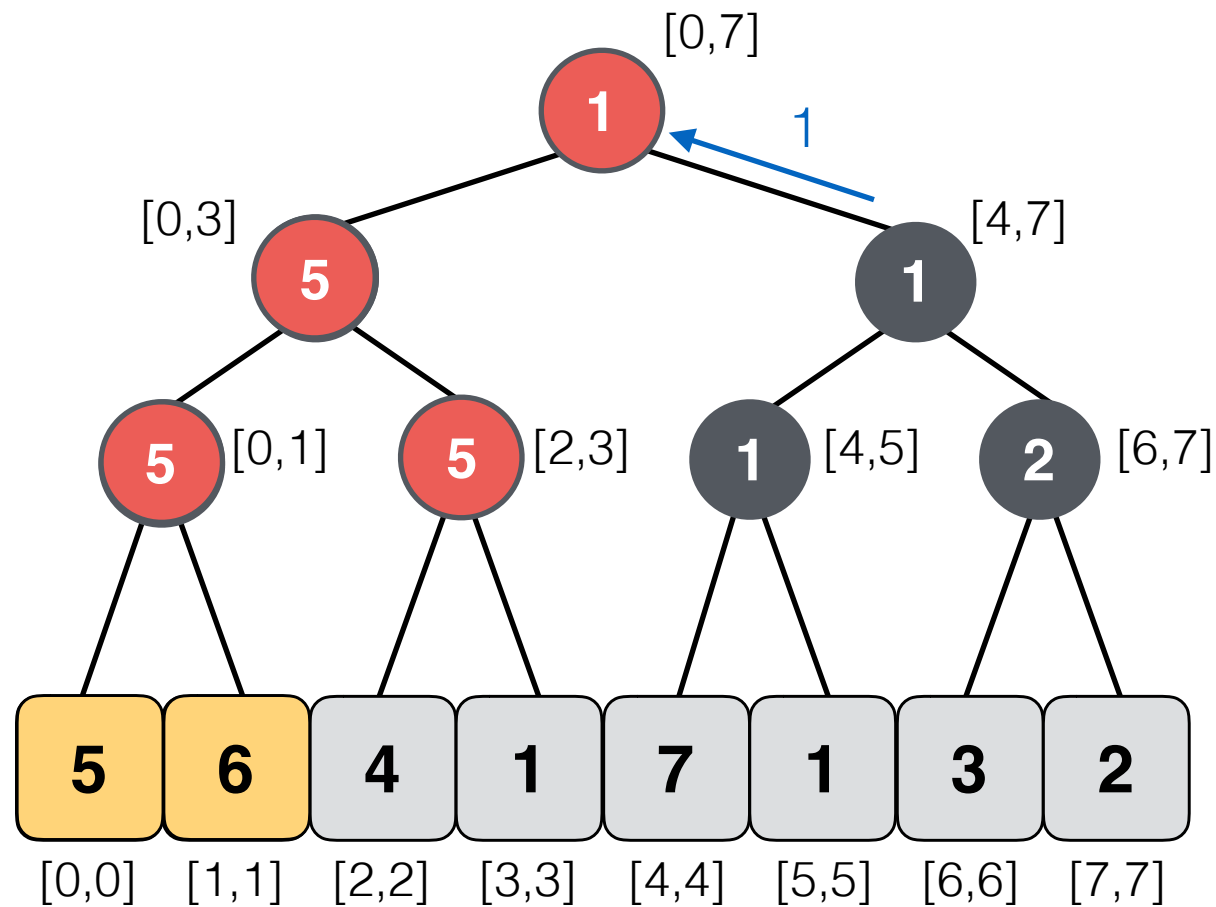


Lazy Tree

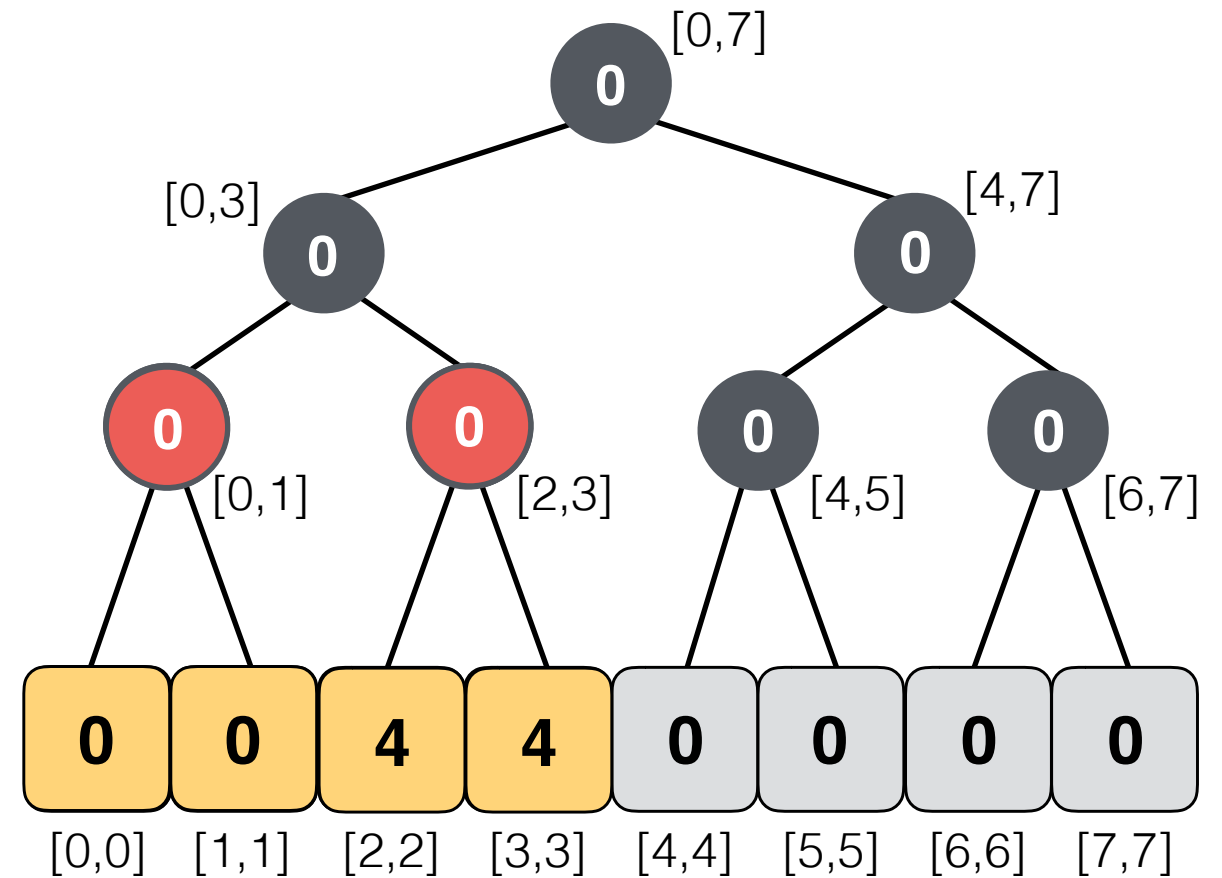
Lazy Propagation in Segment Trees

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Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree

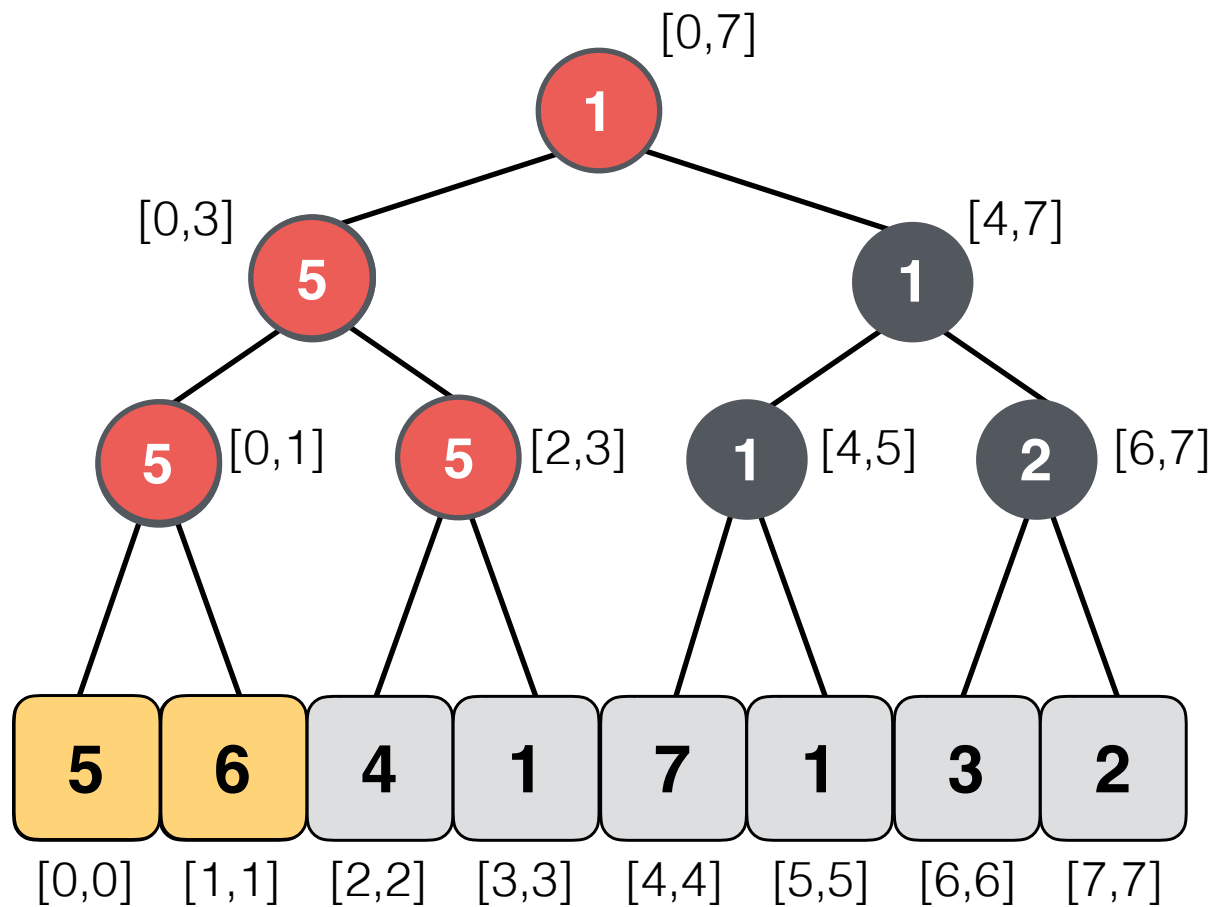


Lazy Tree

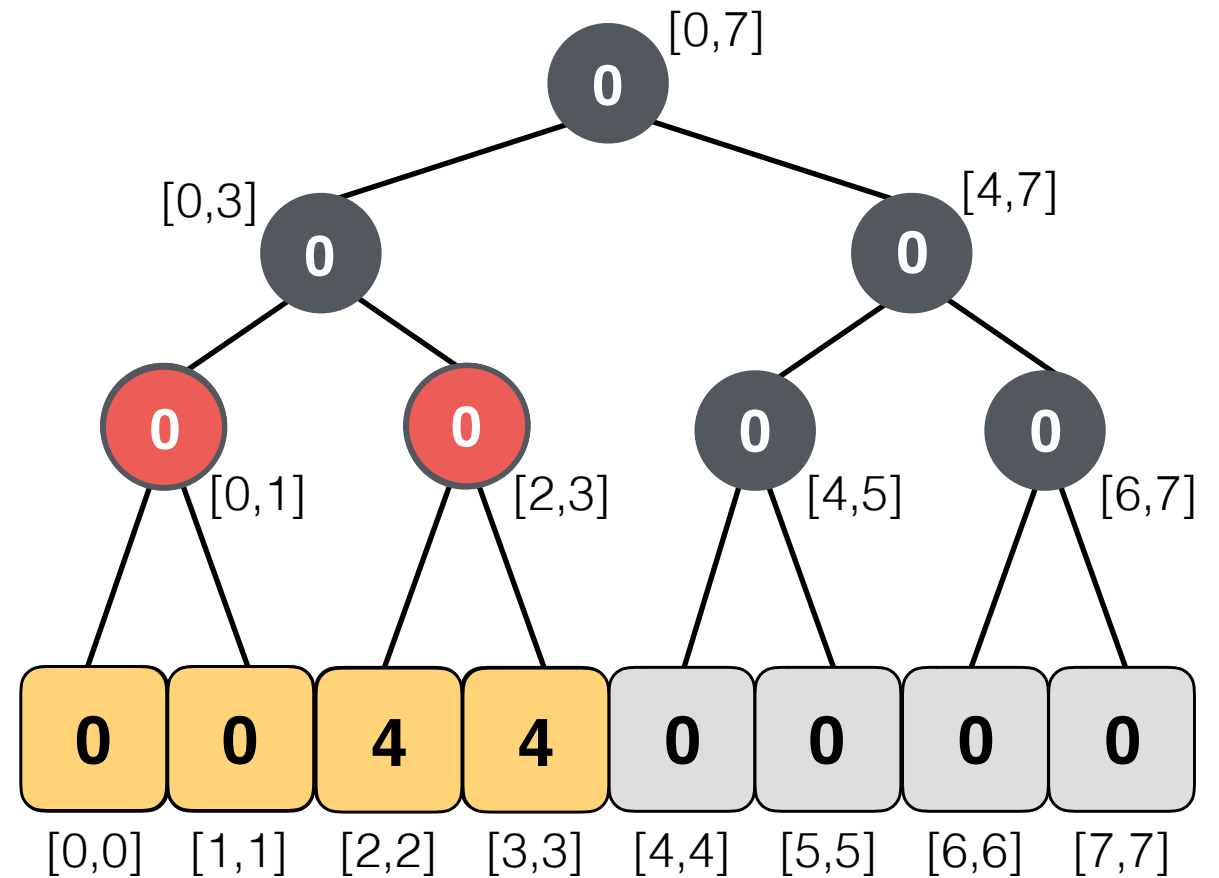
Lazy Propagation in Segment Trees

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Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
→ update_range(0,0,2)
rmq(3,5) = ?



Segment Tree



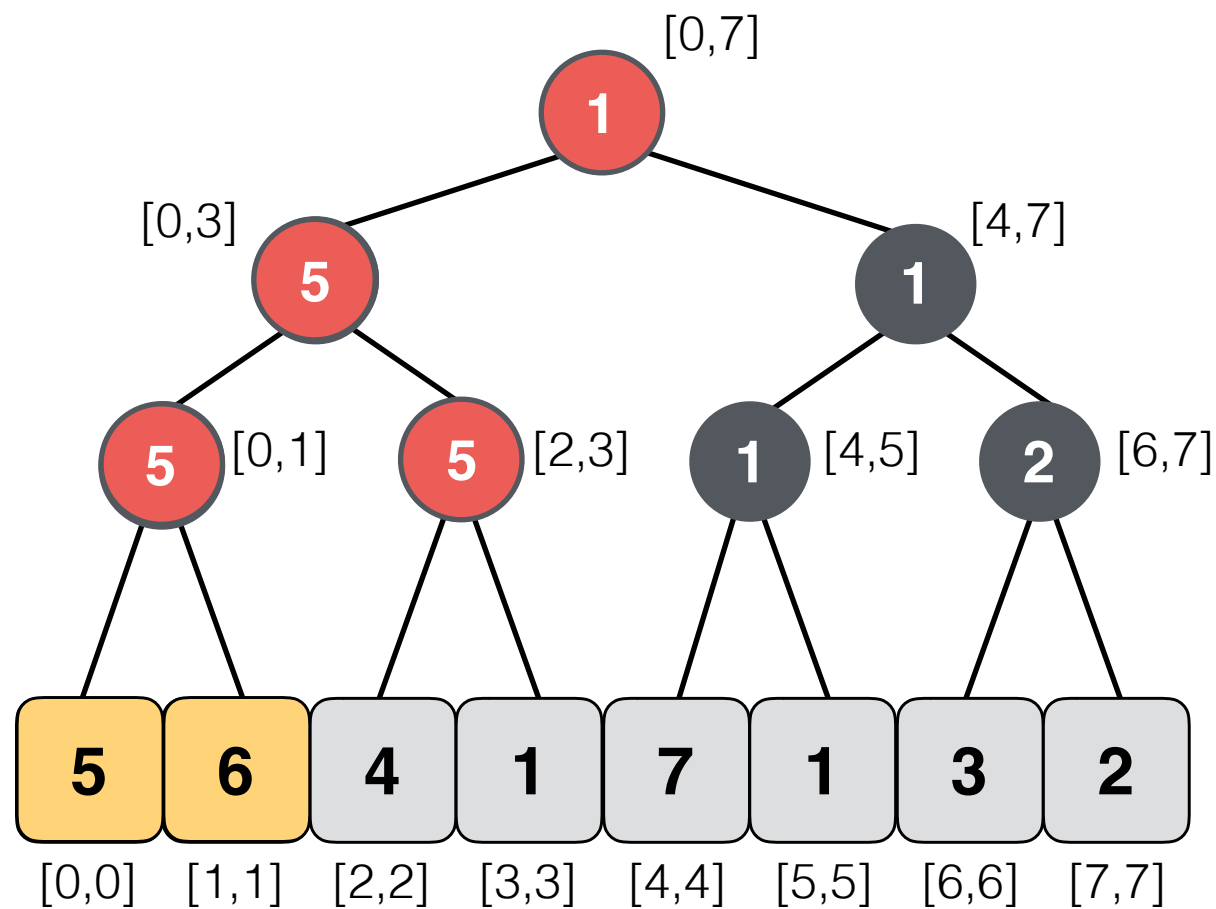
Lazy Tree

Lazy Propagation in Segment Trees

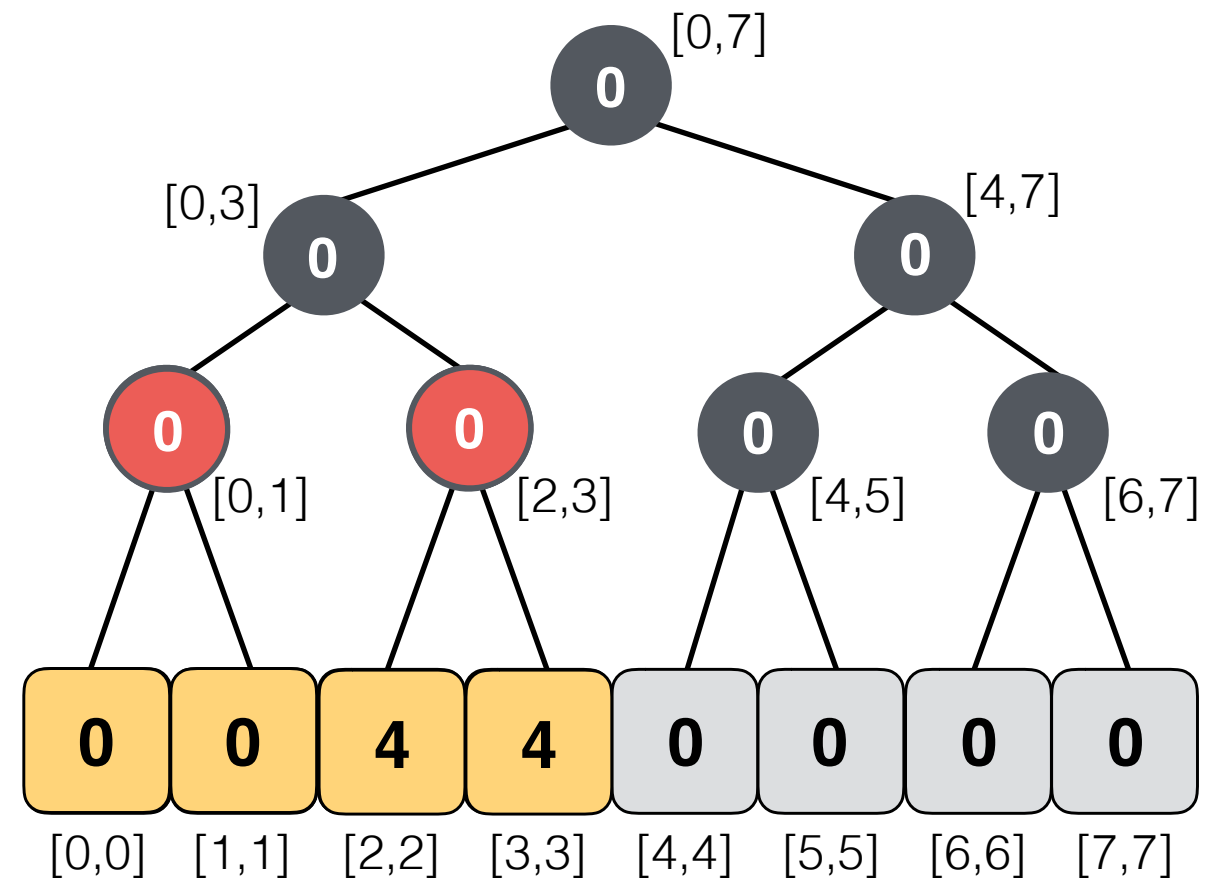
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



