Segment trees

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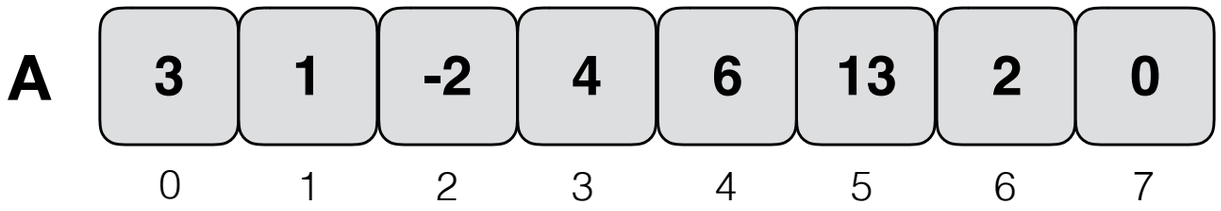
25/10/2017

Problems we would like to solve efficiently



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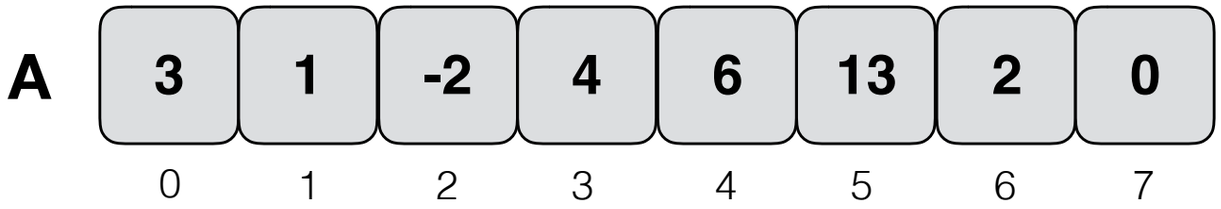

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- `sum`(i) reports the sum of the first i+1 integers
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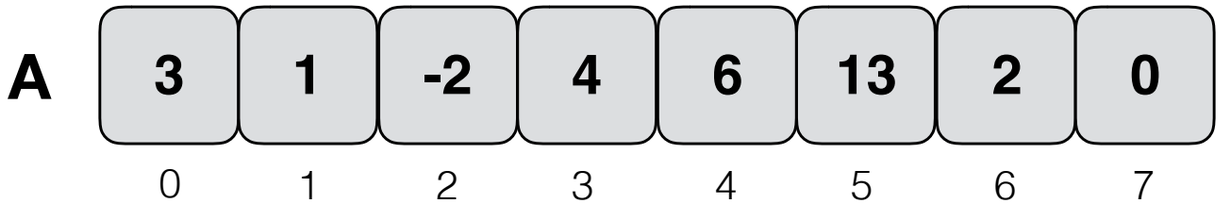

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`sum`(3) = 6
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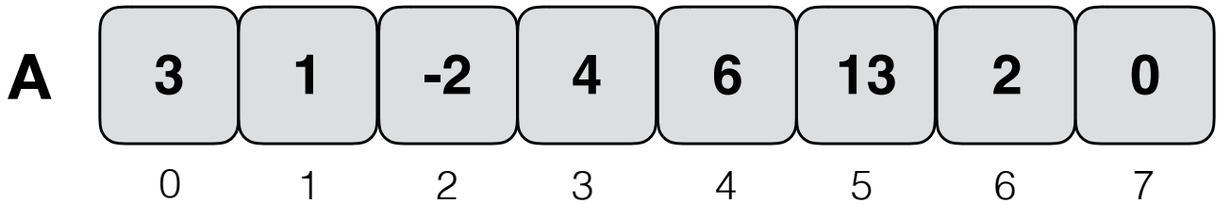

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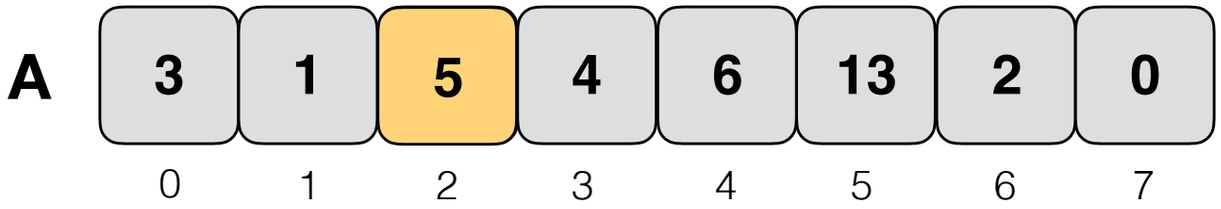

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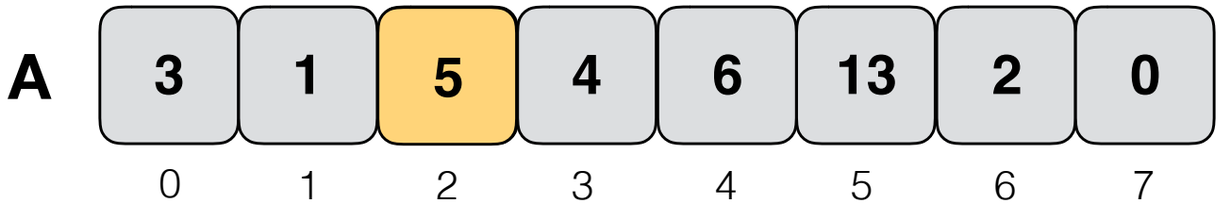

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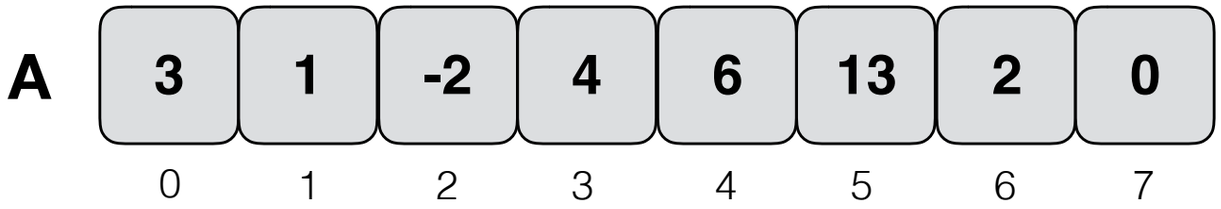

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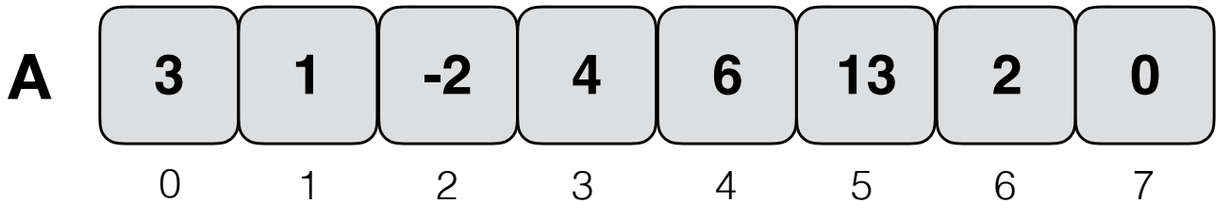

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Range `MIN` (`MAX`) queries

Report the `MIN` (`MAX`) in A[i,j]

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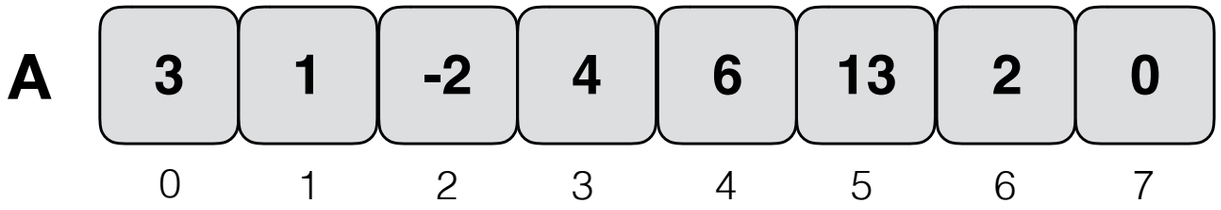

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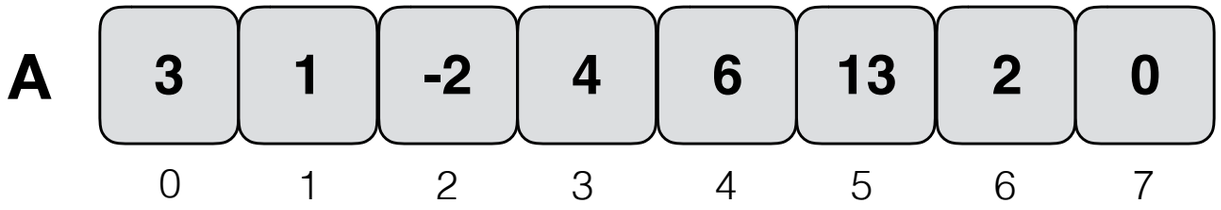
Range `MIN` (`MAX`) queries

Report the `MIN` (`MAX`) in A[i,j]

`min`(1,3) = -2
`max`(4,7) = 0

Problems we would like to solve efficiently



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Range `SUM` queries

Report the sum of the elements in A[i,j]

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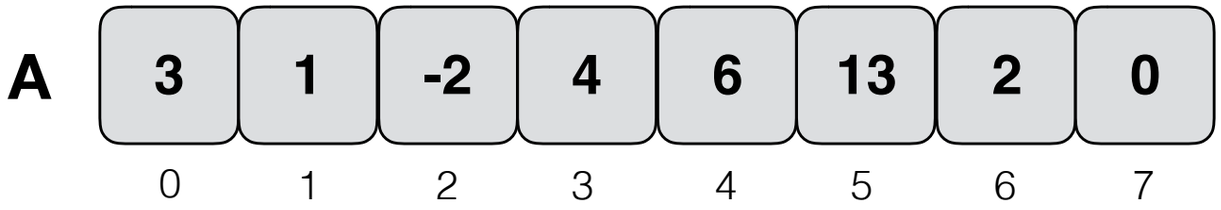

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Report the `MIN` (`MAX`) in A[i,j]

`min`(1,3) = -2
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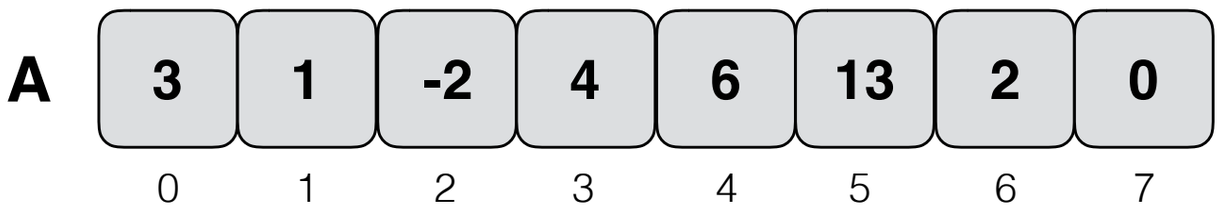
Range `SUM` queries

Report the sum of the elements in A[i,j]

`sum`(1,3) = 3
`sum`(4,7) = 21

Any solutions?



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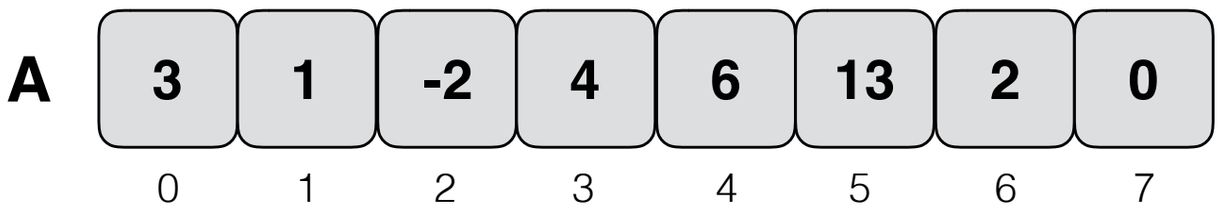
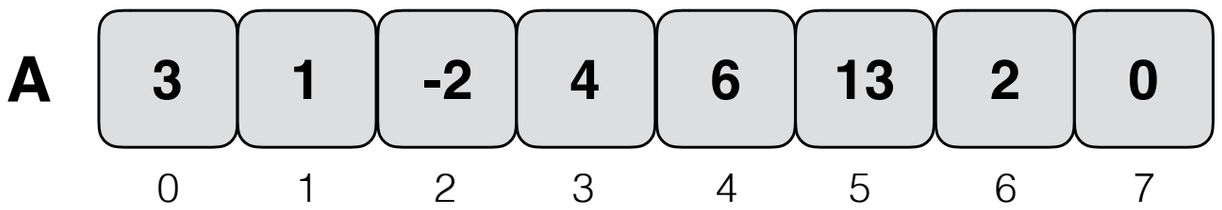


1. Do nothing
2. Pre-calculate all queries

Any solutions?



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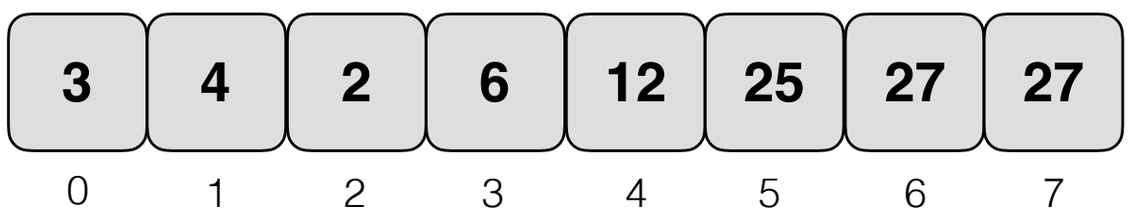
(Static) Prefix sums

1. `update`: O(1)
   `sum`: O(n)
   Space: no auxiliary space
2. `update`: O(n)
   `sum`: O(1)
   Space: no auxiliary space



Any solutions?

A

3	1	-2	4	6	13	2	0
0	1	2	3	4	5	6	7

1. Do nothing
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(Static) Prefix sums

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   `sum`: O(n)
   Space: no auxiliary space
2. `update`: O(n)
   `sum`: O(1)
   Space: no auxiliary space

3	4	2	6	12	25	27	27
0	1	2	3	4	5	6	7

Range `MIN` (`MAX`) and `SUM` queries

1. Query time: O(n)
   Space: no auxiliary space
2. Query time: O(1)
   Space: $O(n^2)$
   Building time: $O(n^2)$

	0	1	2	3	4	5	6	7
0	3	1	-2	-2	-2	-2	-2	-2
1		1	-2	-2	-2	-2	-2	-2
2			-2	-2	-2	-2	-2	-2
3				4	4	4	2	0
4					6	6	2	0
5						13	2	0
6							2	0
7								0

An efficient solution

Remember

An efficient solution is the one that gives *guaranteed good running times* for **all** operations.

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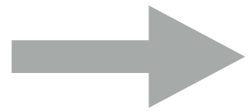

Idea
Impose a complete (static) binary tree over the array: a **segment tree**.

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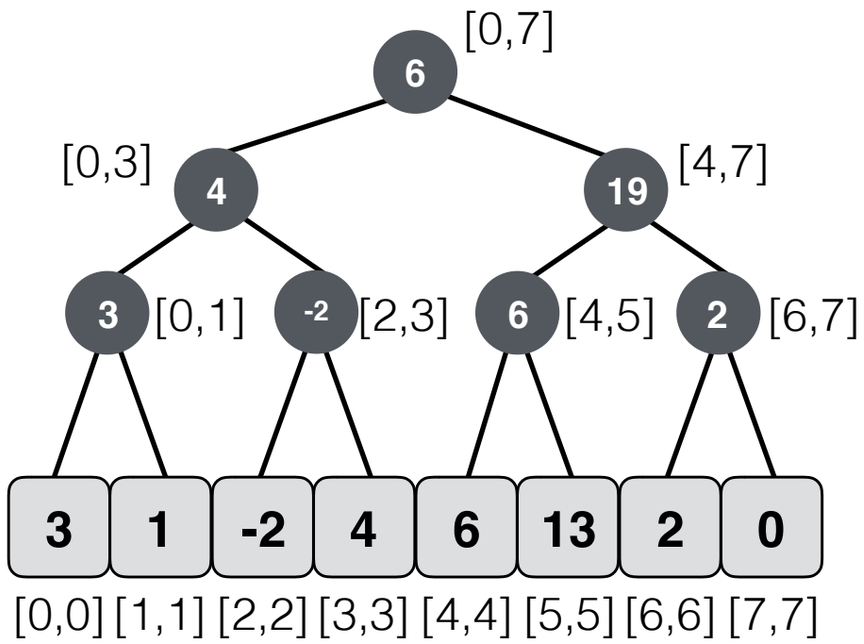

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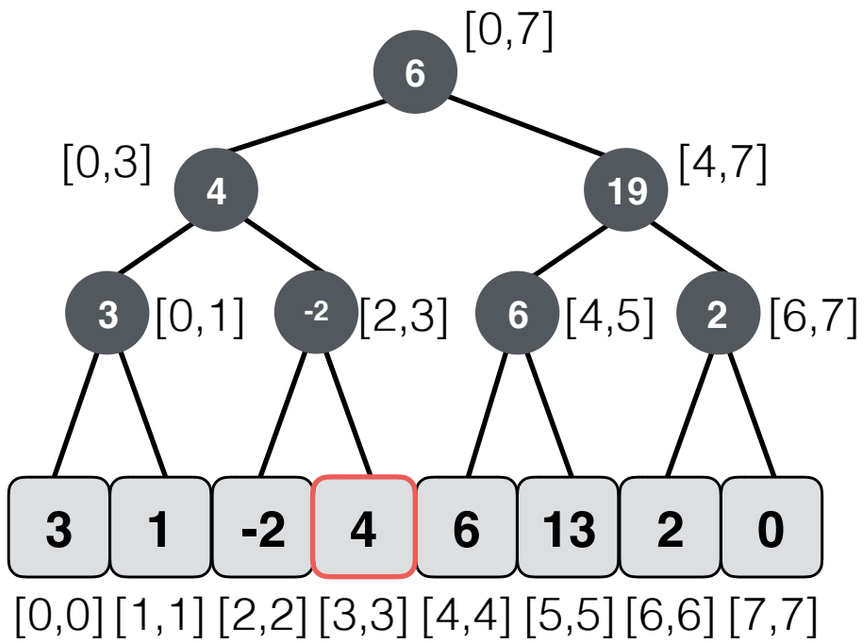

`sum`(3) = (4) +

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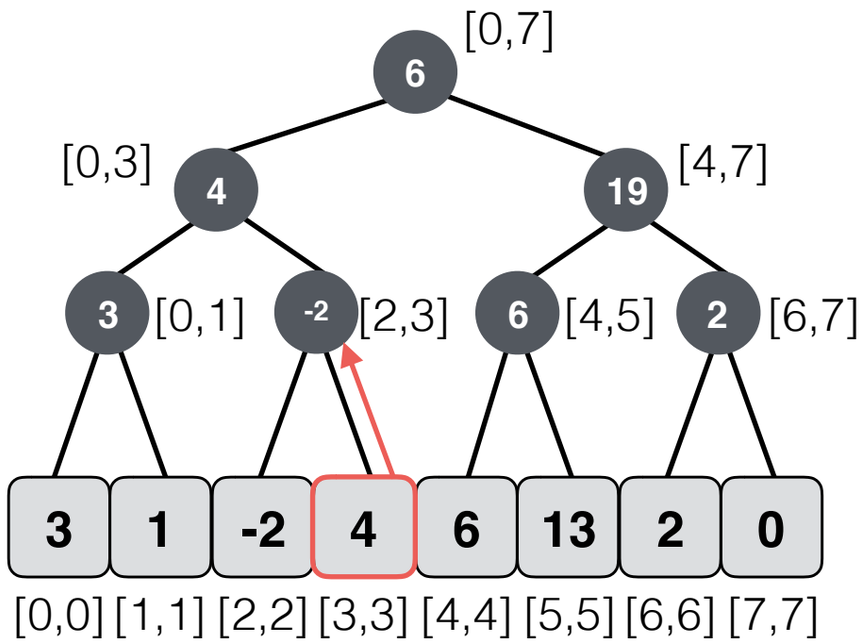

`sum`(3) = (4) + (-2) +

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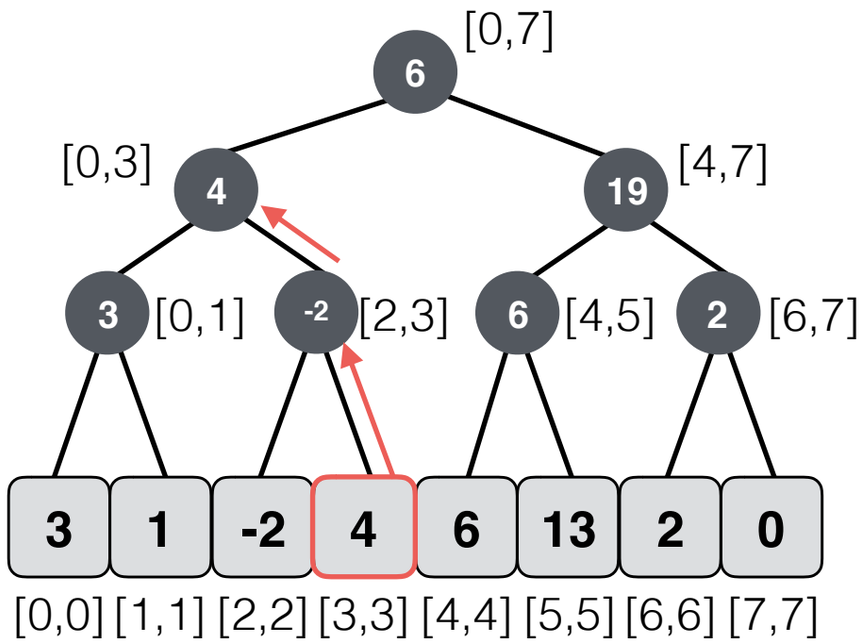

`sum`(3) = (4) + (-2) + (4) = 6

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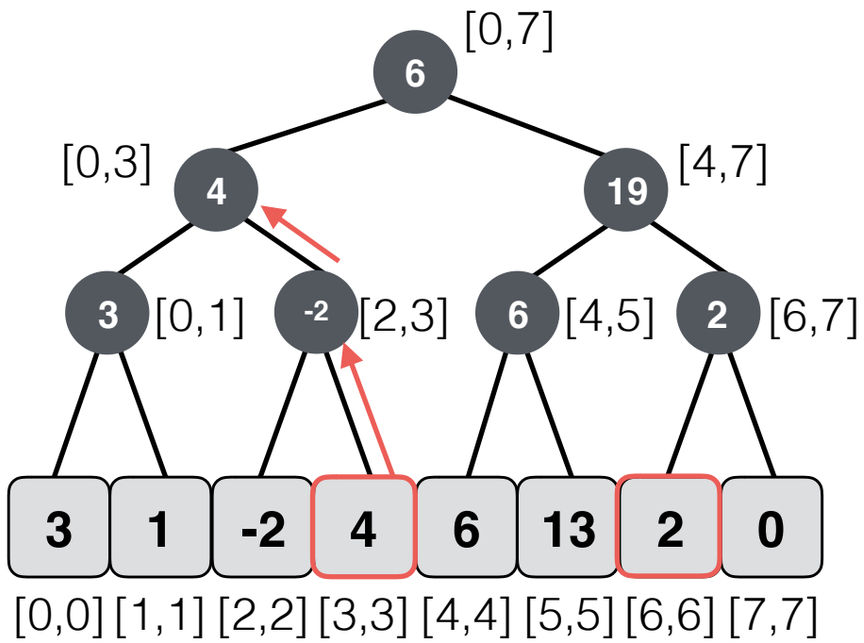

`sum`(3) = (4) + (-2) + (4) = 6
`sum`(6) = (2) +

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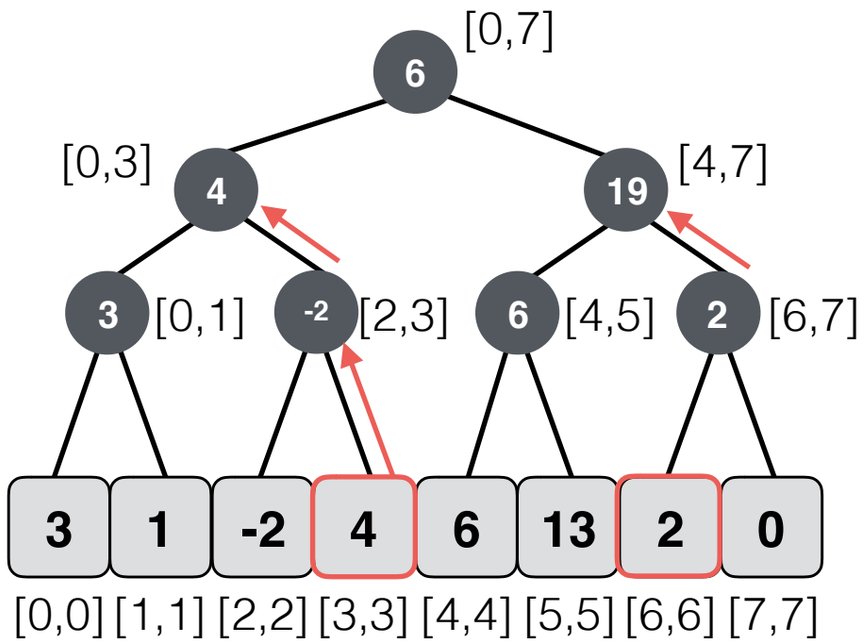

`sum`(3) = (4) + (-2) + (4) = 6
`sum`(6) = (2) + (19)+

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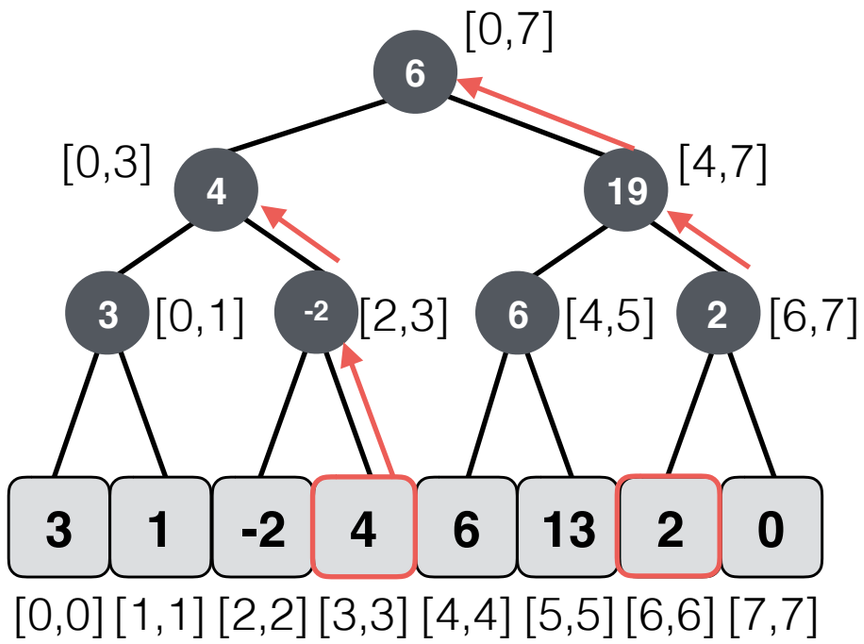

`sum`(3) = (4) + (-2) + (4) = 6
`sum`(6) = (2) + (19)+ (6) = 27

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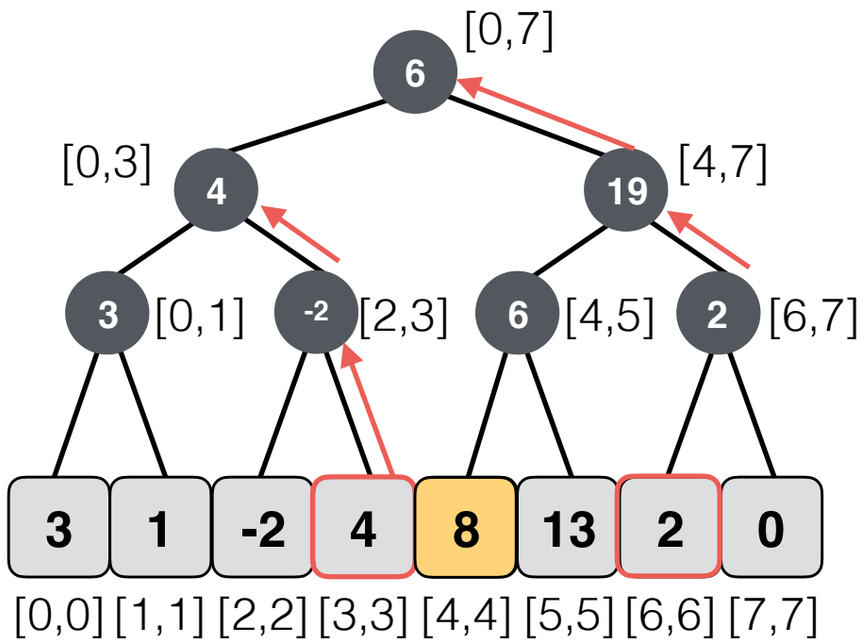

`sum`(3) = (4) + (-2) + (4) = 6
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`update`(4, 8)

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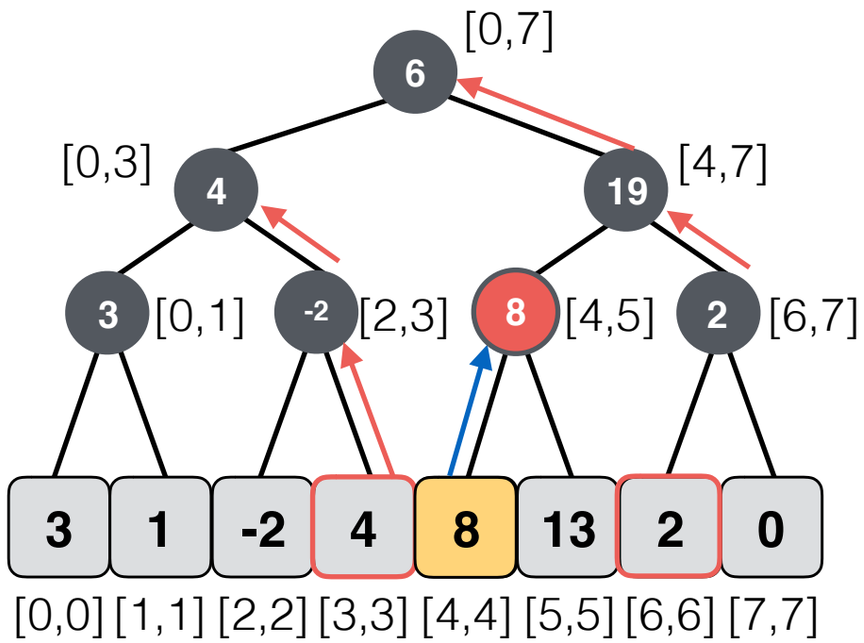

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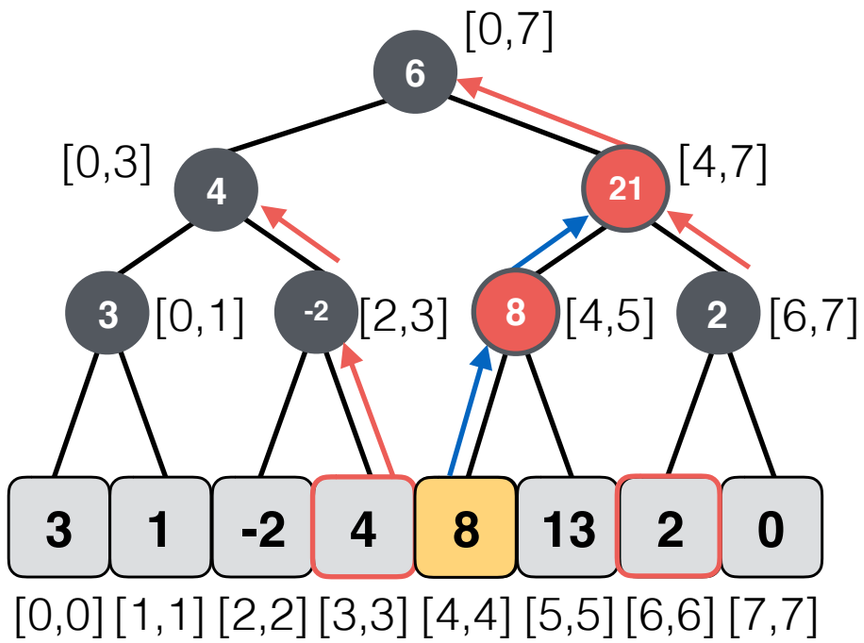

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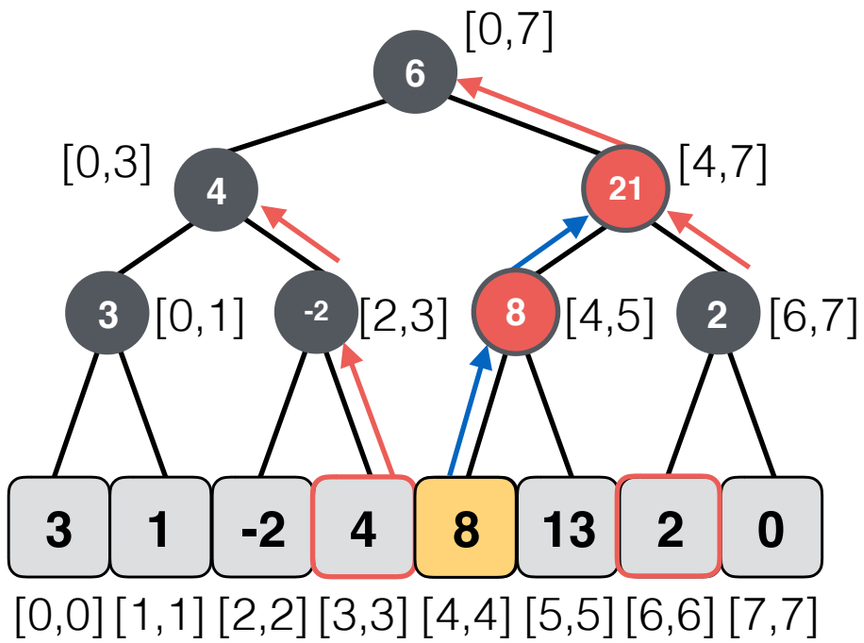

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`sum` and `update` in O(log n)

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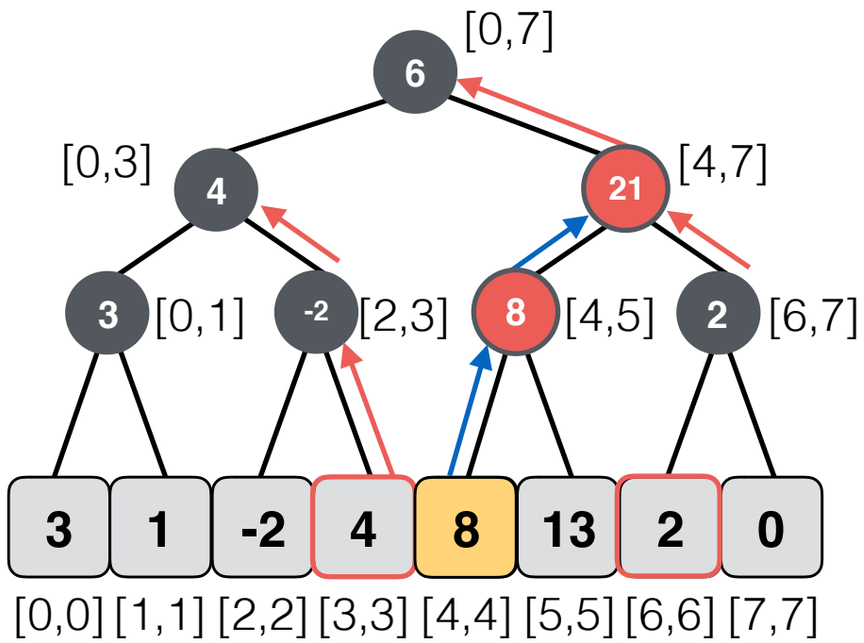

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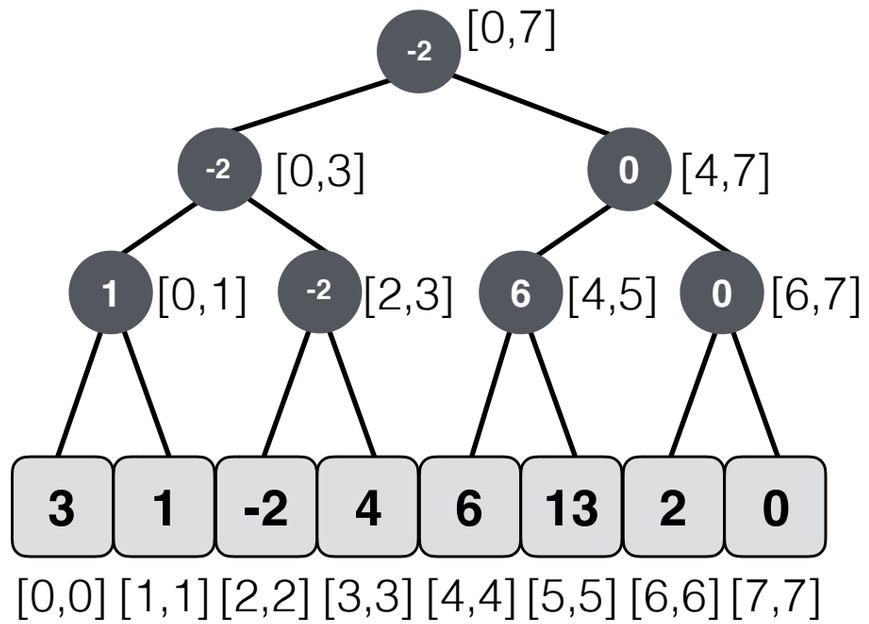
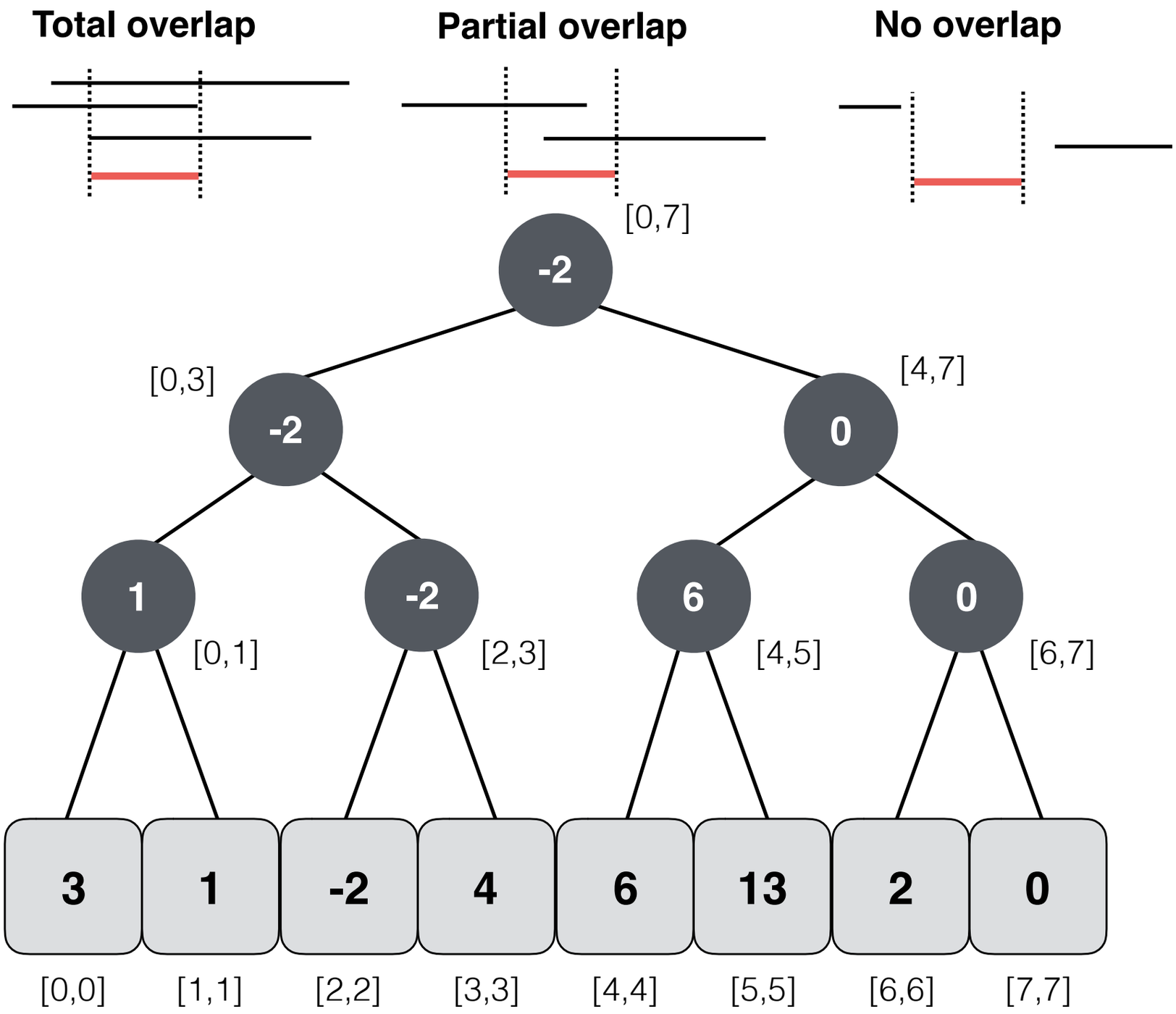
What we consider next, stay tuned!

Range MIN queries



Range `MIN` Queries with Segment Trees

Consider a segment tree with n leaves (2n - 1 nodes in total).



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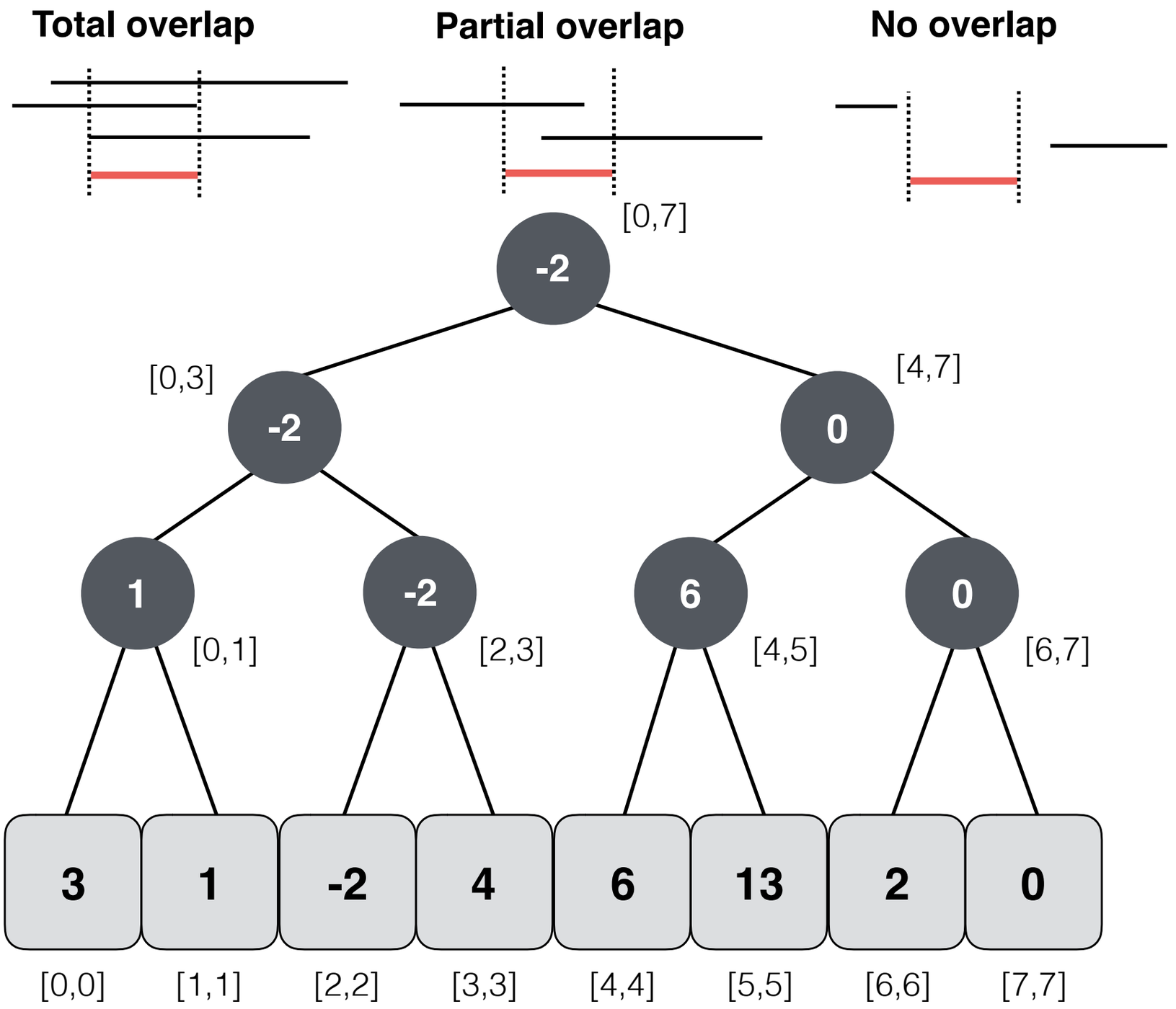
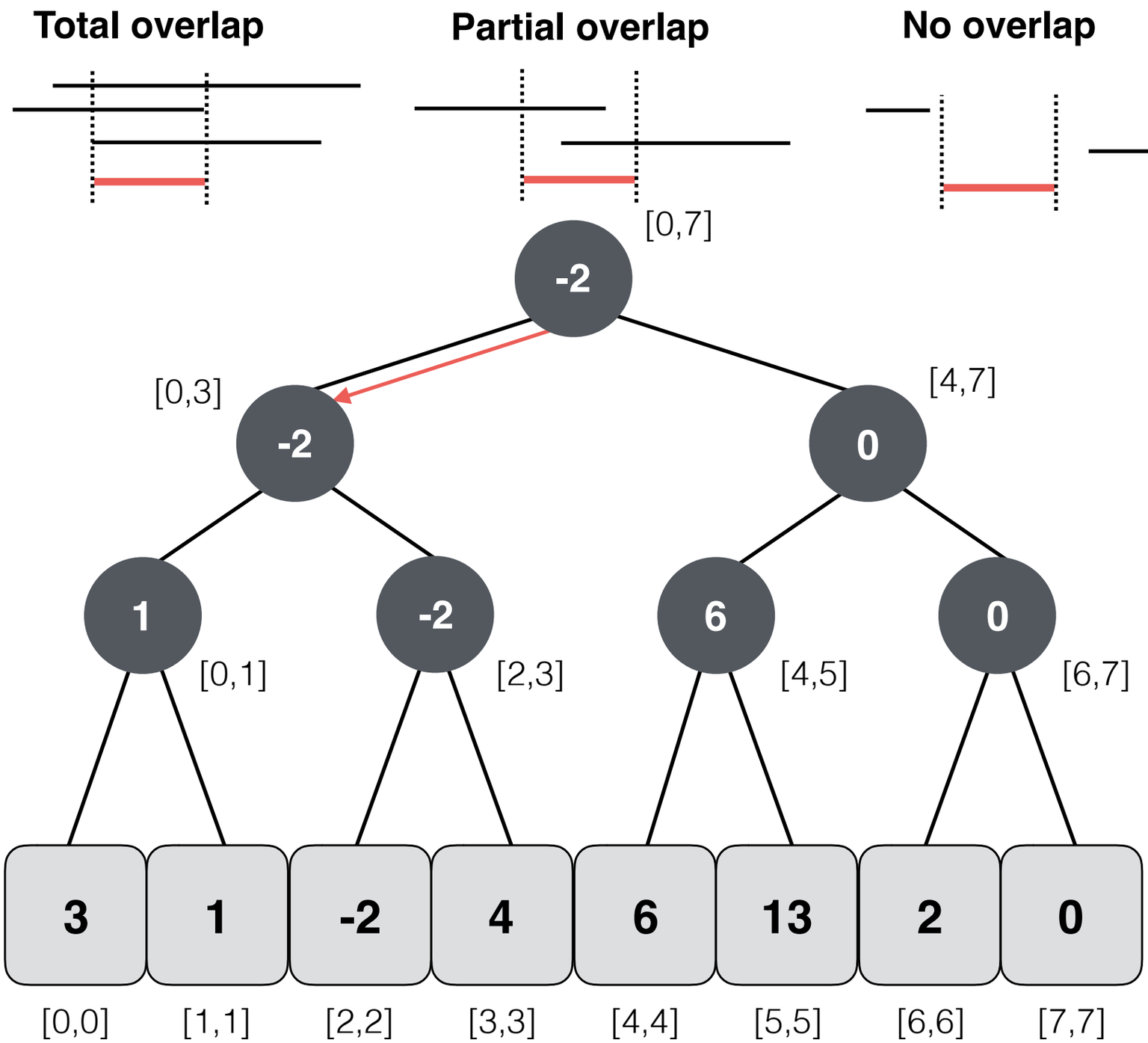


`min`(1,3)

Range `MIN` Queries with Segment Trees

Consider a segment tree with n leaves (2n - 1 nodes in total).



`min`(1,3)

Range `MIN` Queries with Segment Trees

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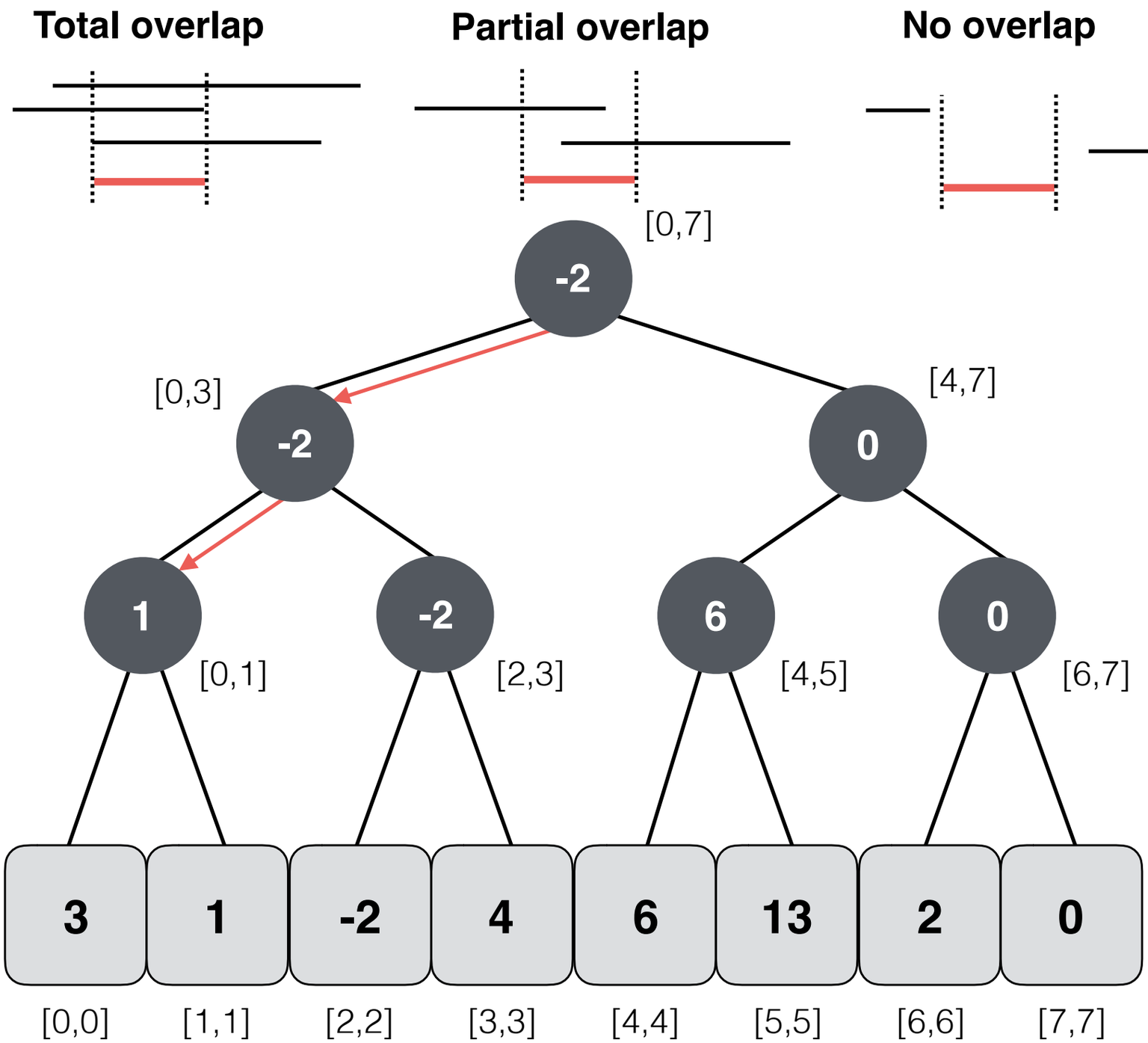

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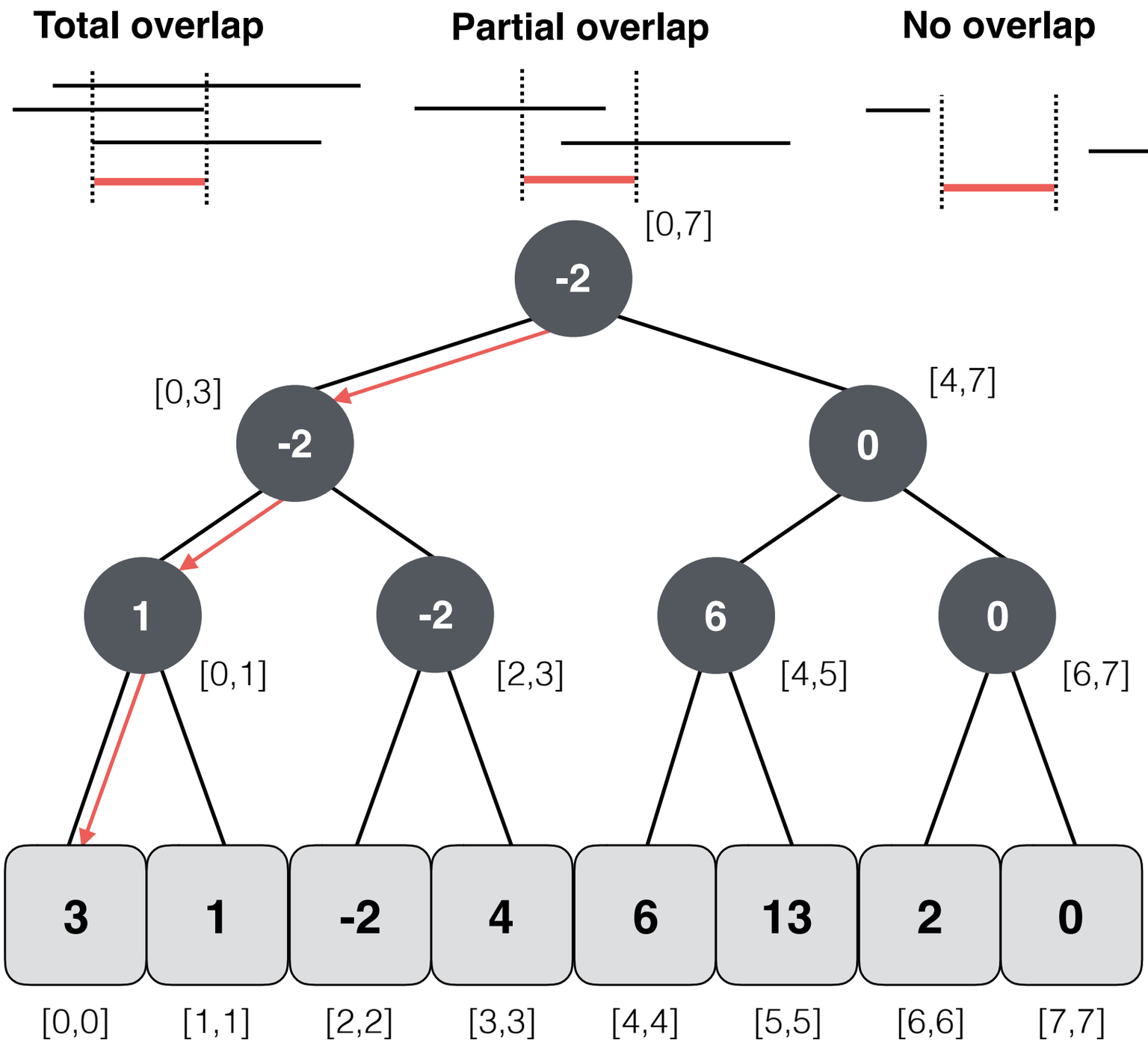

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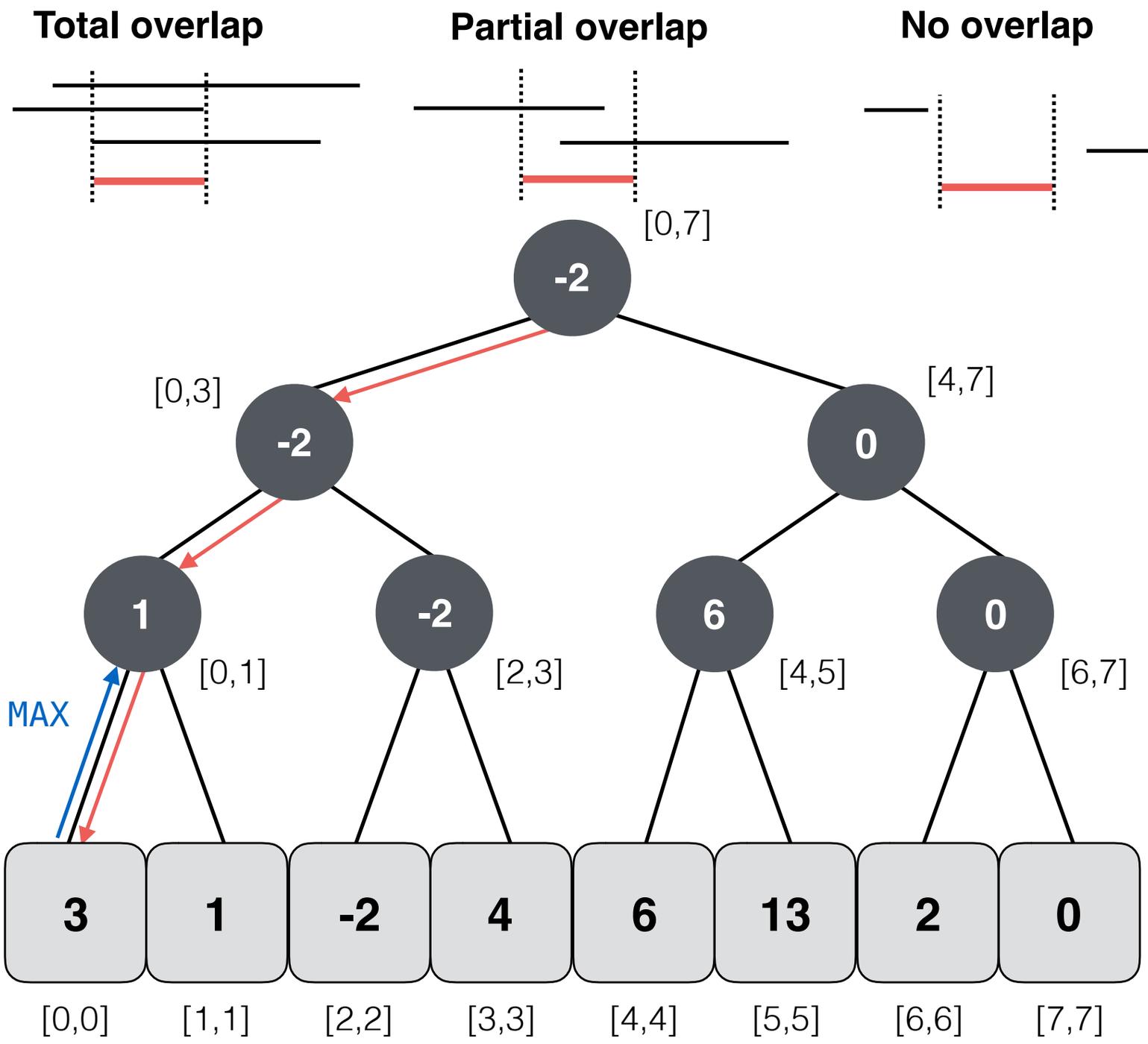

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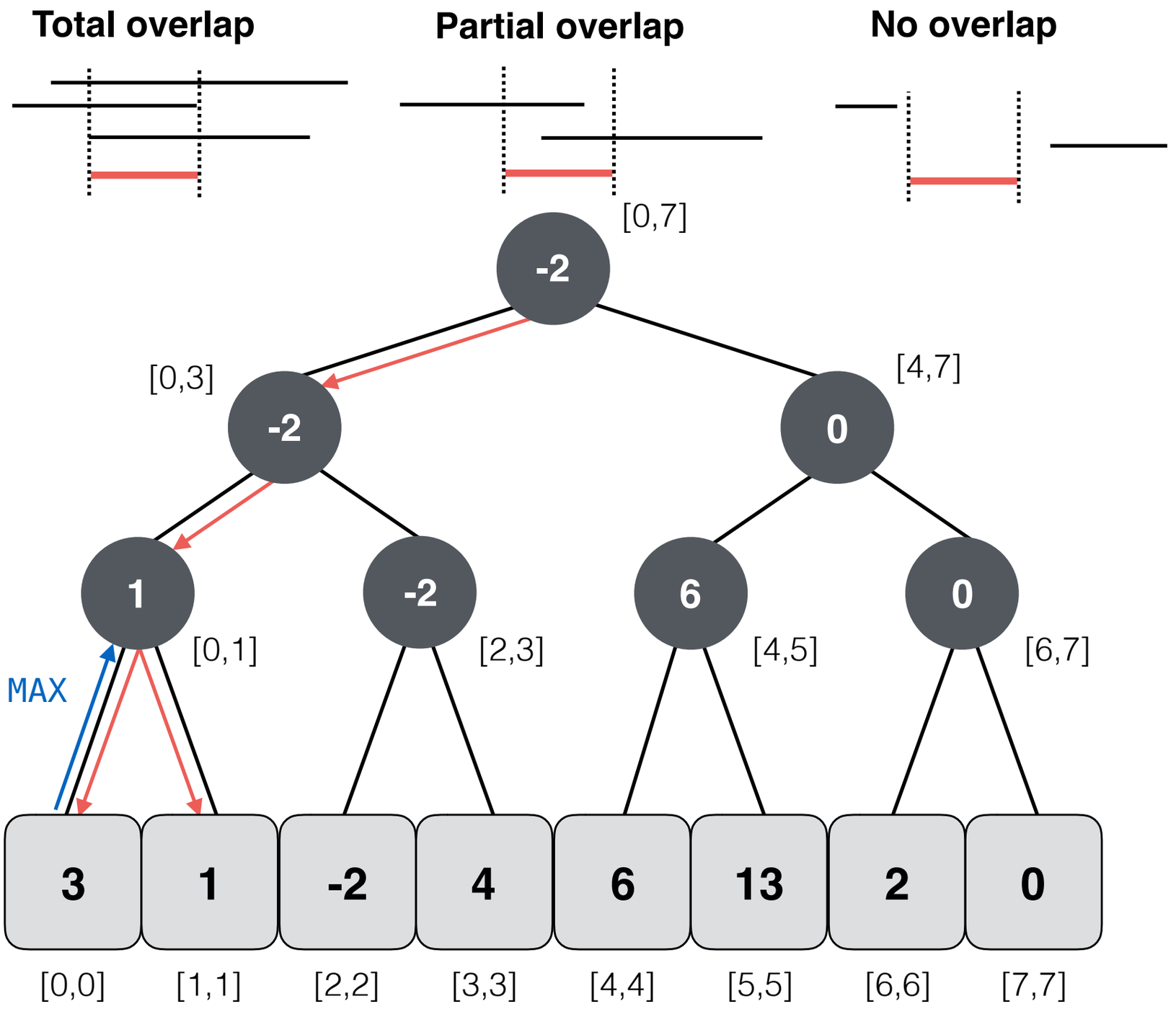

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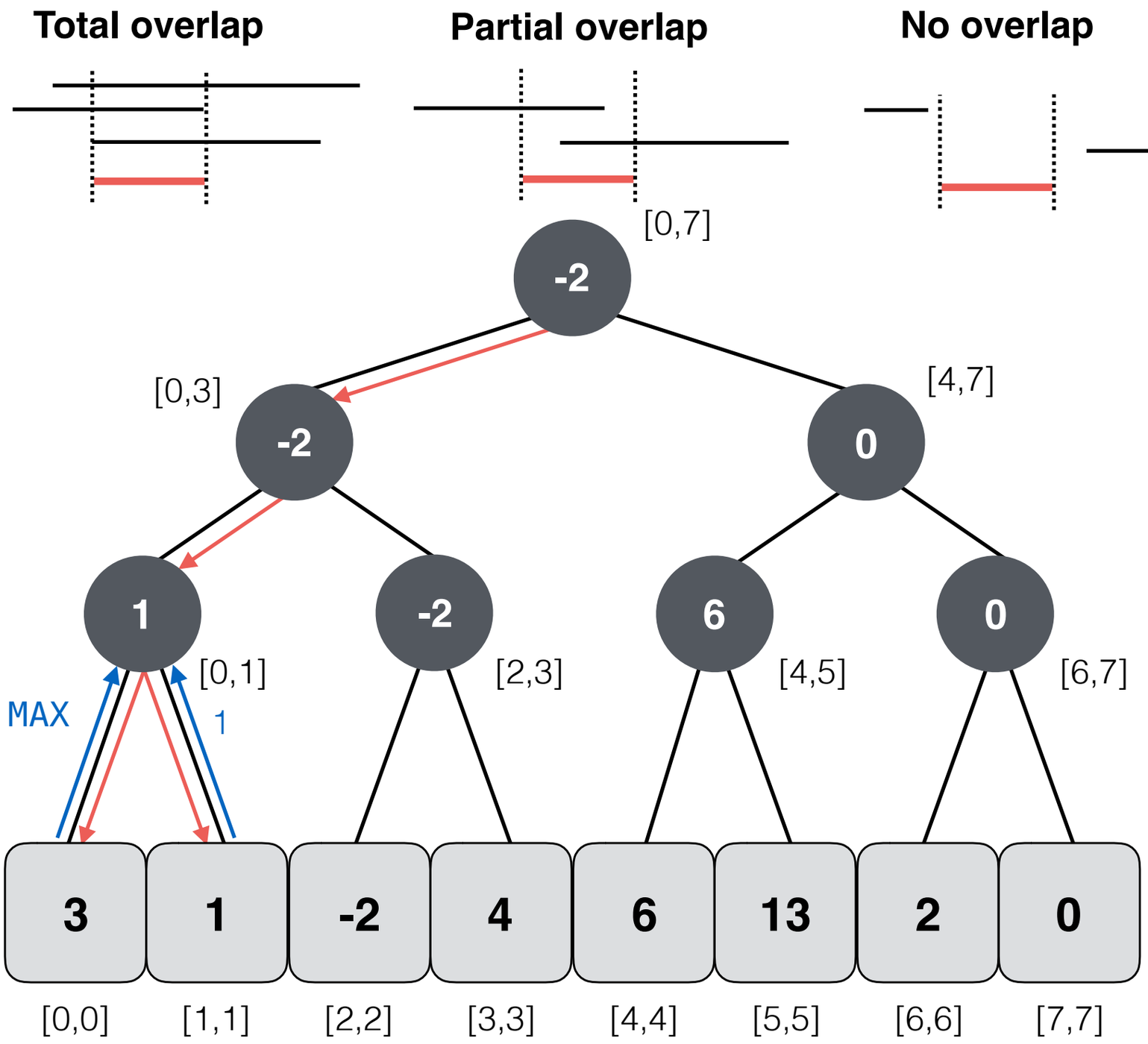

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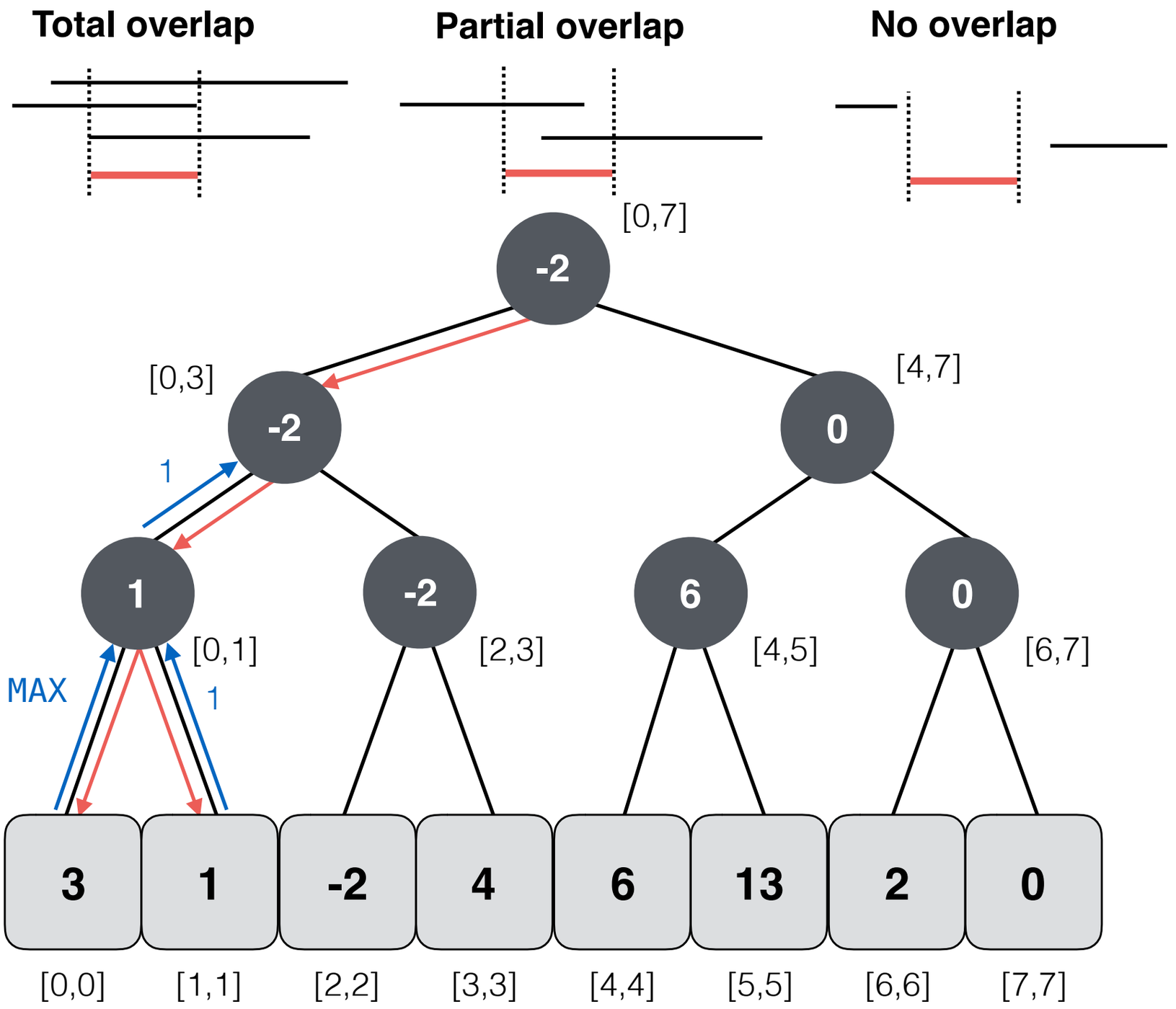

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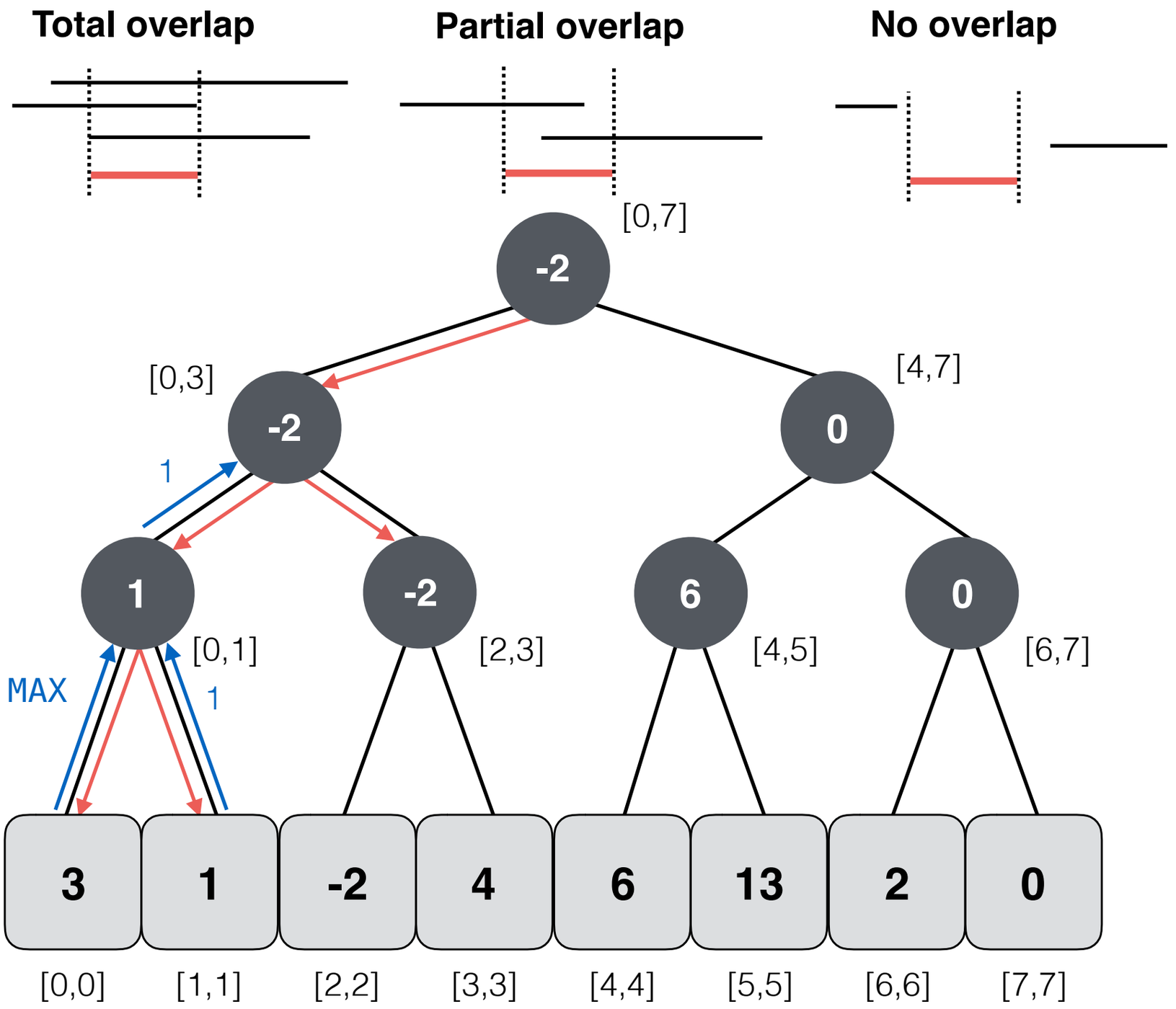

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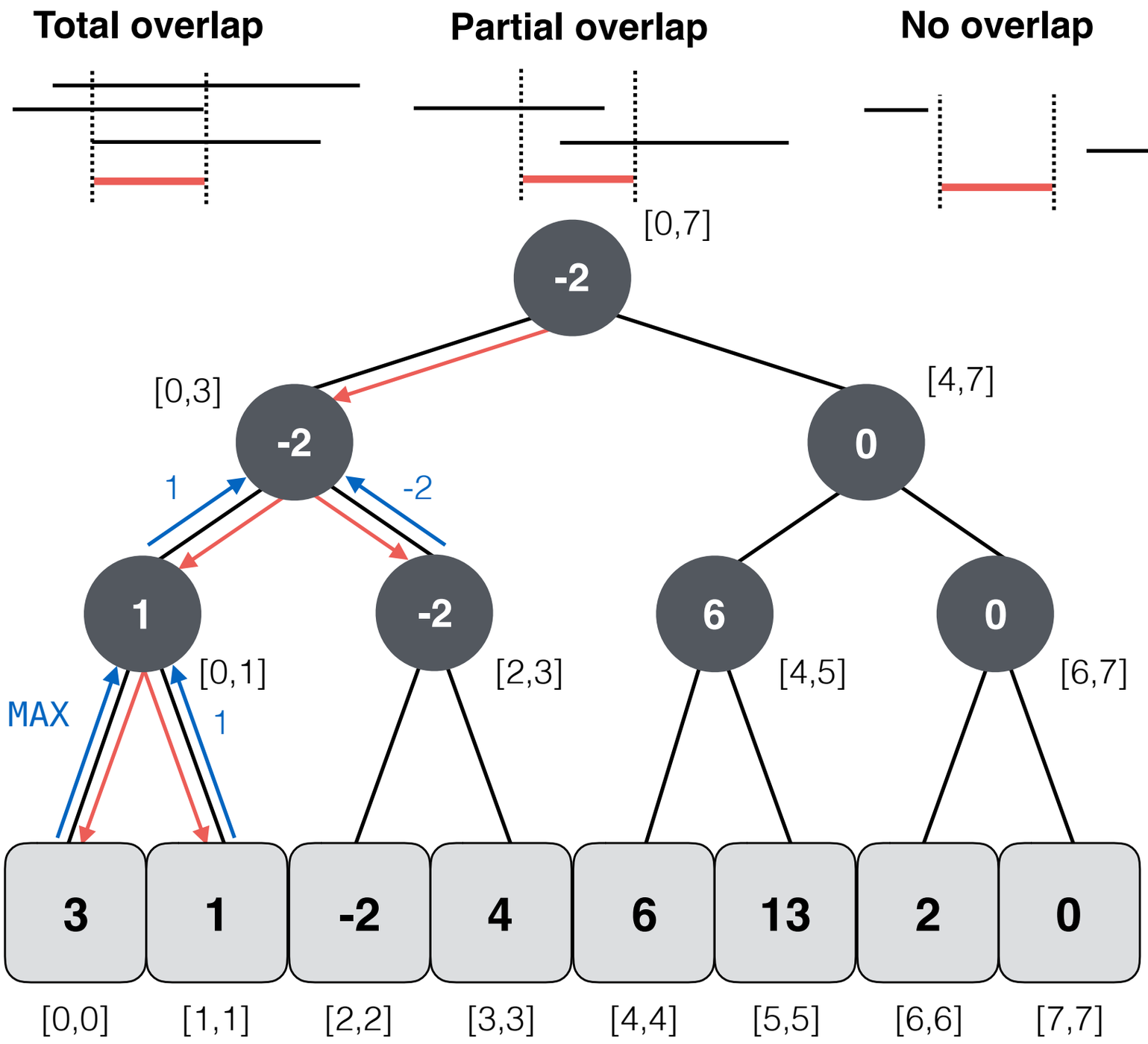

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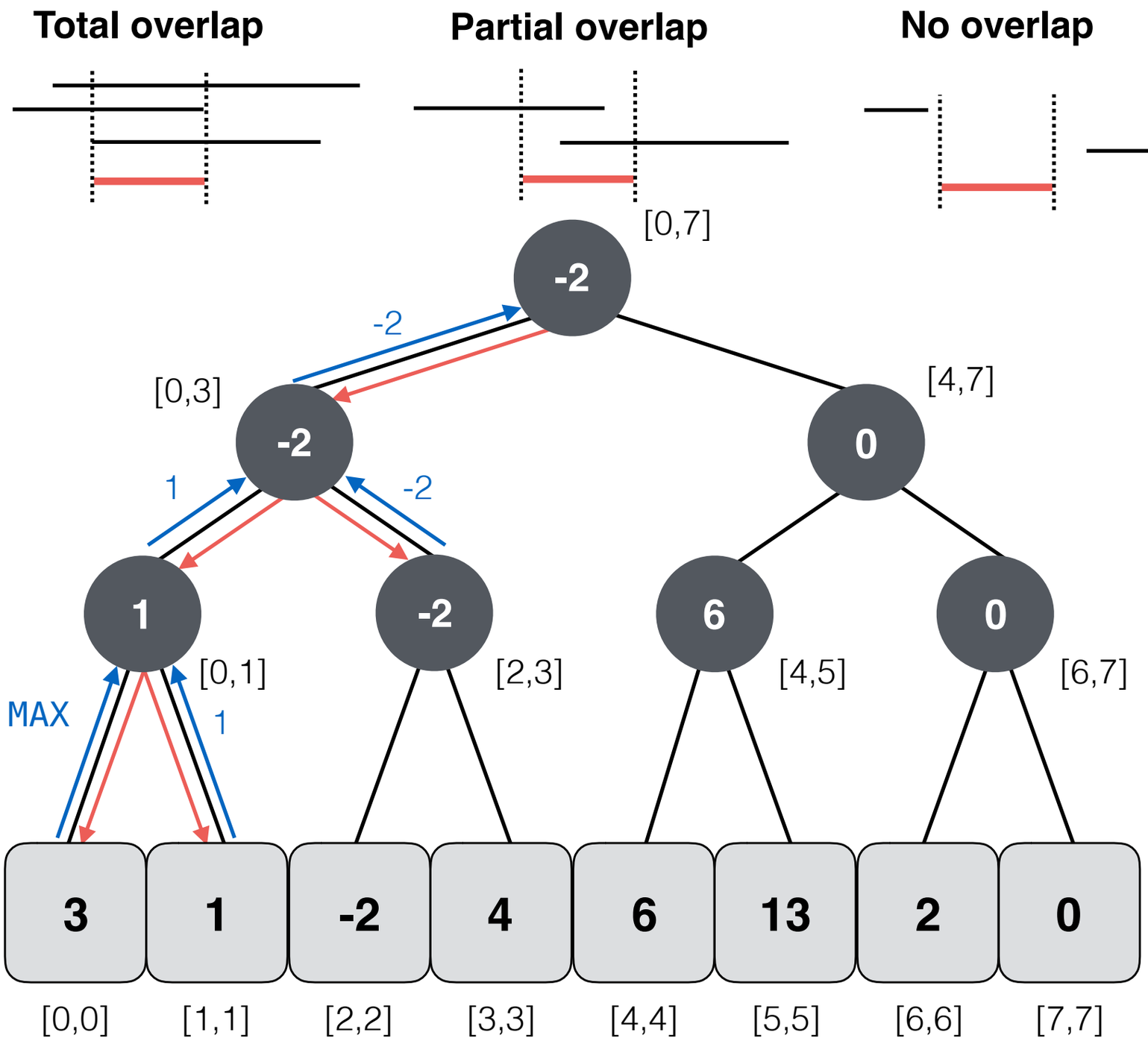

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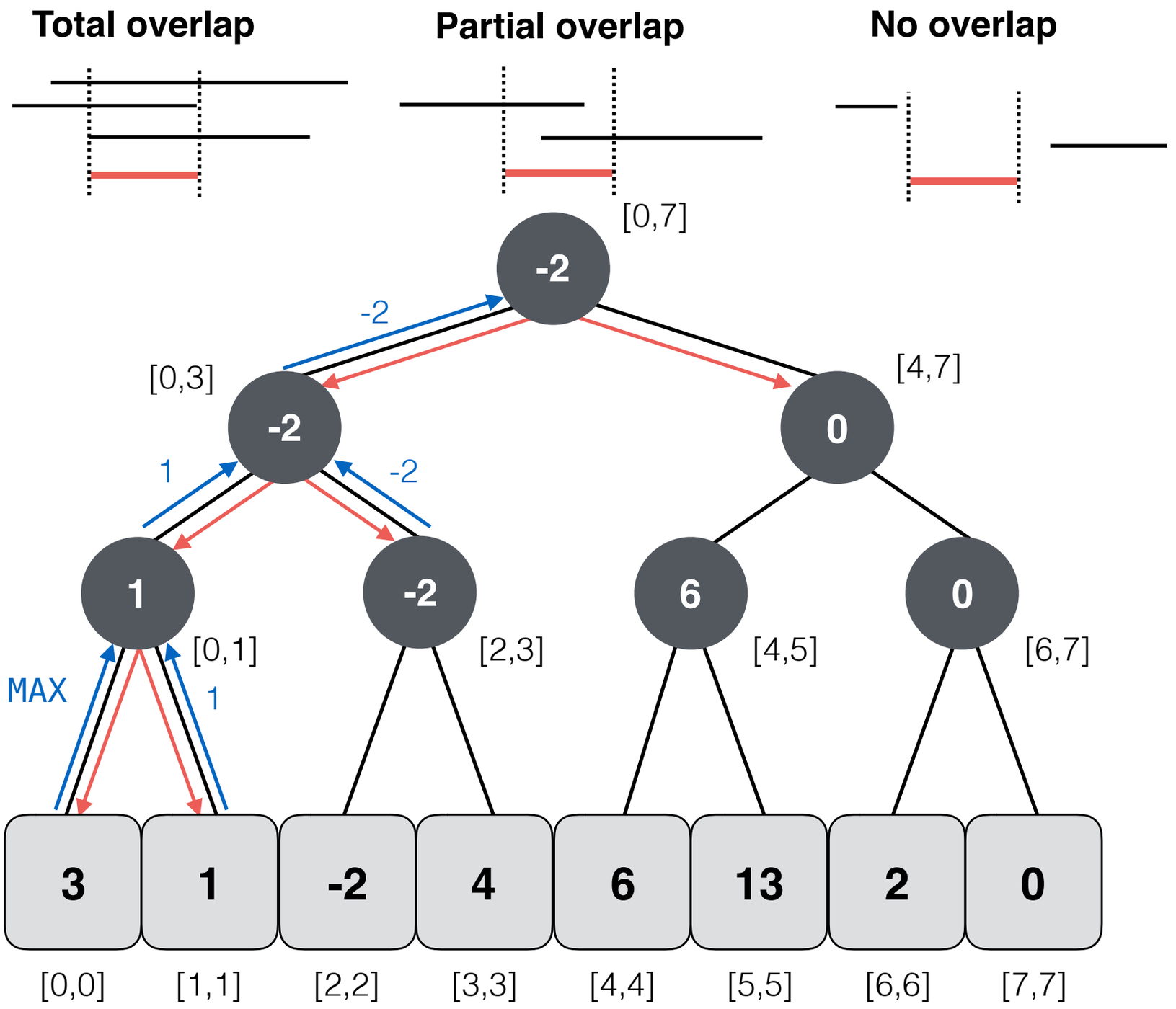

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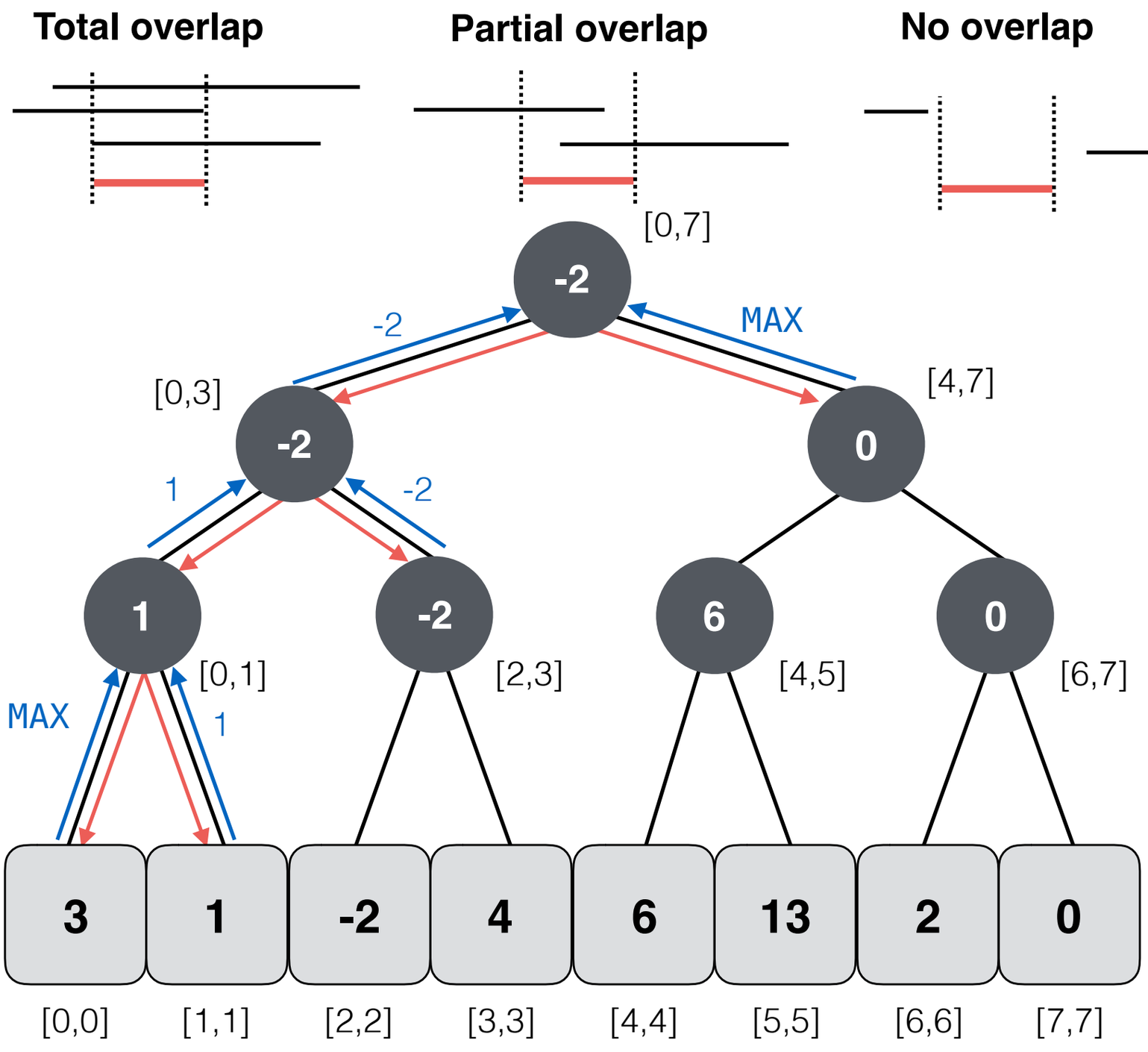

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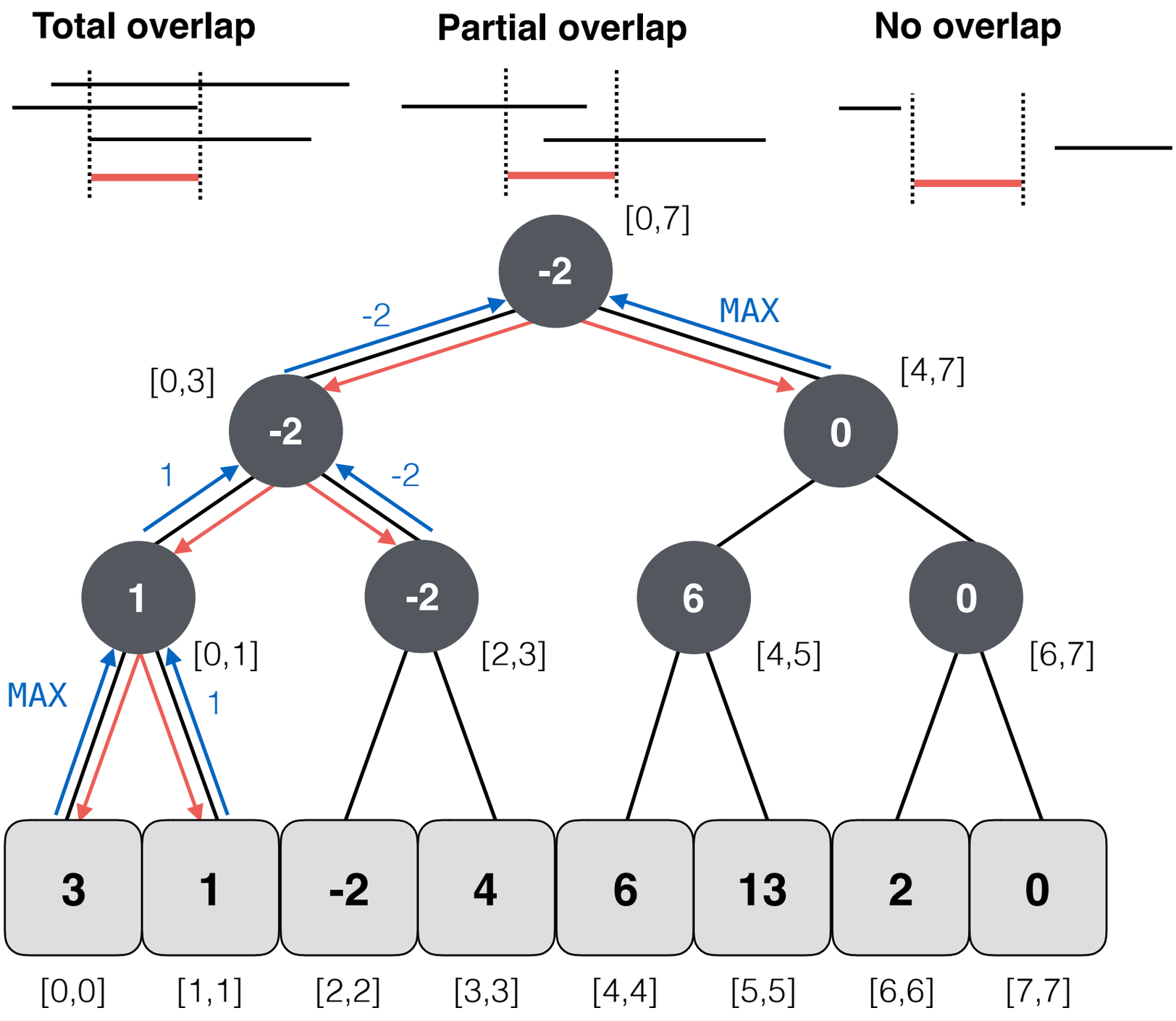

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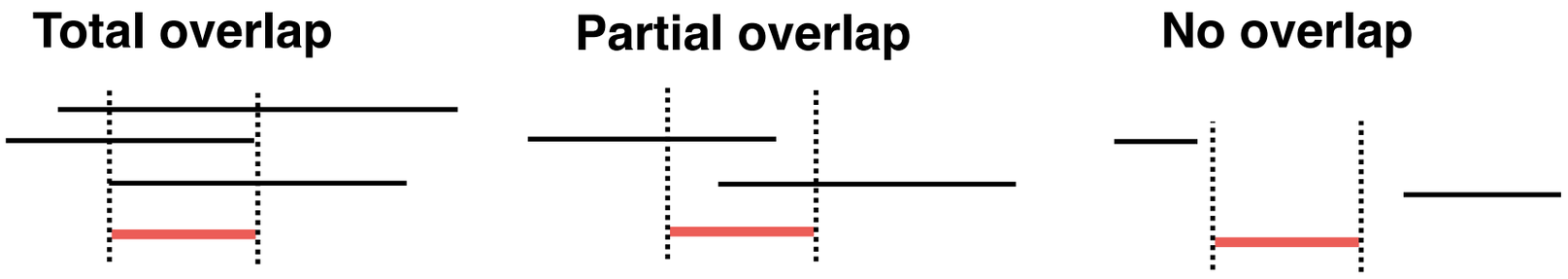

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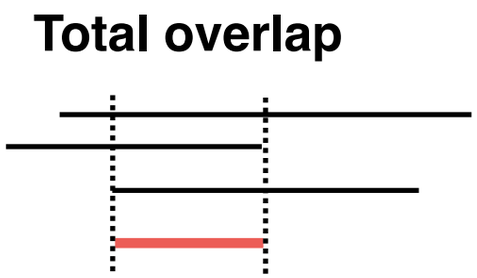

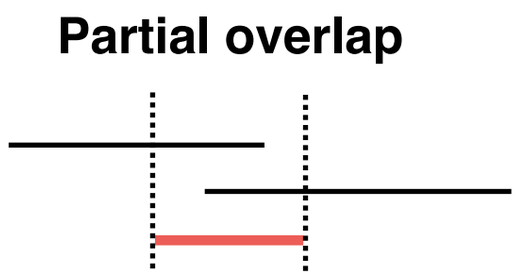

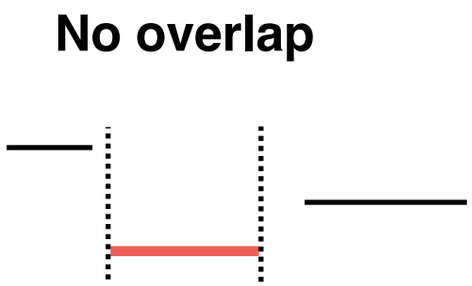

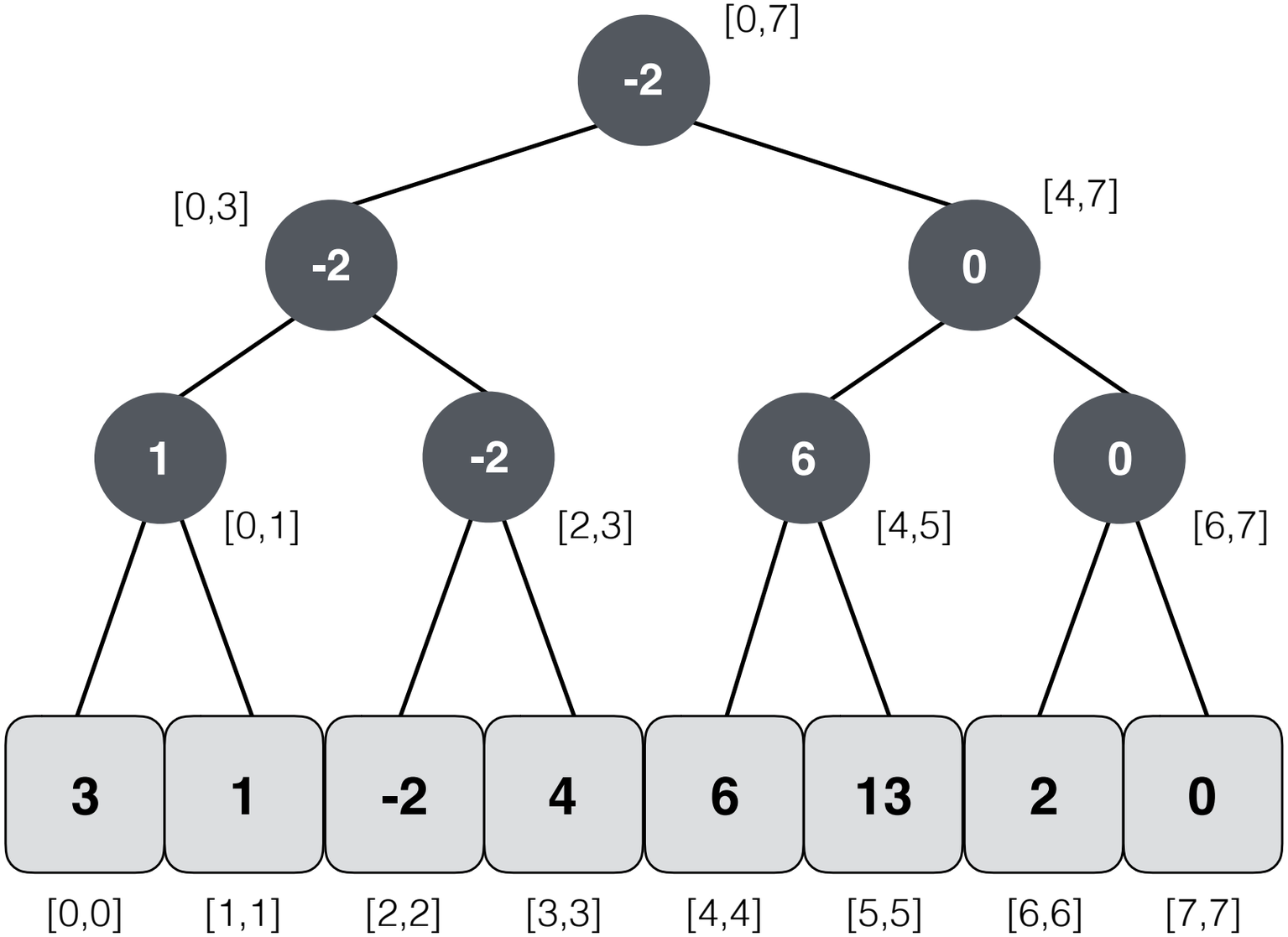

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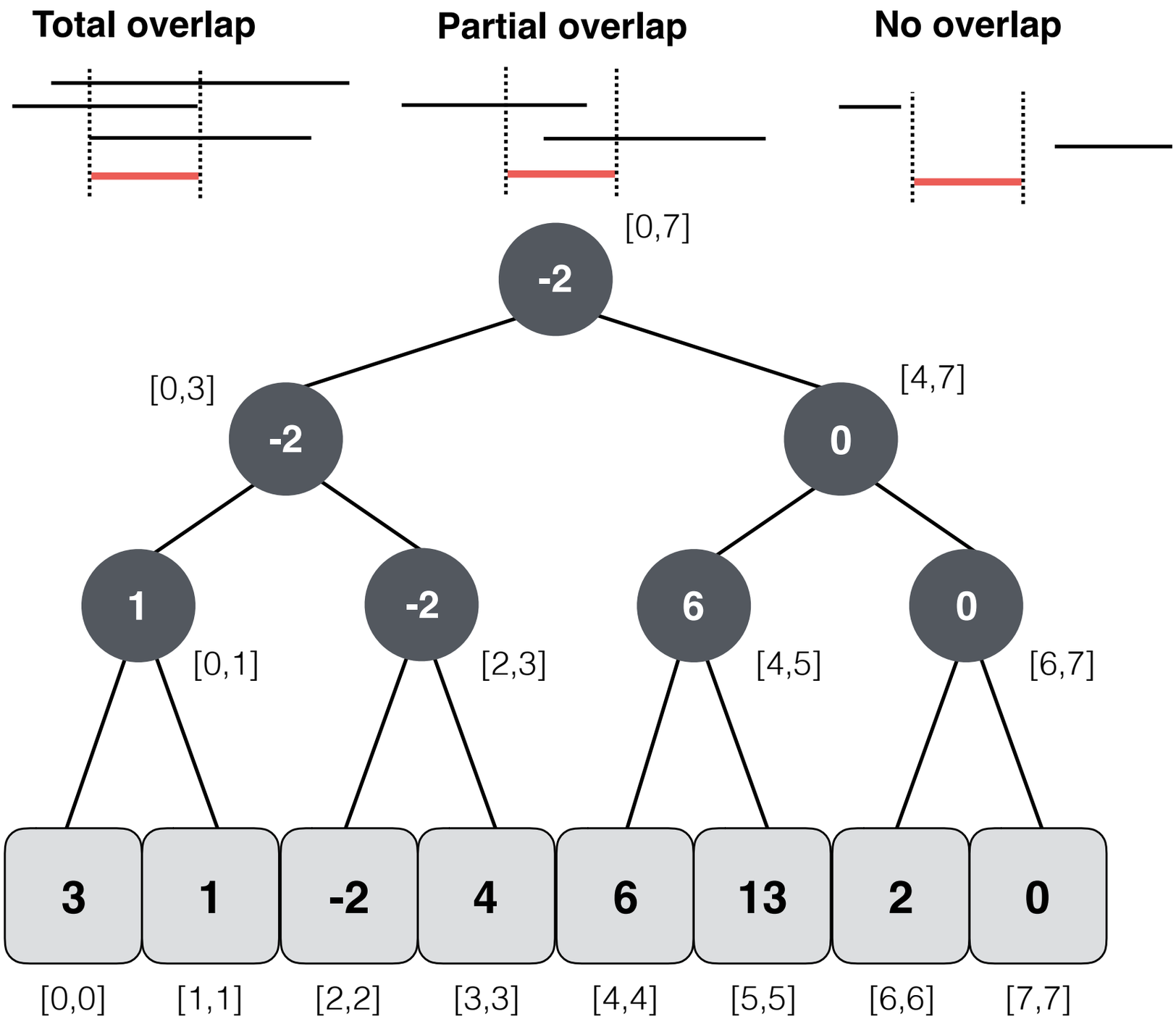

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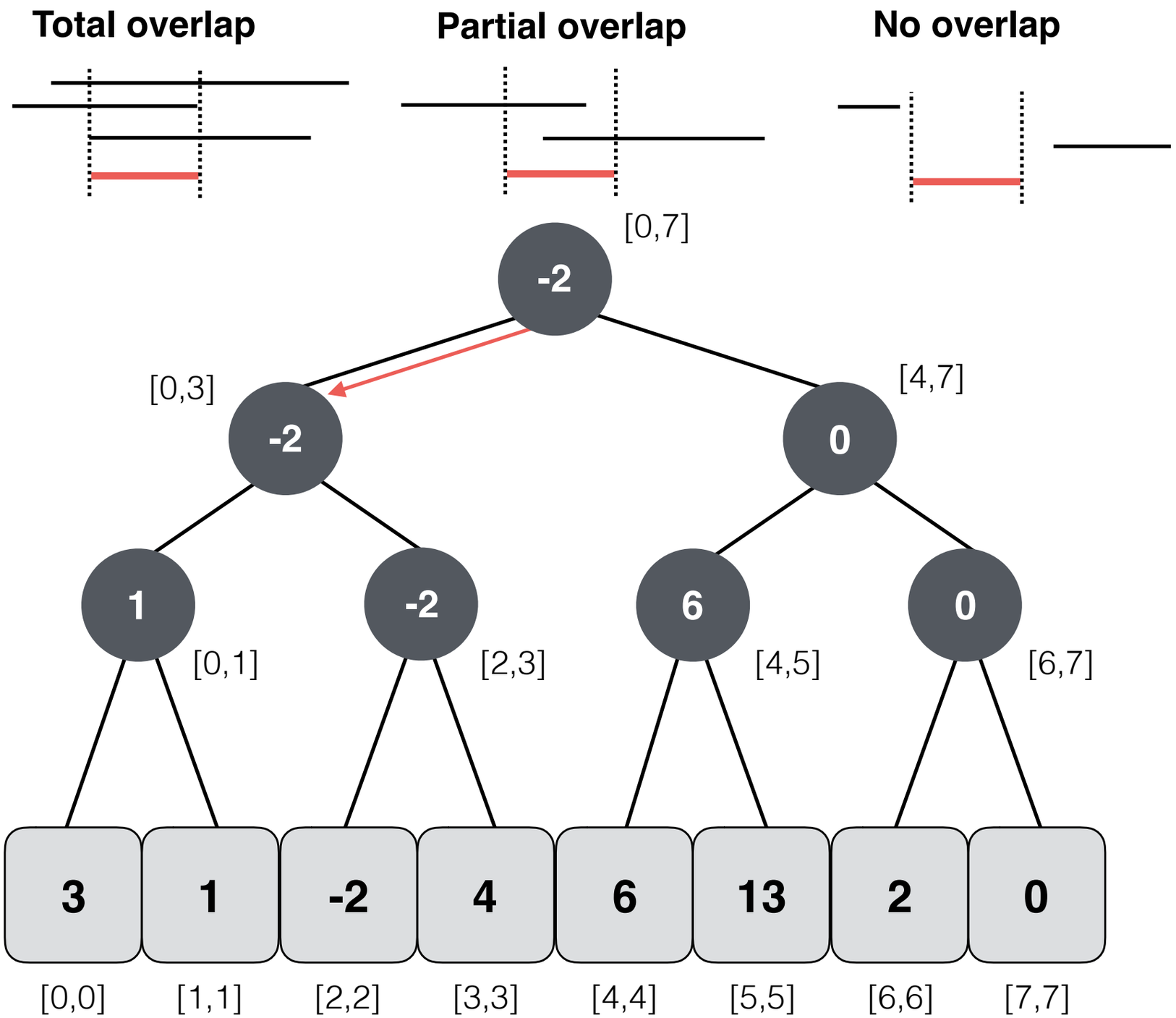

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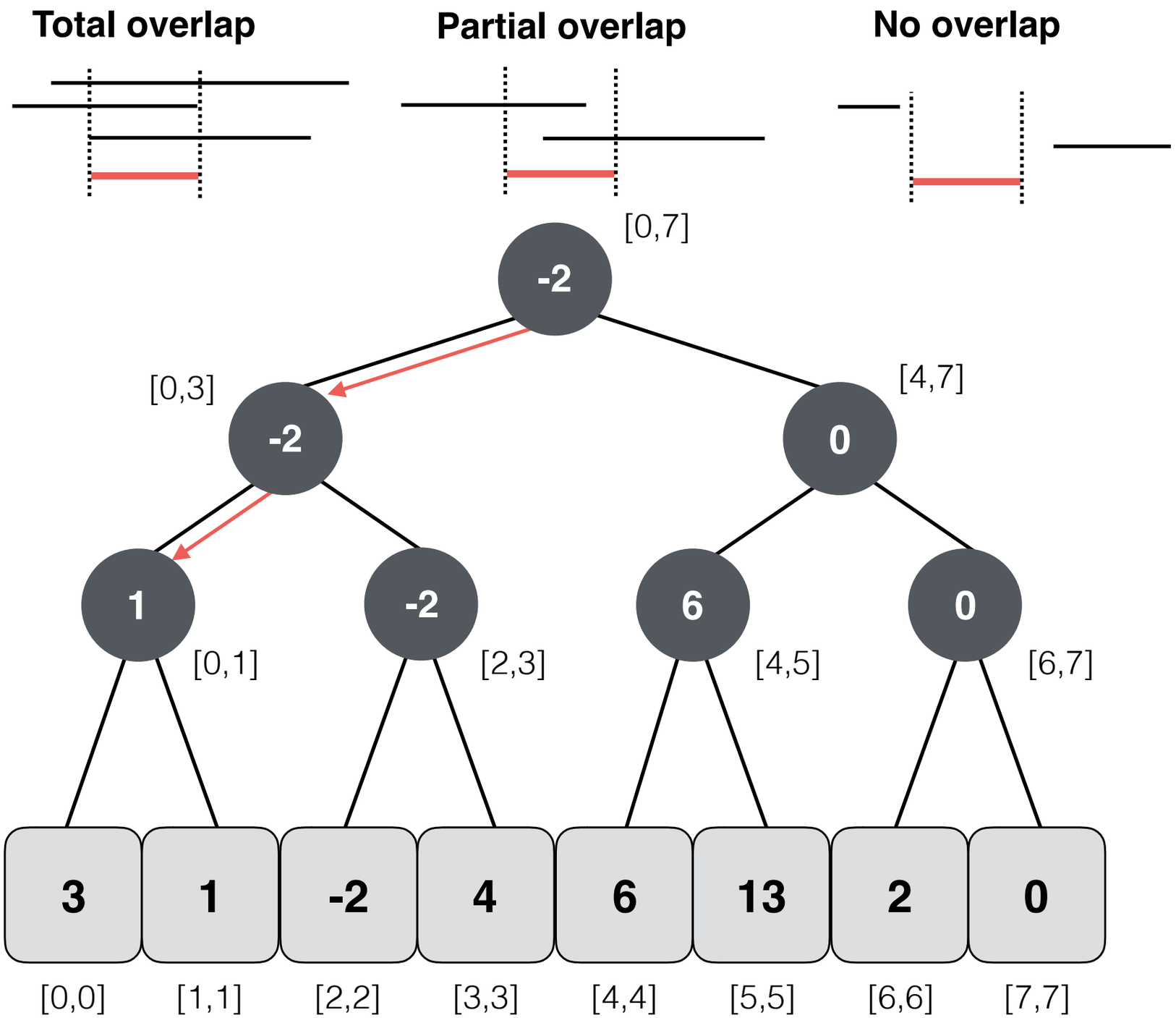

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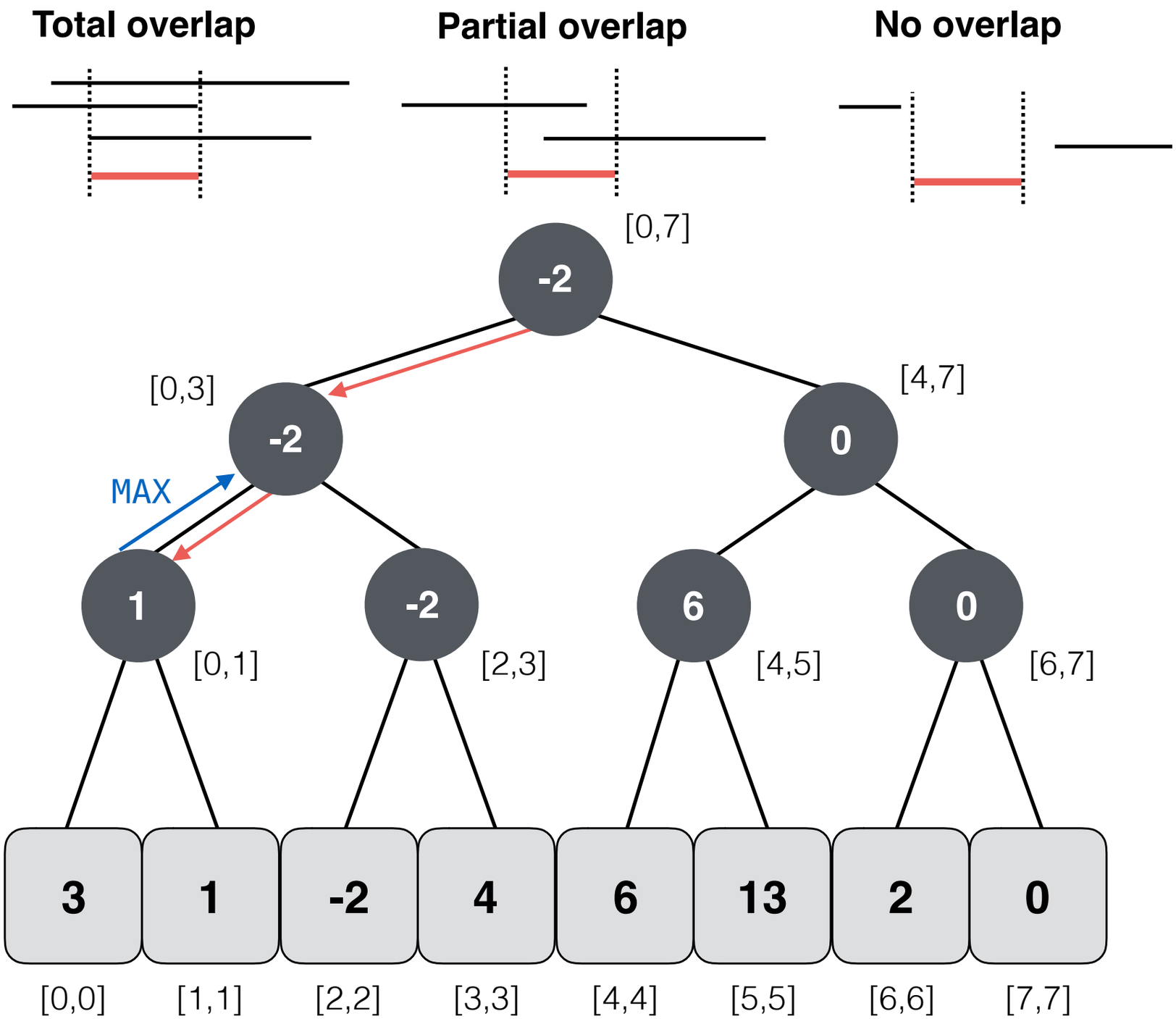

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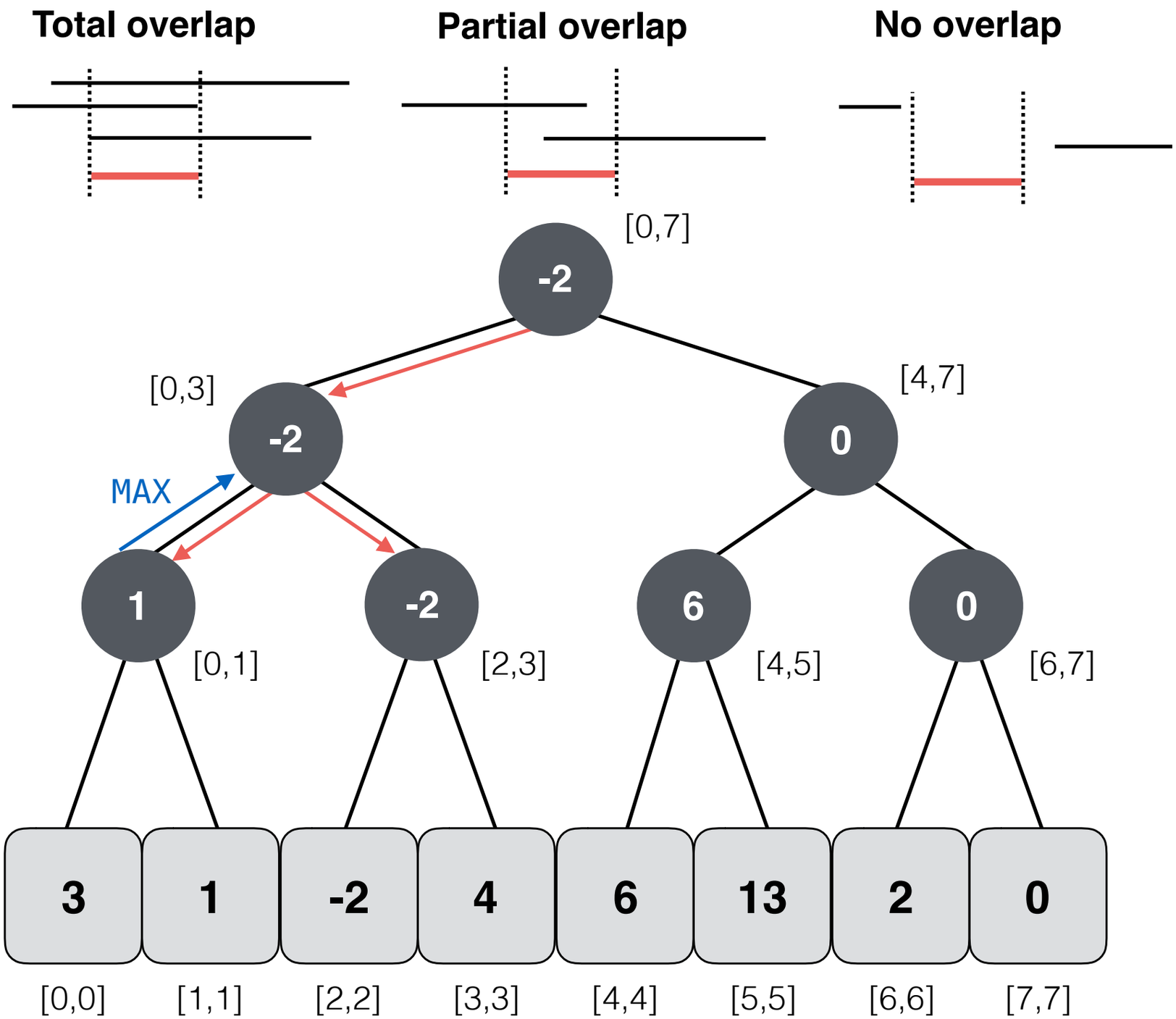

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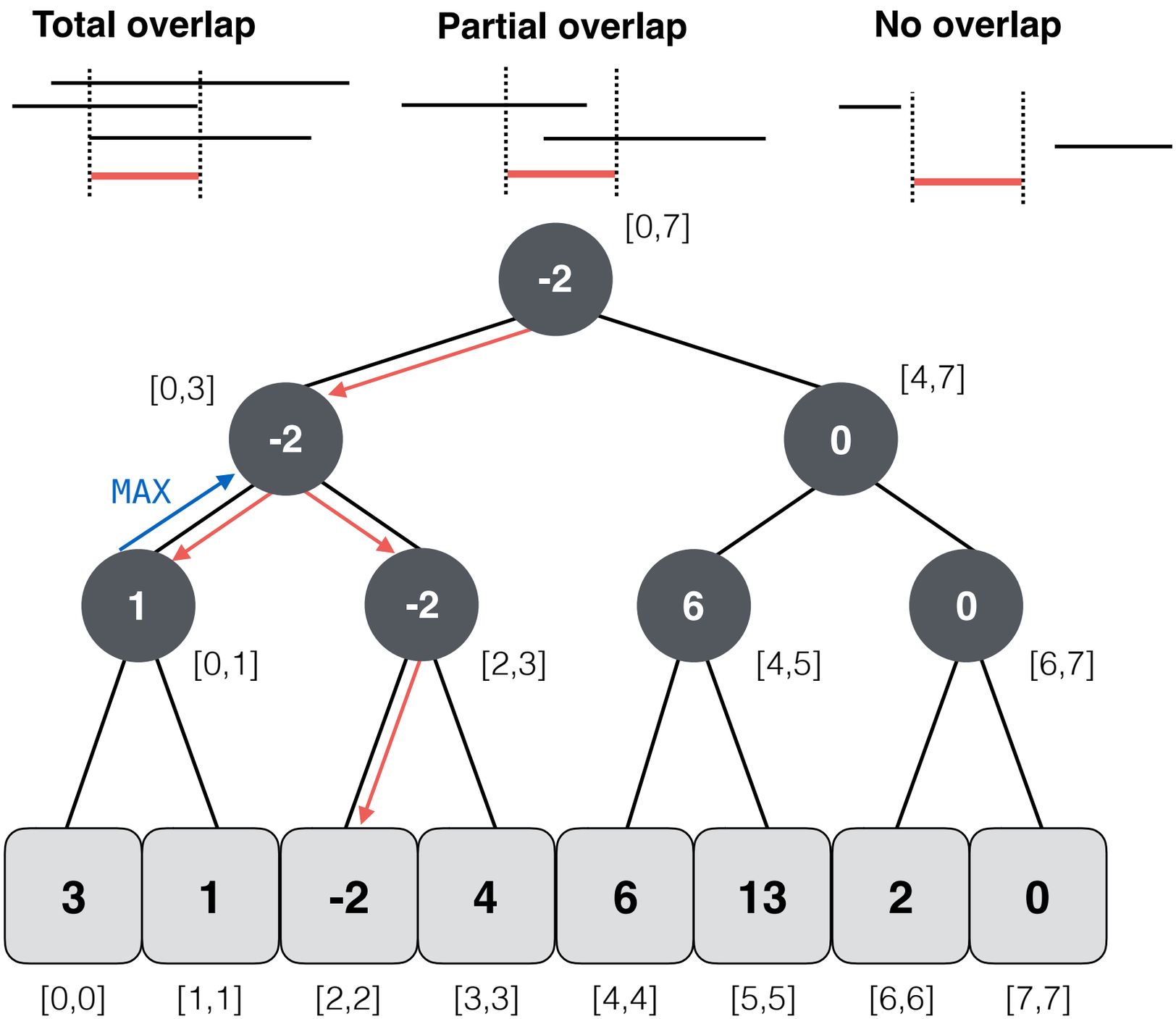

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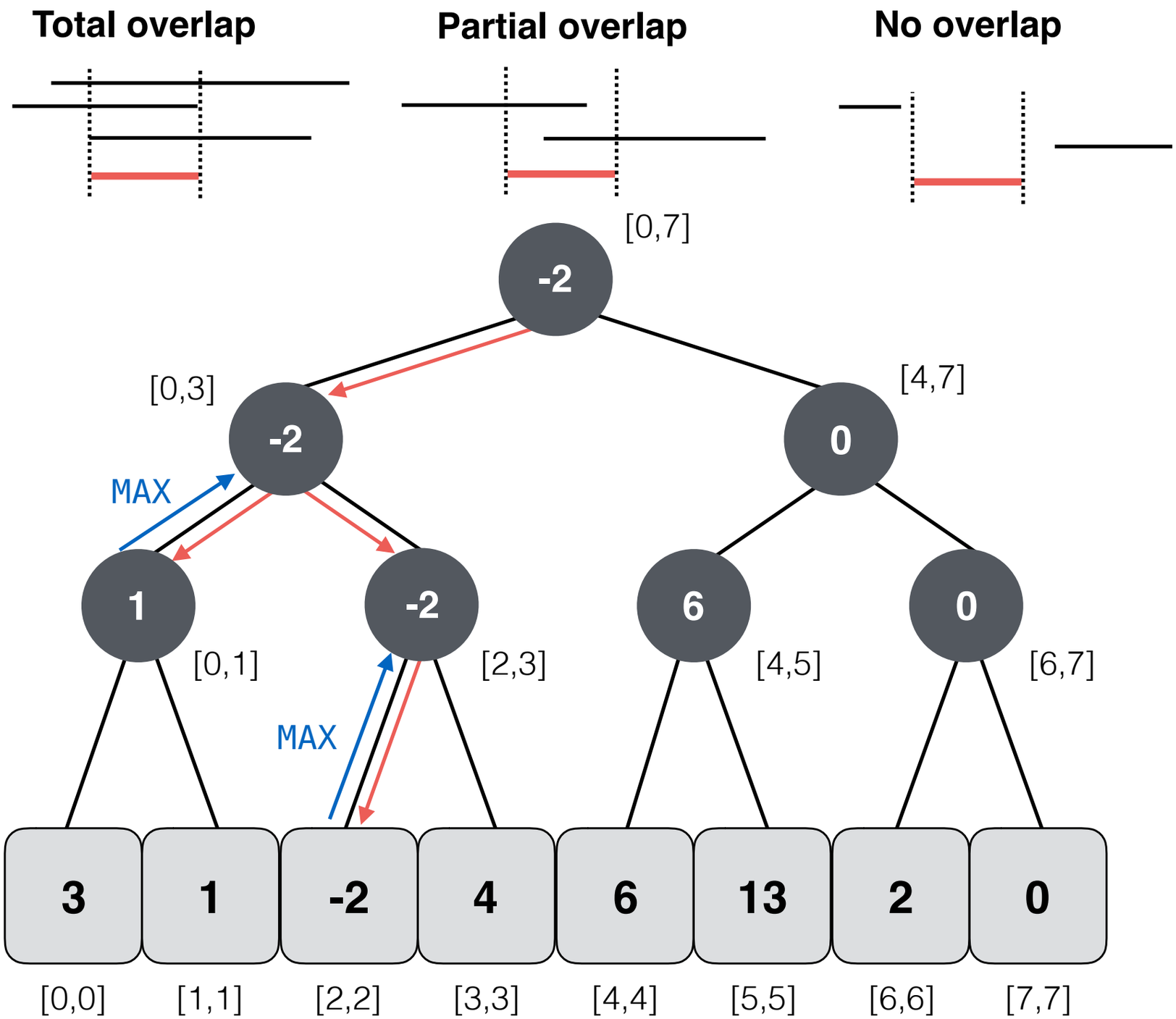

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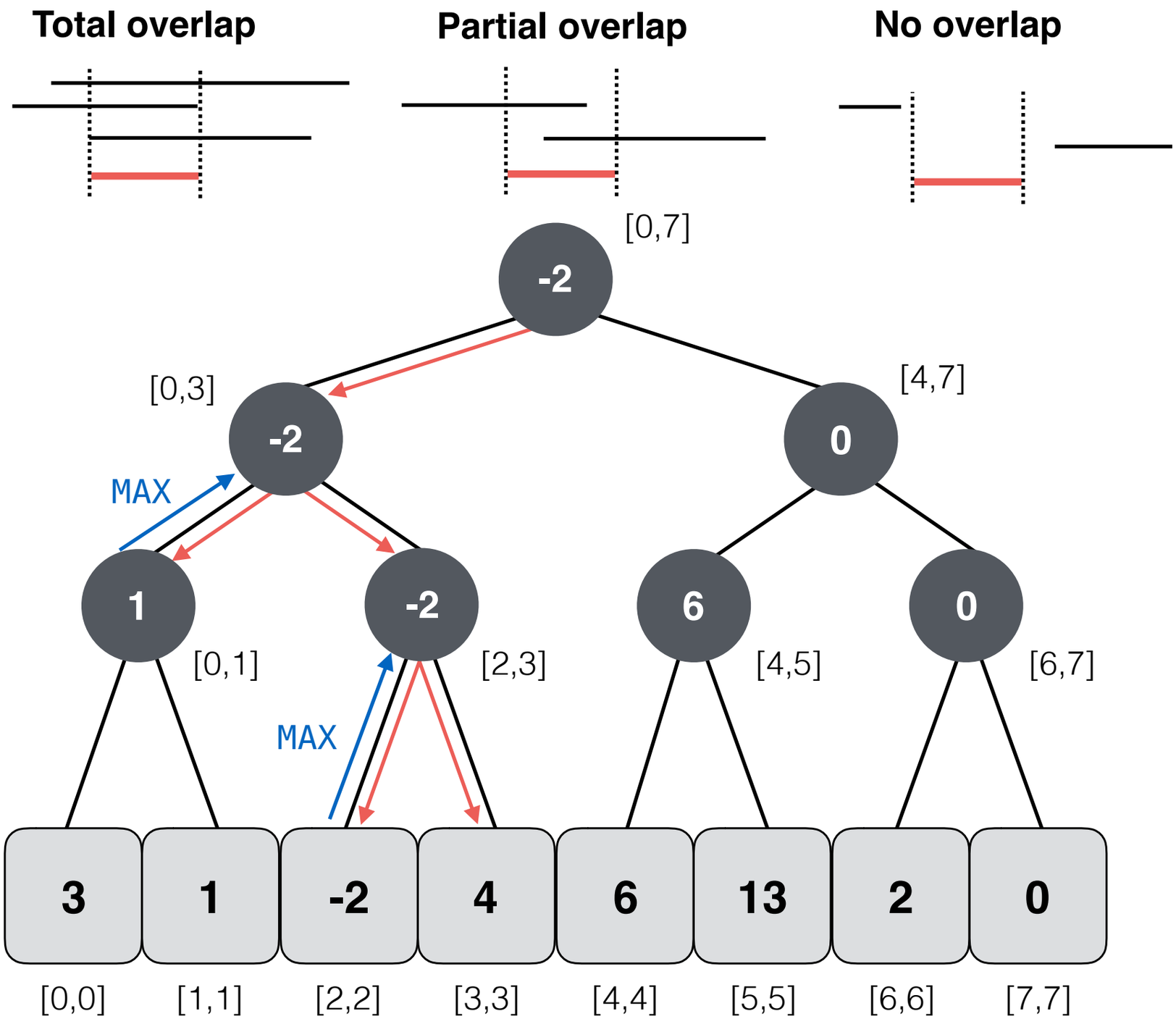

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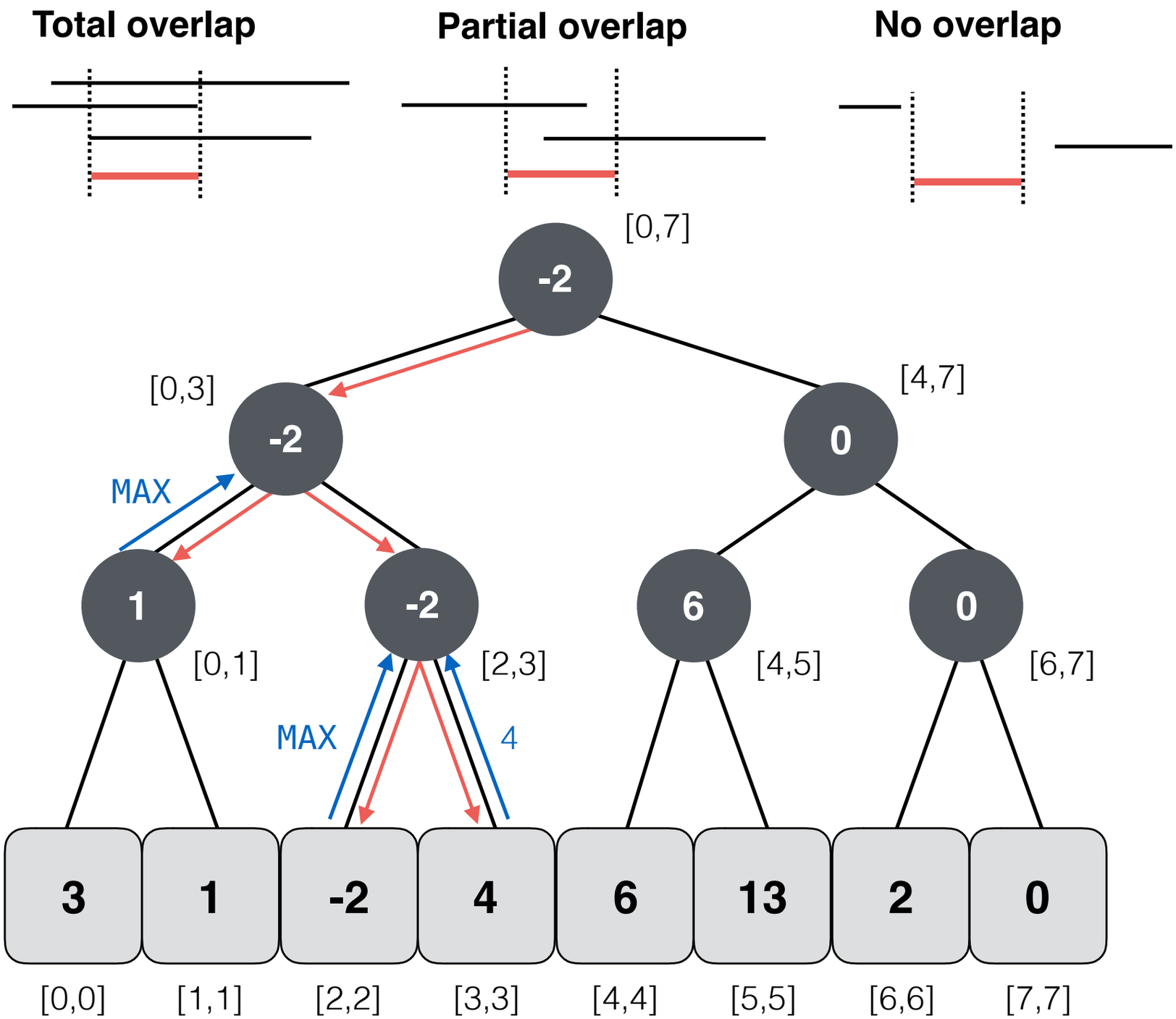

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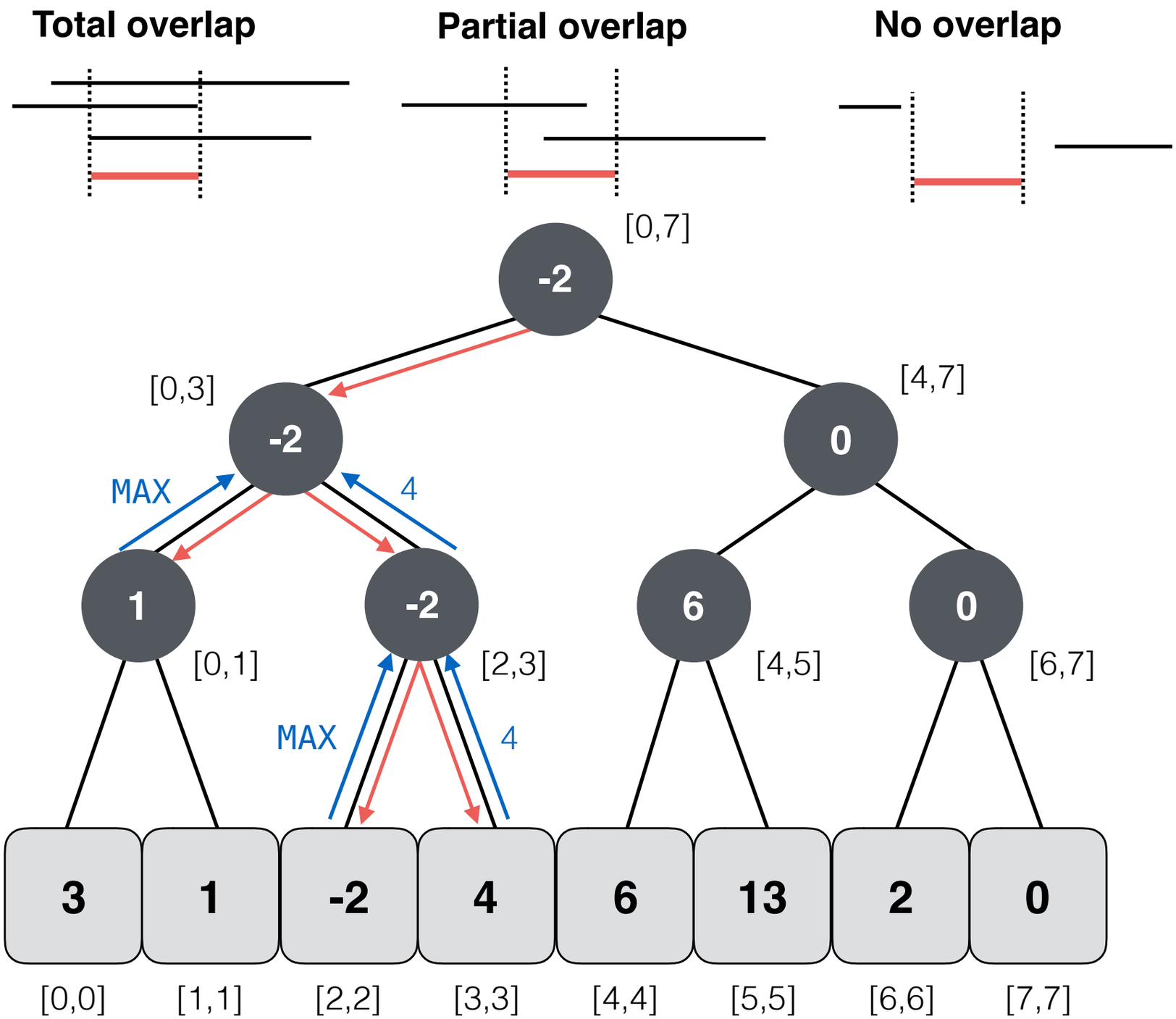

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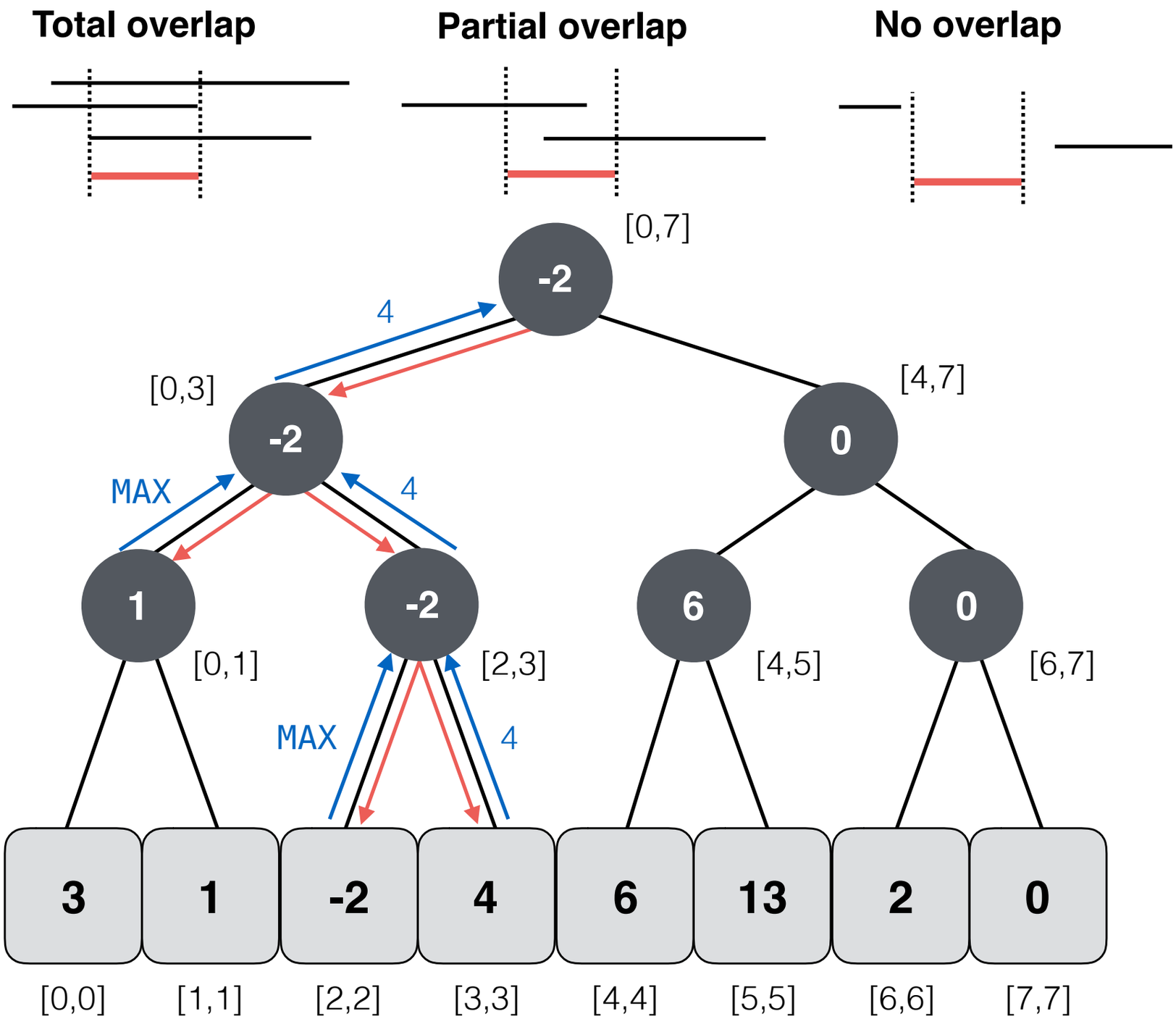

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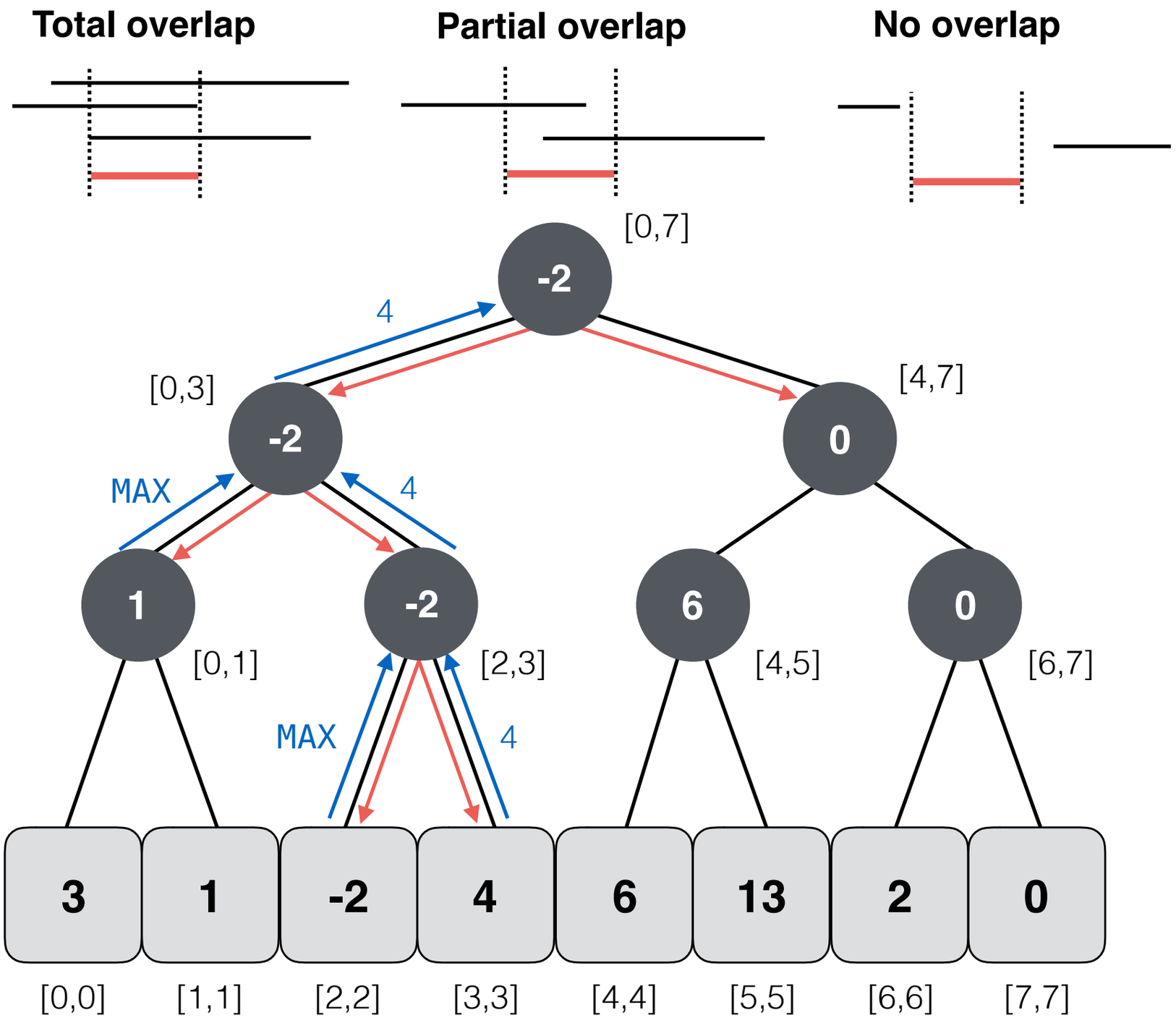

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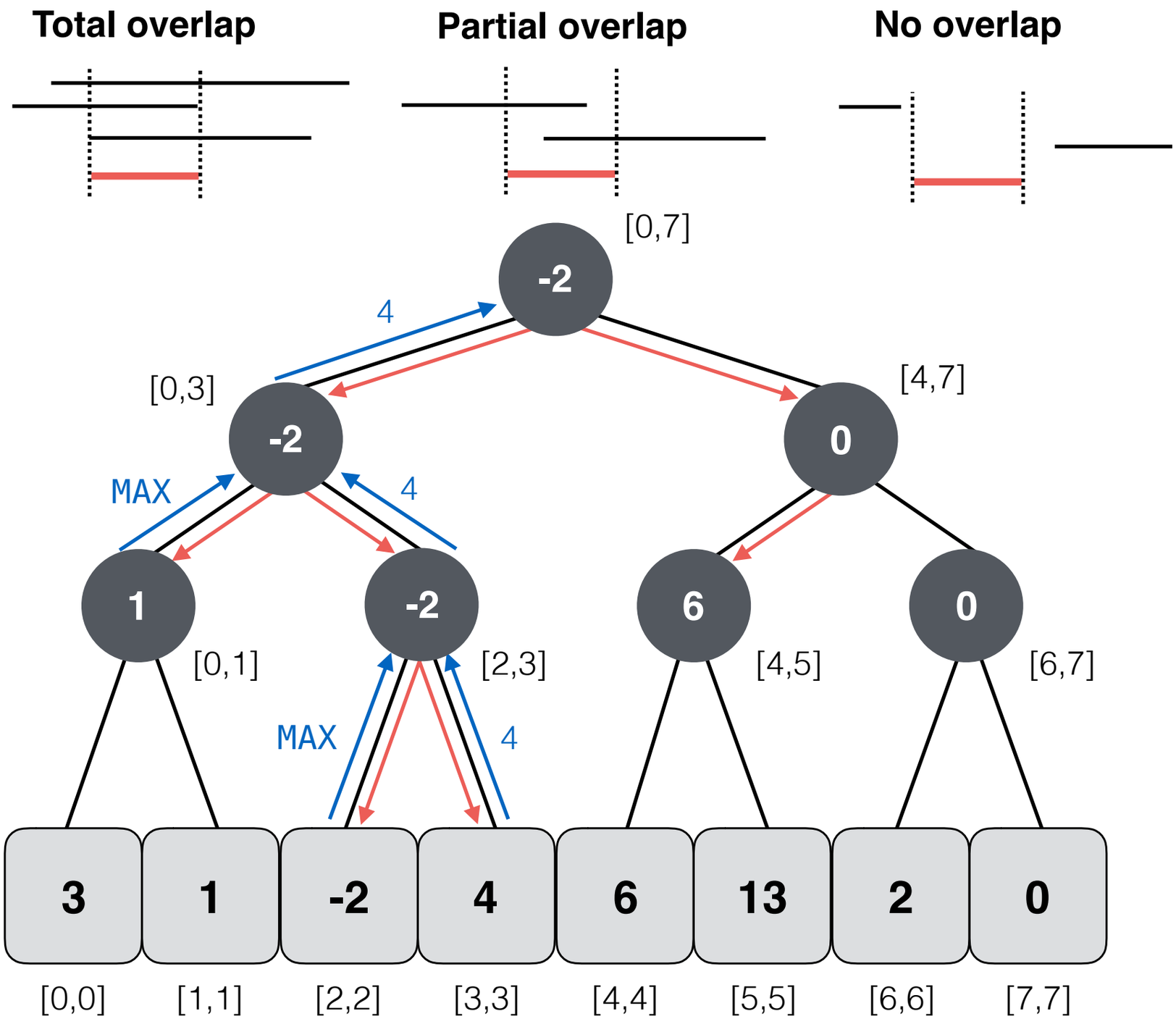

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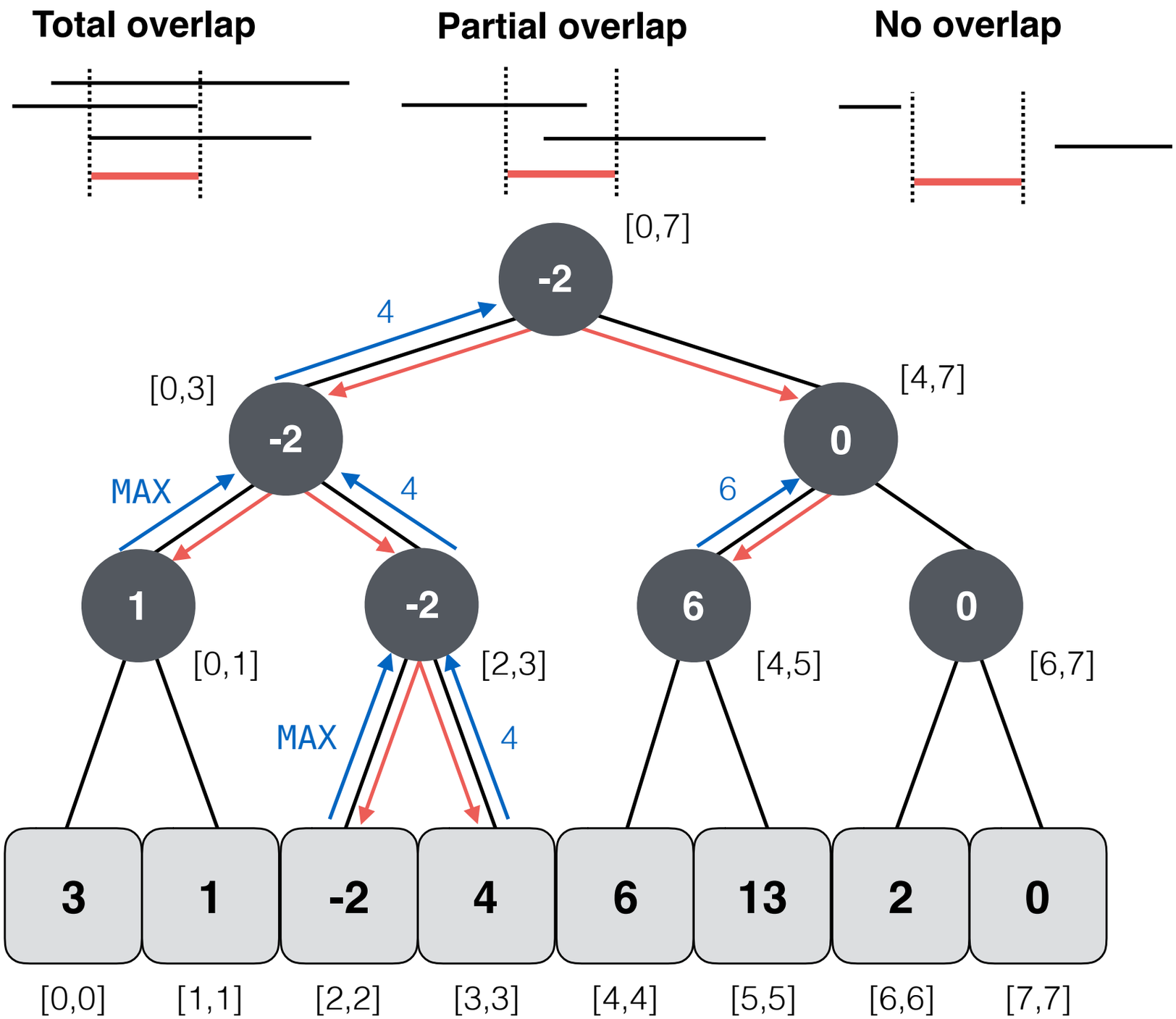

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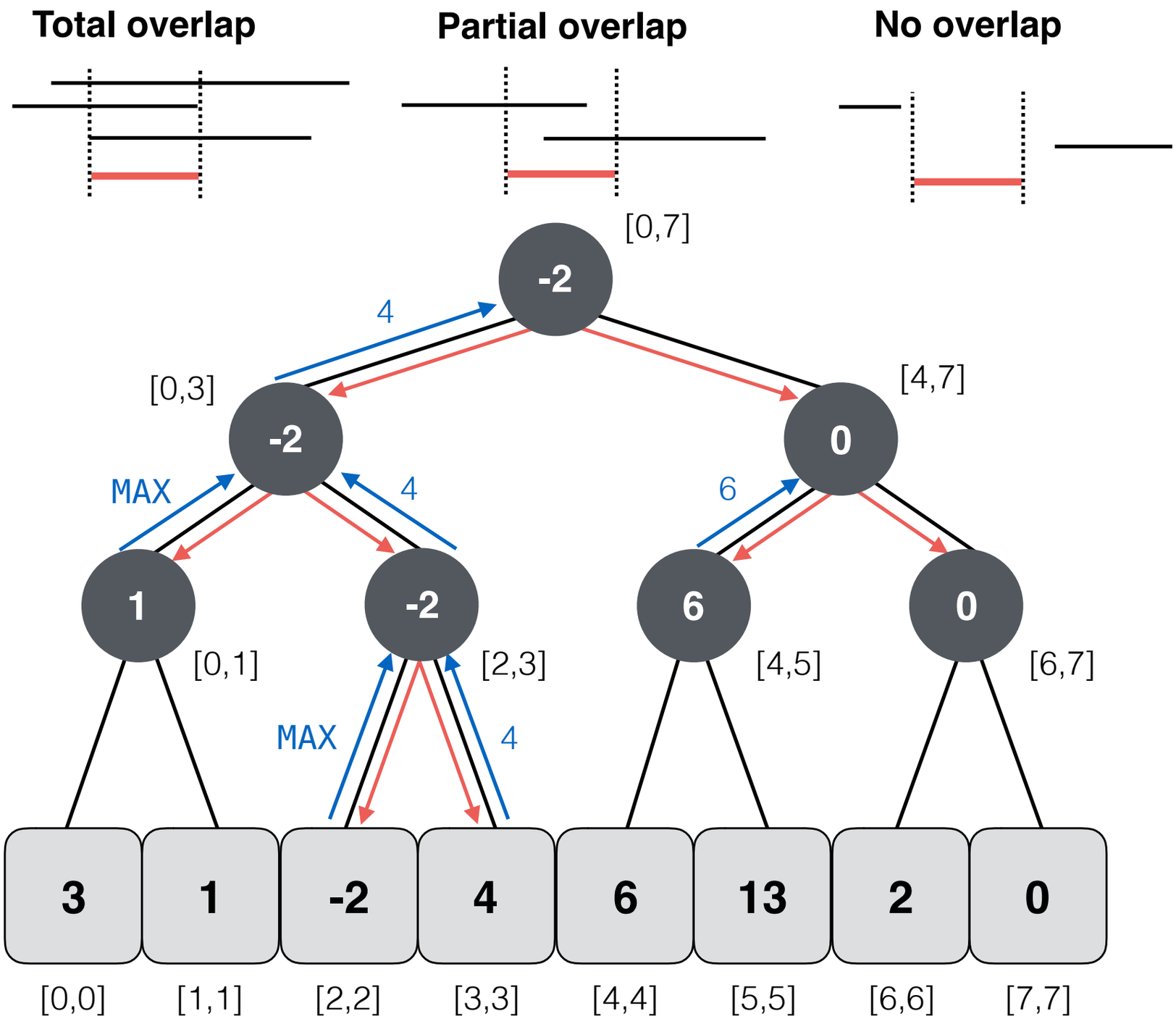

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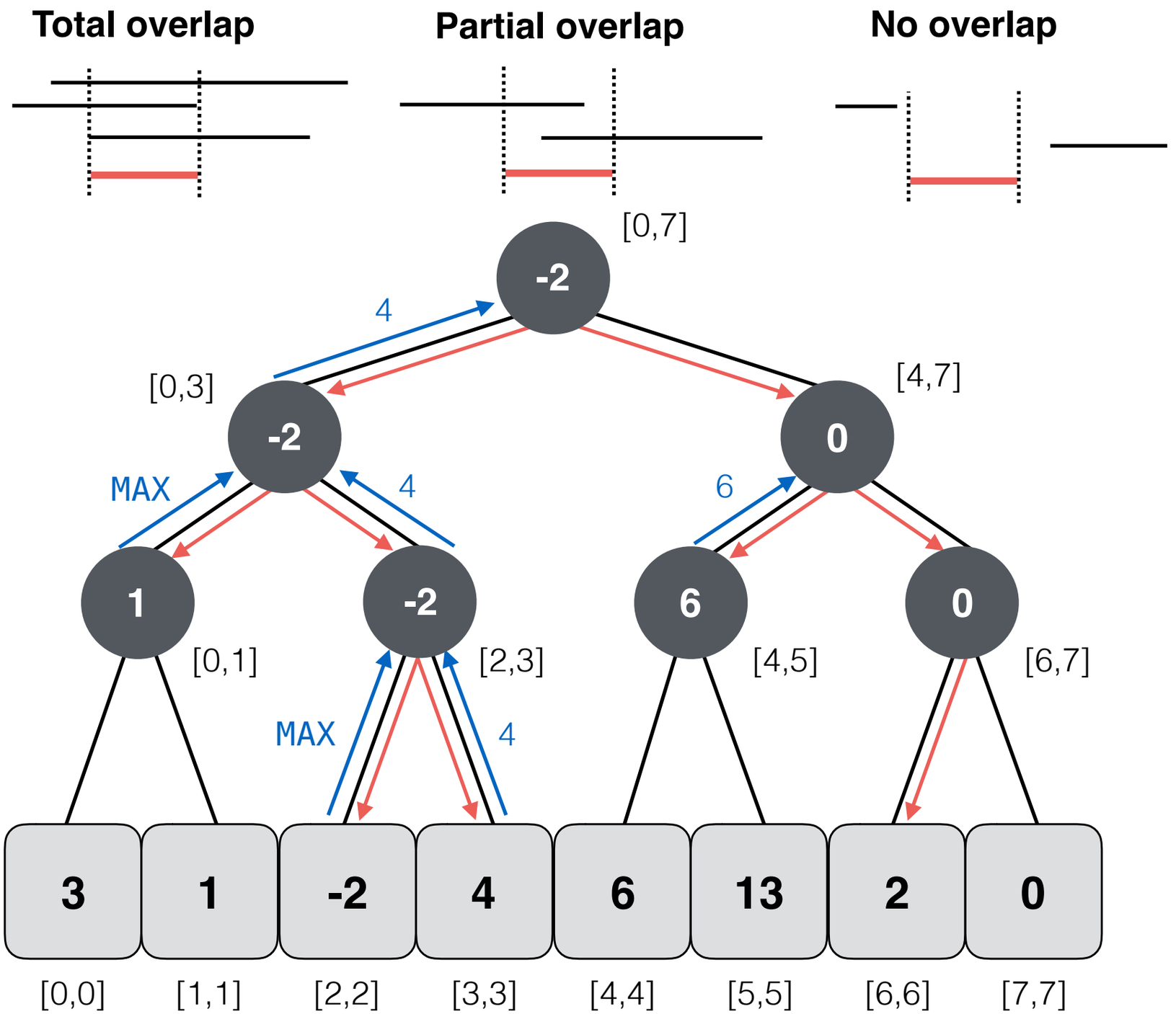

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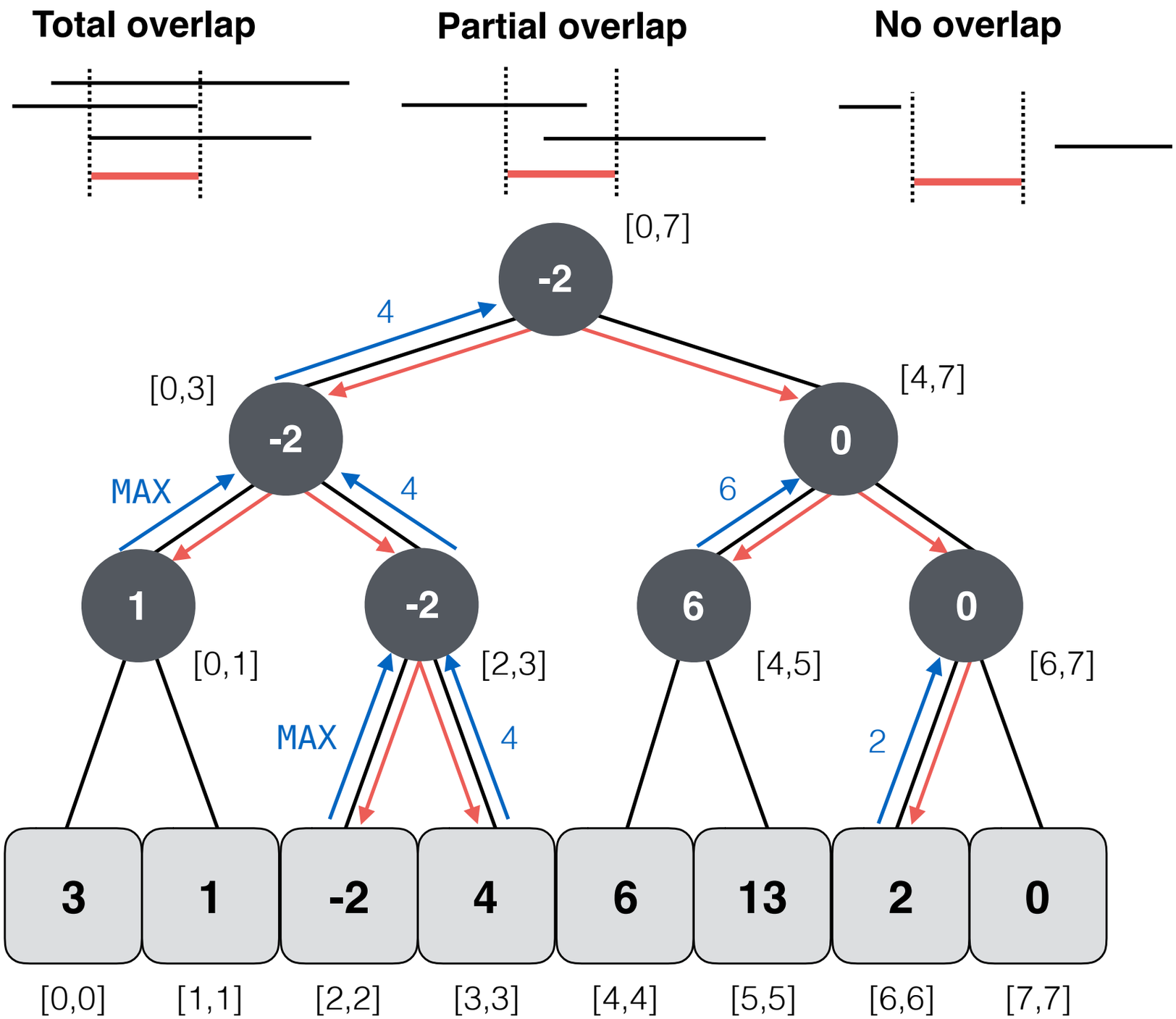

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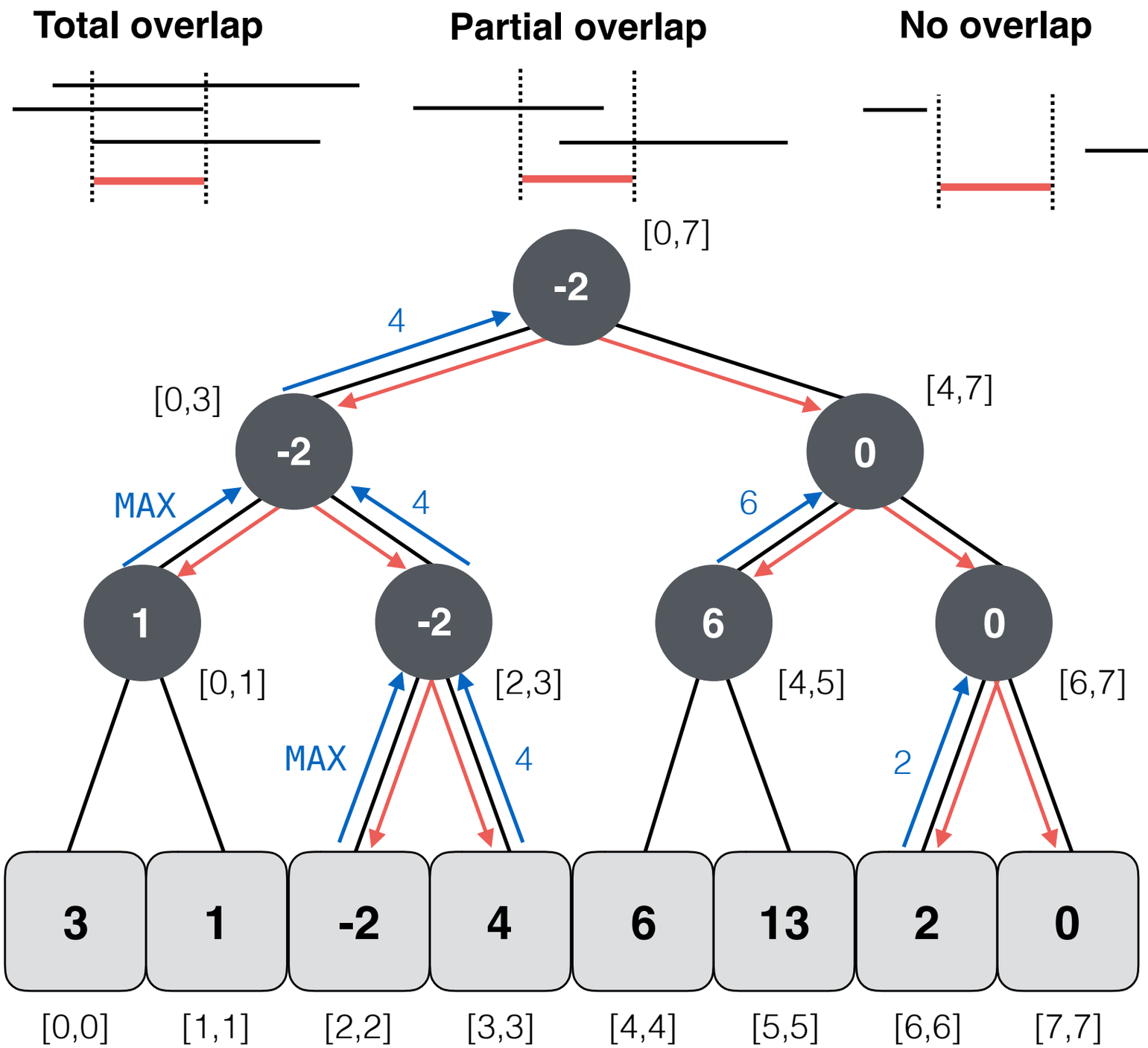

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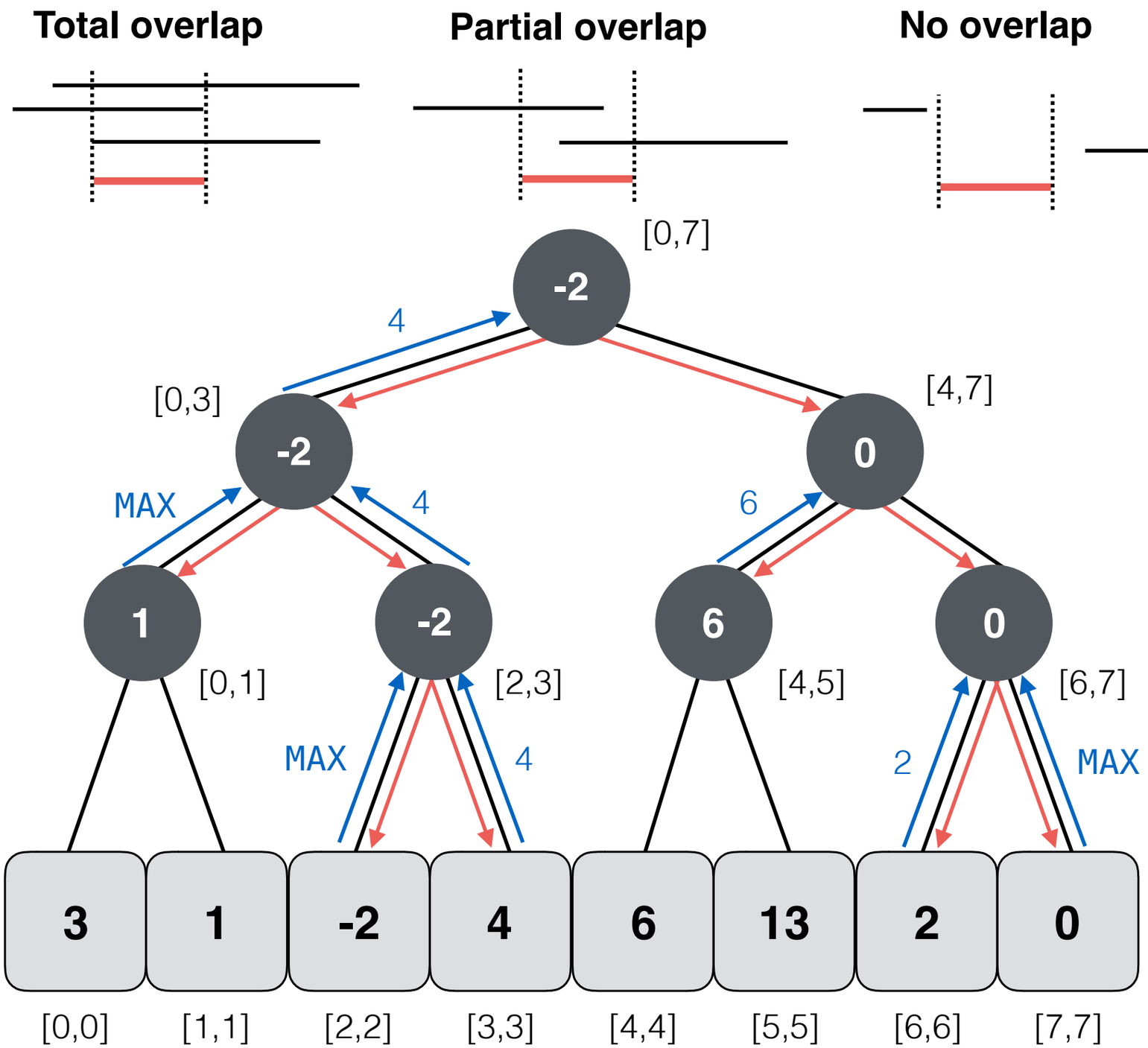

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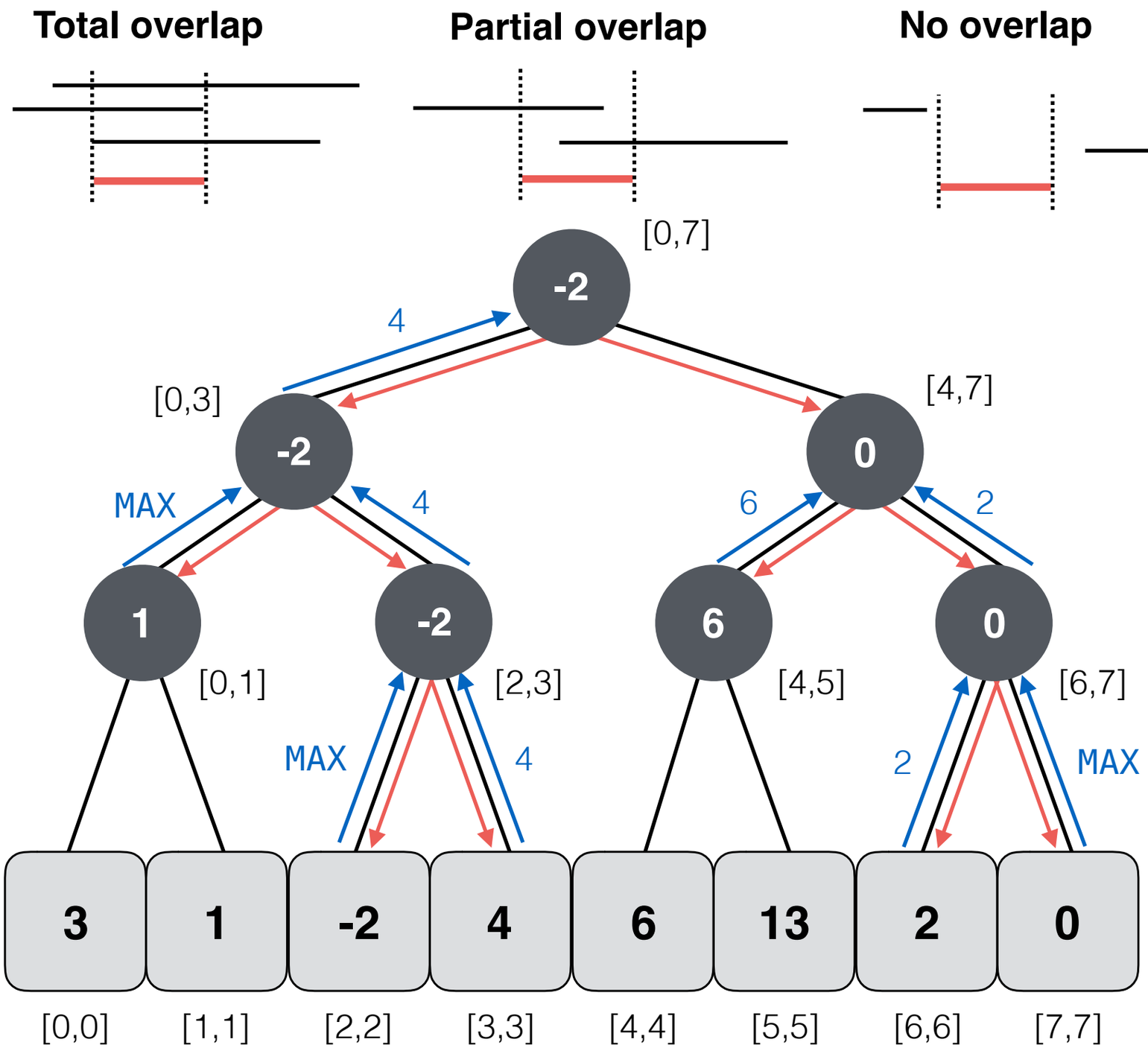