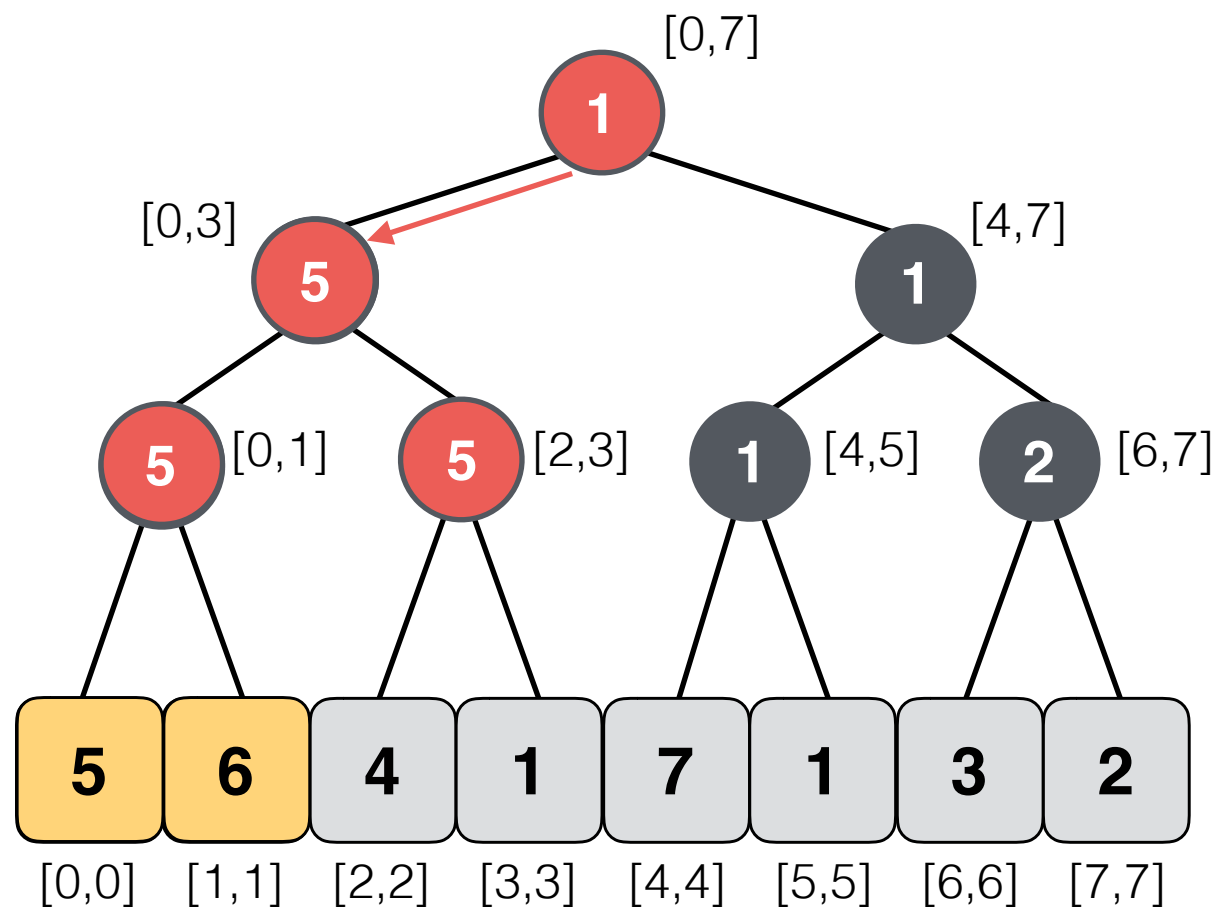
Lazy Tree

Lazy Propagation in Segment Trees

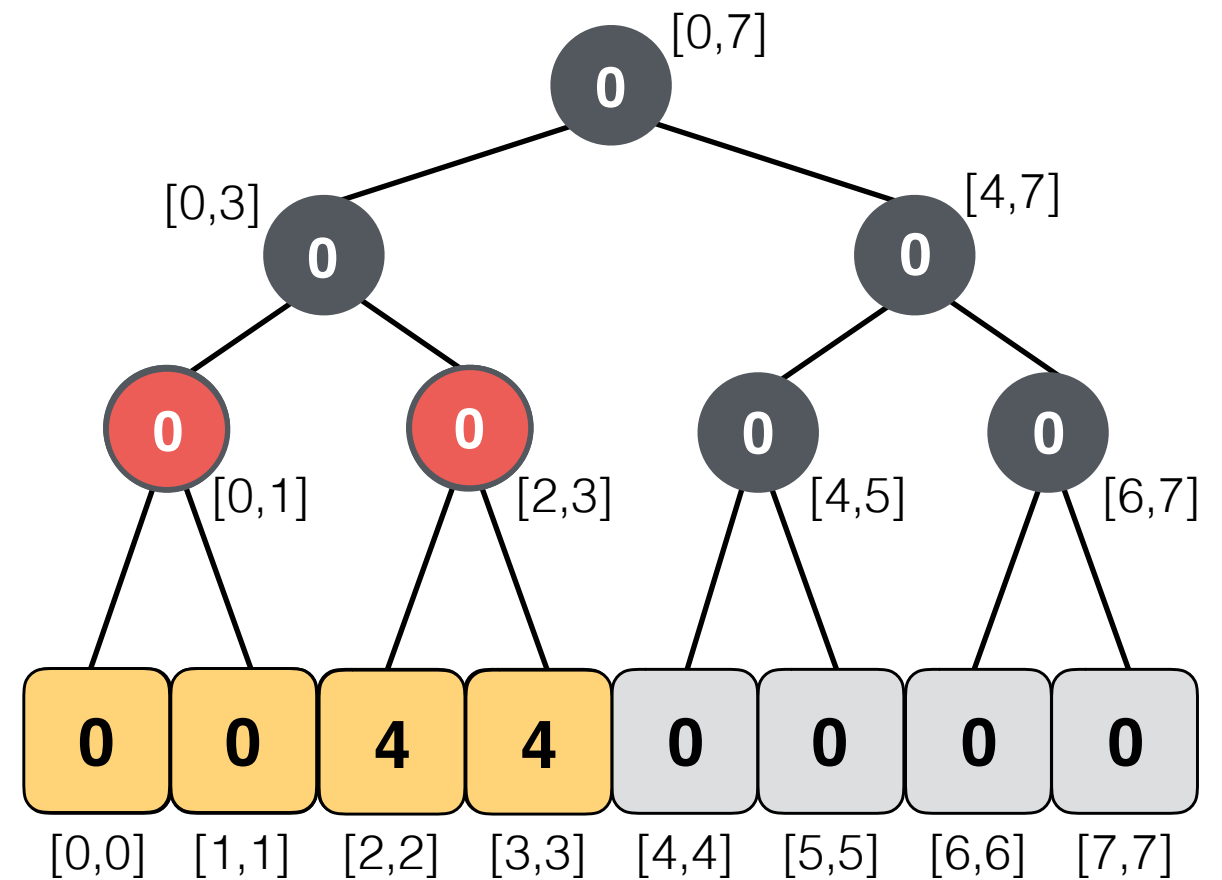
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



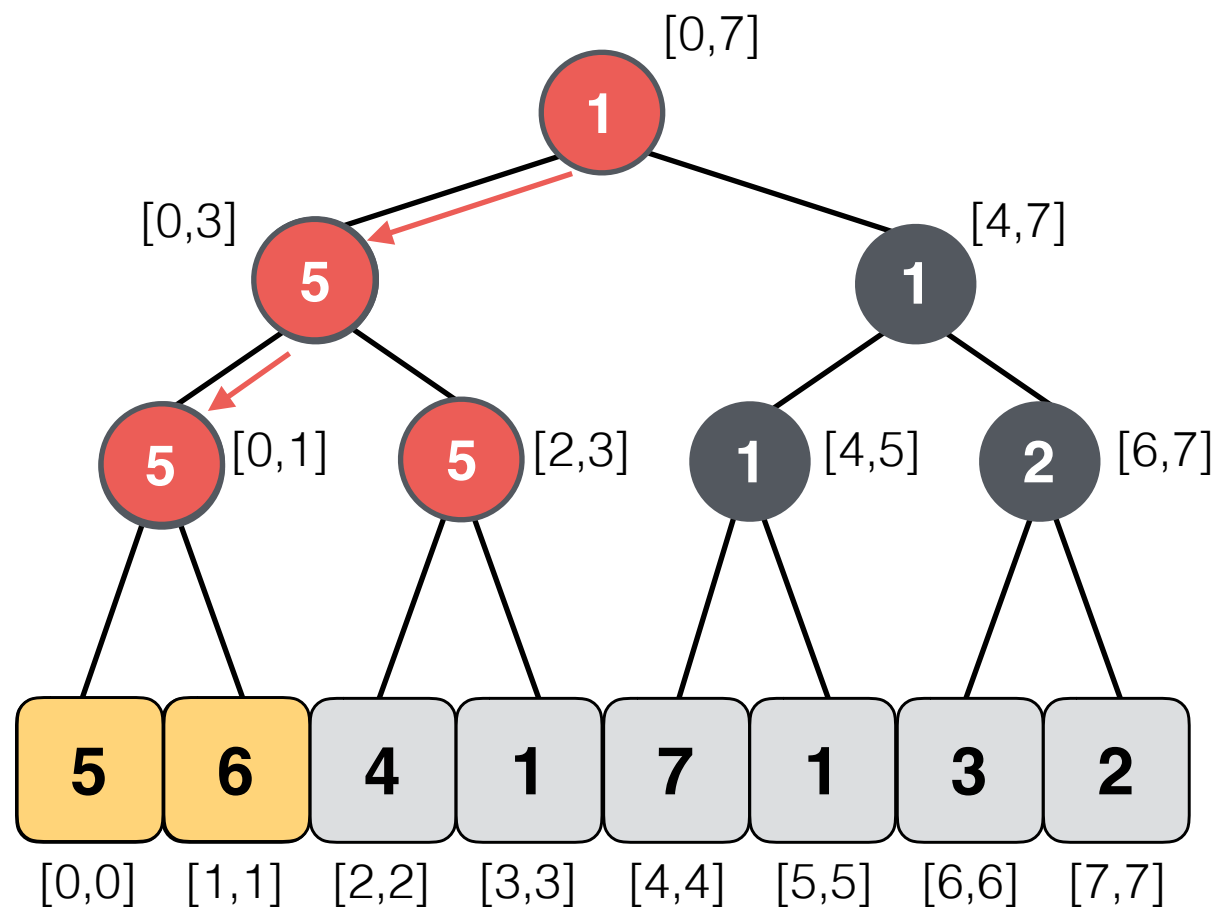
Lazy Tree

Lazy Propagation in Segment Trees

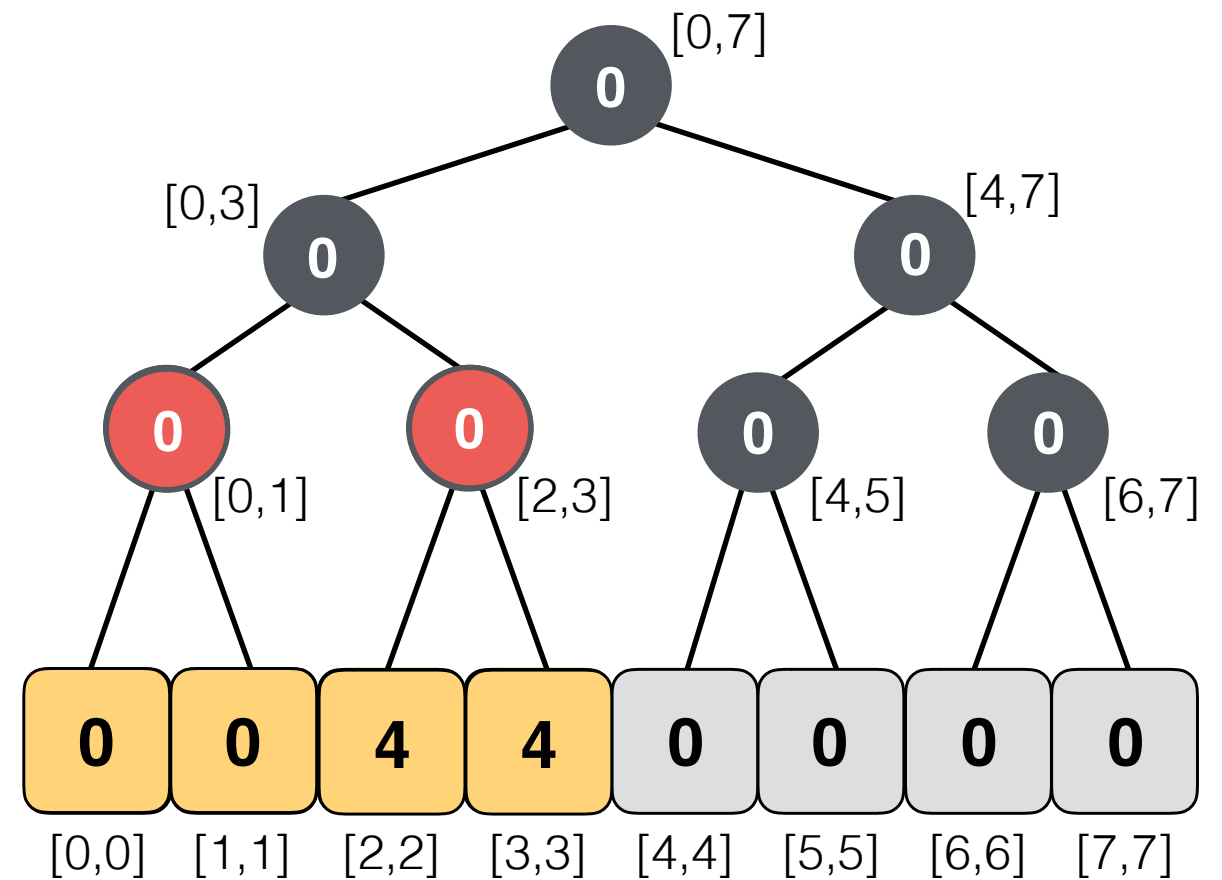
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



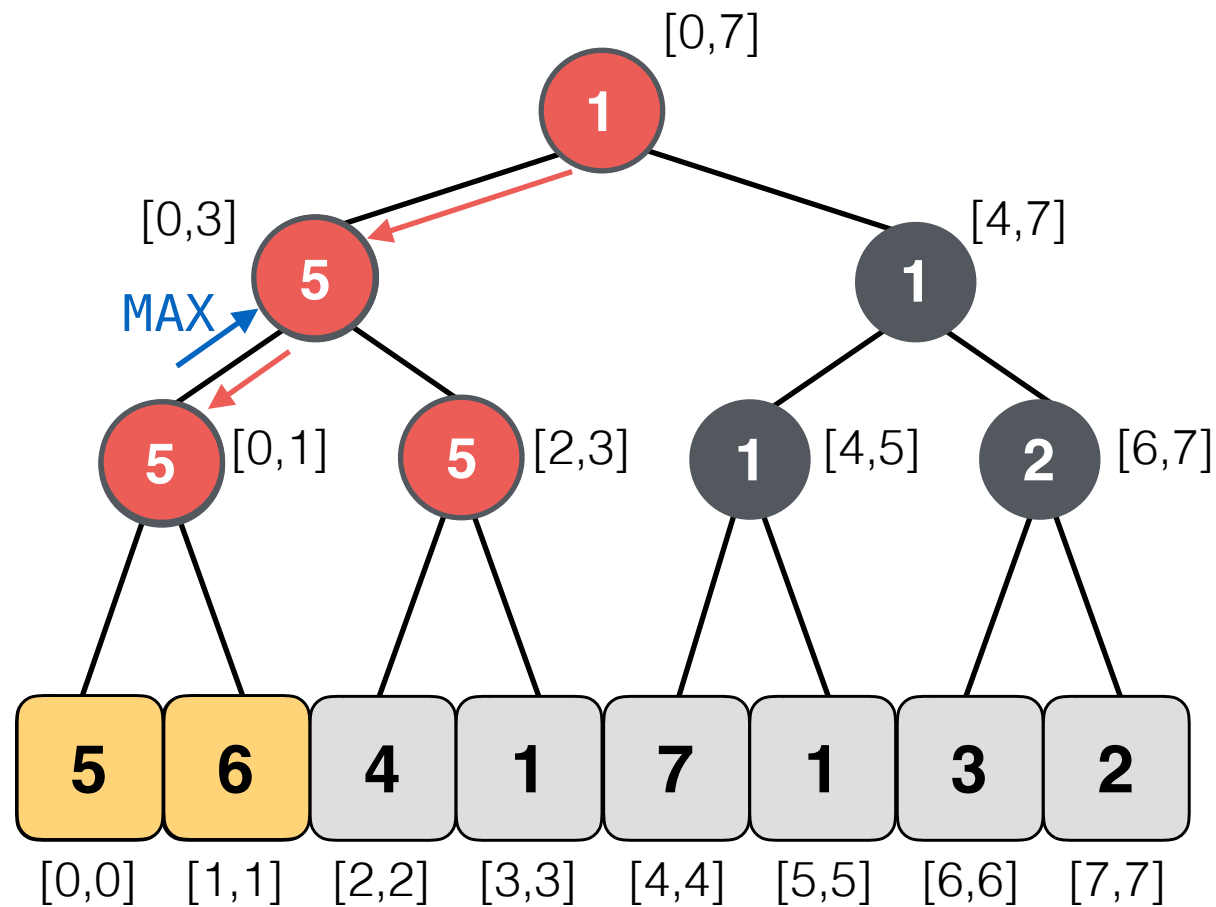
Lazy Tree

Lazy Propagation in Segment Trees

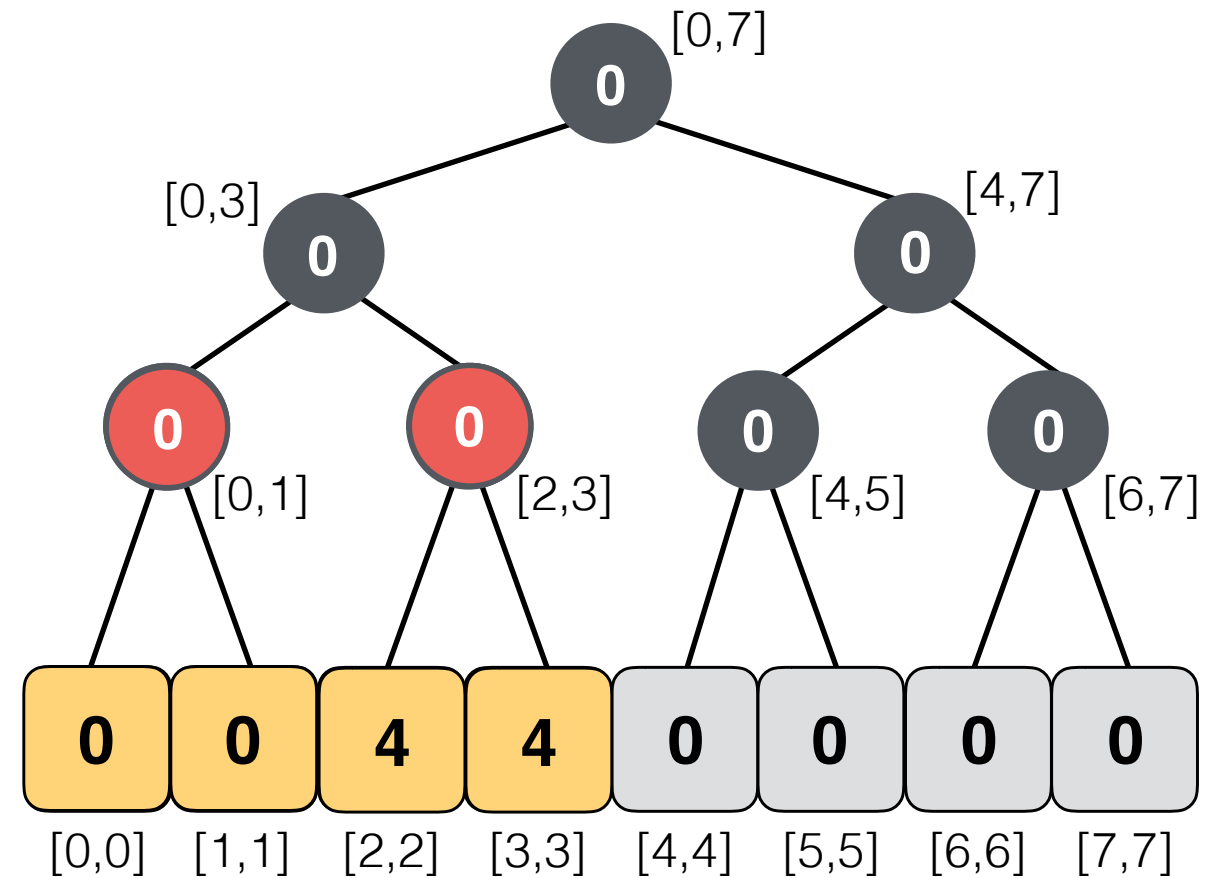
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



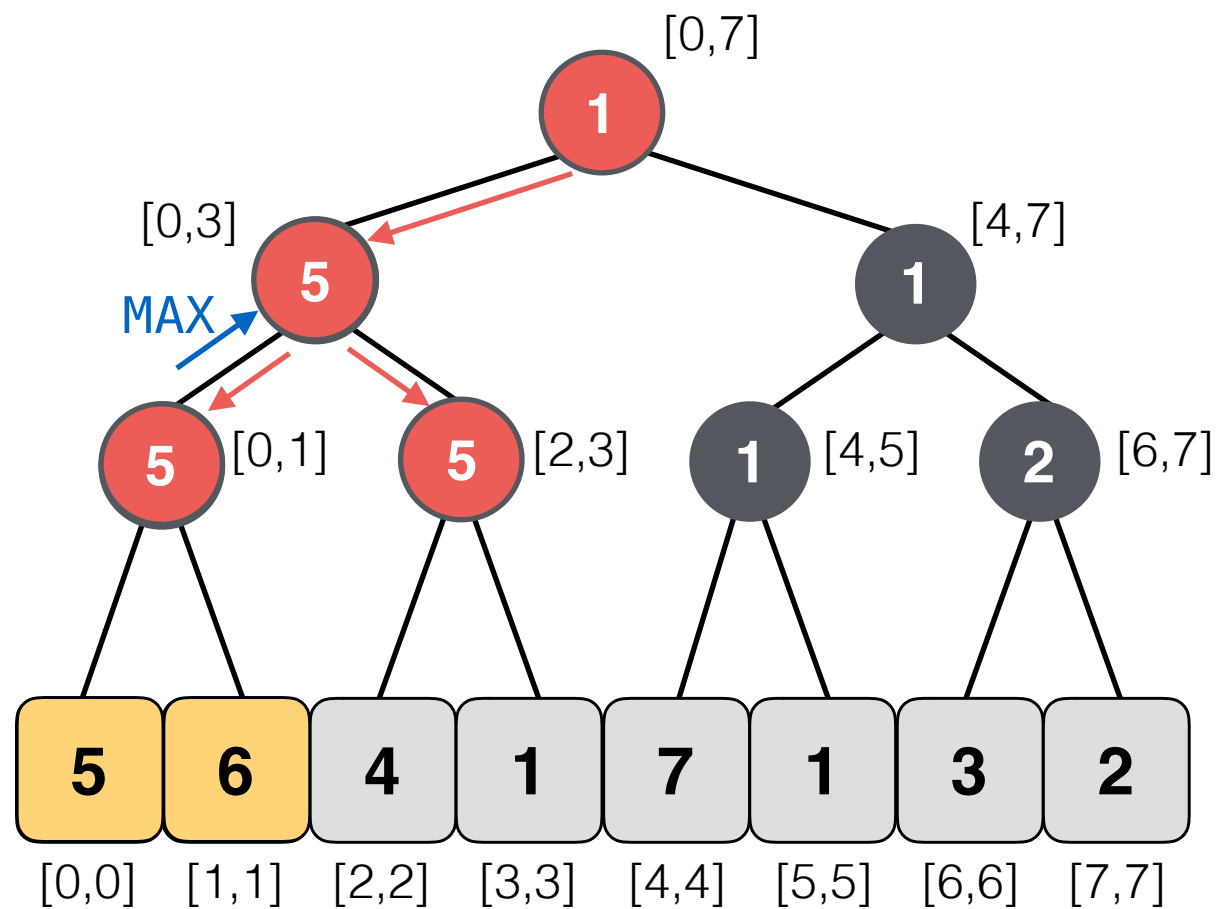
Lazy Tree

Lazy Propagation in Segment Trees

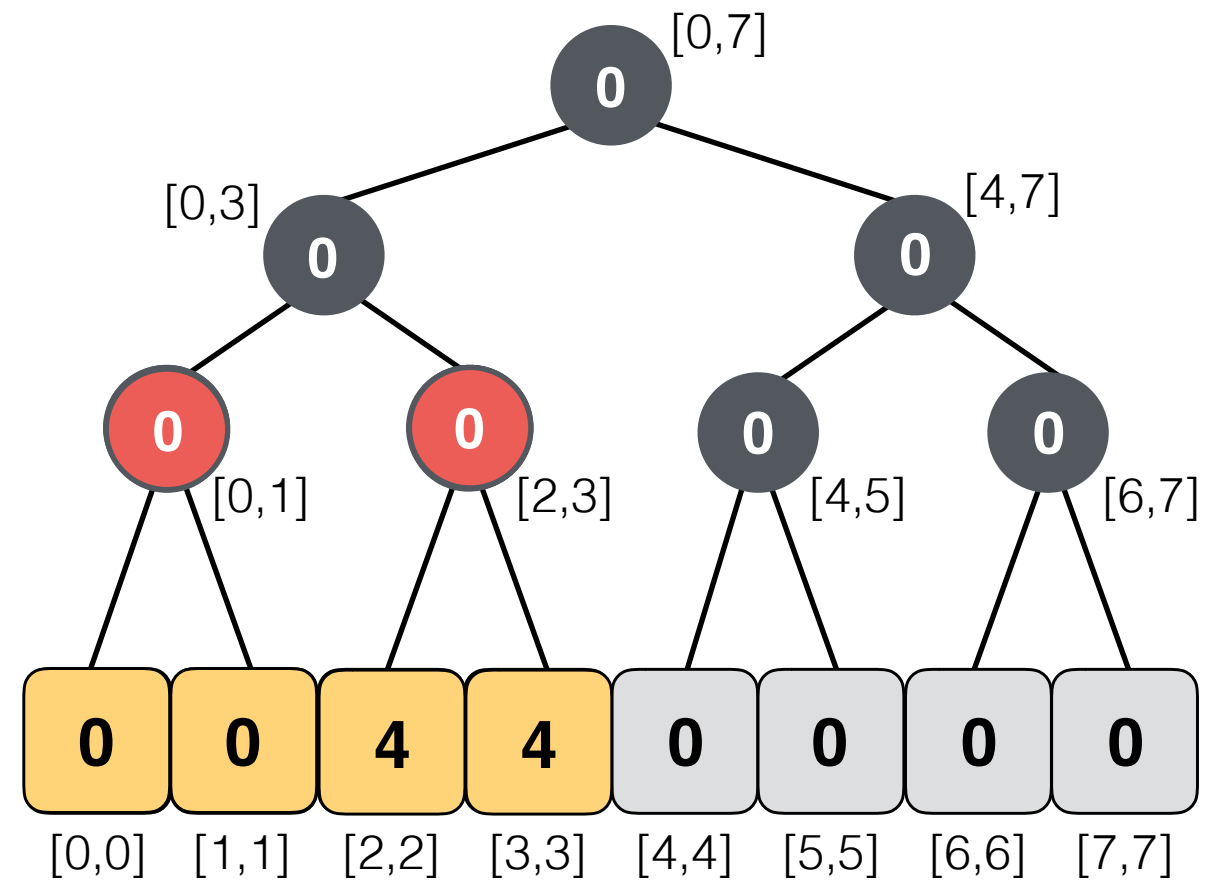
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



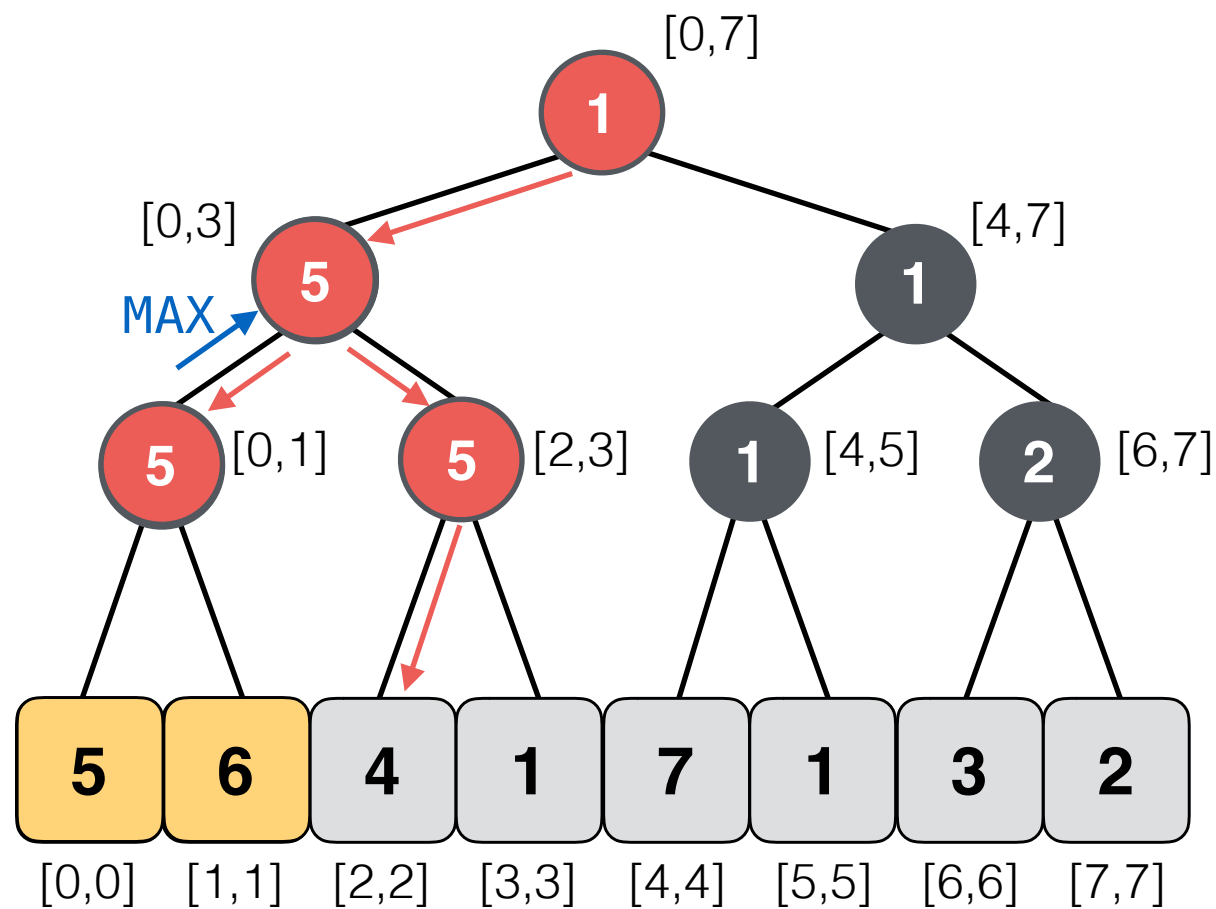
Lazy Tree

Lazy Propagation in Segment Trees

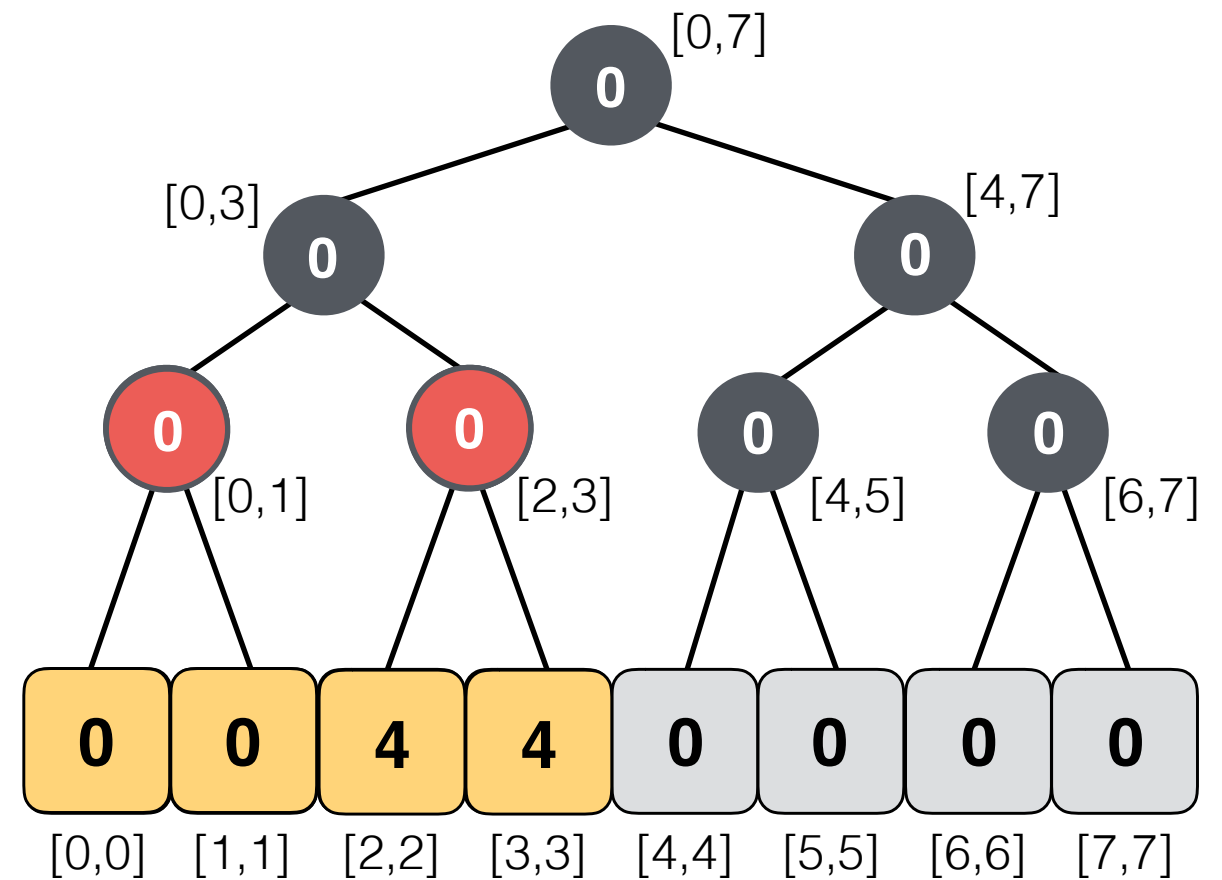
Avoid going down to the leaves and then up updating the internal nodes.
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update_range(0,0,2)

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Segment Tree



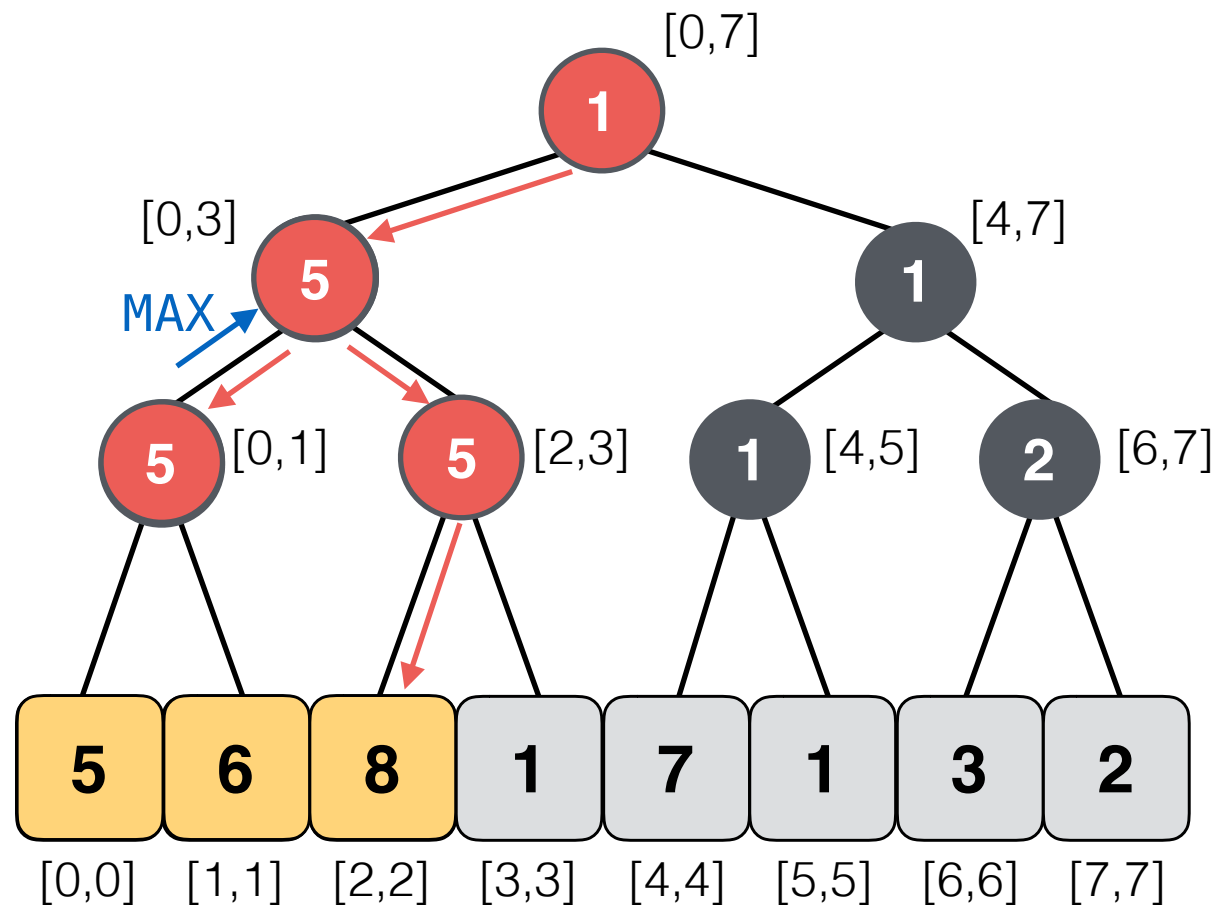
Lazy Tree

Lazy Propagation in Segment Trees

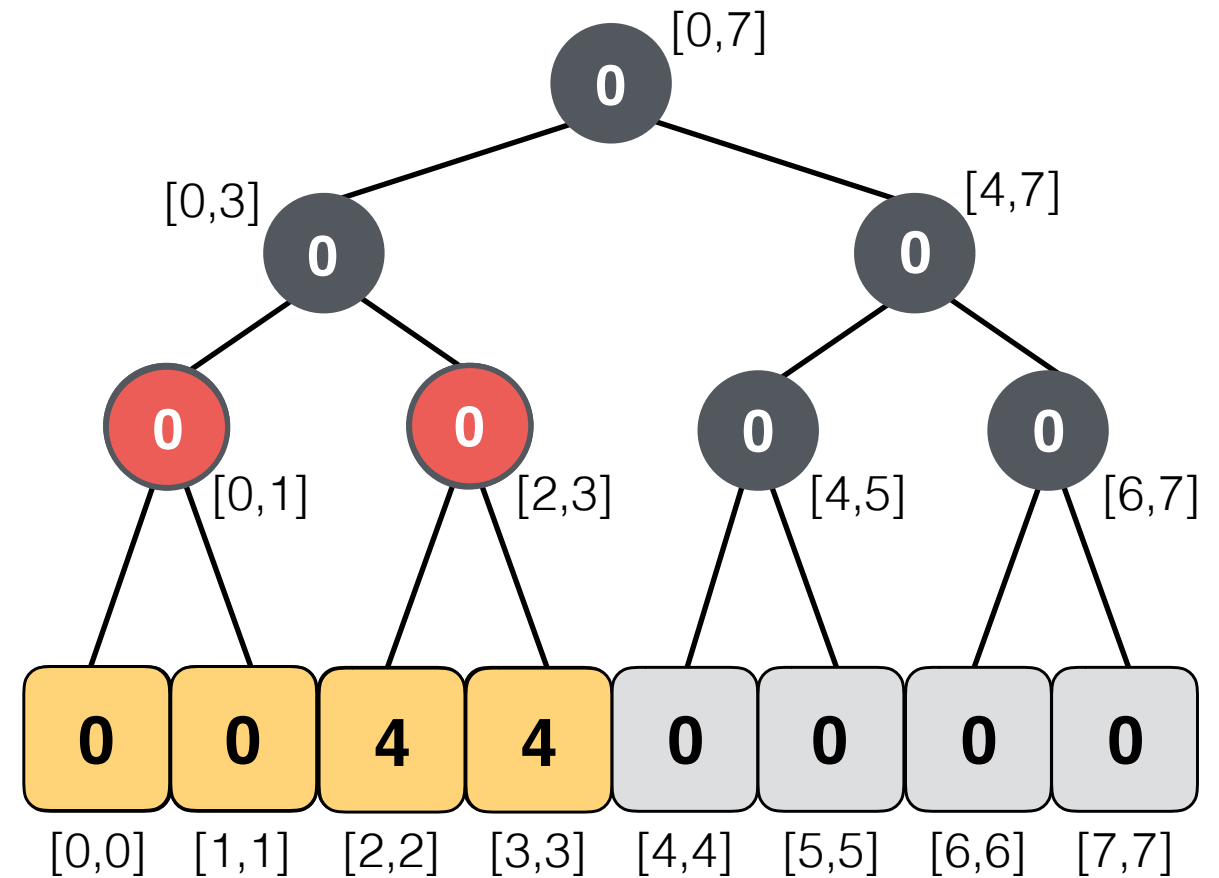
Avoid going down to the leaves and then up updating the internal nodes.
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update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