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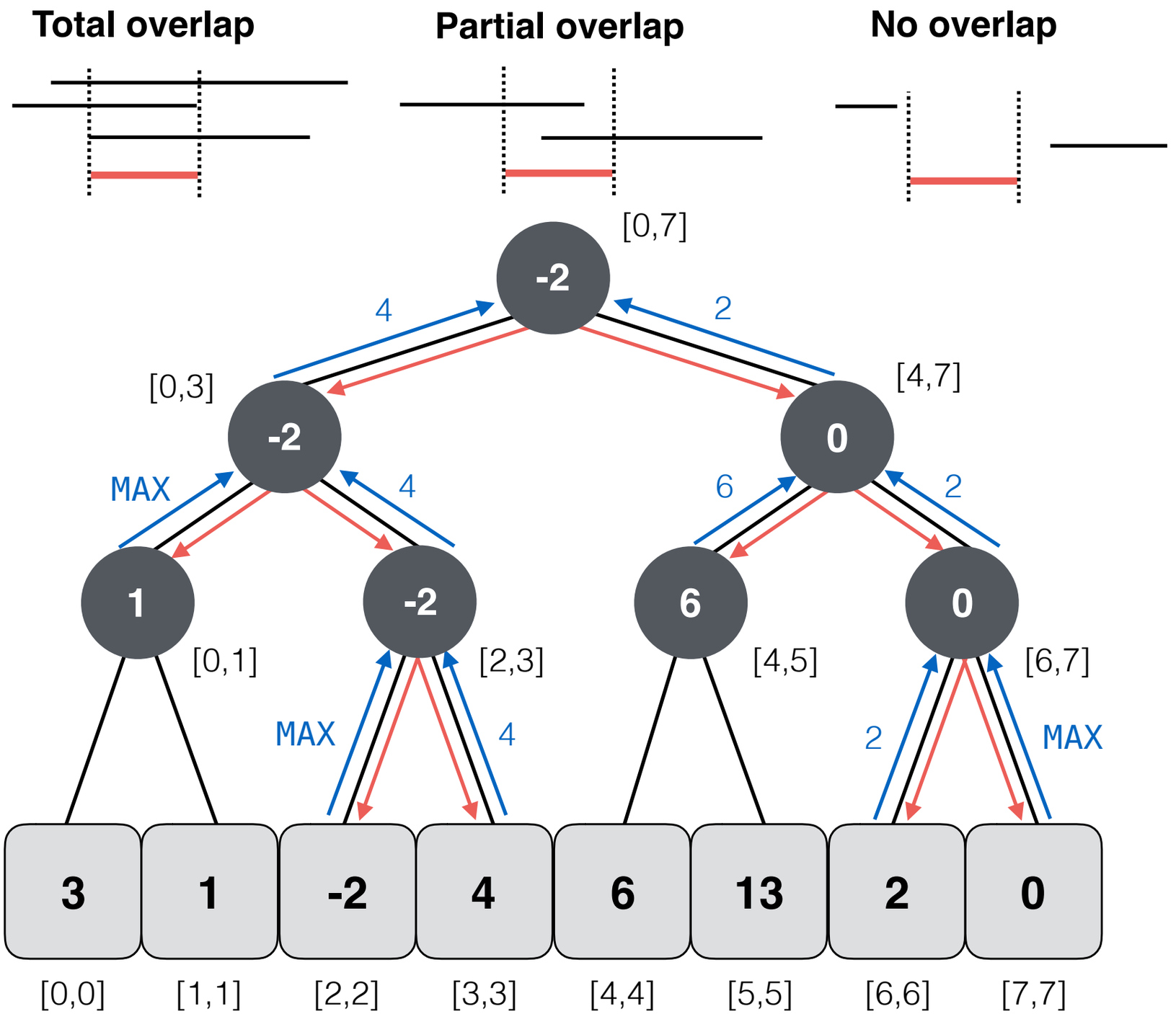

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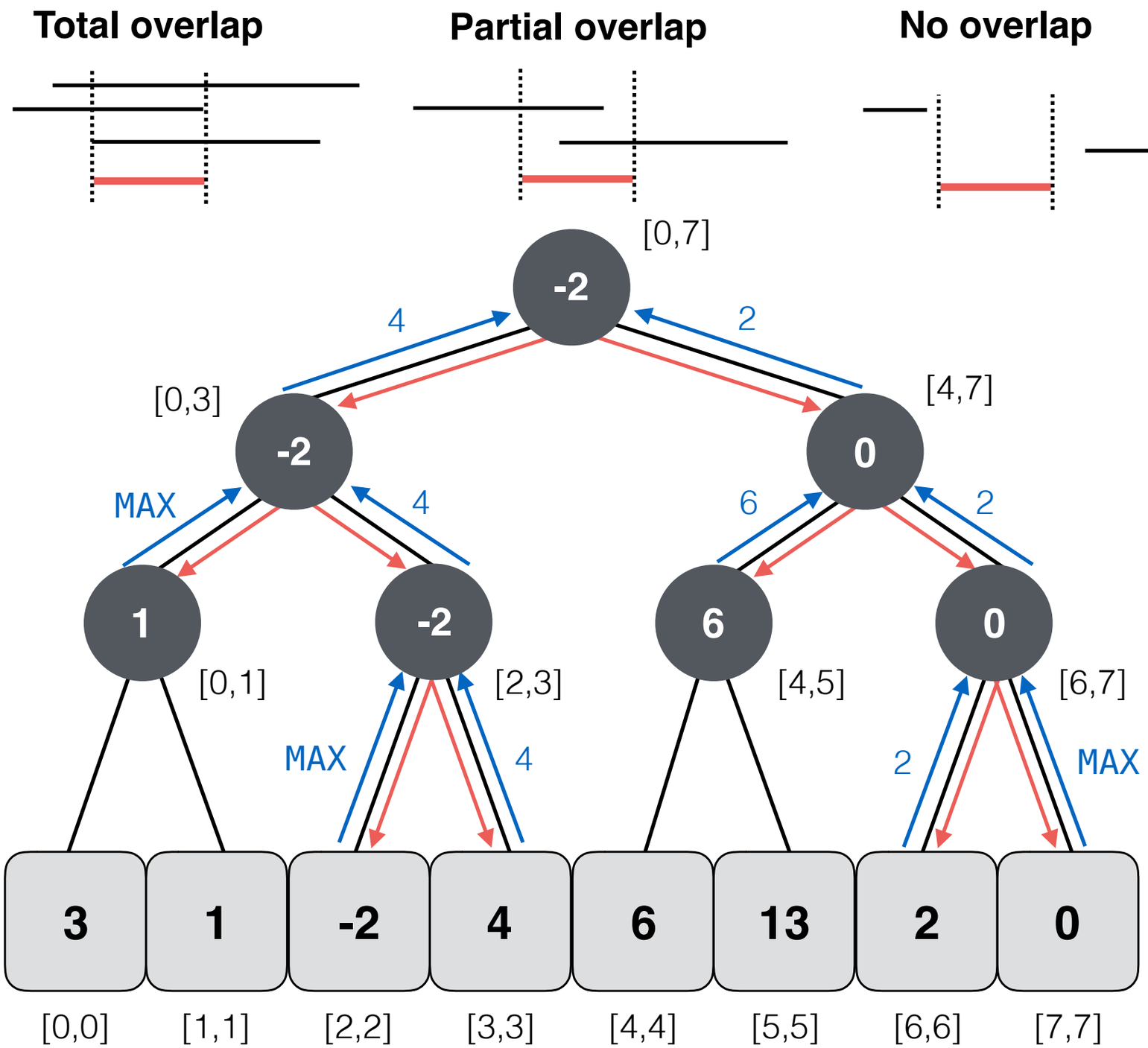

`min`(1,3) = -2

`min`(3,6) = 2

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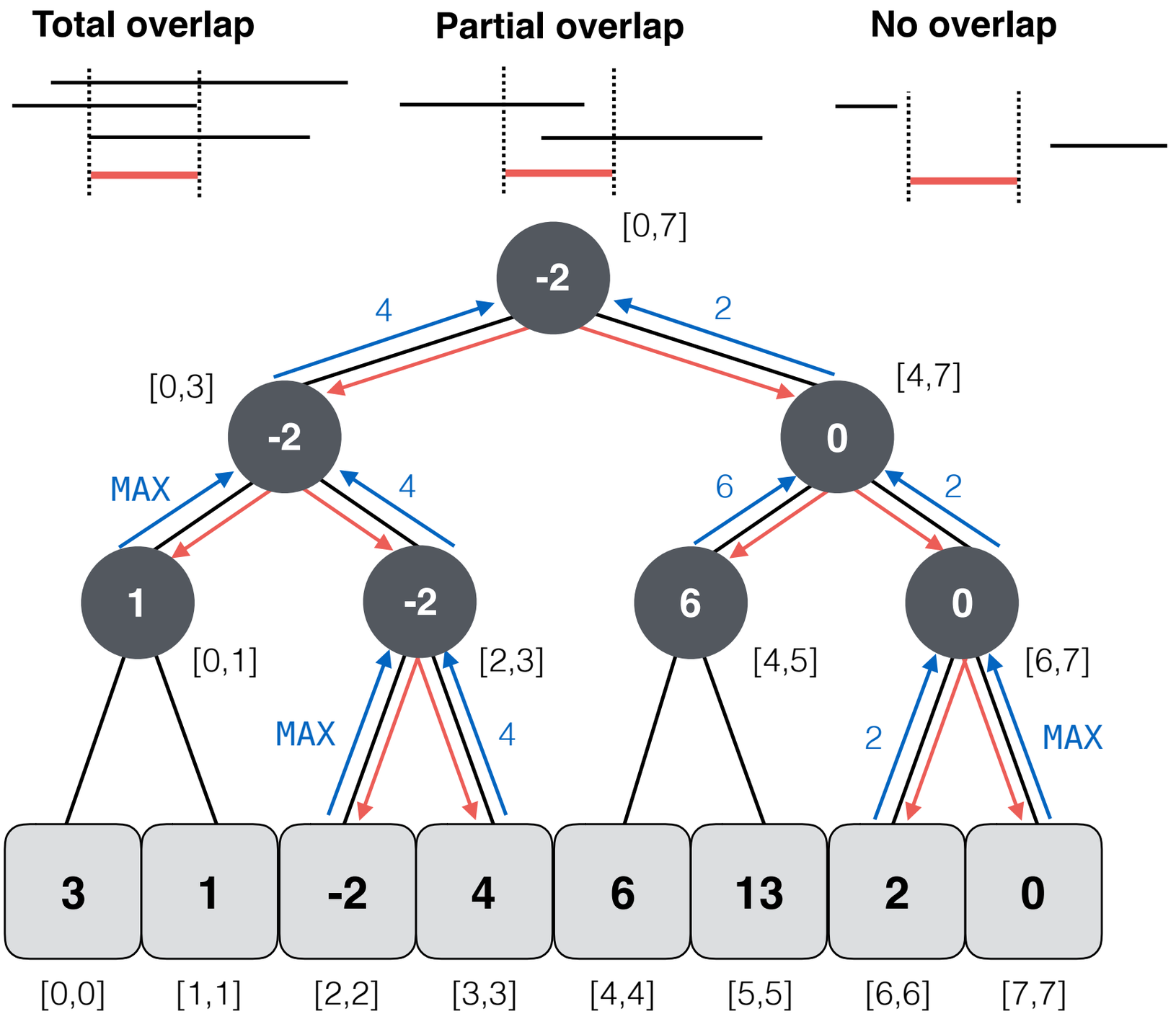

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Query time: O(log n)
Space: O(n)
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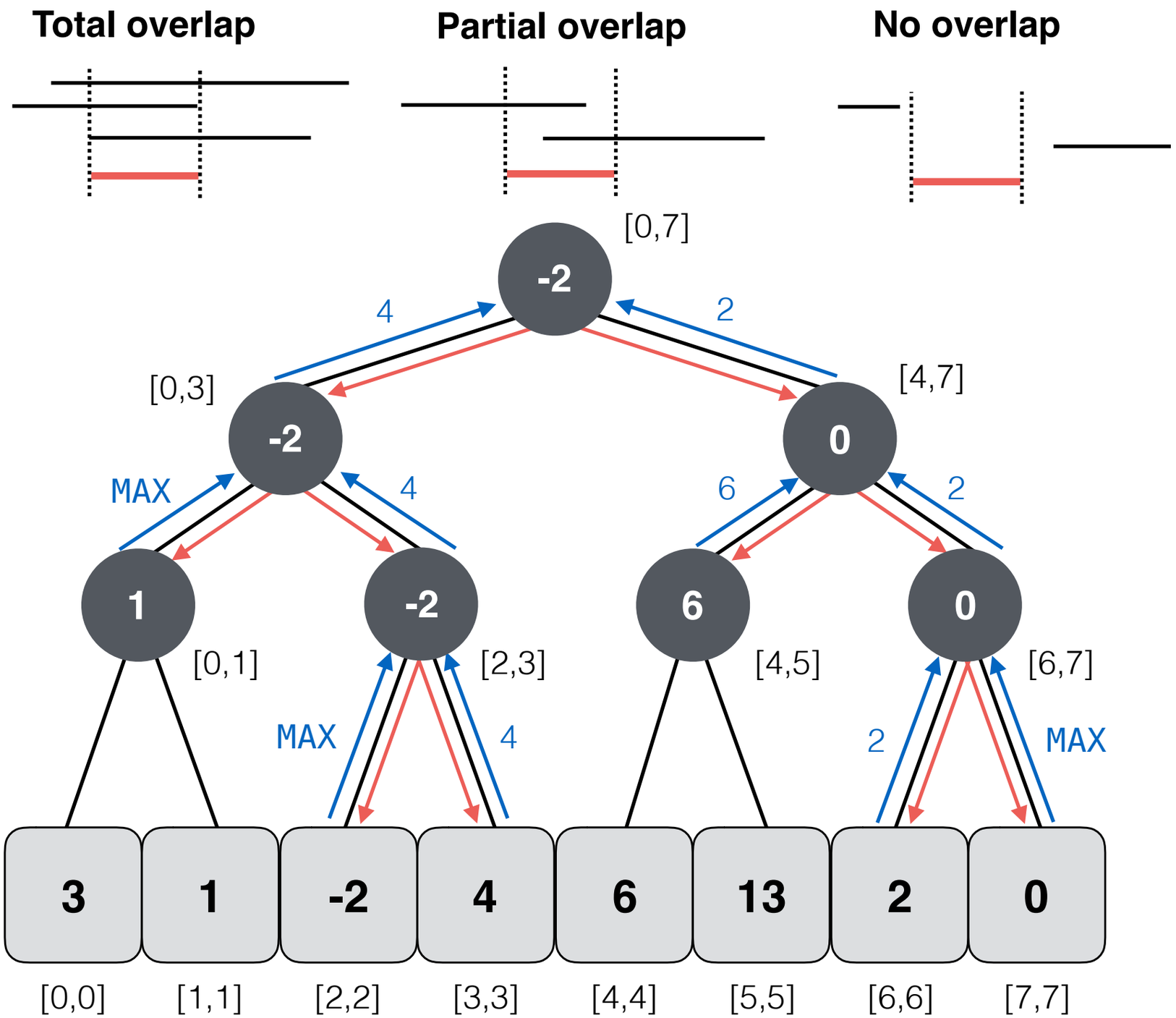


`min`(1,3) = -2

`min`(3,6) = 2

Query time: O(log n)
Space: O(n)
Building time: O(n)

How do we represent trees?

```
node* root = nullptr;
std::deque<node*> q;
int n = 0;
std::cin >> n;

for (int i = 0; i < n; ++i) {
    int x = 0;
    std::cin >> x;
    node* n = new node(x);
    q.push_back(n);
}

node* last = nullptr;
if (n % 2) {
    last = q.back();
    q.pop_back();
}

auto min_parent = [&](node* left, node* right) {
    int min = std::min<int>(left->key, right->key);
    node* parent = new node(min, left, right);
    q.push_back(parent);
};

while (q.size() != 1) {
    min_parent(q[0], q[1]);
    q.pop_front();
    q.pop_front();
}

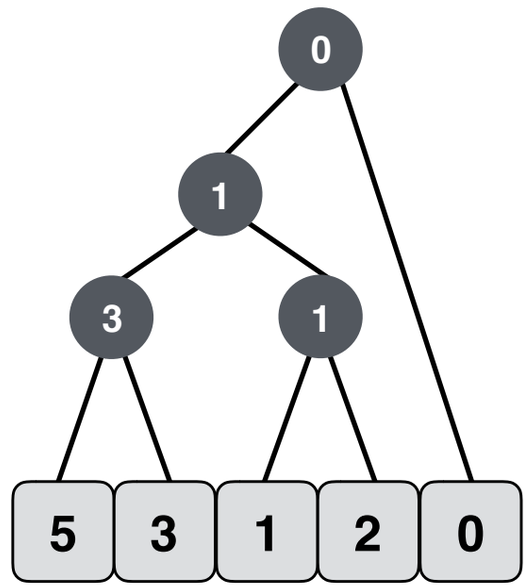
if (last != nullptr) {
    min_parent(q.front(), last);
    q.pop_front();
}

root = q.front();
```

Pointers

```
struct node {
    node(int k,
         node* l = nullptr,
         node* r = nullptr)
        : key(k), left(l), right(r)
    {}

    int key;
    node* left;
    node* right;
};
```



How do we represent trees?

Arrays

```
std::vector<int> tree;

int n = 0;
std::cin >> n;
int tree_size = 2 * n - 1;
tree.resize(tree_size);

int h = ceil(log2(n));
 // left-most node id following level order
int left_most_node = (int(1) << (h - 1)) - 1;
int offset = LEFT(left_most_node);

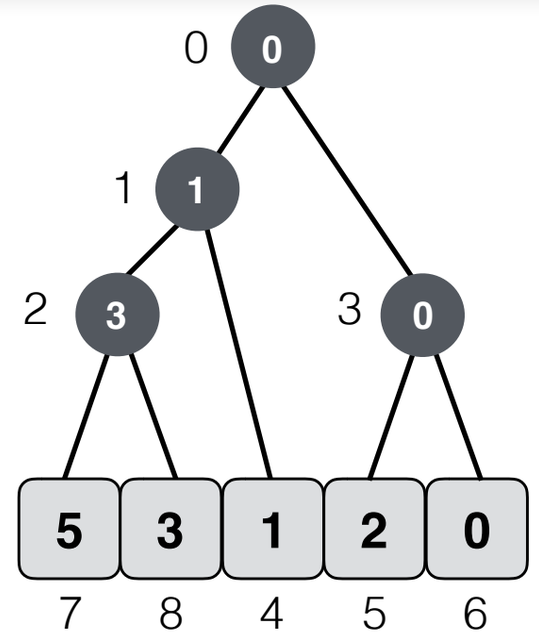
// set leaves circularly

// 1. go forward
int i = 0;
for (int j = offset; j != tree_size; ++i, ++j) {
    int x = 0;
    std::cin >> x;
    tree[j] = x;
}

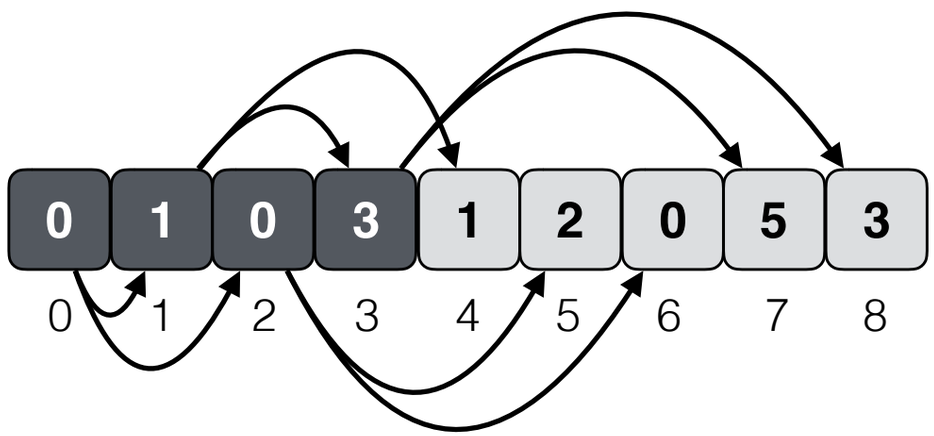
// 2. fall back
for (int j = 0; i != n; ++i, ++j) {
    int x = 0;
    std::cin >> x;
    tree[n - 1 + j] = x;
}

// set internal nodes
for (int i = tree_size - 1; i != 0; i -= 2) {
    int min = std::min<int>(tree[i], tree[i - 1]);
    tree[PARENT(i)] = min;
}
```

```
#define LEFT(i)     2 * i + 1
#define RIGHT(i)    2 * i + 2
#define PARENT(i) (i - 1) / 2
```



Pointers are implicit!



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}

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for (int i = tree_size - 1; i != 0; i -= 2) {
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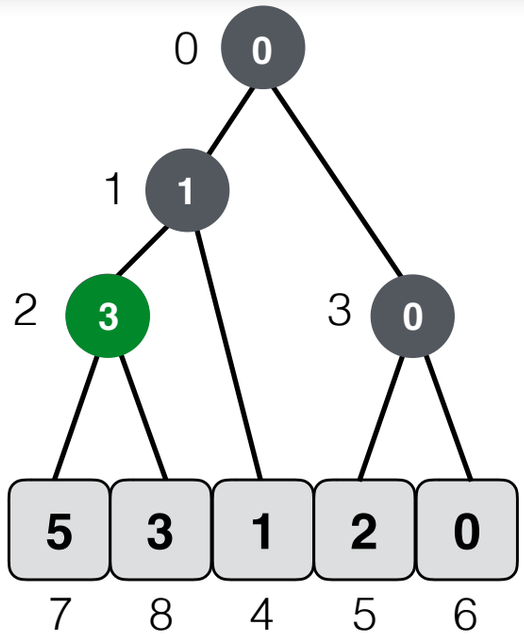

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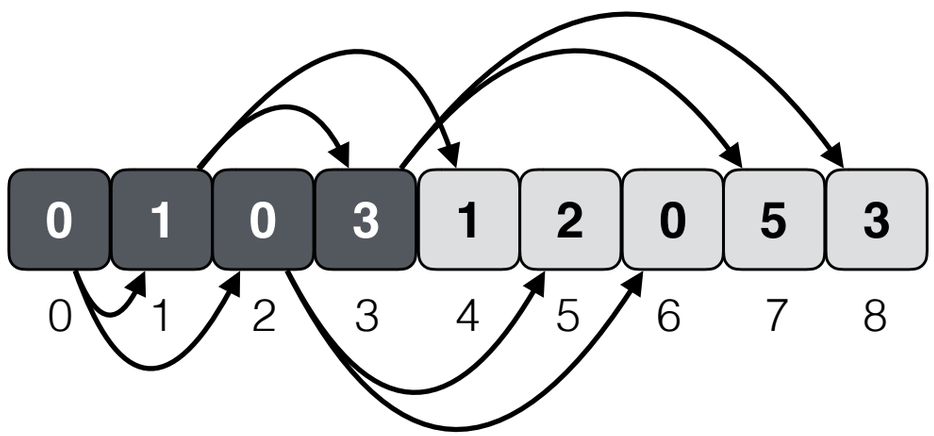

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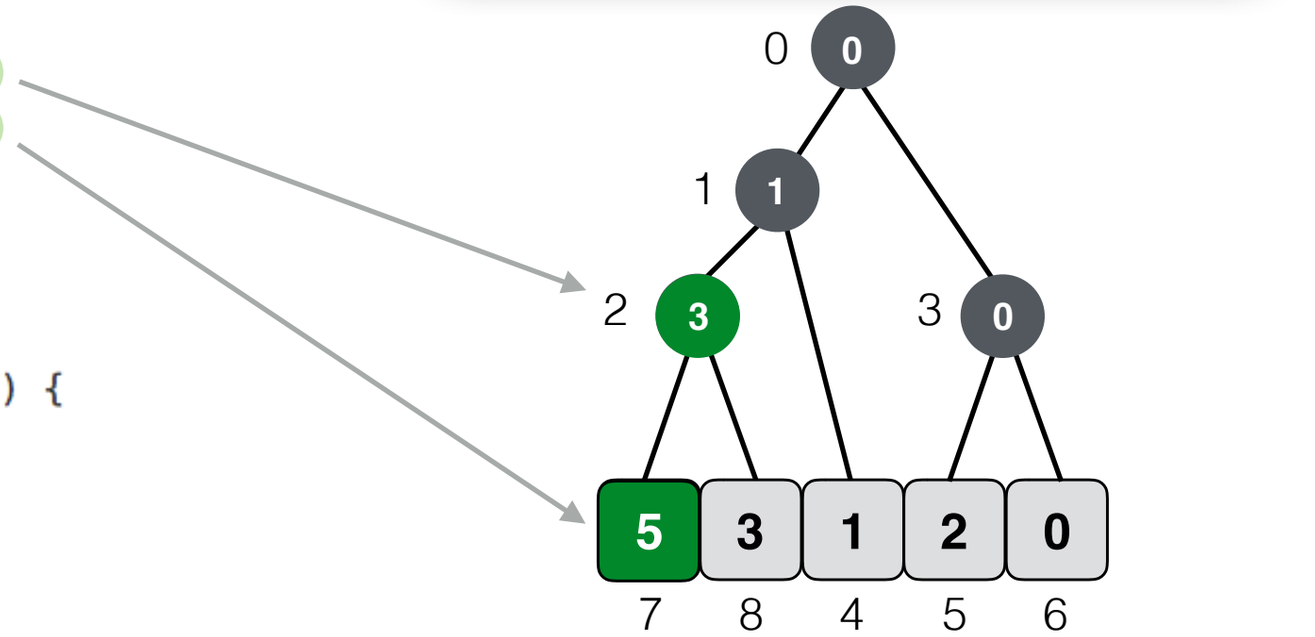

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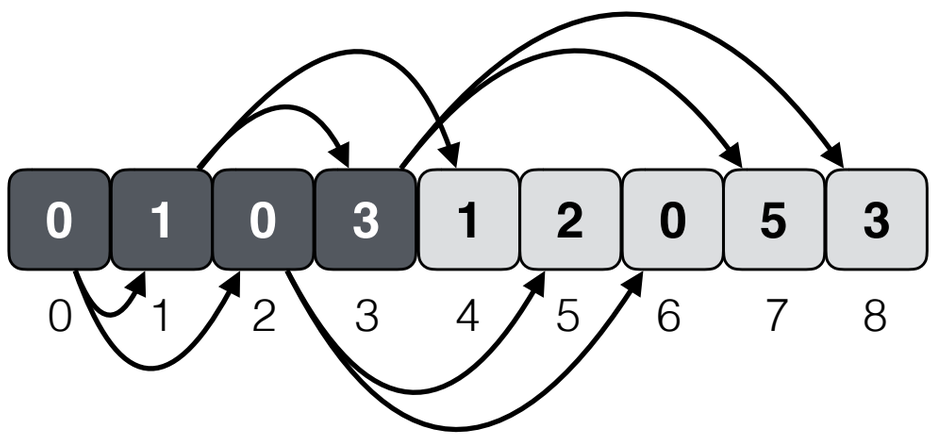

How do we represent trees?

Remember

Be skeptic: *measure* first and then conclude.

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Pointers VS. **Arrays**

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Pointers VS. **Arrays**

Experiment over 5 million nodes

Visit the tree and compute the sum of all nodes.

```
[→  segment_trees git:(master) ✗ g++ -std=c++11 -O3 trees_with_pointers.cpp -o trees_with_pointers
[→  segment_trees git:(master) ✗ ./trees_with_pointers < input7
building took: 19.3895 [sec]
sum is: 3676408720
sum took: 0.118645 [sec]
[→  segment_trees git:(master) ✗ g++ -std=c++11 -O3 trees_with_arrays.cpp -o trees_with_arrays
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building took: 18.0724 [sec]
sum is: 3676408920
sum took: 0.0118351 [sec]
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10X

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→  segment_trees git:(master) ✗
```

10X

OK, we are going to adopt the array-based representation!

Building Segment Trees recursively

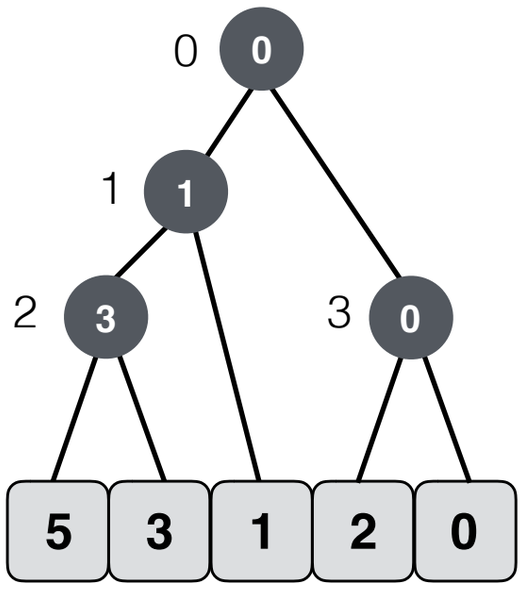
```
size_t n = leaves.size();
// round up to the next power of 2
size_t m = size_t(1) << static_cast<size_t>(ceil(log2(n)));
m_tree.resize(2 * m - 1, m_traits.invalid);

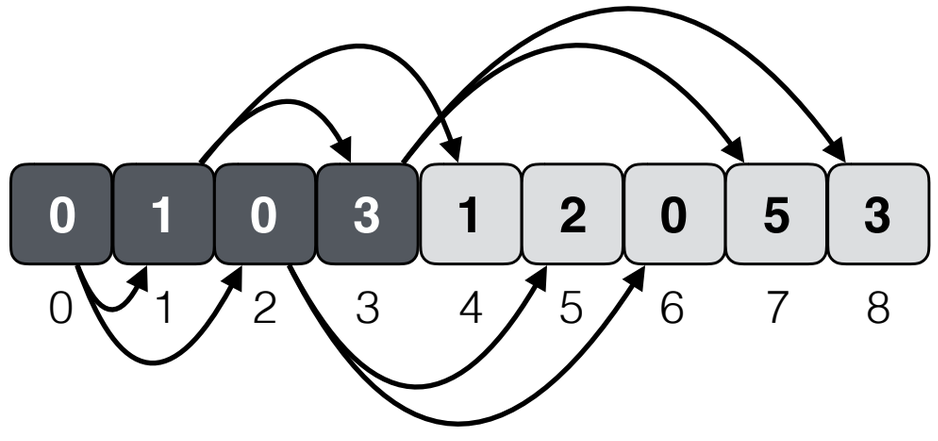
build(leaves, 0, m - 1, 0);
```

```
struct type_traits {
    IntType invalid;
    BinaryFunct funct;
};
```

```
void build(std::vector<IntType> const& leaves, size_t lo, size_t hi, size_t pos) {
    if (lo == hi) {
        m_tree[pos] = leaves[lo];
        return;
    }
    size_t mid = (lo + hi) / 2;
    build(leaves, lo, mid, LEFT(pos));
    build(leaves, mid + 1, hi, RIGHT(pos));
    m_tree[pos] = m_traits.funct(m_tree[LEFT(pos)], m_tree[RIGHT(pos)]);
}
```

```
#define LEFT(i)     2 * i + 1
#define RIGHT(i)    2 * i + 2
#define PARENT(i) (i - 1) / 2
```





Range (MIN) Queries with Segment Trees

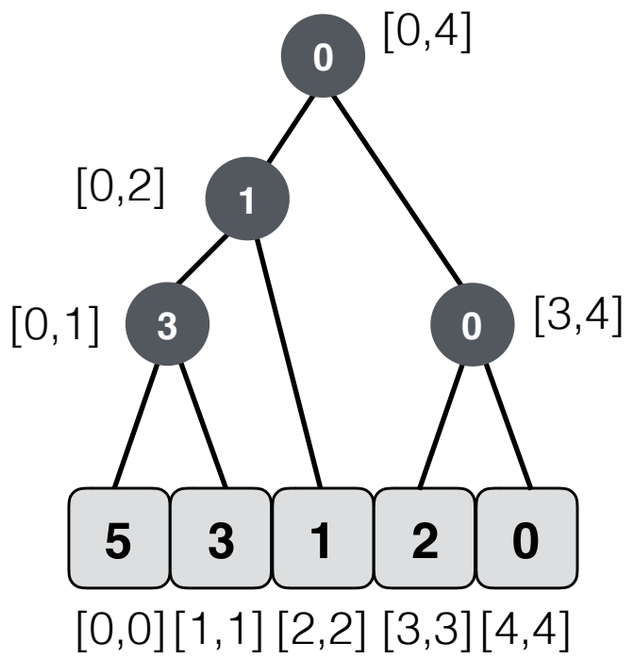
```
IntType rmq(range const& query, range node_segment, size_t pos) {

    if (query.lo <= node_segment.lo
    and query.hi >= node_segment.hi) { // total overlap
        return m_tree[pos];
    }
    if (query.lo > node_segment.hi
    or  query.hi < node_segment.lo) { // no overlap
        return m_traits.invalid;
    }

    // partial overlap
    size_t mid = (node_segment.lo + node_segment.hi) / 2;
    return m_traits.funct(
        rmq(query, {node_segment.lo, mid}, LEFT(pos)),
        rmq(query, {mid + 1, node_segment.hi}, RIGHT(pos))
    );
}
```

```
struct range {
    range(size_t l, size_t h)
        : lo(l), hi(h)
    {}

    size_t lo, hi;
};
```



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```

```
struct range {
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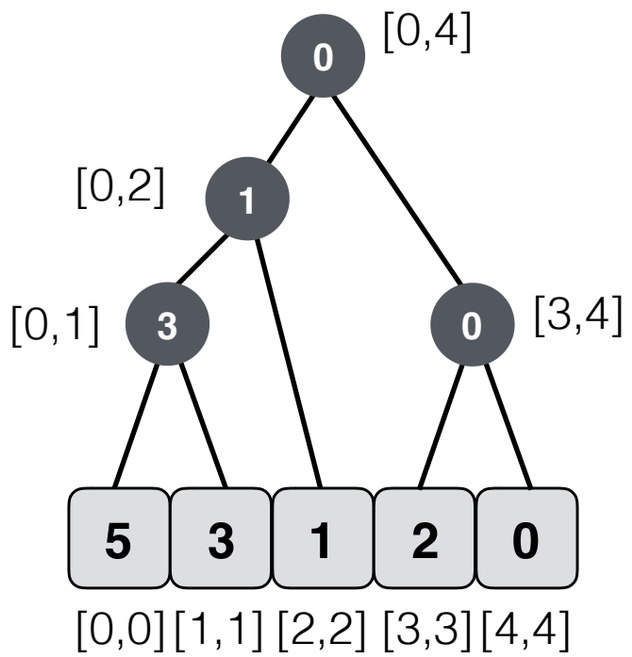
```

min(1,3) = ?



Updating Segment Trees

Let's add two new operations (updates):

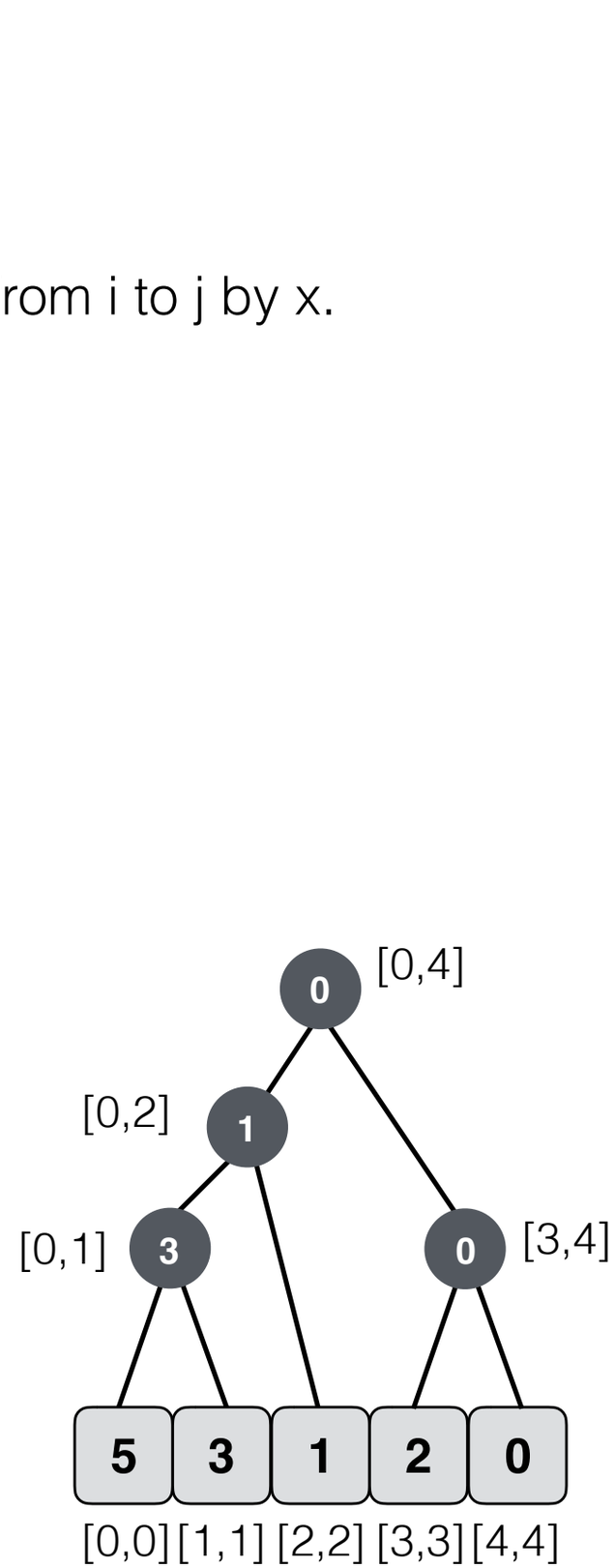
- `update`(i, x) which increments the i-th leaf by x;
- `update_range`(i, j ,x) which increments all leaves from i to j by x.

```
void update(size_t i, IntType delta, range node_segment, size_t pos) {

    if (i > node_segment.hi
    or  i < node_segment.lo) {
        return;
    }

    if (node_segment.lo == node_segment.hi) { // leaf
        m_tree[pos] += delta;
        return;
    }

    size_t mid = (node_segment.lo + node_segment.hi) / 2;
    update(i, delta, {node_segment.lo, mid}, LEFT(pos));
    update(i, delta, {mid + 1, node_segment.hi}, RIGHT(pos));
    m_tree[pos] = m_traits.funct(m_tree[LEFT(pos)], m_tree[RIGHT(pos)]);
}
```



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    update(i, delta, {mid + 1, node_segment.hi}, RIGHT(pos));
    m_tree[pos] = m_traits.funct(m_tree[LEFT(pos)], m_tree[RIGHT(pos)]);
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if (query.lo > node_segment.hi
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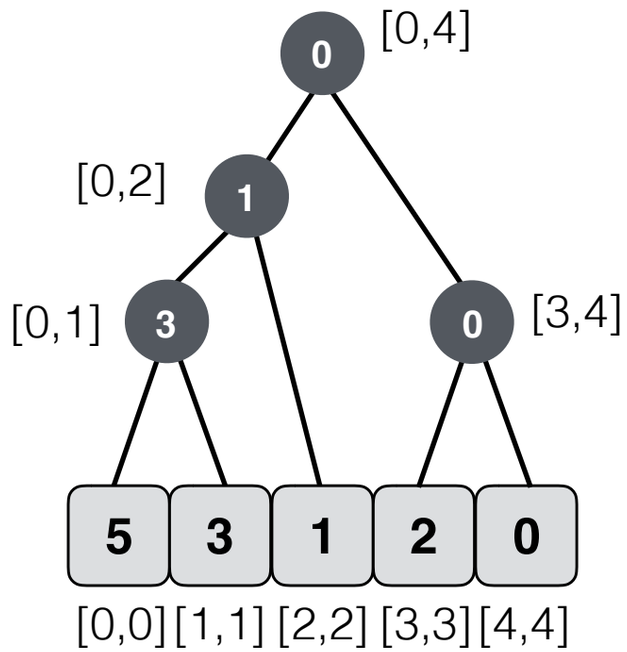

Benchmarking Segment Trees

```
→  segment_trees git:(master) ✗ python gen_data.py 5000000 100000 10000 > input13
→  segment_trees git:(master) ✗ ./rmq_segment_tree < input13
parsing the input took: 18.5193 [sec]
building tree with 5000000 leaves
building took: 0.314939 [sec]
executing 100000 range queries
average query time: 1.74382 [musec]
executing 10000 updates
average update time: 0.561733 [musec]
executing 10000 range updates
average range update time: 2.55461 [musec]
→  segment_trees git:(master) ✗
```

Lazy Propagation in Segment Trees

Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

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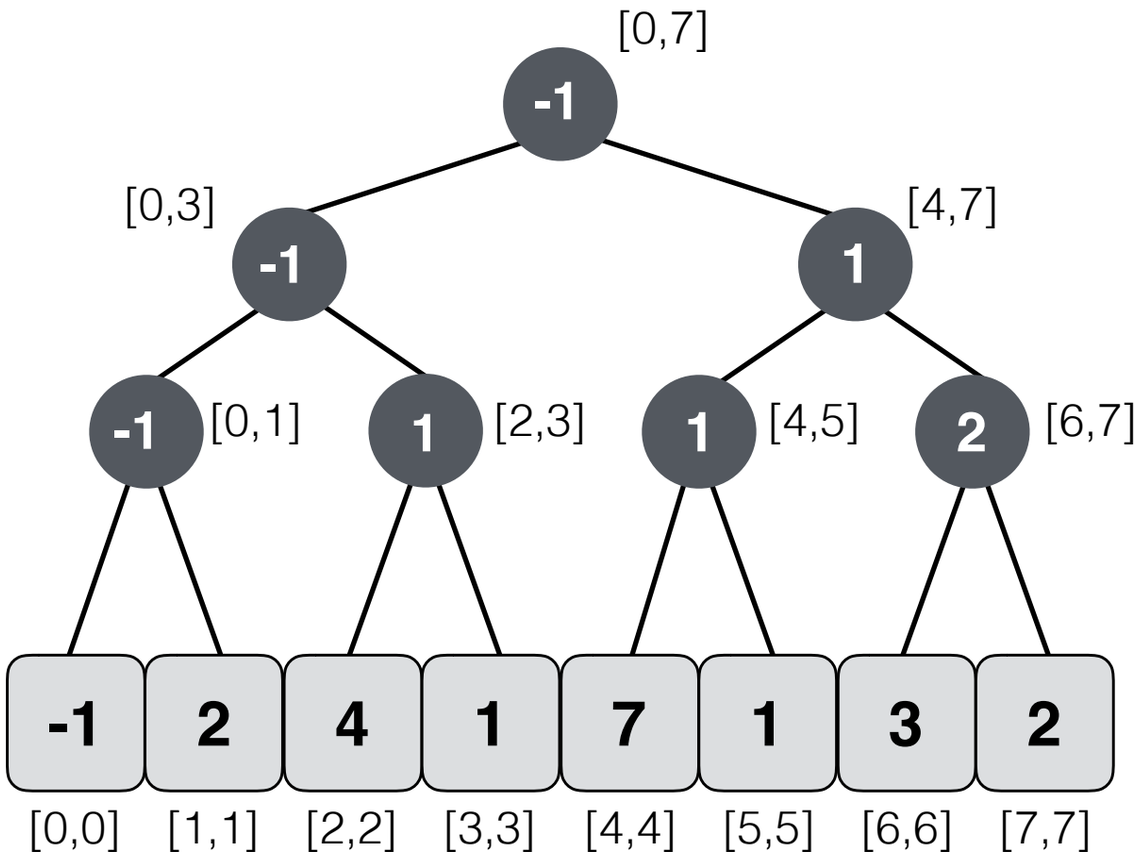
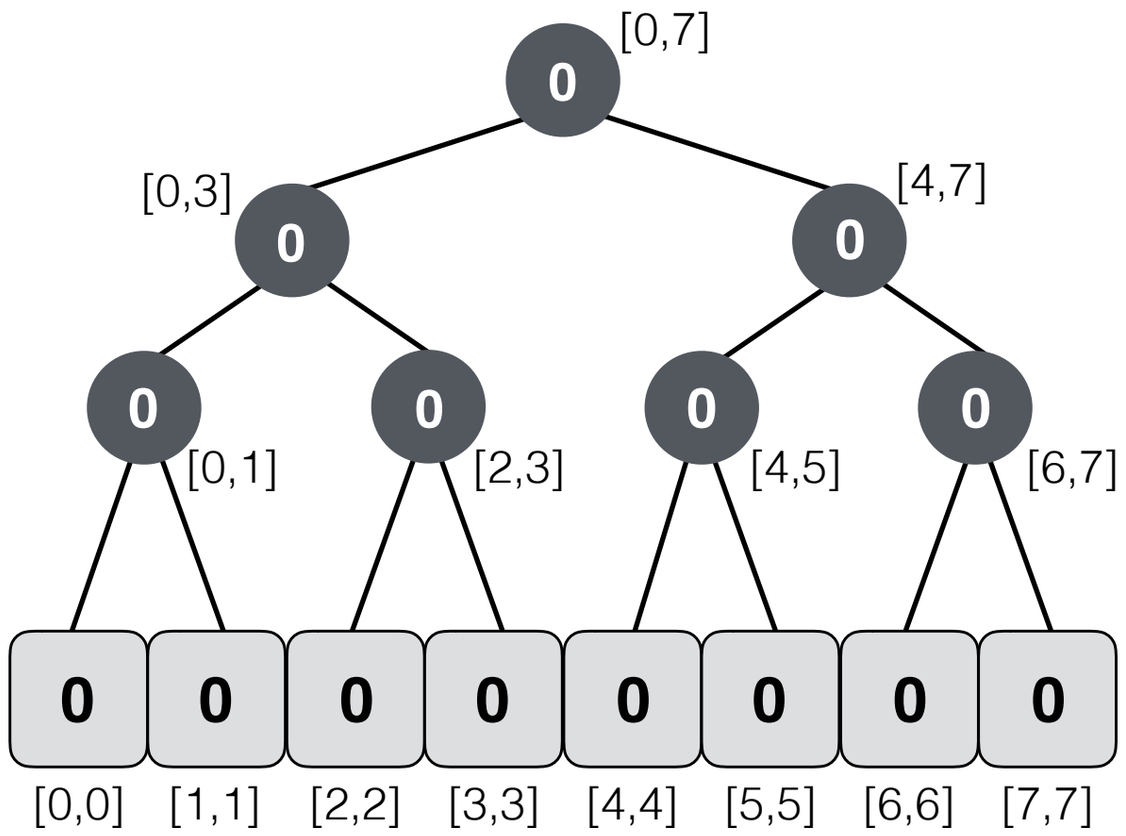


Segment Tree



Lazy Tree

Lazy Propagation in Segment Trees

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`update_range`(0,3,3)
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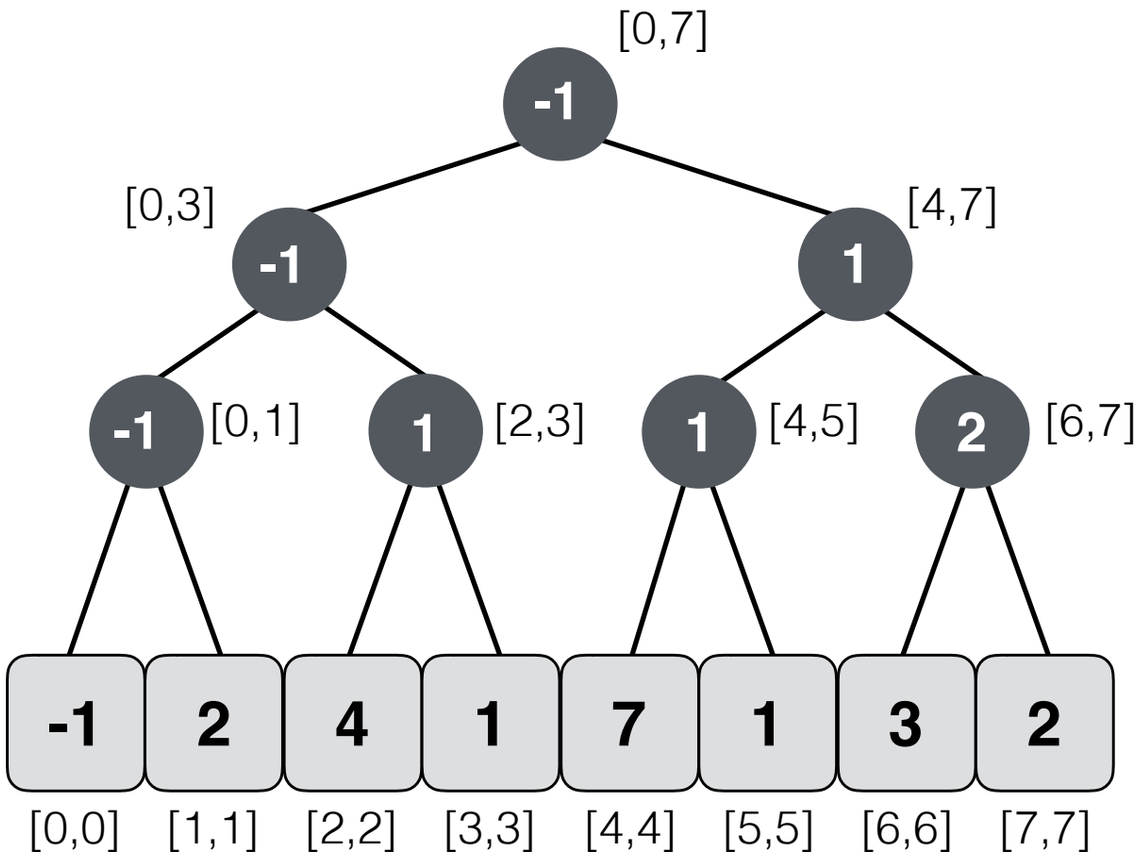
`update_range`(0,0,2)
`rmq`(3,5) = ?



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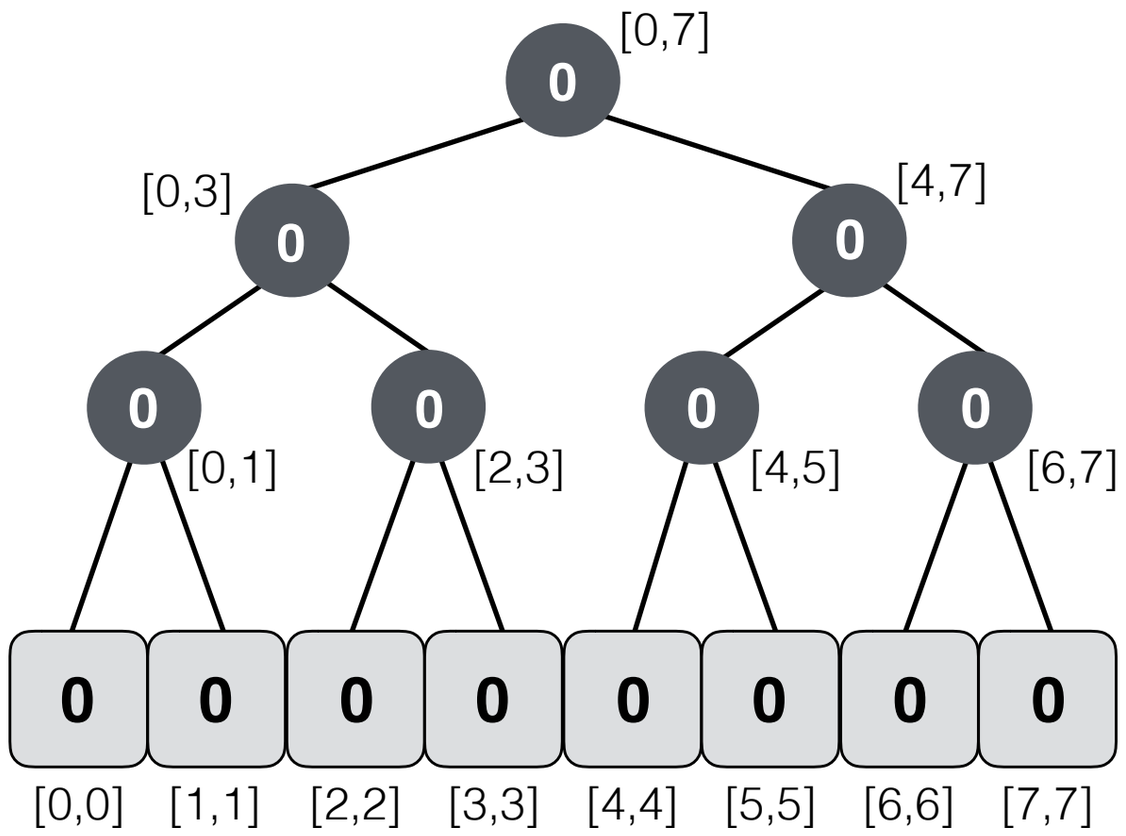

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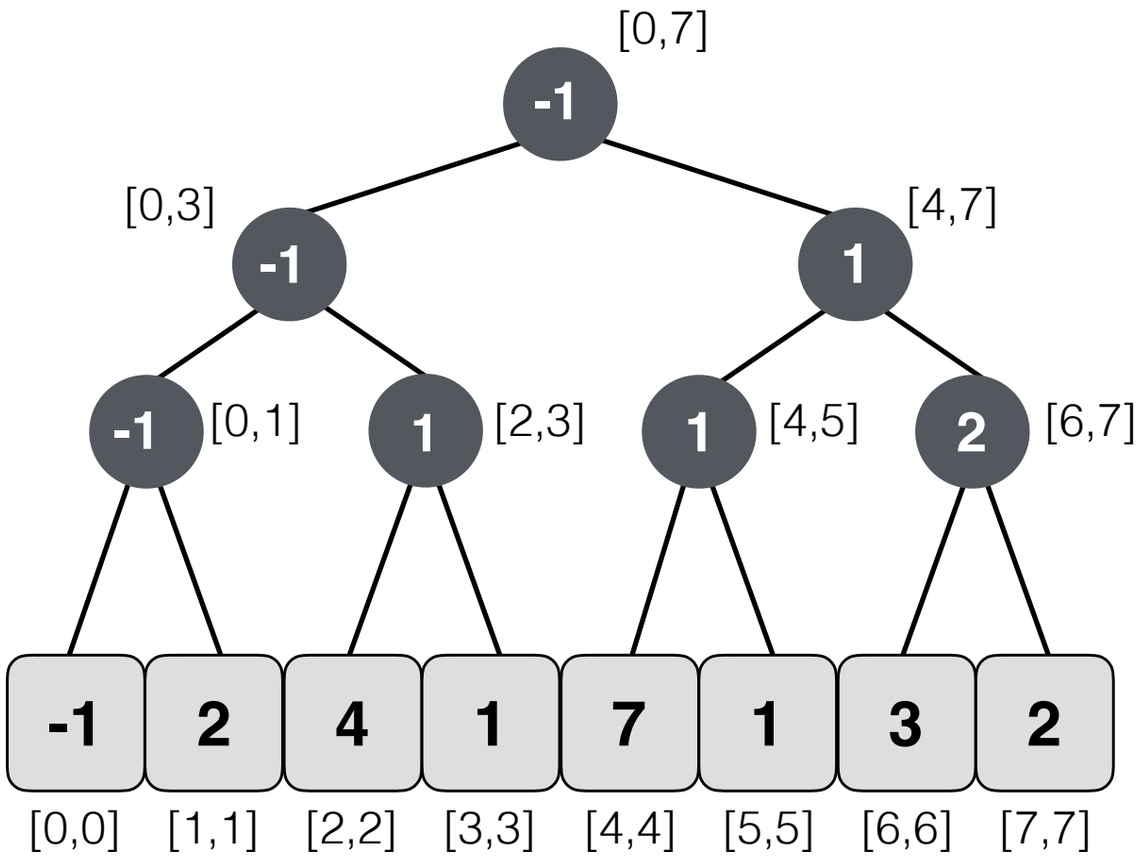
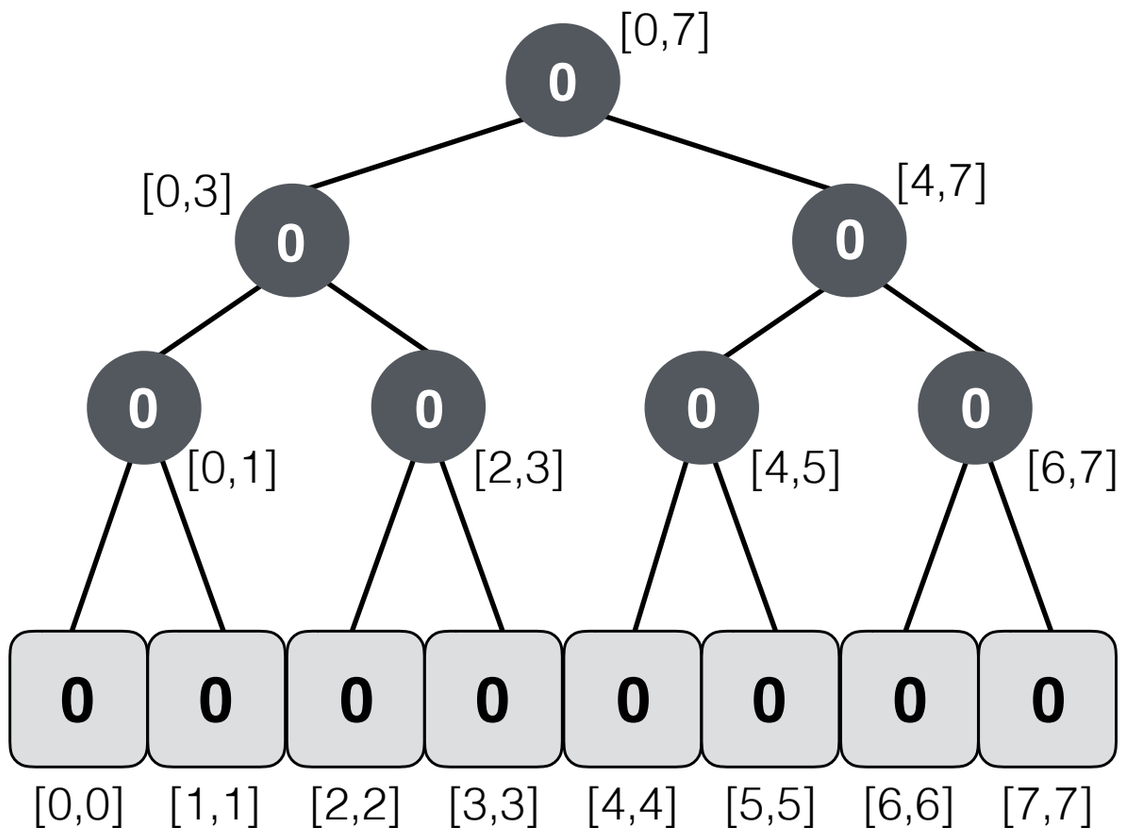


Segment Tree



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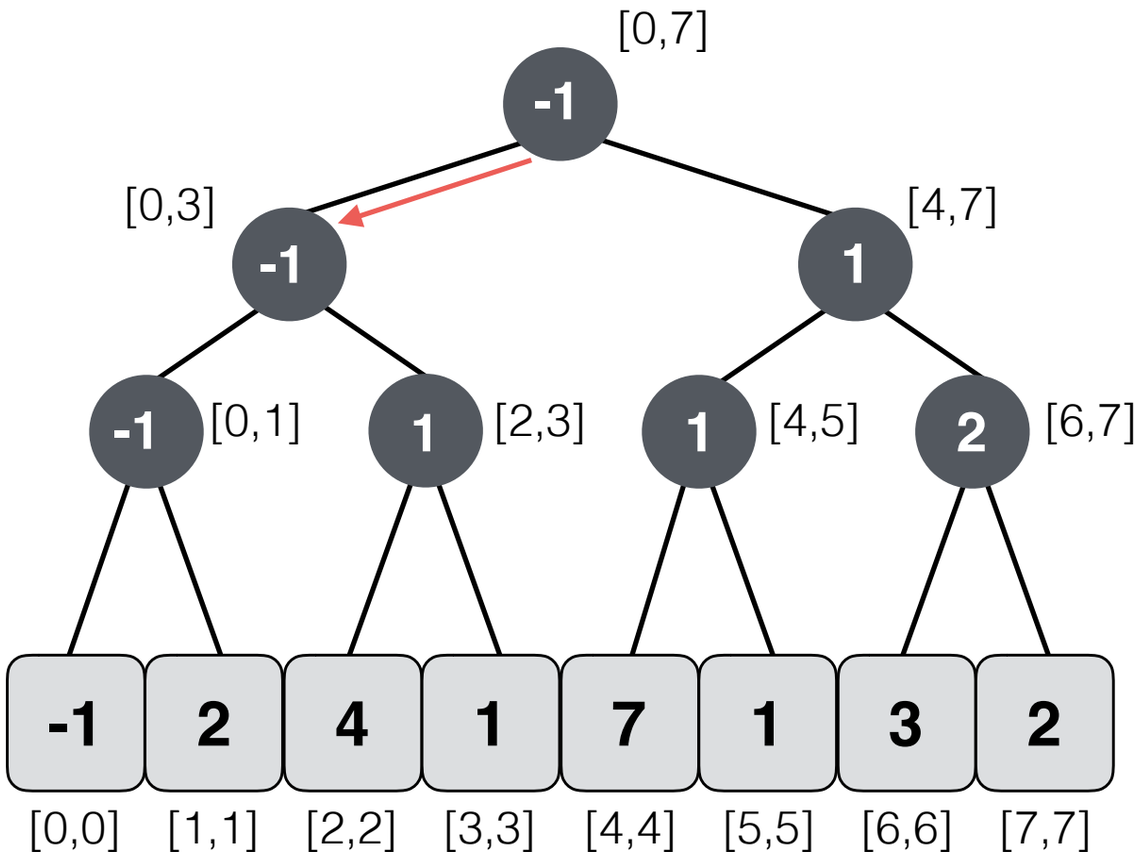

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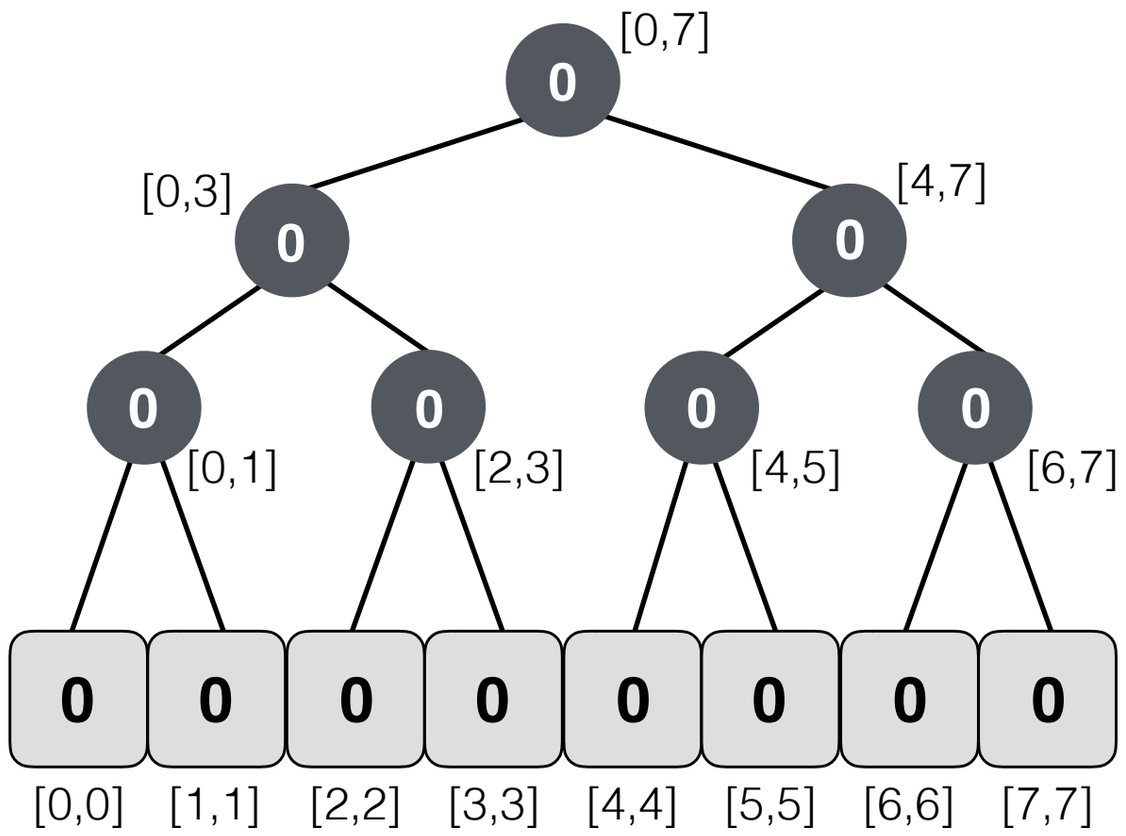

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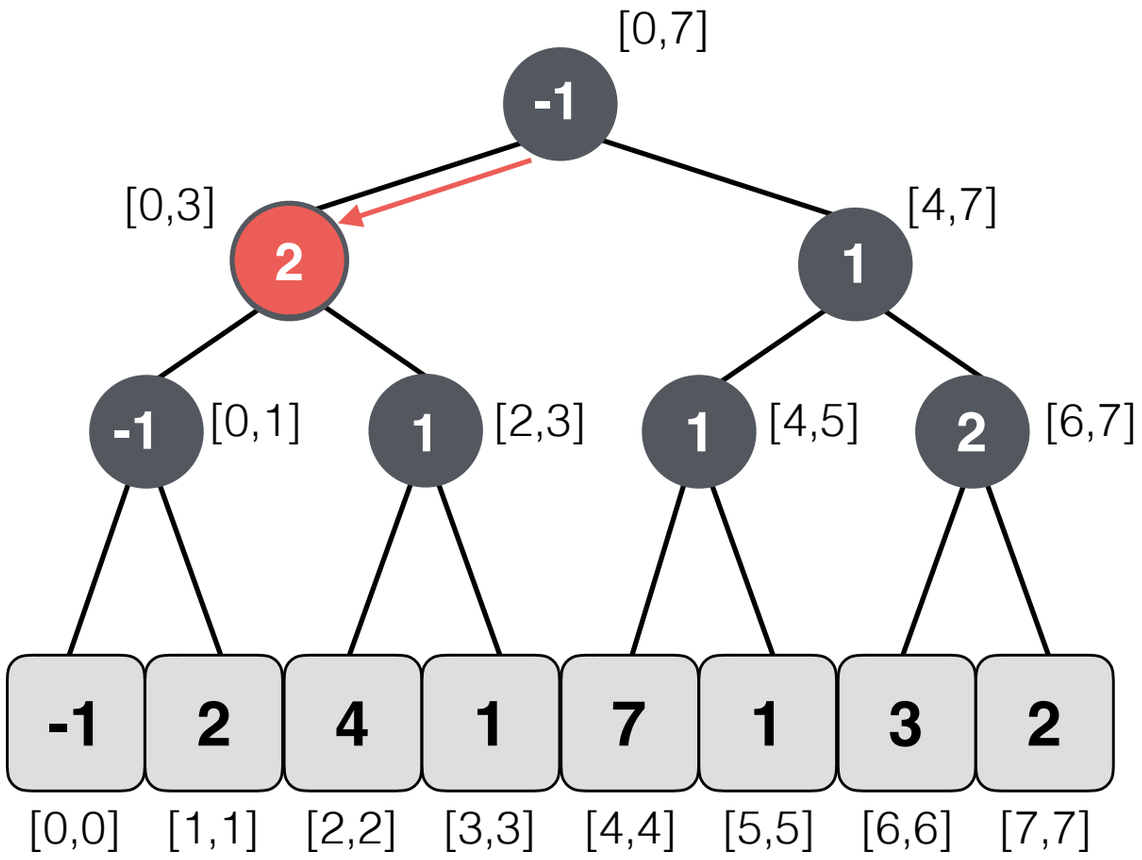

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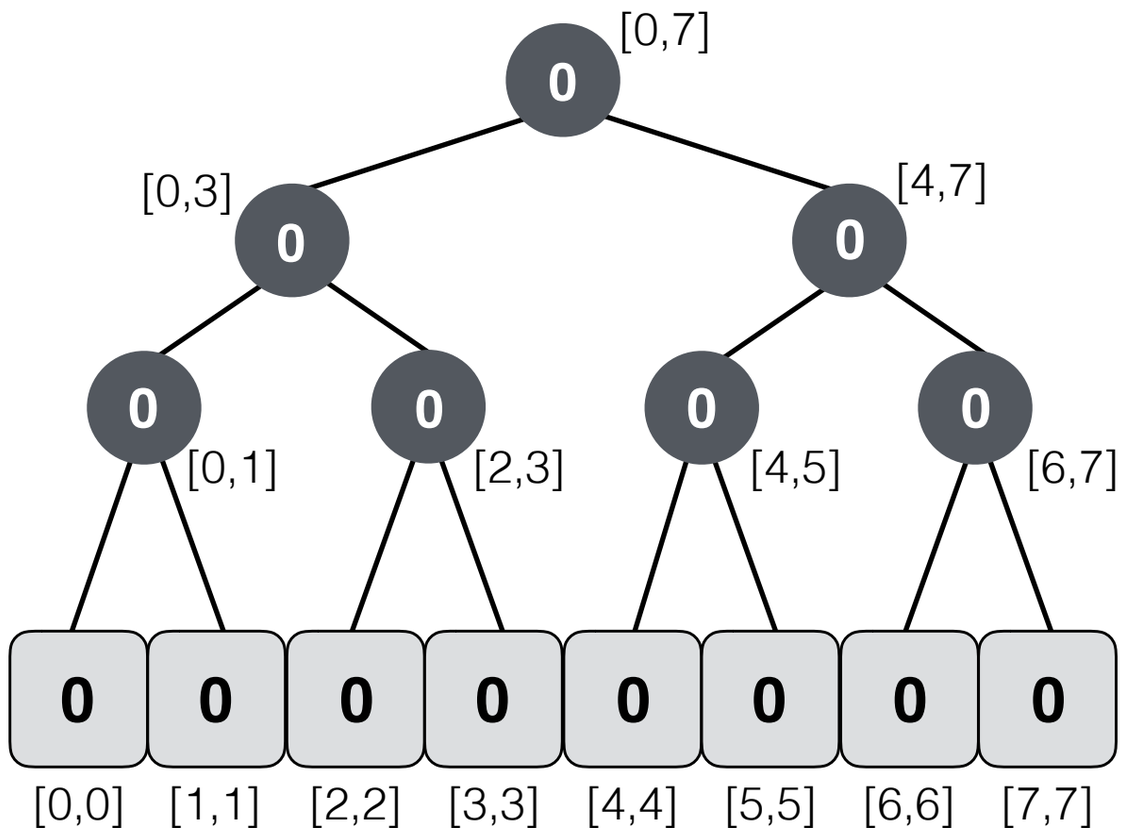

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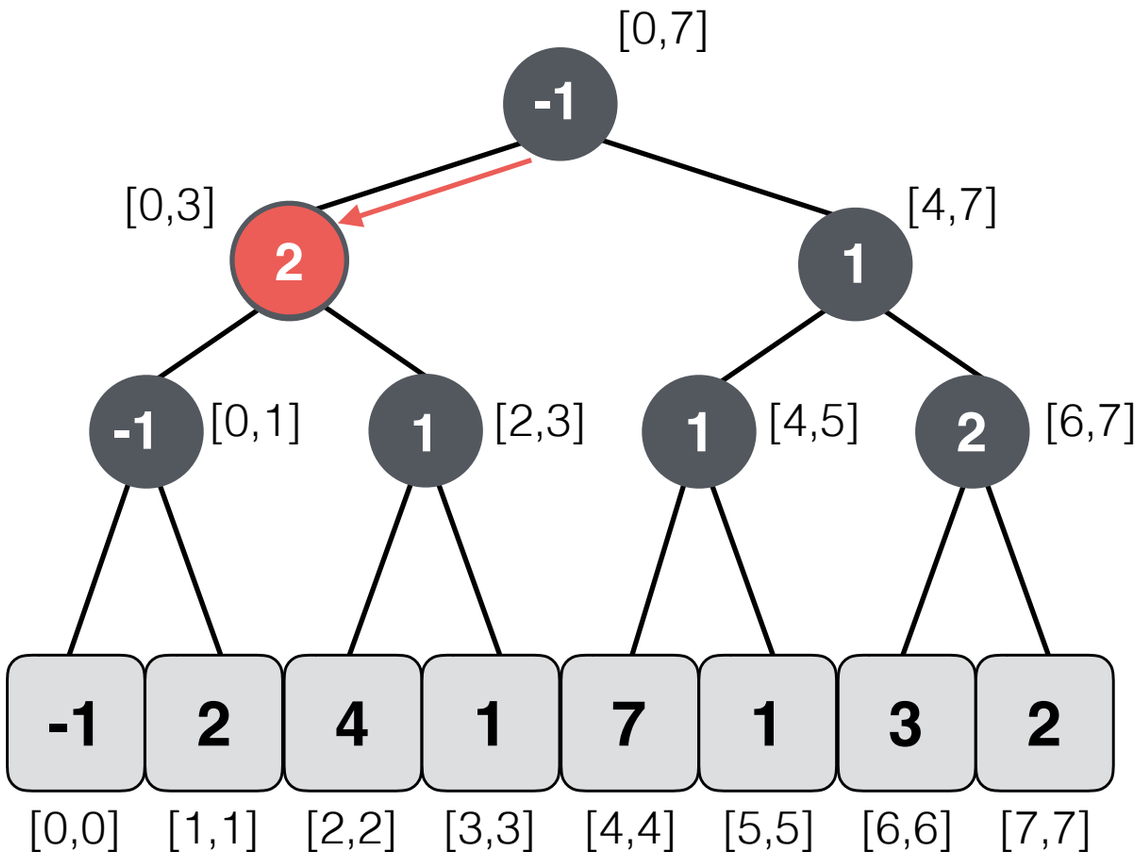

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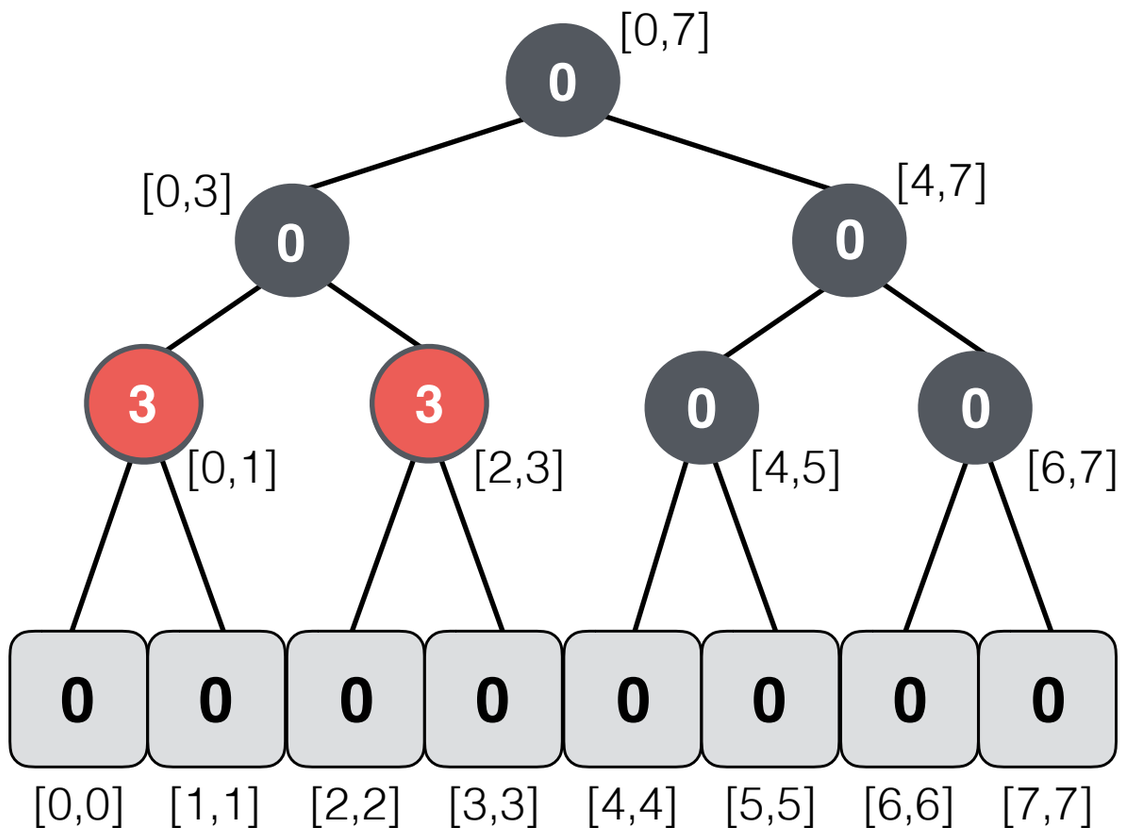

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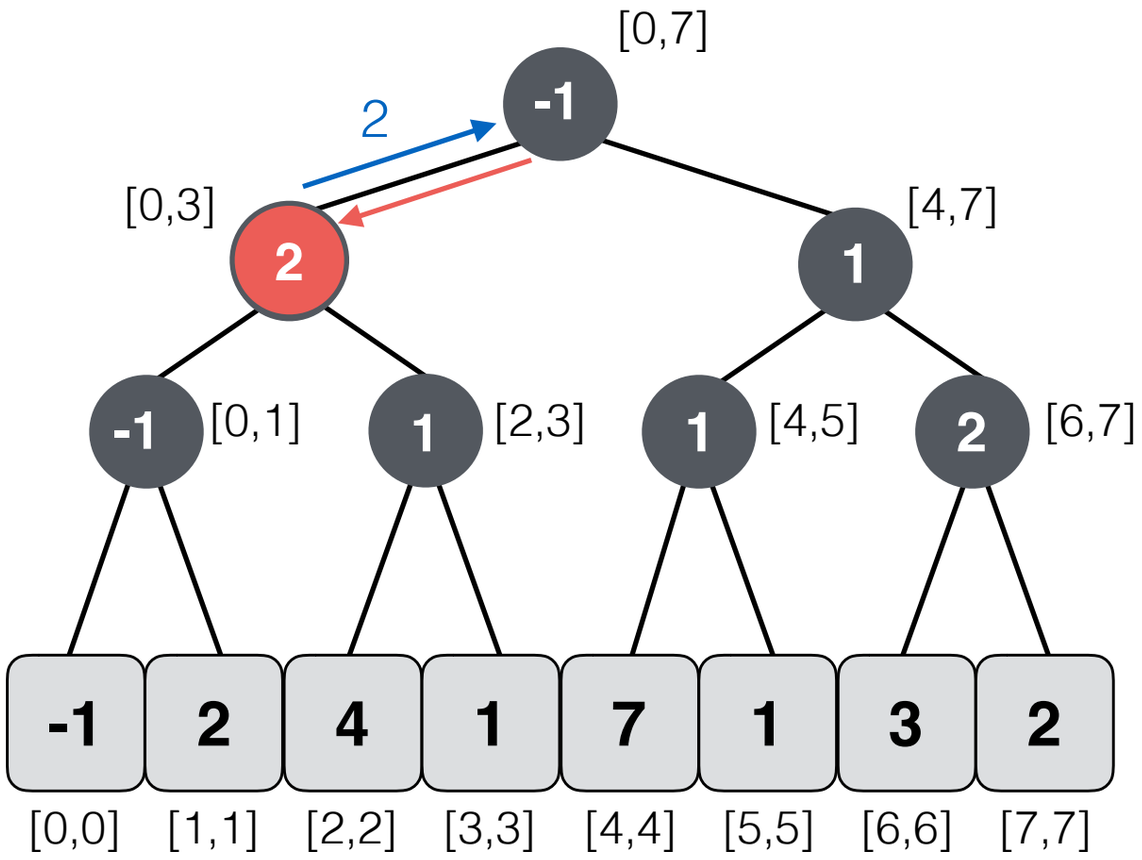

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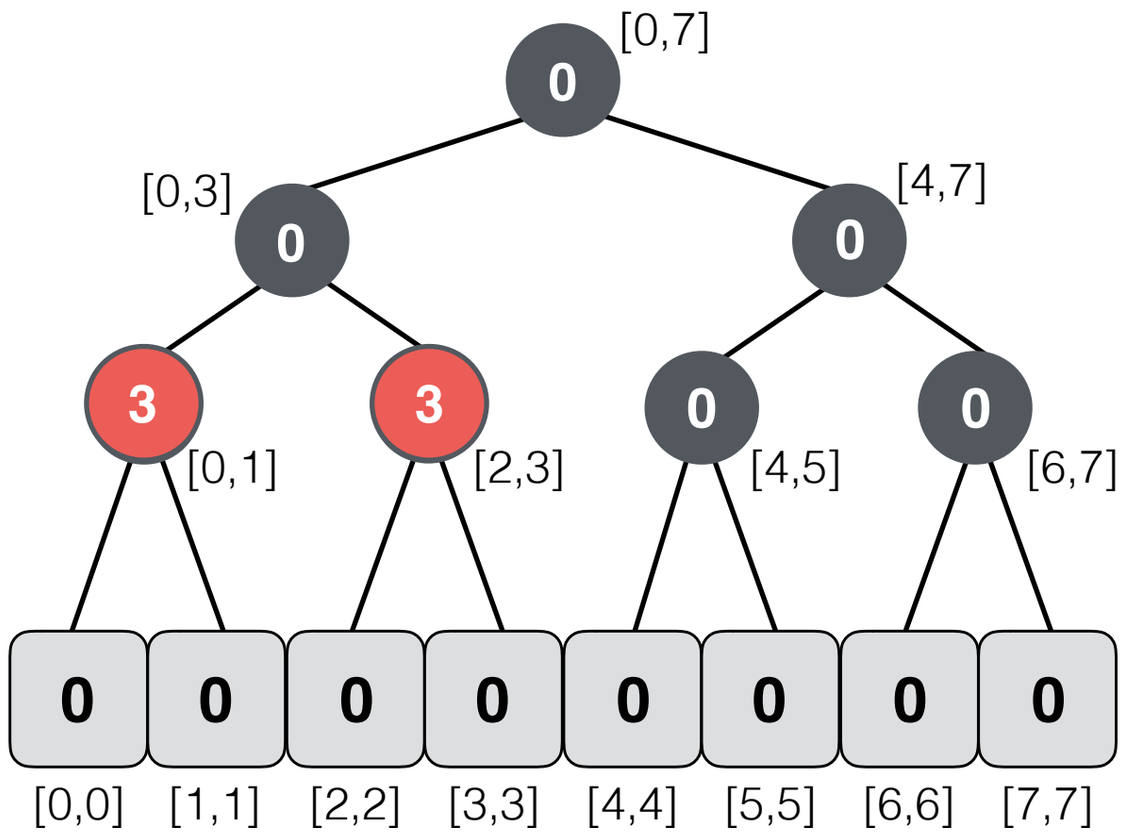

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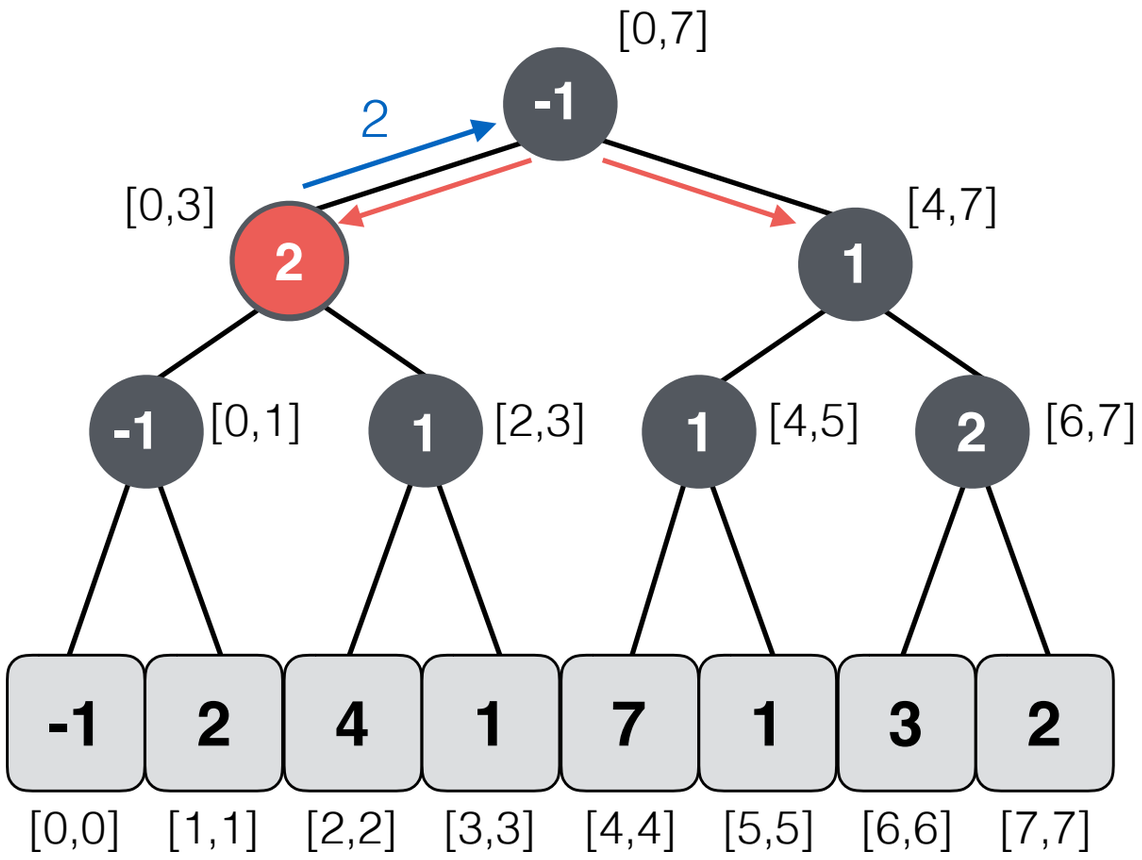

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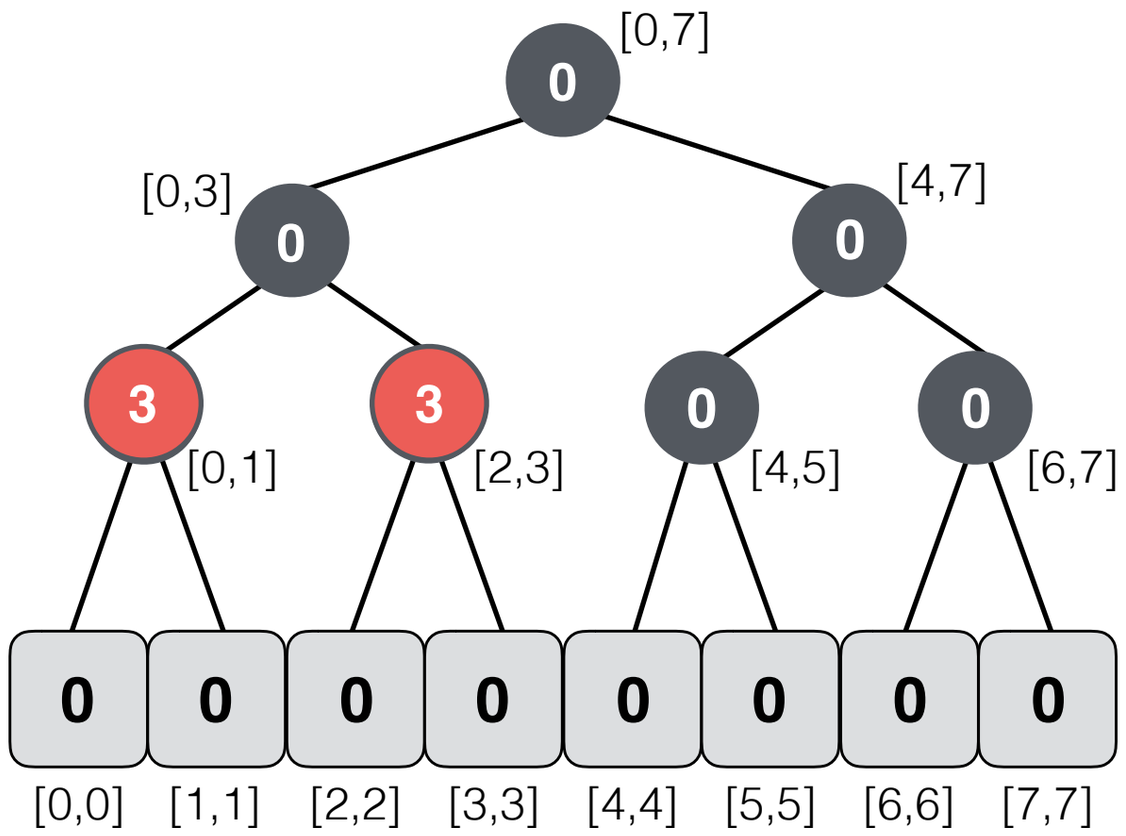