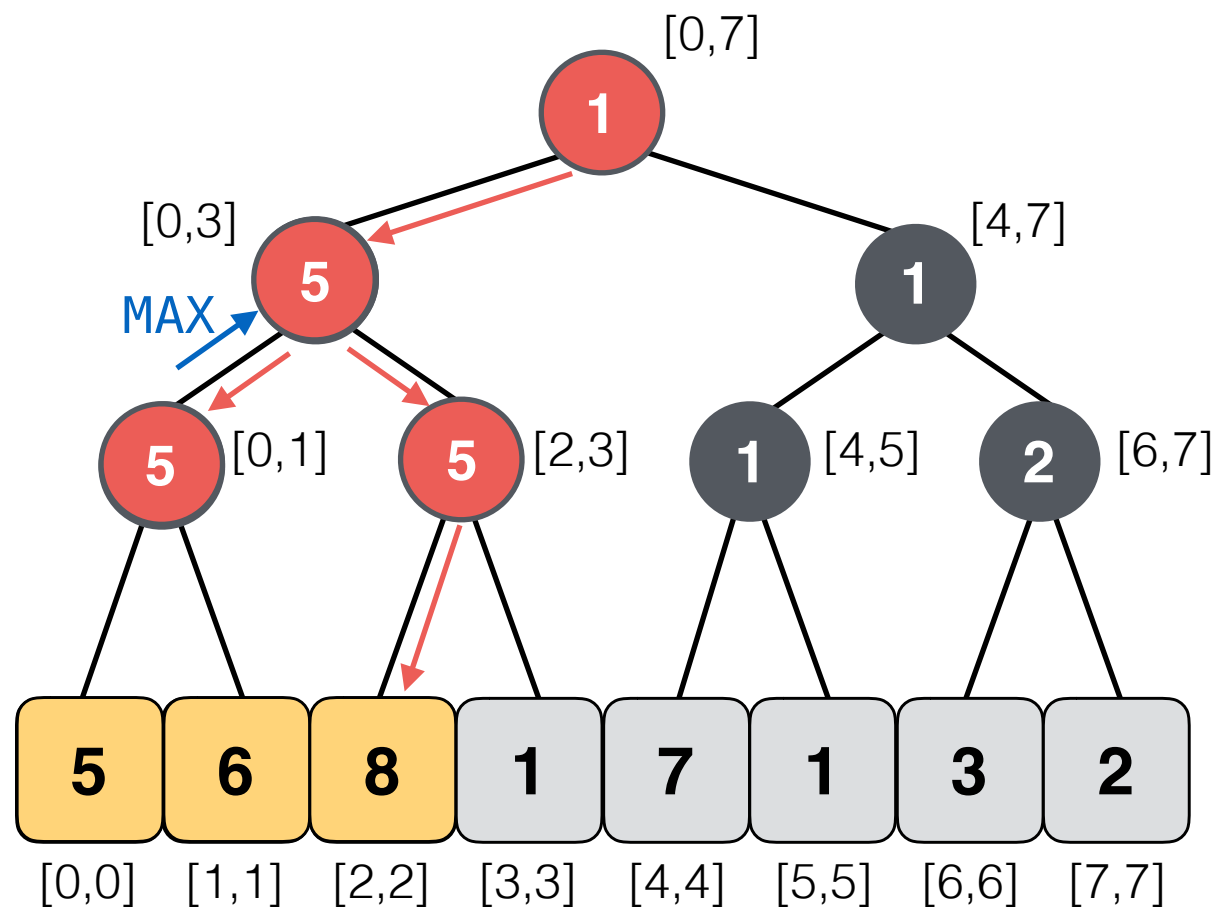
Lazy Tree

Lazy Propagation in Segment Trees

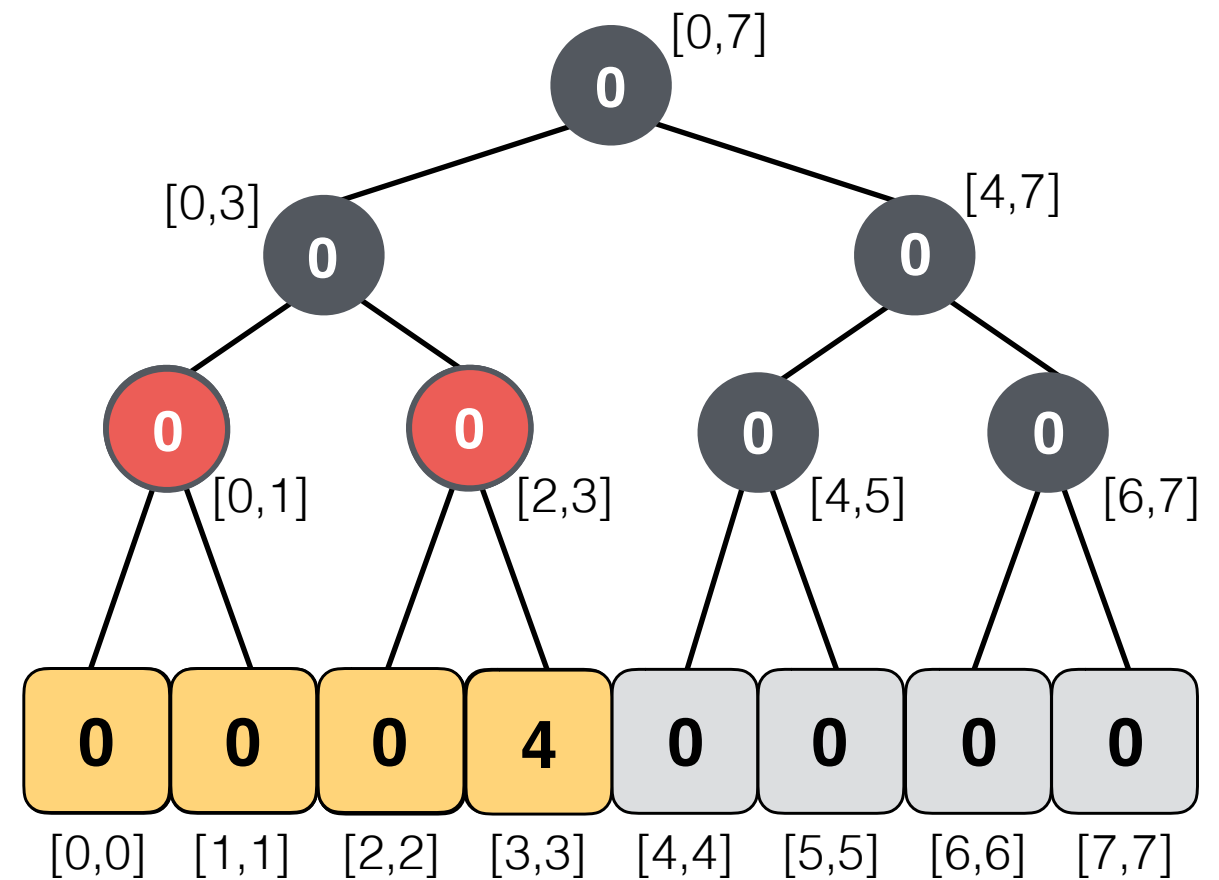
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



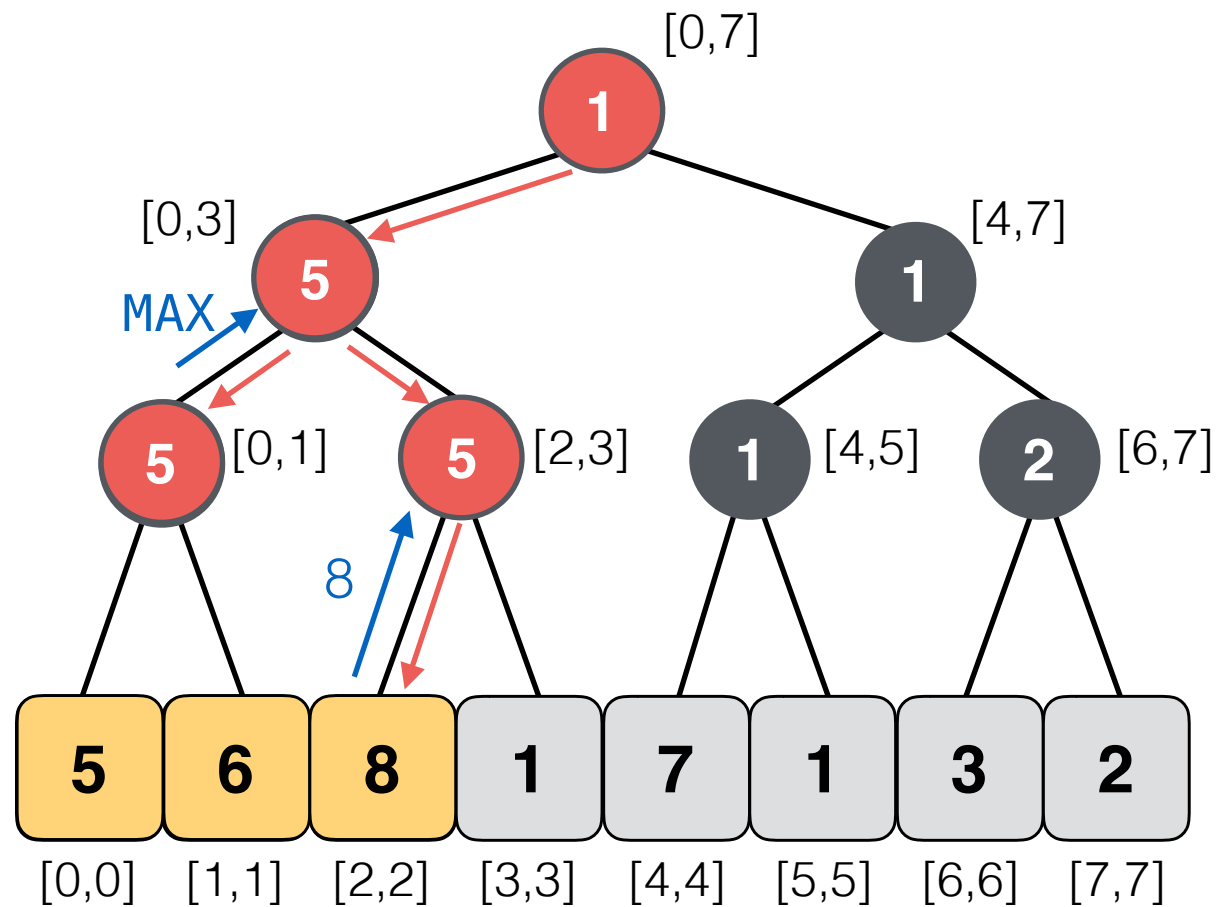
Lazy Tree

Lazy Propagation in Segment Trees

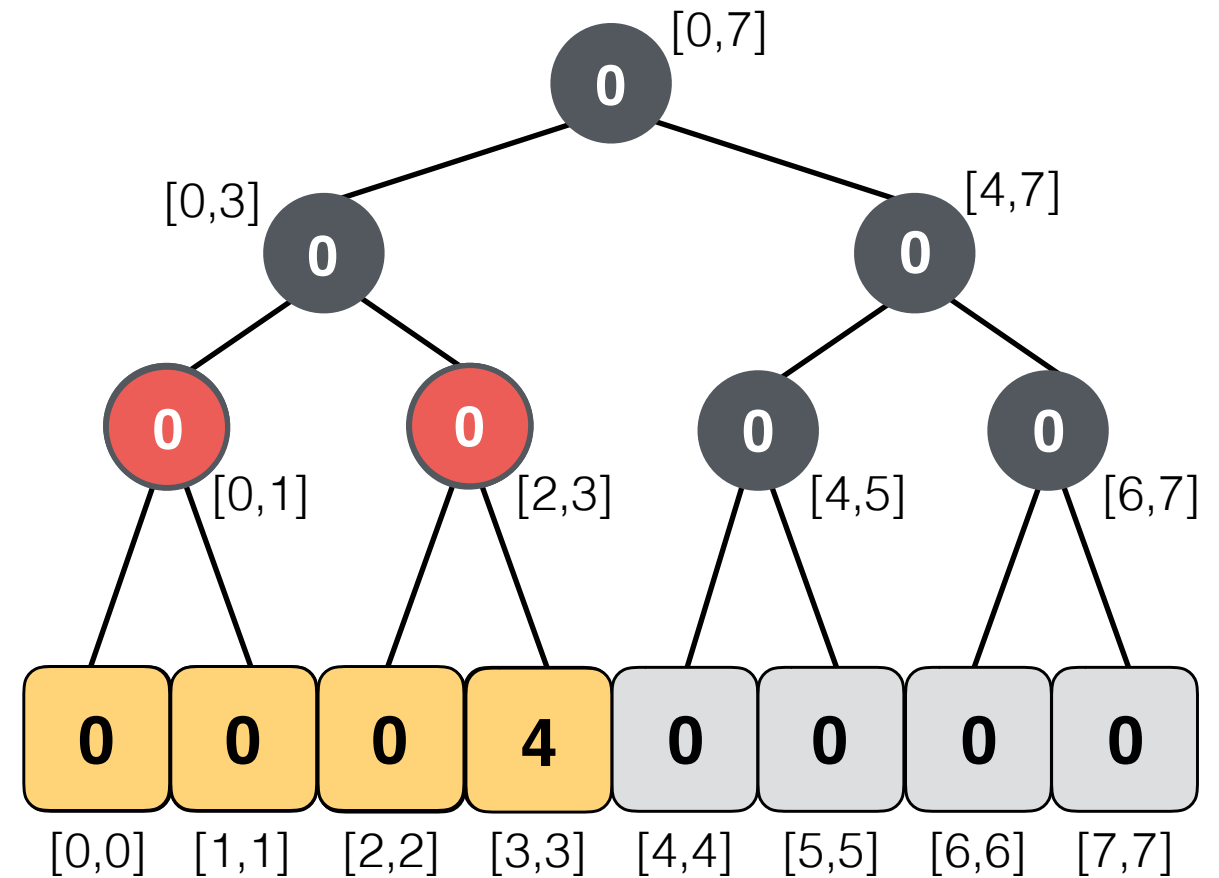
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



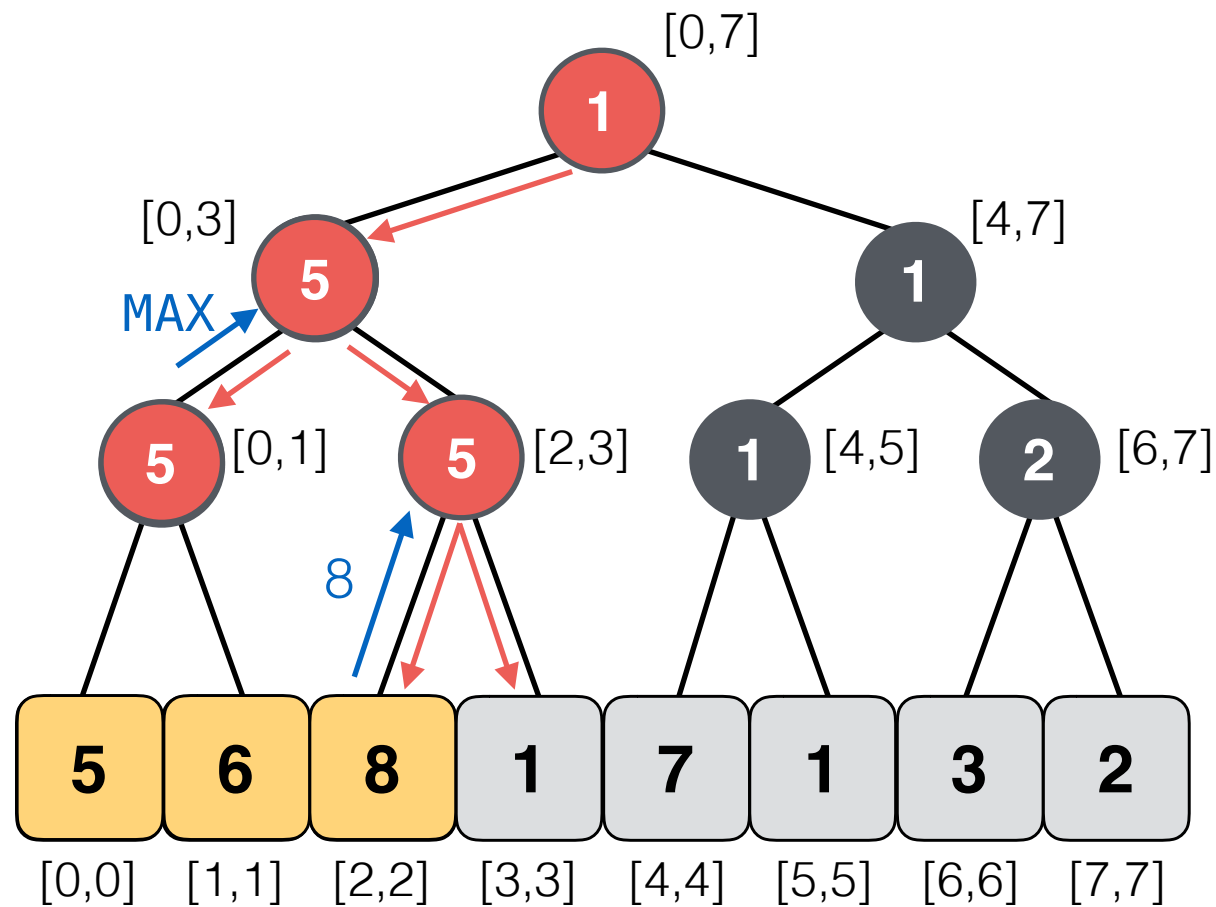
Lazy Tree

Lazy Propagation in Segment Trees

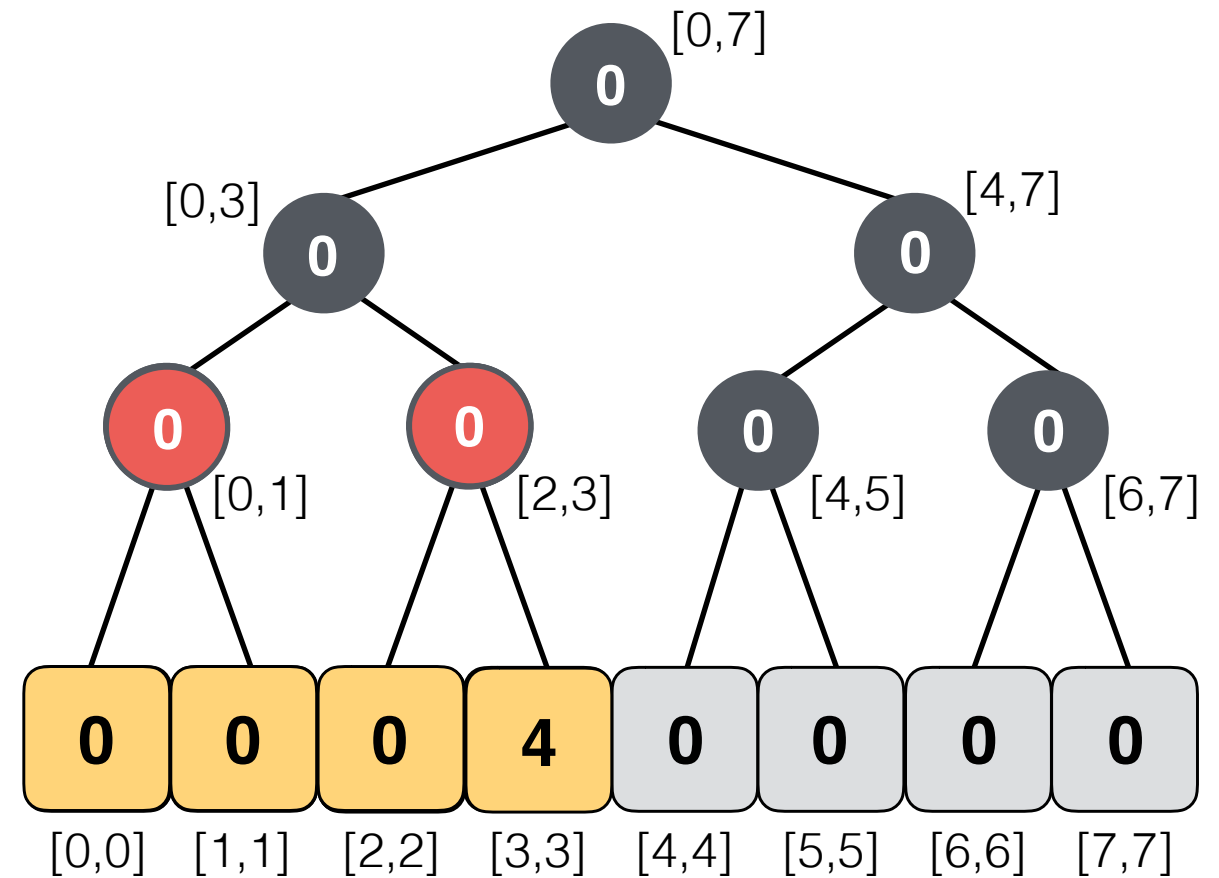
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



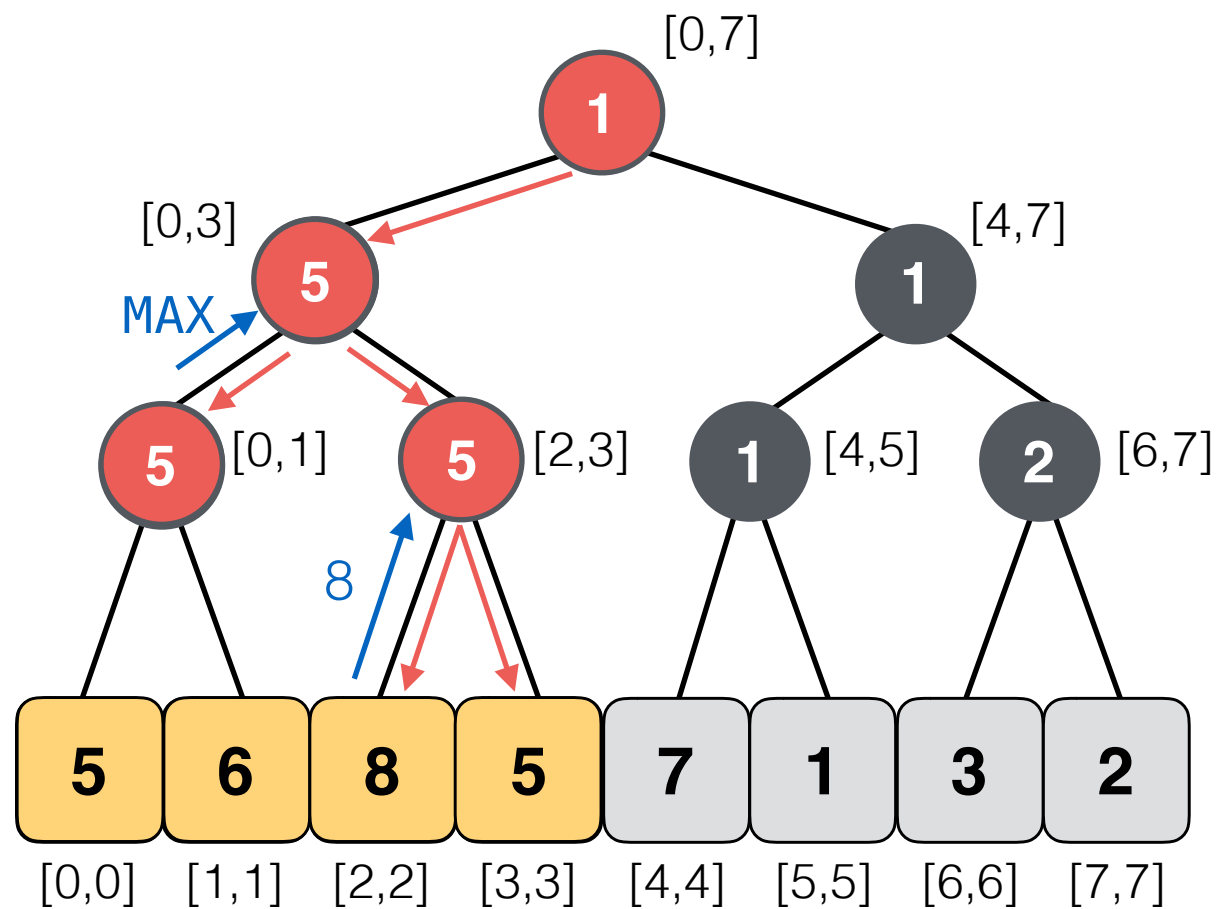
Lazy Tree

Lazy Propagation in Segment Trees

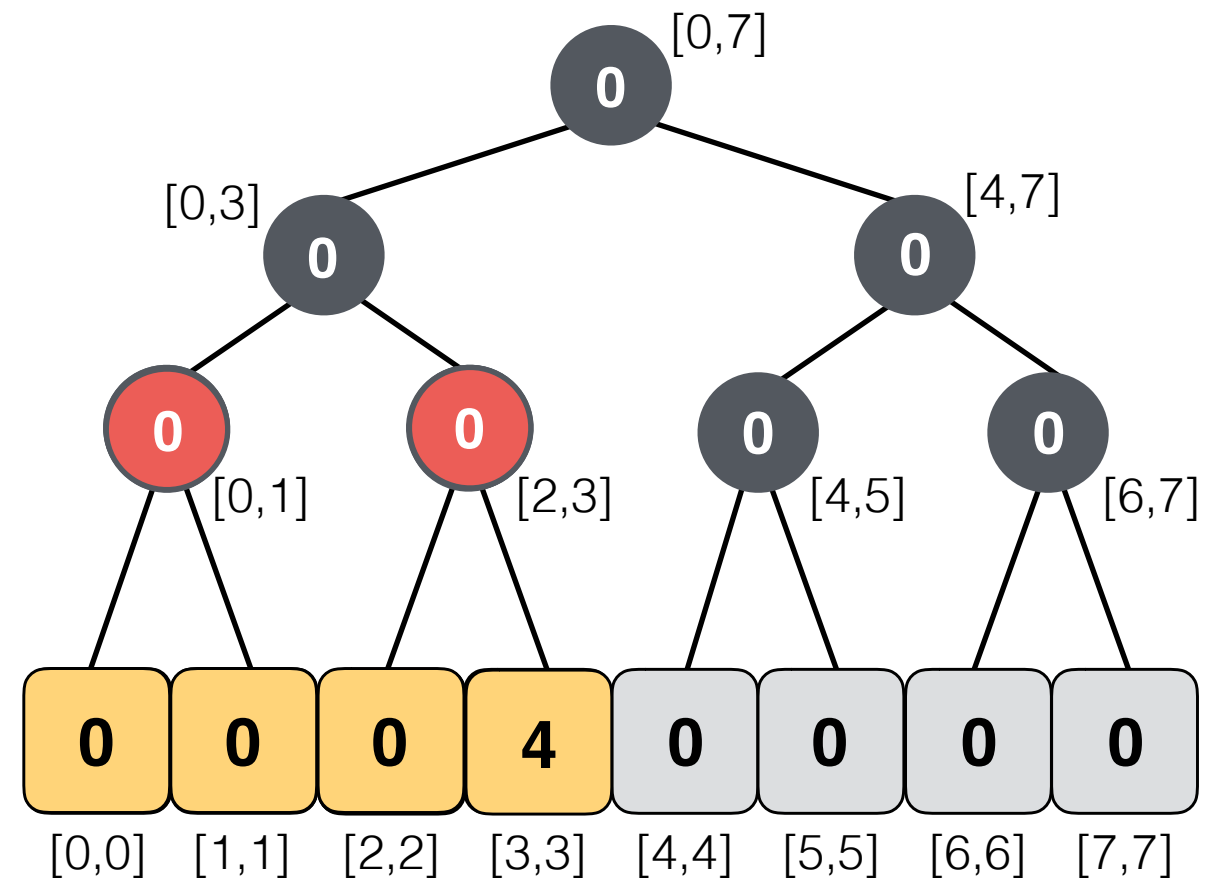
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



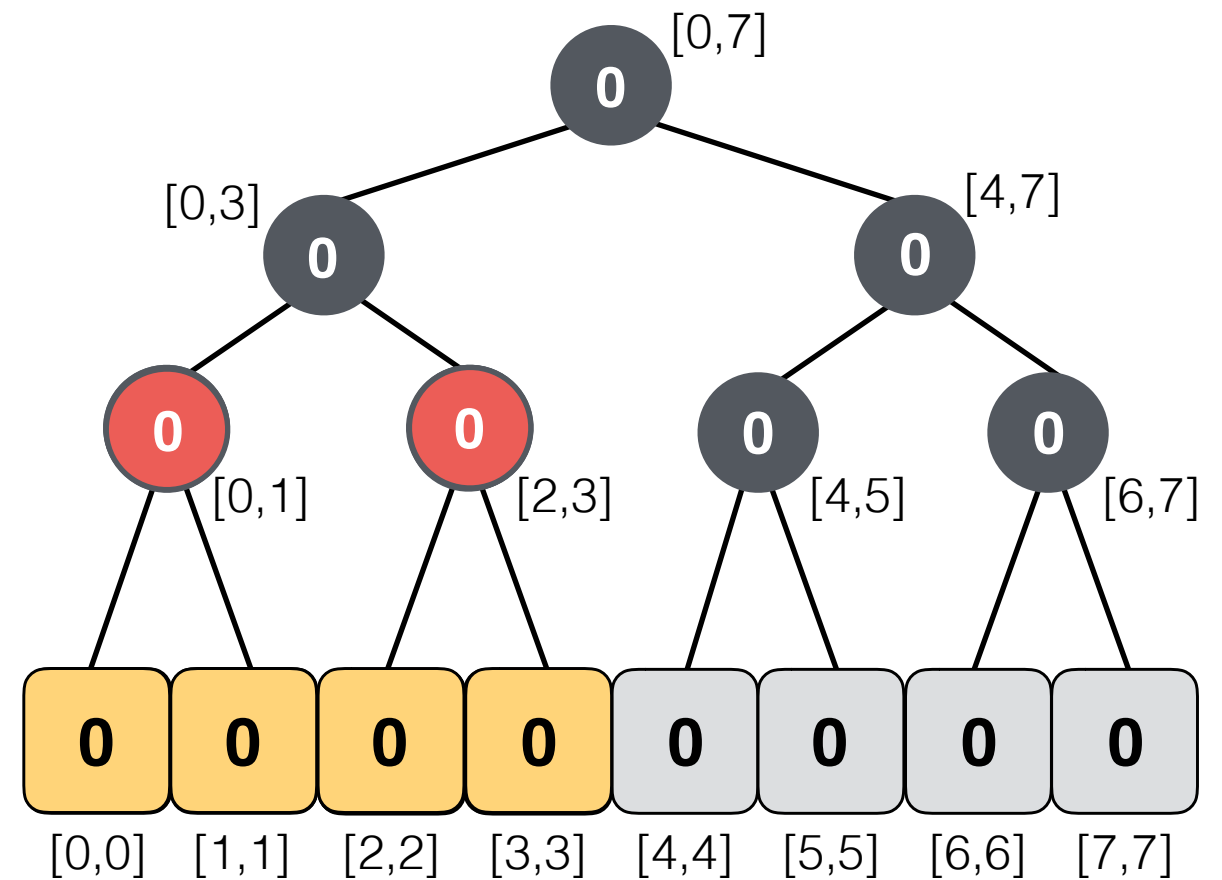
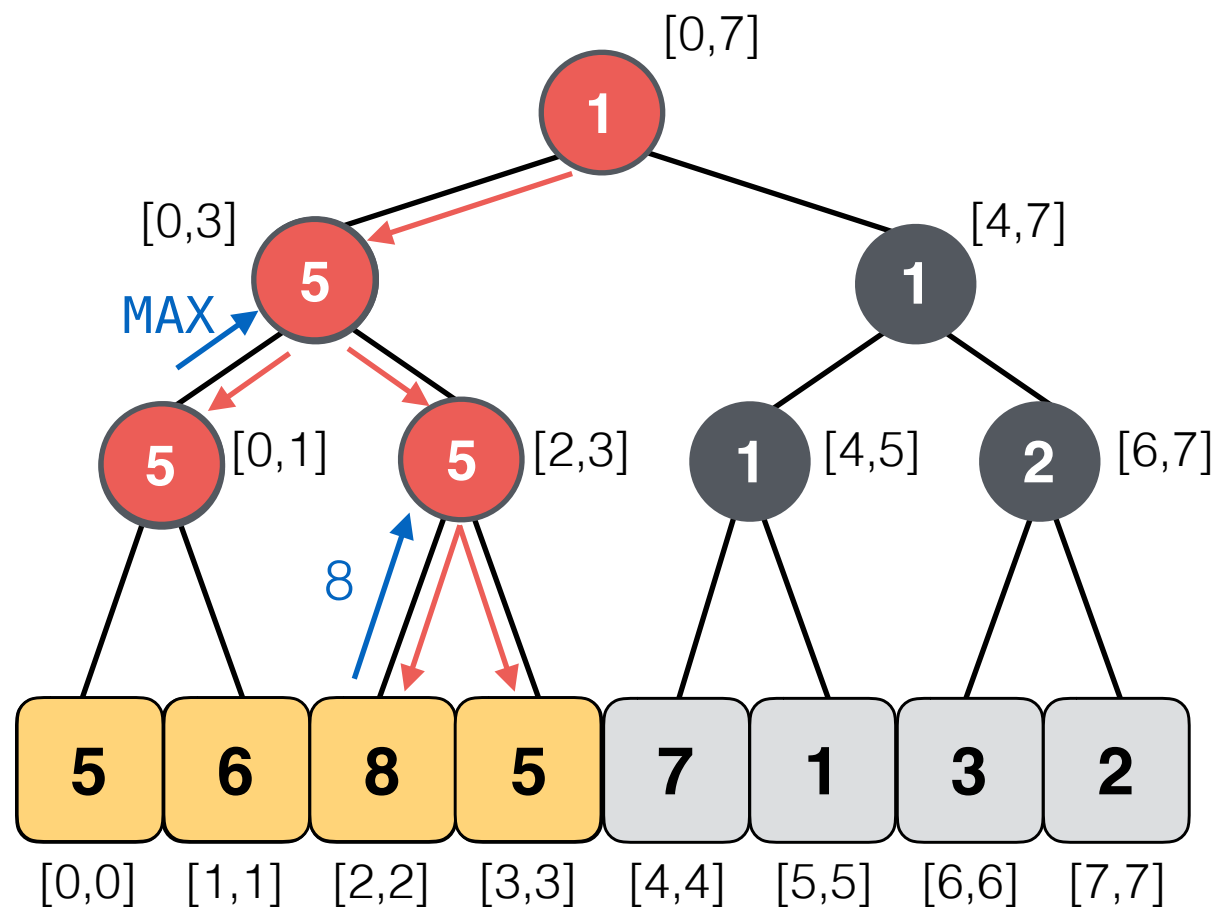
Lazy Tree

Lazy Propagation in Segment Trees

Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?