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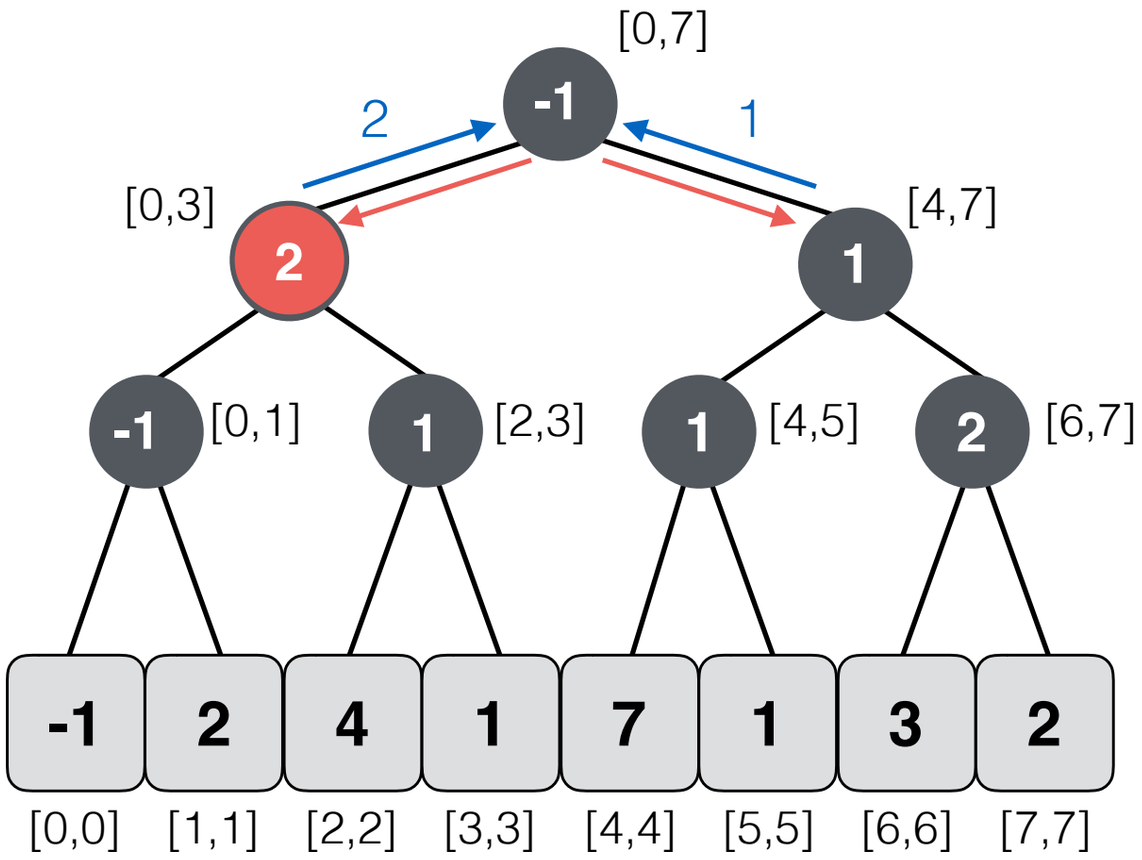

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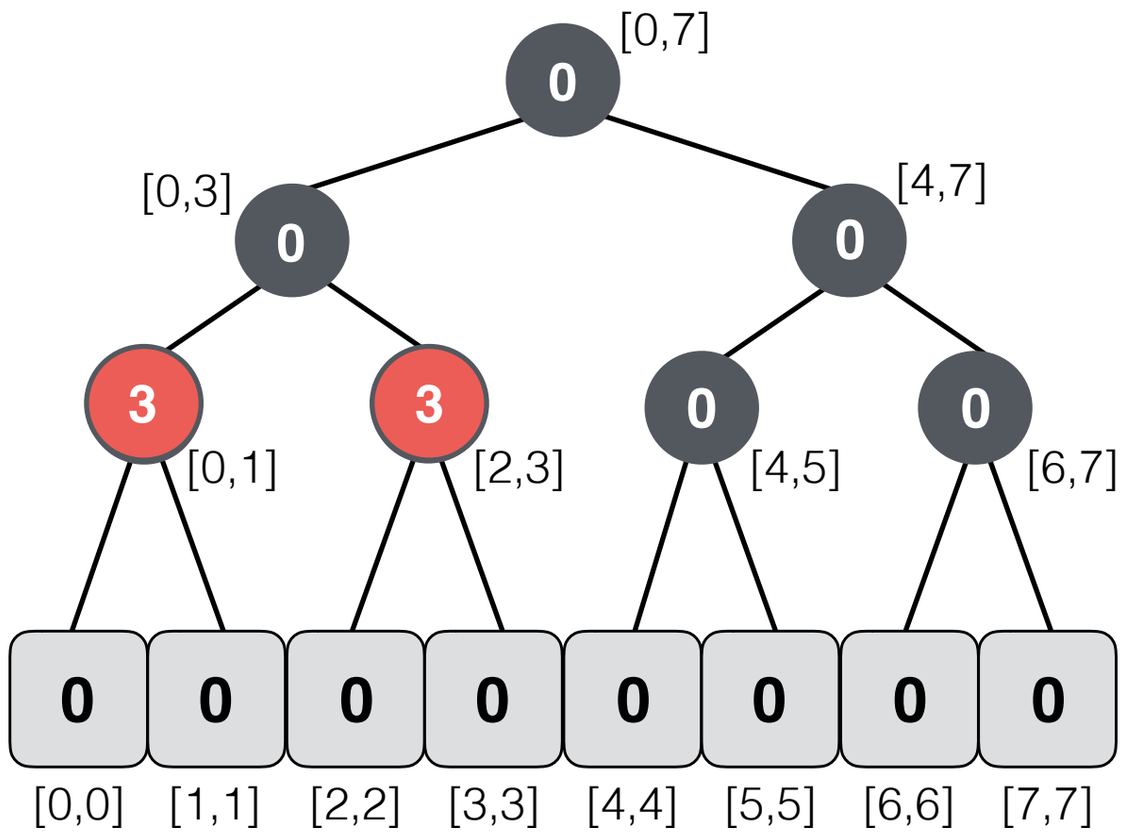

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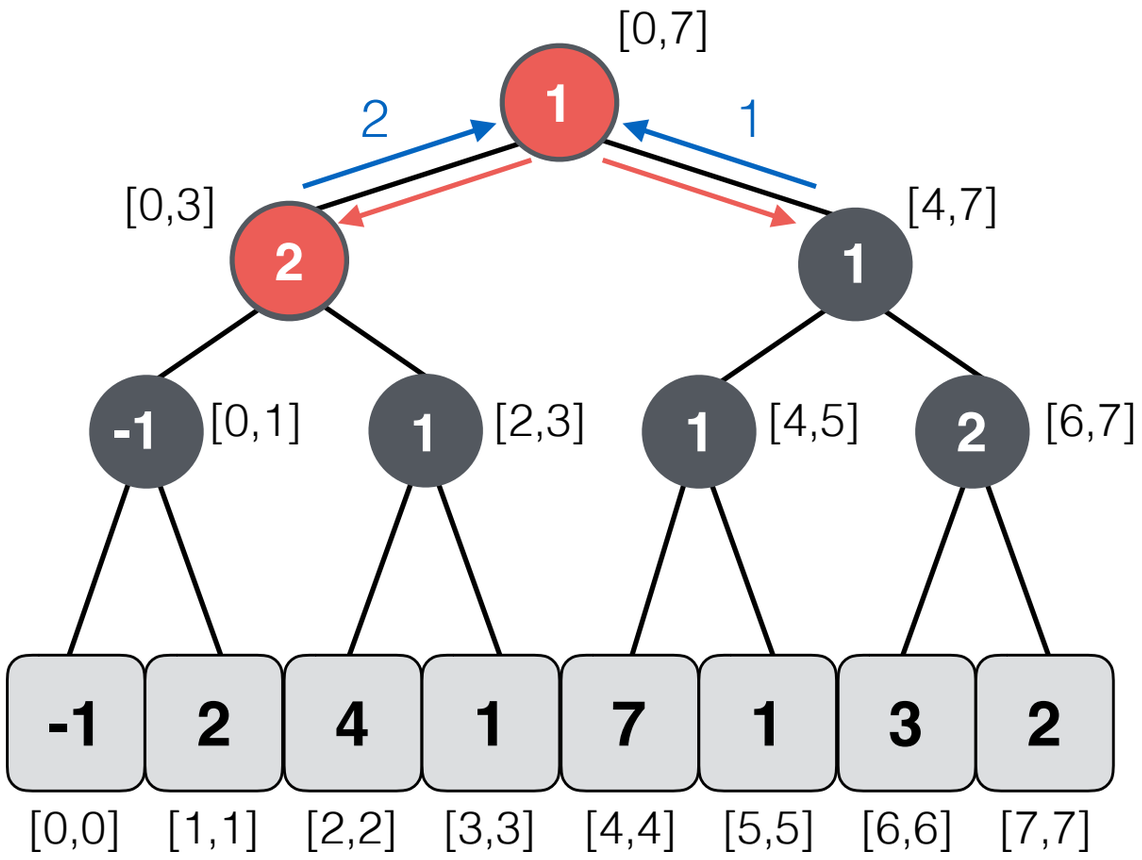

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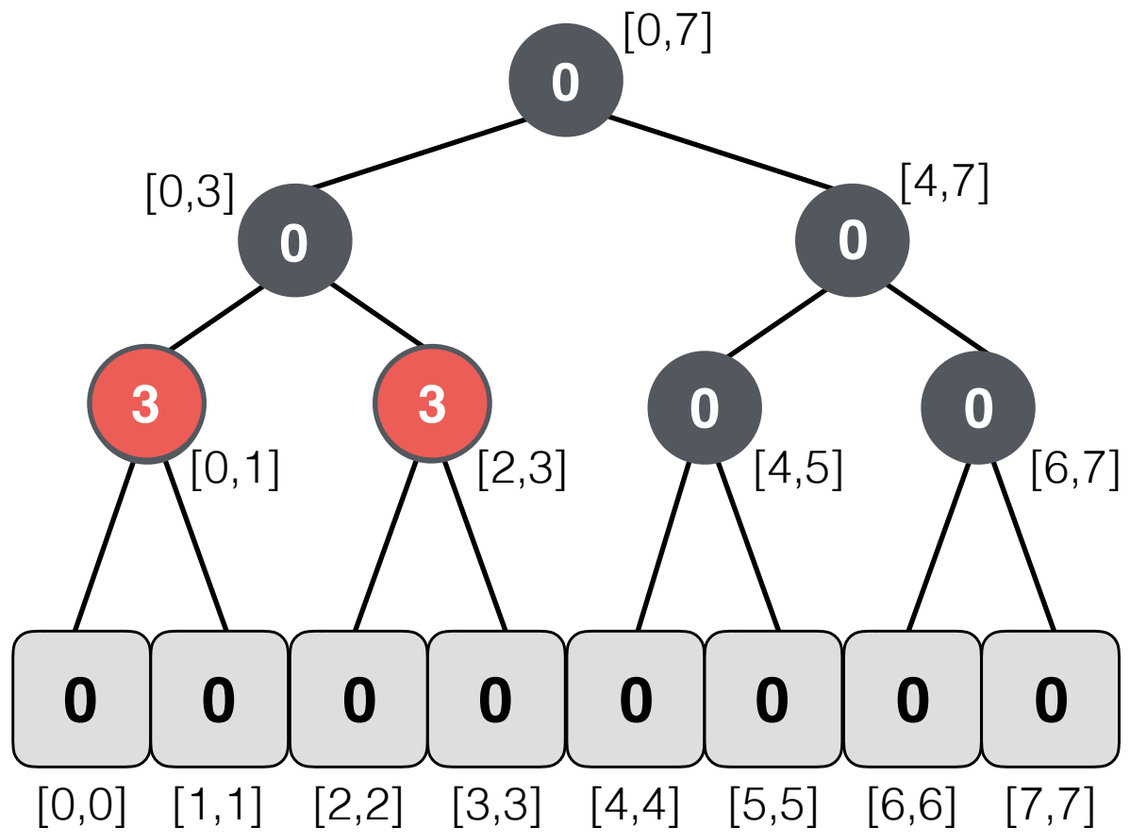

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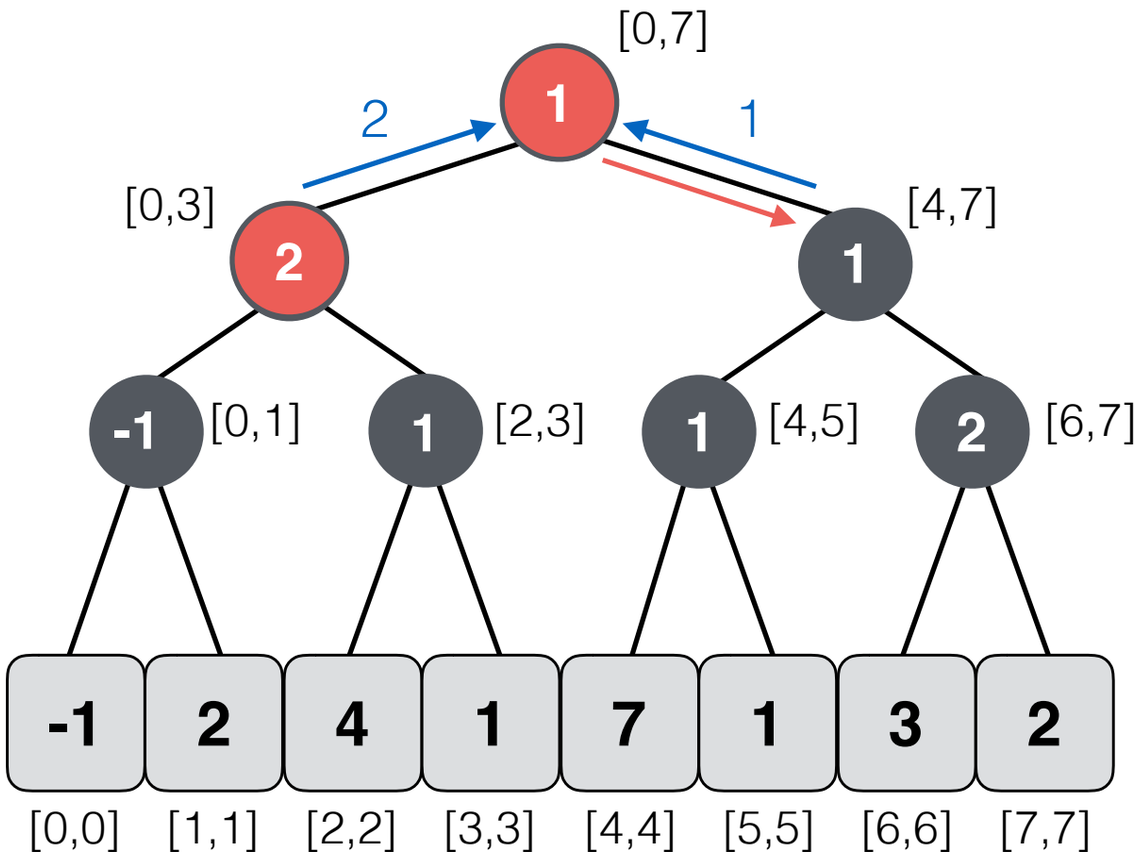

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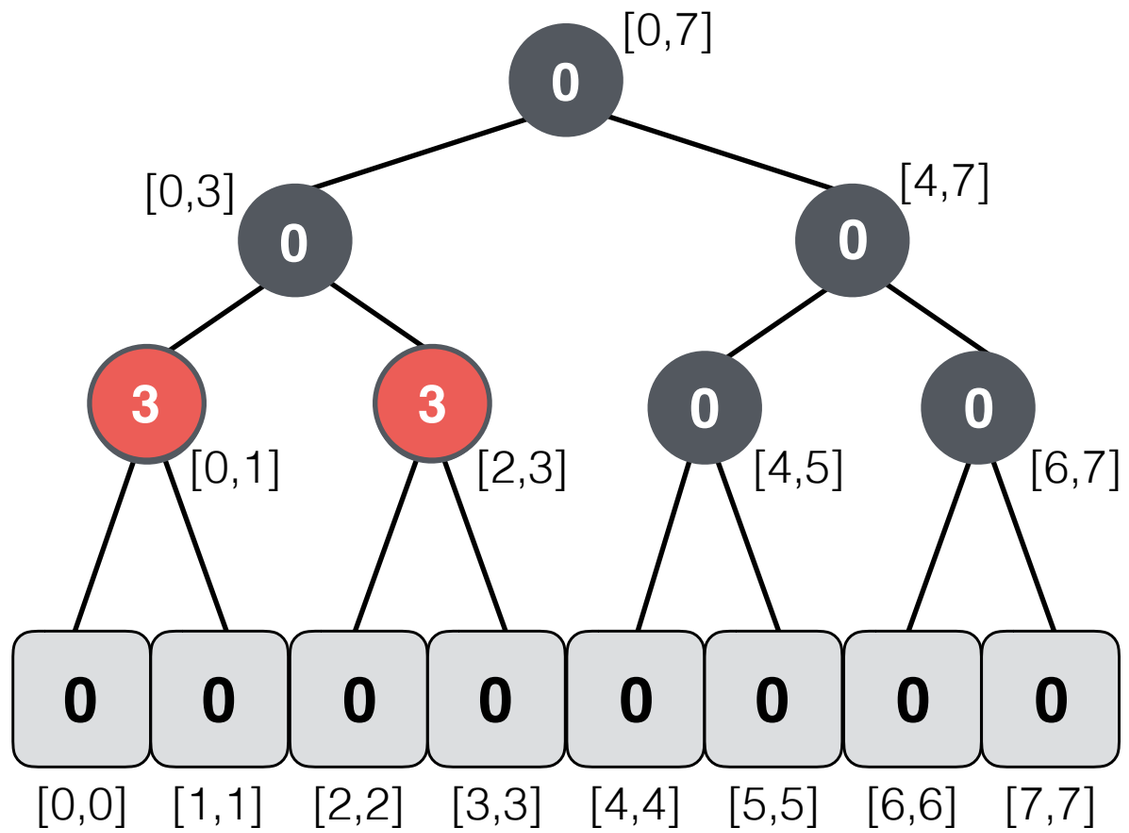

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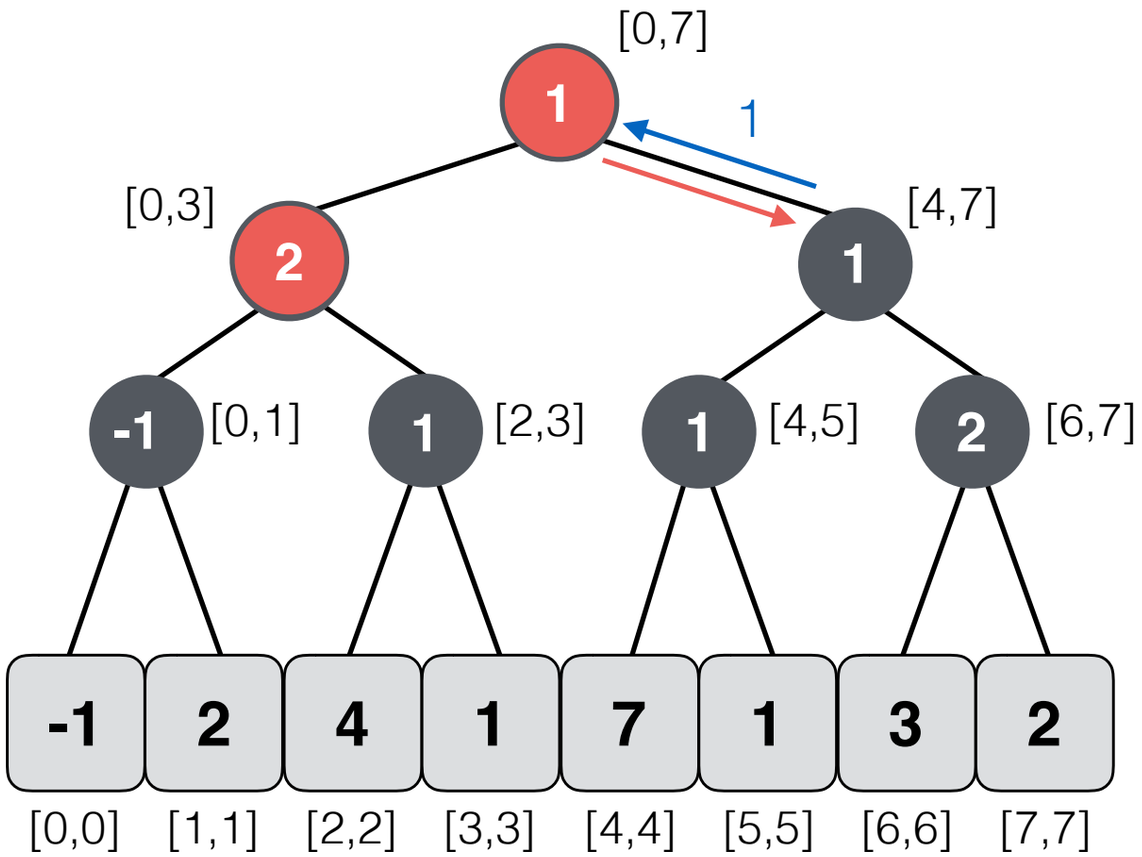

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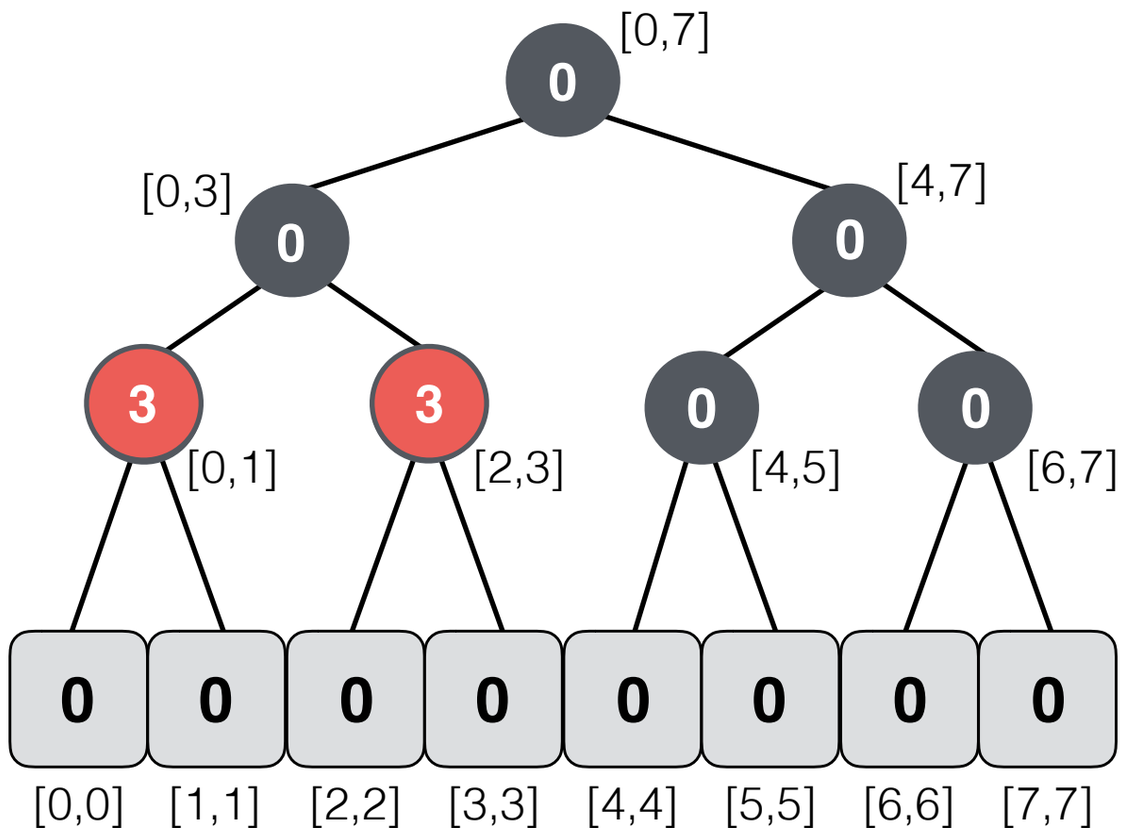

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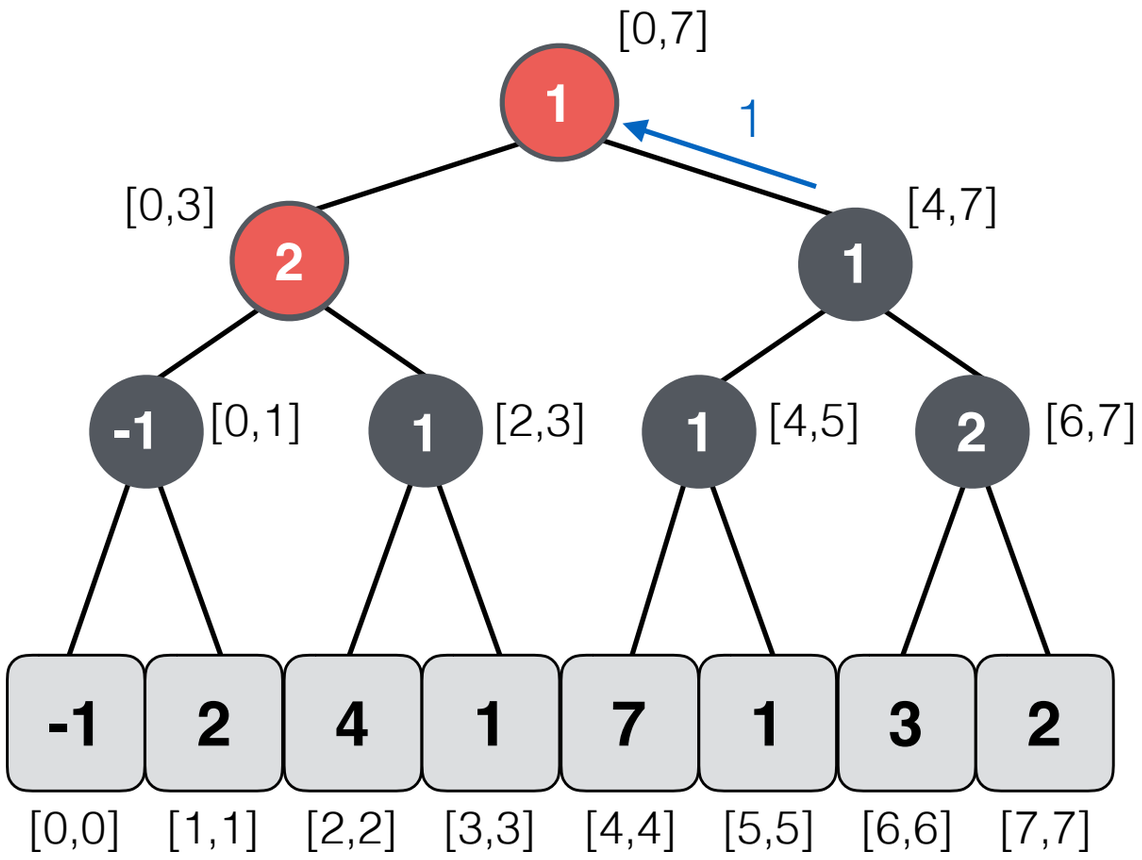

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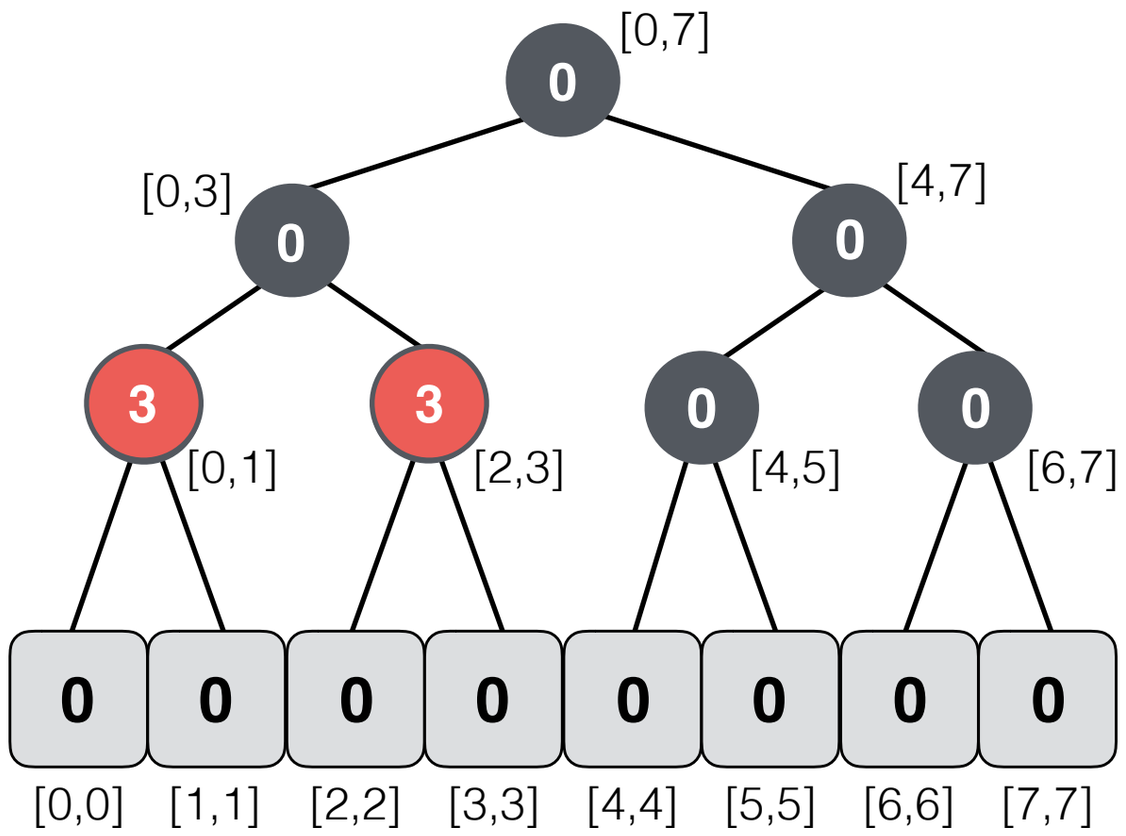

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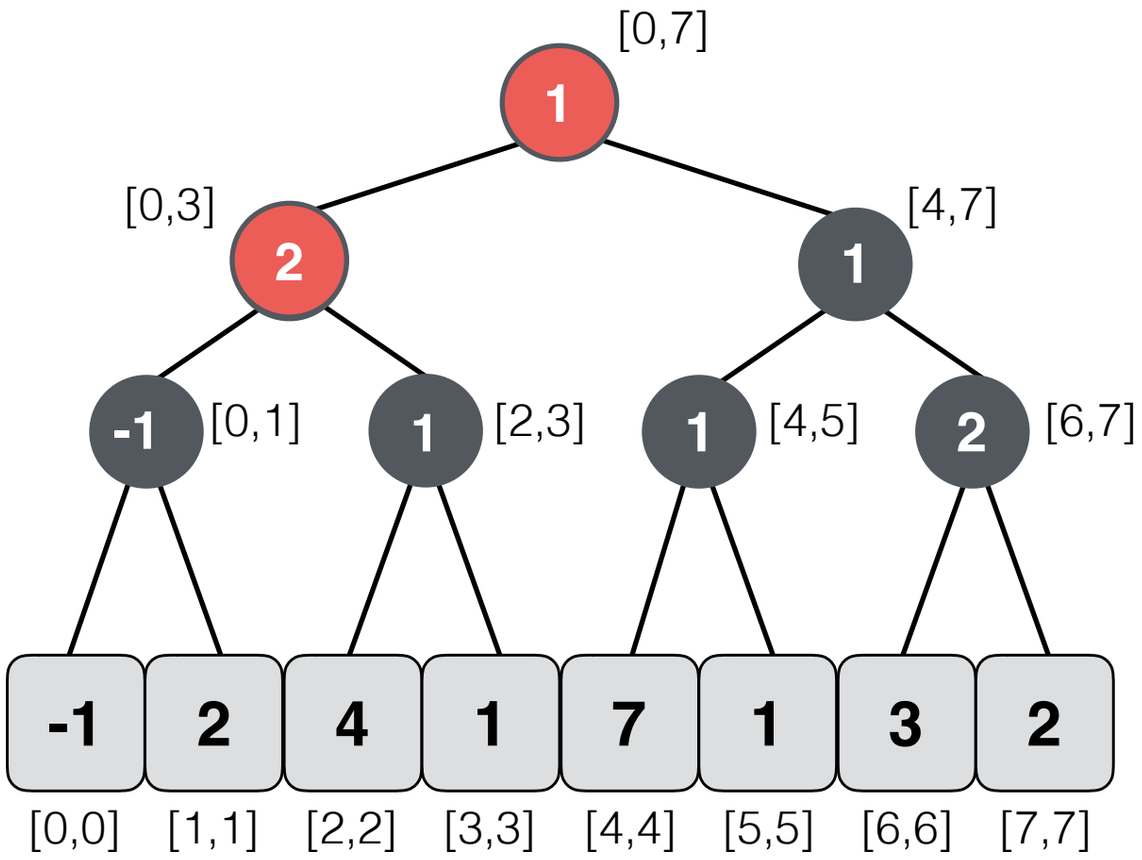

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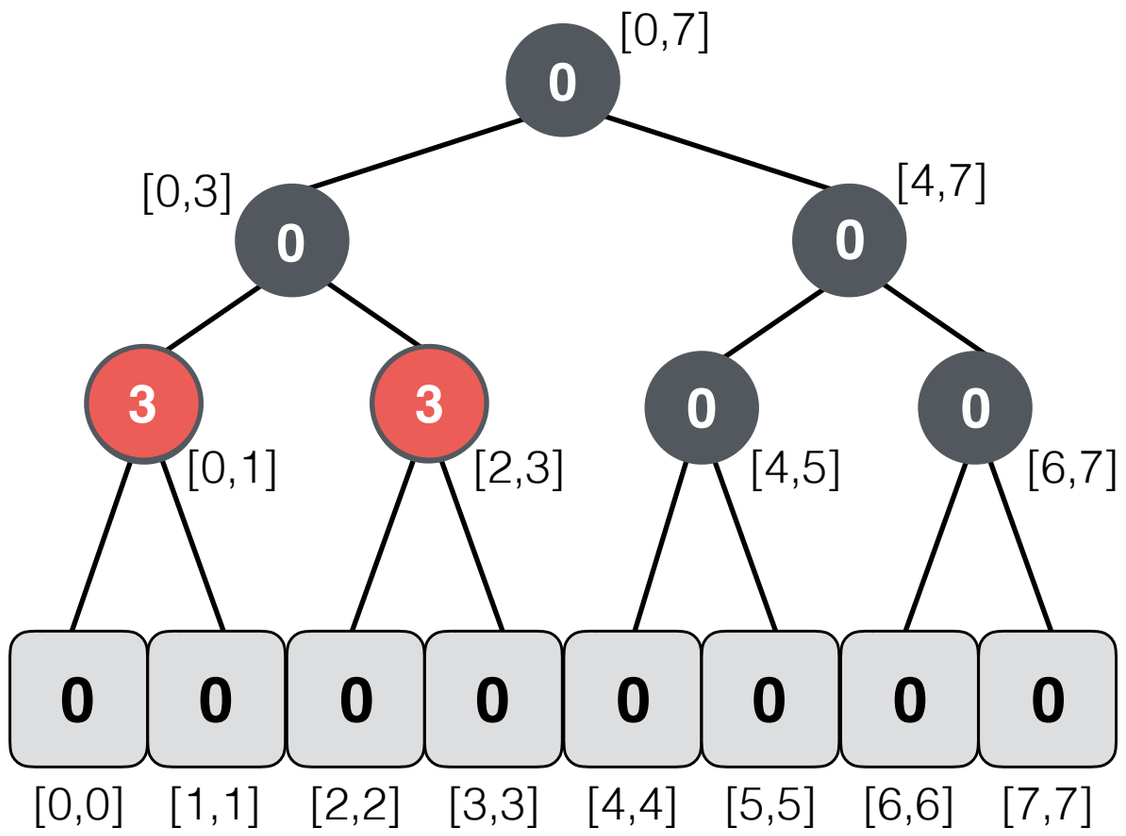

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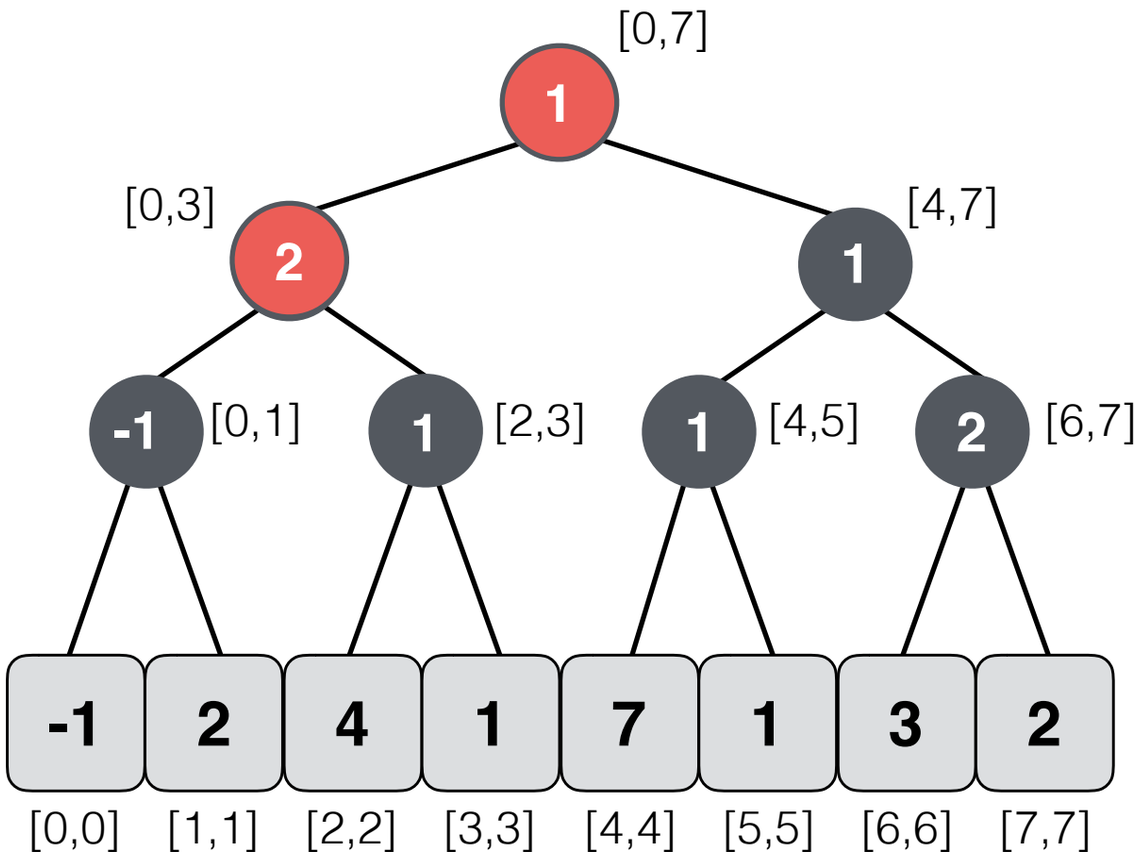

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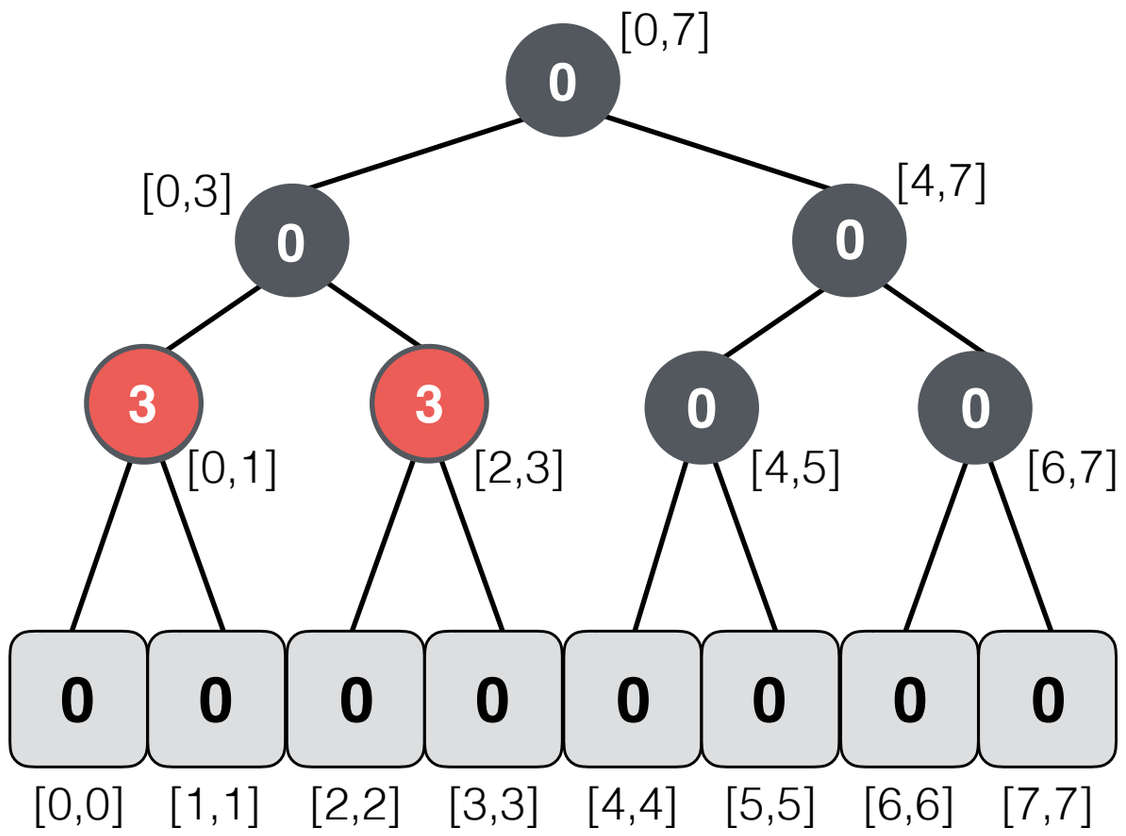

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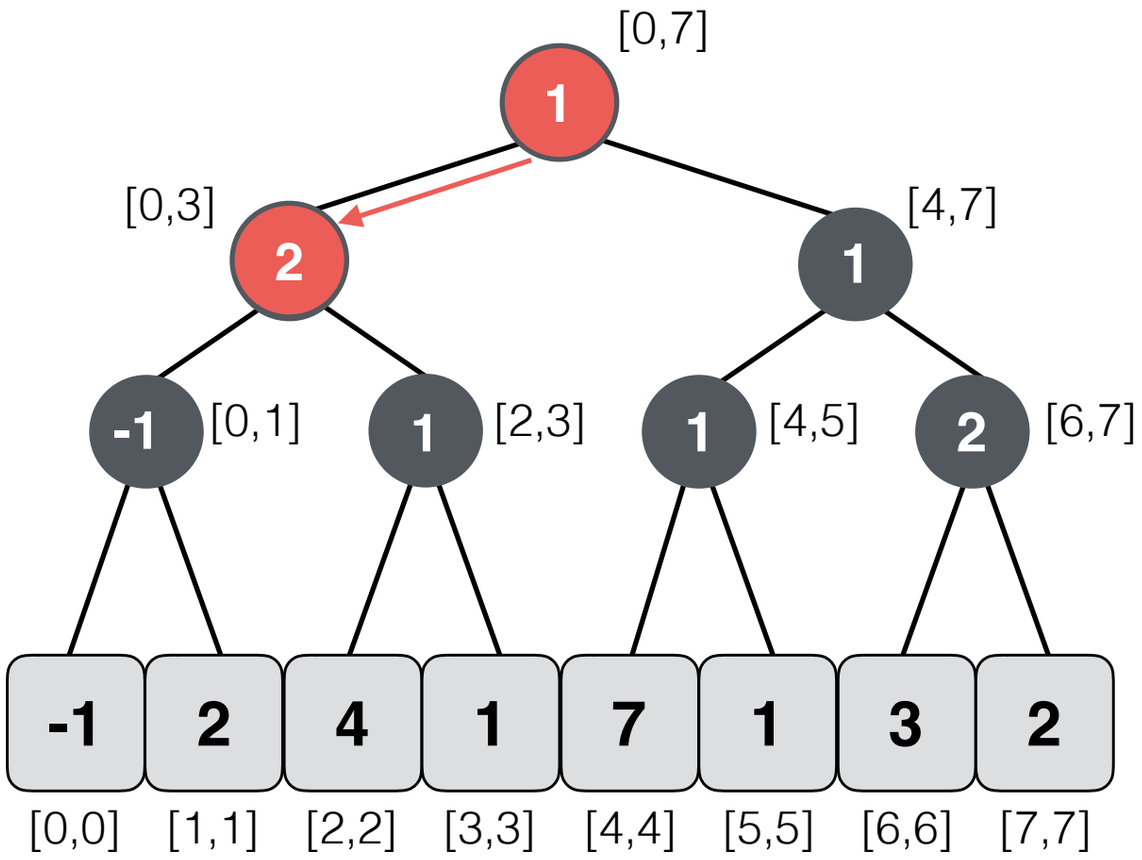

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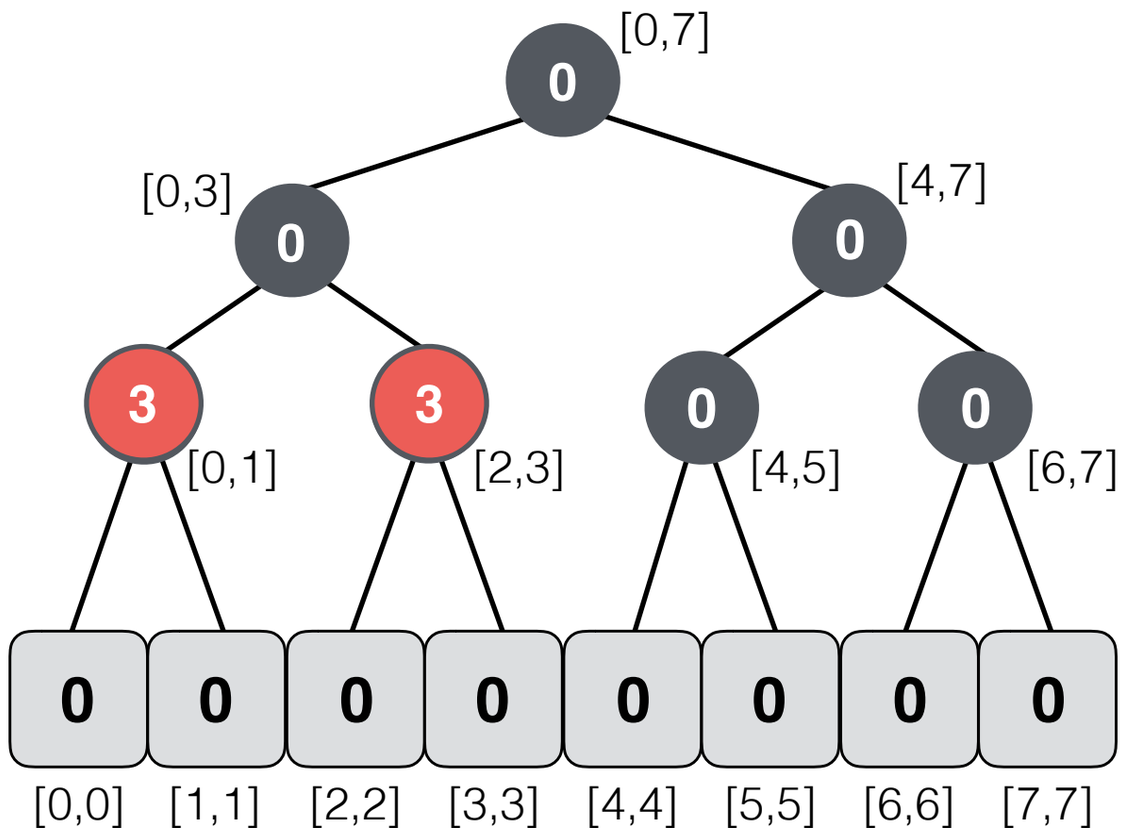

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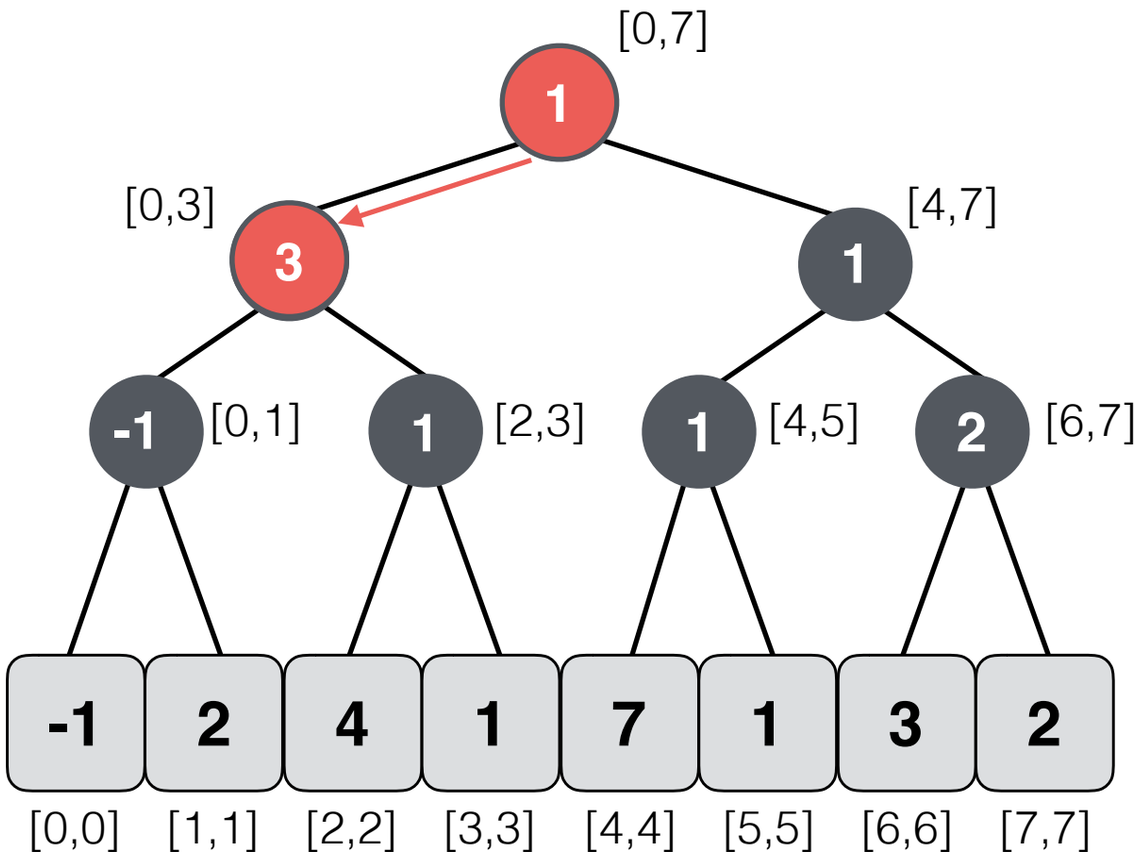

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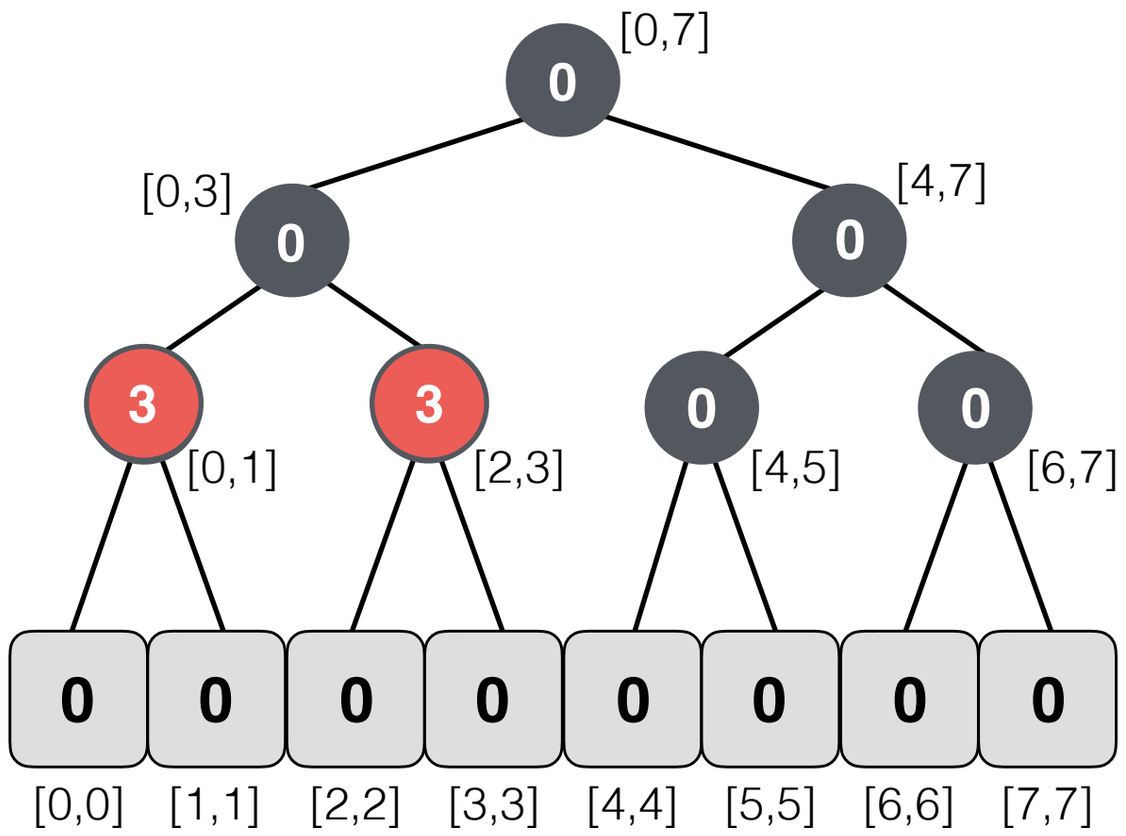

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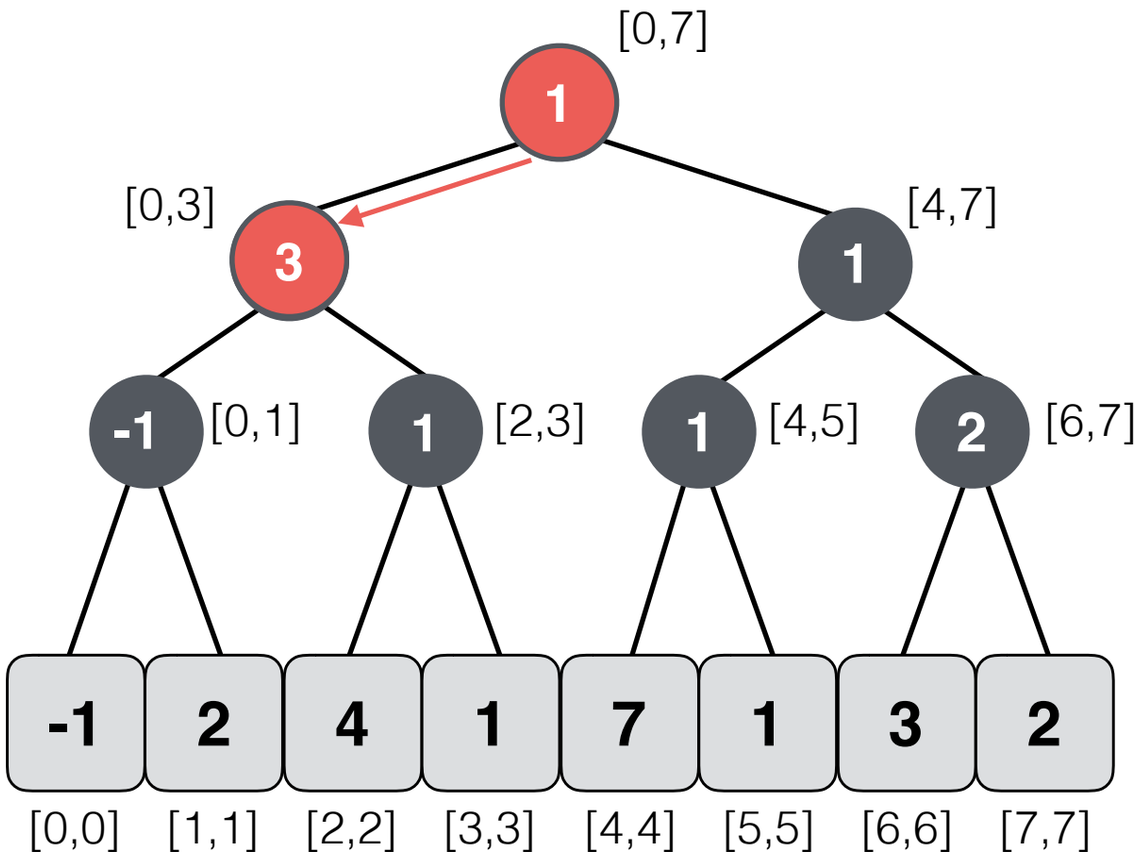

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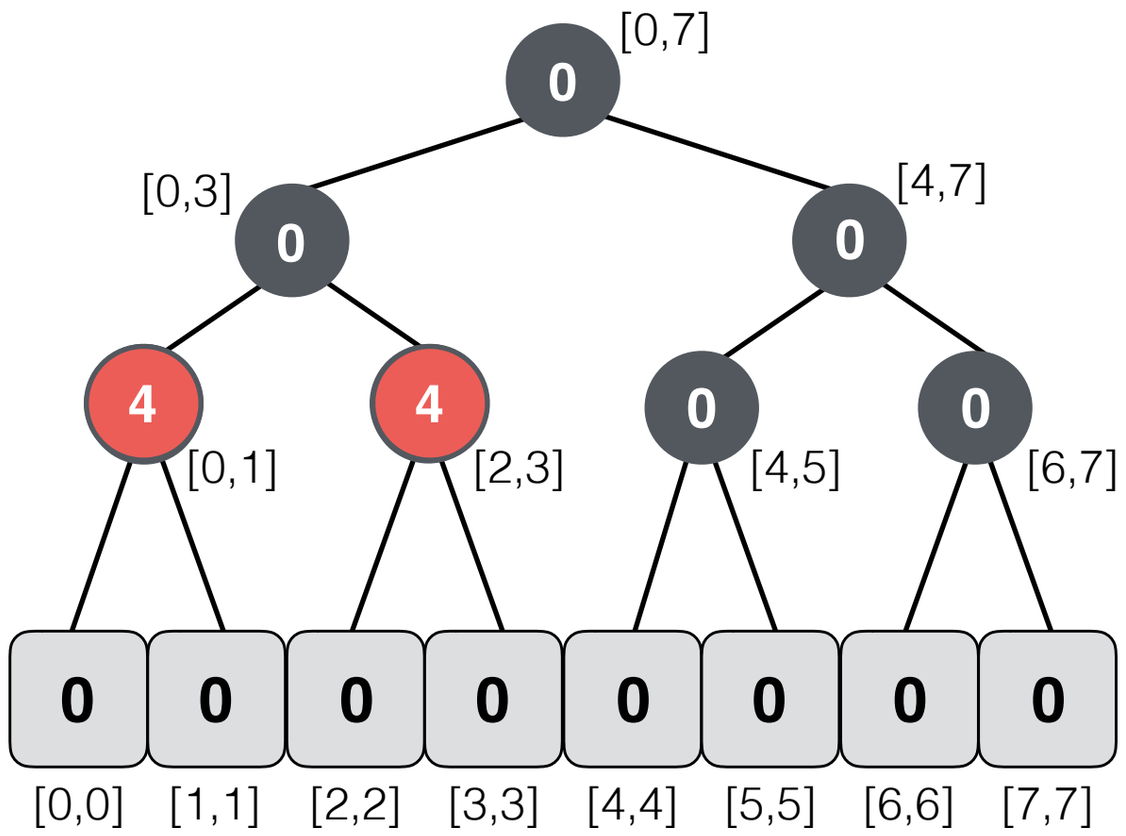

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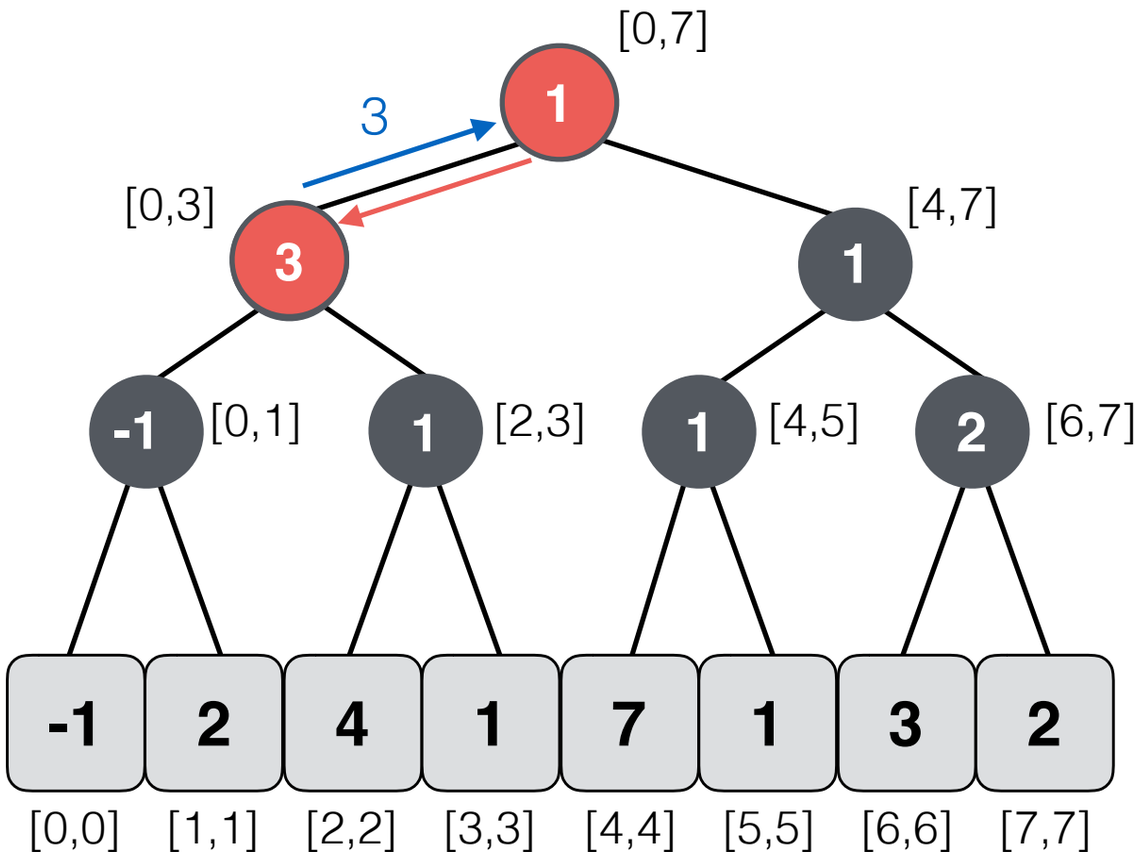

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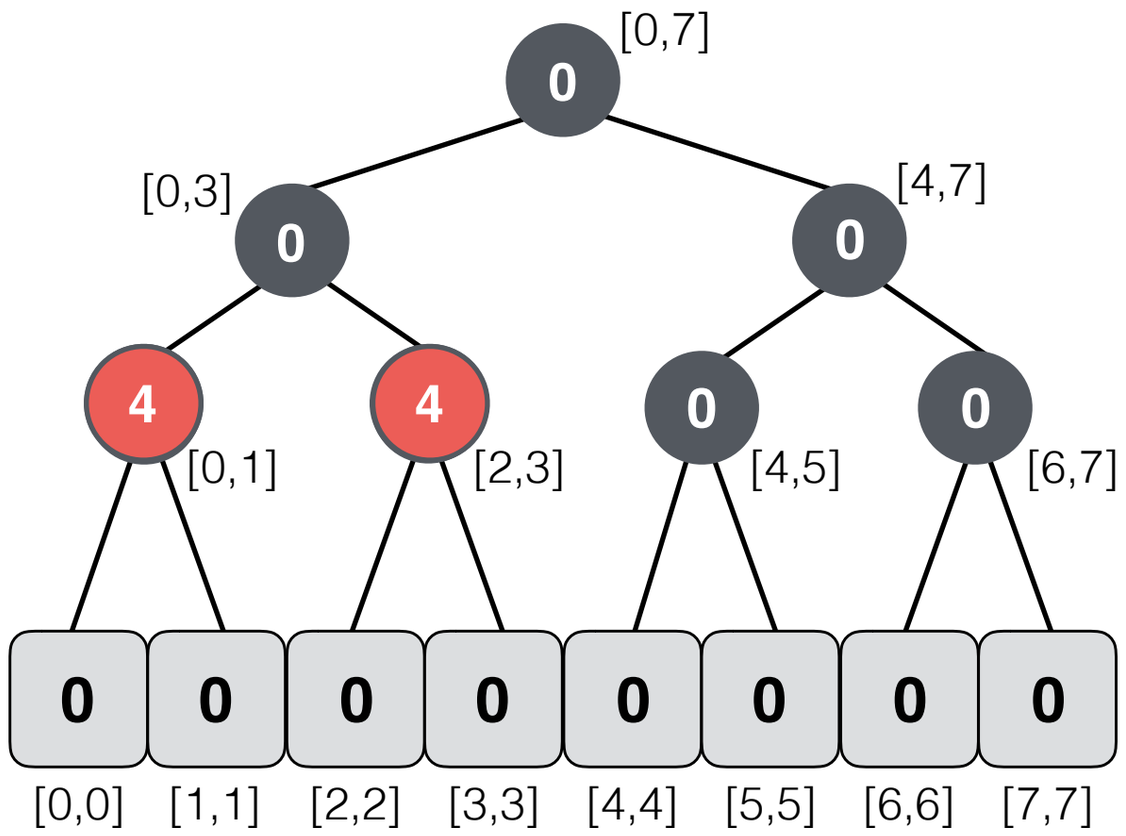

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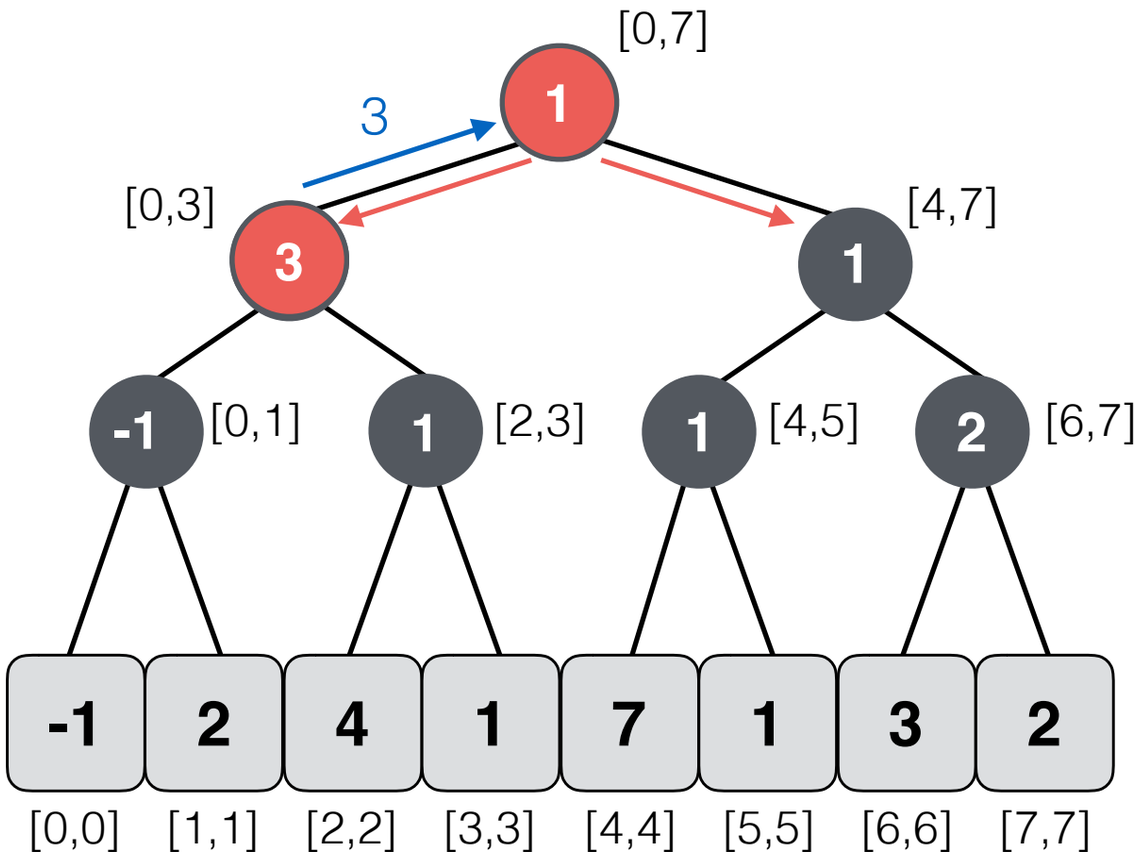

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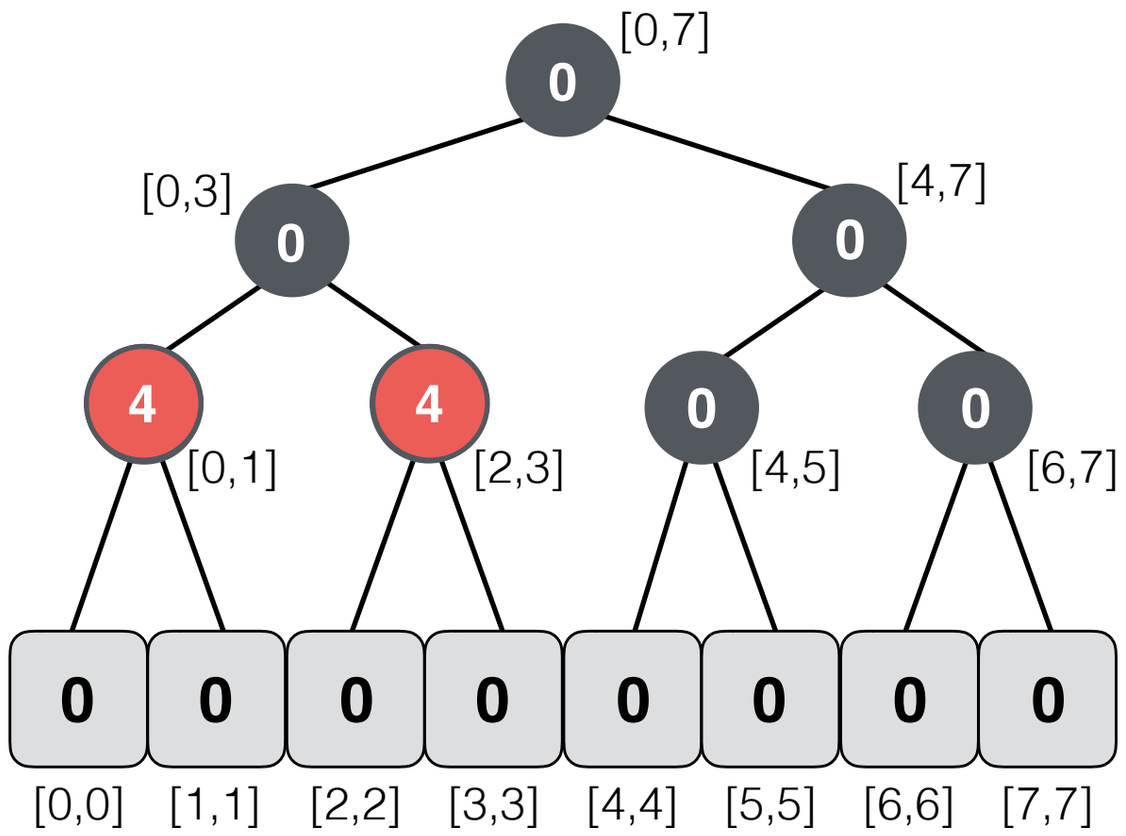

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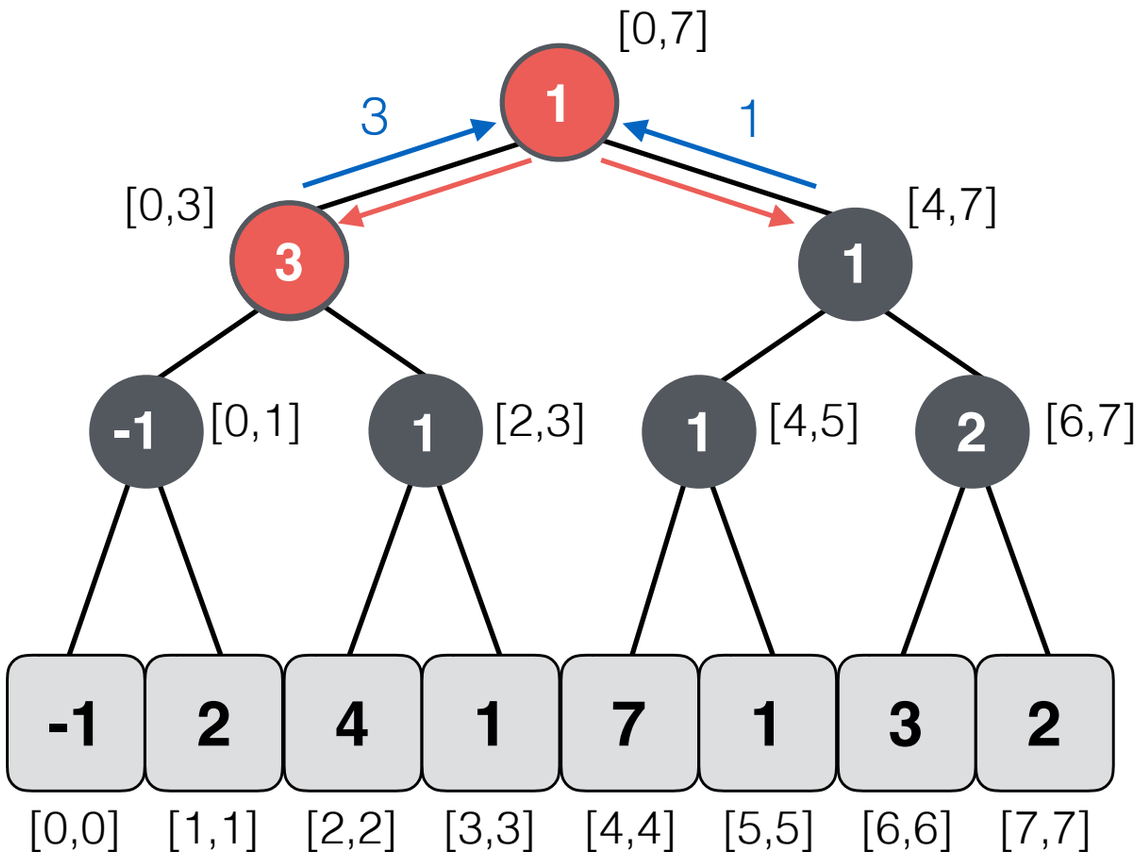

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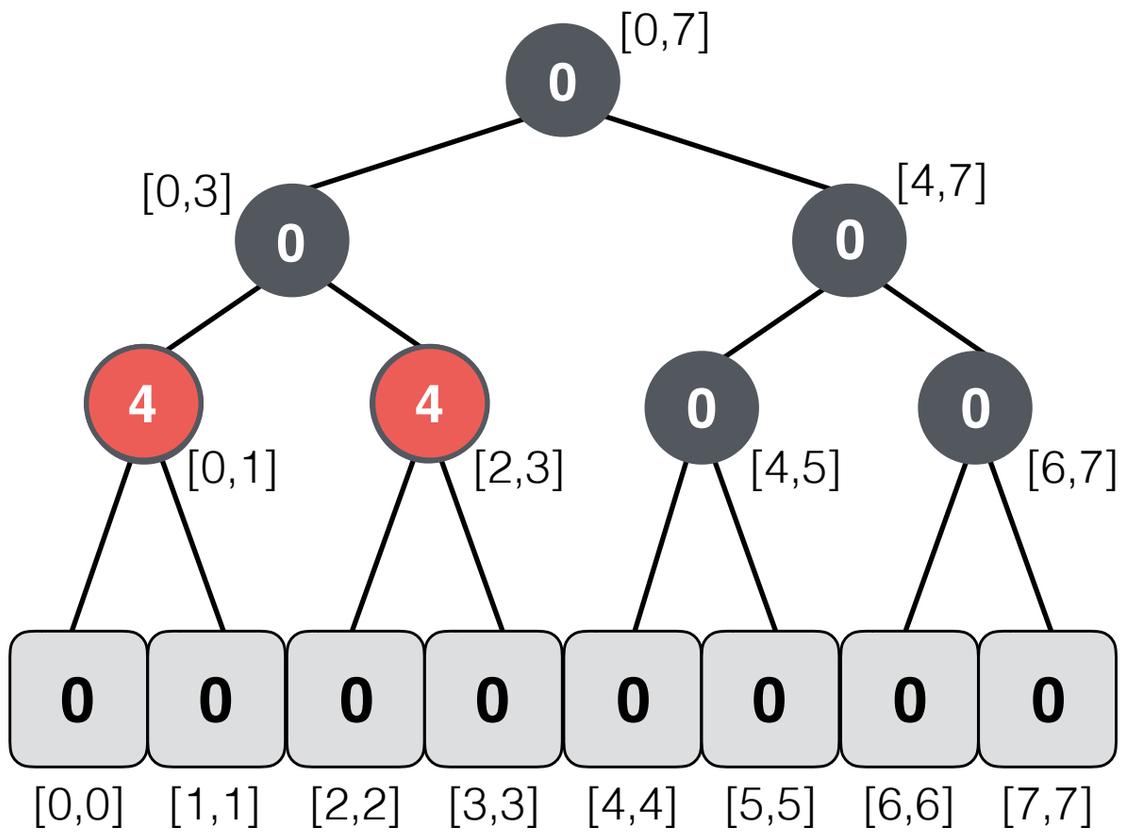

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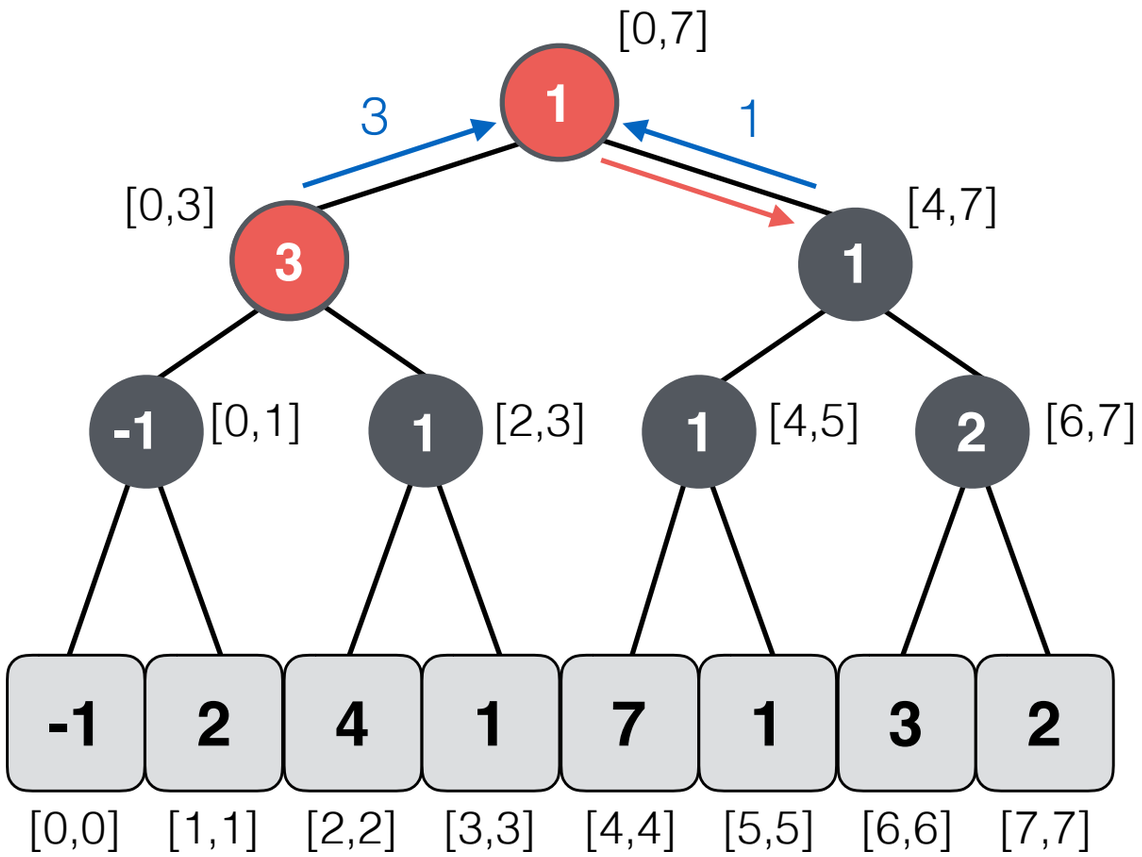

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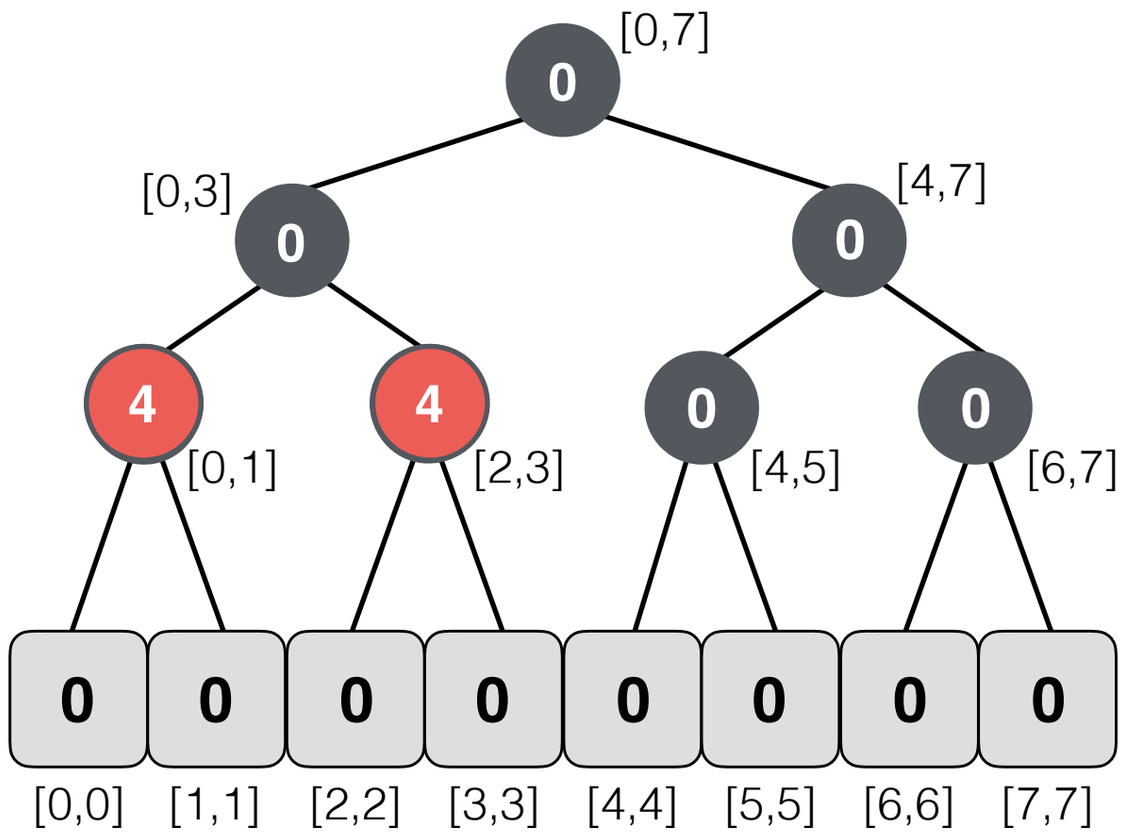

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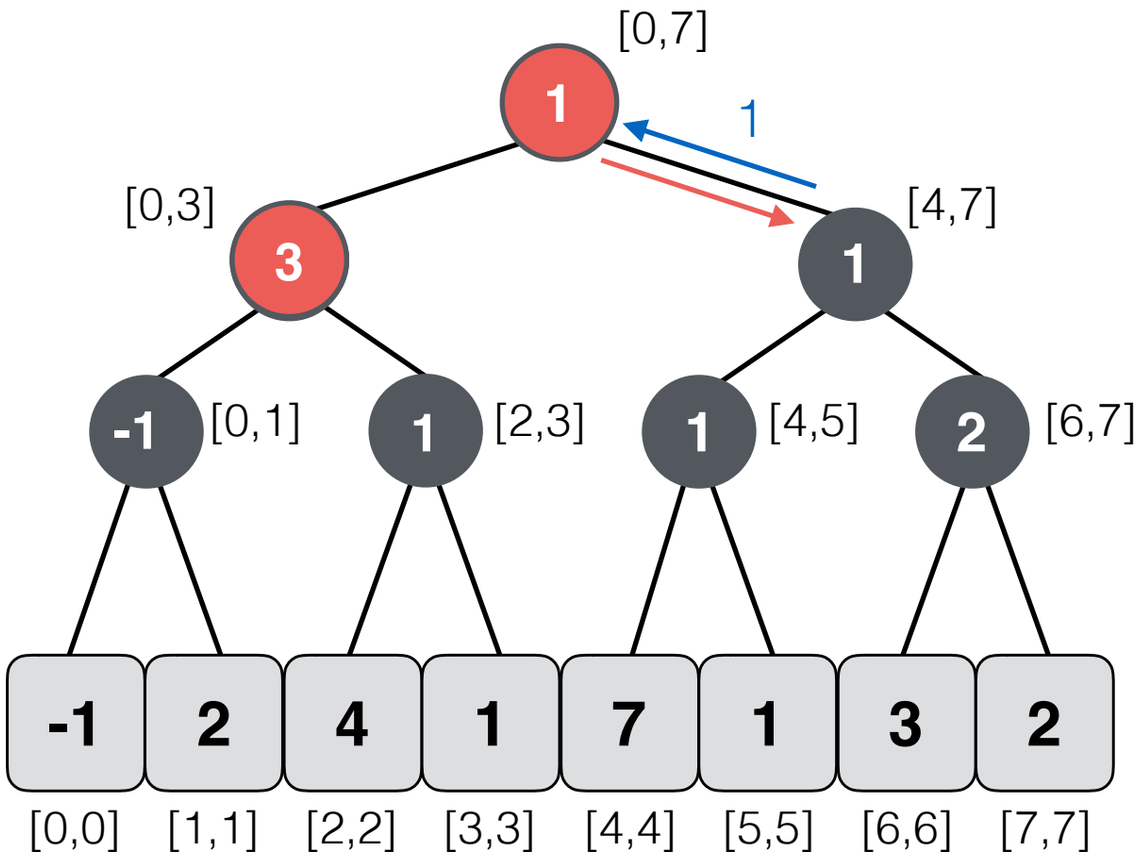