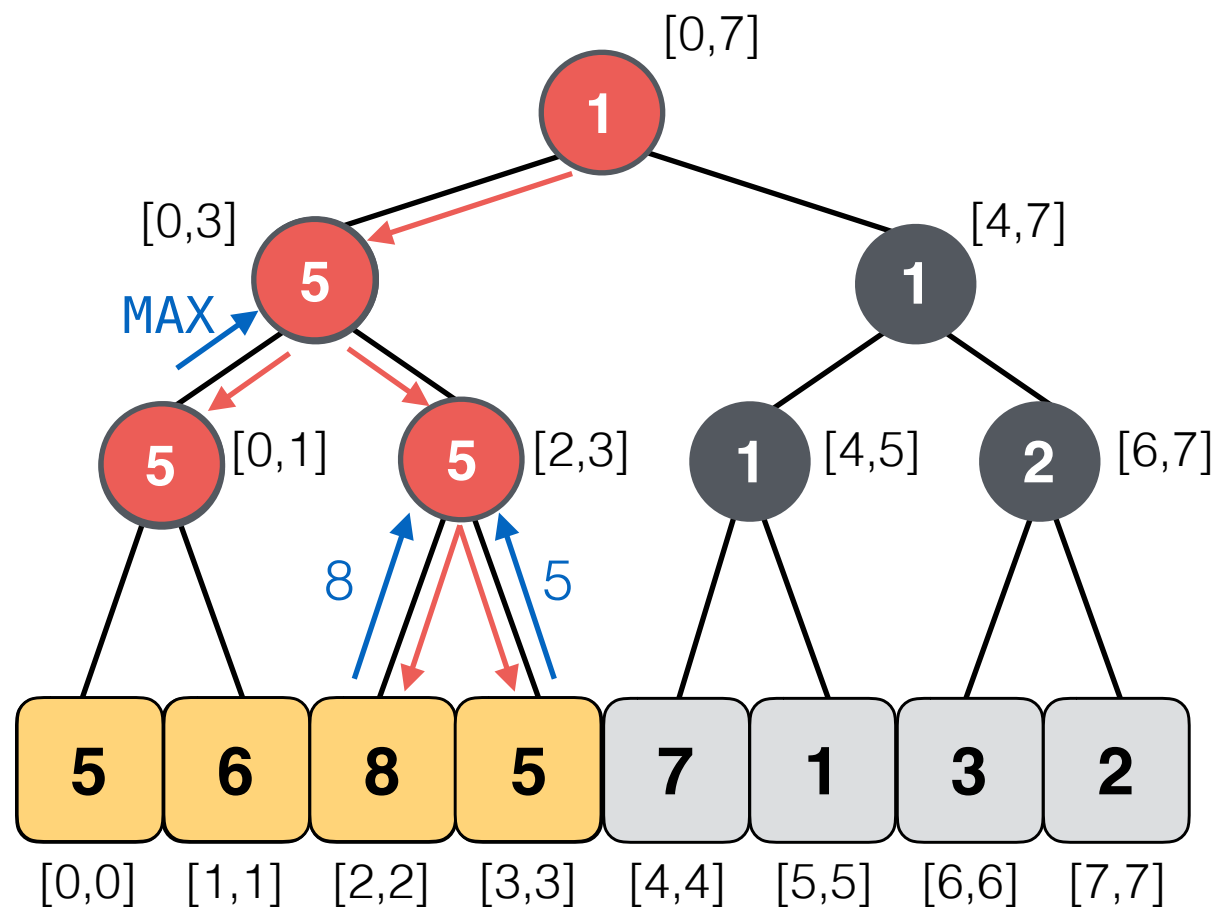


Lazy Propagation in Segment Trees

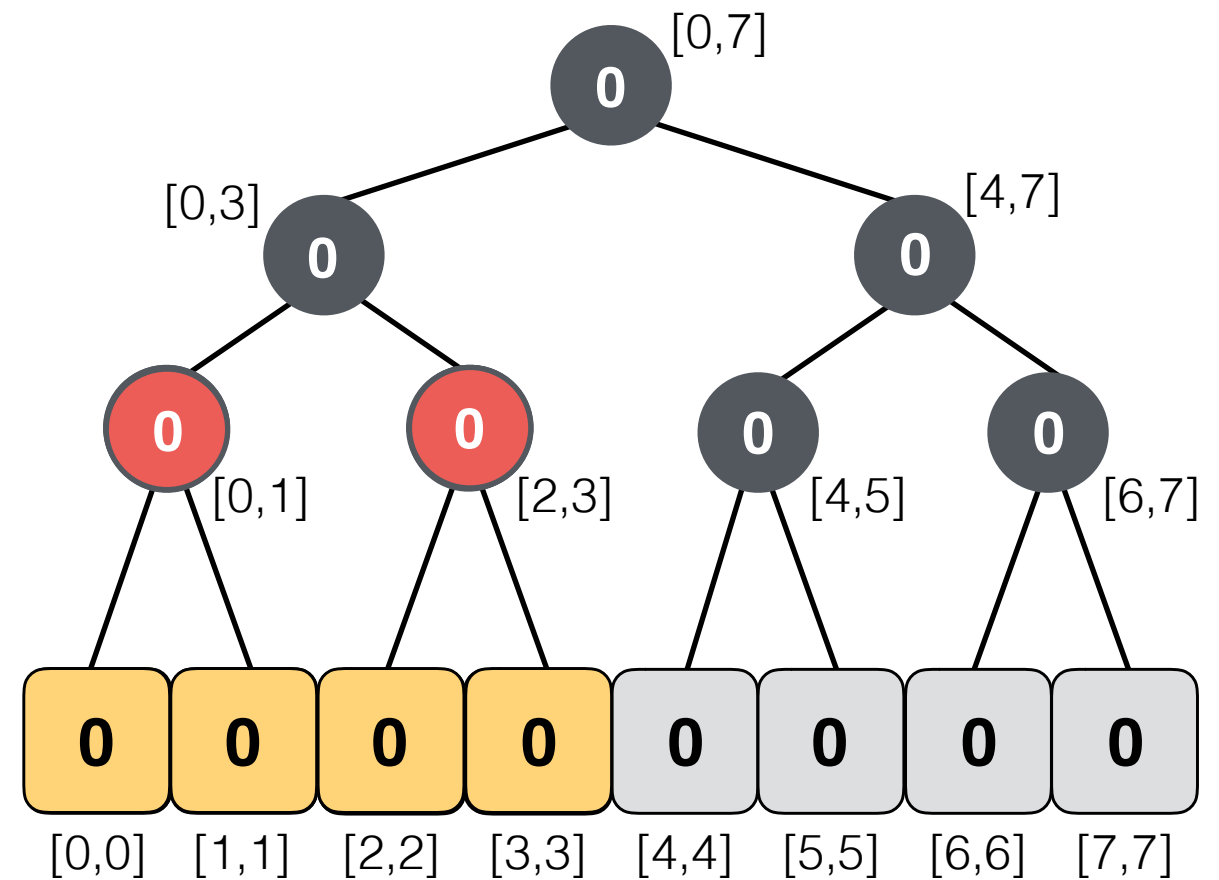
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



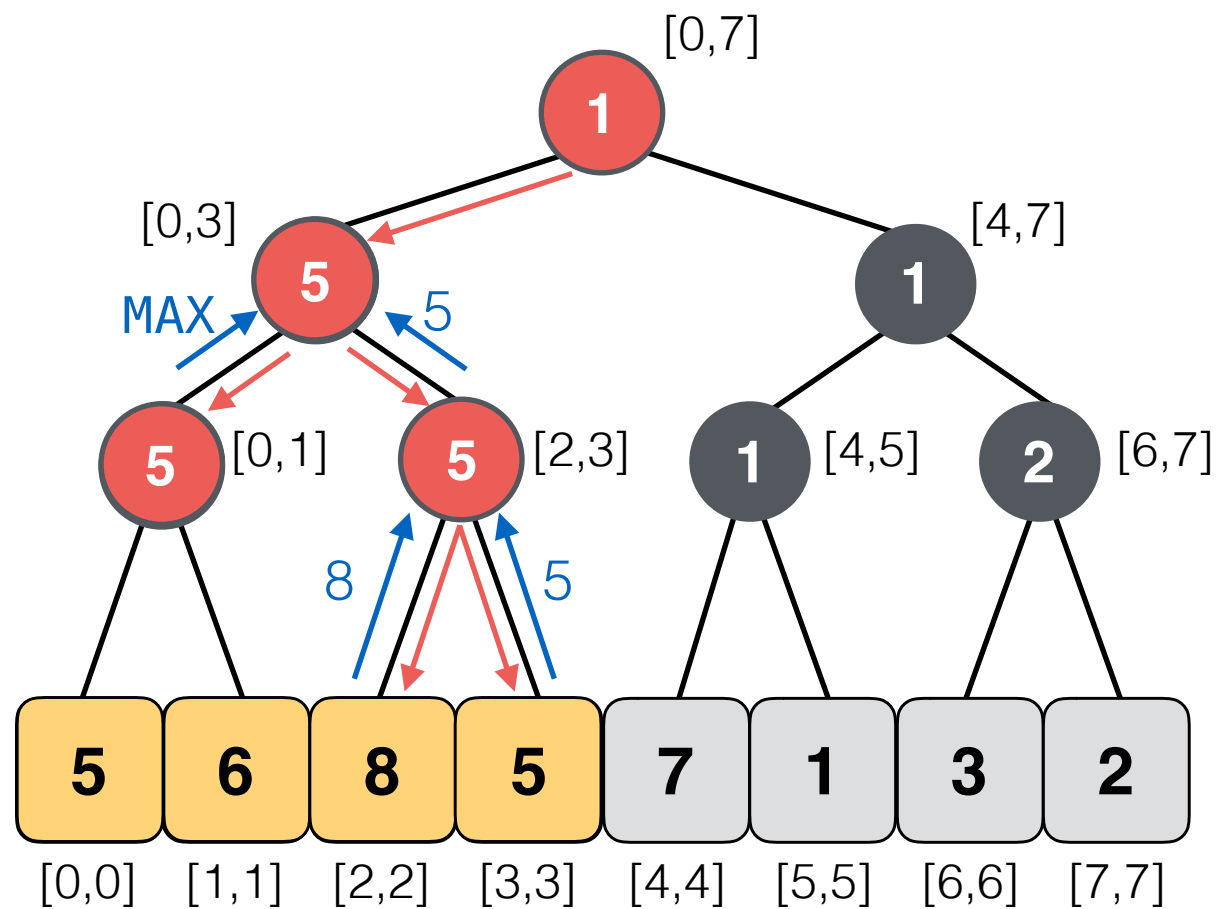
Lazy Tree

Lazy Propagation in Segment Trees

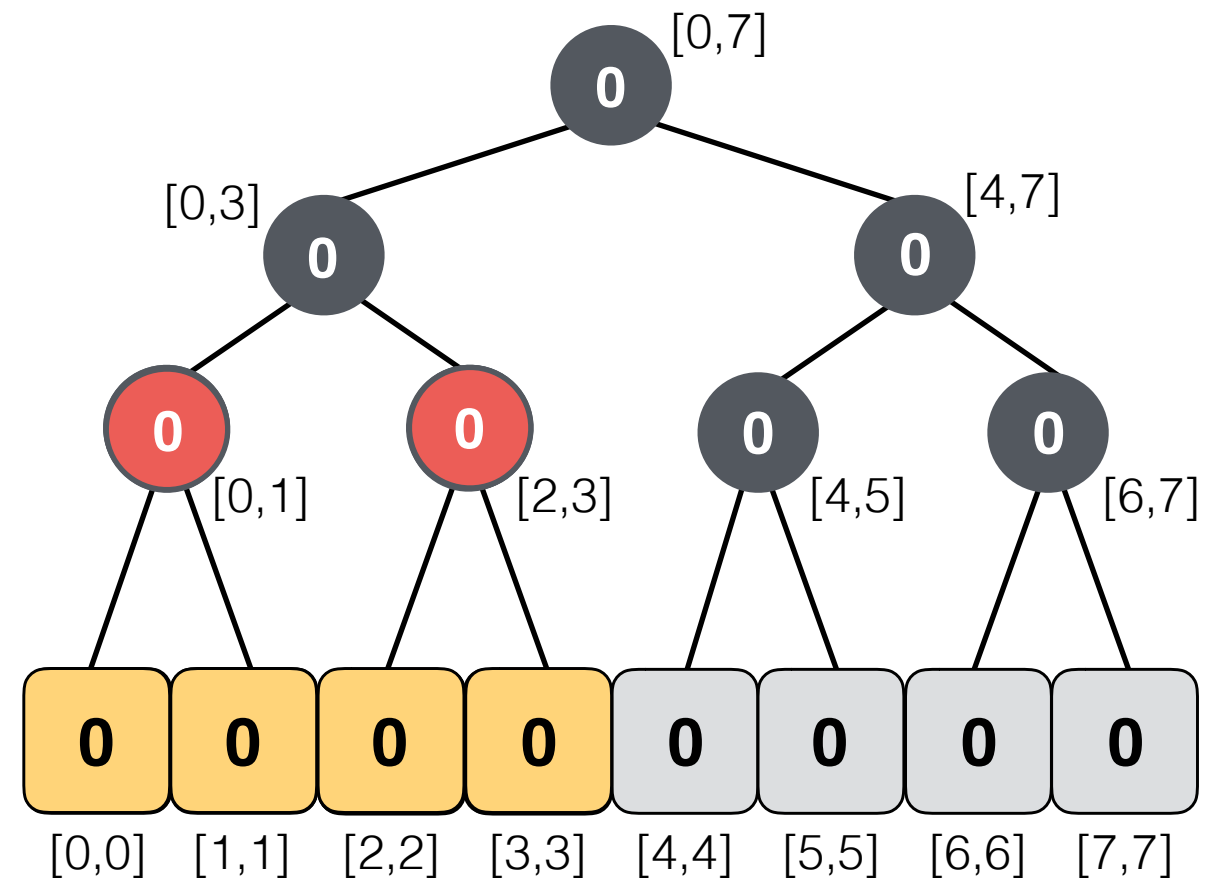
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



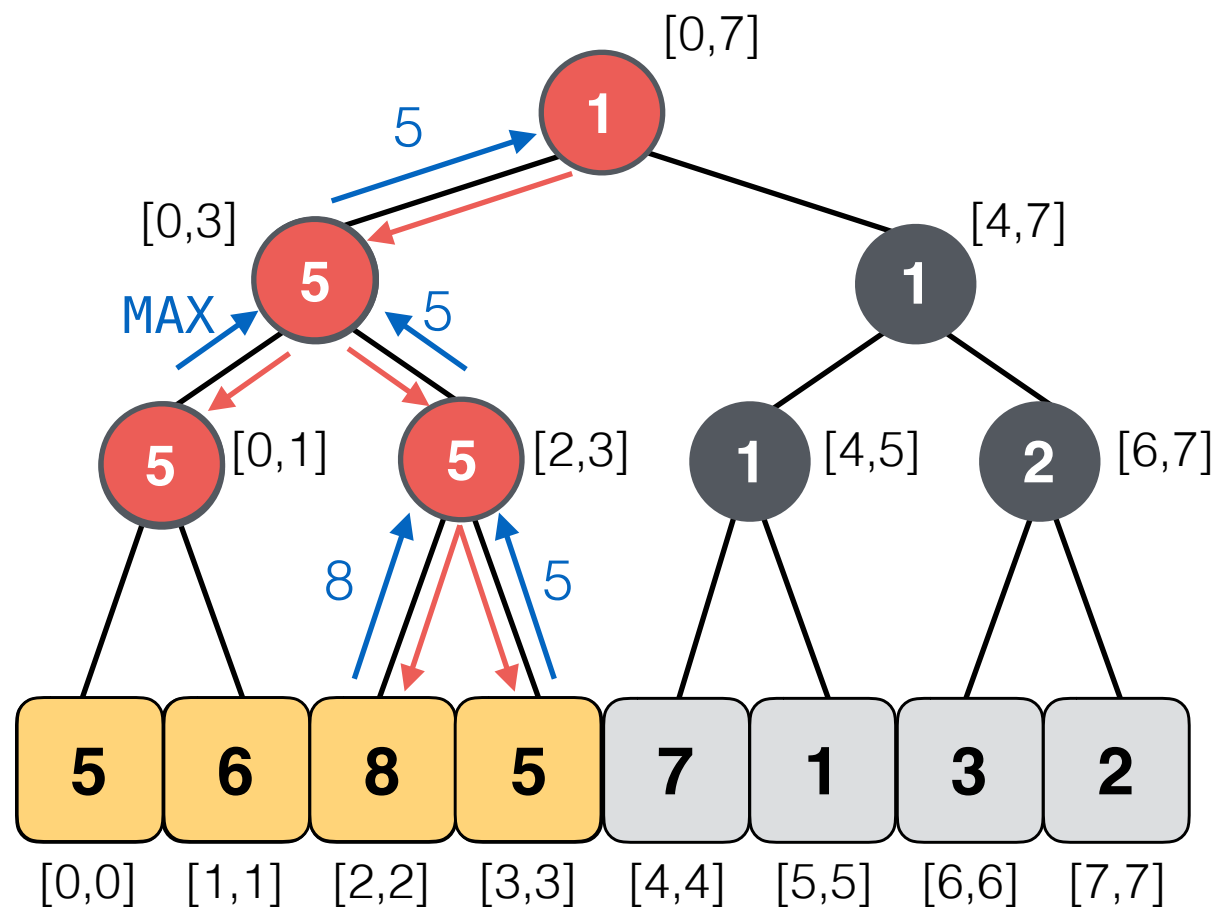
Lazy Tree

Lazy Propagation in Segment Trees

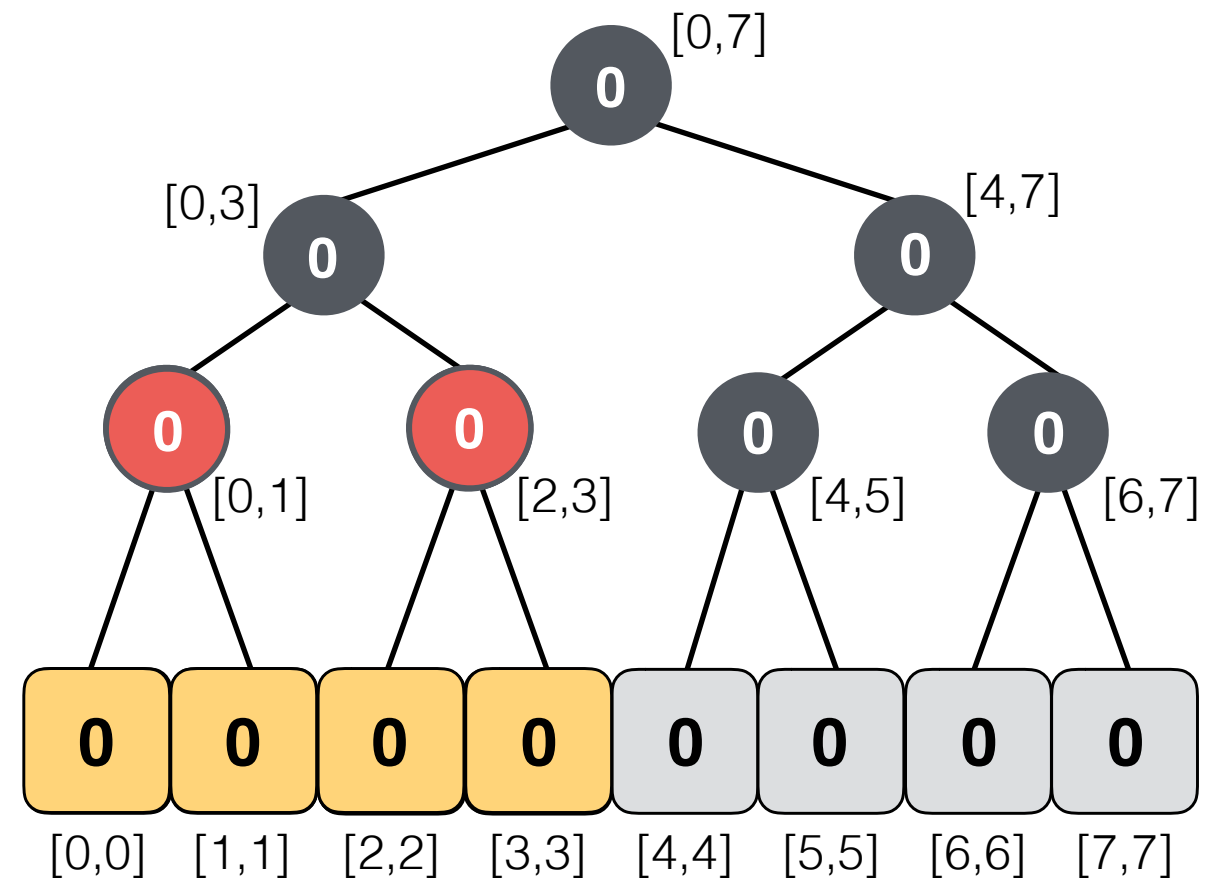
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



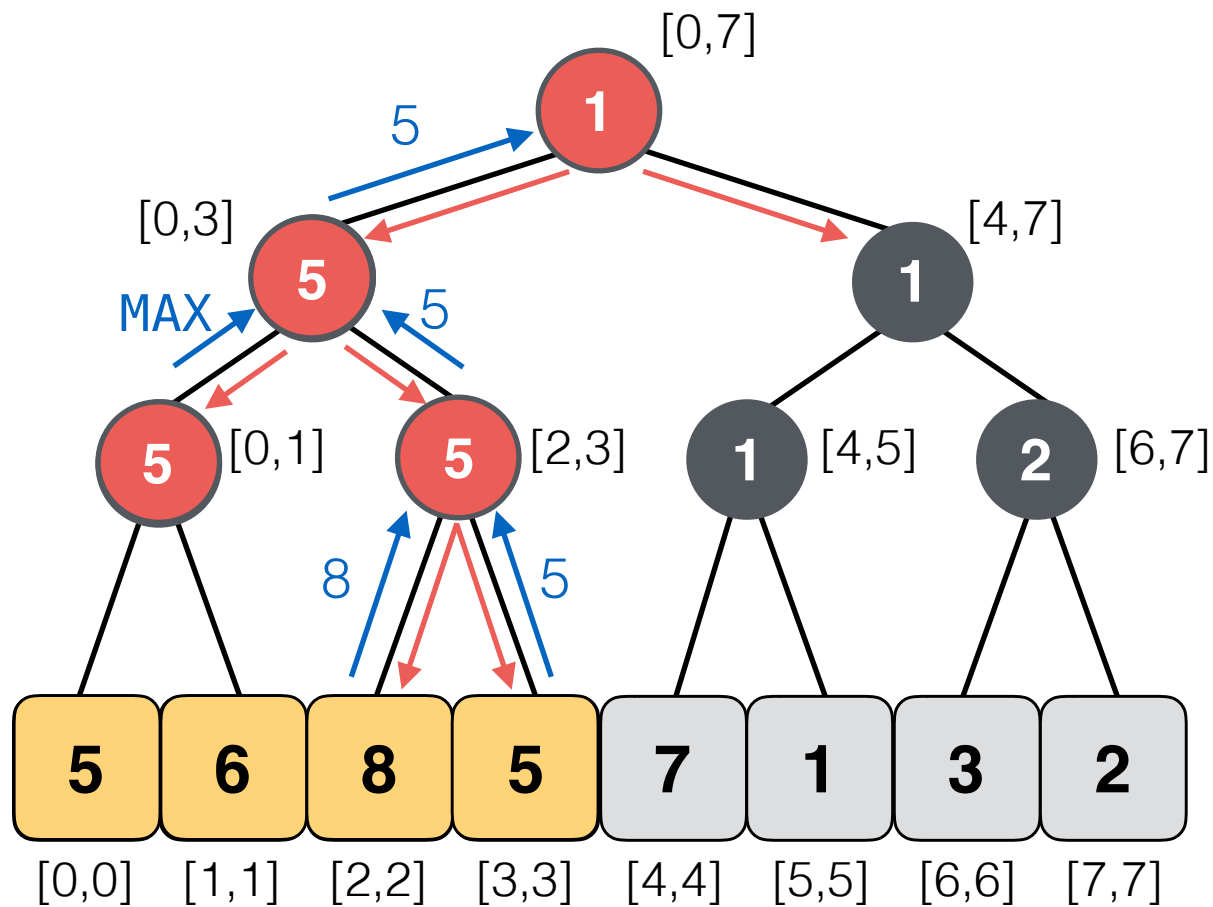
Lazy Tree

Lazy Propagation in Segment Trees

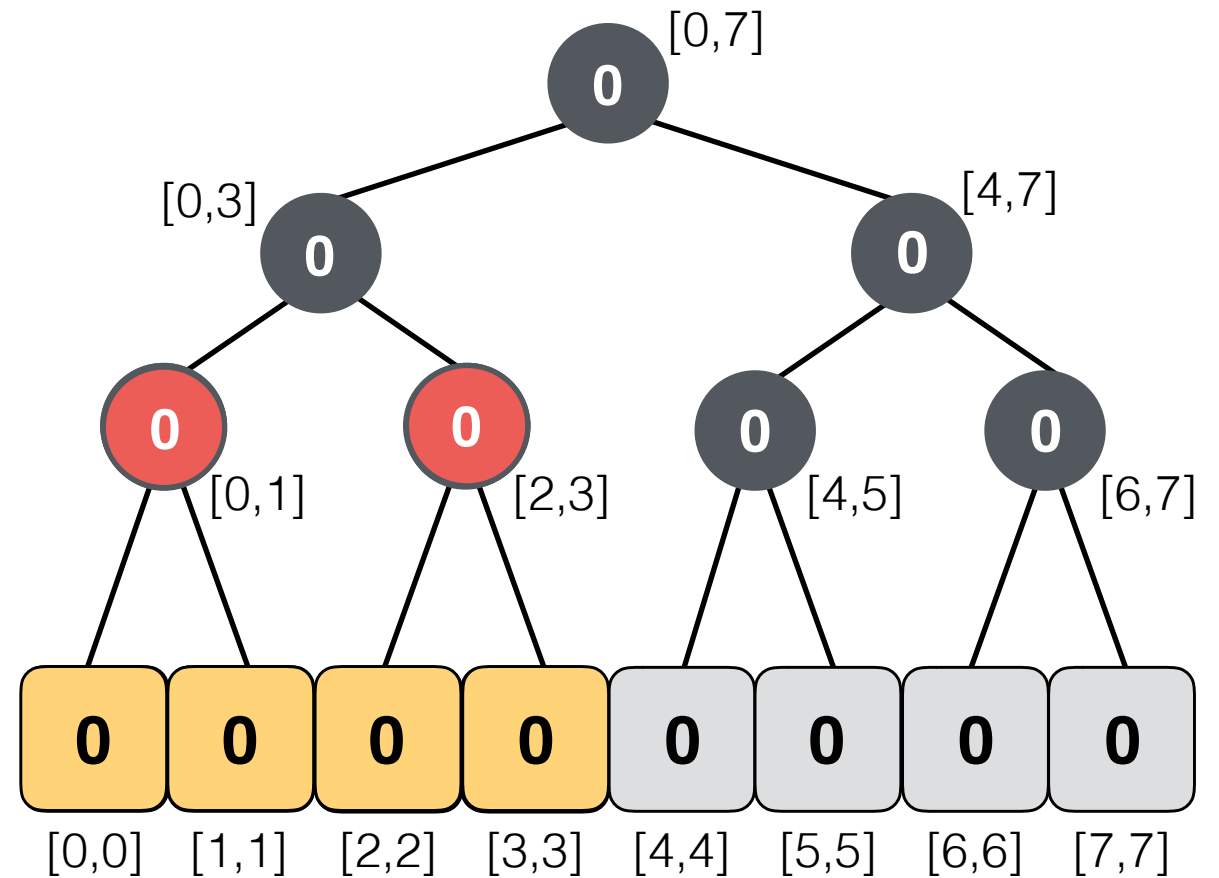
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



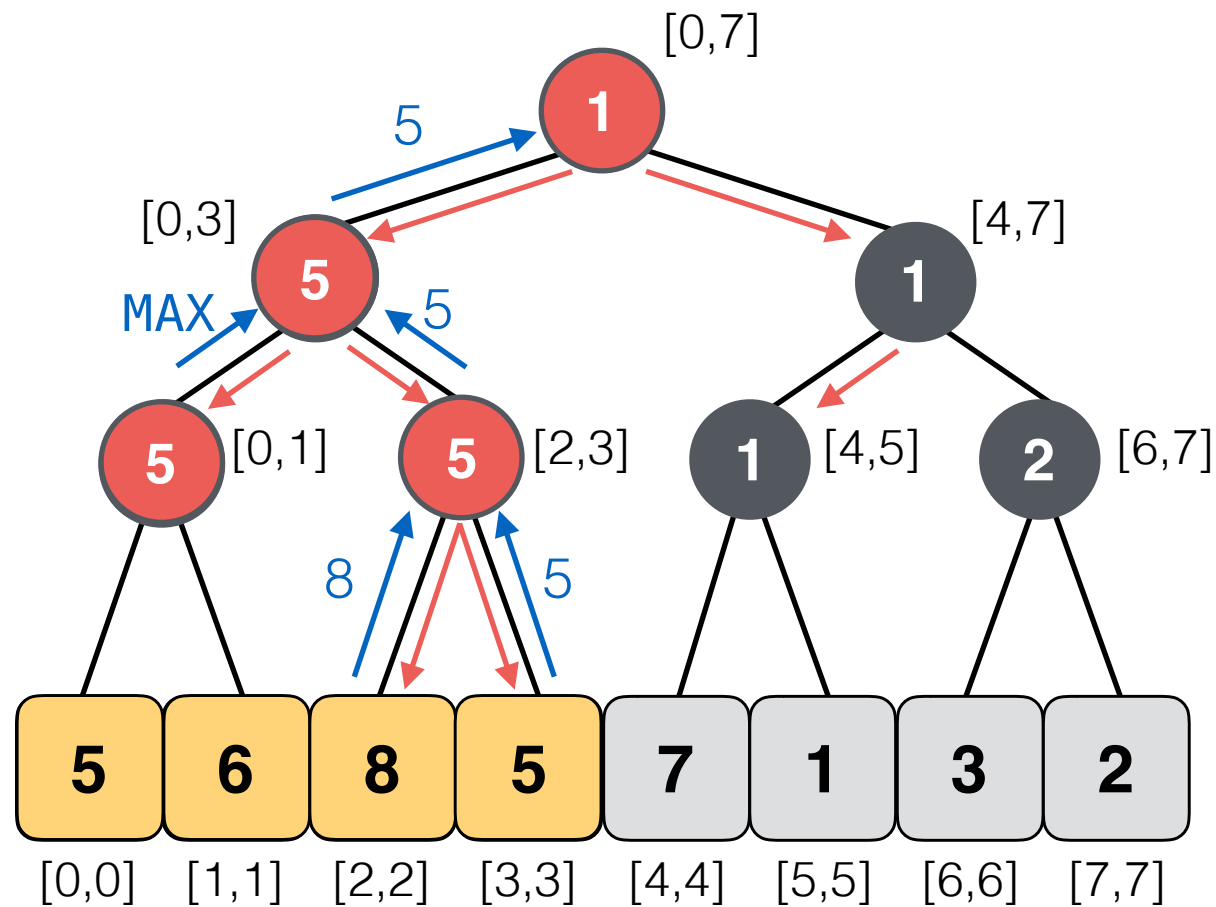
Lazy Tree

Lazy Propagation in Segment Trees

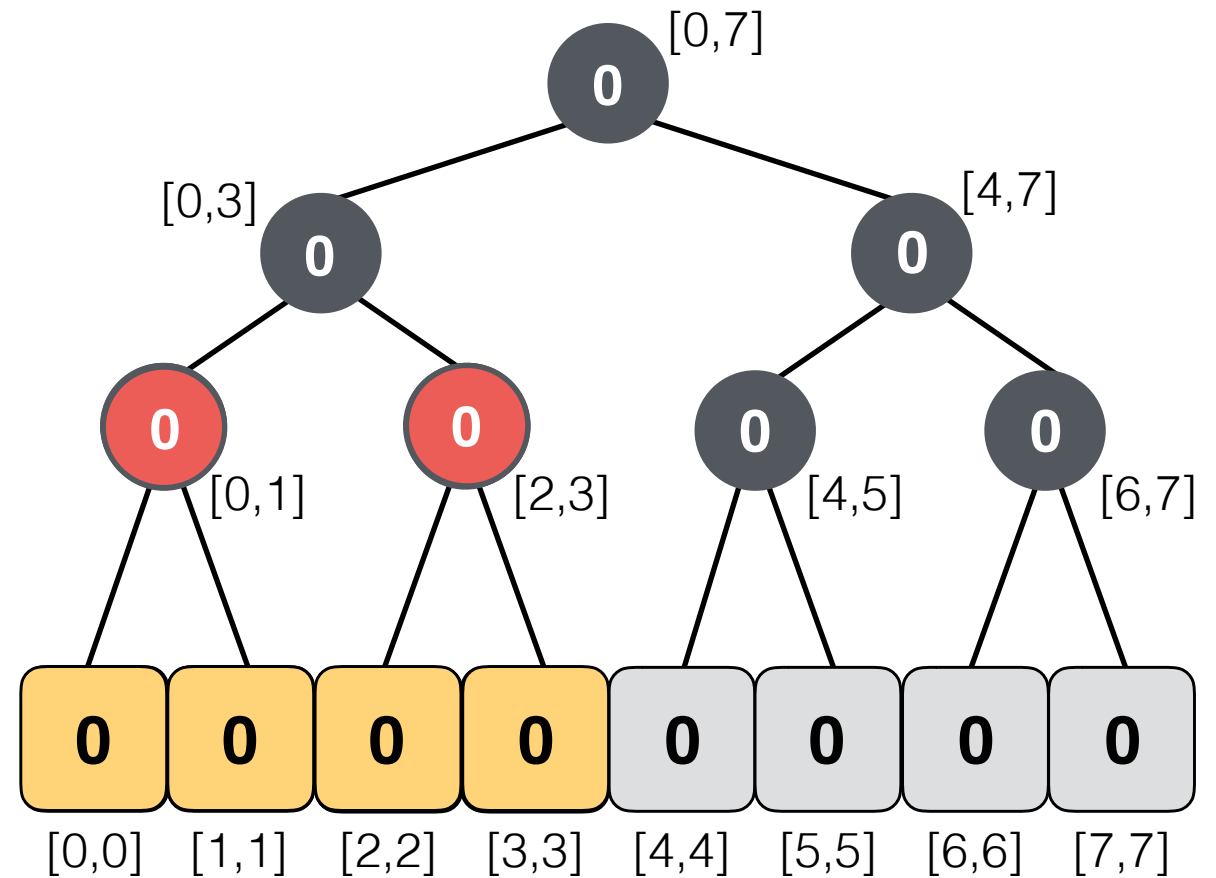
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



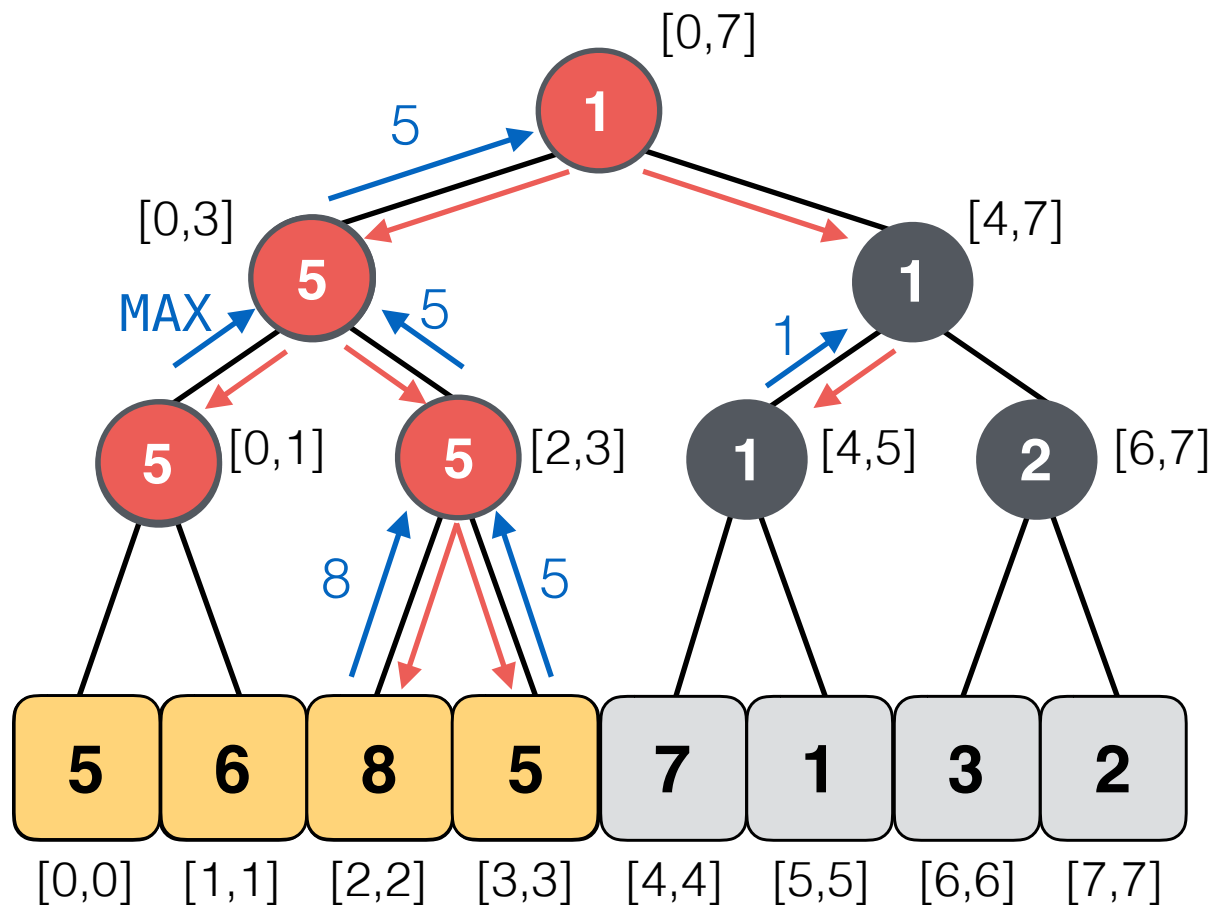
Lazy Tree

Lazy Propagation in Segment Trees

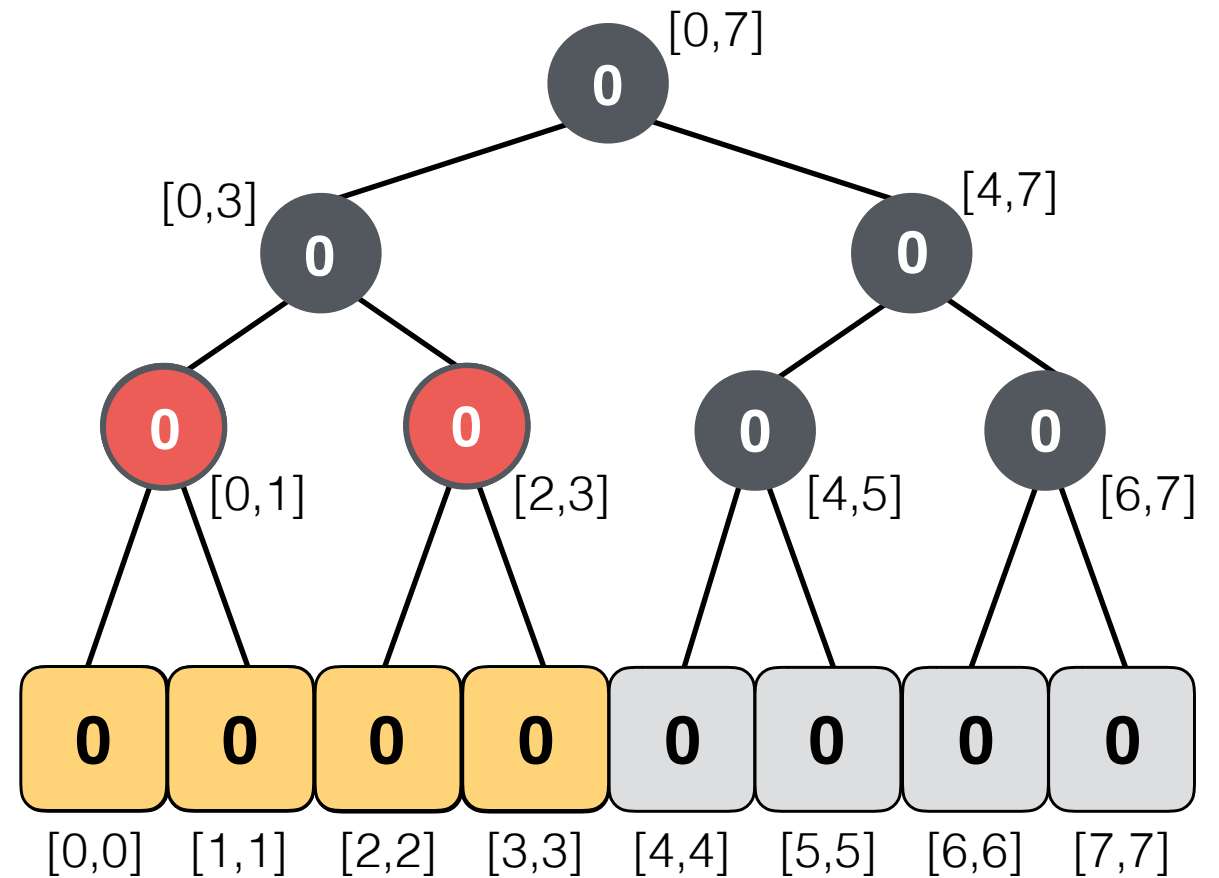
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