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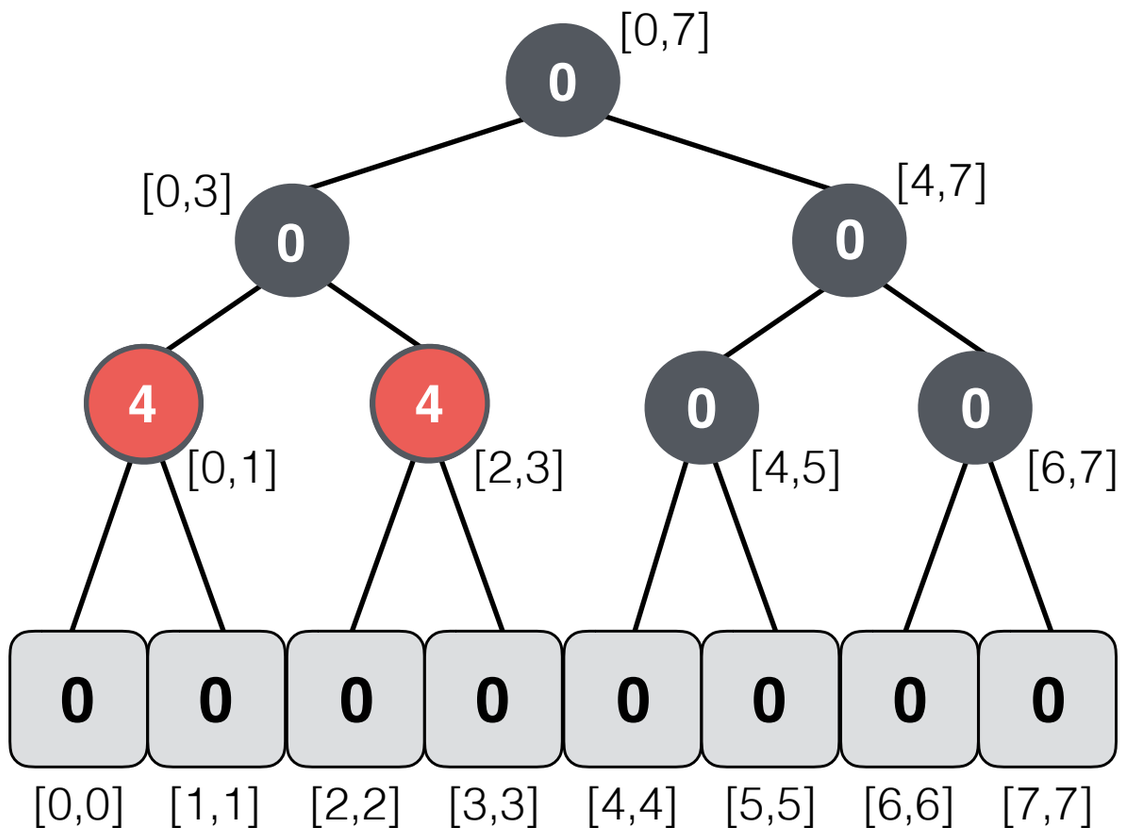

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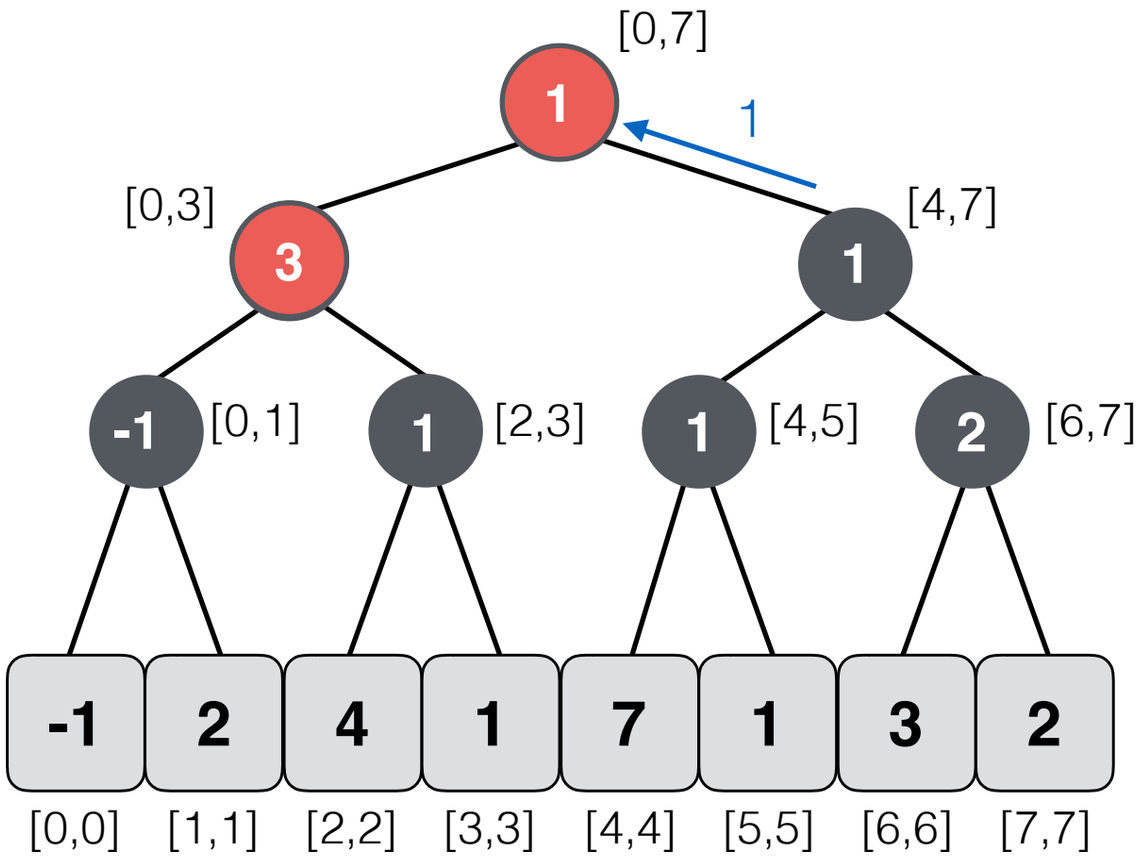

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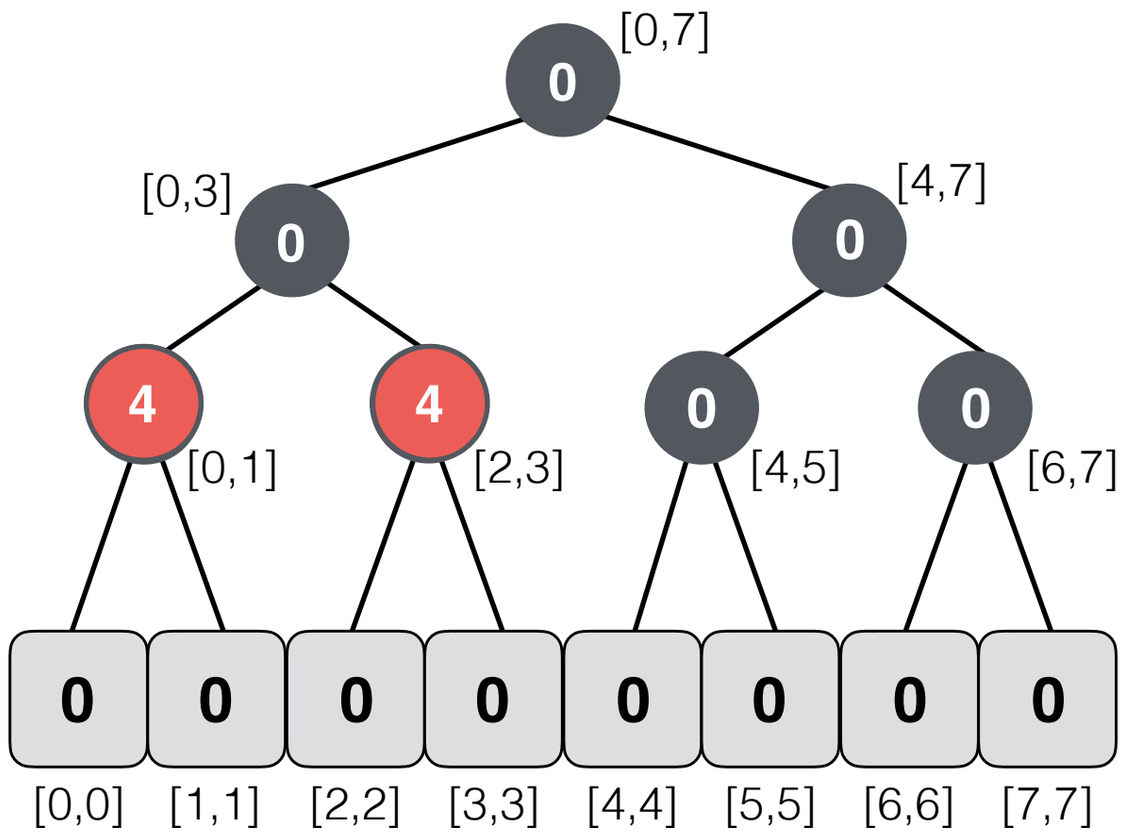

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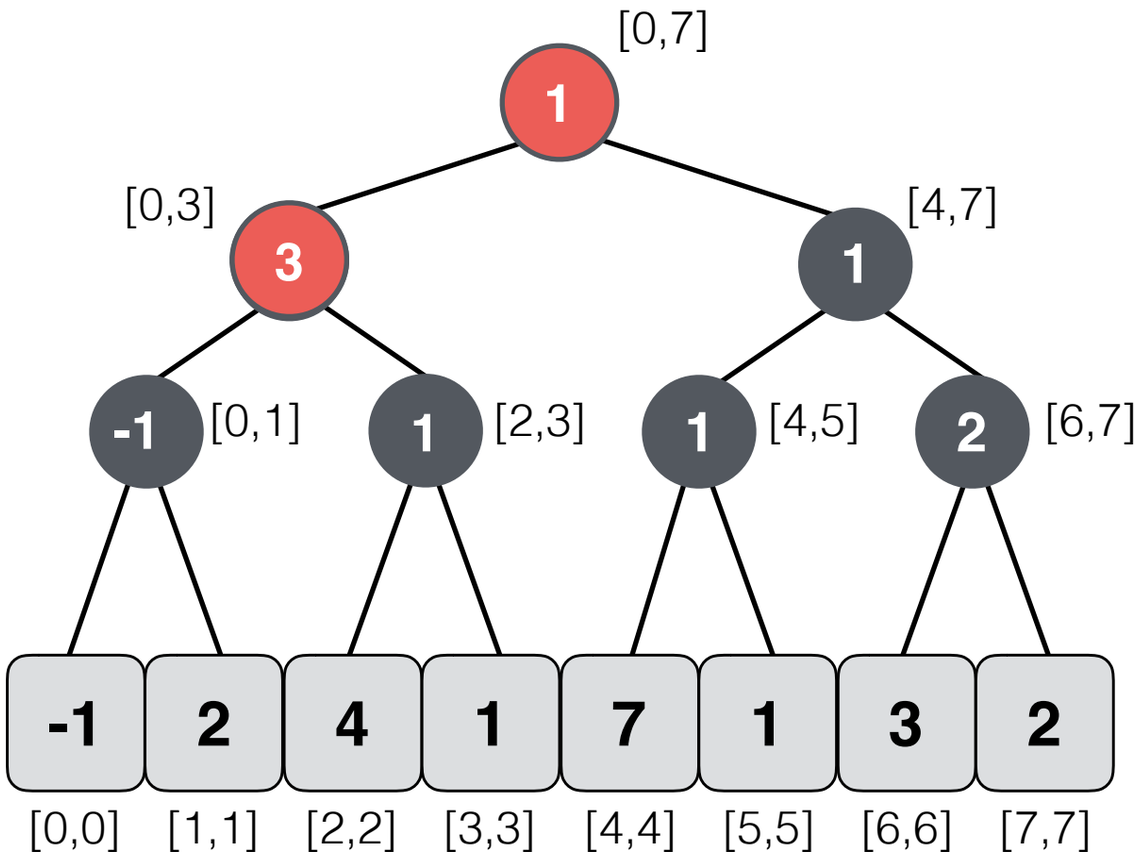

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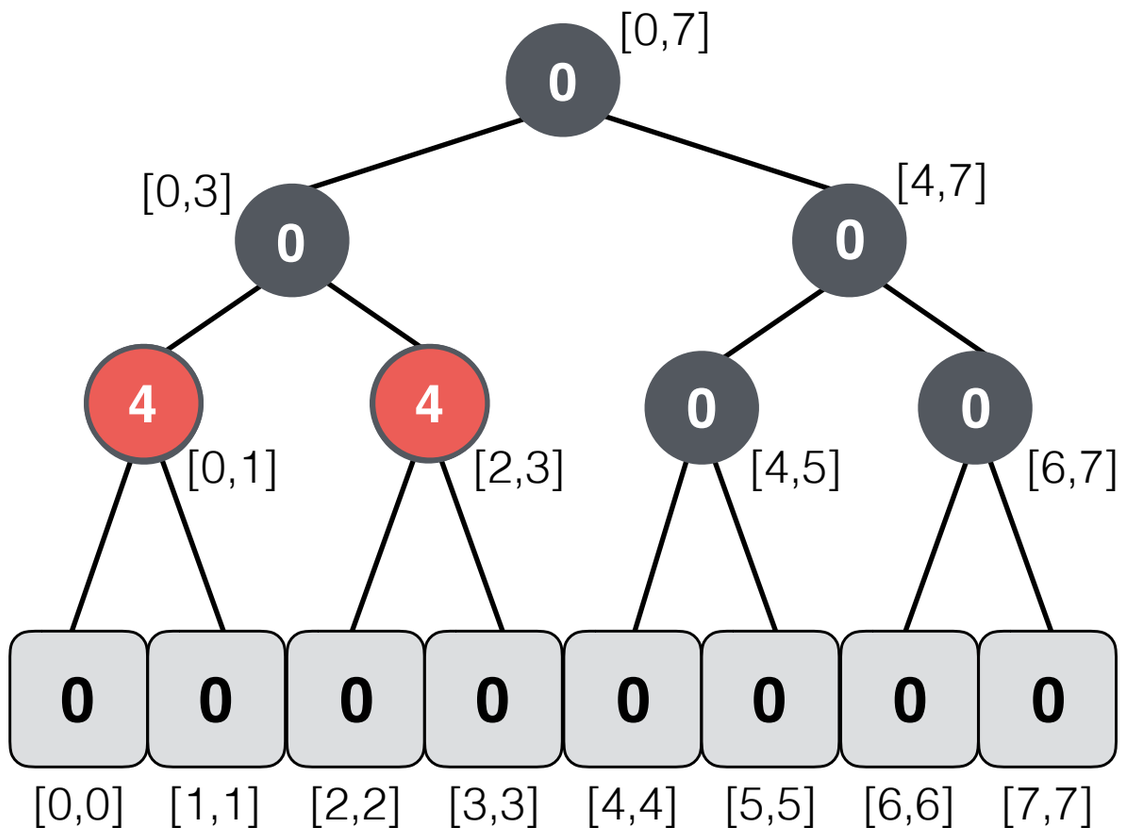

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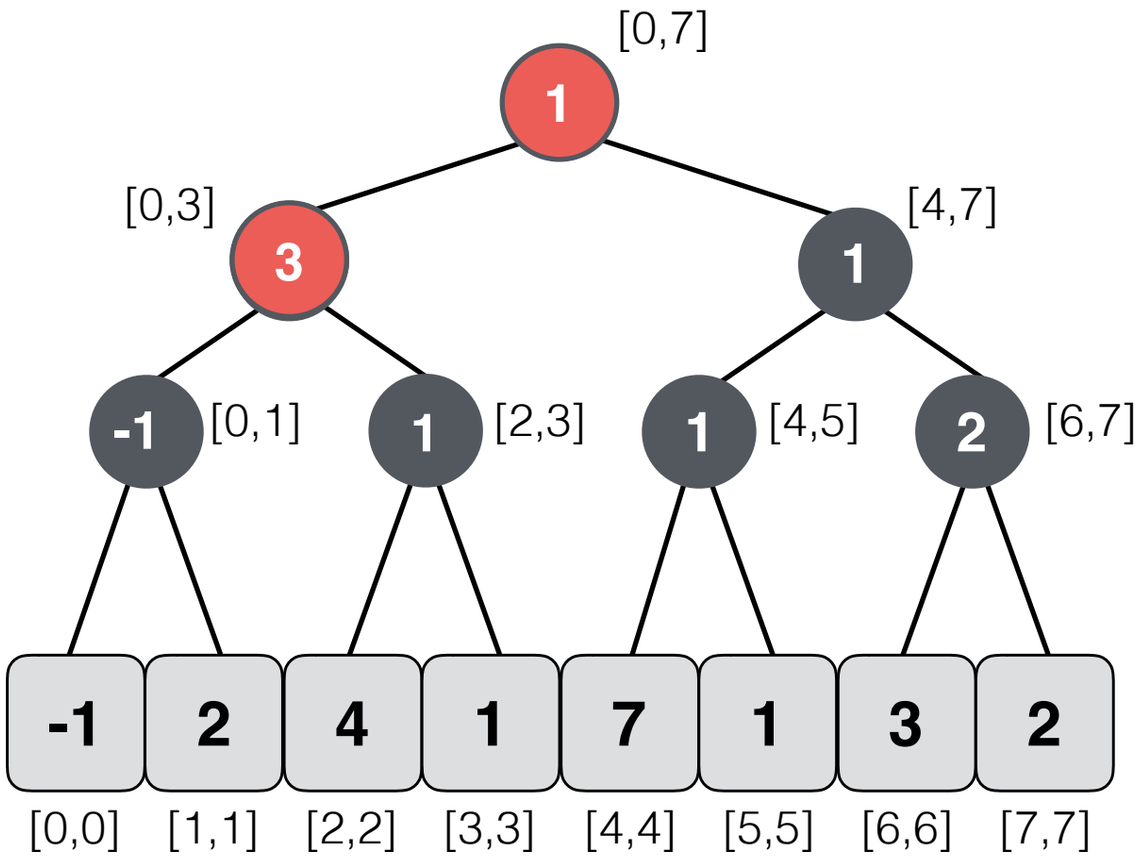

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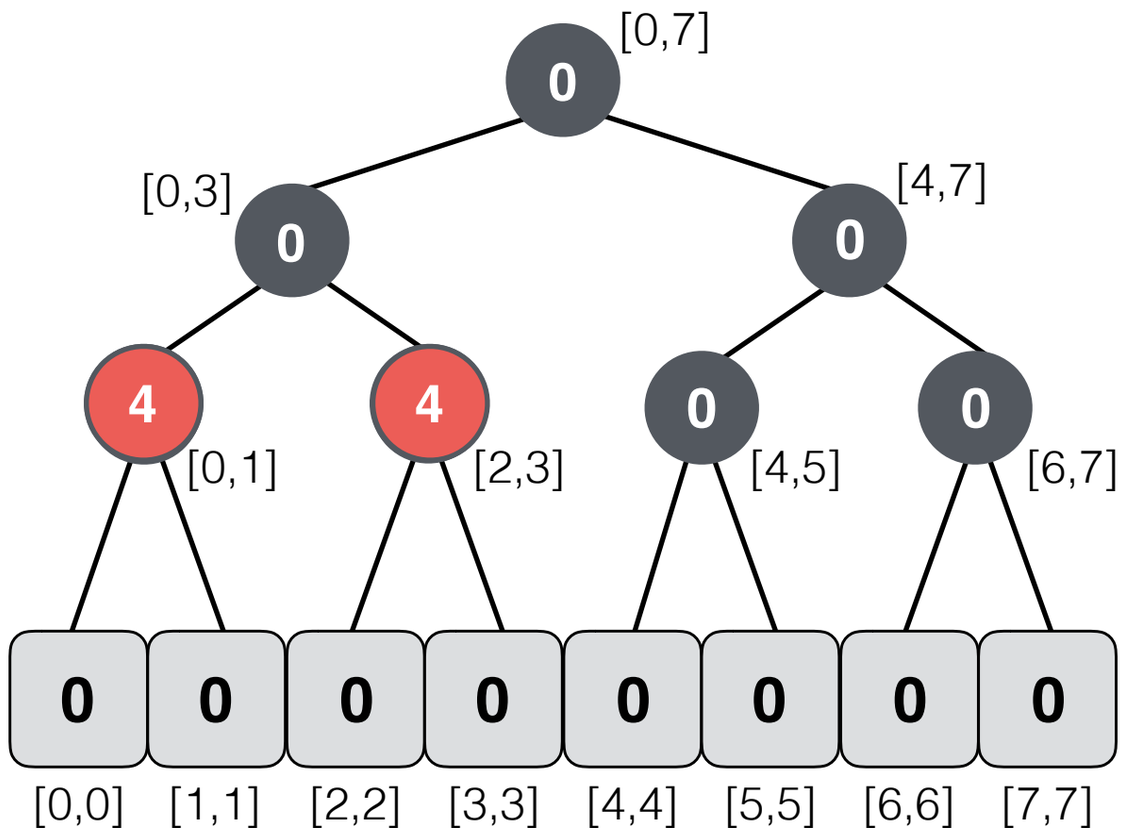

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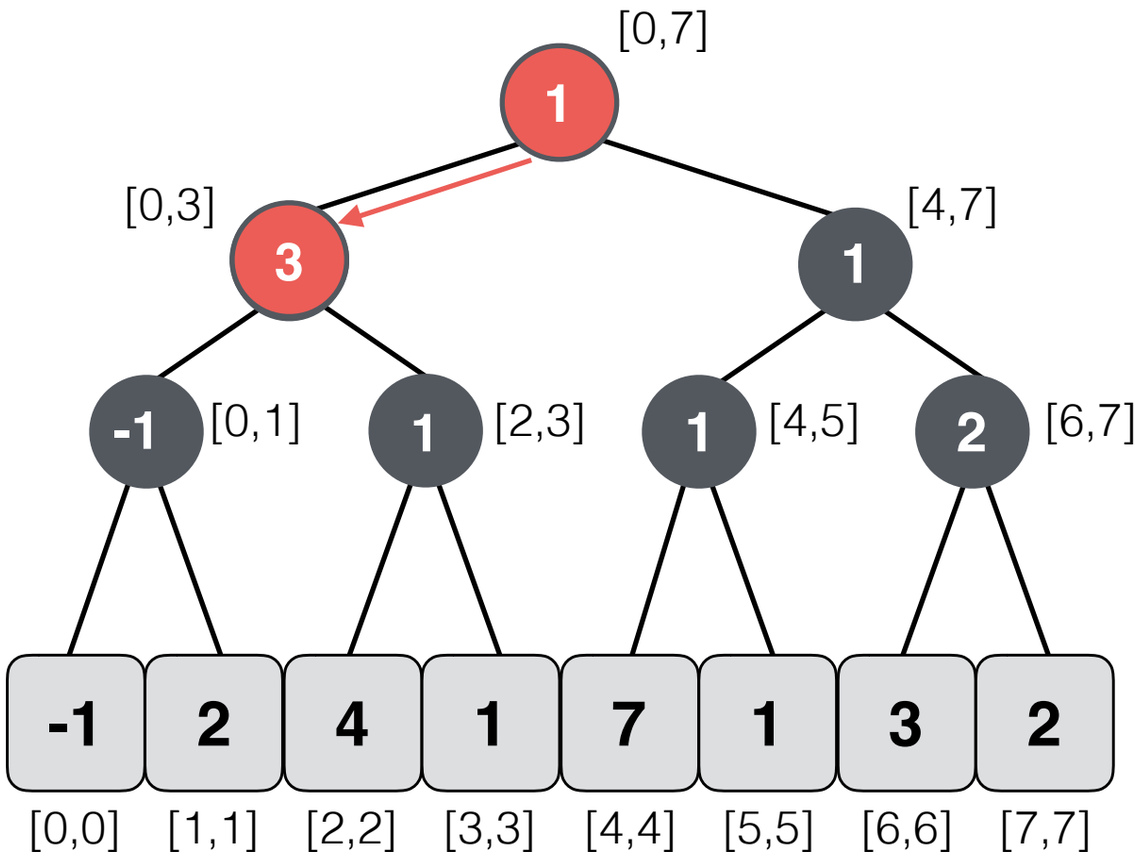

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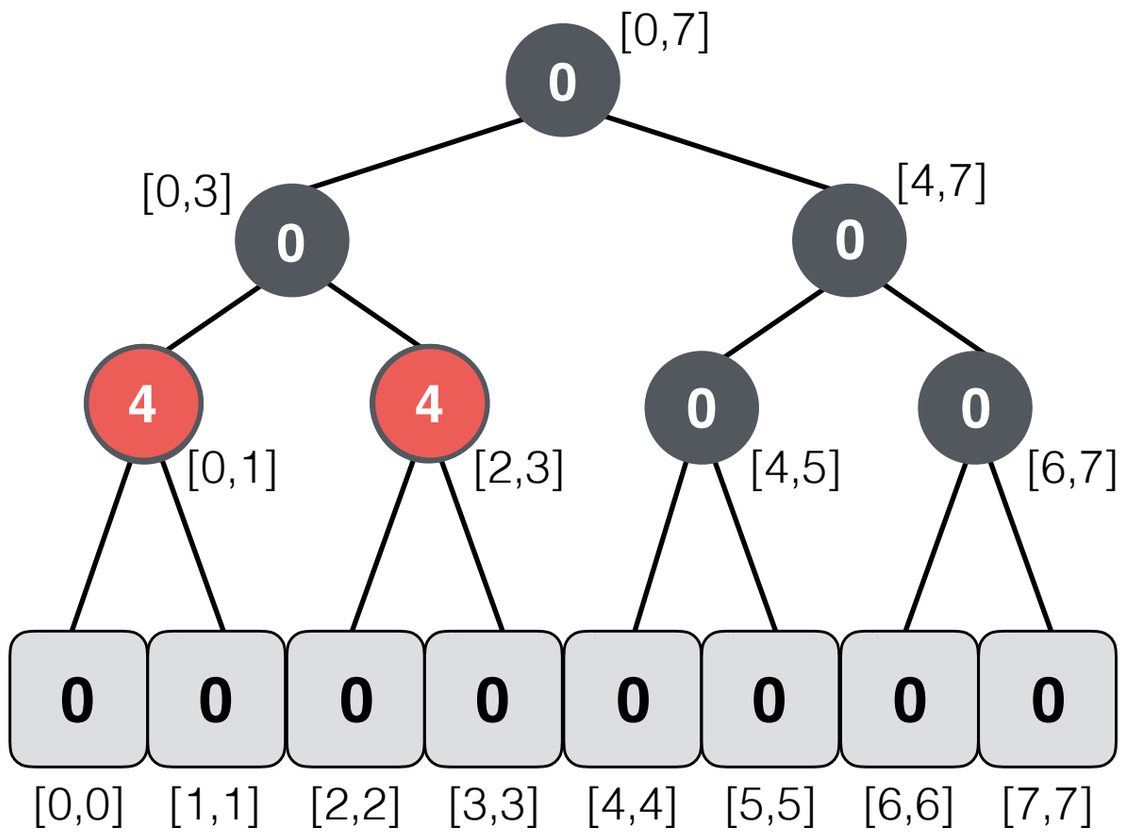

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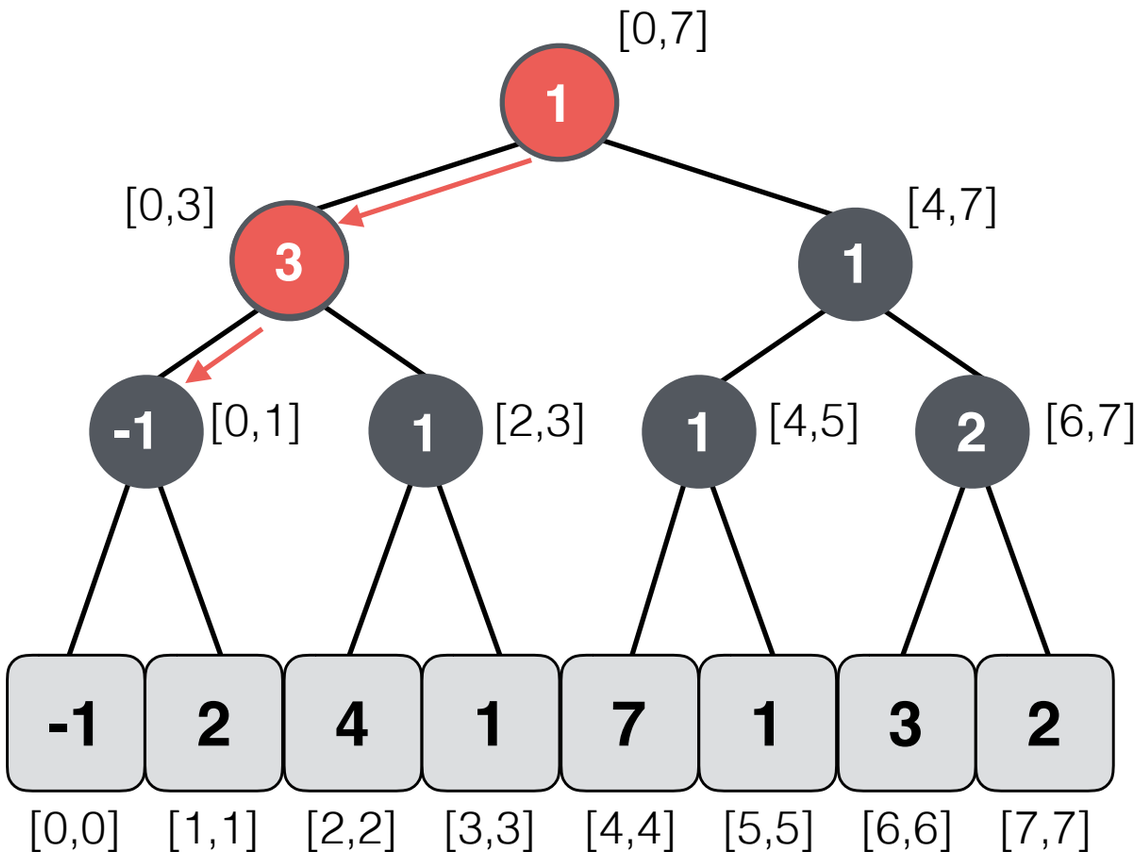

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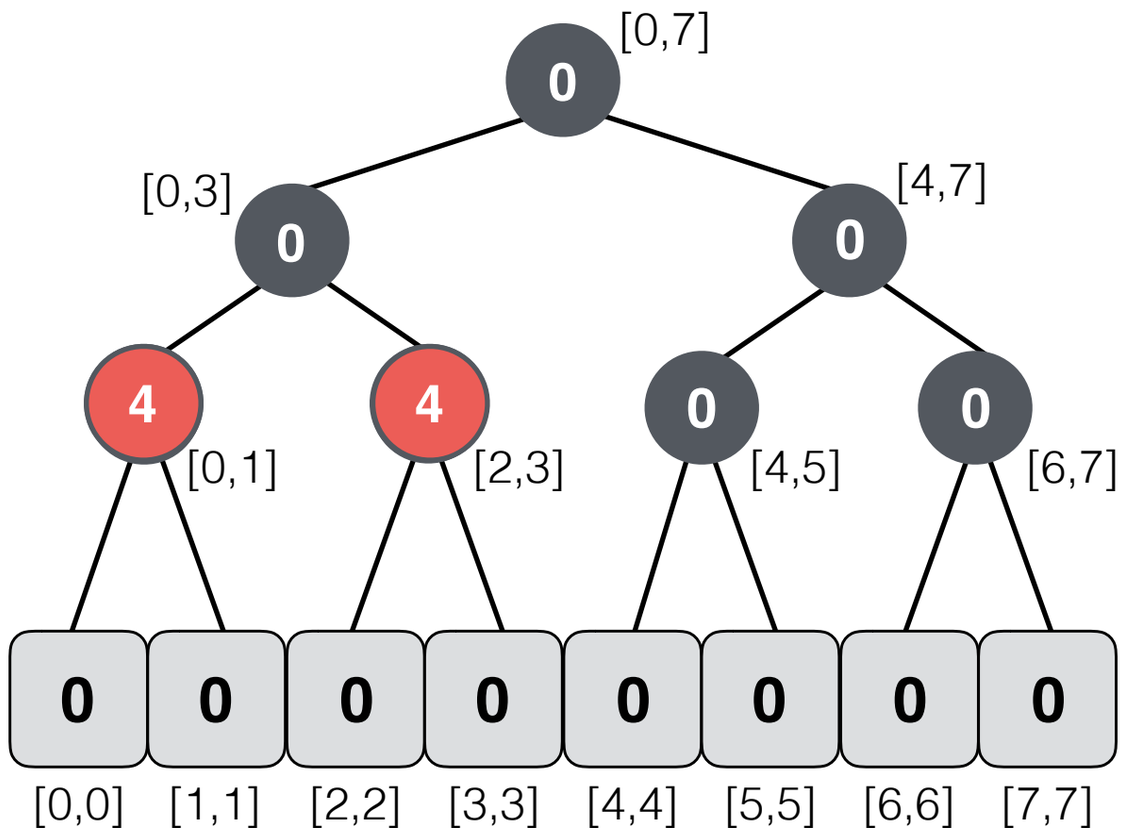

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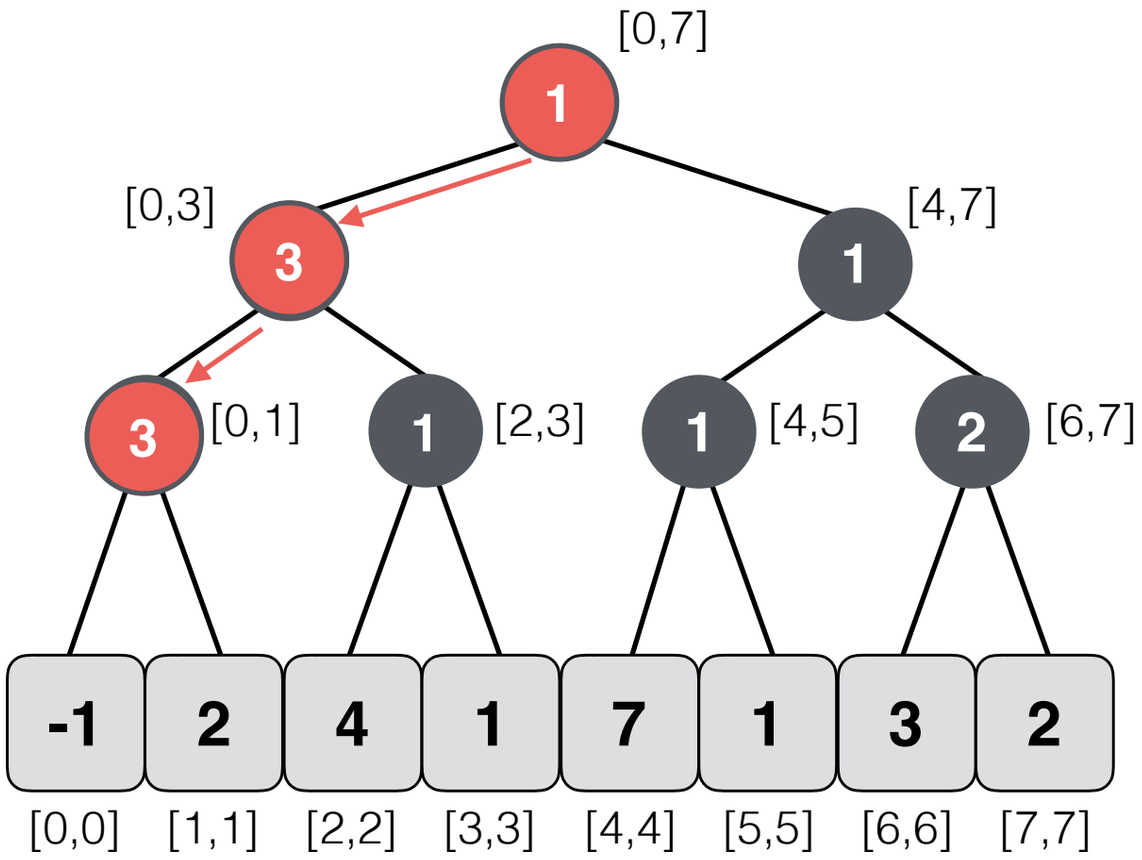

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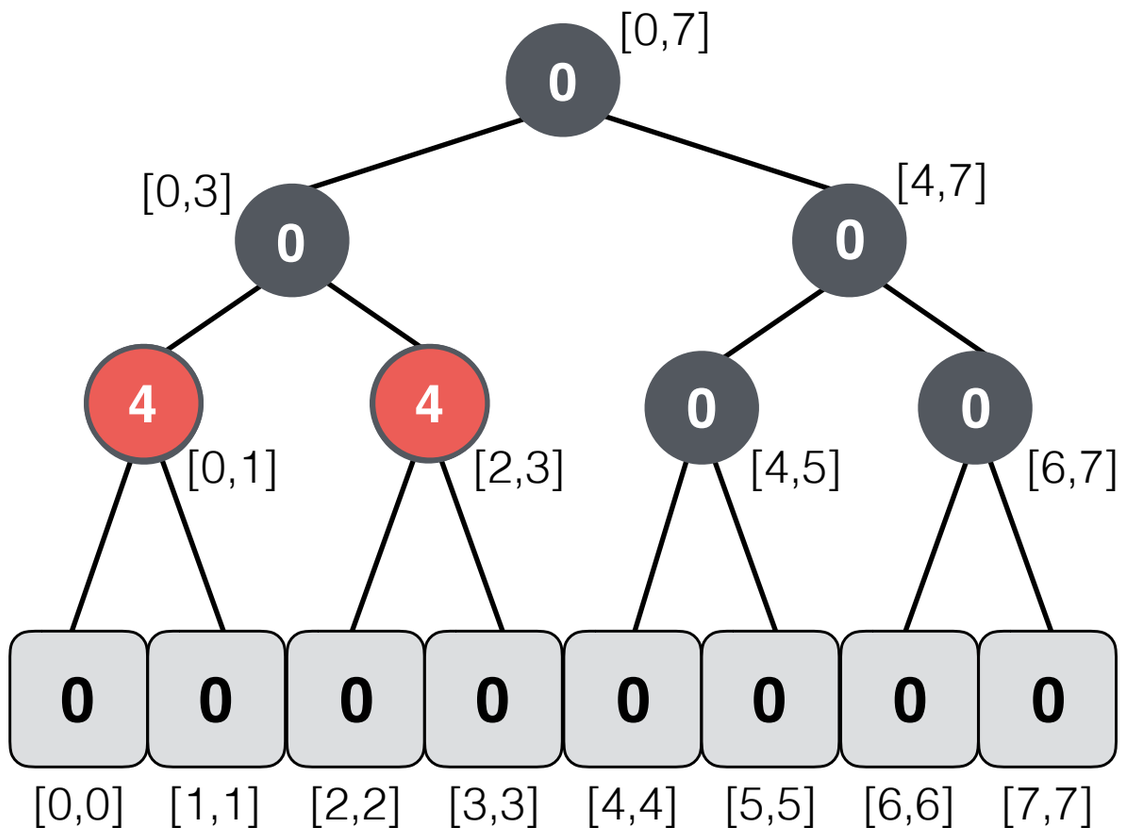

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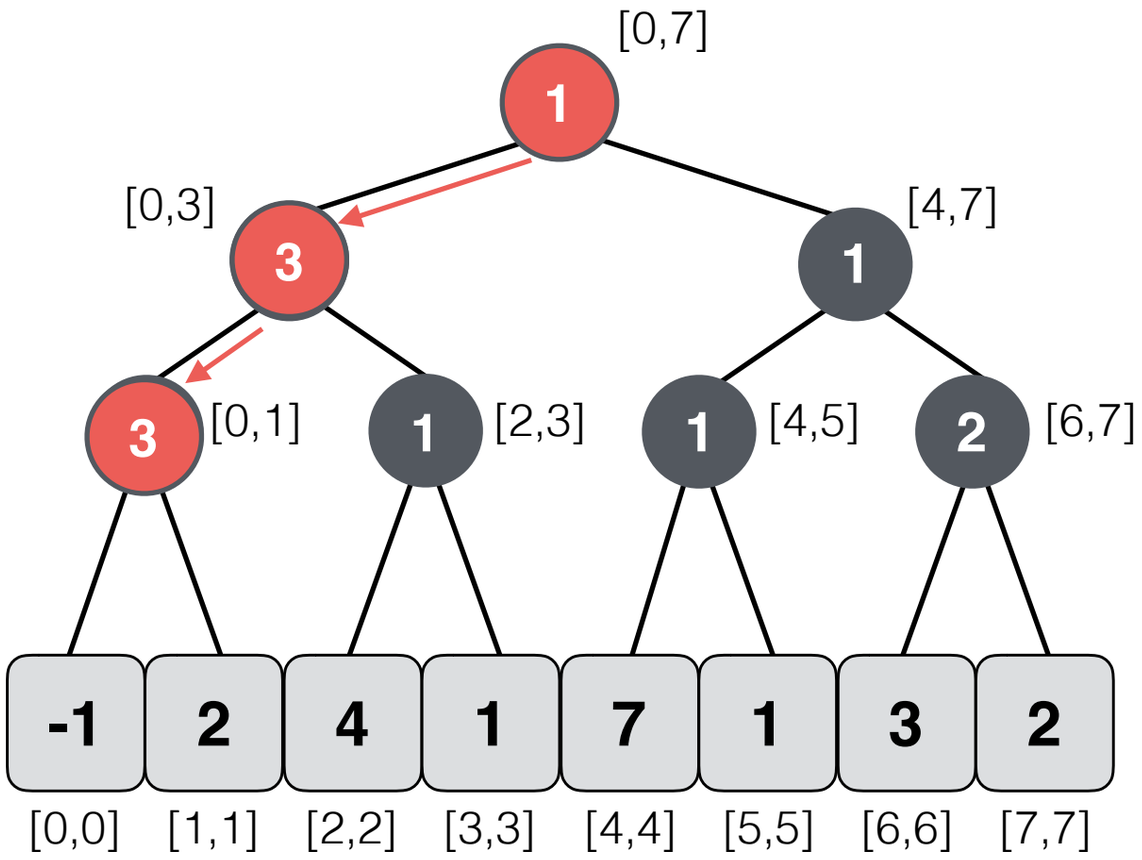

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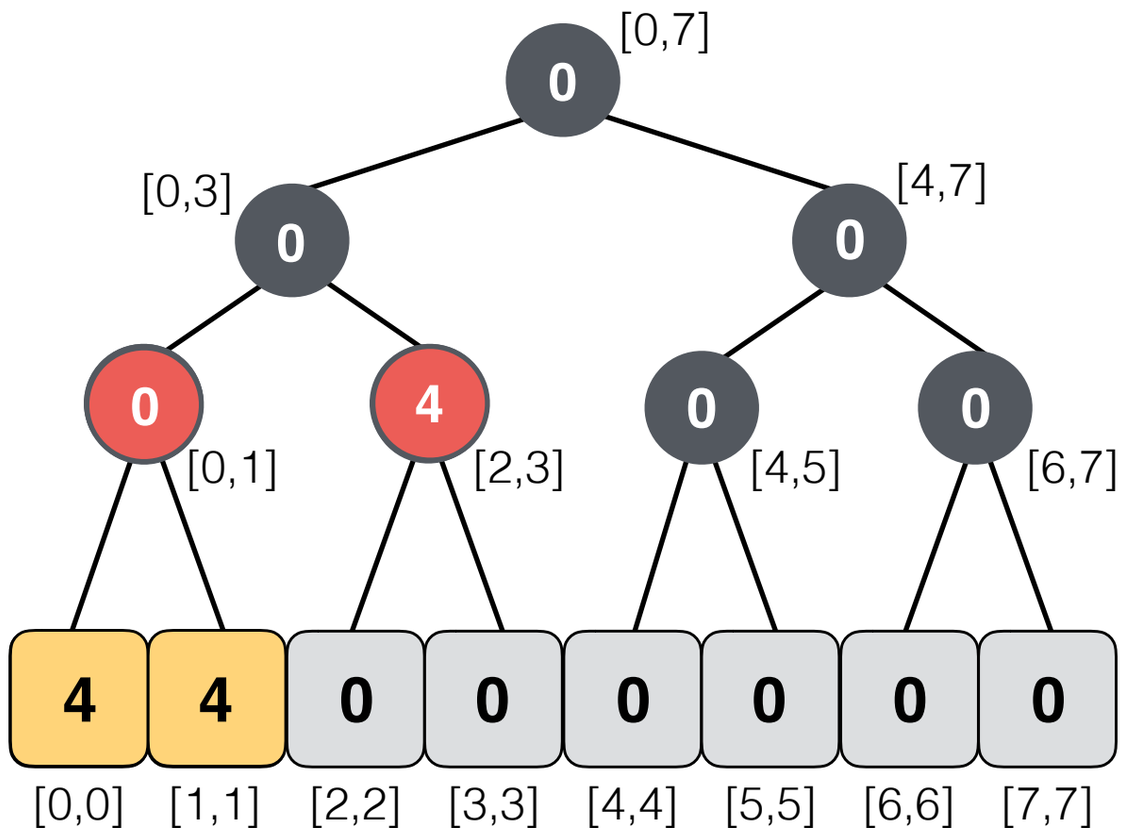

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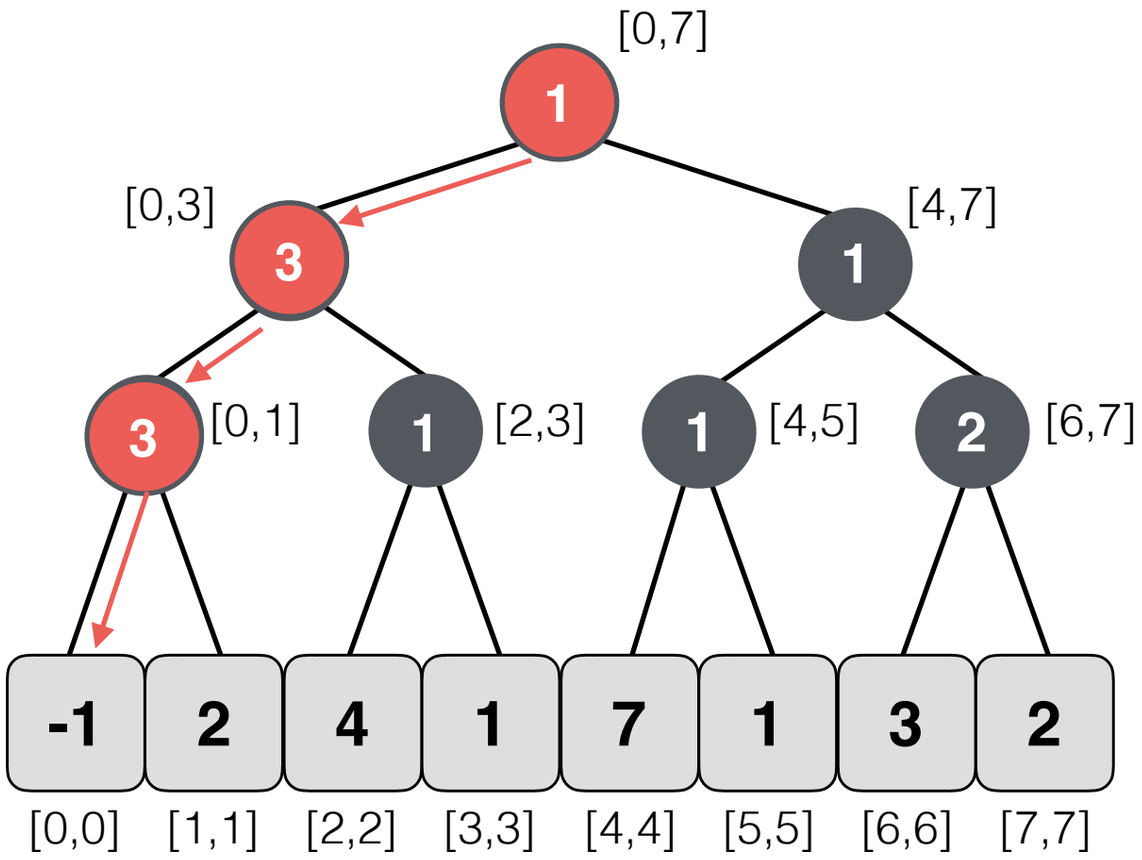

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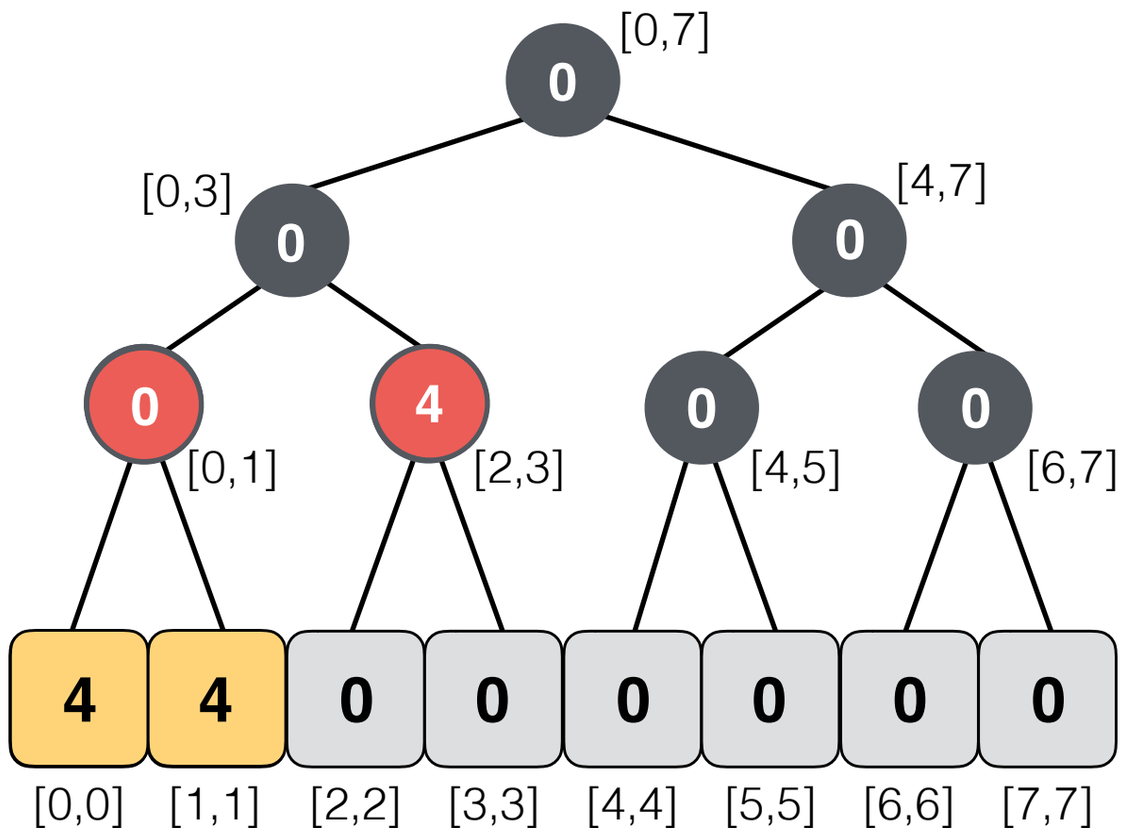

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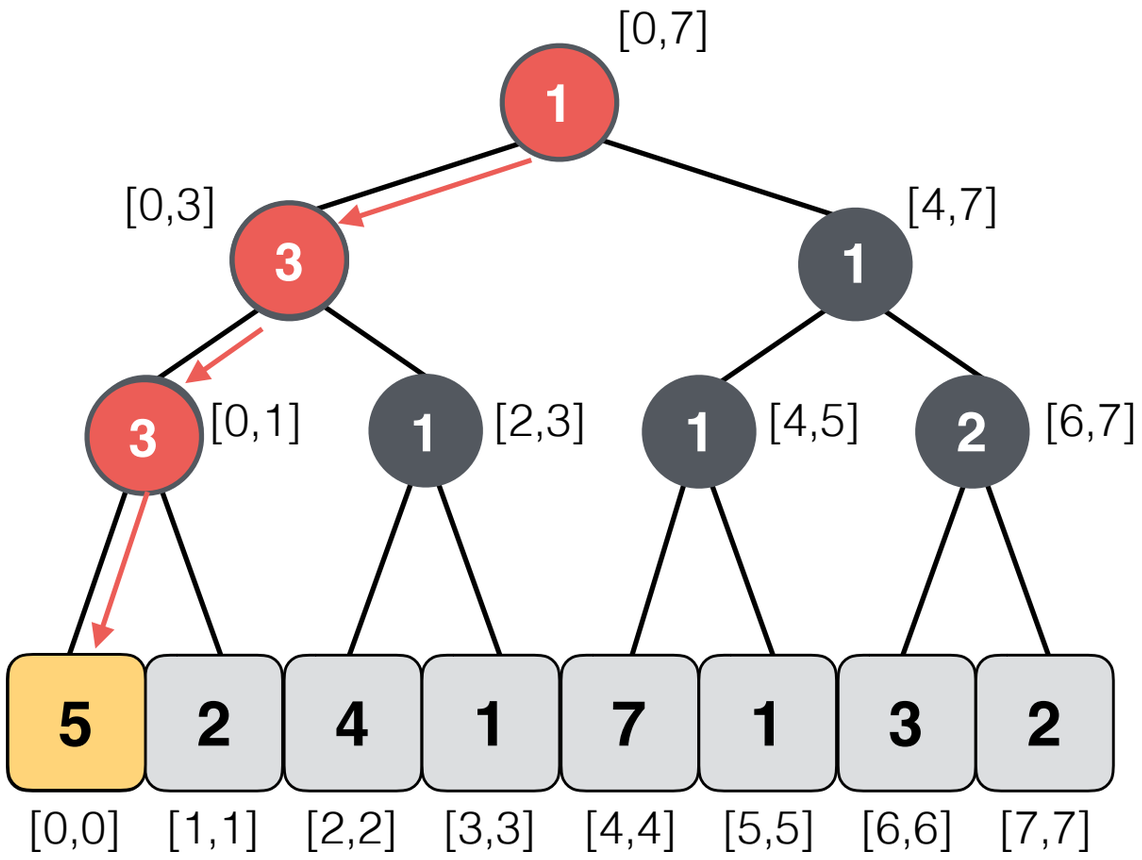

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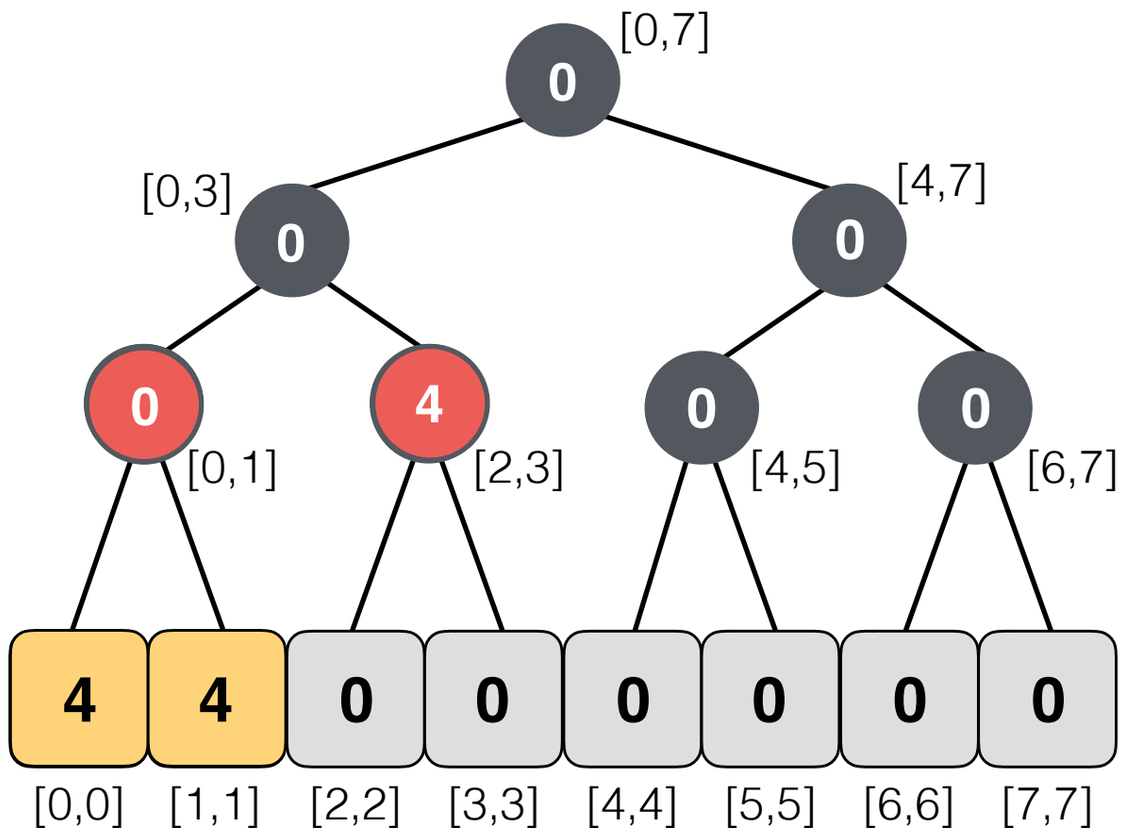

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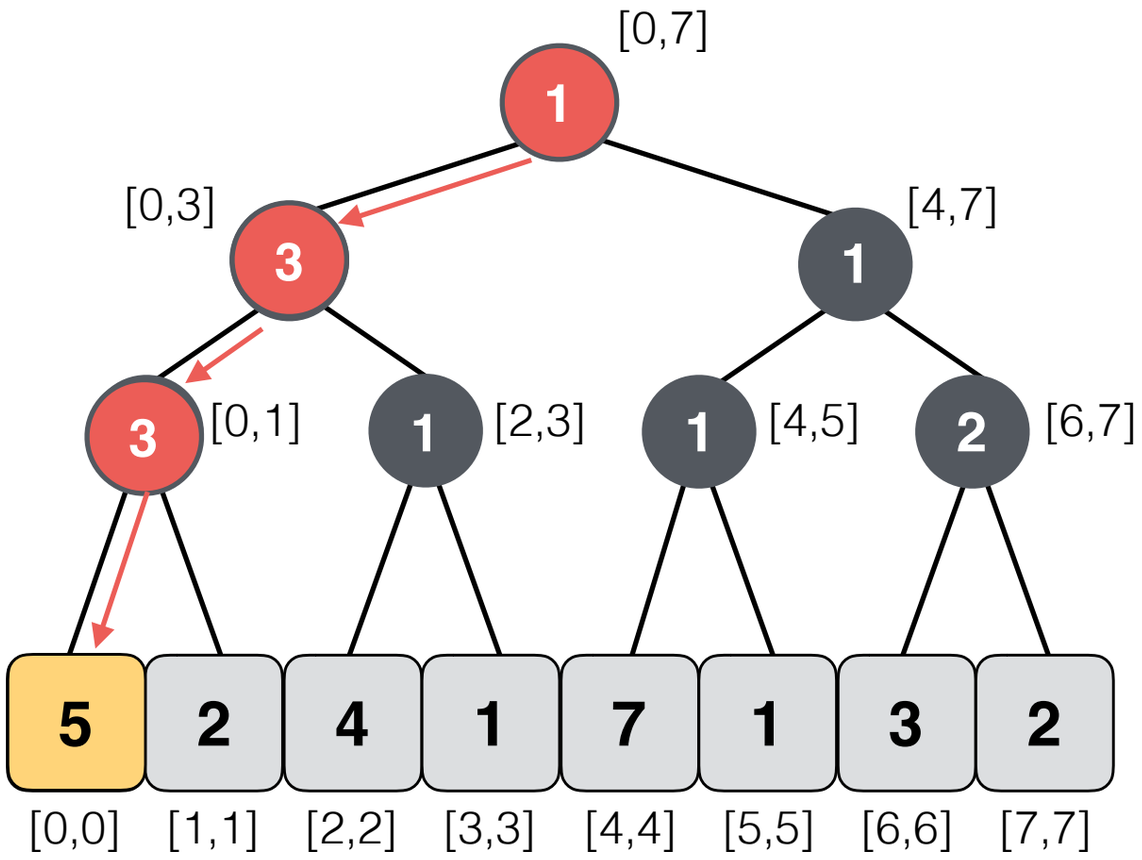

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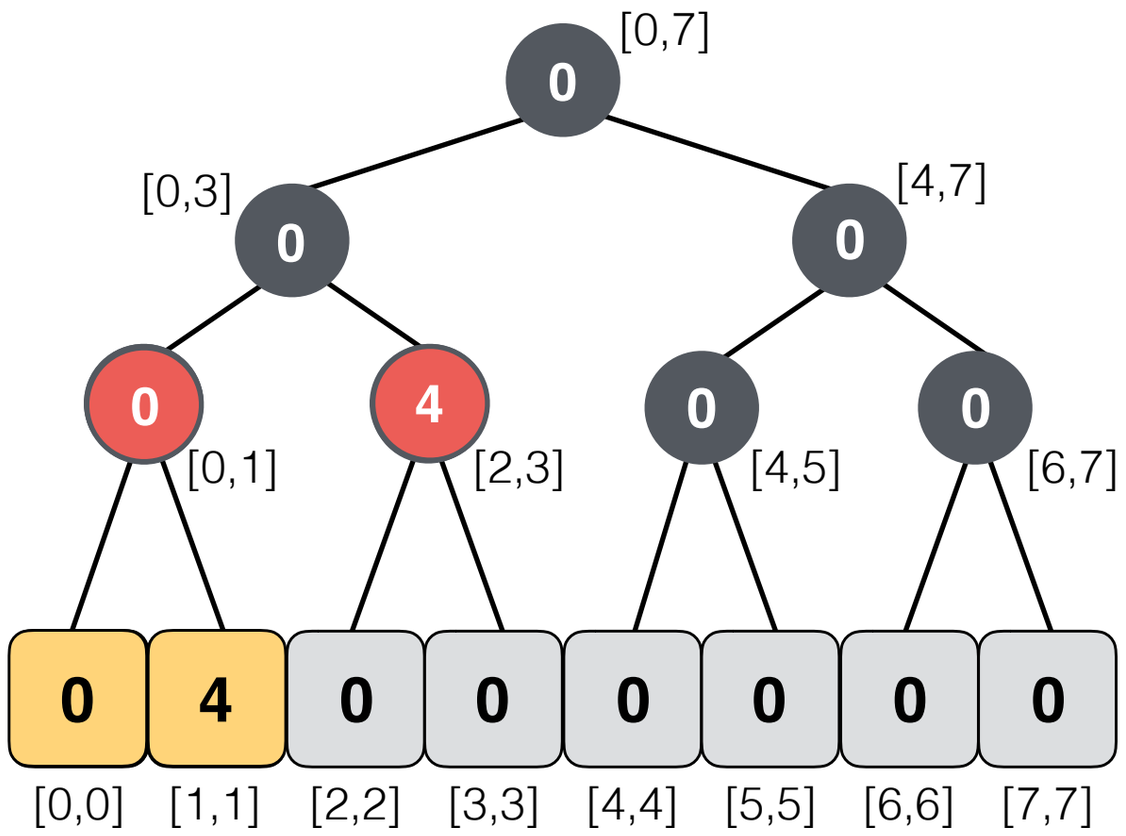

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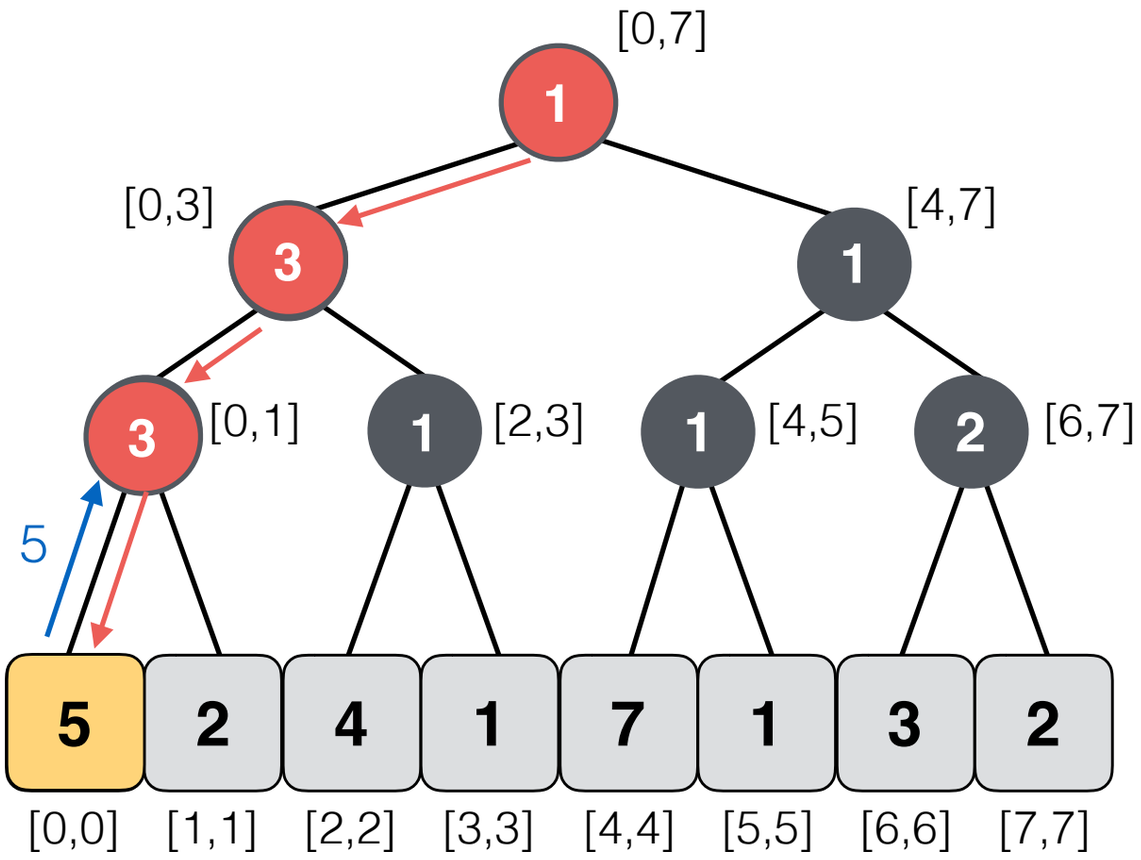

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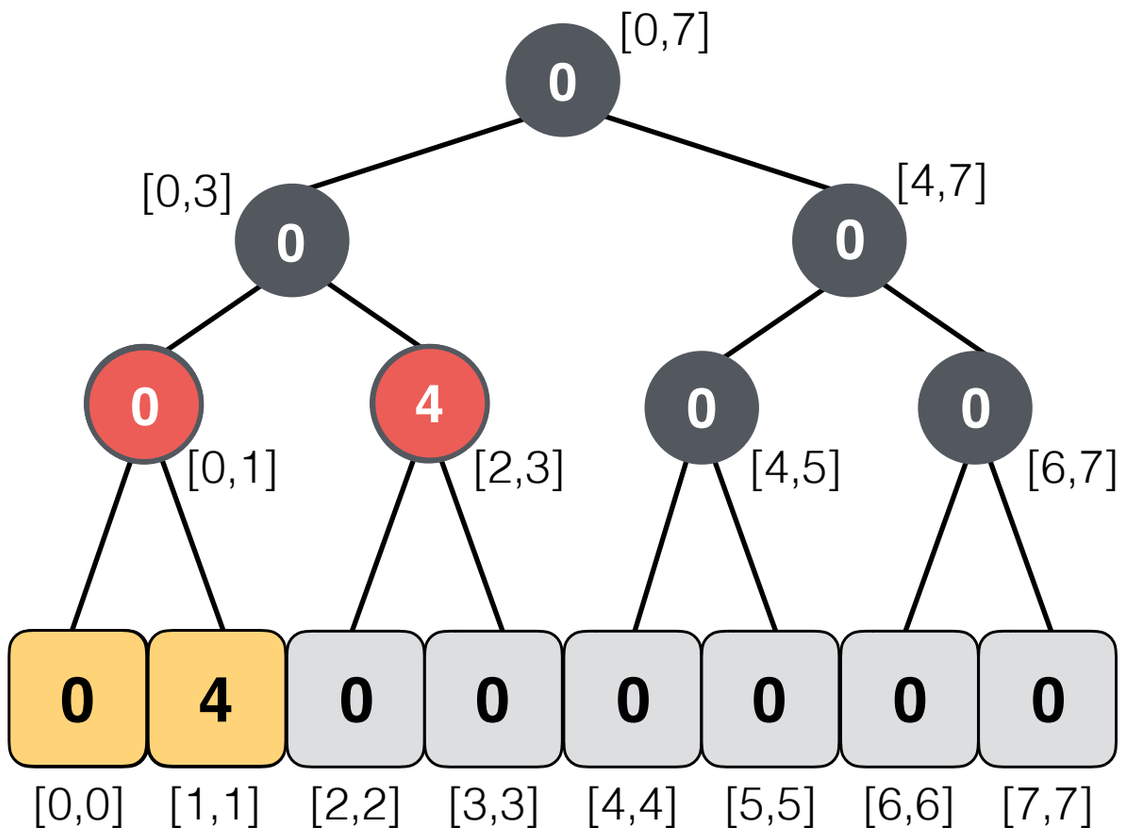

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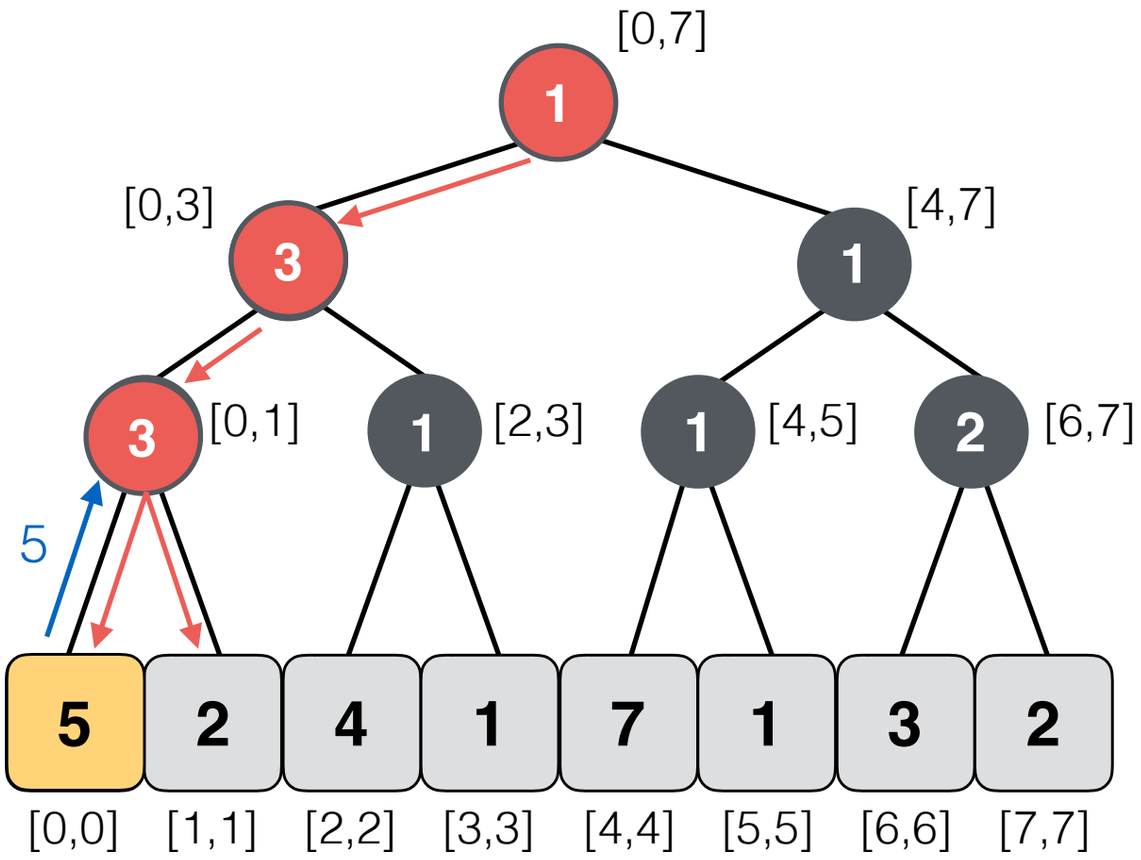

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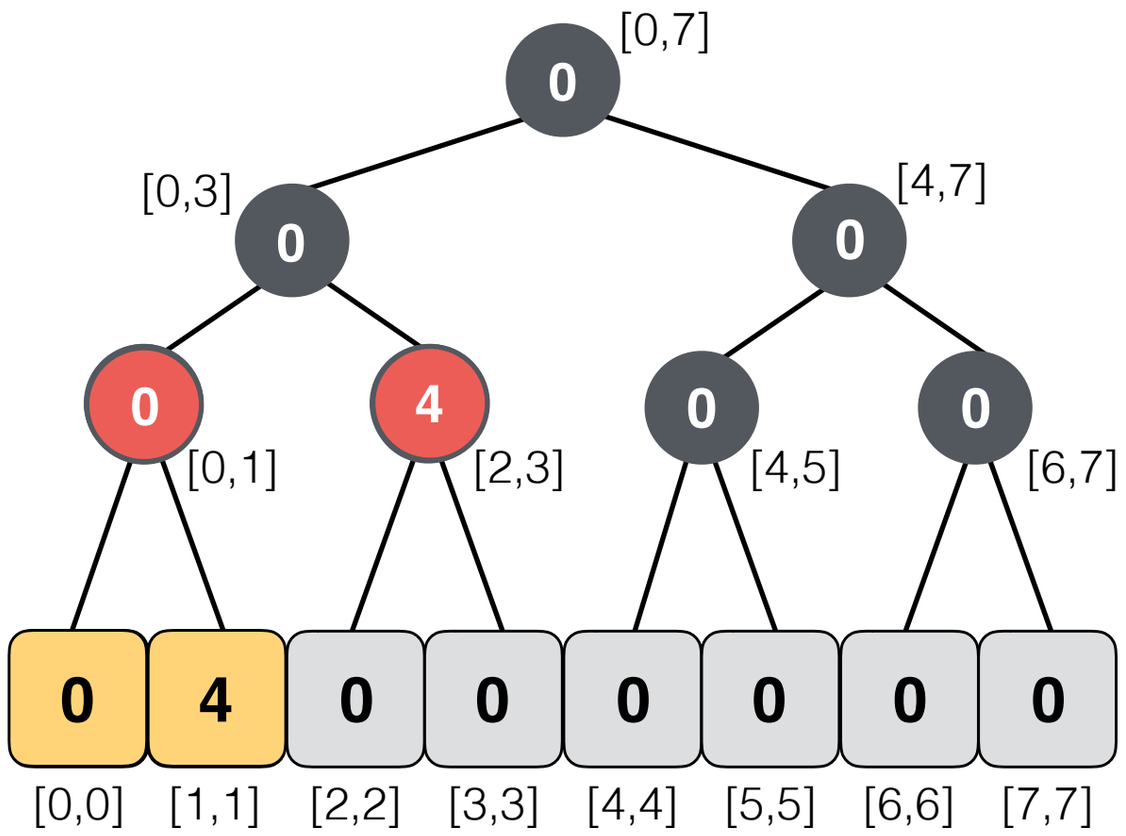

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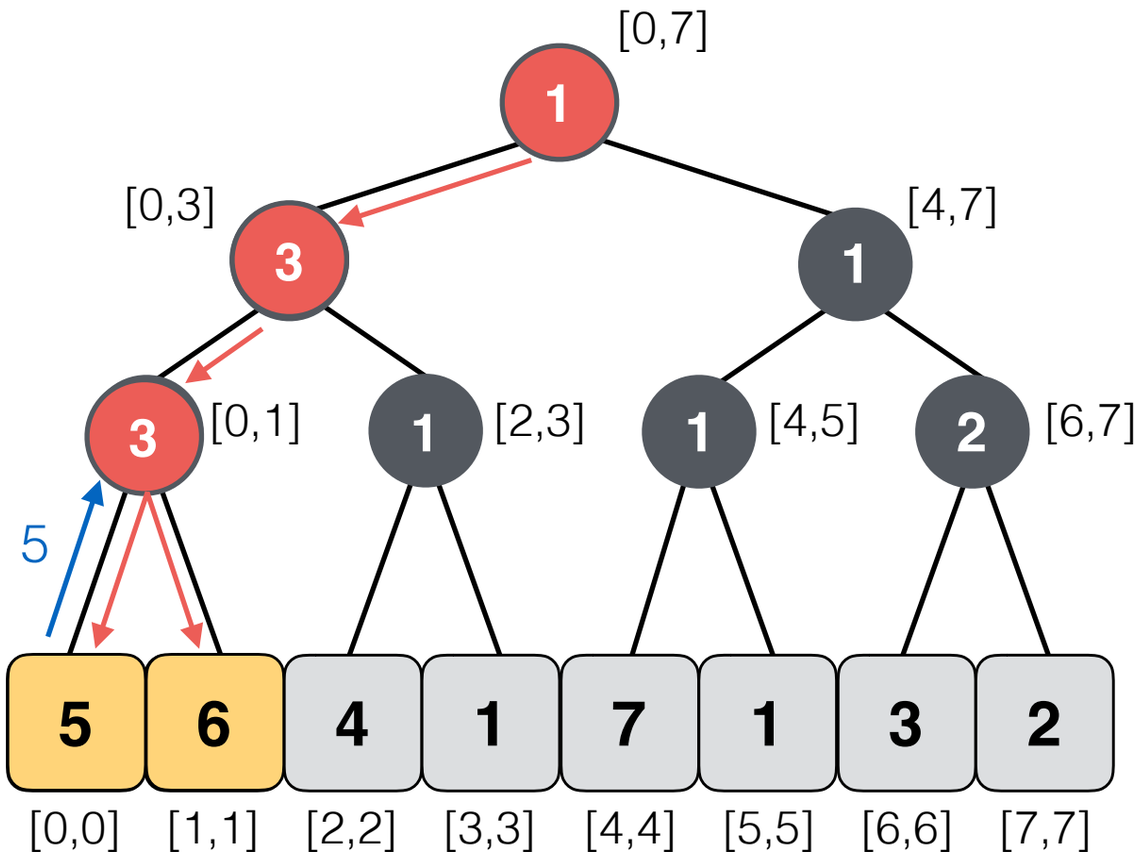

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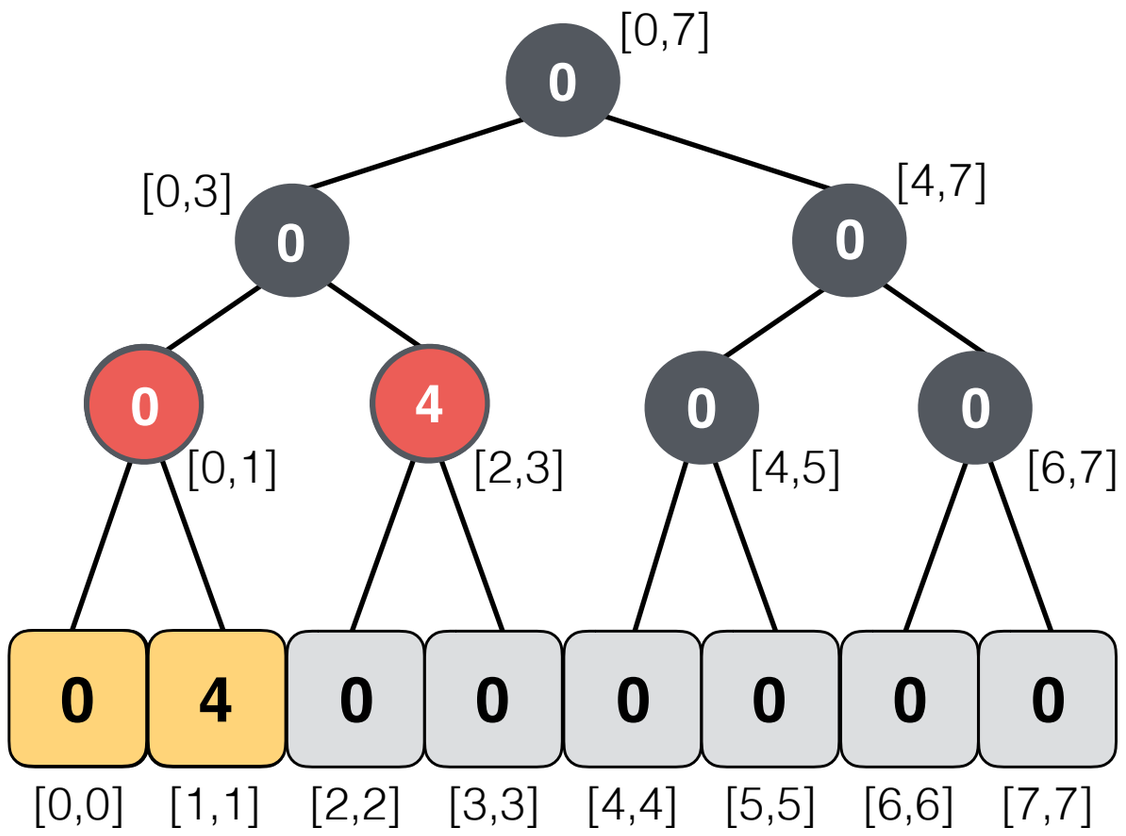

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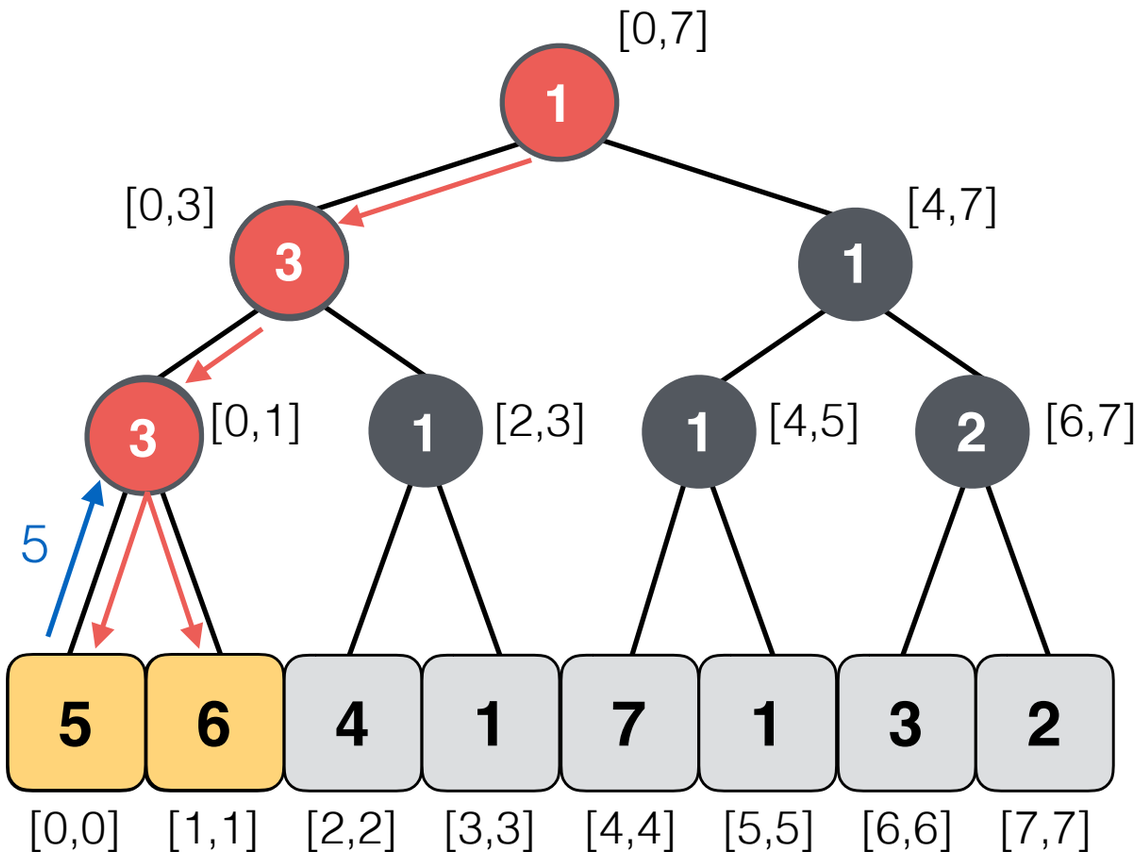

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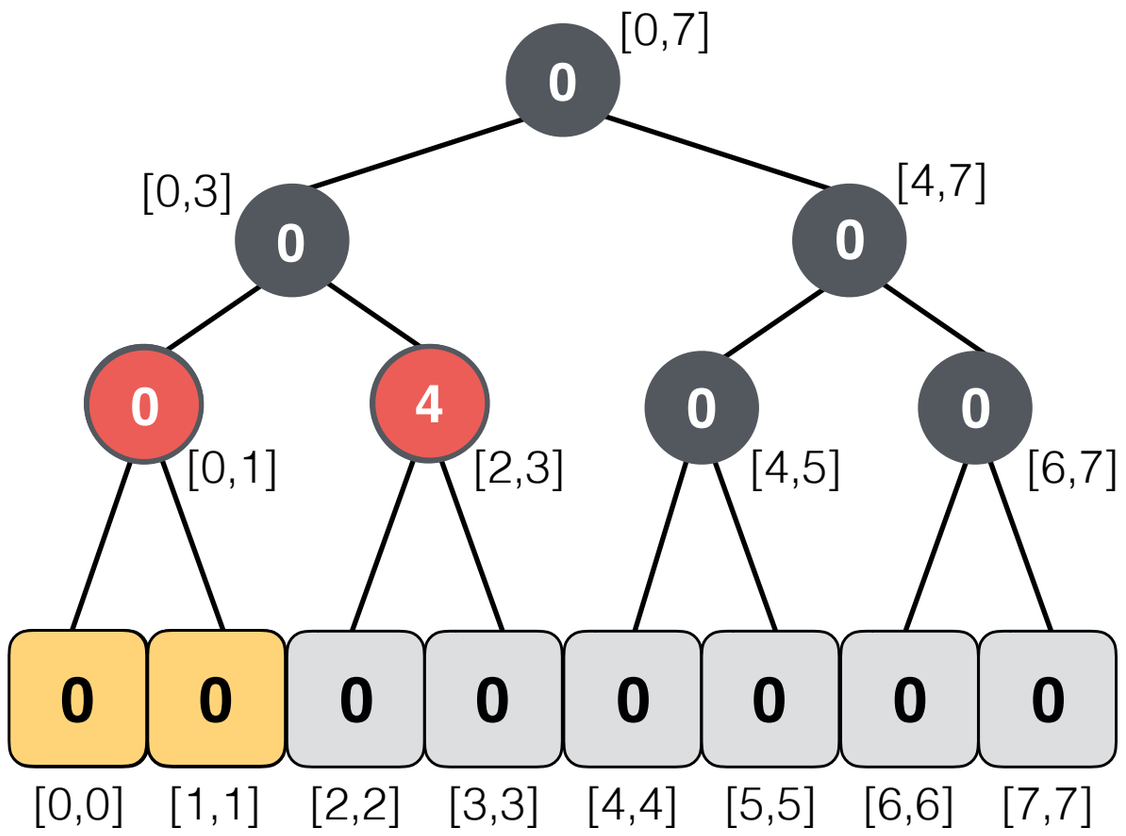

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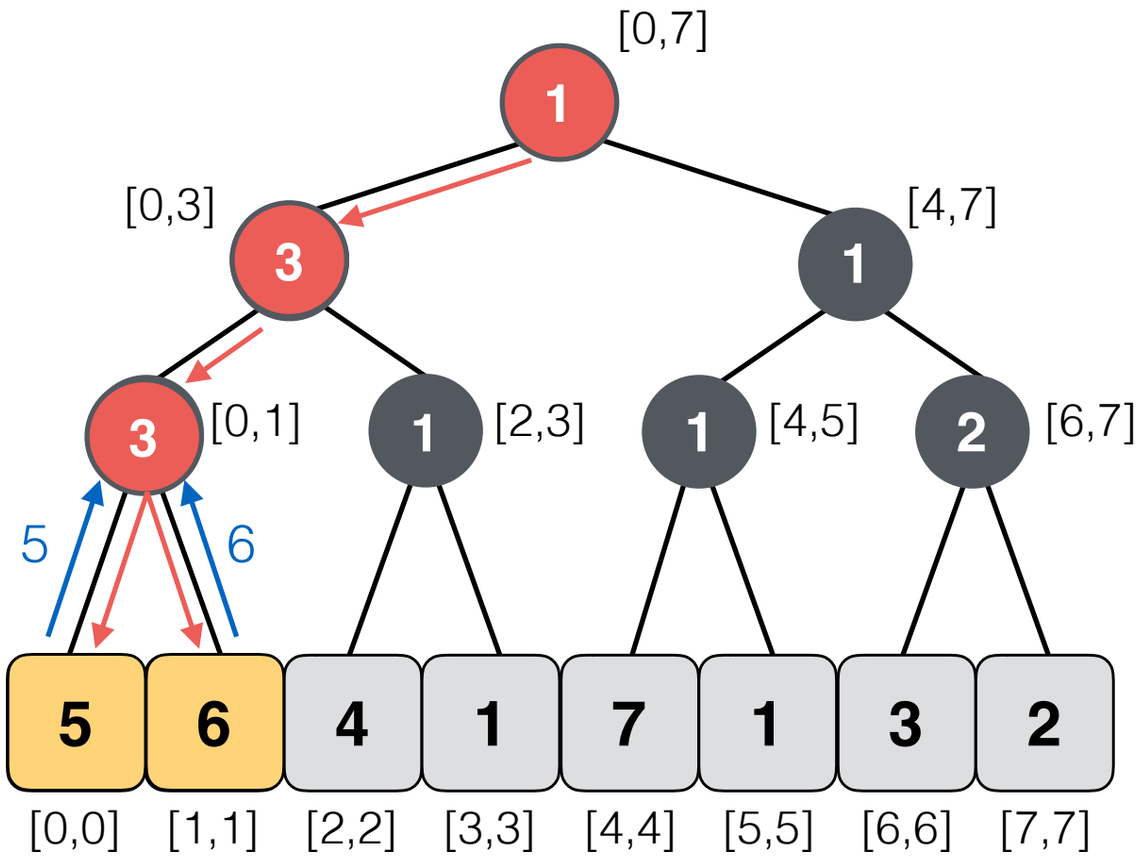

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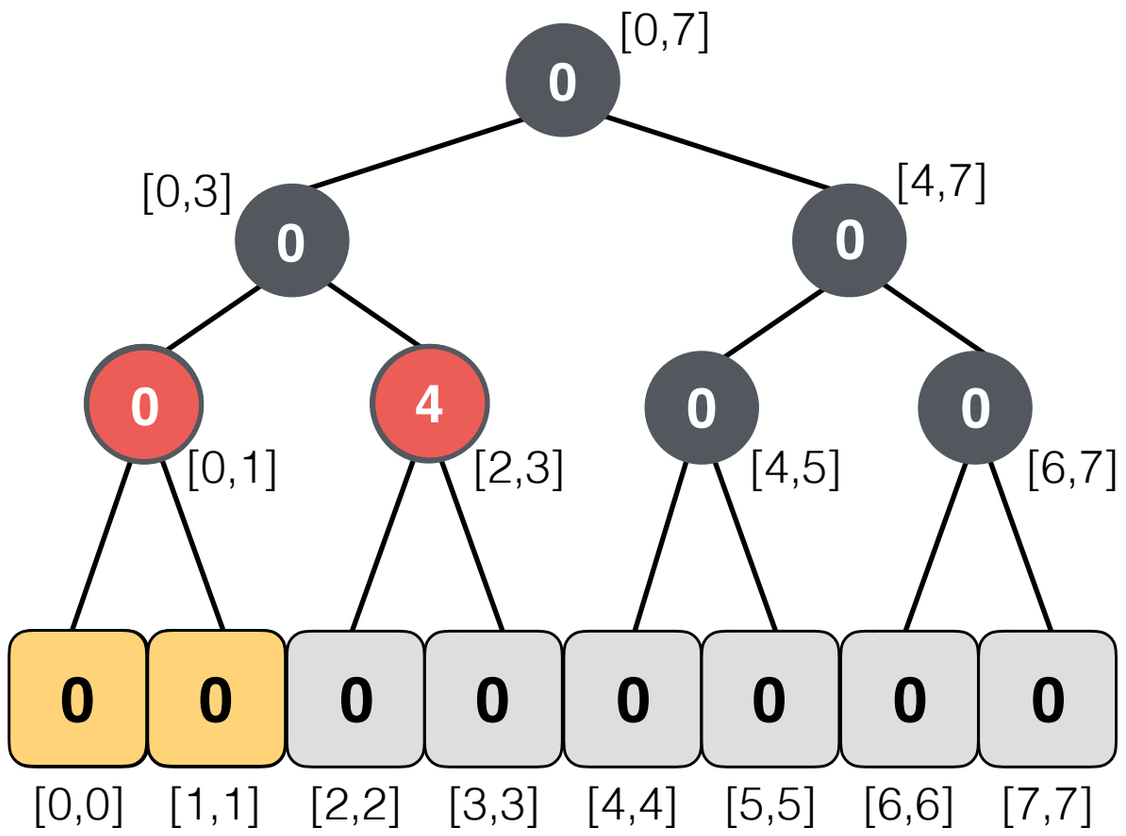

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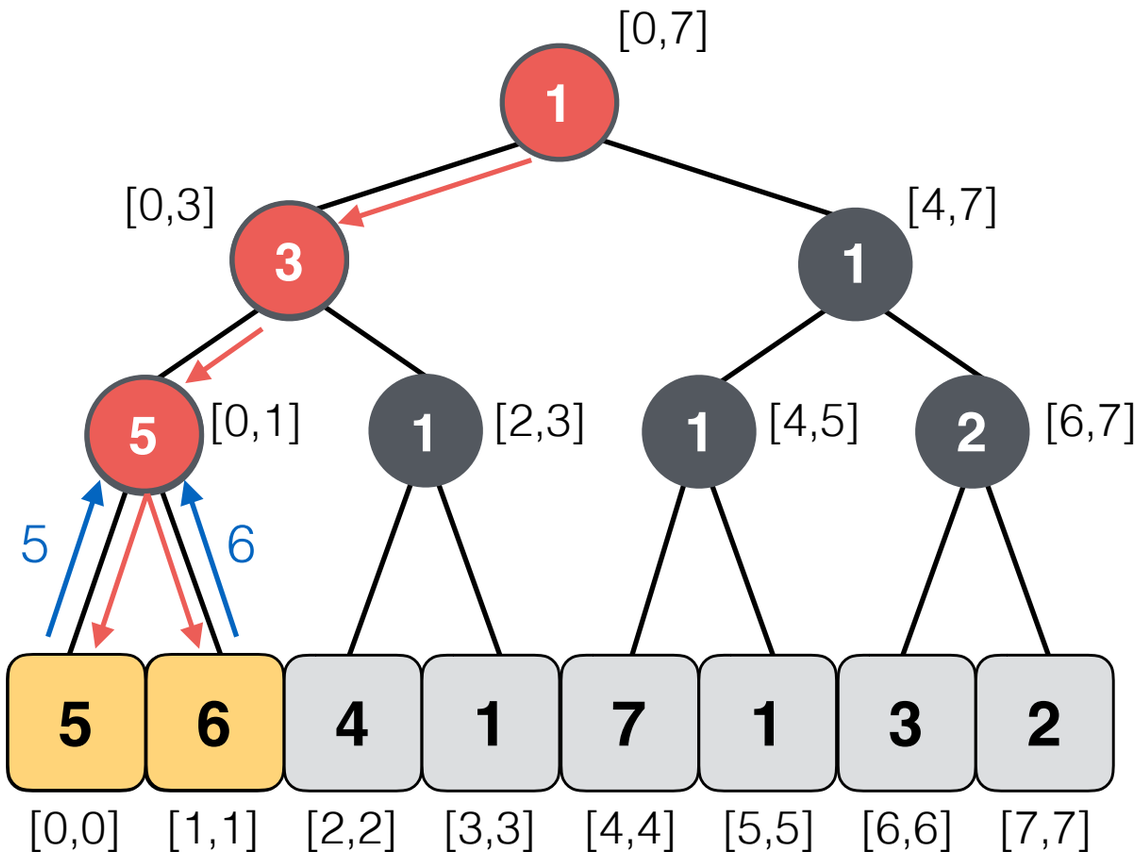

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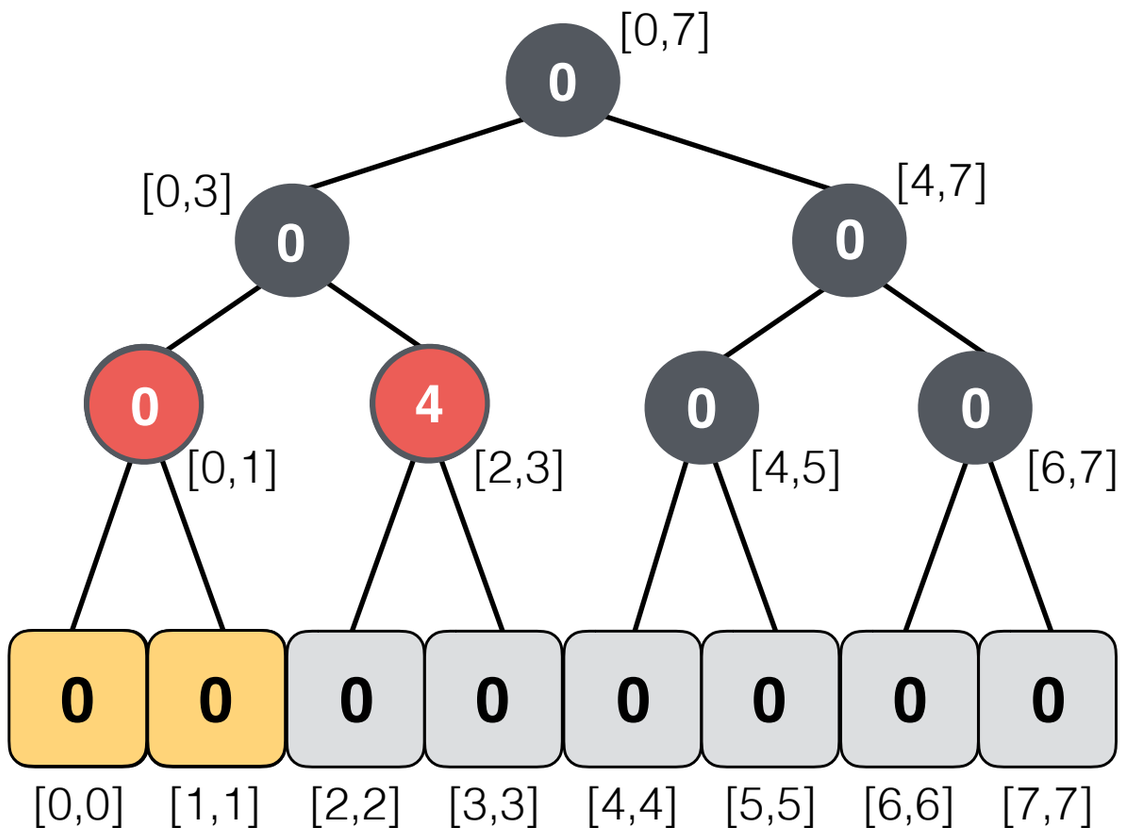

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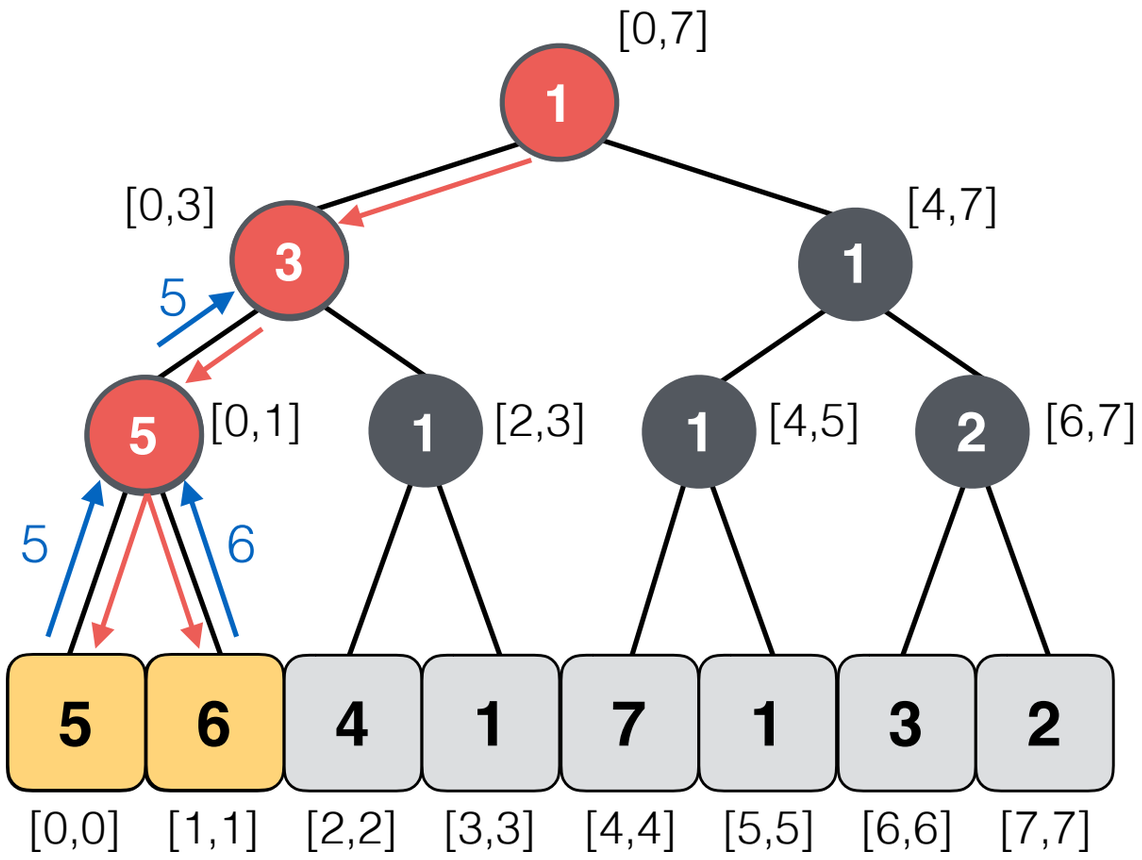

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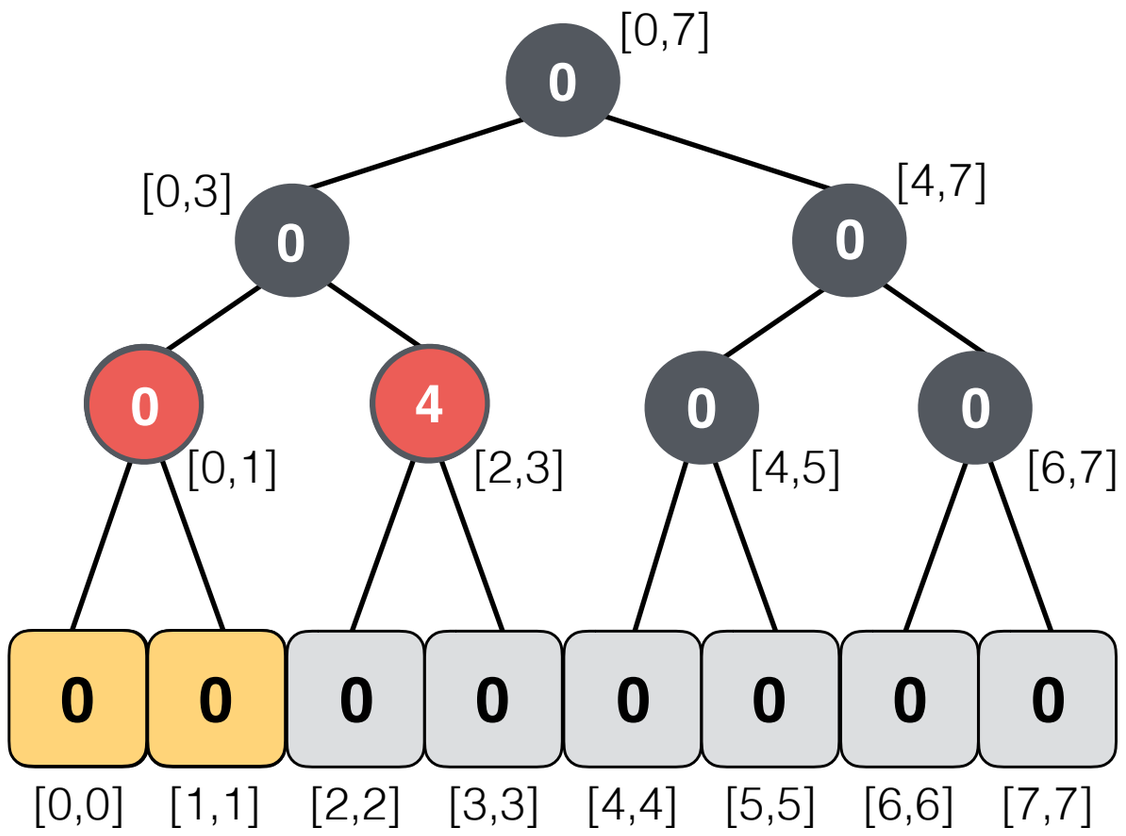

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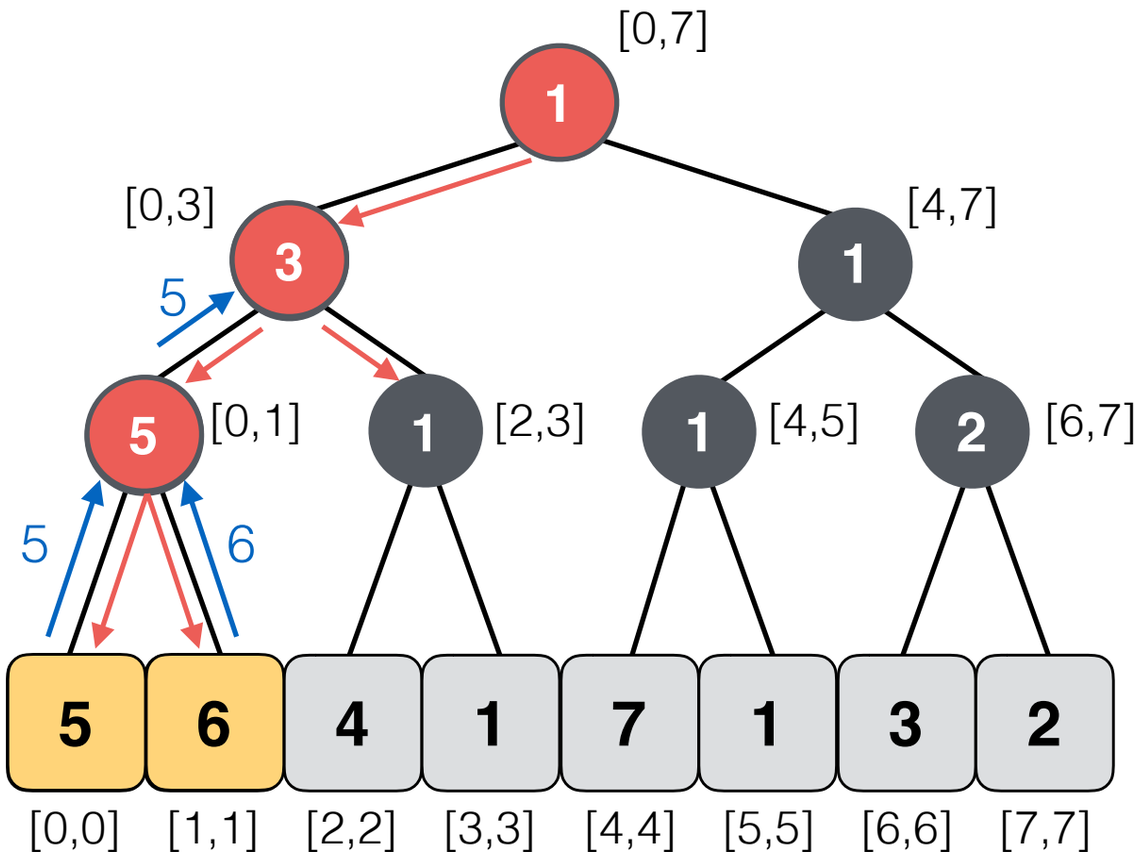

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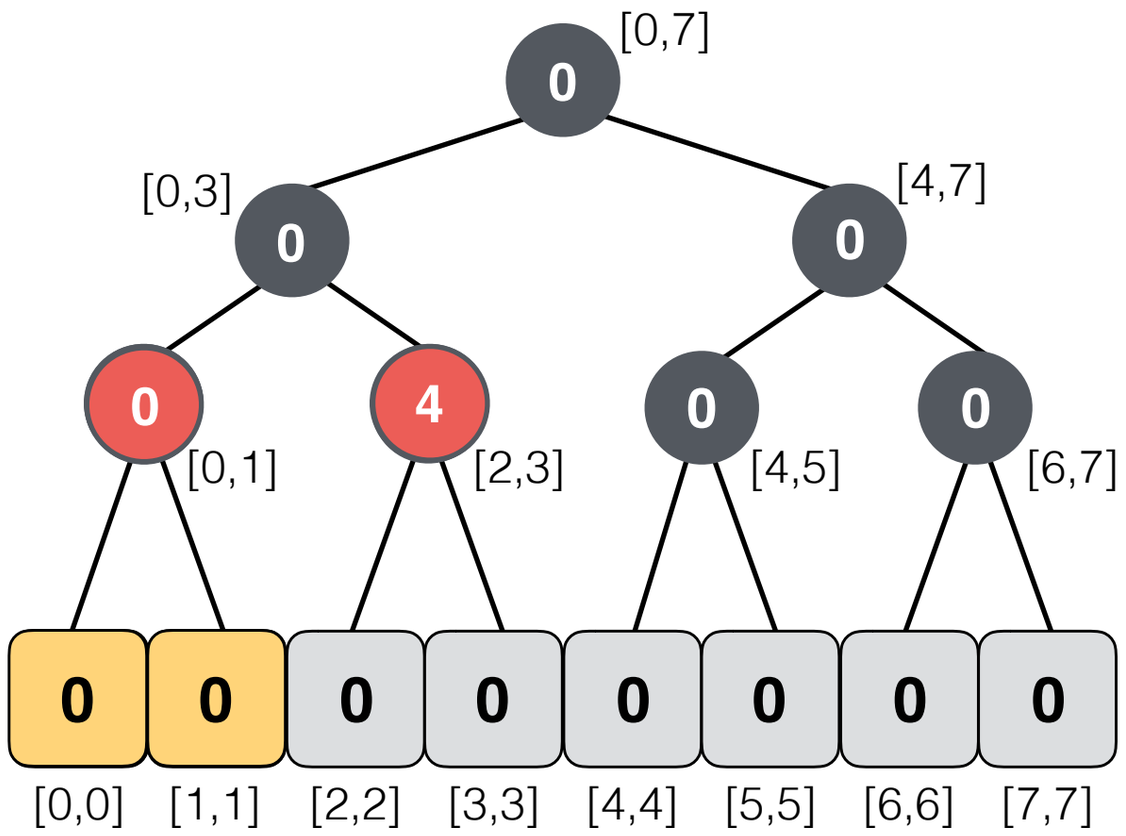

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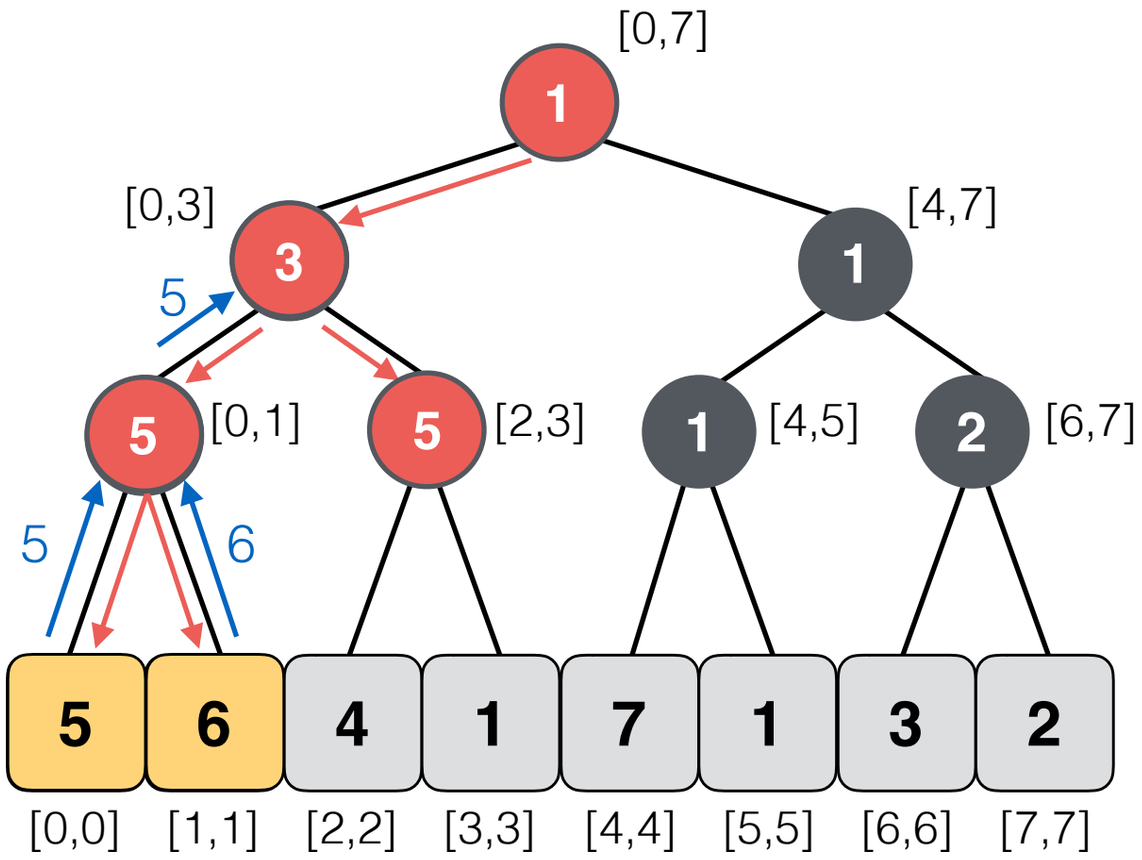

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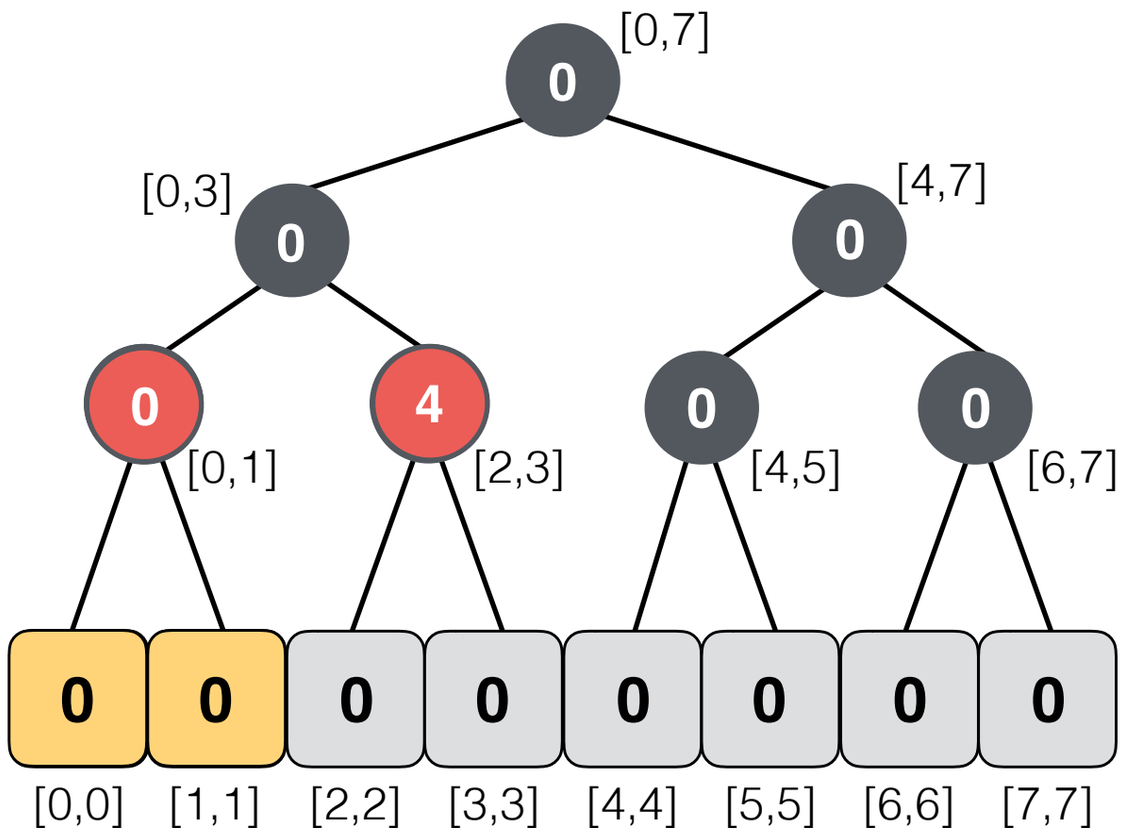

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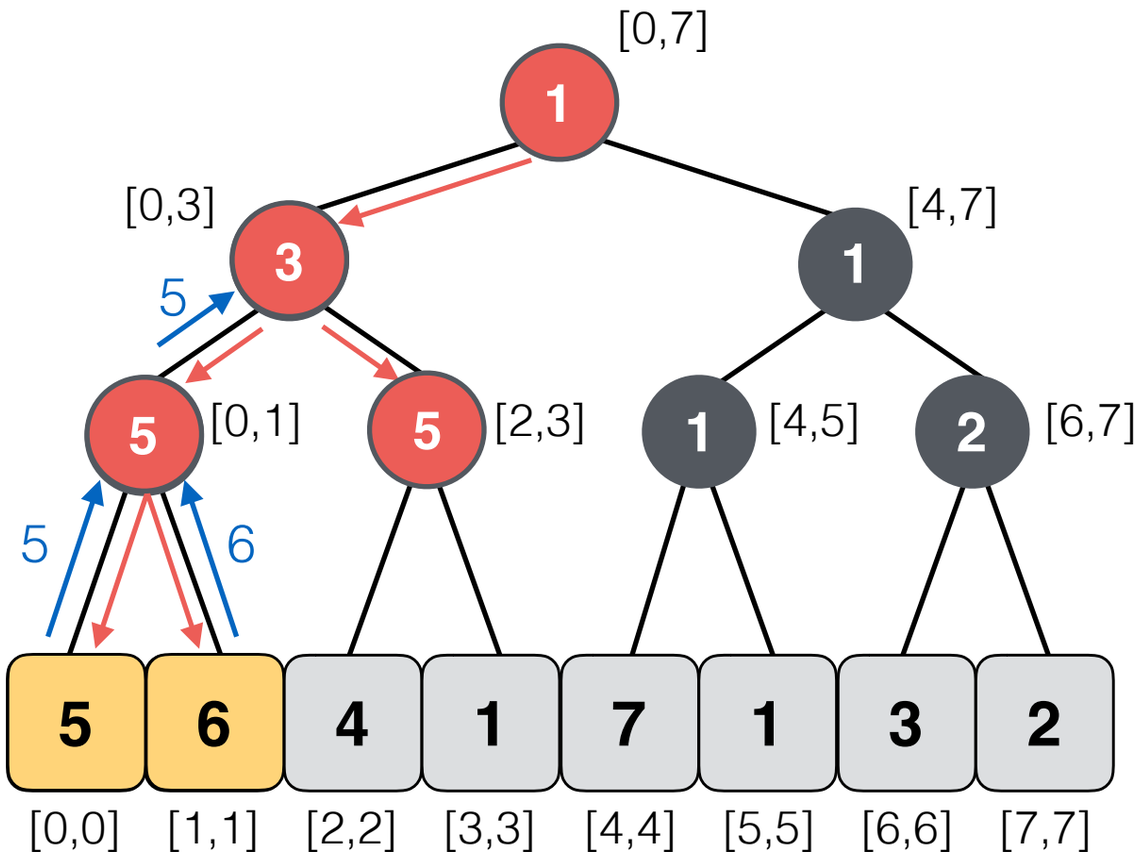

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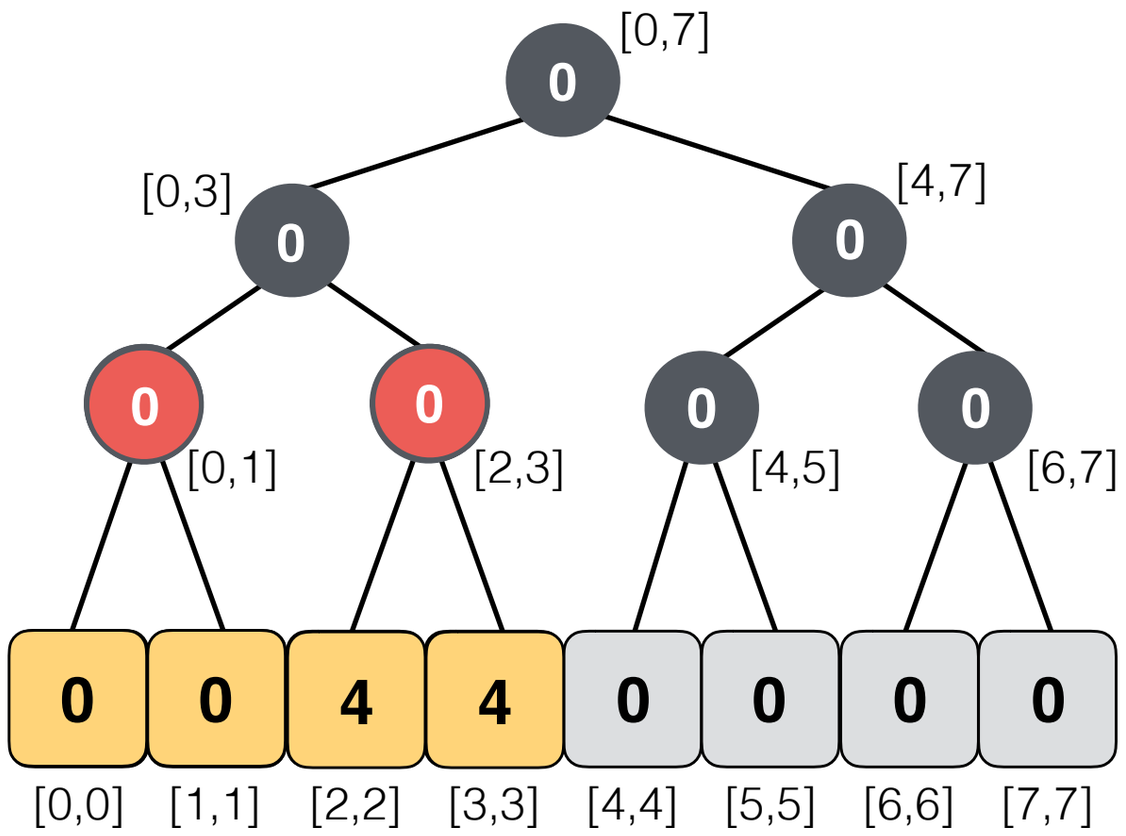

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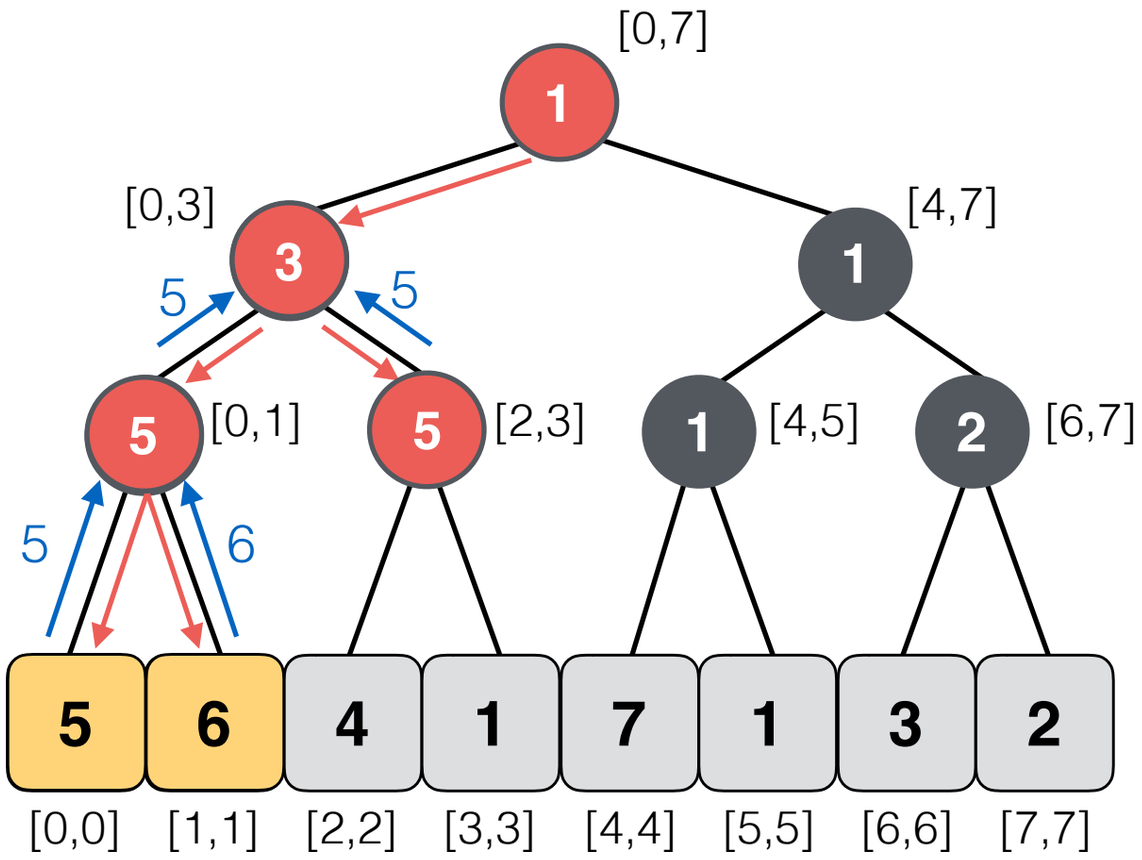

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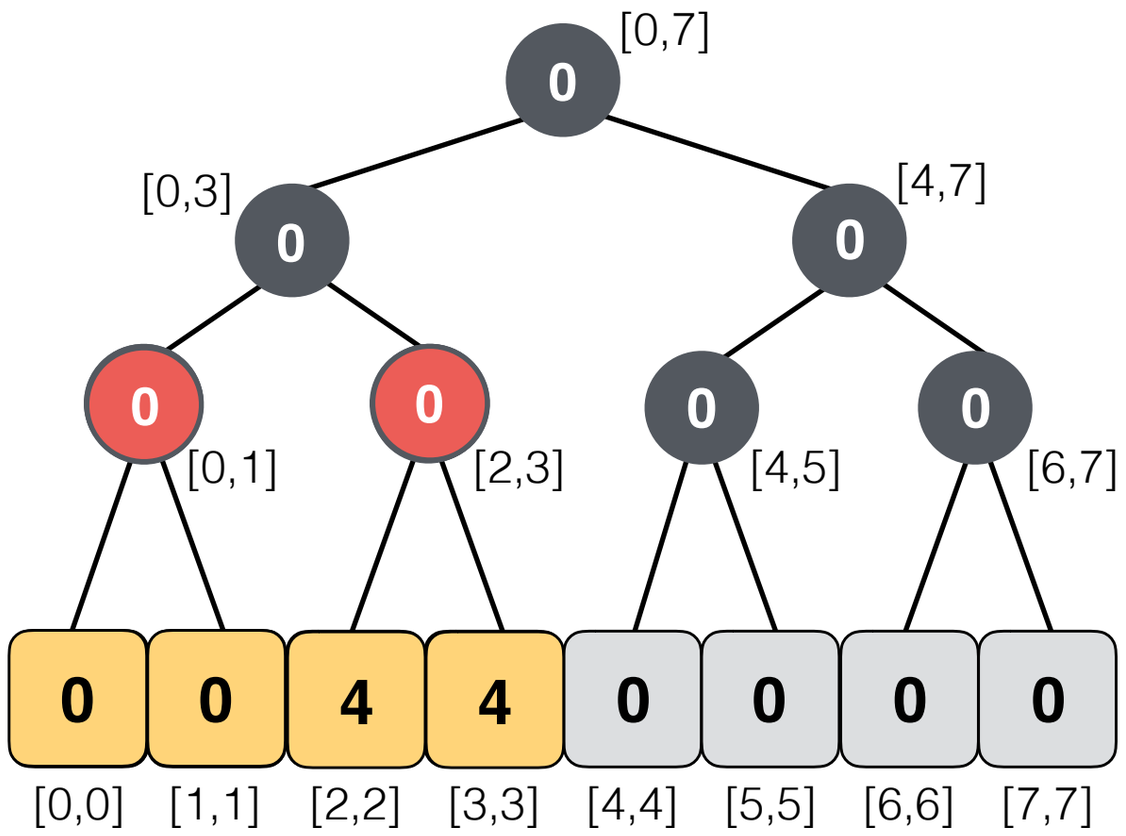

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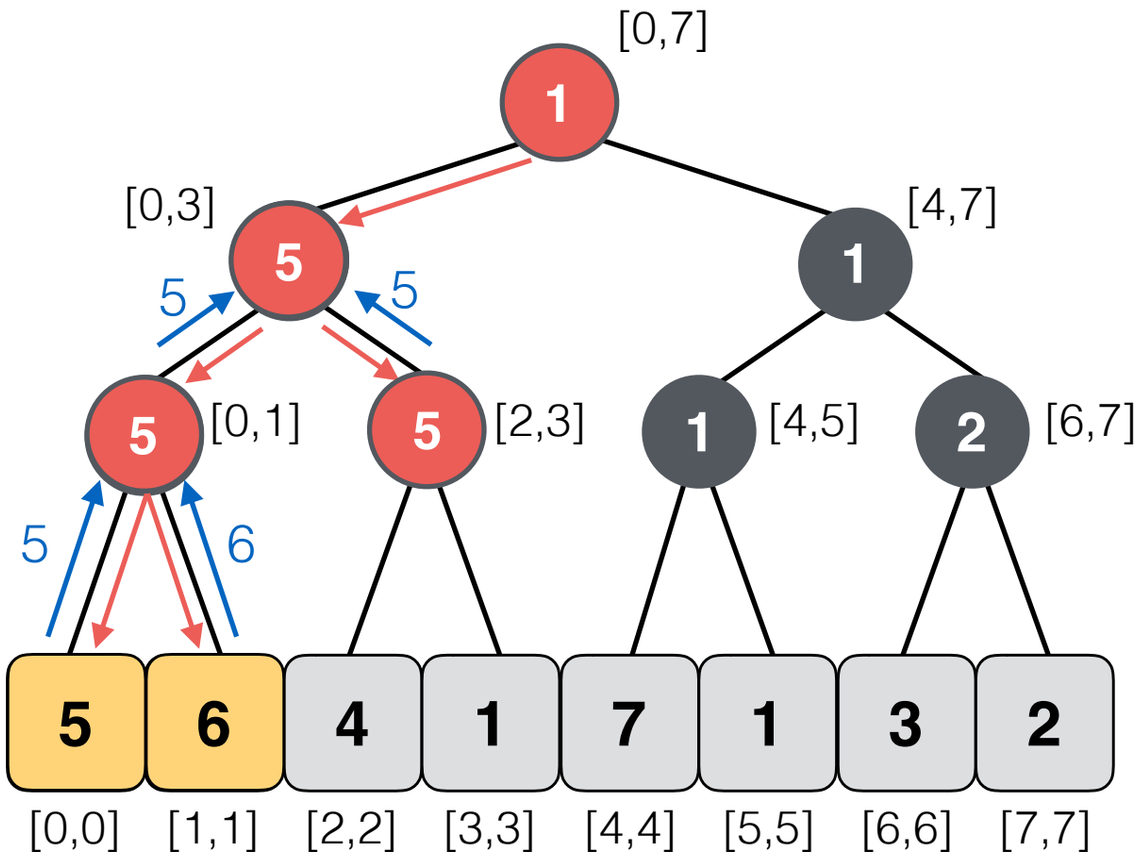

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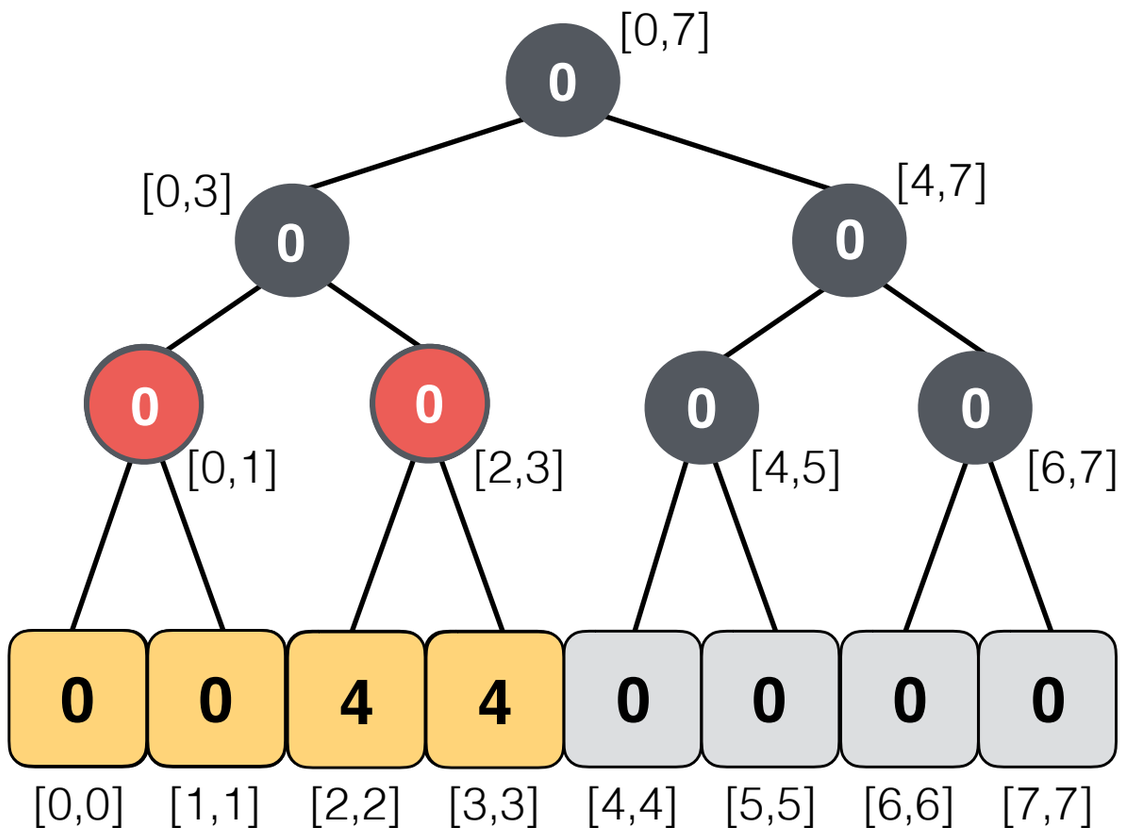

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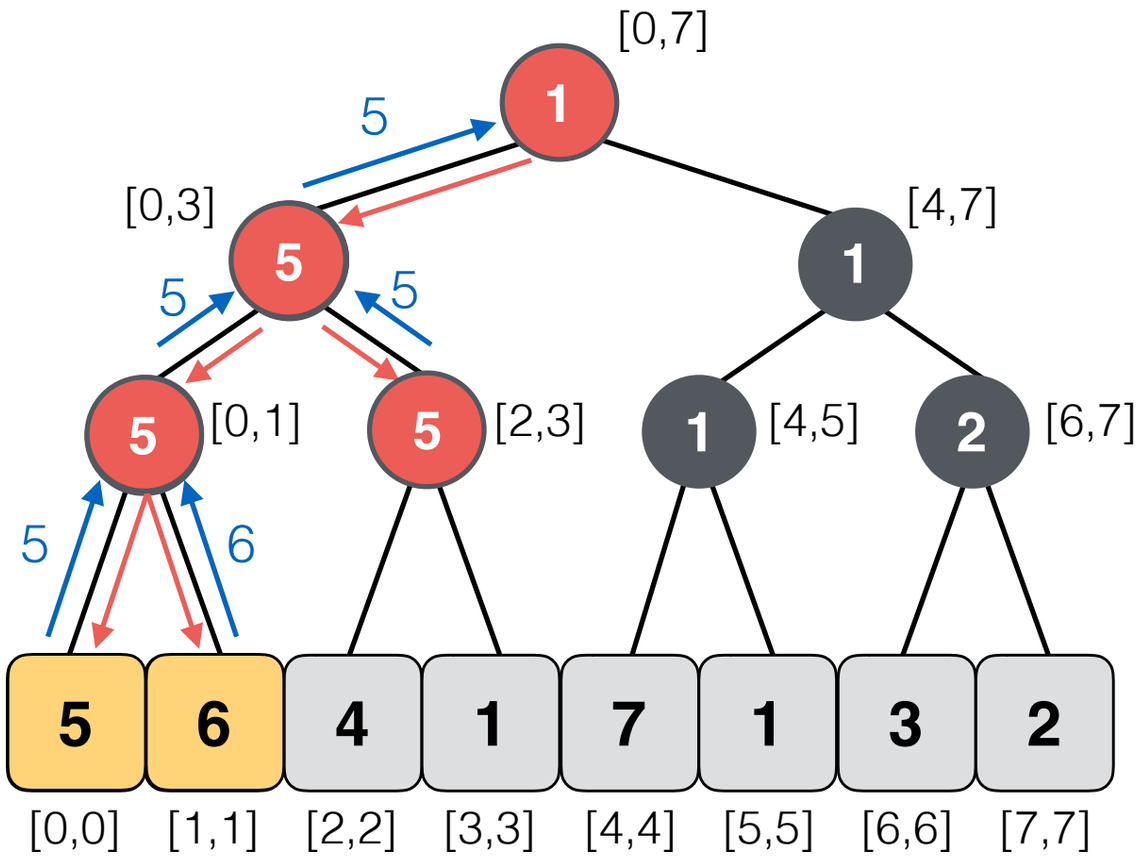

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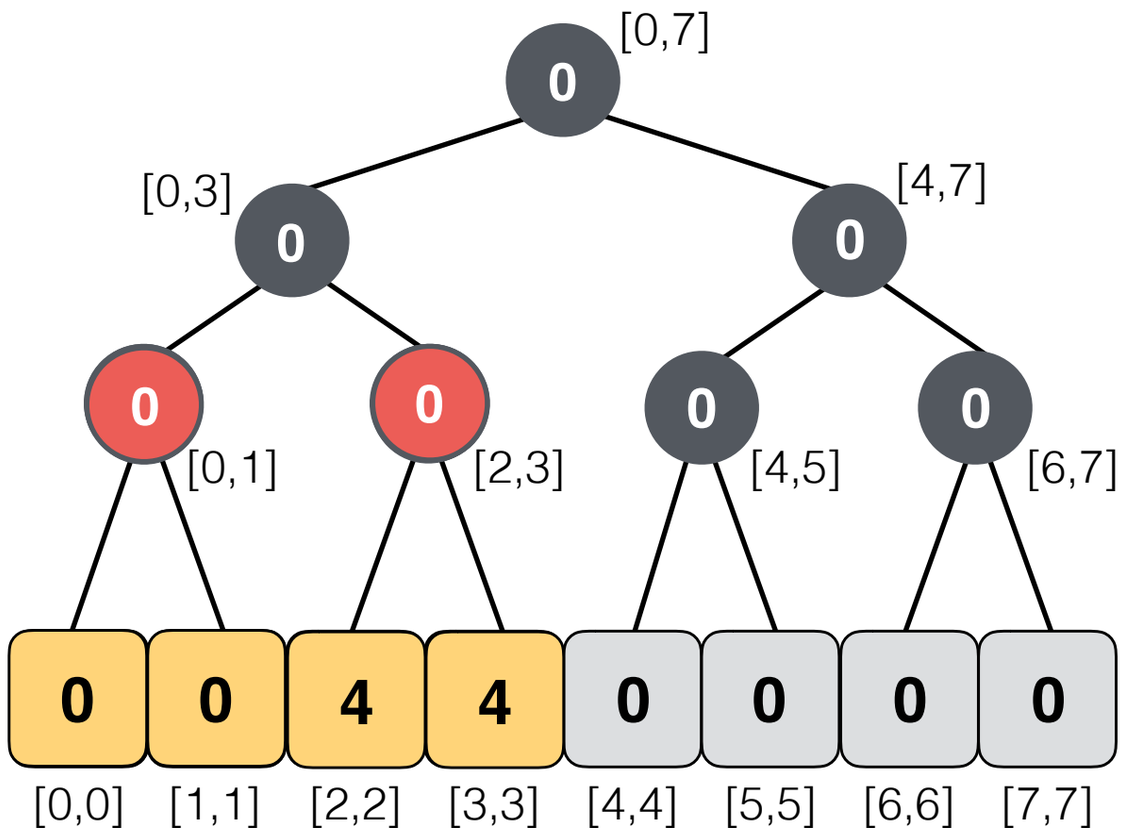

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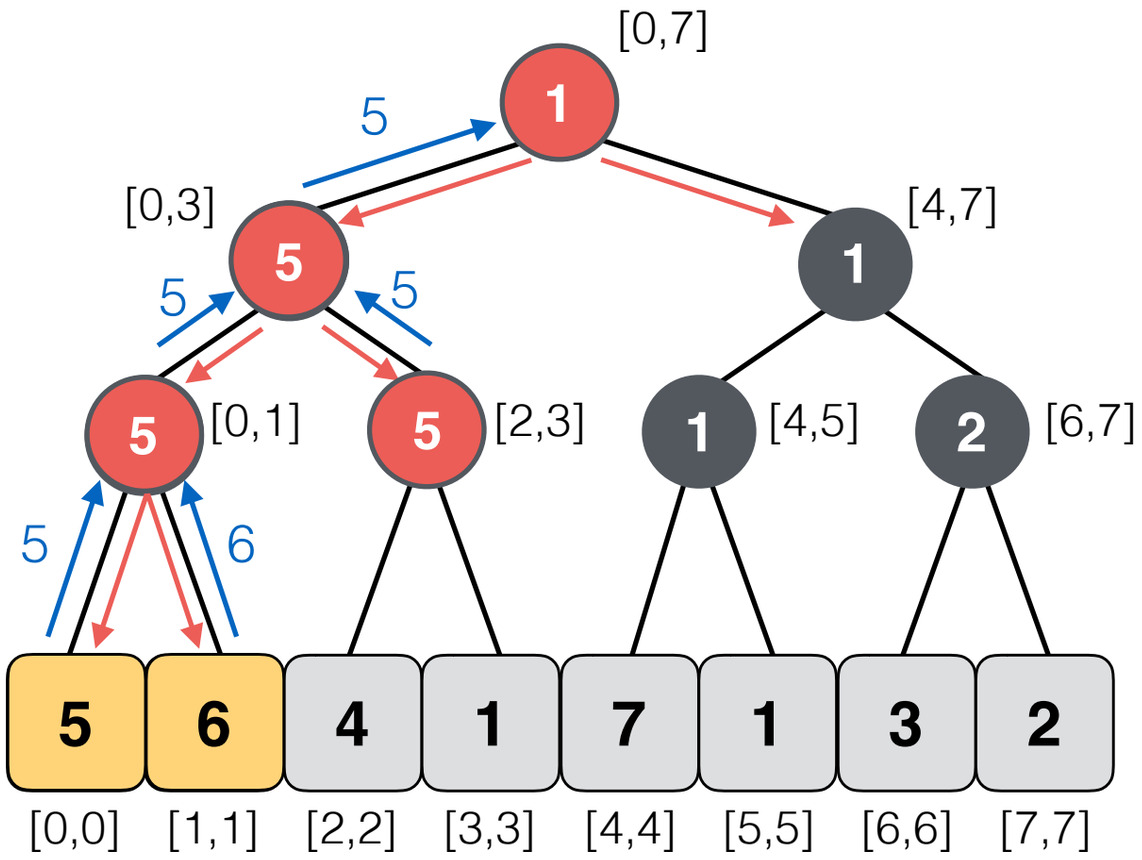

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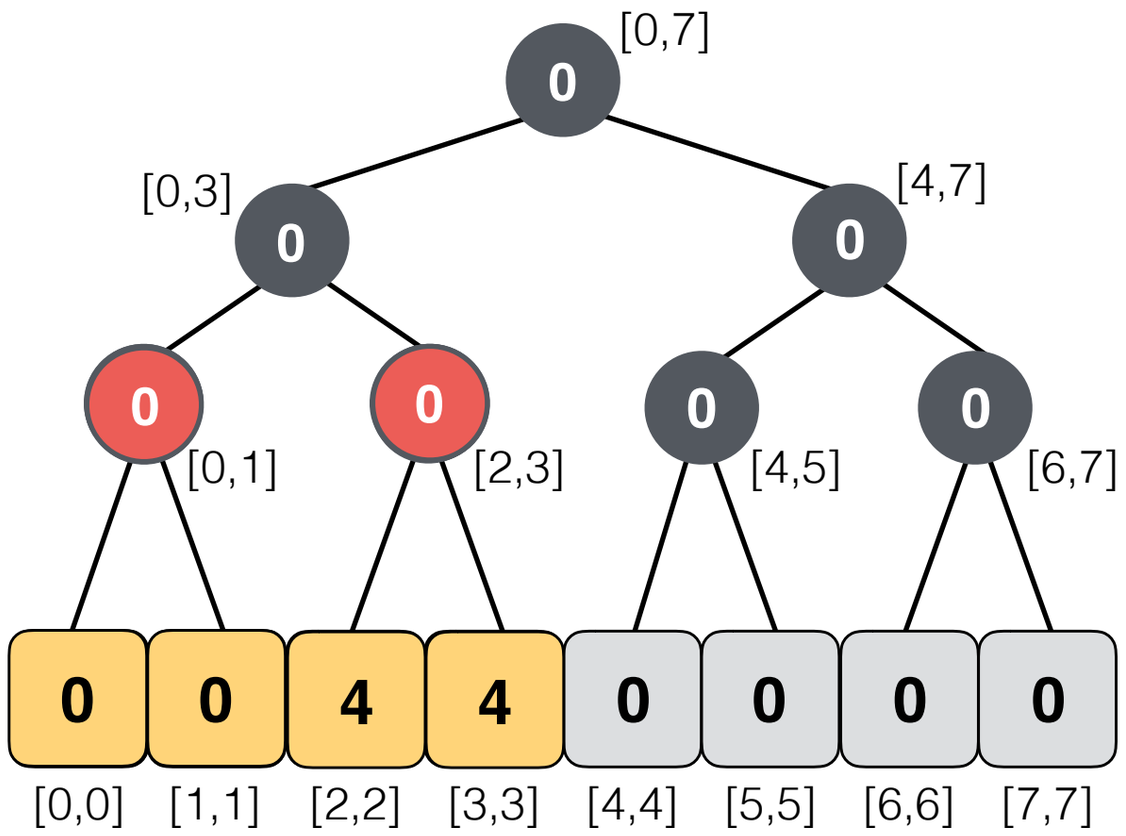

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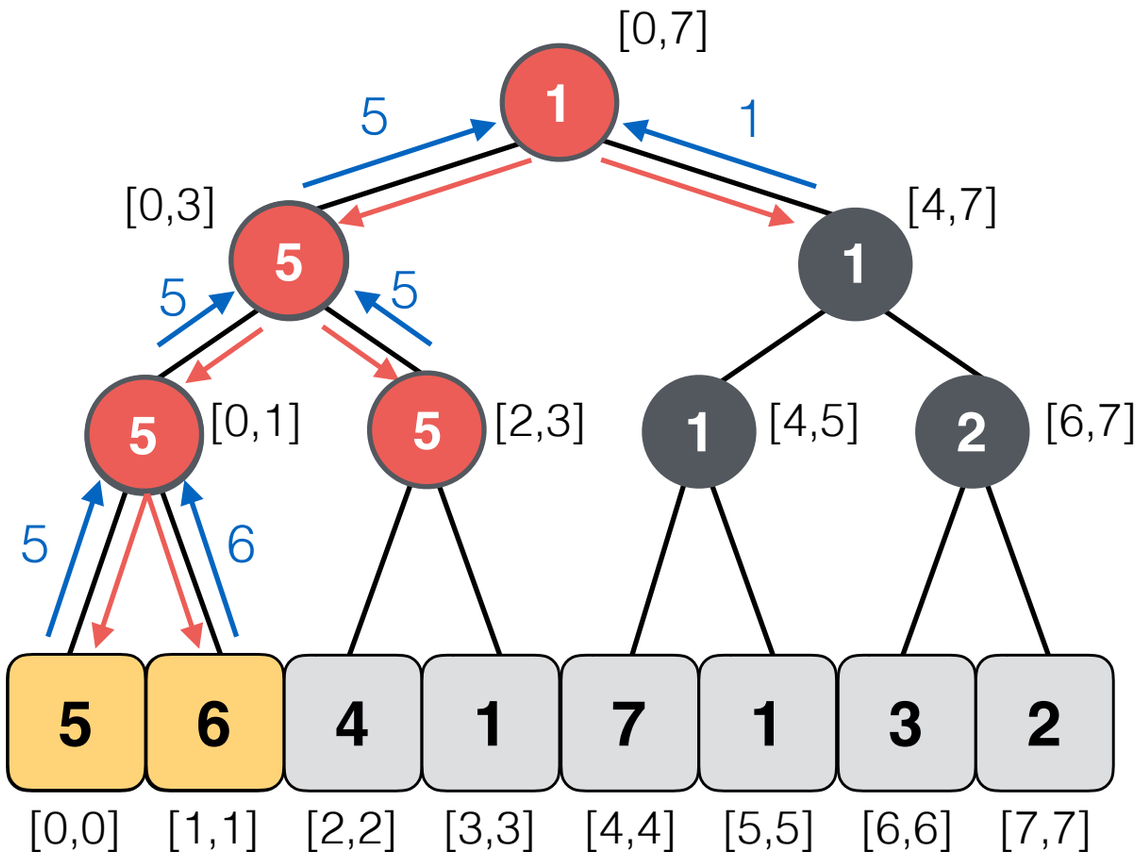

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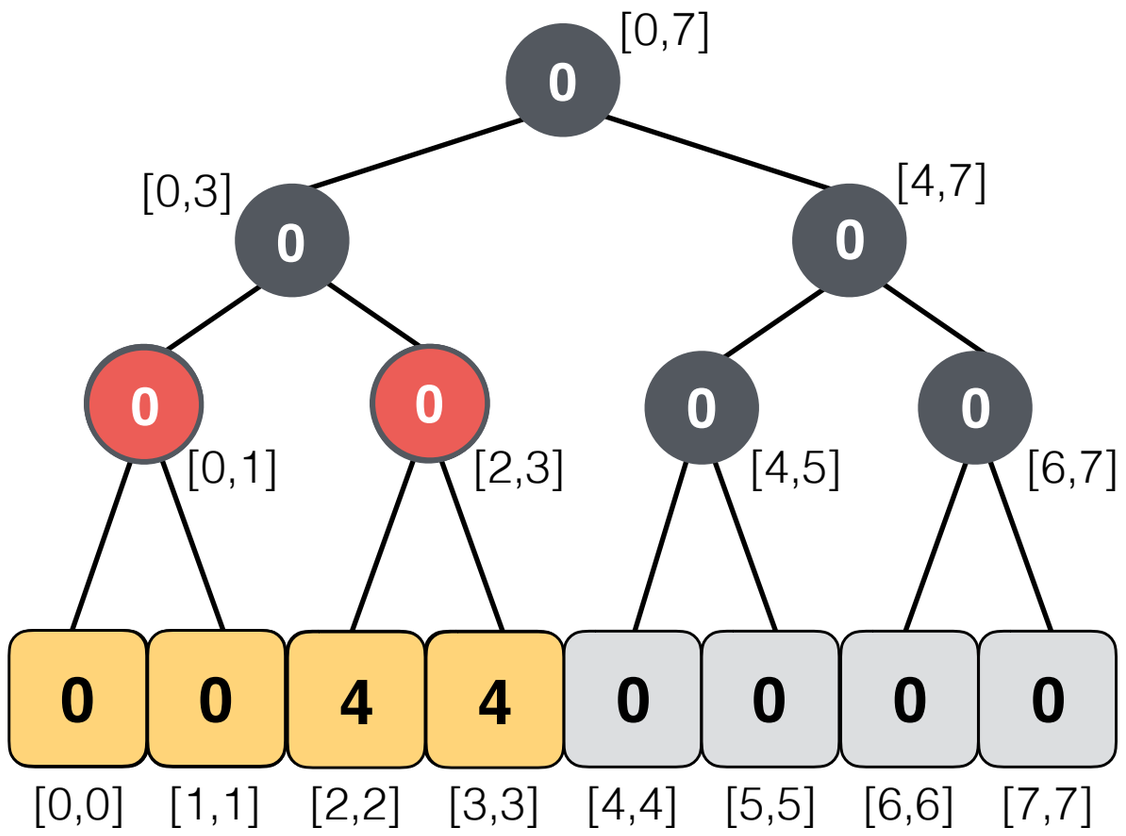

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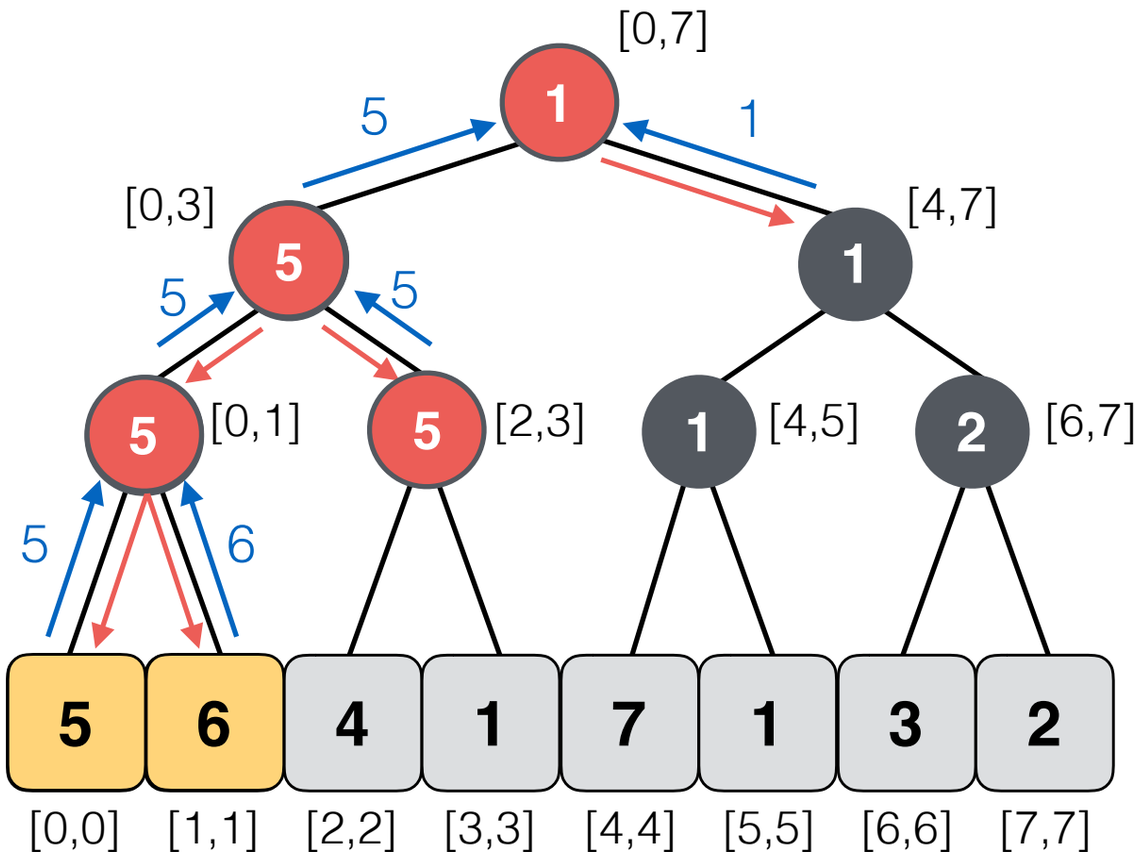

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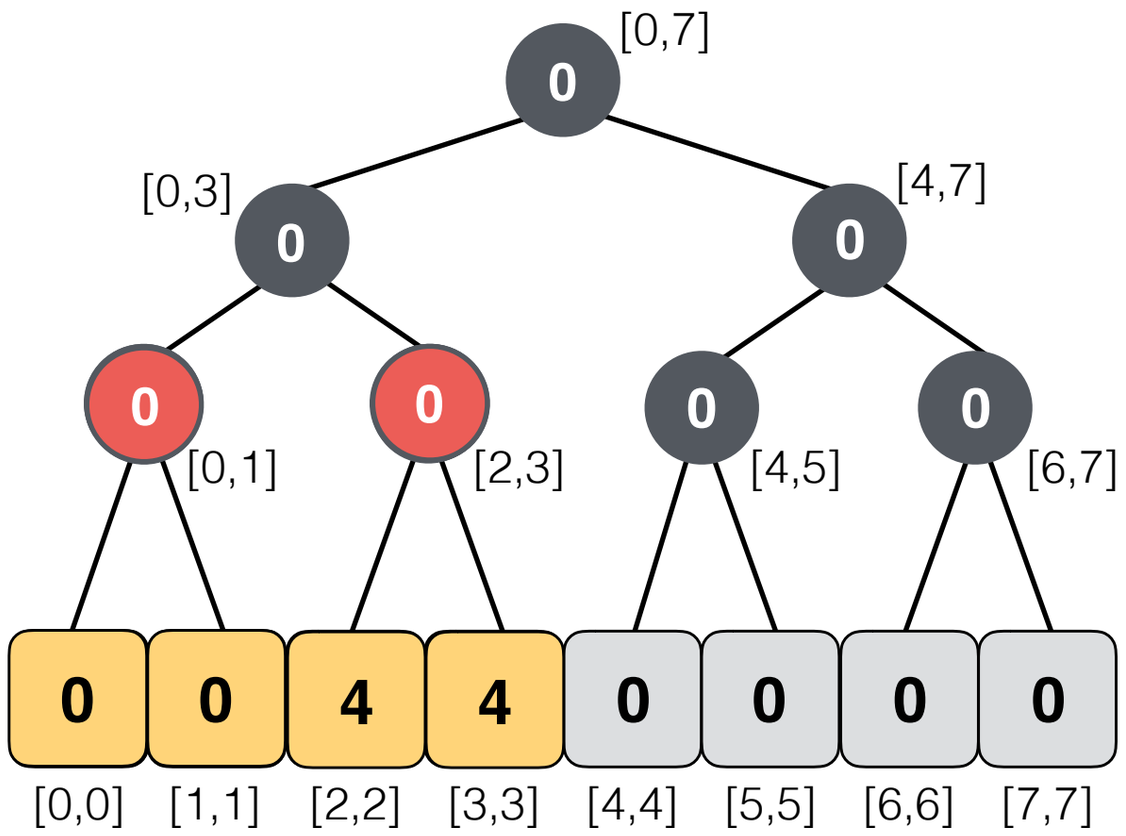

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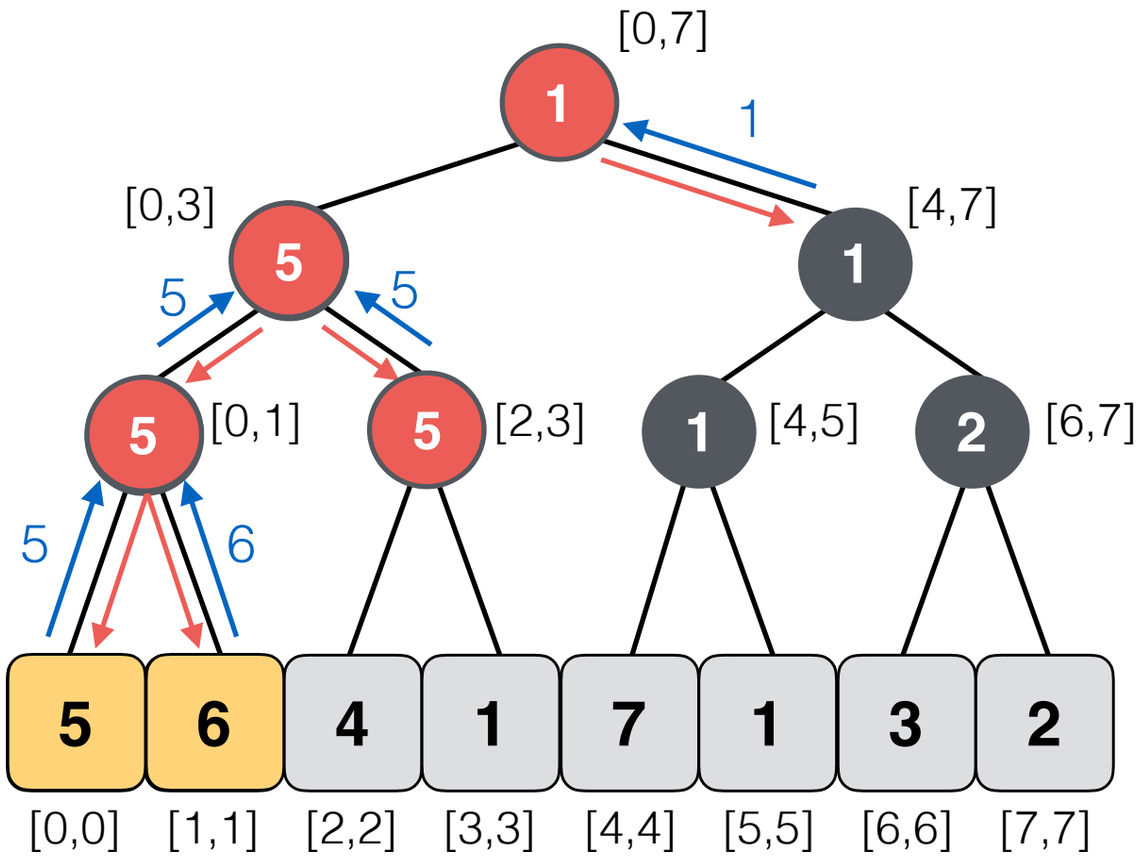

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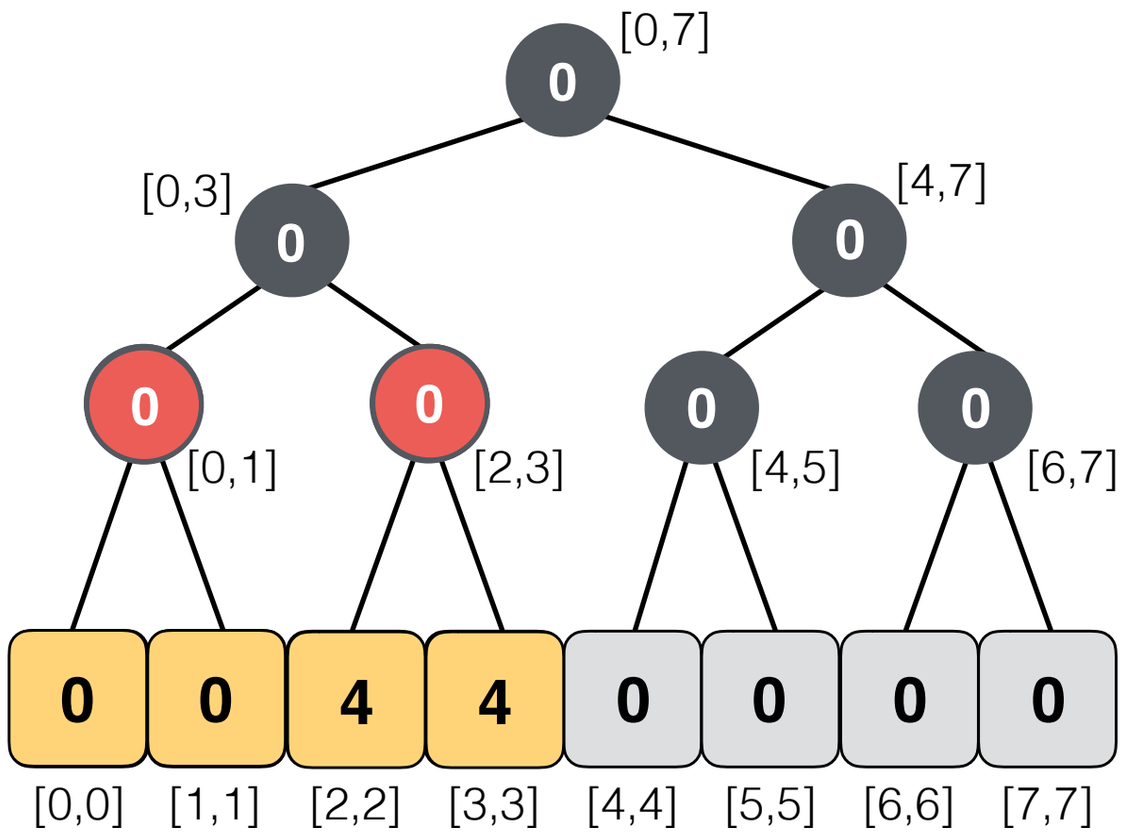

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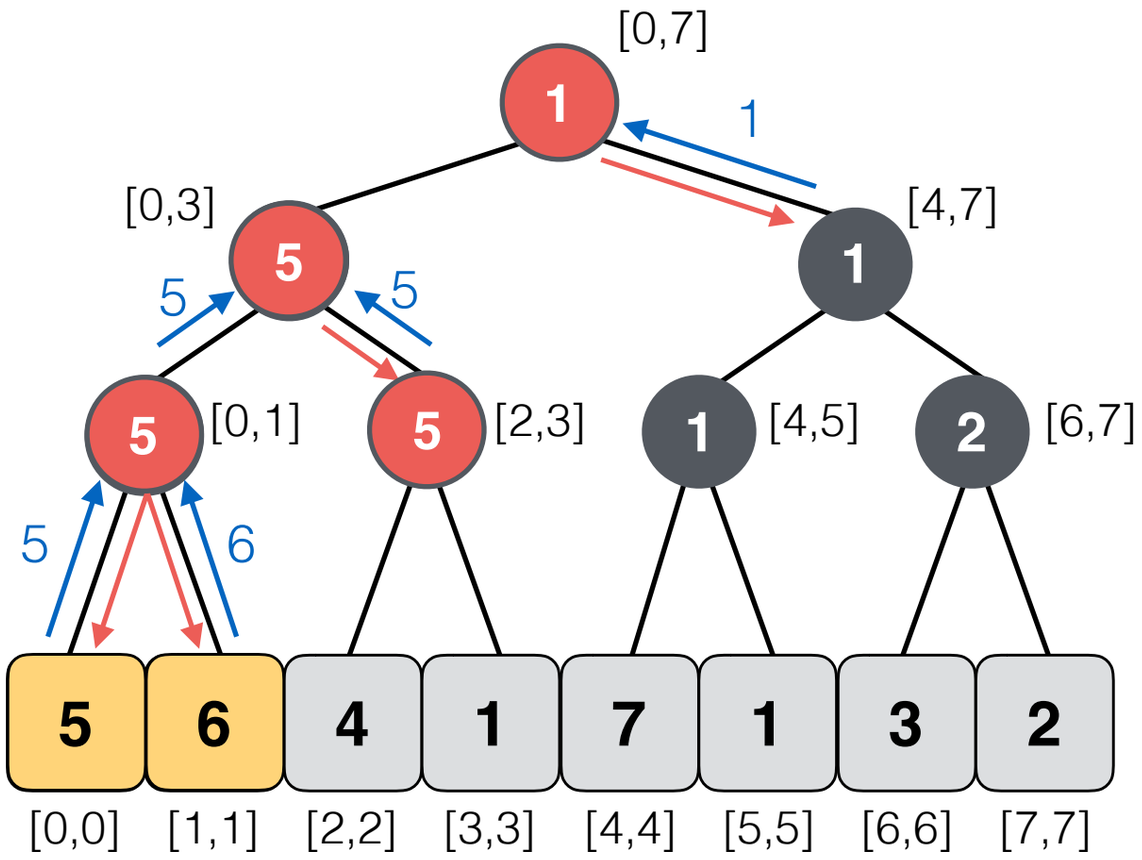

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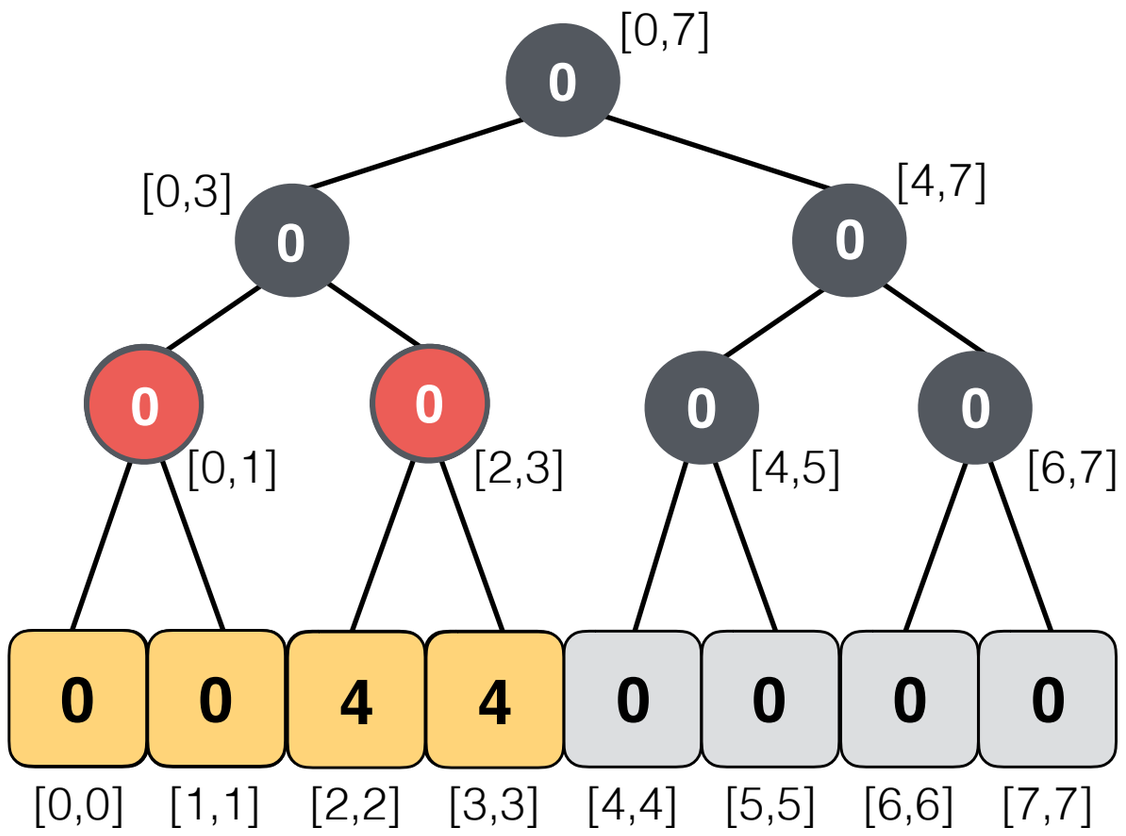

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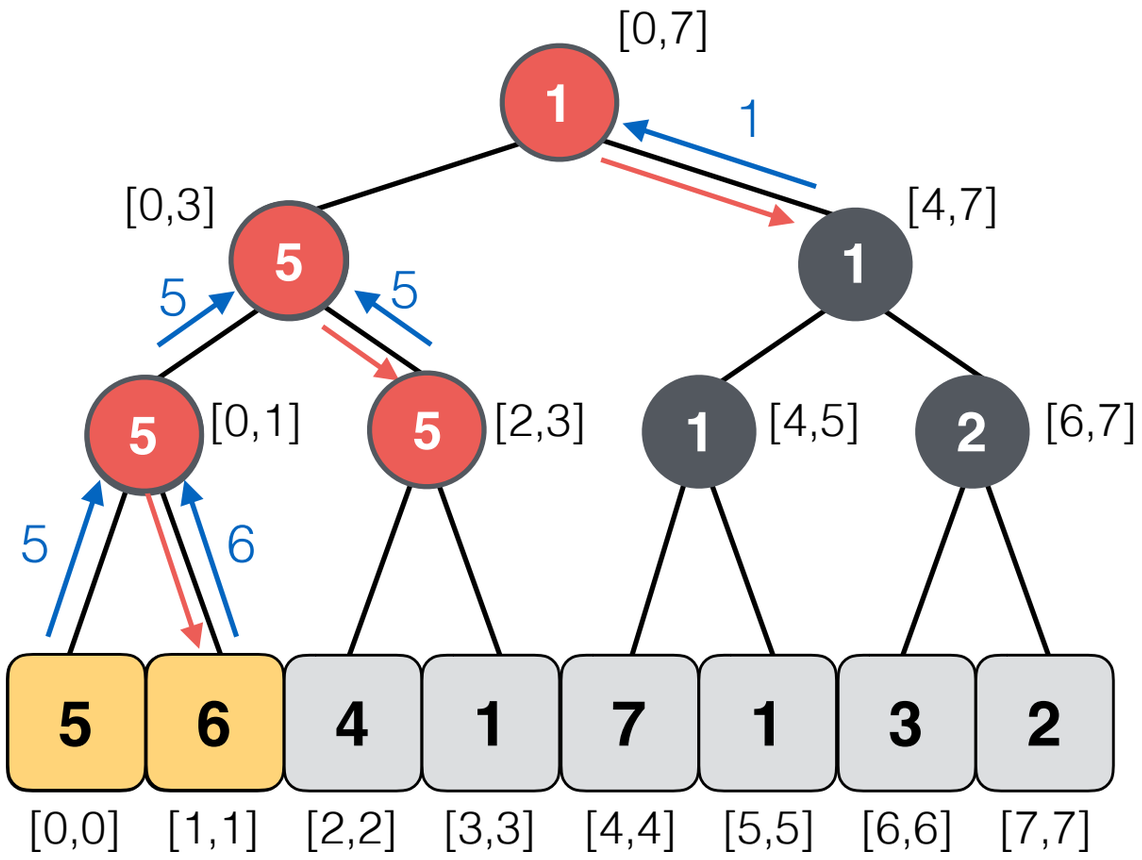

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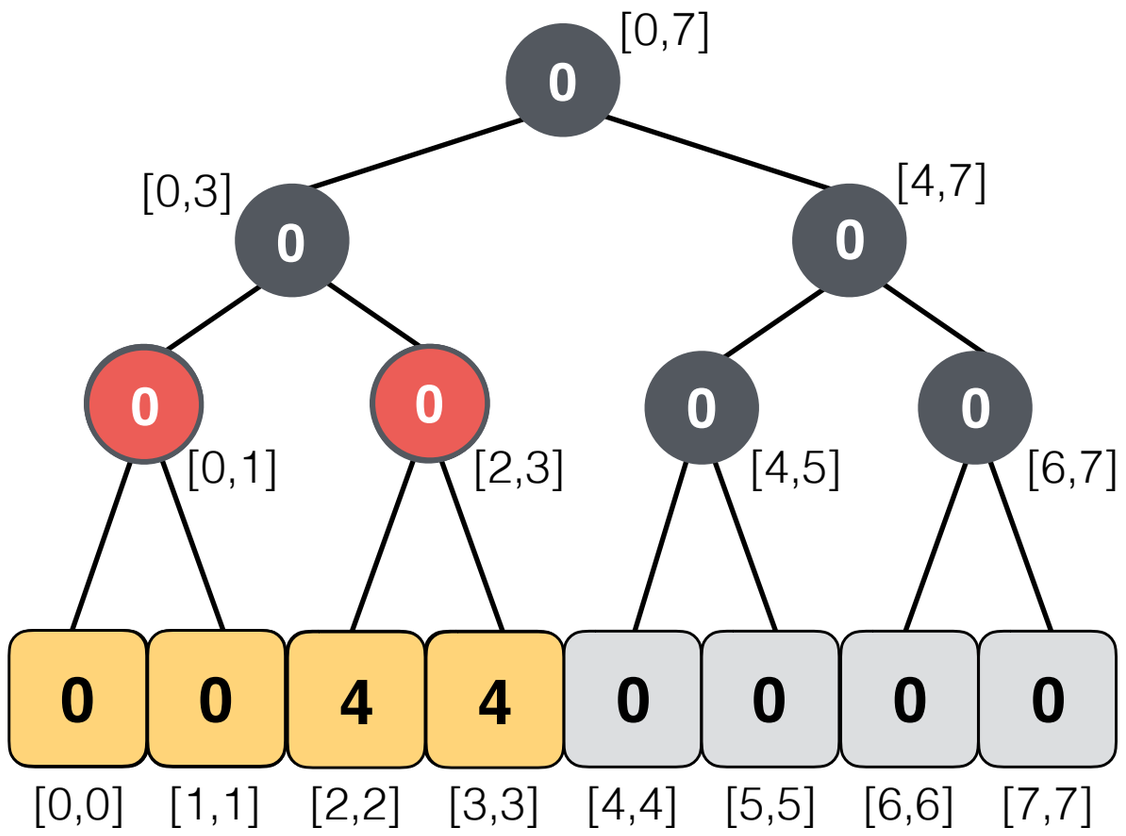