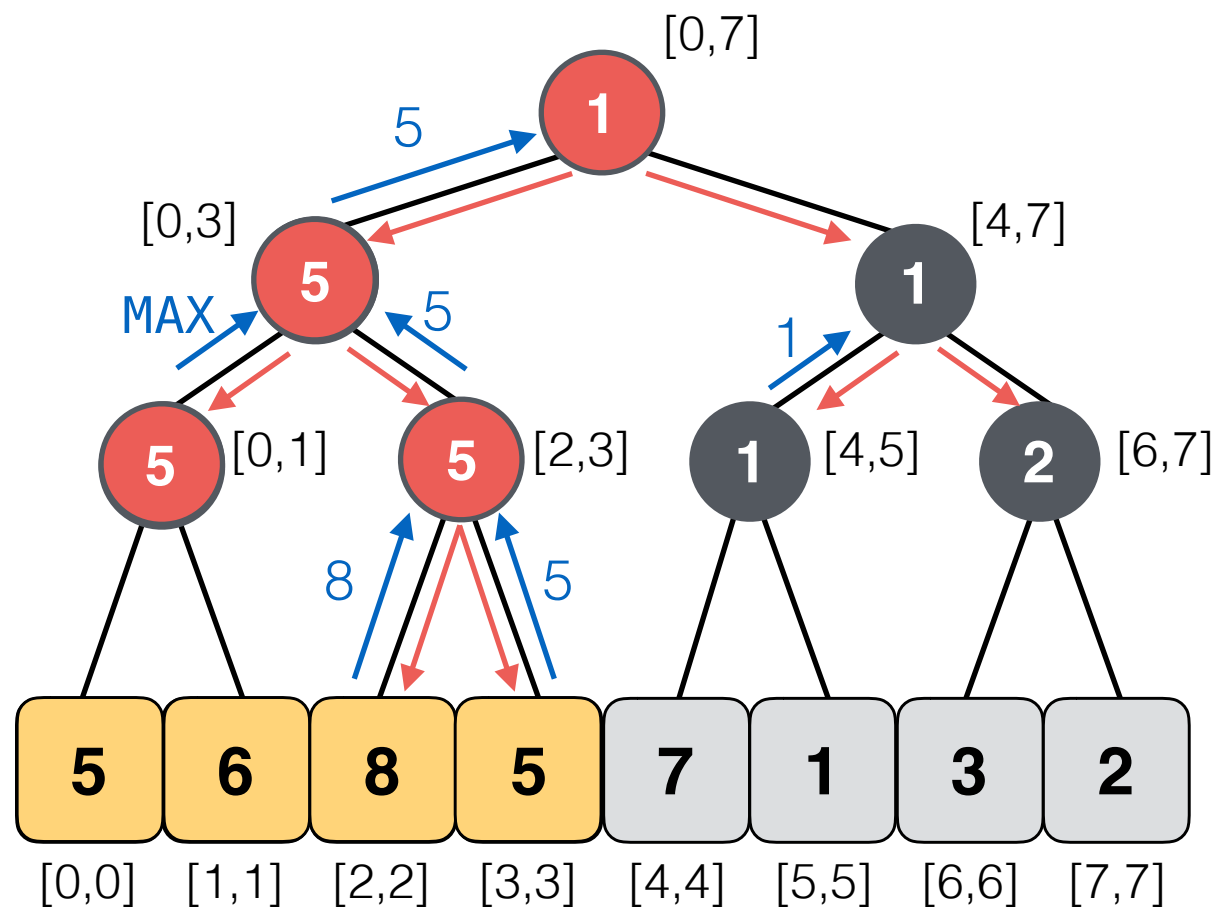
Lazy Tree

Lazy Propagation in Segment Trees

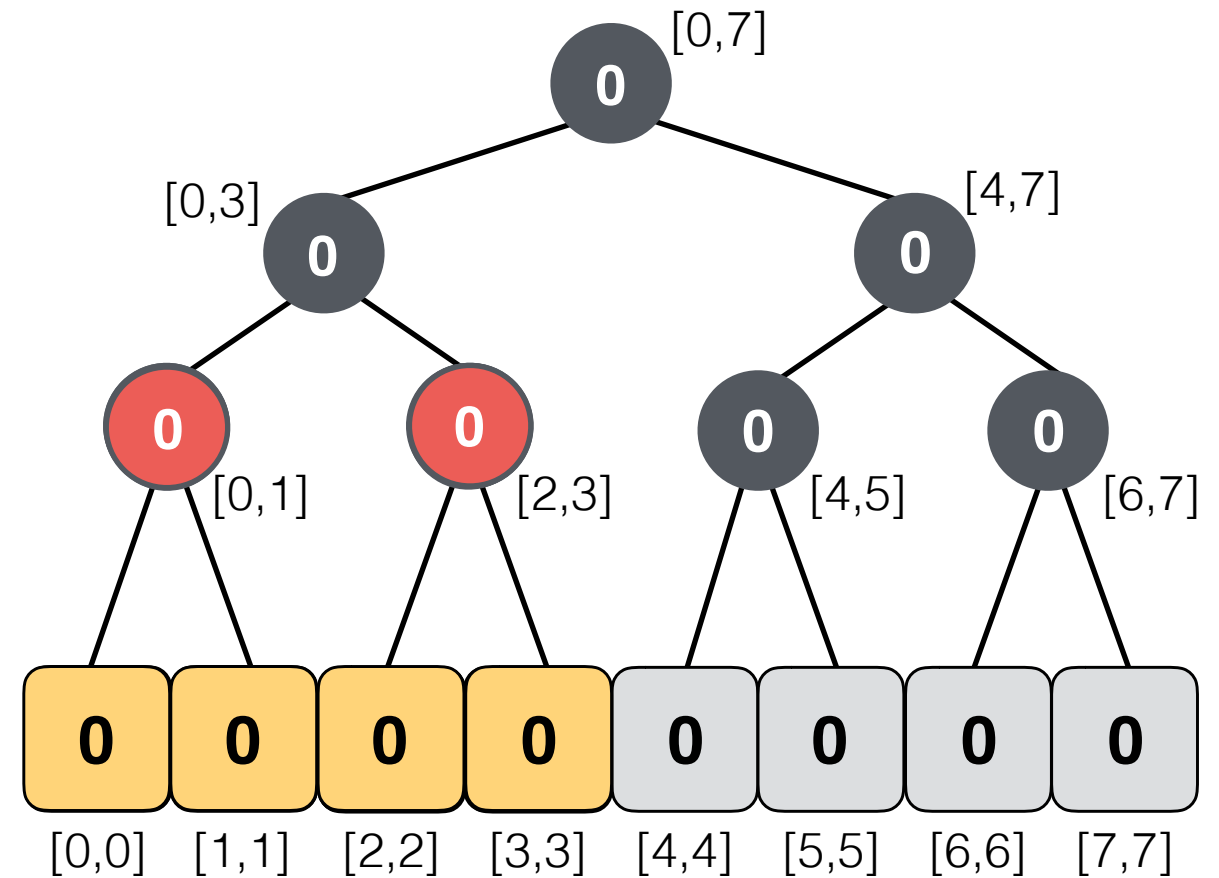
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



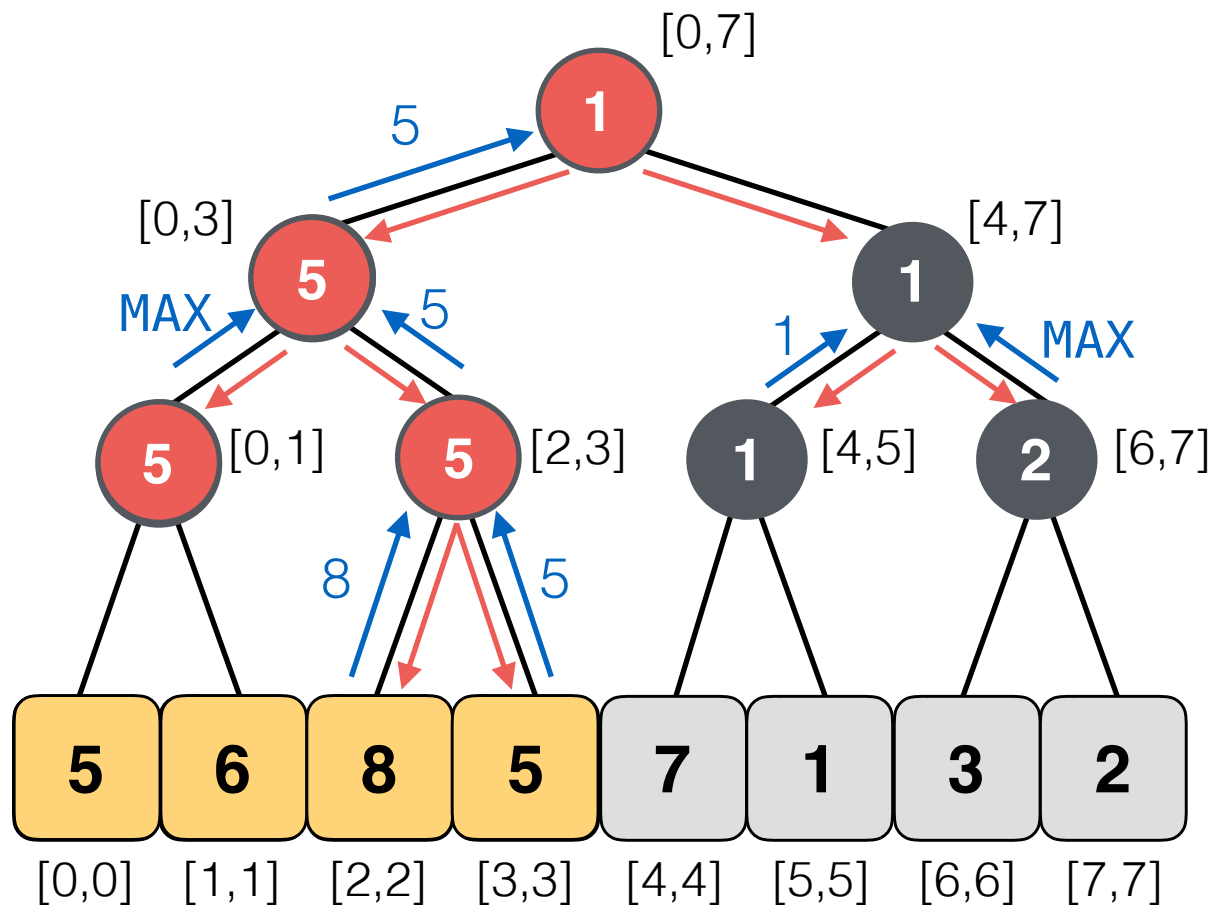
Lazy Tree

Lazy Propagation in Segment Trees

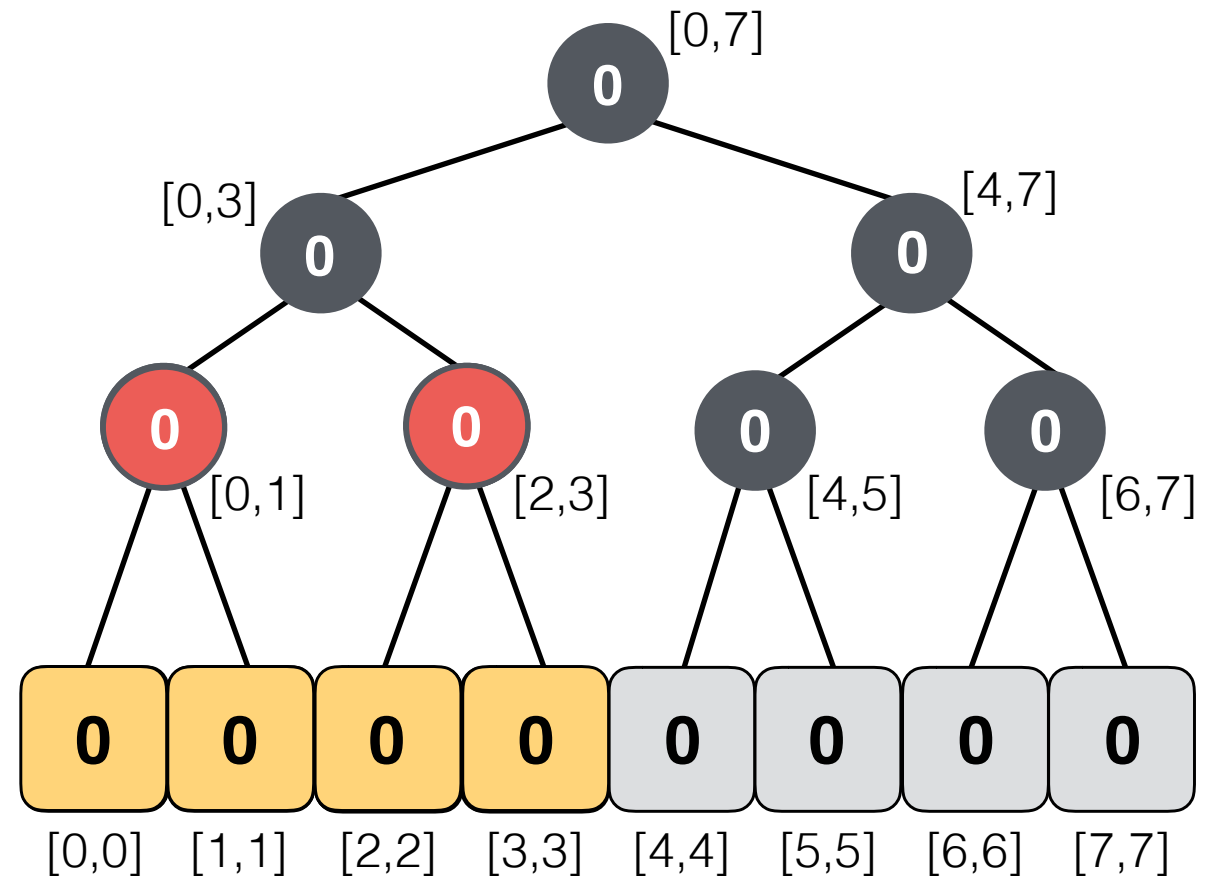
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



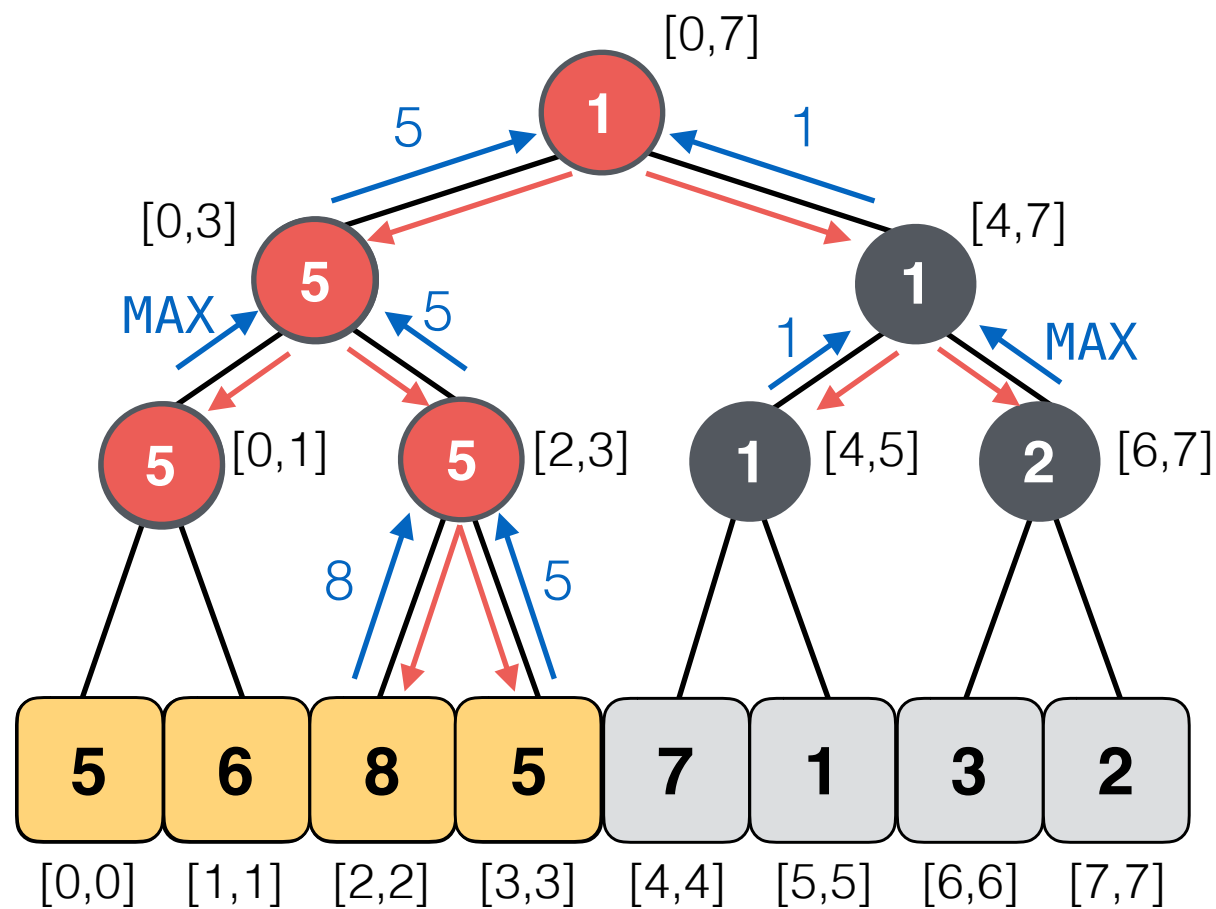
Lazy Tree

Lazy Propagation in Segment Trees

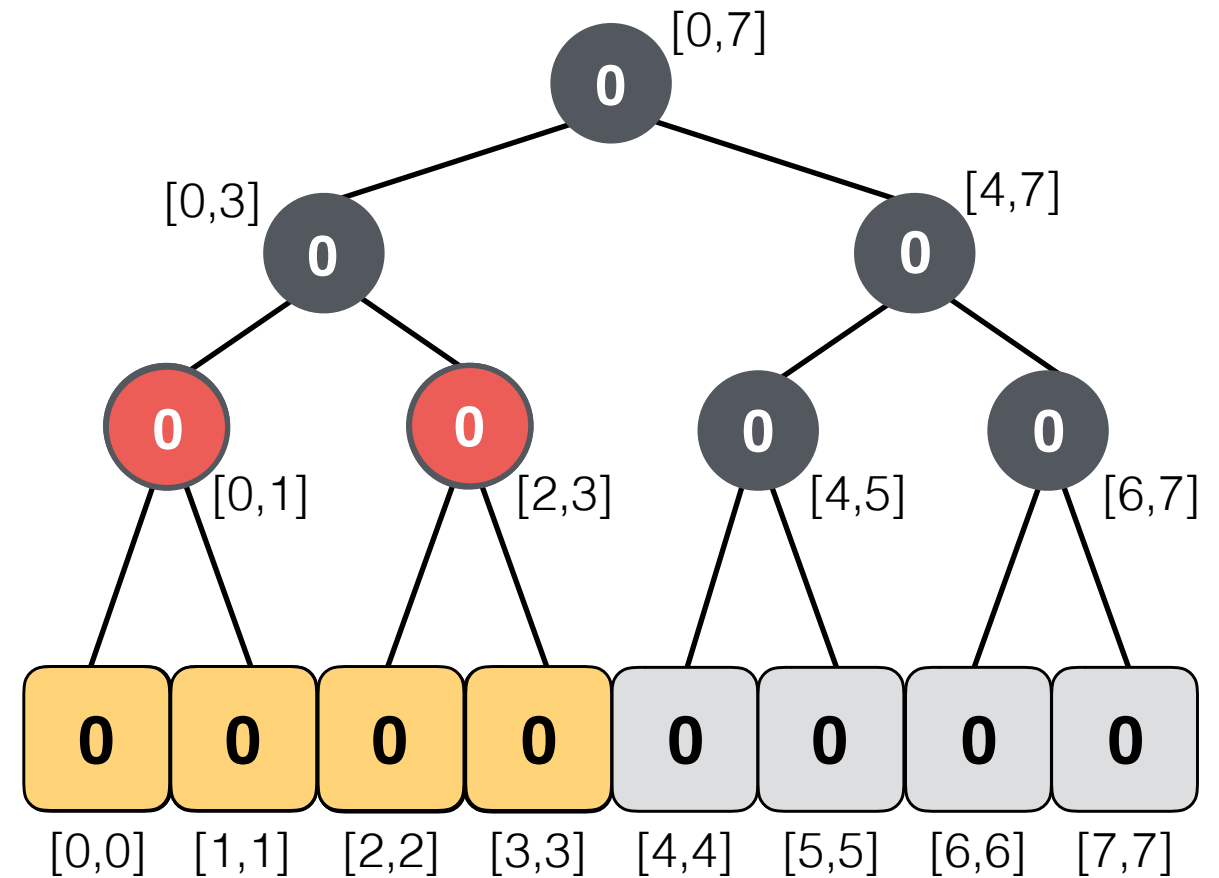
Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

update_range(0,3,3)
update_range(0,3,1)
update_range(0,0,2)

→ rmq(3,5) = ?



Segment Tree



Lazy Tree

Exercises

Implement lazy propagation and test the difference in running time for a mix of updates/queries.

<http://www.geeksforgeeks.org/lazy-propagation-in-segment-tree/>

<http://www.cdn.geeksforgeeks.org/segment-tree-set-1-sum-of-given-range/>

References

Full segment tree code and benchmark at:

https://github.com/rossanoventurini/CompetitiveProgramming/tree/master/code/segment_trees

Video lectures:

https://www.youtube.com/watch?v=ZBHKZF5w4YU&list=PLrmLmBdmIIPv_jNDXtJGYTPNQ2L1gdHxu&index=22

https://www.youtube.com/watch?v=xuoQdt5pHj0&index=23&list=PLrmLmBdmIIPv_jNDXtJGYTPNQ2L1gdHxu