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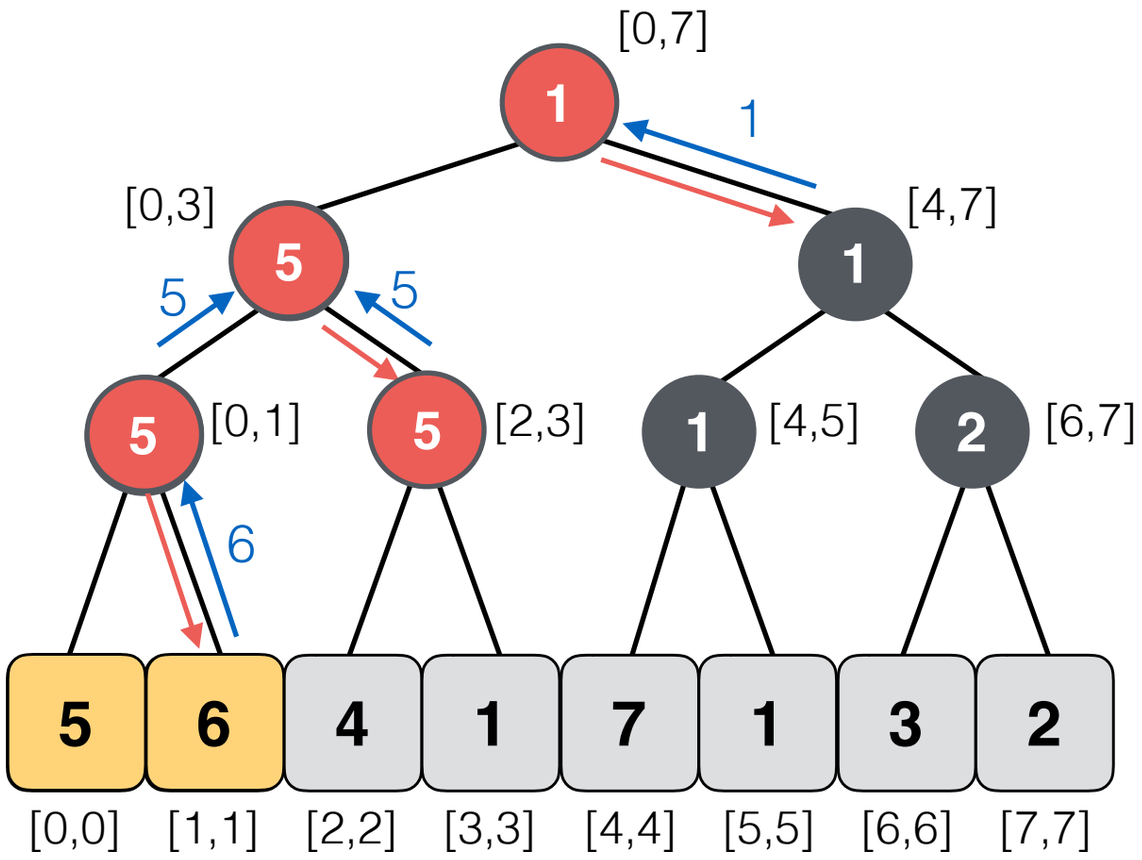

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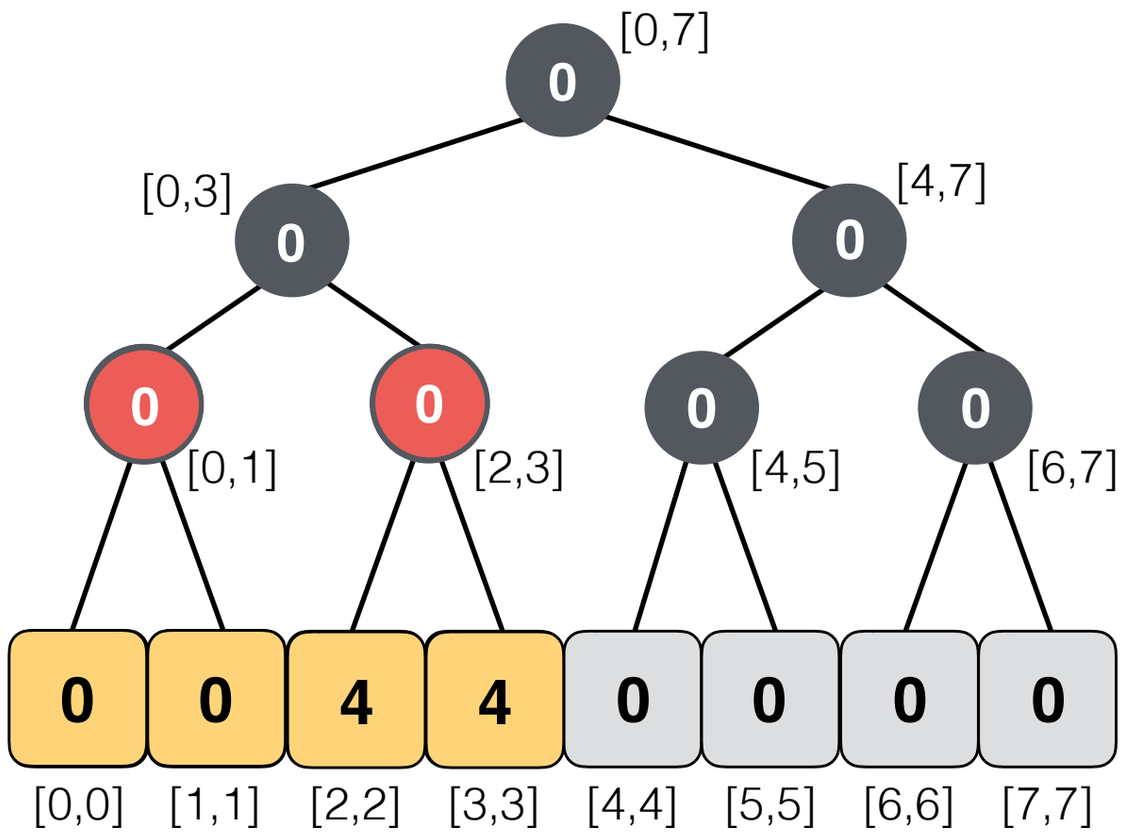

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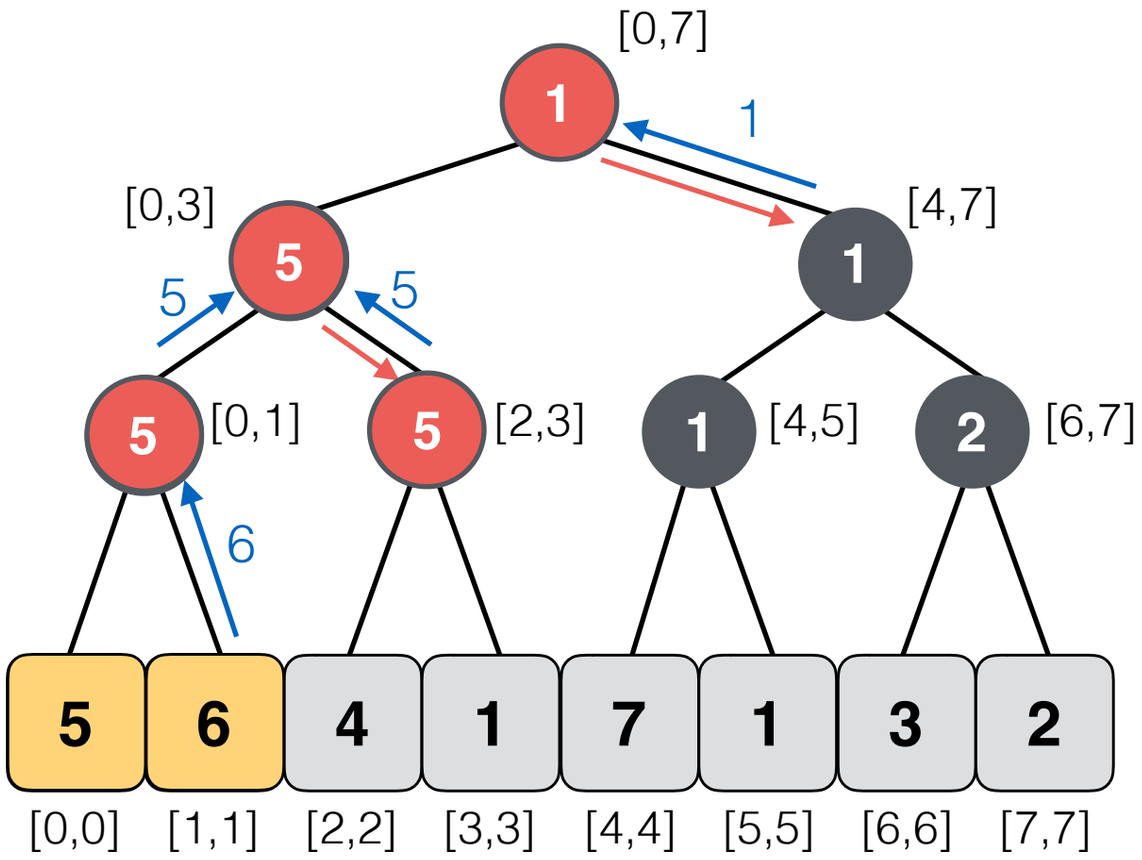

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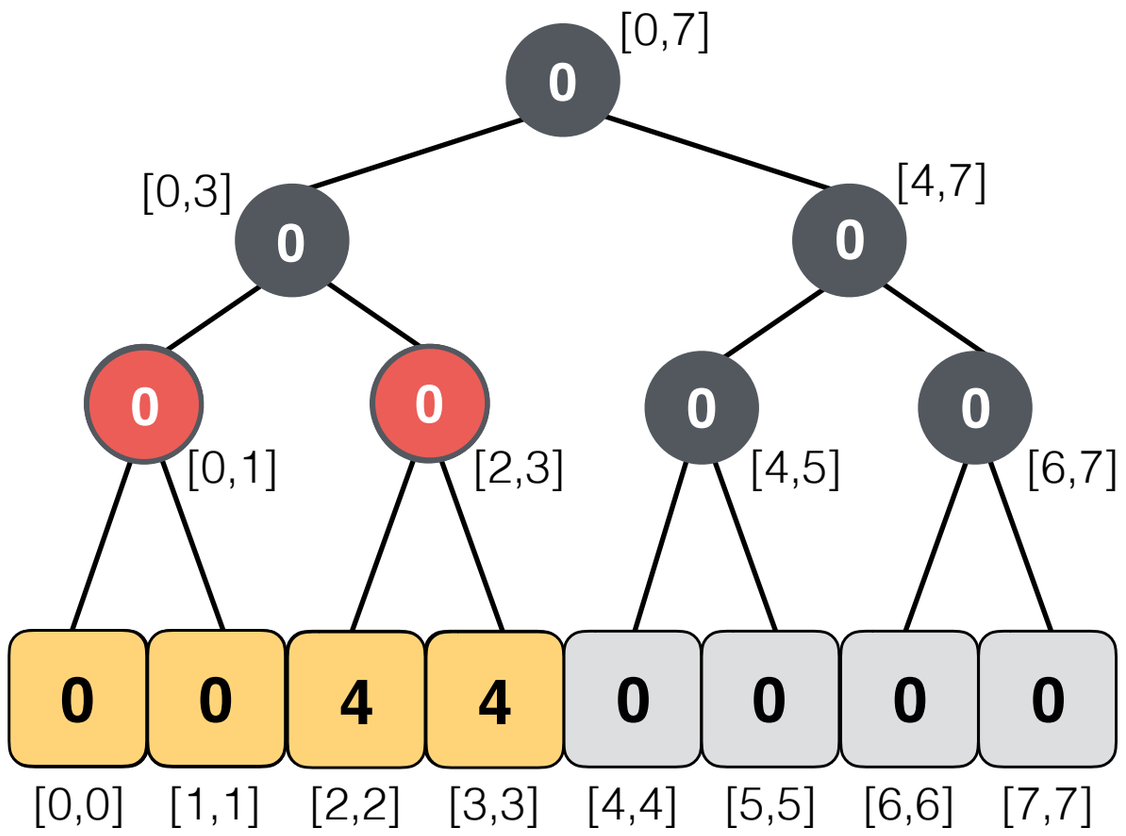

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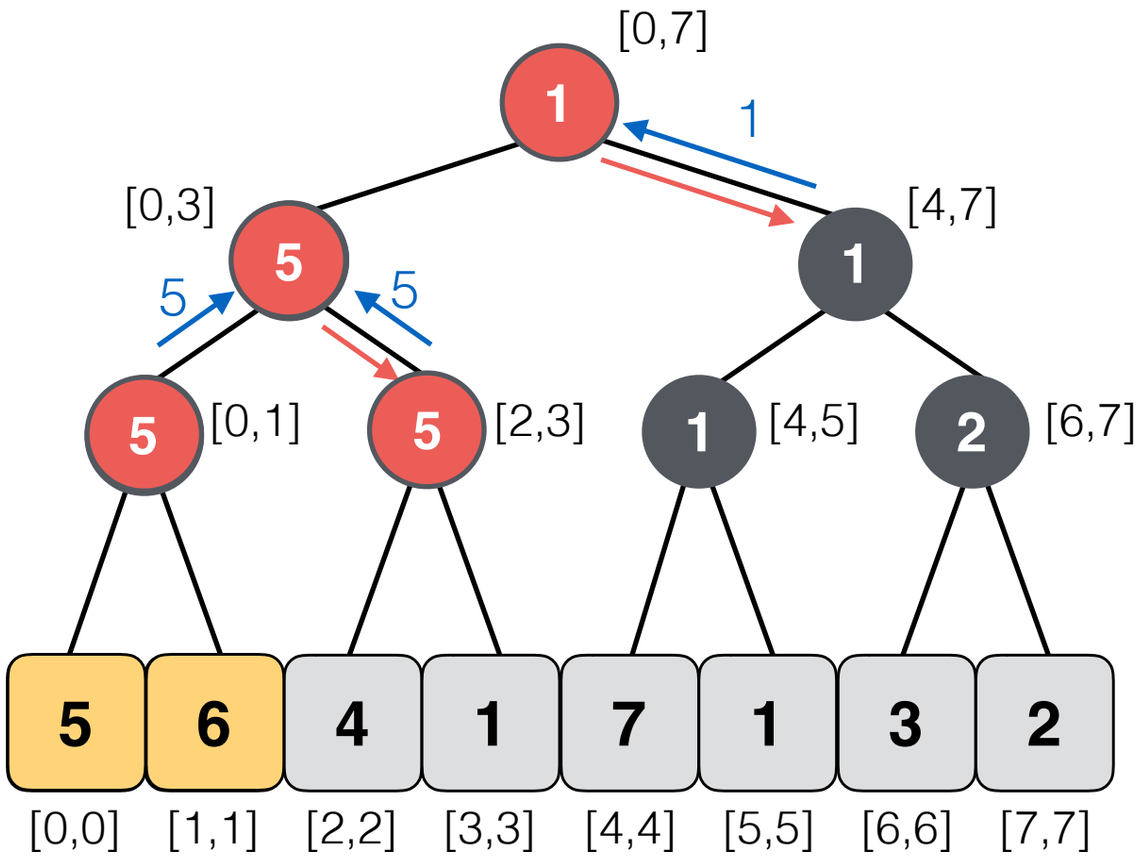

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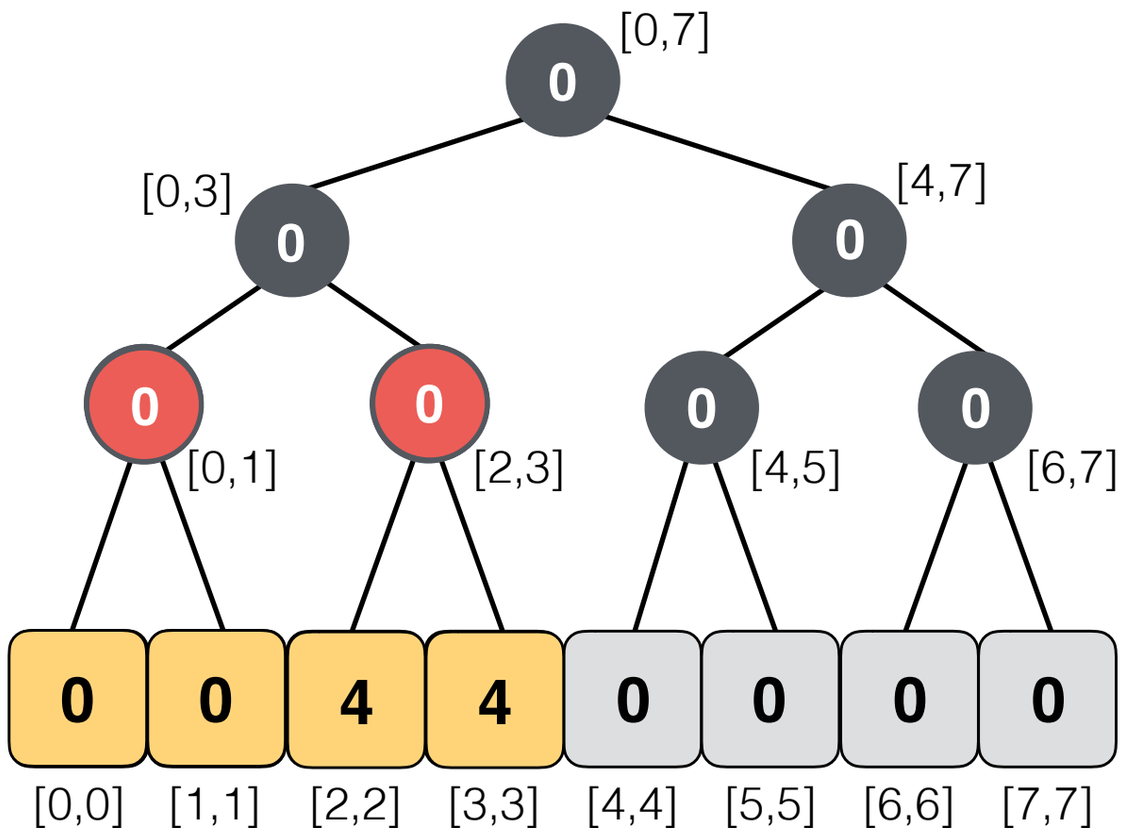

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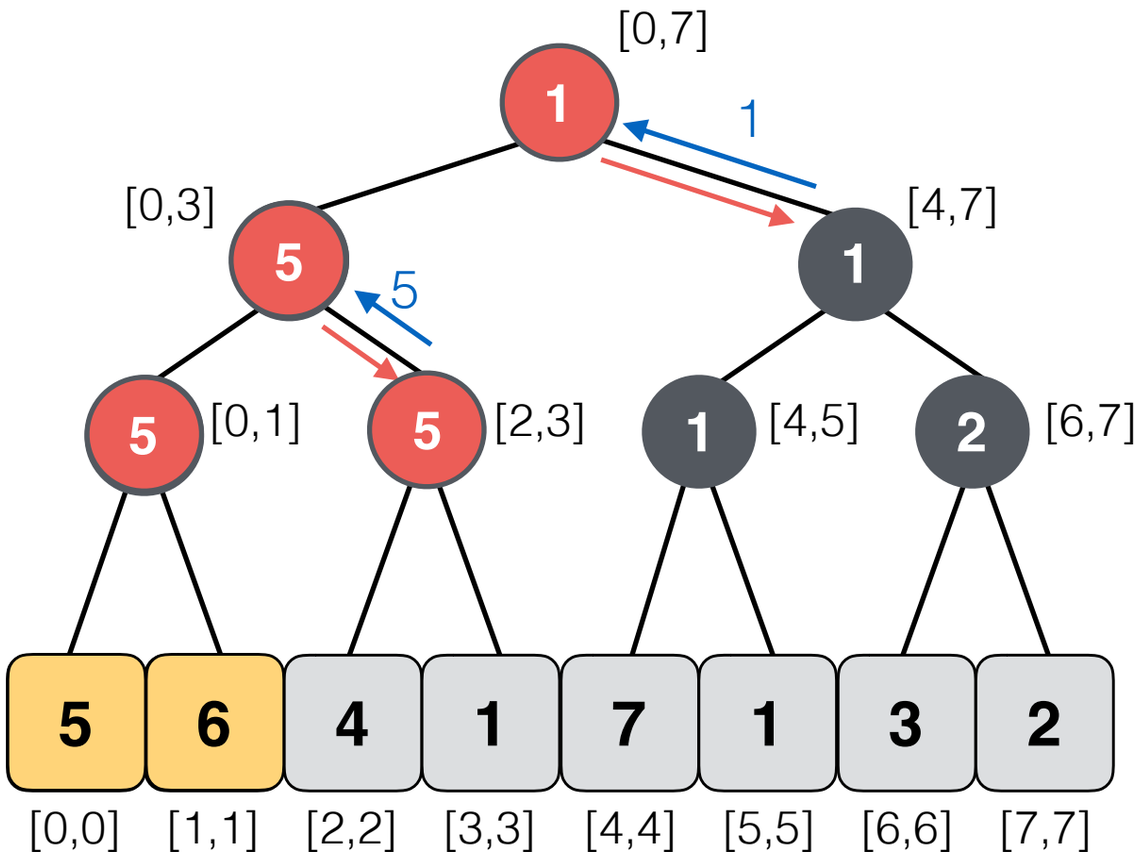

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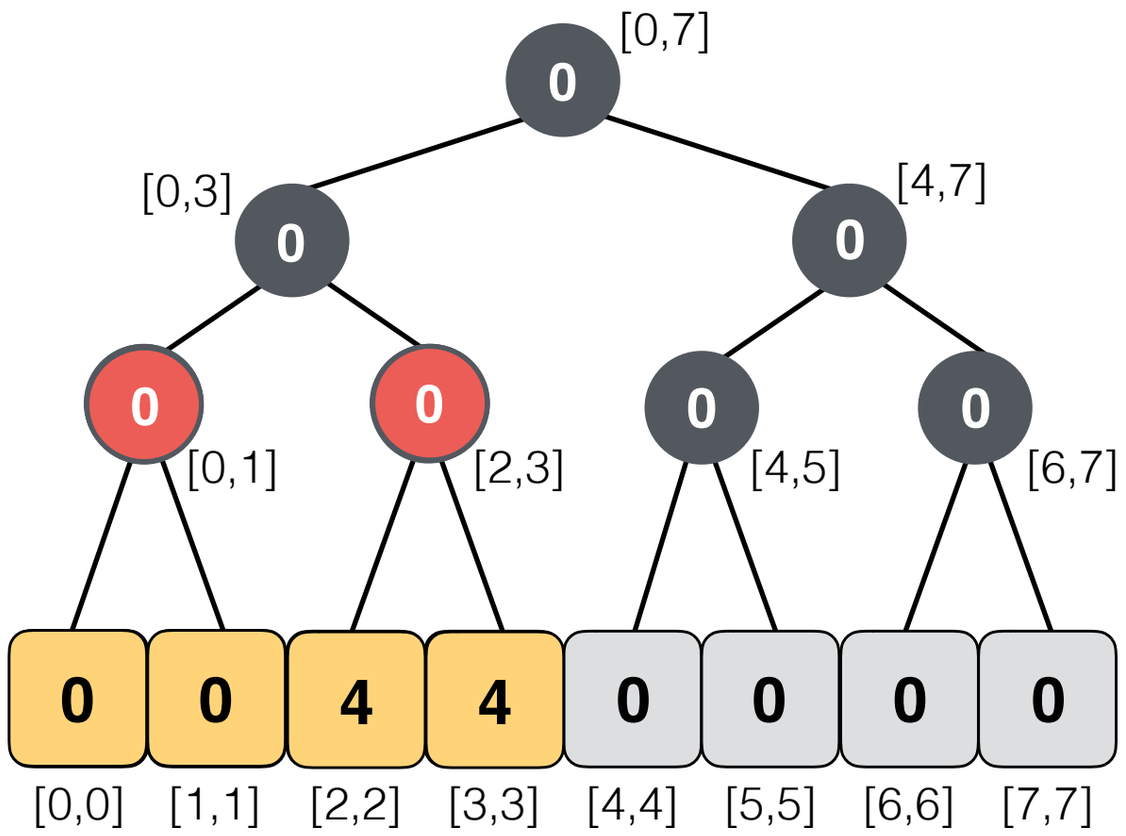

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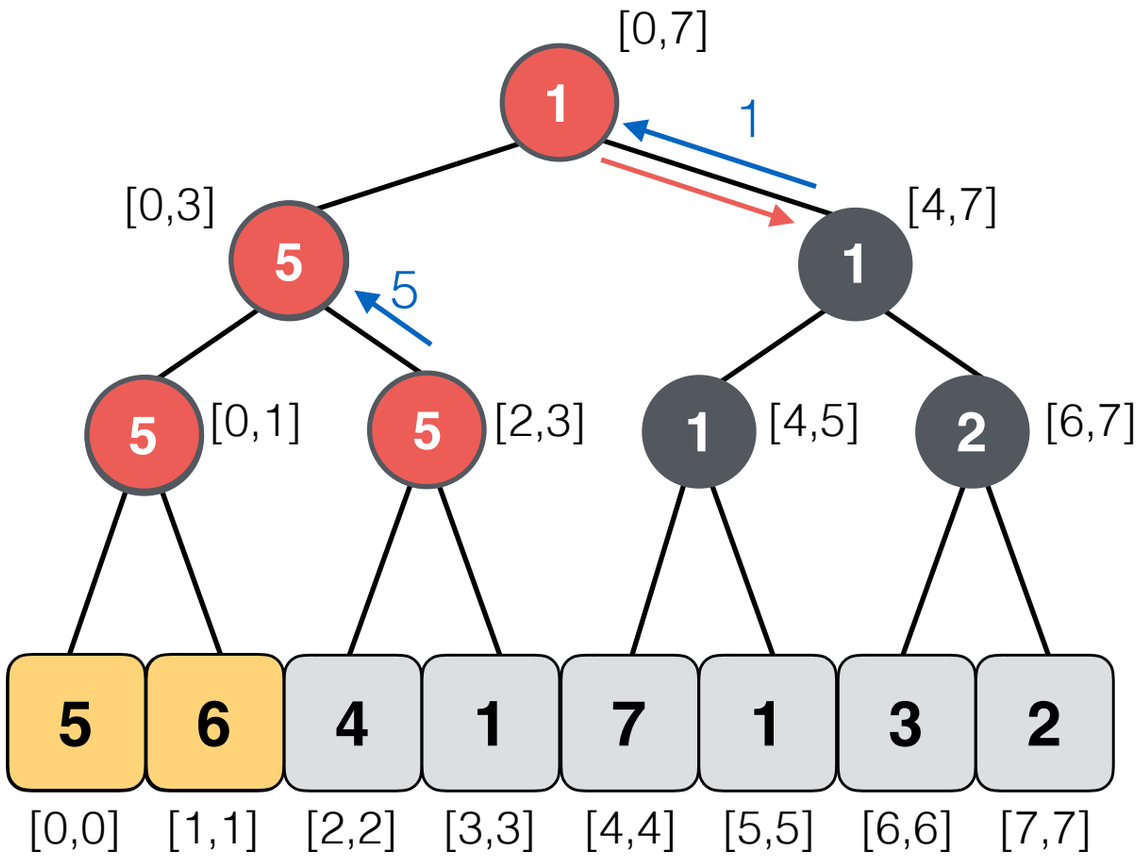

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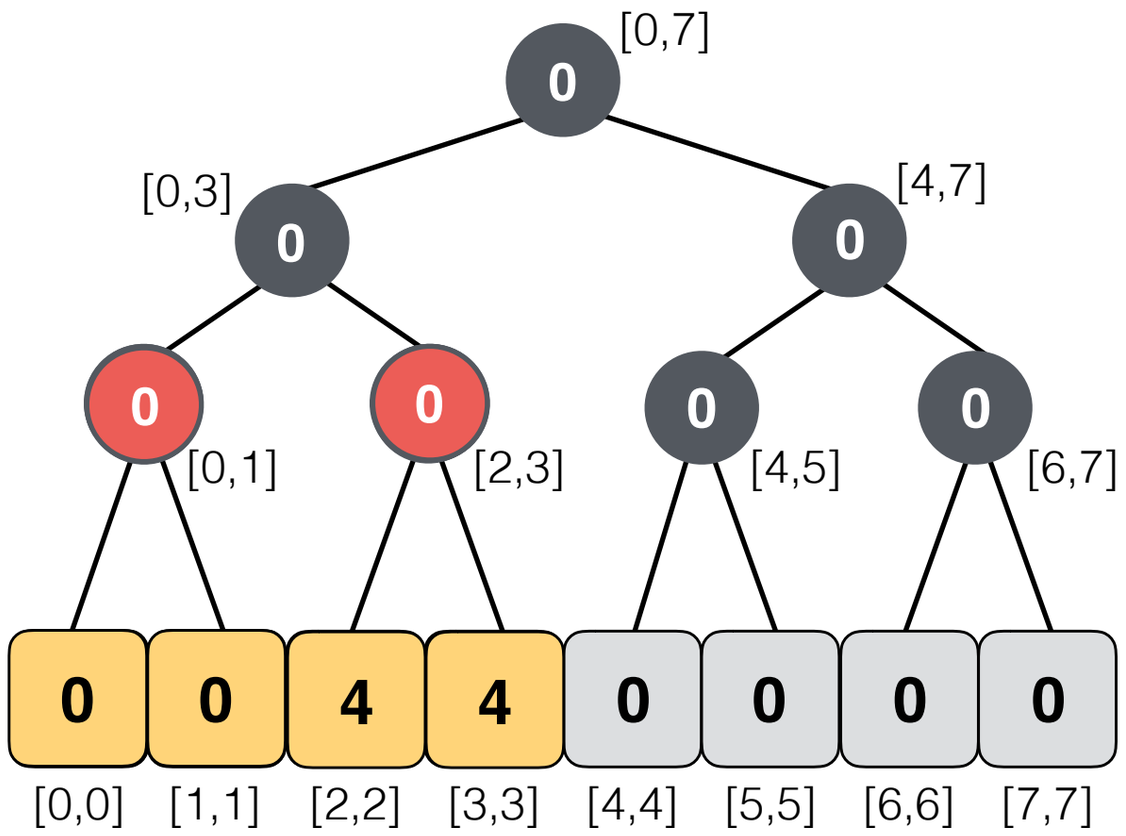

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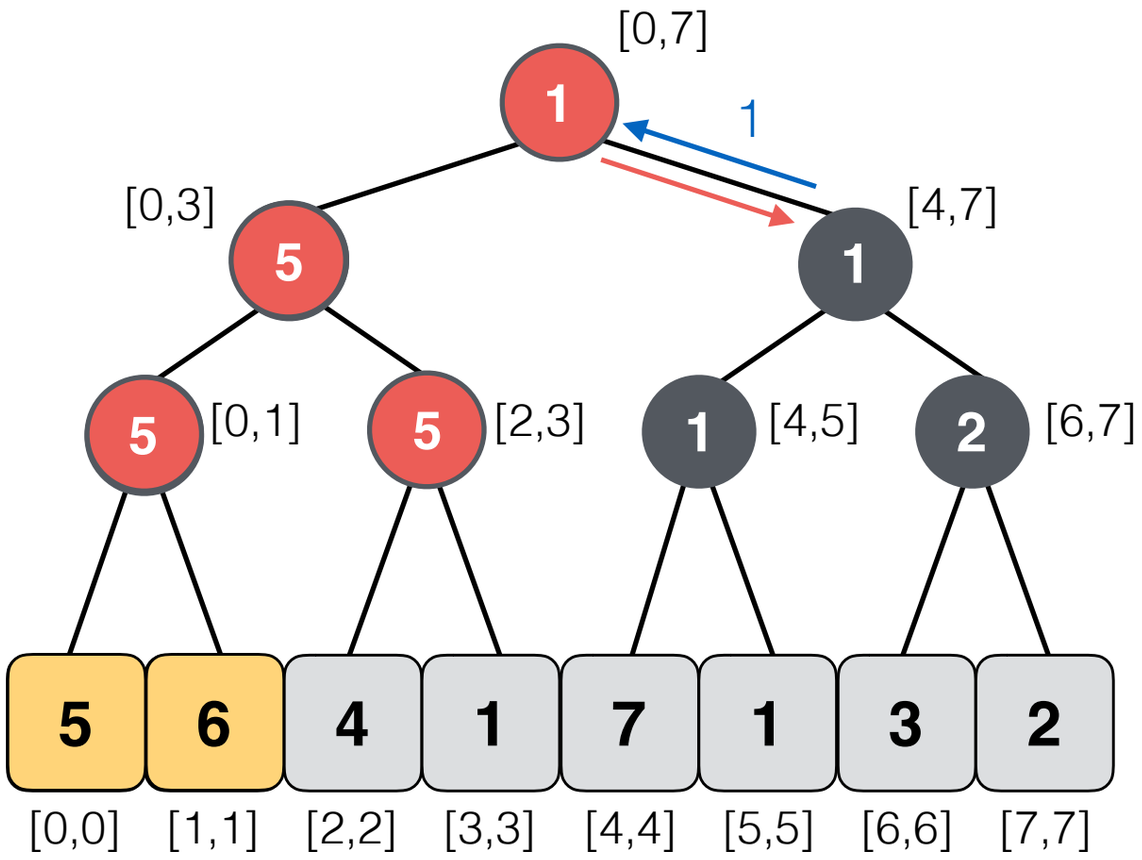

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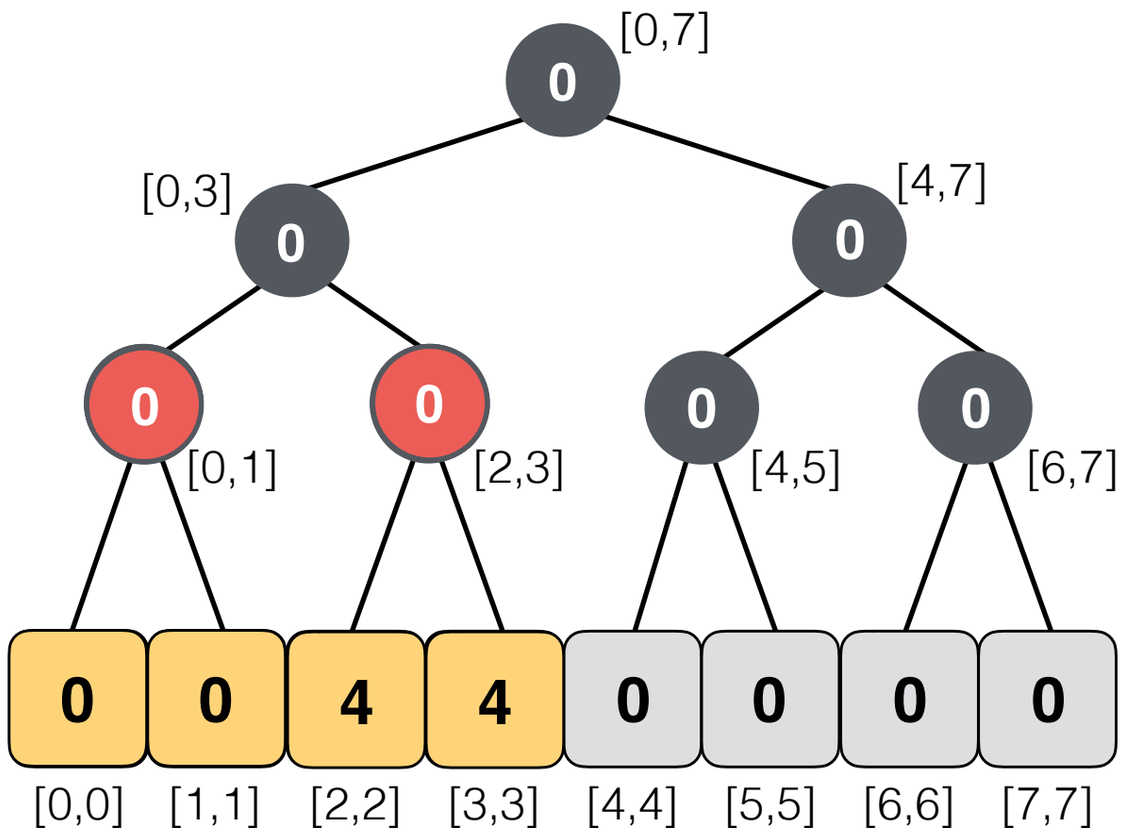

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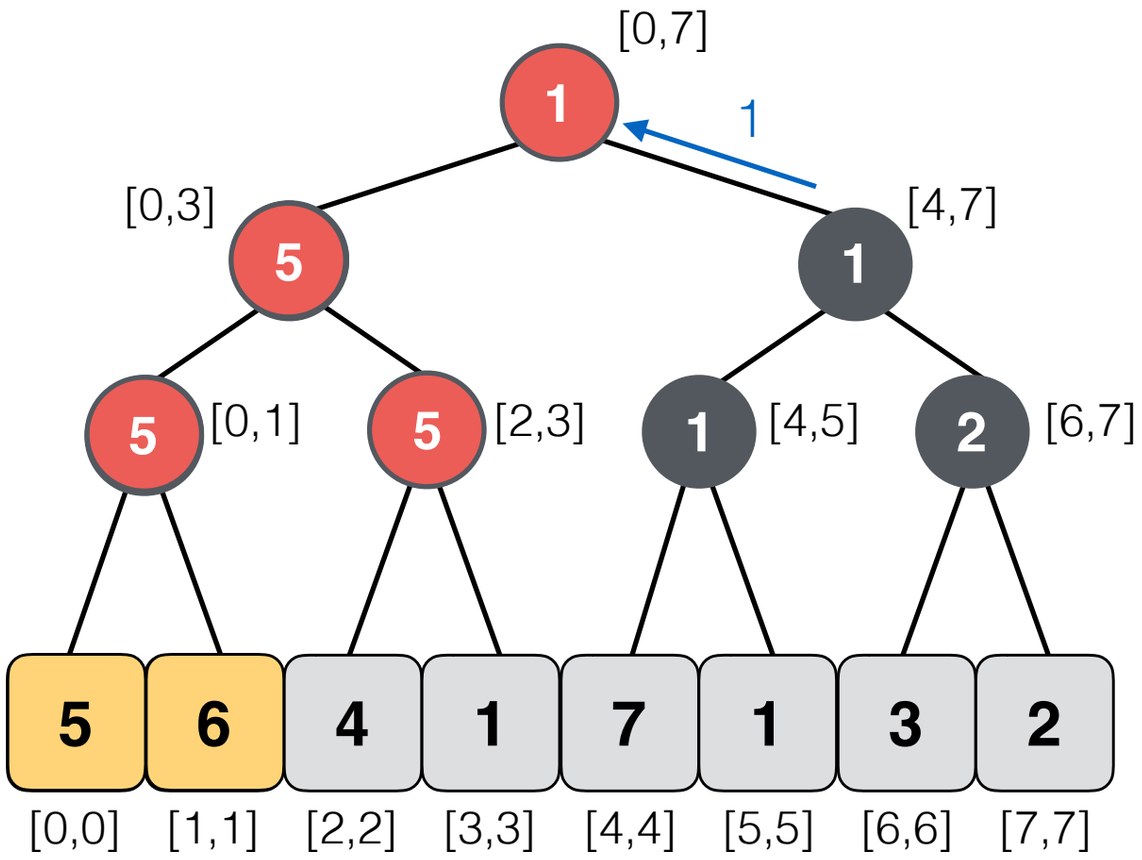

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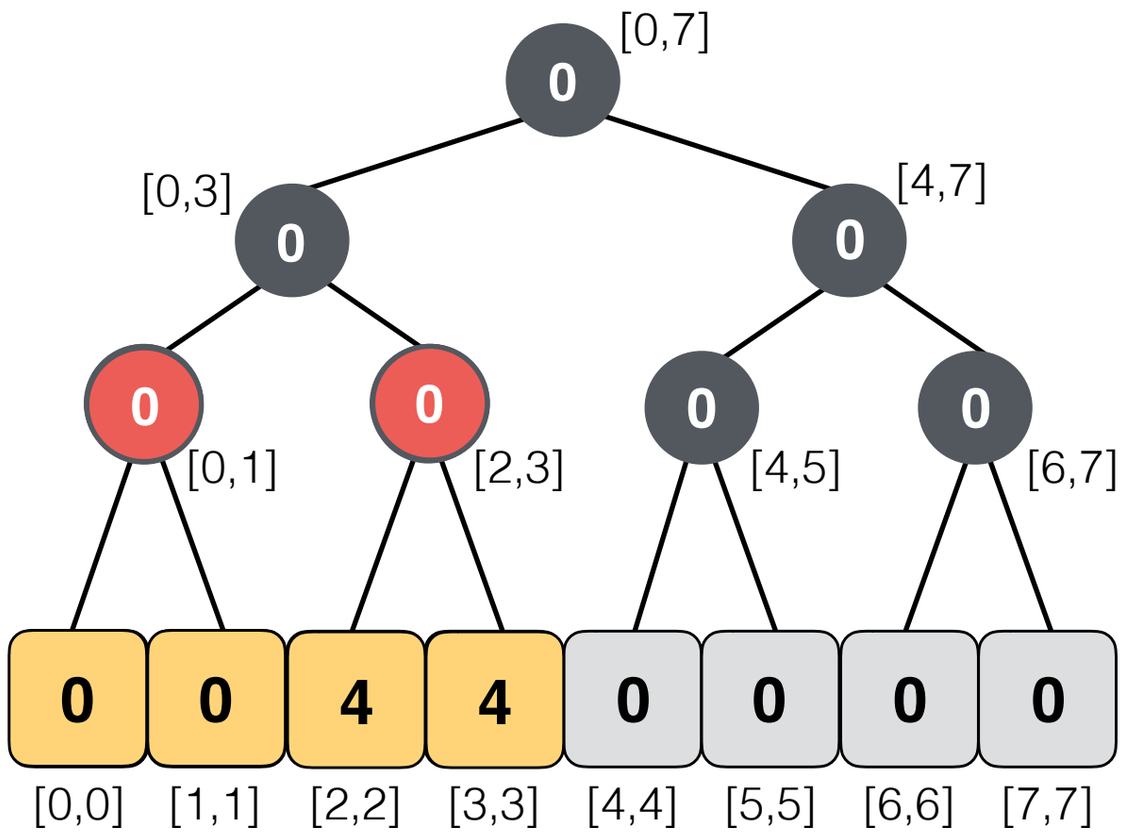

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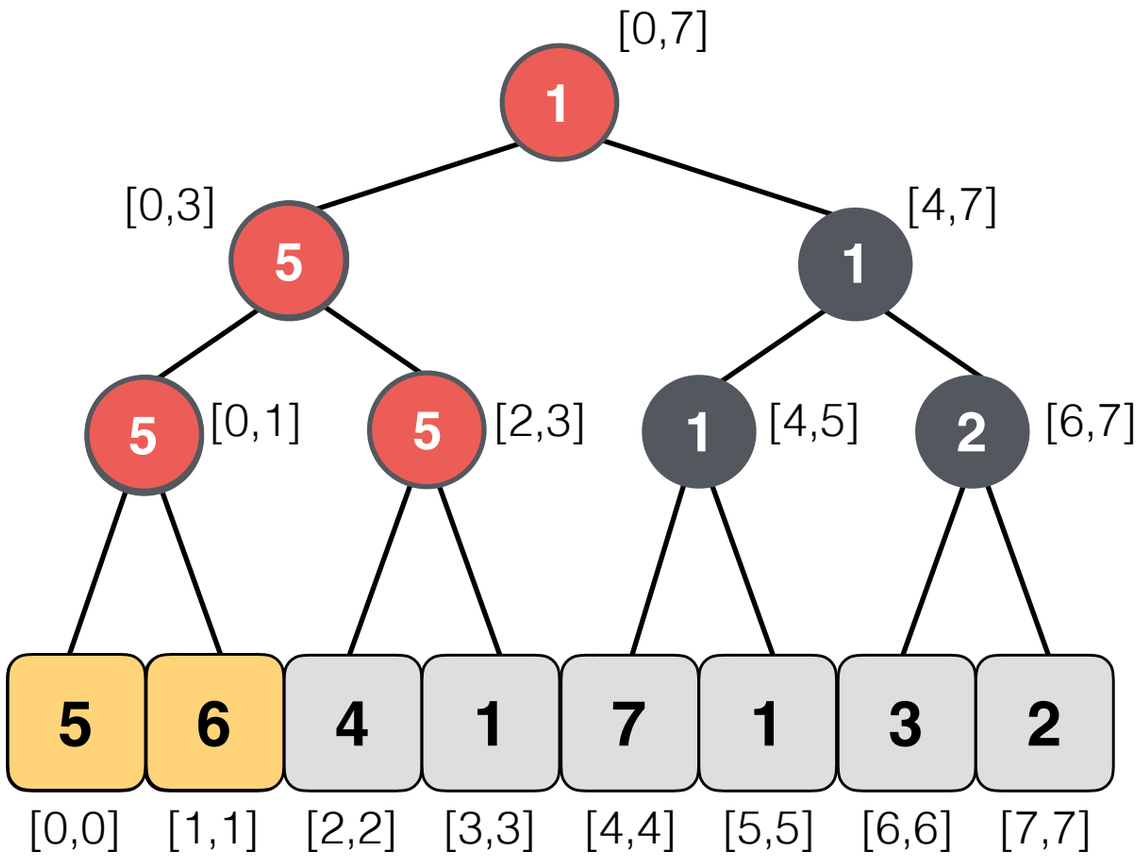

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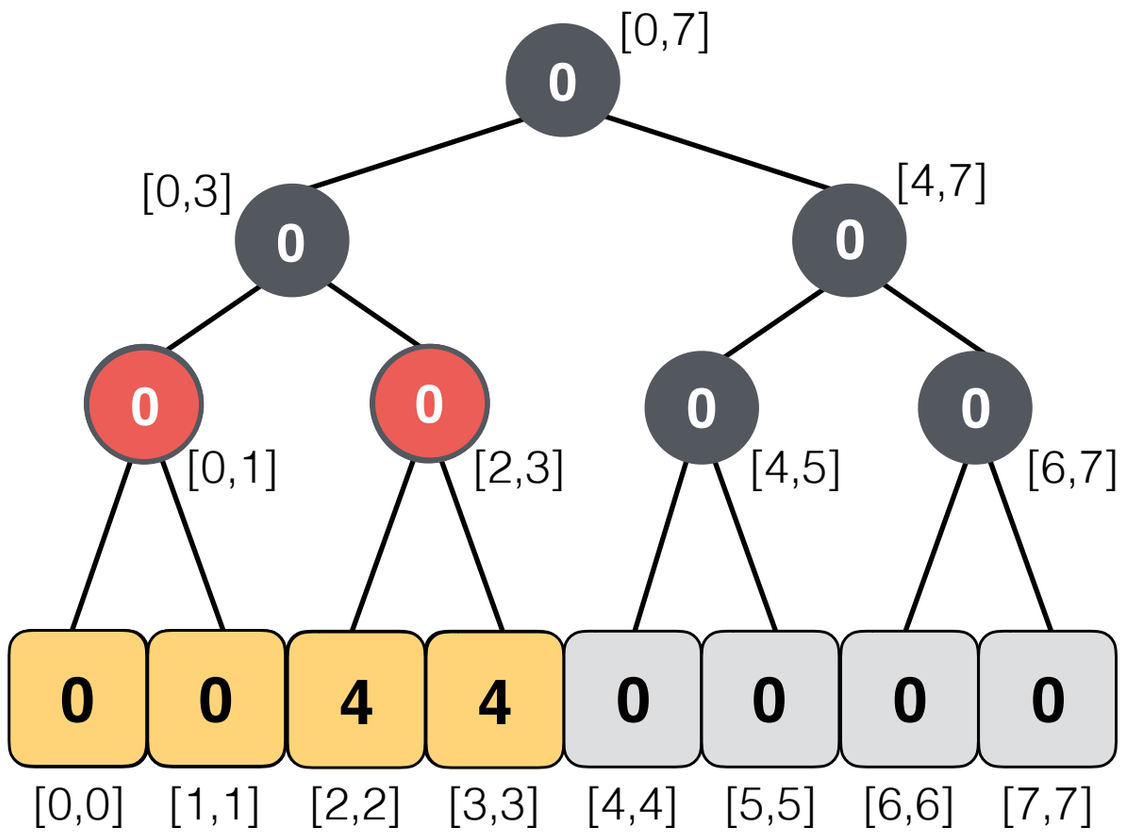

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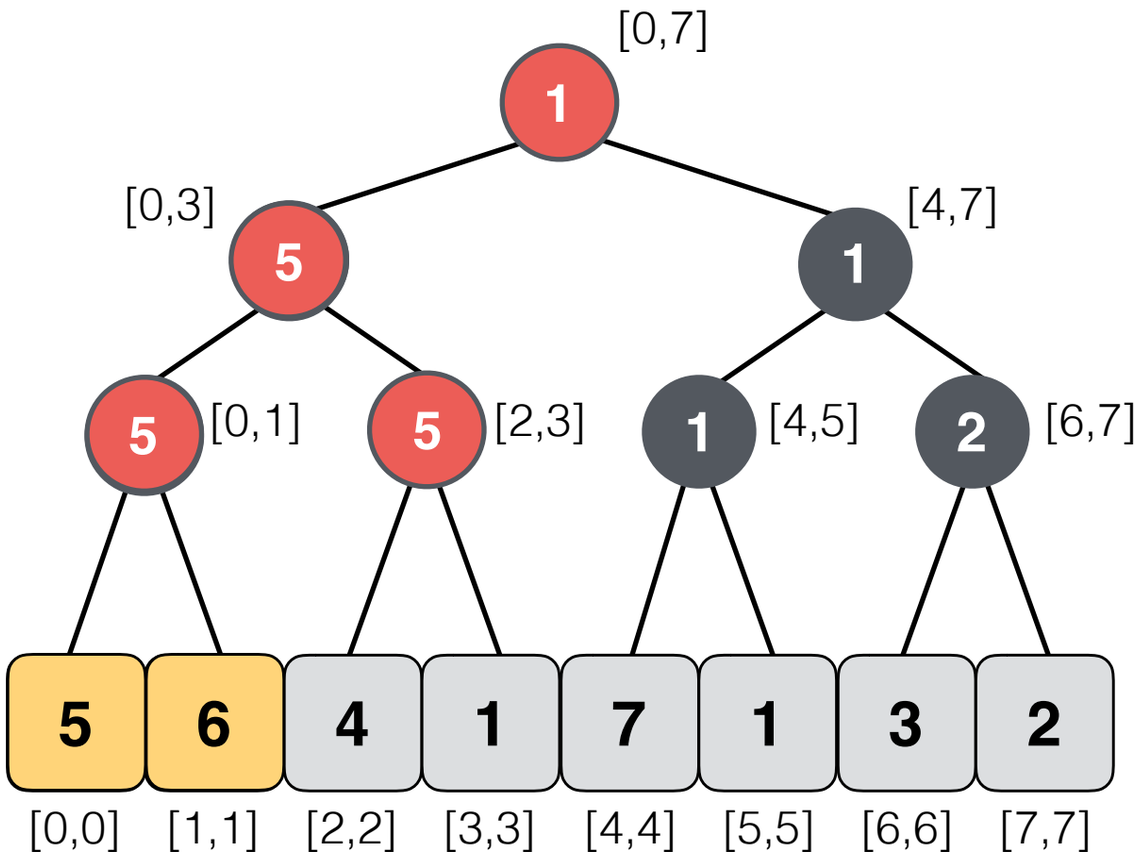

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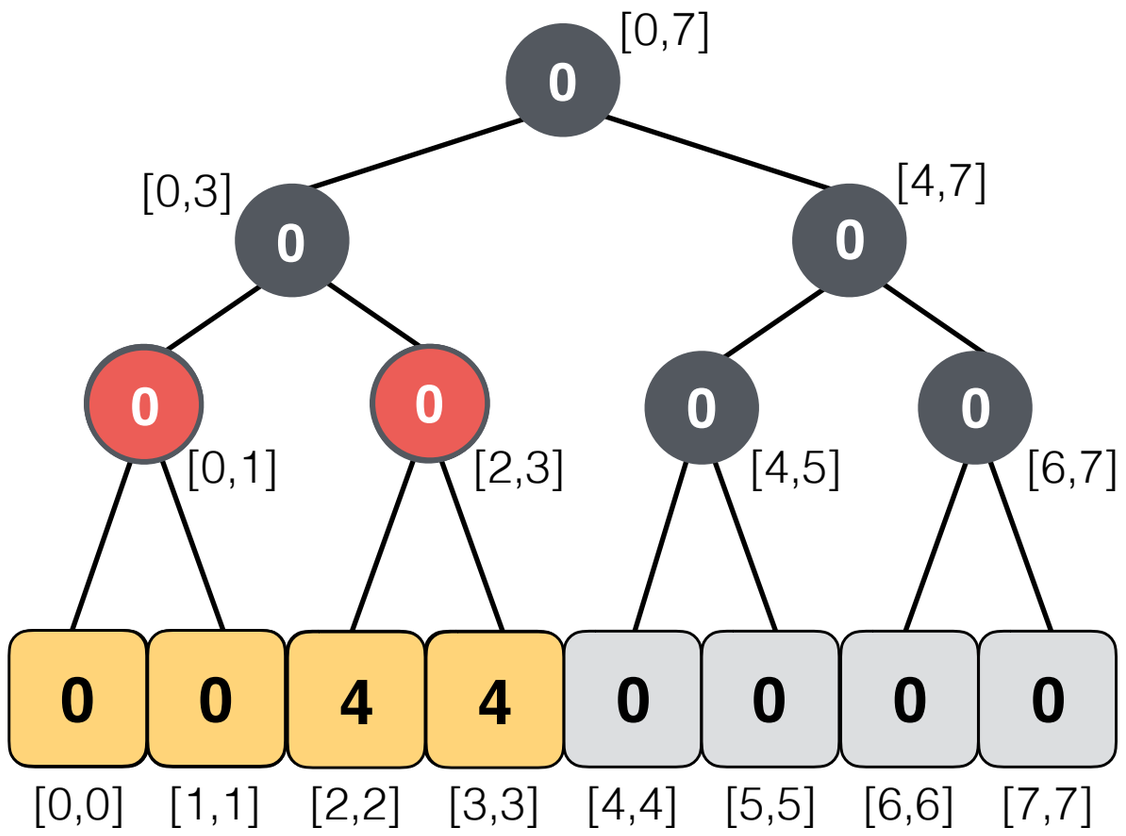

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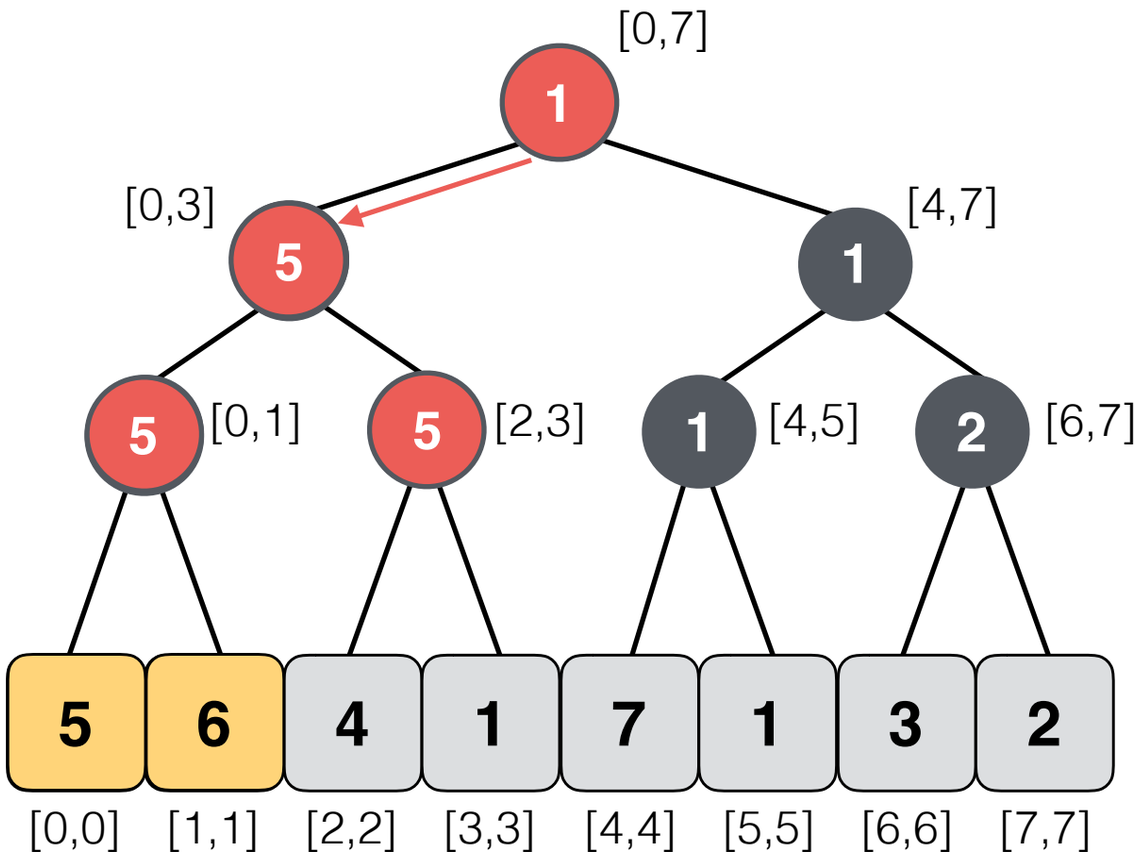

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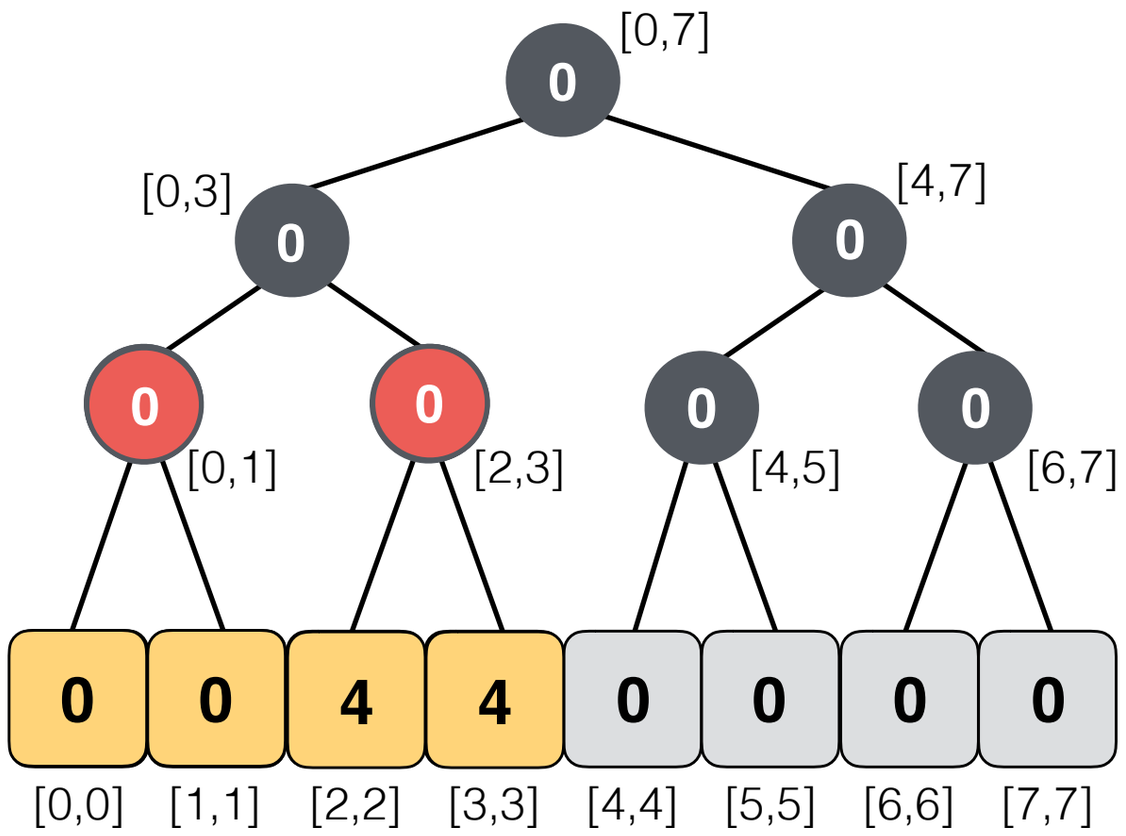

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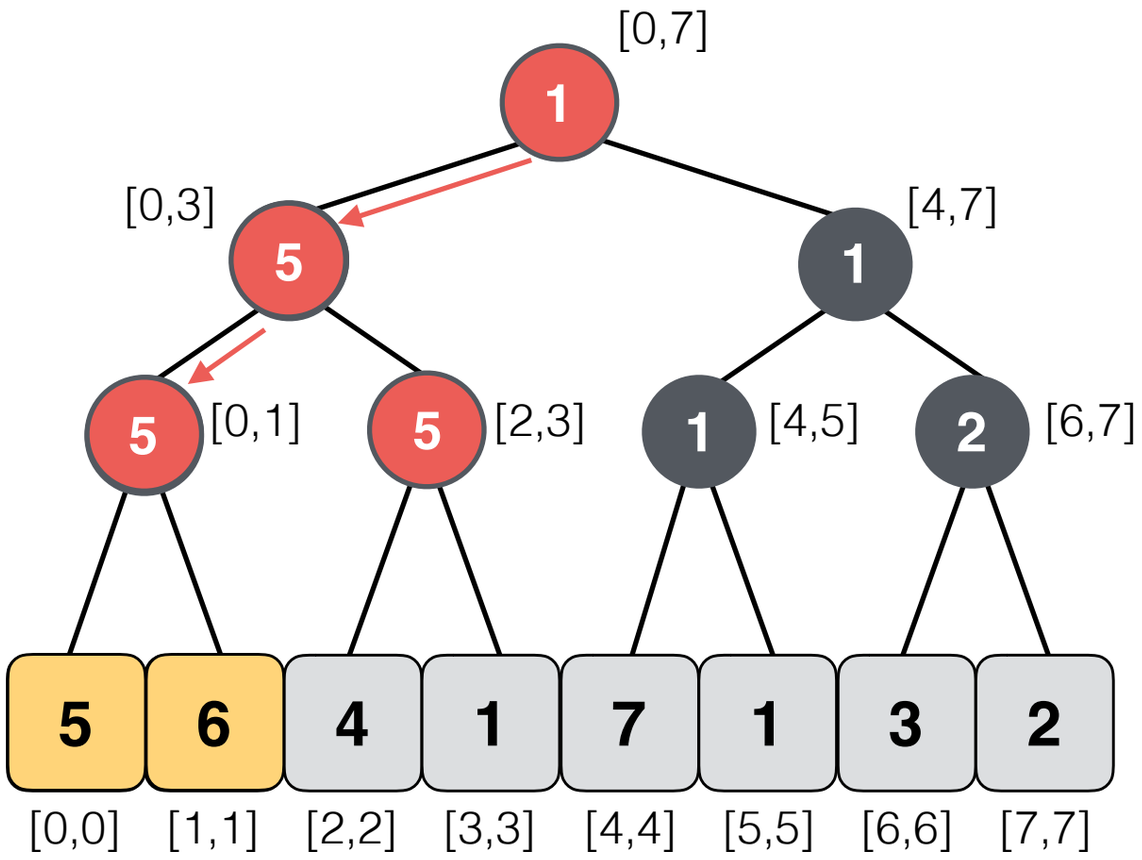

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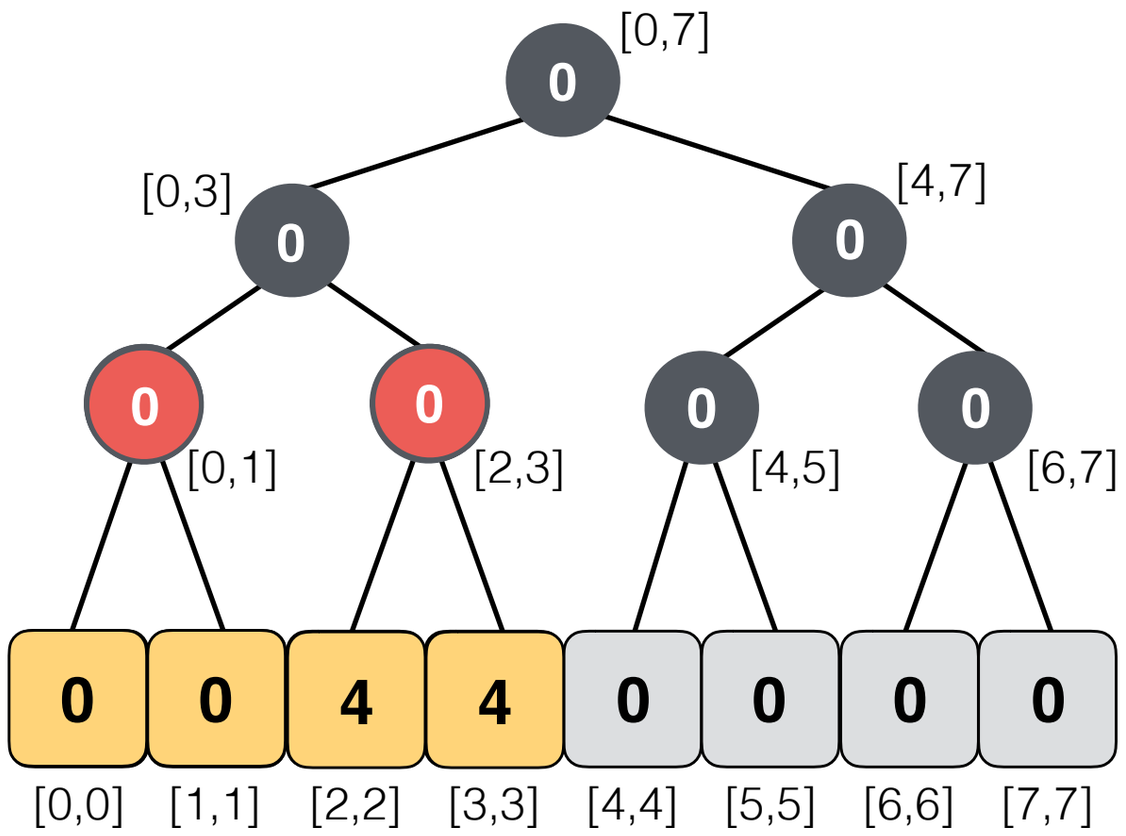

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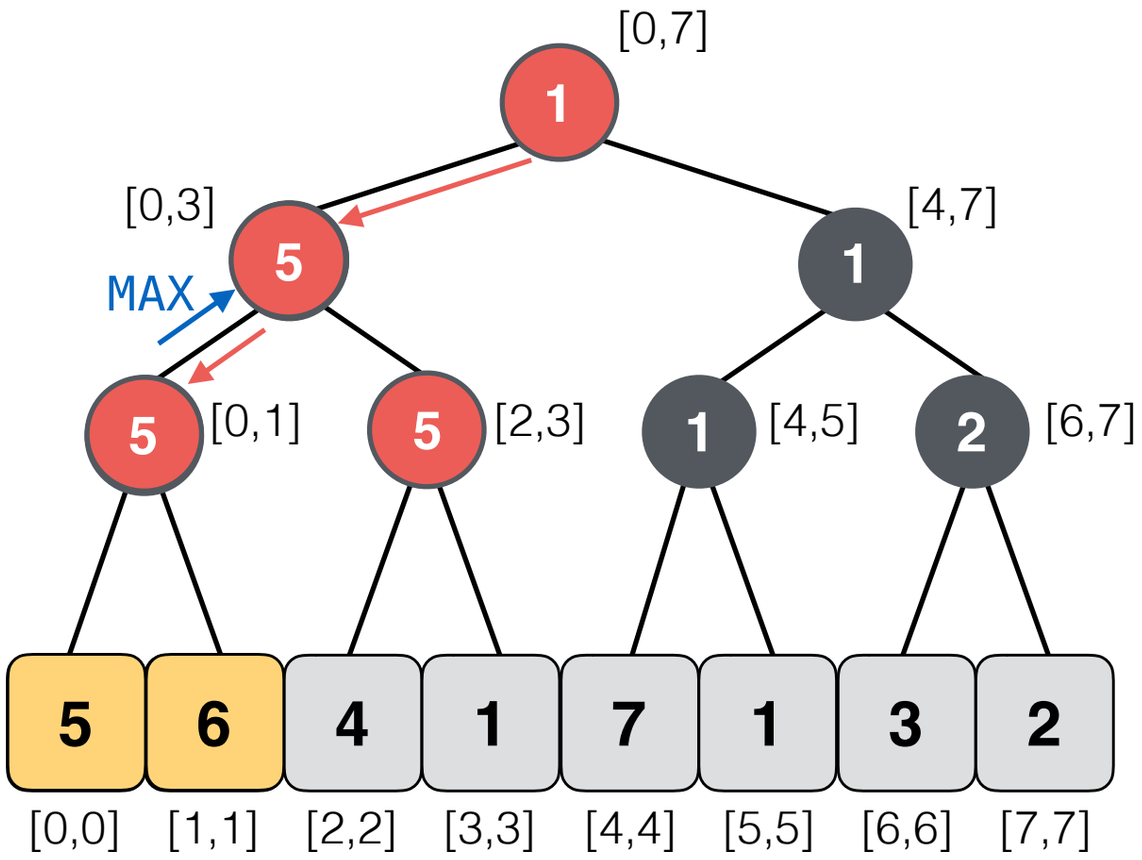

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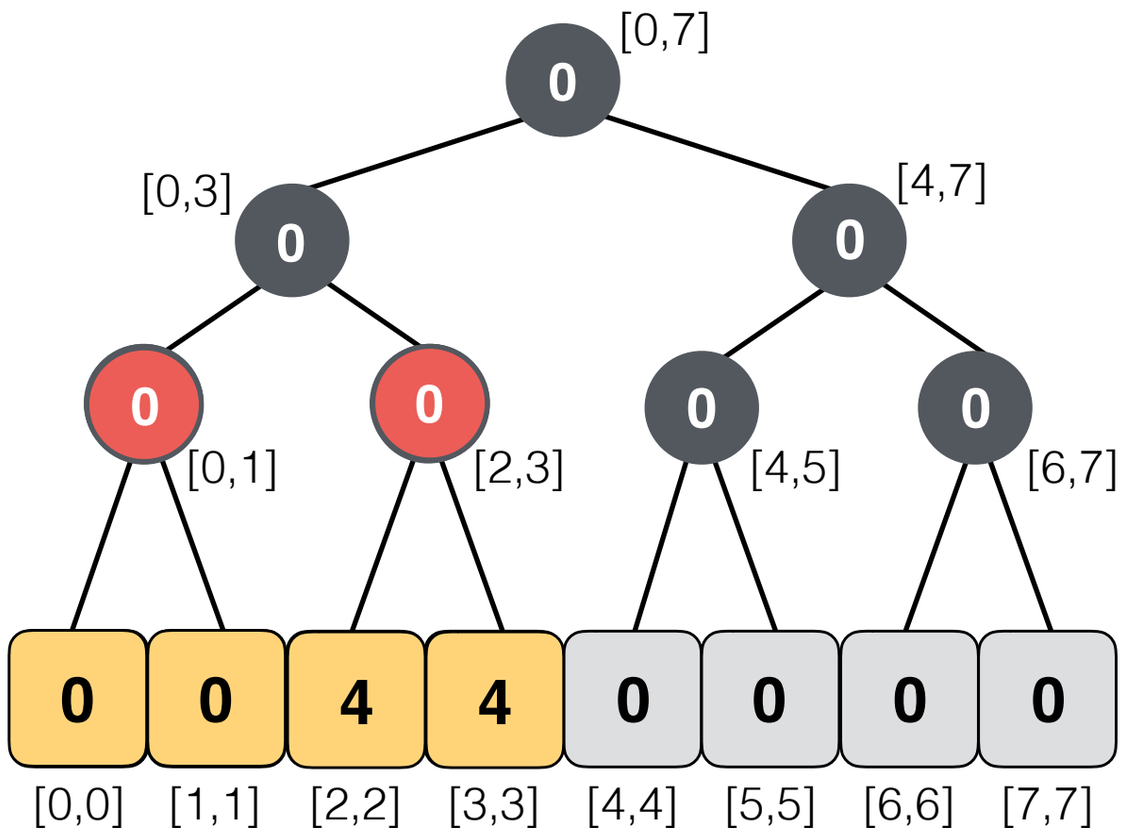

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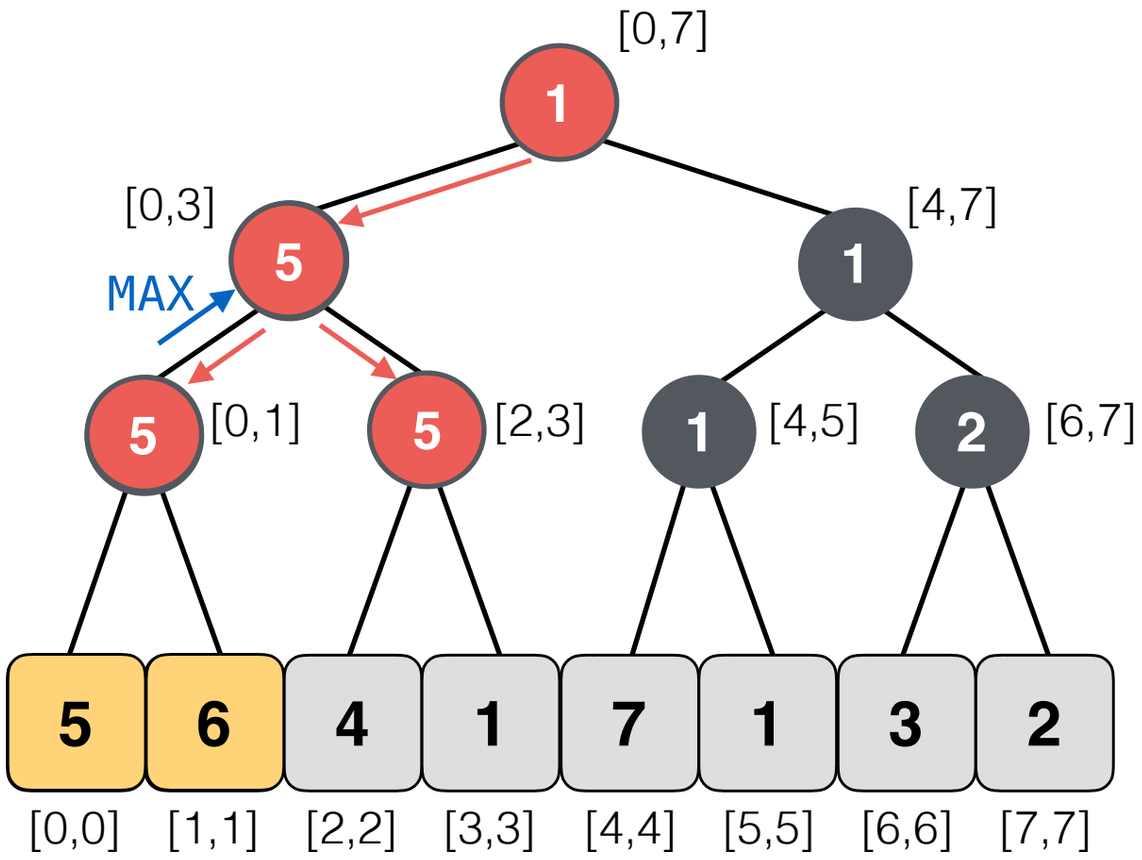

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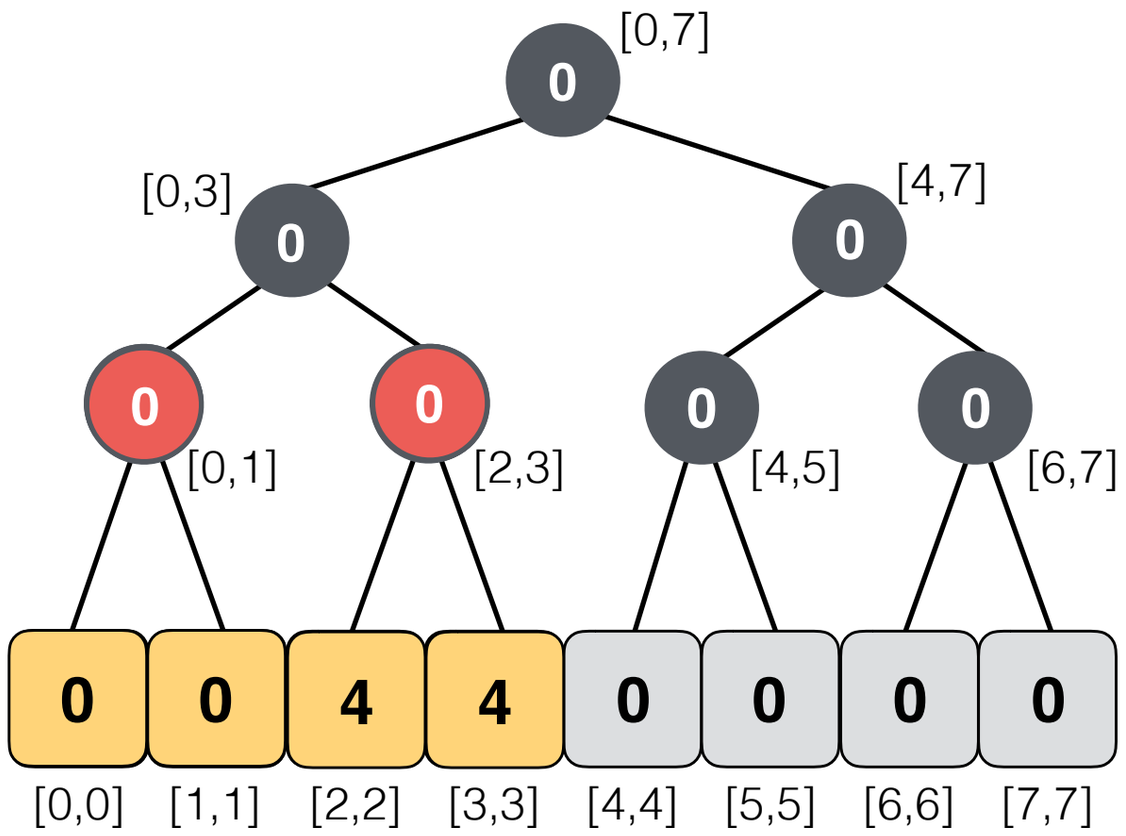

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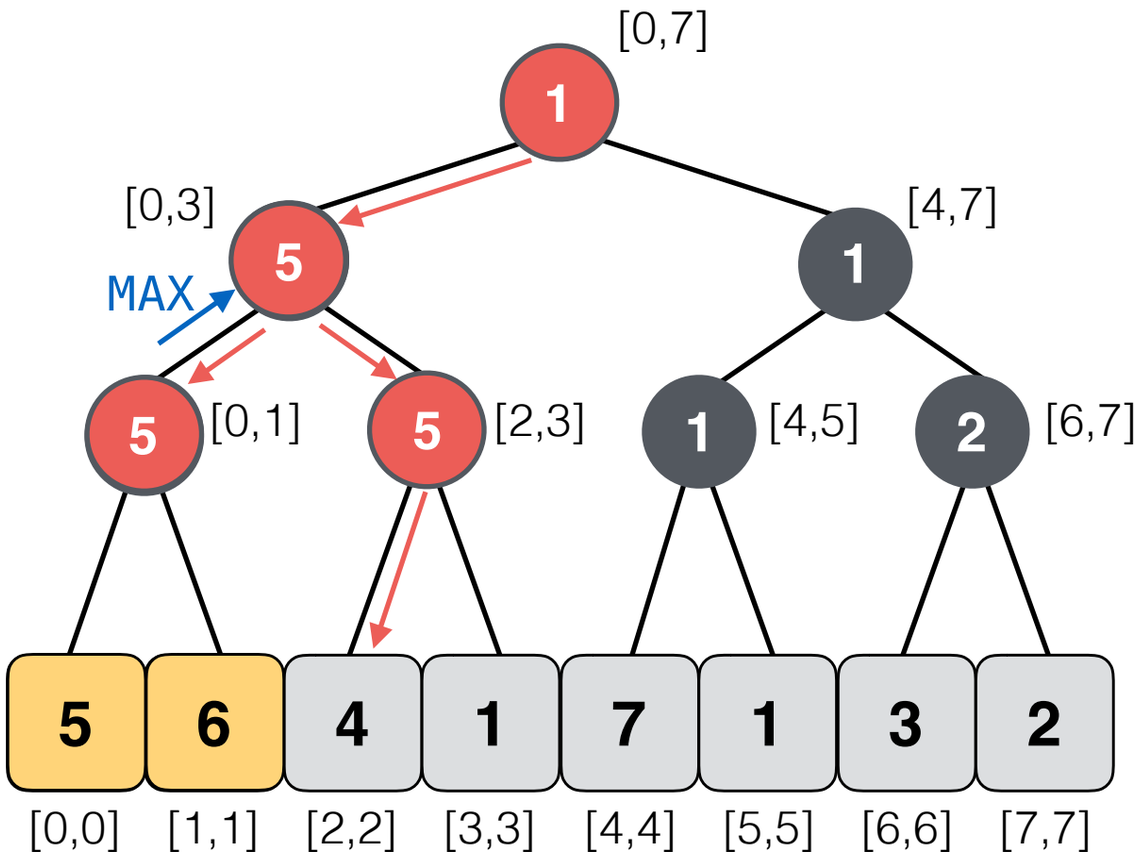

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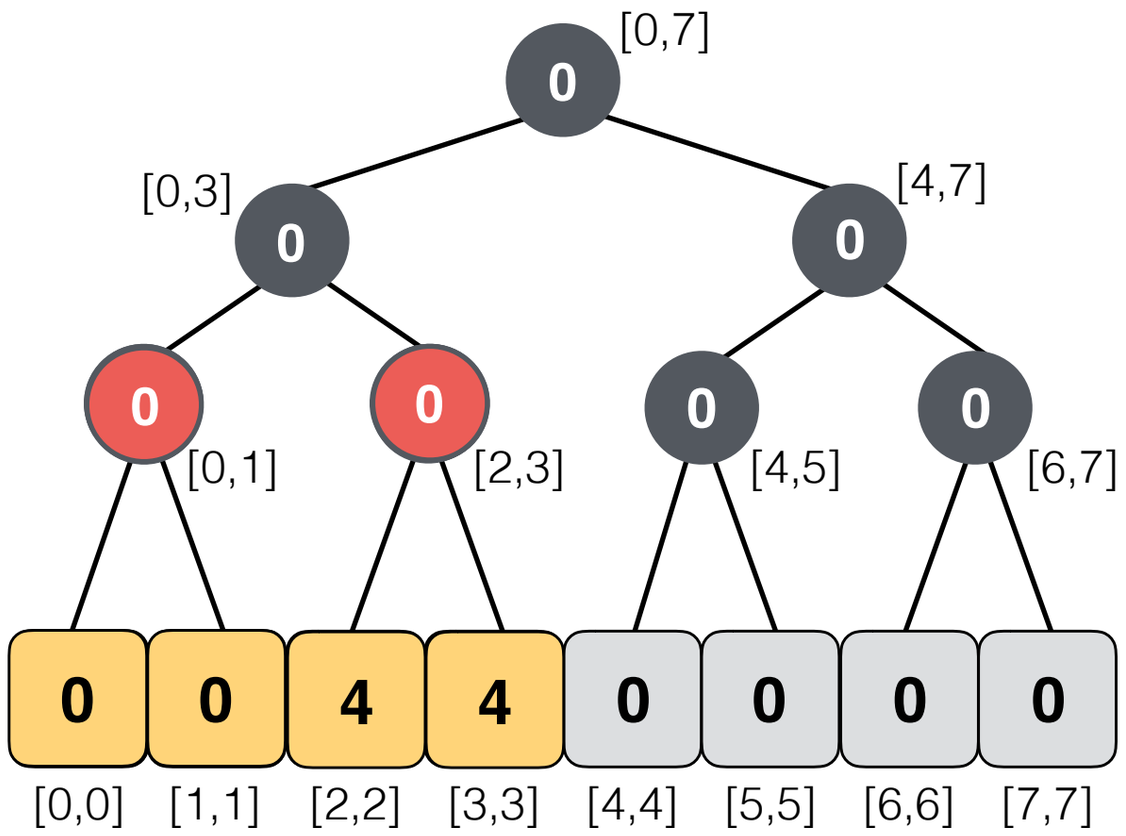

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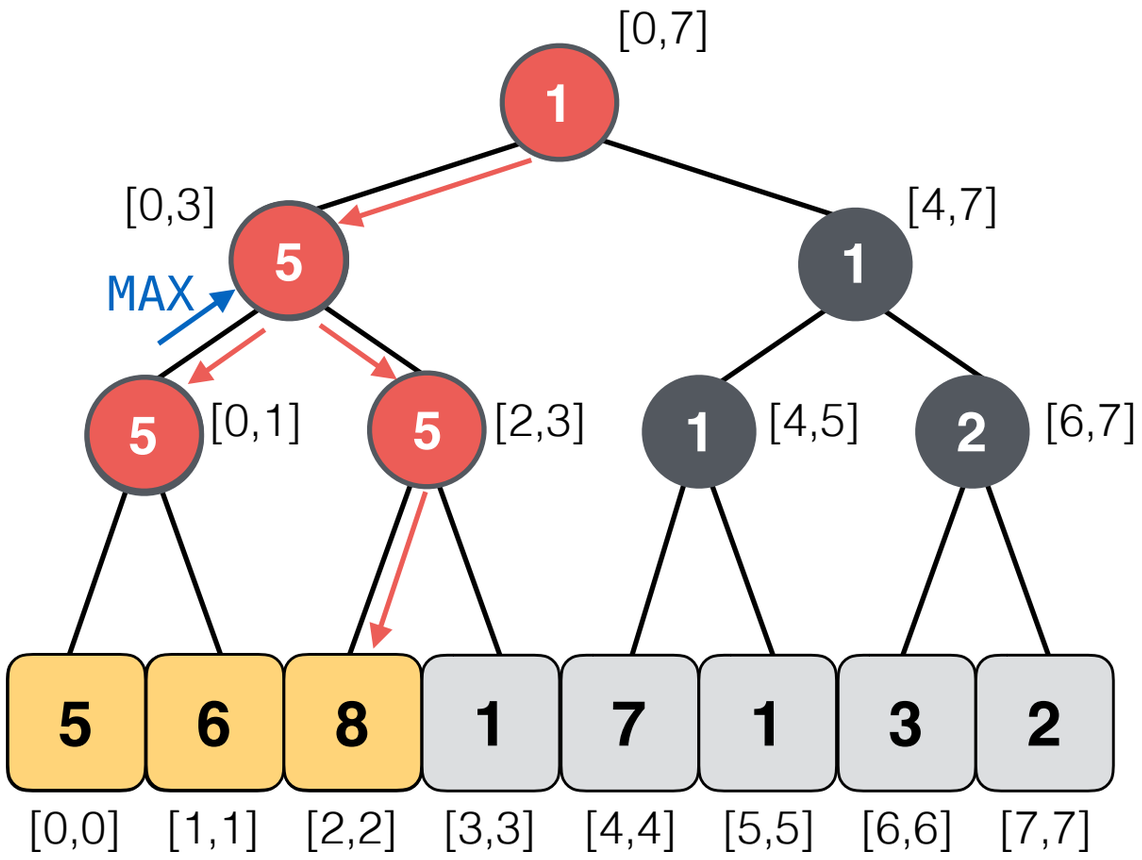

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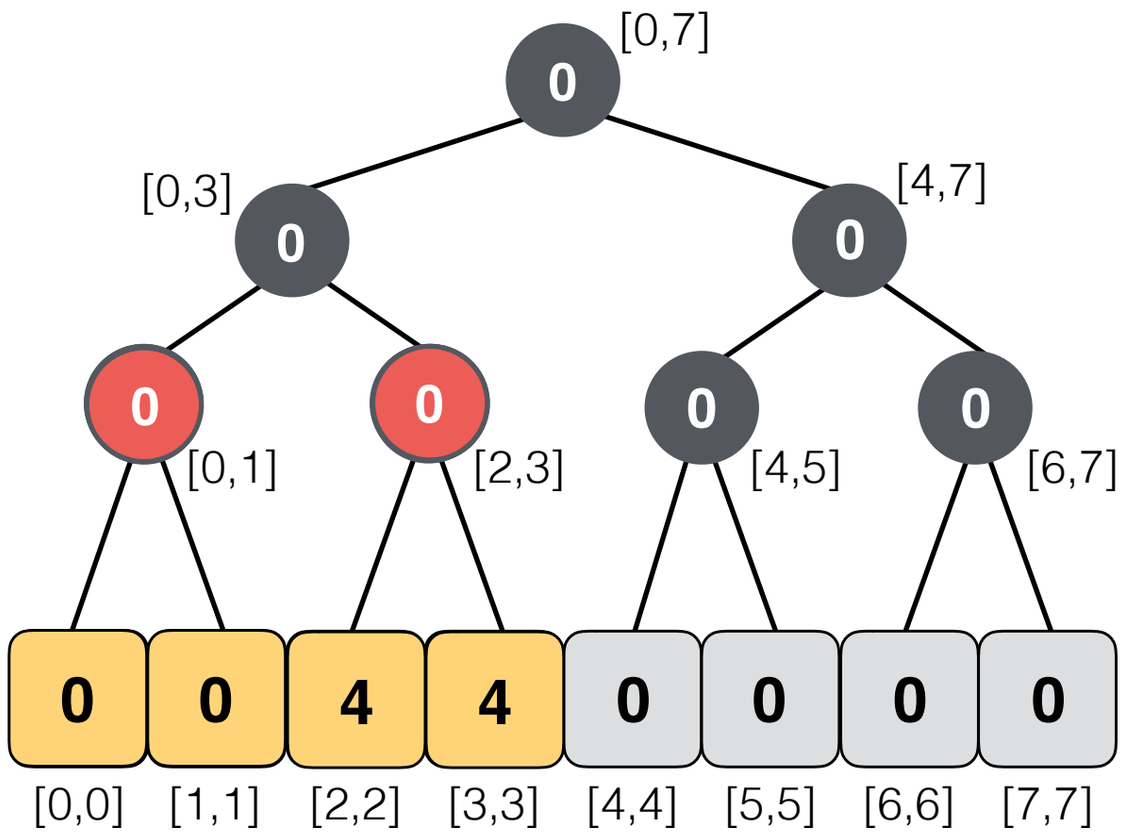

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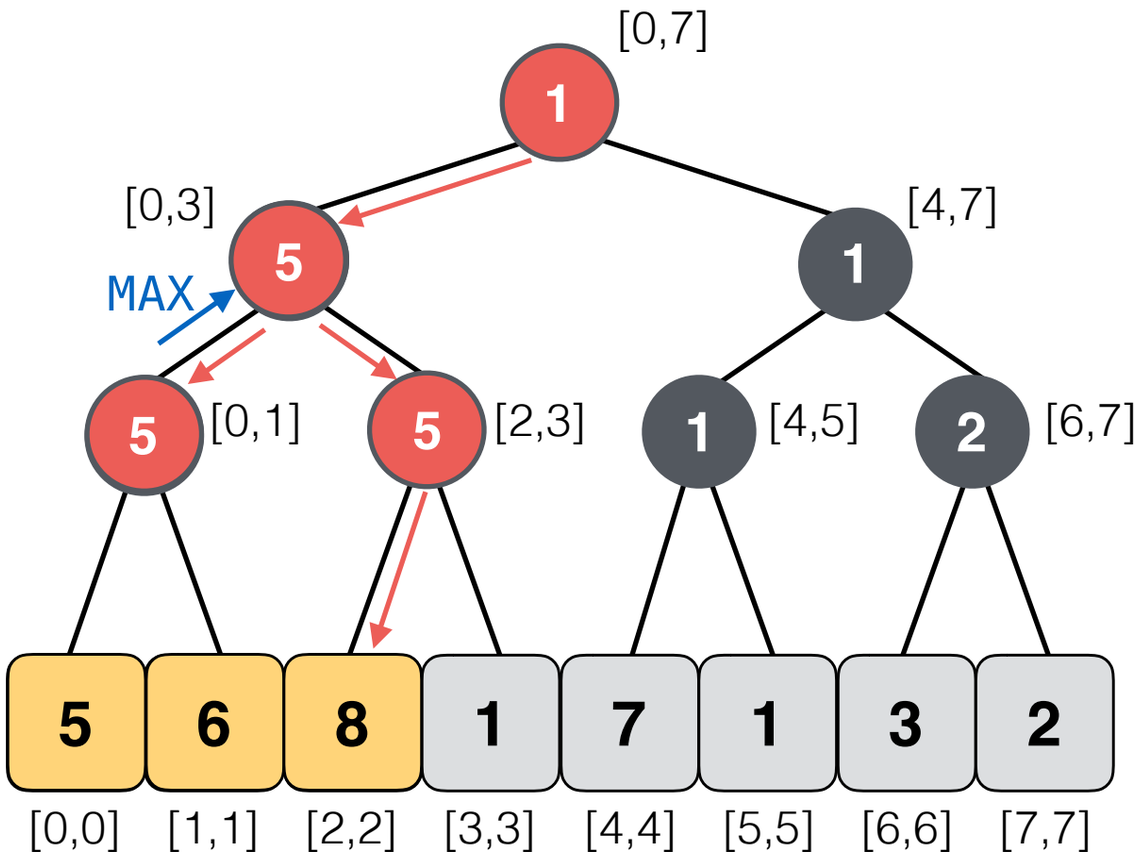

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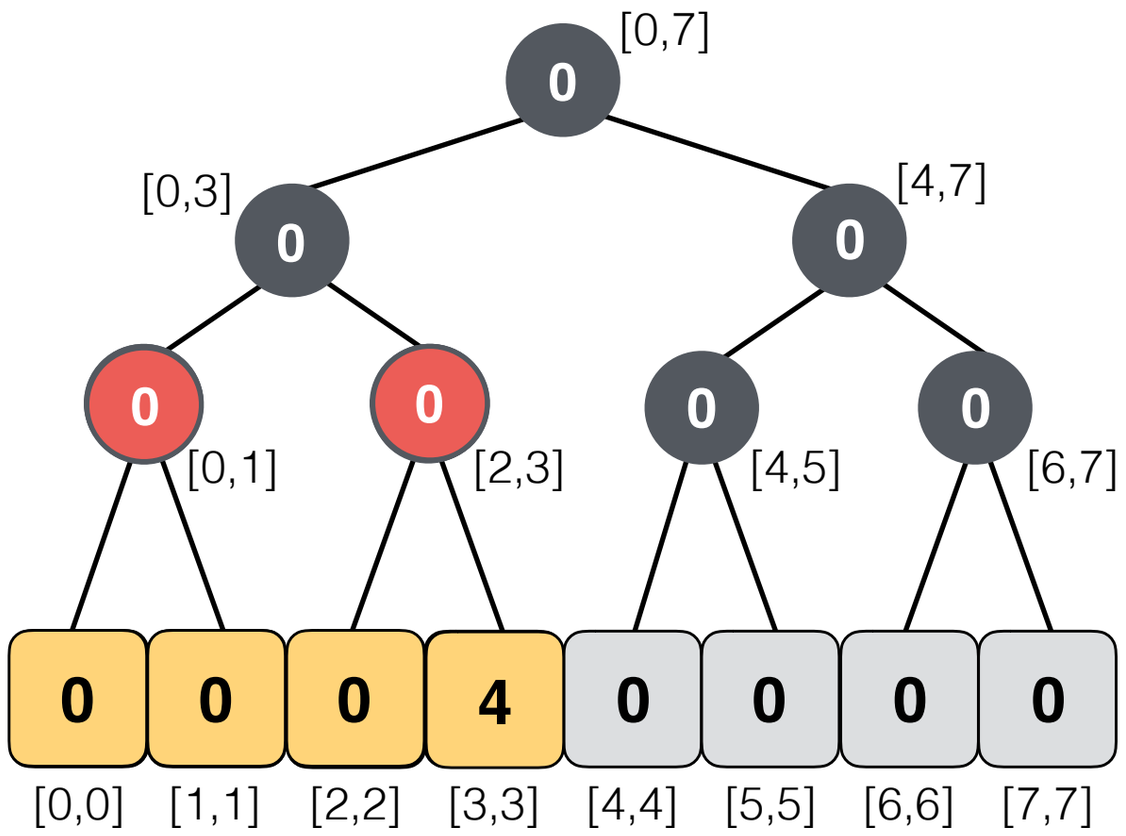

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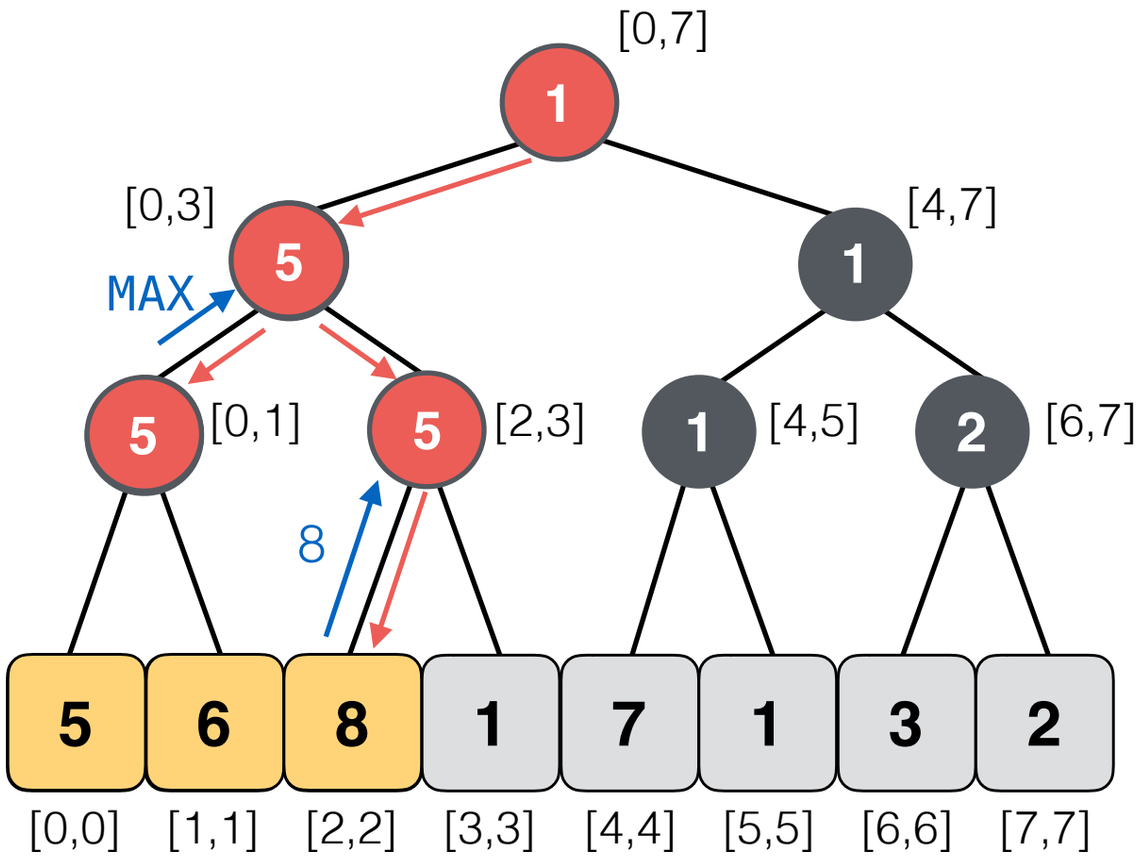

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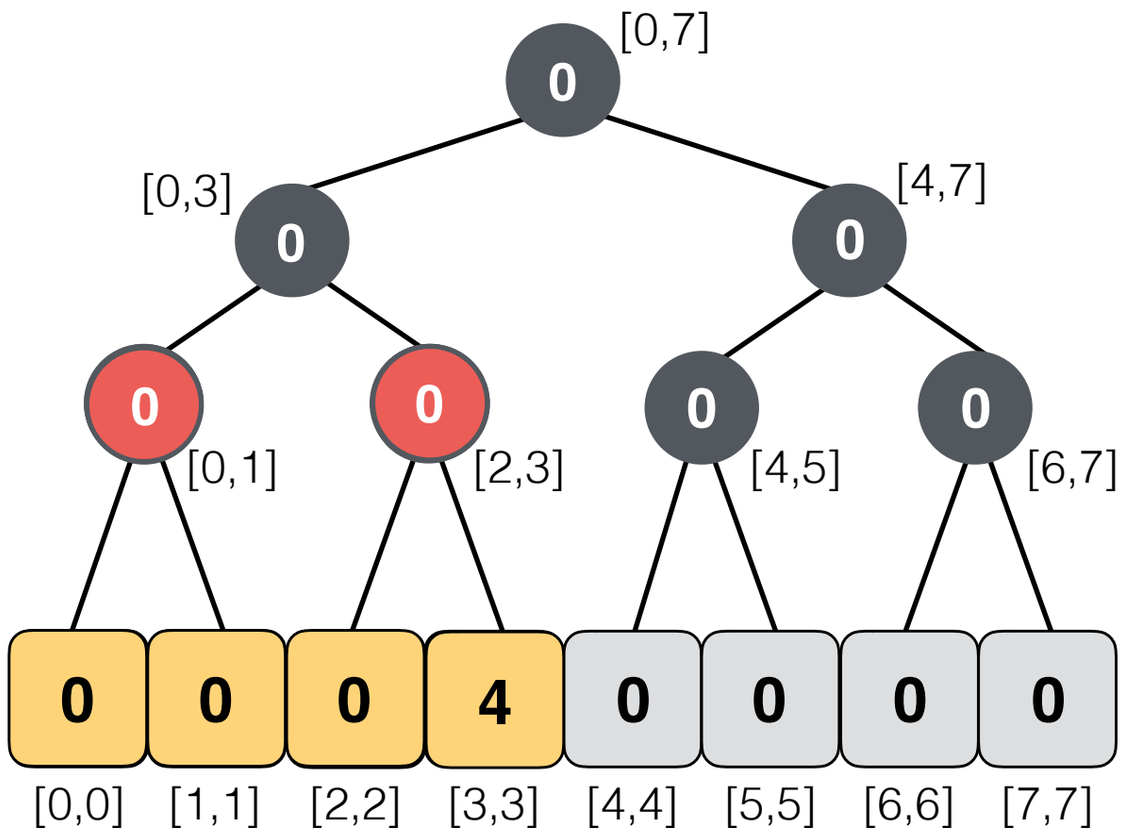

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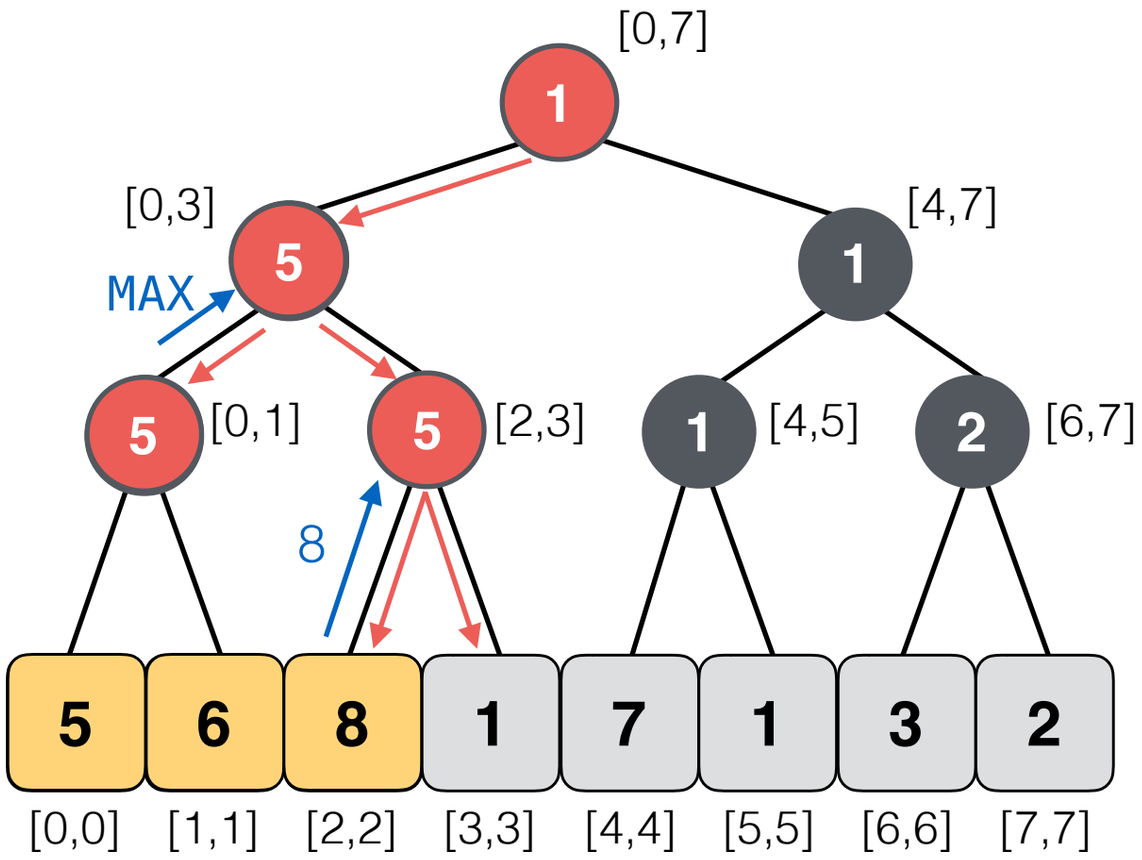

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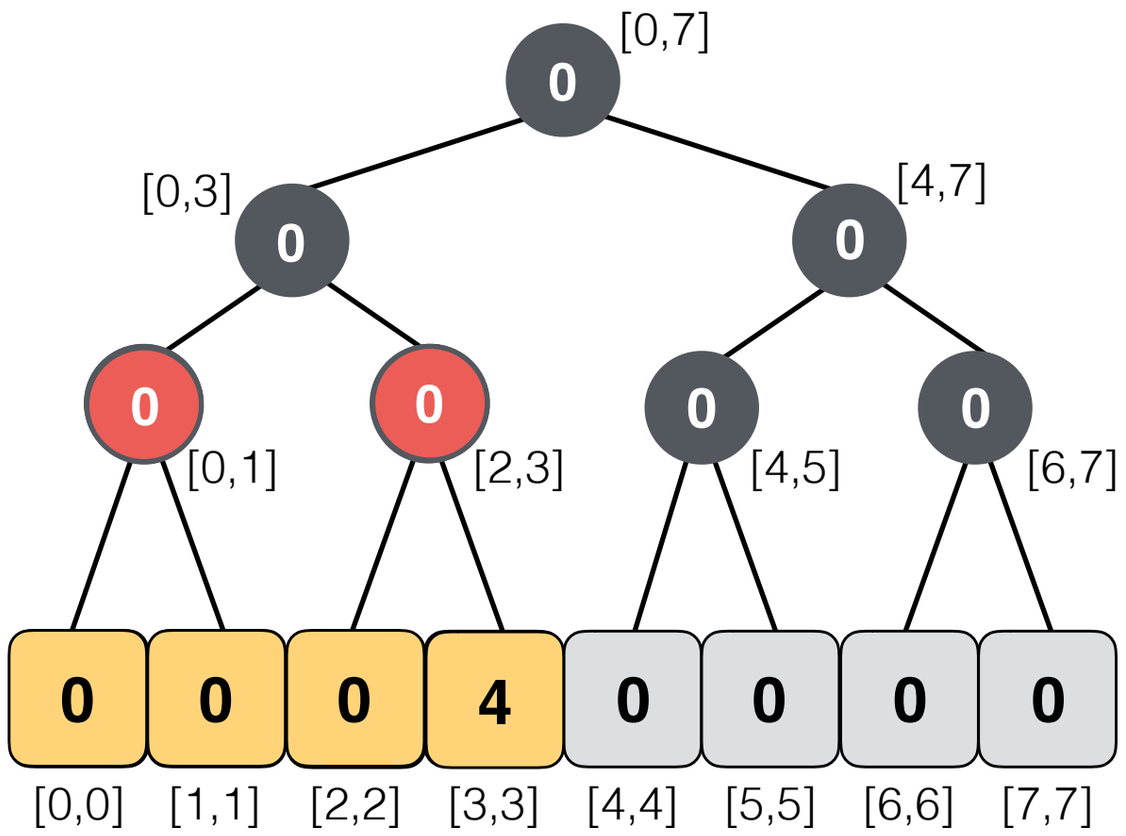

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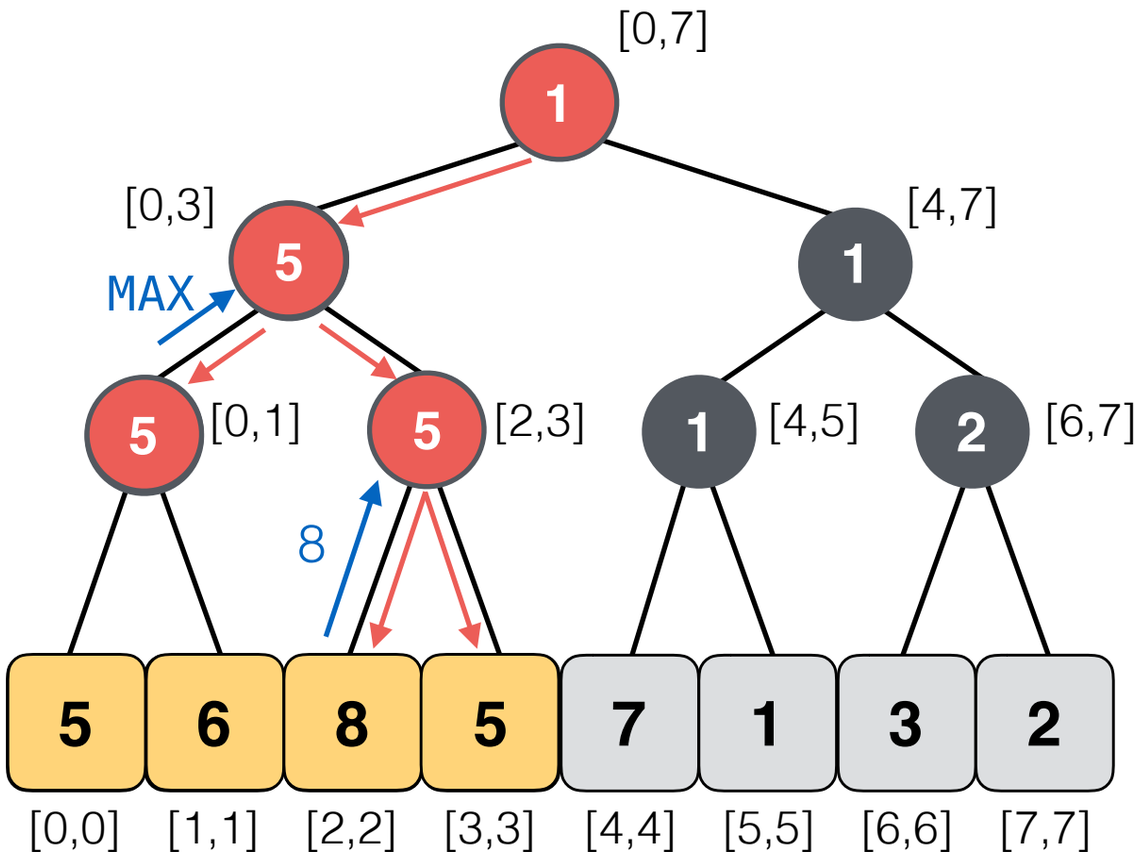

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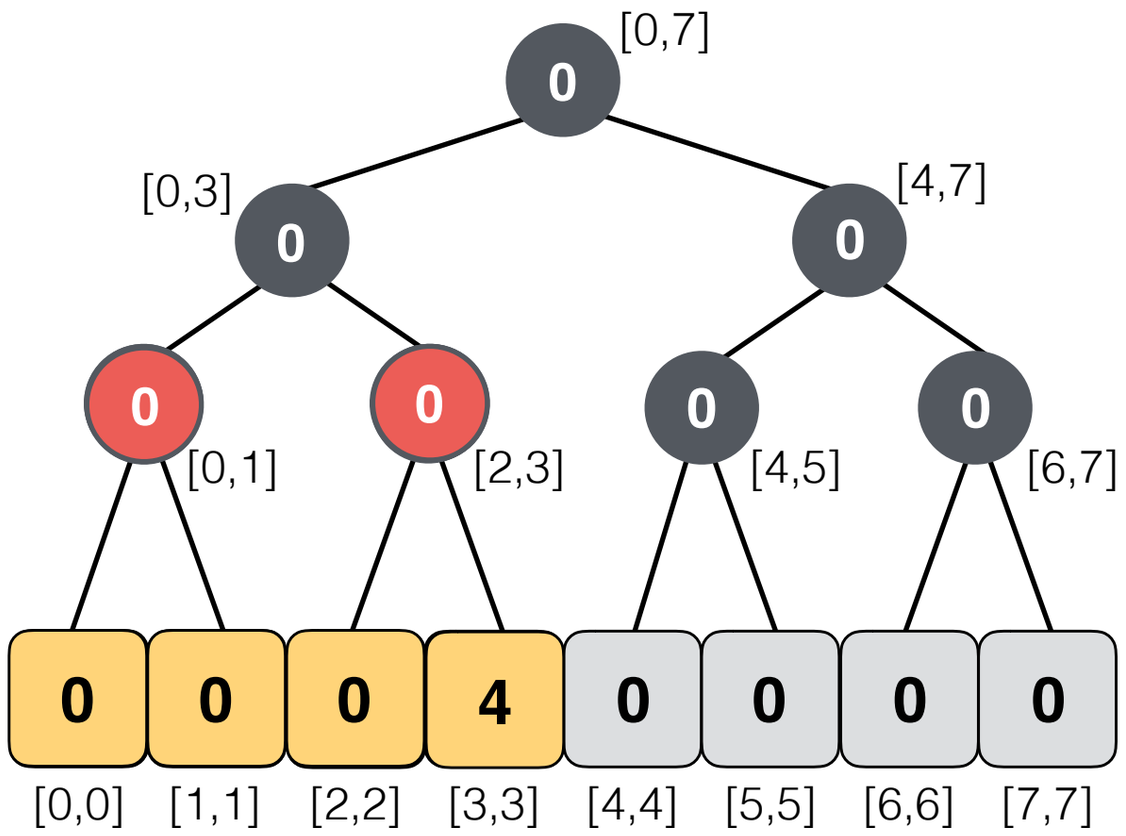

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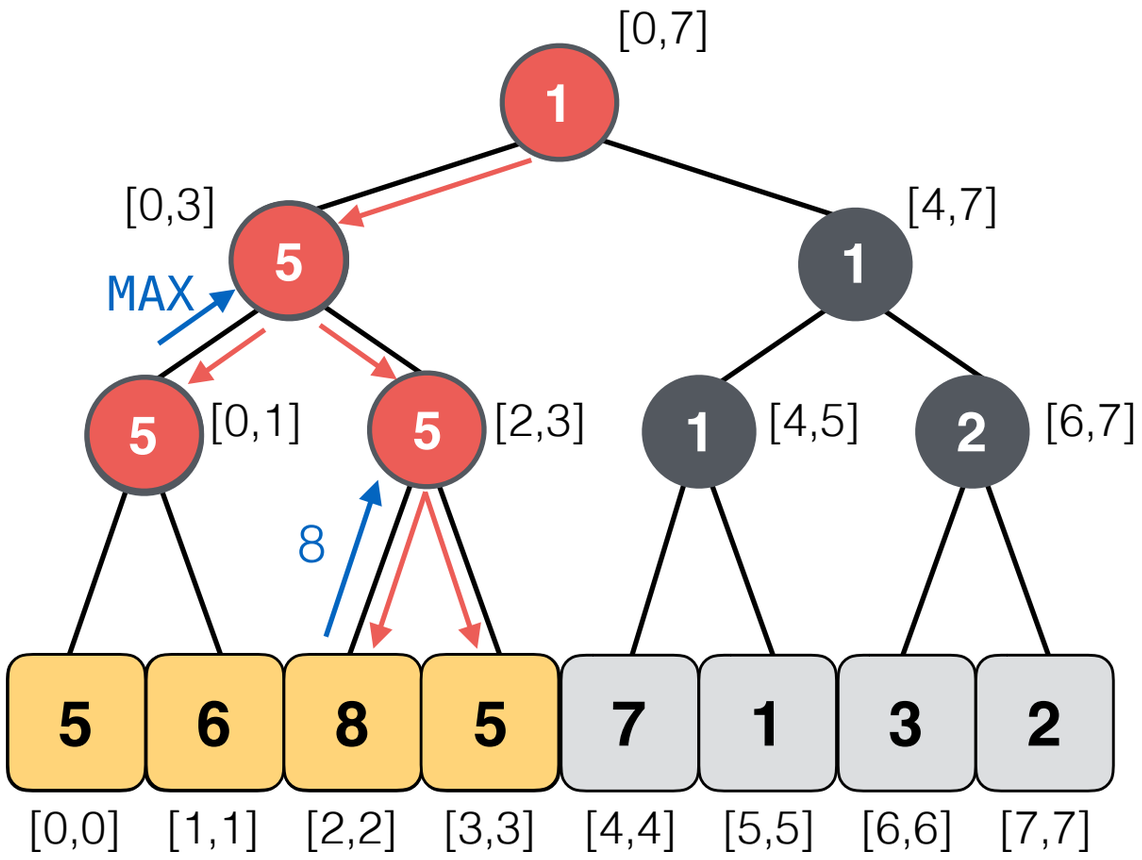

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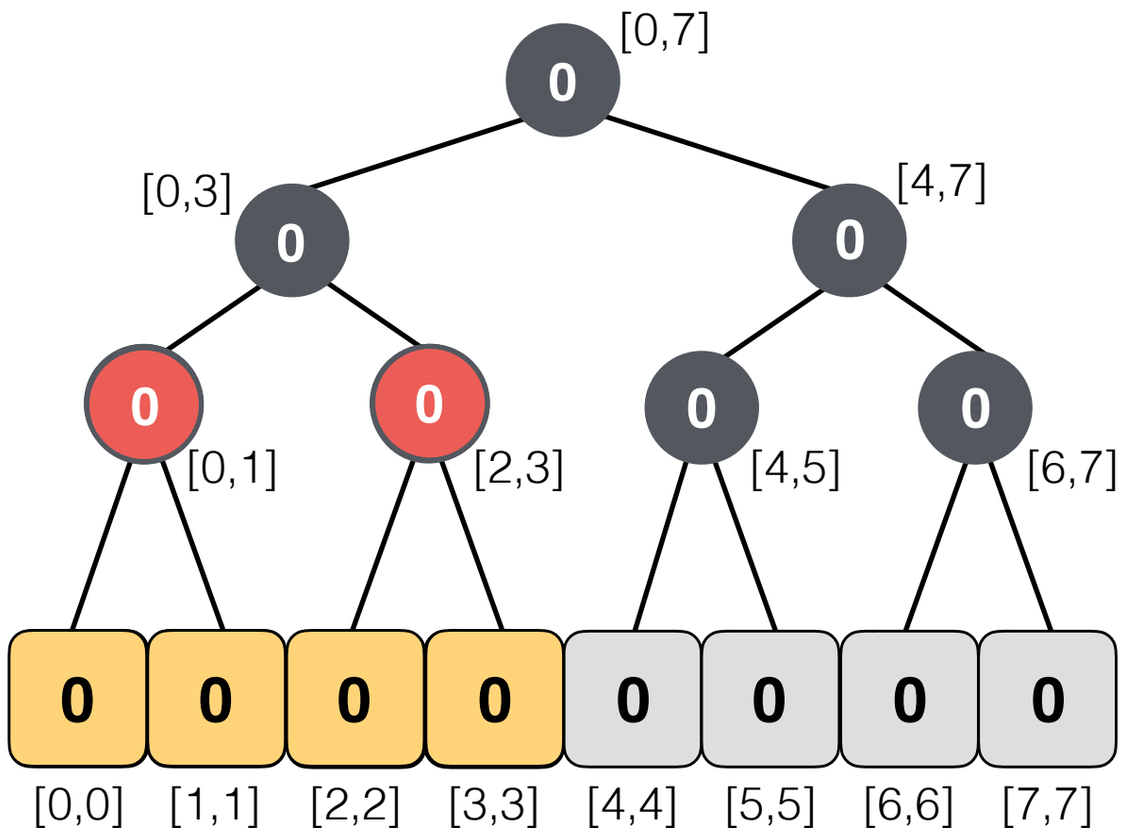

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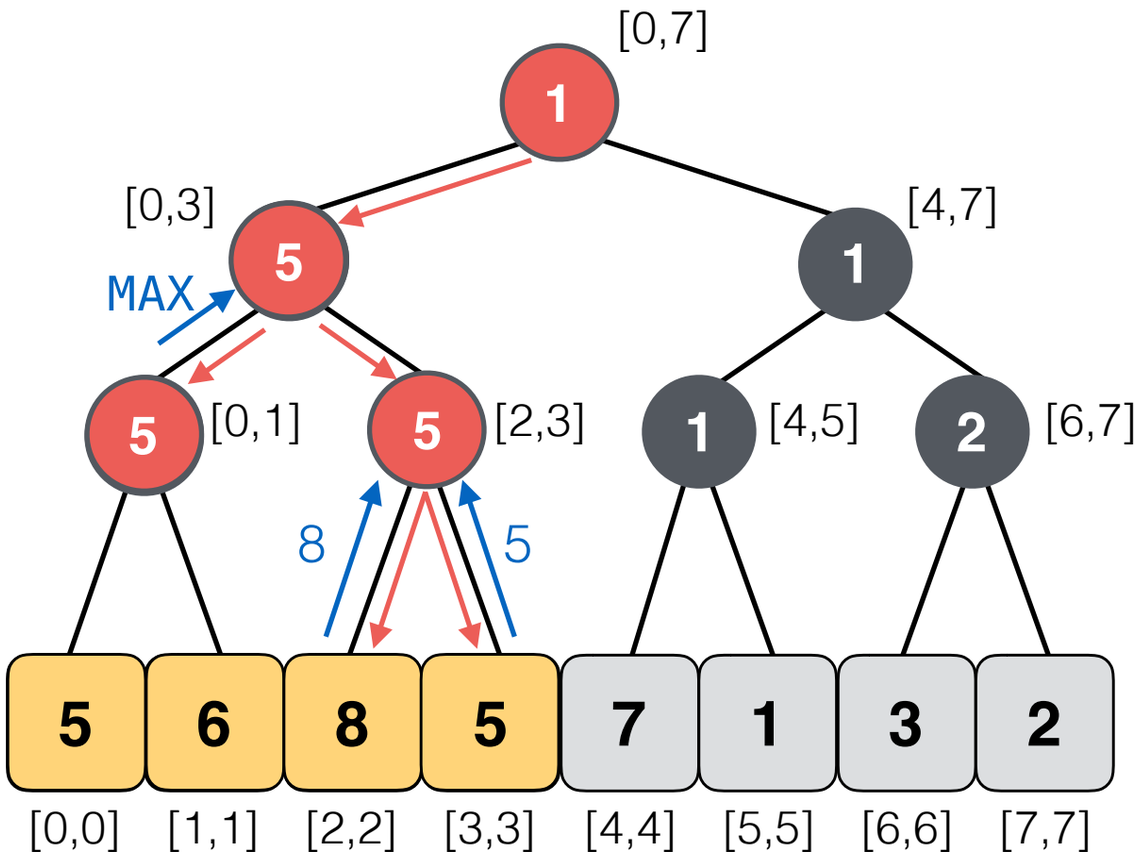

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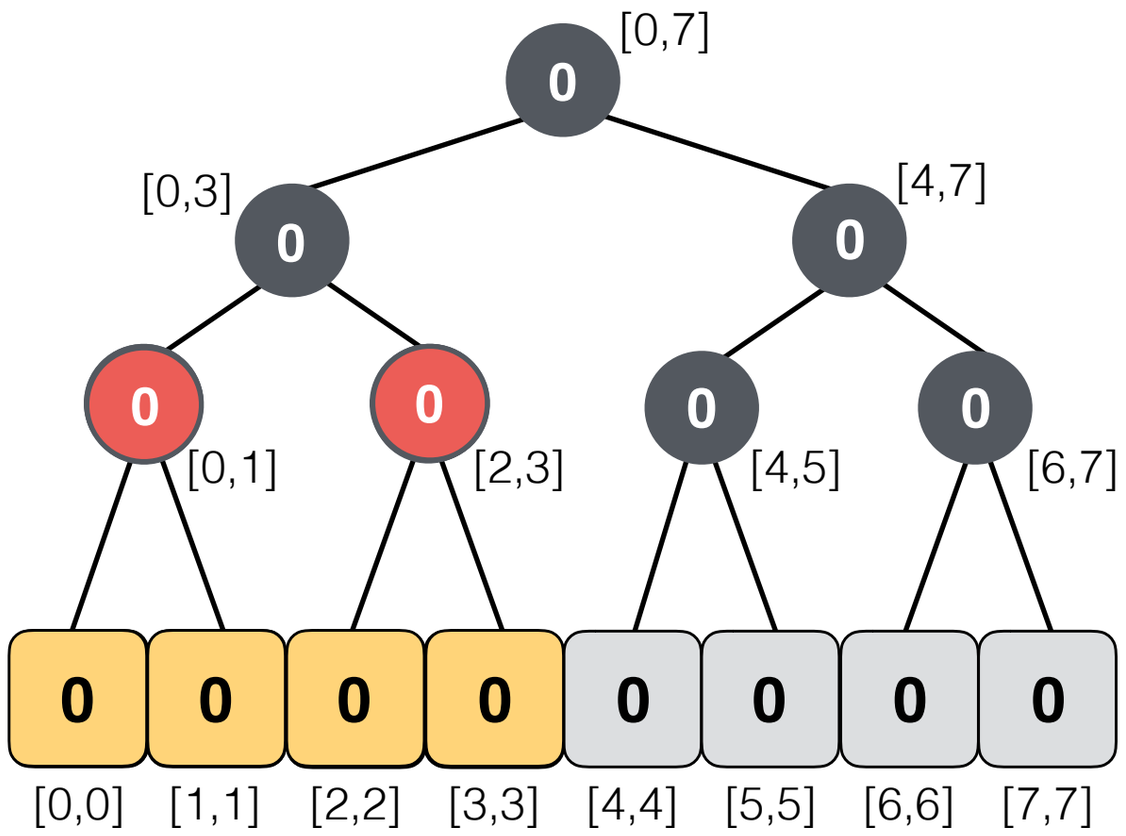

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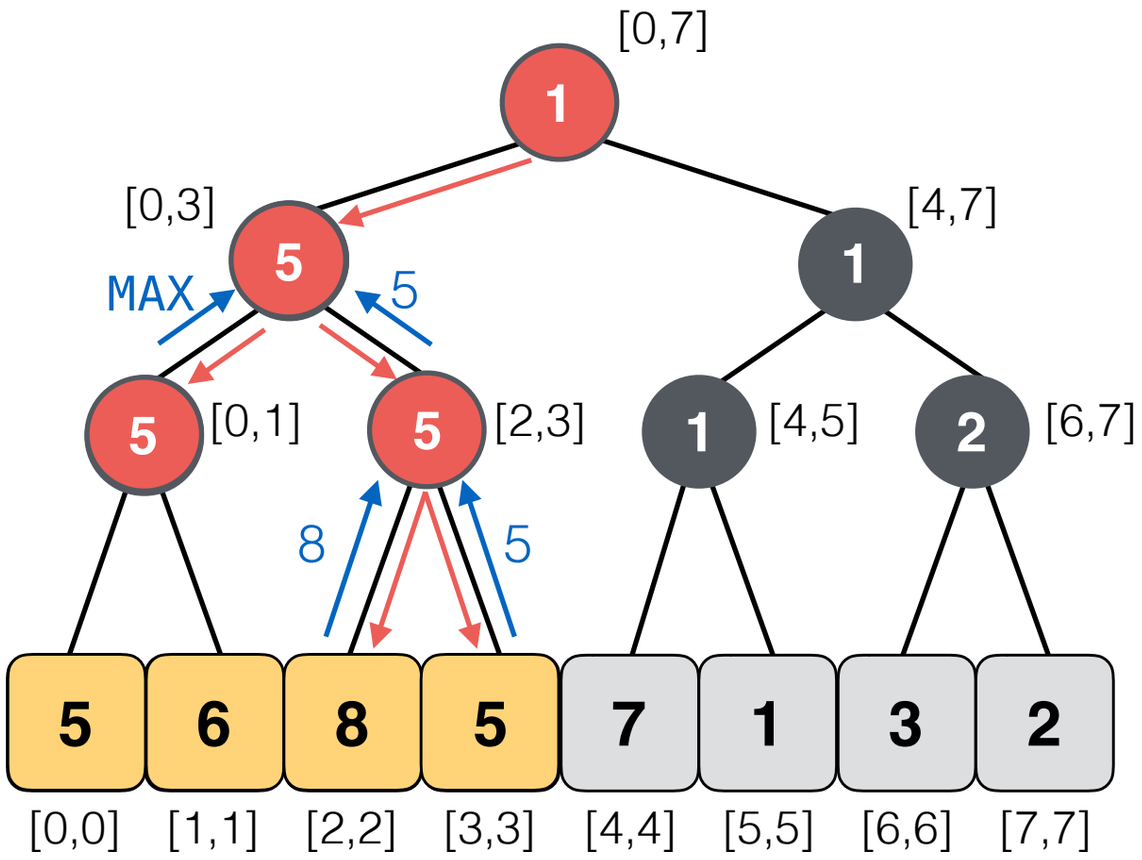

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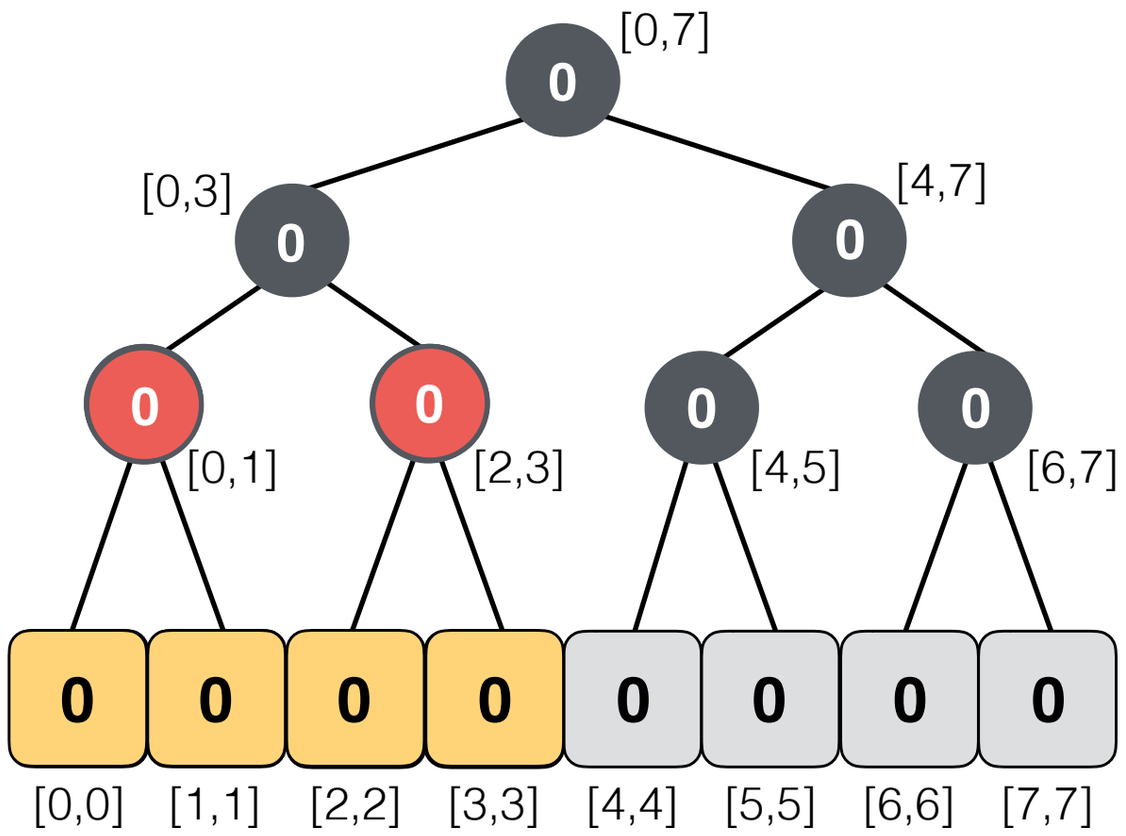

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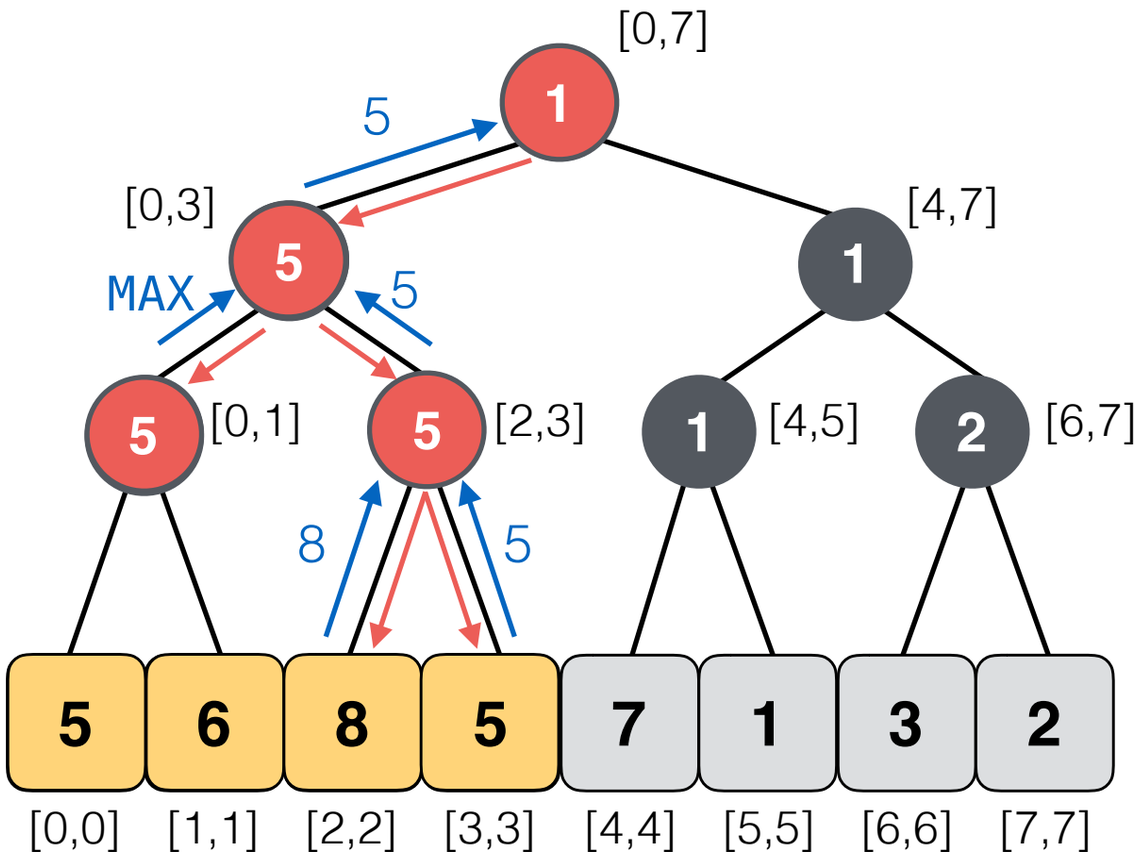

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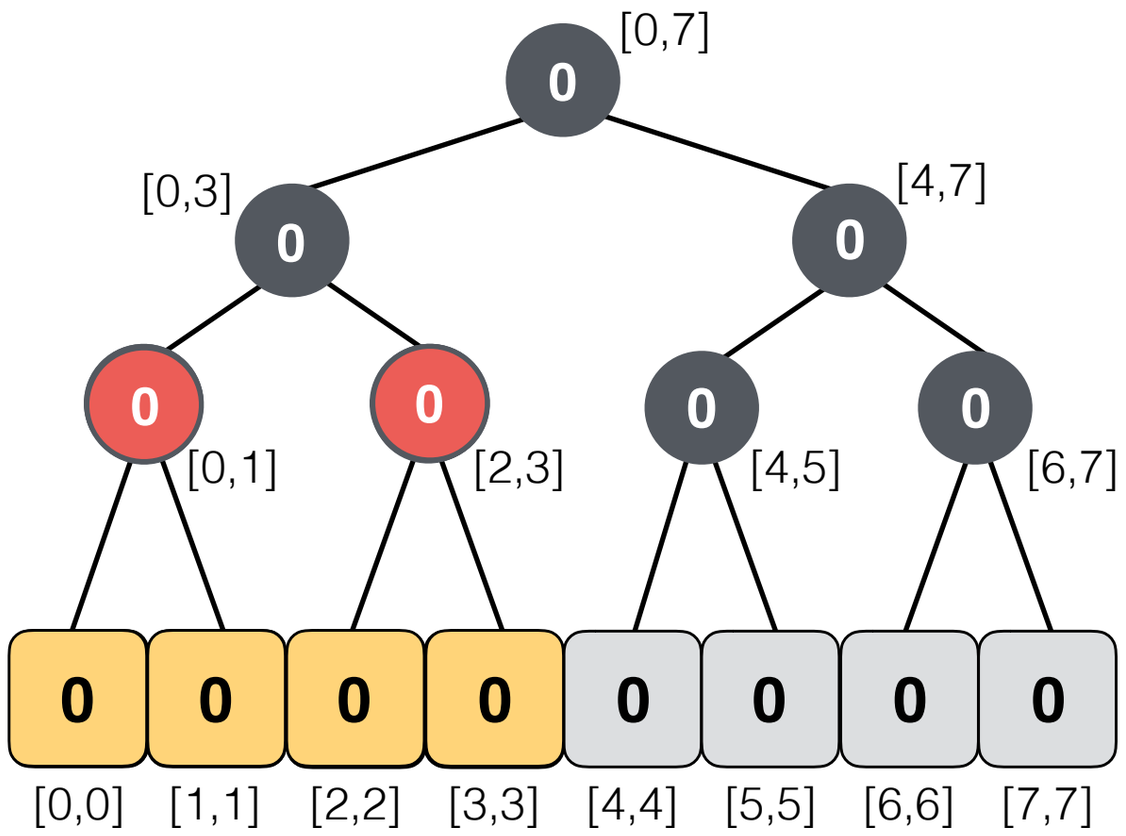

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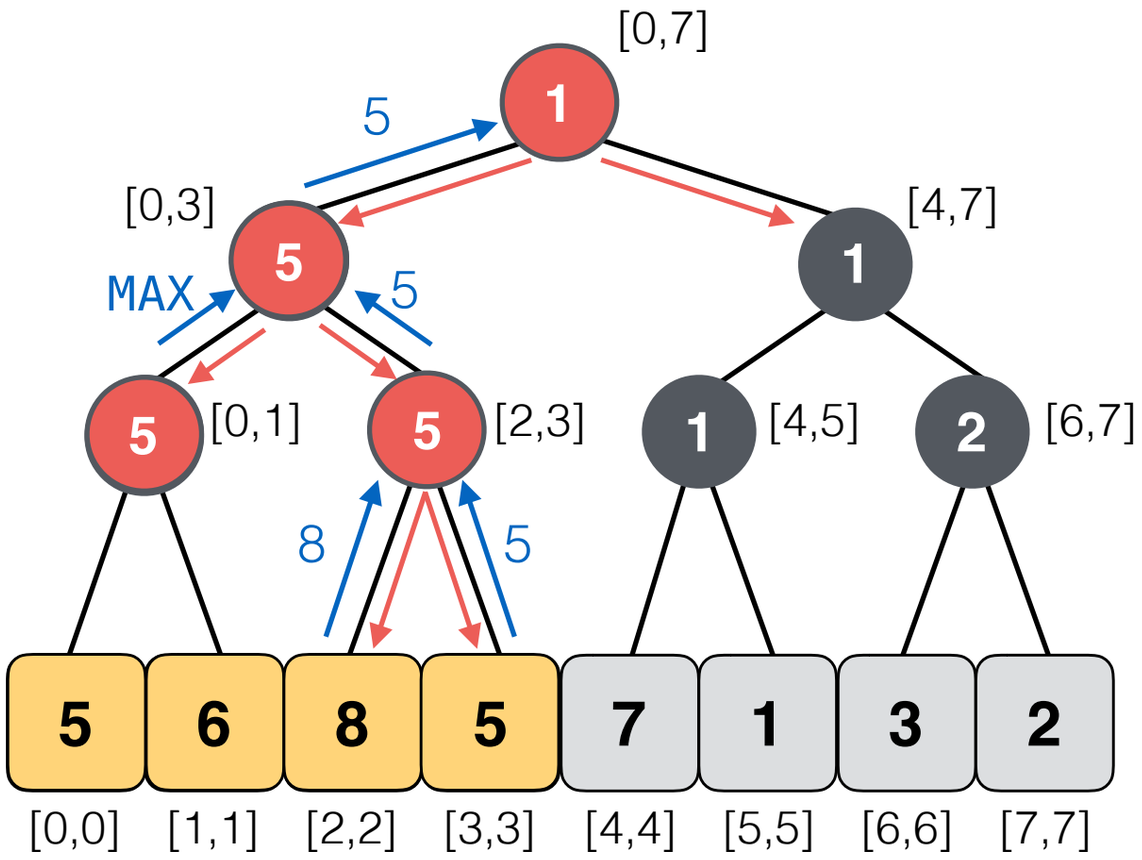

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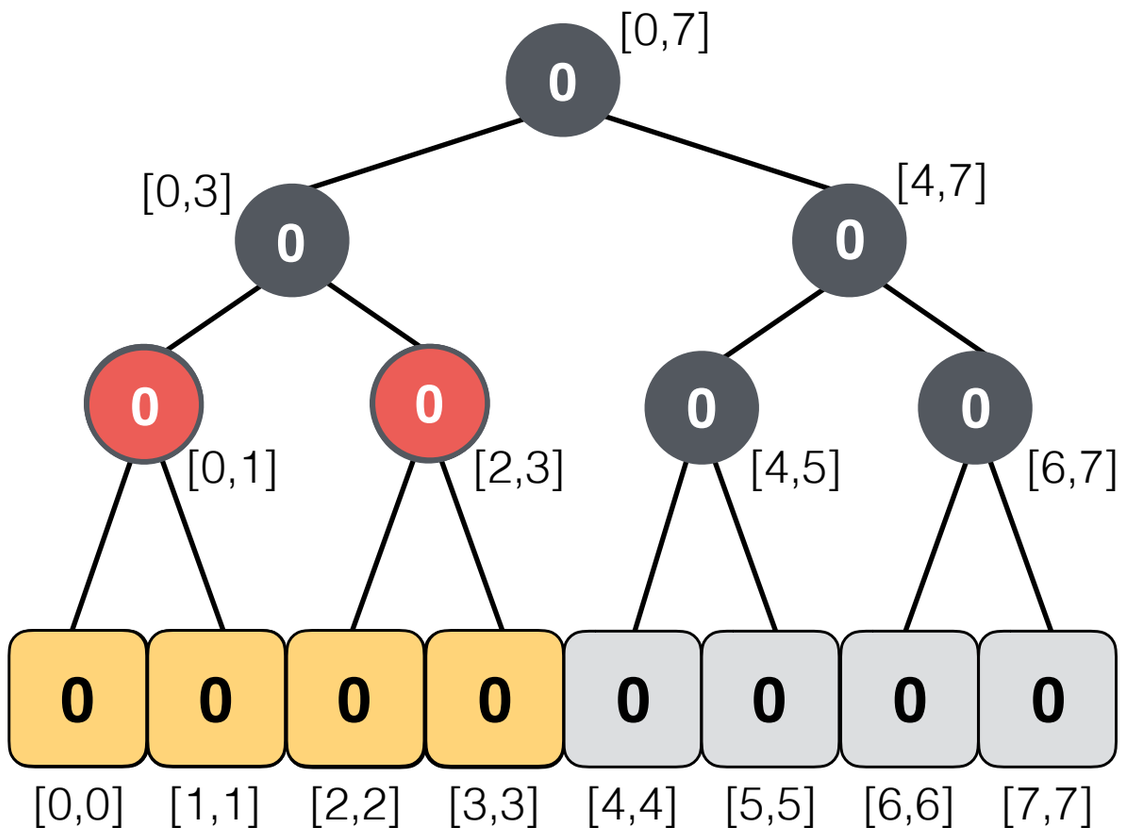

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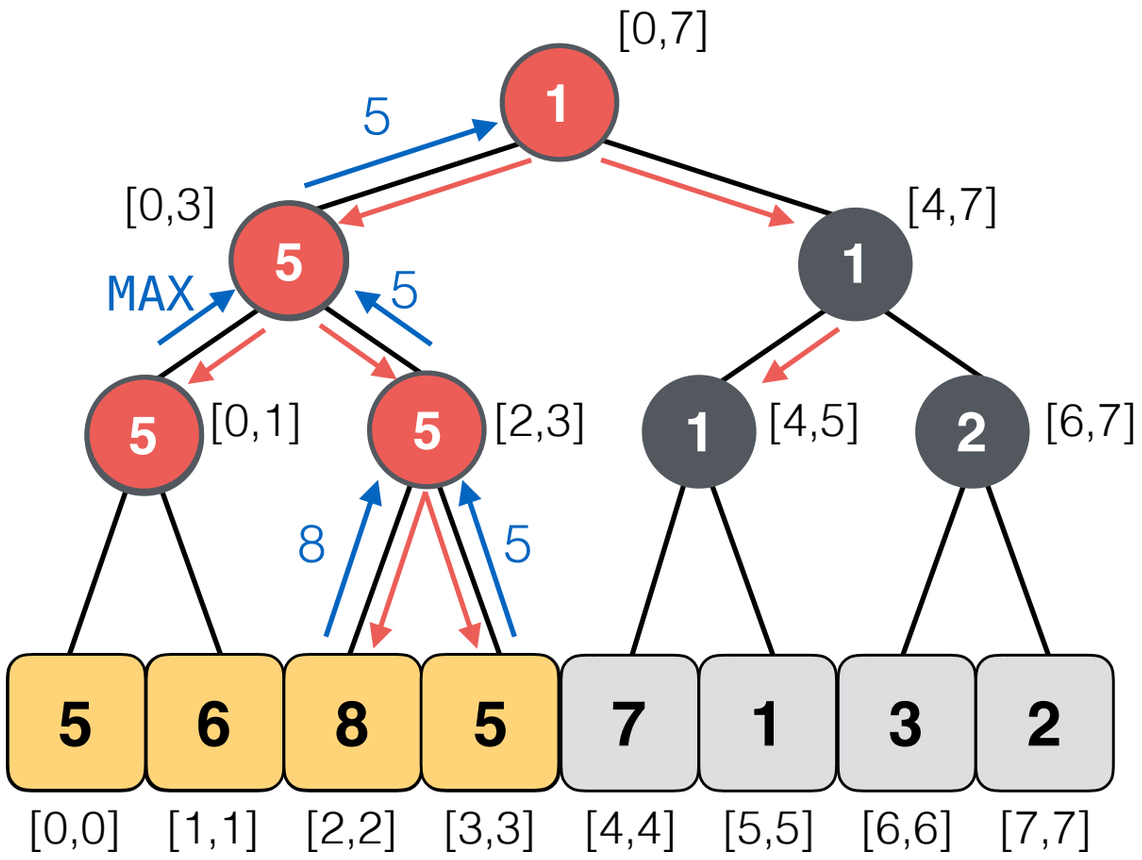

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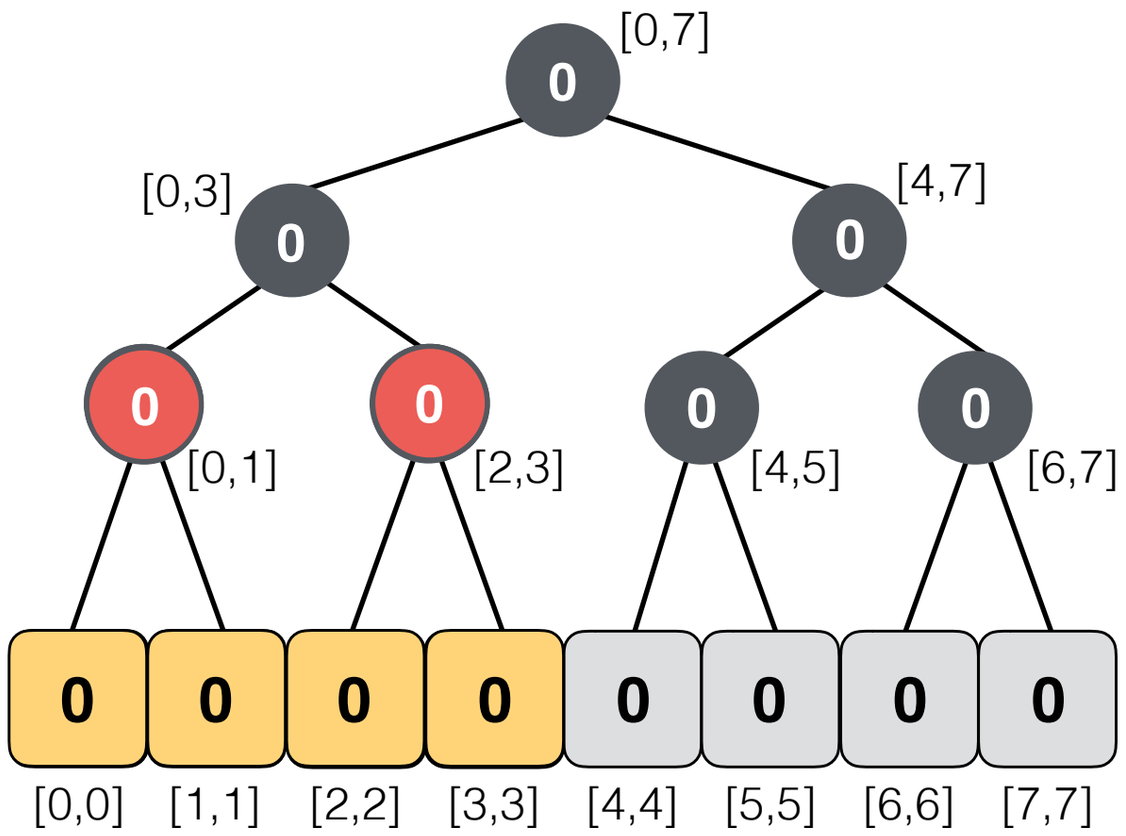

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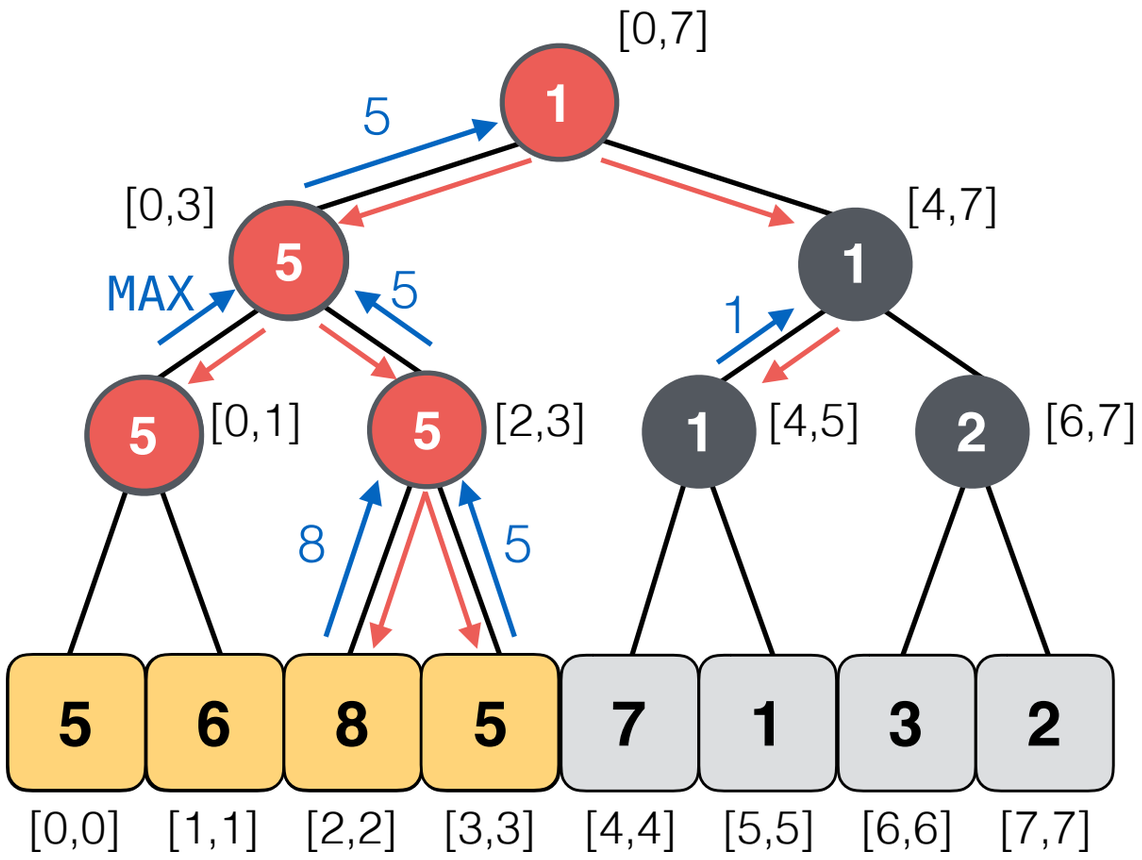

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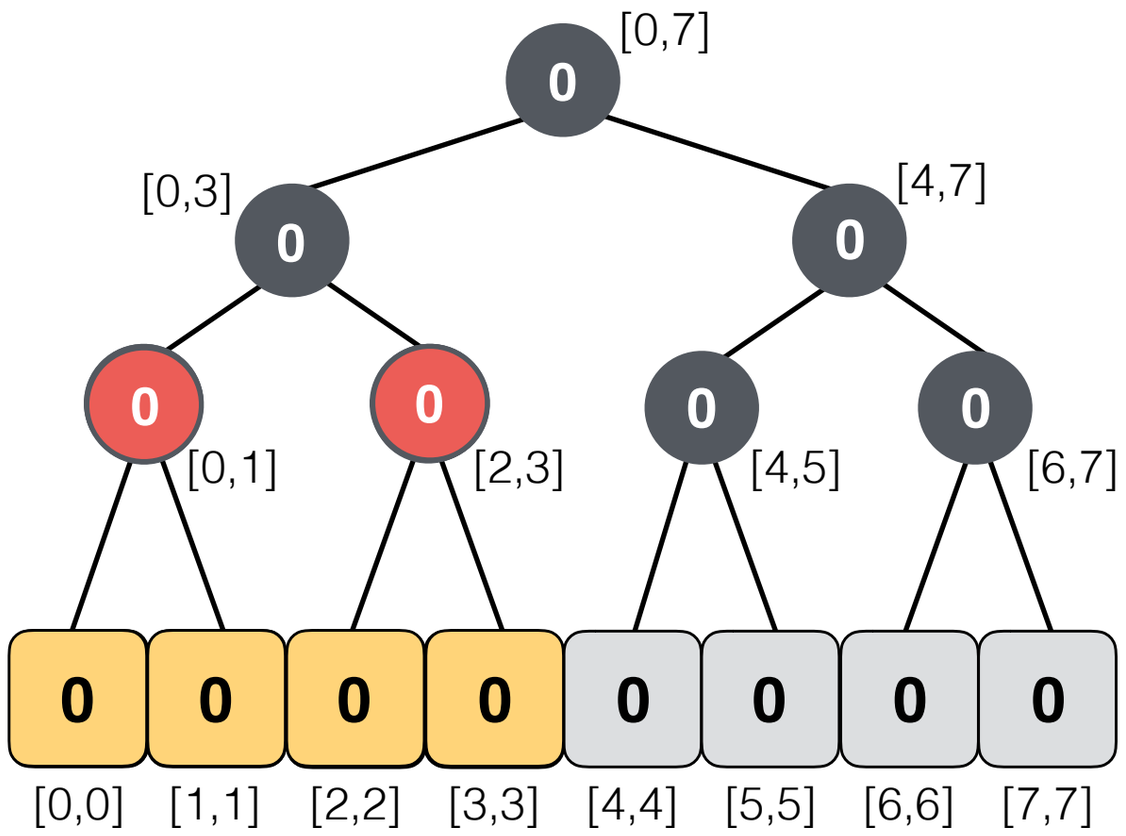

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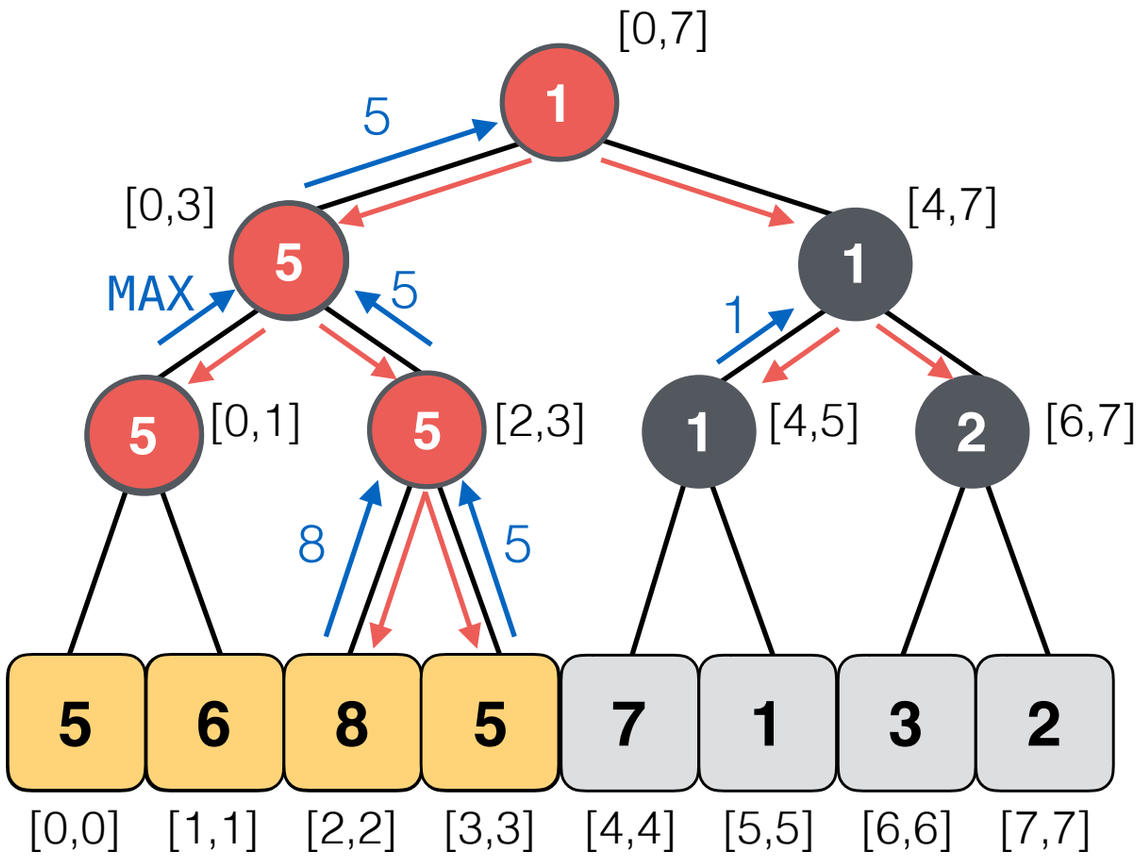
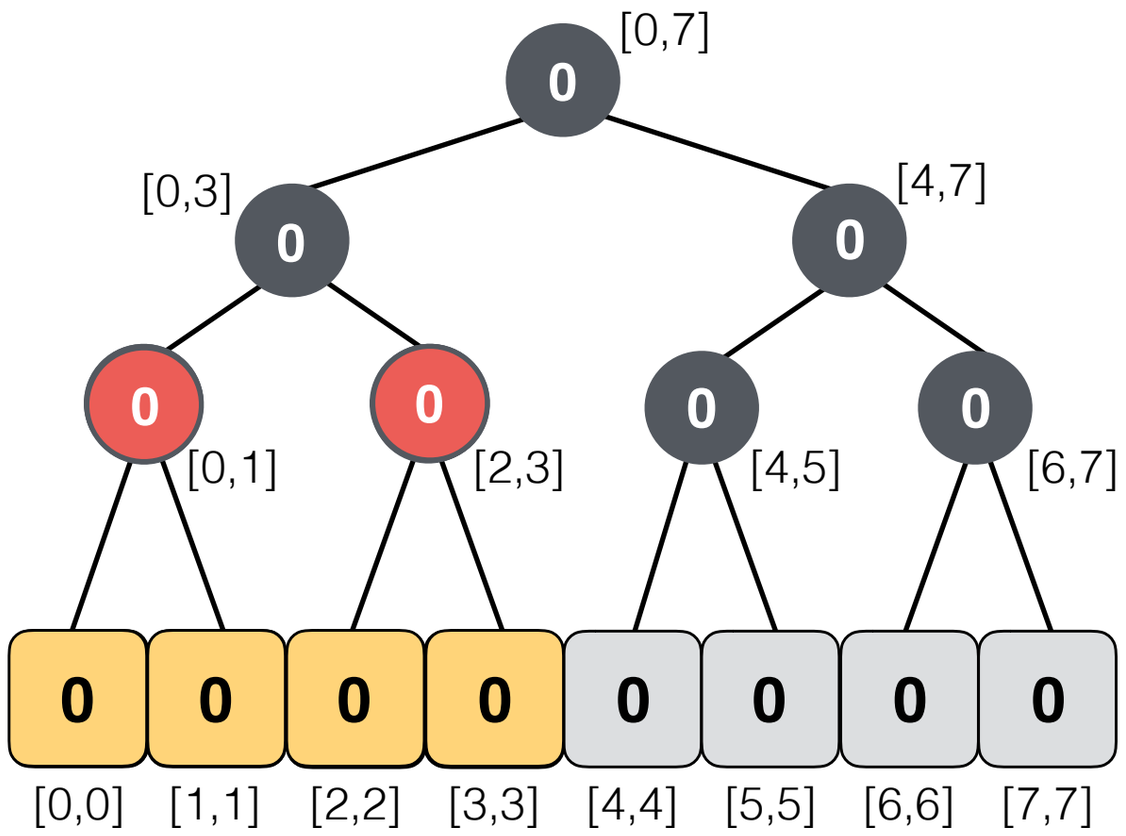


Segment Tree

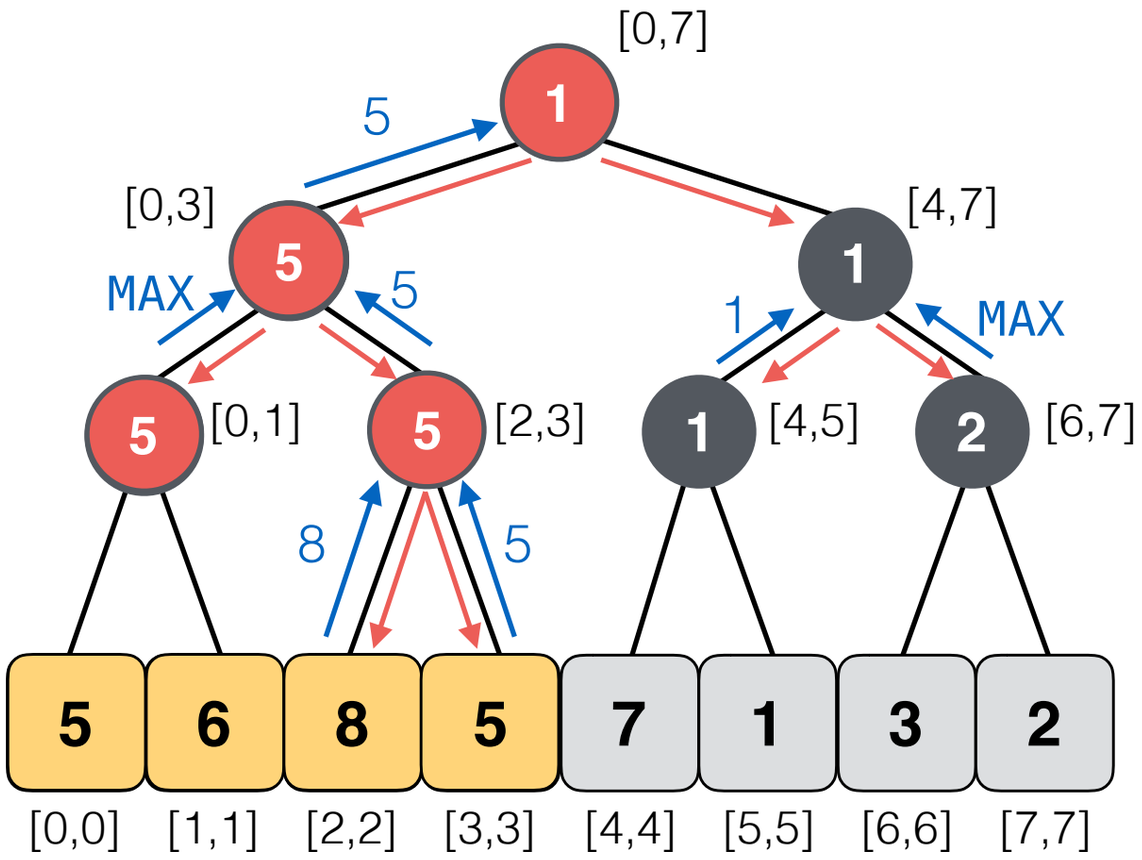


Lazy Tree

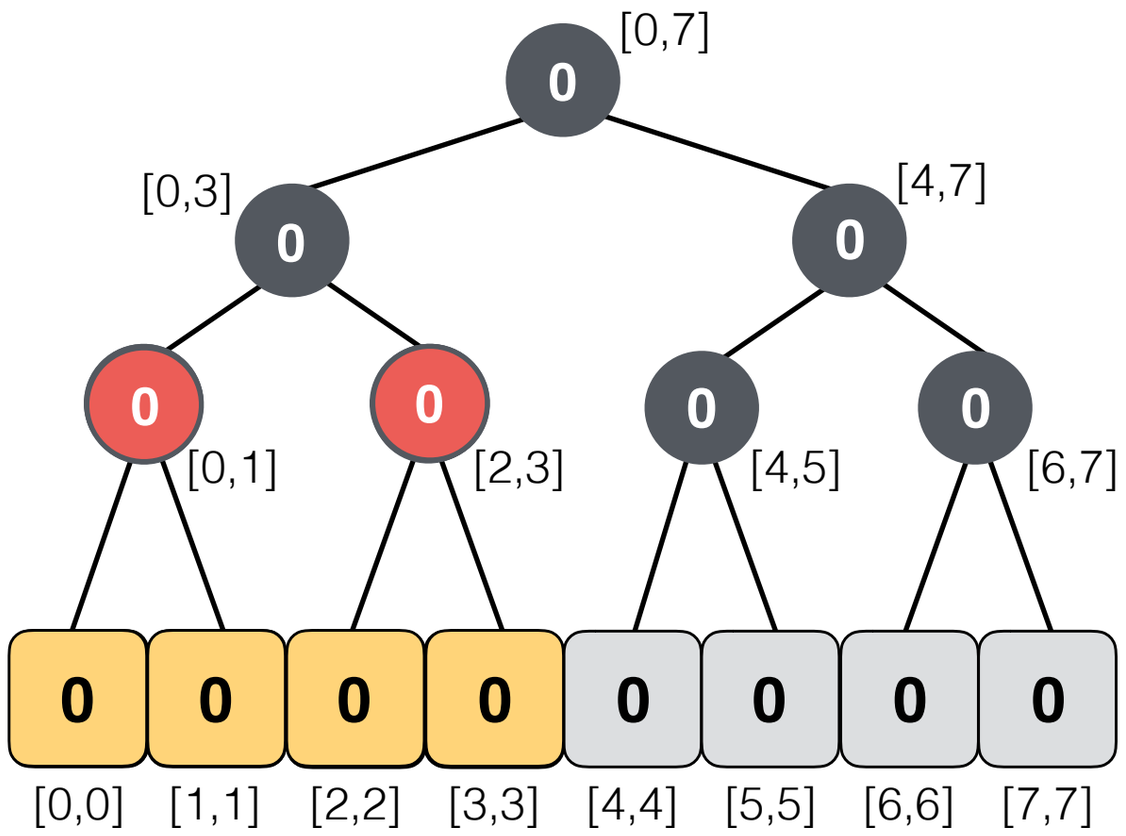
Lazy Propagation in Segment Trees

Avoid going down to the leaves and then up updating the internal nodes.
Only update when needed.

`update_range`(0,3,3)
`update_range`(0,3,1)
`update_range`(0,0,2)
→ `rmq`(3,5) = ?



Segment Tree



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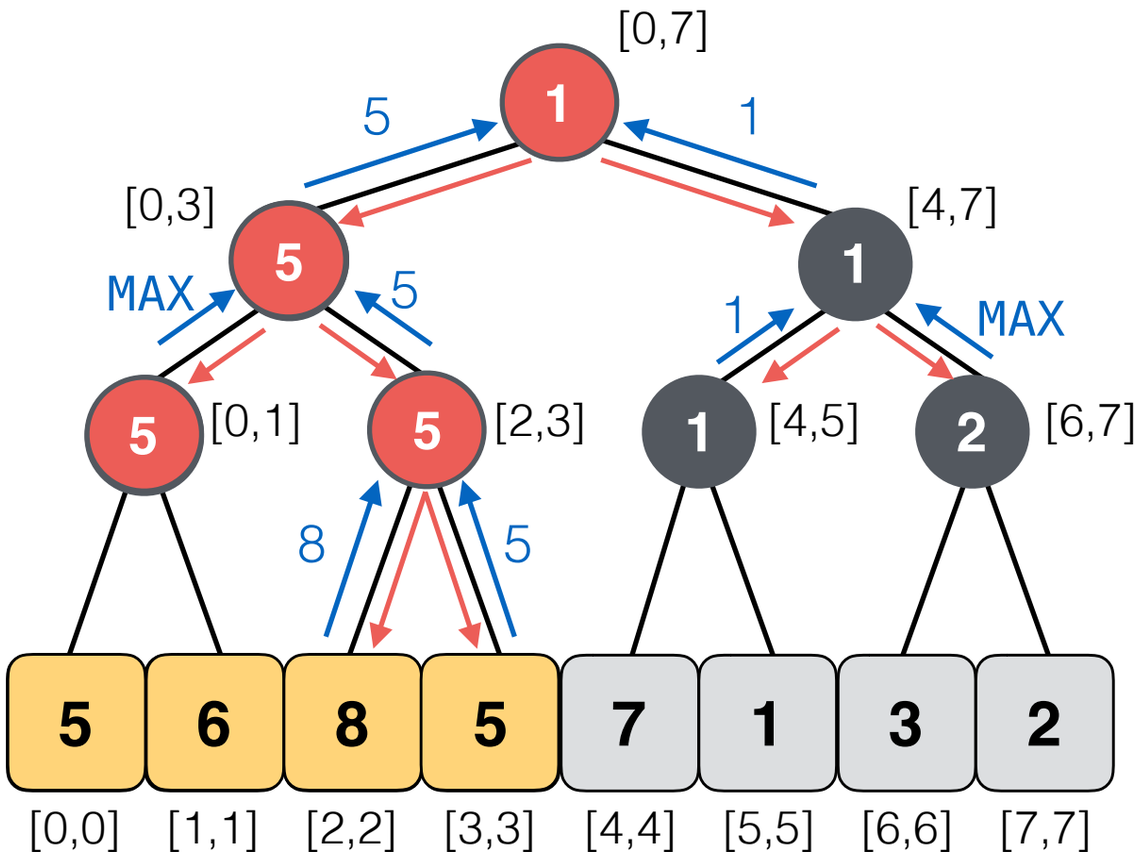
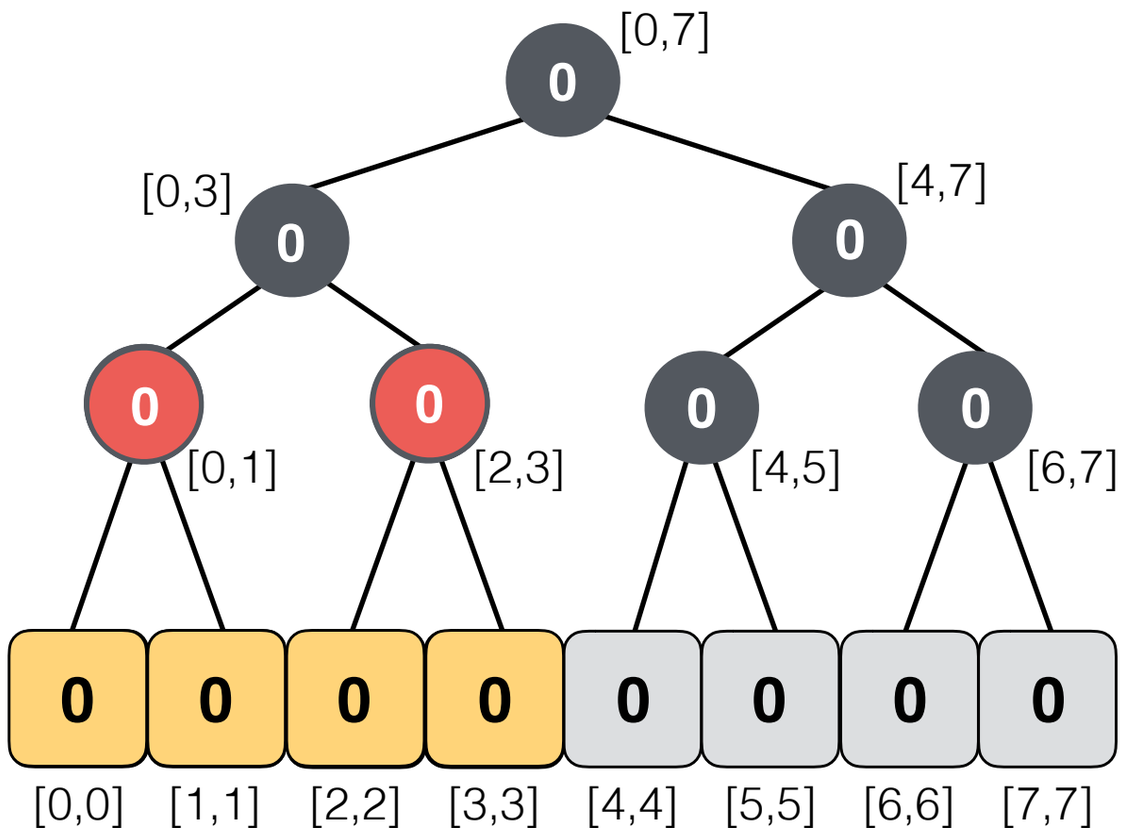


Segment Tree



Lazy Tree

Exercises

Implement lazy propagation and test the difference in running time for a mix of updates/queries.

http://www.geeksforgeeks.org/lazy-propagation-in-segment-tree/

http://www.cdn.geeksforgeeks.org/segment-tree-set-1-sum-of-given-range/

References

Full segment tree code and benchmark at:
https://github.com/rossanoventurini/CompetitiveProgramming/tree/master/code/segment_trees

Video lectures:
https://www.youtube.com/watch?v=ZBHKZF5w4YU&list=PLrmLmBdmIlpv_jNDXtJGYTPNQ2L1gdHxu&index=22
https://www.youtube.com/watch?v=xuoQdt5pHj0&index=23&list=PLrmLmBdmIlpv_jNDXtJGYTPNQ2L1gdHxu