

The set of production rules we have initially is:

$$\begin{aligned} S &\rightarrow aTa \\ T &\rightarrow bTb \mid aCa \\ C &\rightarrow C+C \mid C-C \mid C*C \mid C/C \mid (C) \mid H \\ H &\rightarrow Y.Y \mid Y. \mid .Y \\ Y &\rightarrow NY \mid N \\ N &\rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \end{aligned}$$

Remove unit production $C \rightarrow H$ and $Y \rightarrow N$, we have:

$$\begin{aligned} S &\rightarrow aTa \\ T &\rightarrow bTb \mid aCa \\ C &\rightarrow C+C \mid C-C \mid C*C \mid C/C \mid (C) \mid Y.Y \mid Y. \mid .Y \\ H &\rightarrow Y.Y \mid Y. \mid .Y \\ Y &\rightarrow NY \mid 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \\ N &\rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \end{aligned}$$

Before going into the formal definition of this grammar, let's denote:

- $\Sigma_N = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ because these digit terminal symbols are created equally by variable N .
- $\Sigma_{\text{OPERATOR}} = \{+, -, *, /\}$ because operators have the same purpose.

The **formal definition of Pushdown Automata** (7-tuples) to this grammar is:

- **Finite set of states** $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}\}$
- **Finite set of input alphabet** $\Sigma = \{., 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, -, *, /, (,), a, b\}$
- **Finite stack alphabet** $\Gamma = \{Z_0, a, b, ()\}$
- **Start state** $q_0 = q_0$
- **Initial stack top symbol** $Z_0 = Z_0$
- **Set of accepted states** $F = \{q_{10}\}$
- **Transition function** δ :

$$\delta(q_0, \epsilon, \epsilon) \vdash (q_1, Z_0)$$

$$\delta(q_1, a, \epsilon) \vdash (q_2, a)$$

$$\delta(q_2, a, \epsilon) \vdash (q_3, a)$$

$$\delta(q_2, b, \epsilon) \vdash (q_2, b)$$

$$\delta(q_3, (, \epsilon) \vdash (q_3, ($$

$$\delta(q_3, \Sigma_N, \epsilon) \vdash (q_4, \epsilon)$$

$\delta(q_3, \cdot, \epsilon) \vdash (q_5, \epsilon)$
 $\delta(q_4, \Sigma_N, \epsilon) \vdash (q_4, \epsilon)$
 $\delta(q_4, \cdot, \epsilon) \vdash (q_6, \epsilon)$
 $\delta(q_5, \Sigma_N, \epsilon) \vdash (q_6, \epsilon)$
 $\delta(q_6, \Sigma_N, \epsilon) \vdash (q_6, \epsilon)$
 $\delta(q_6,), () \vdash (q_7, \epsilon)$
 $\delta(q_6, a, a) \vdash (q_8, \epsilon)$
 $\delta(q_6, \Sigma_{\text{OPERATOR}}, \epsilon) \vdash (q_3, \epsilon)$
 $\delta(q_7, \Sigma_{\text{OPERATOR}}, \epsilon) \vdash (q_3, \epsilon)$
 $\delta(q_7,), () \vdash (q_7, \epsilon)$
 $\delta(q_7, a, a) \vdash (q_8, \epsilon)$
 $\delta(q_8, b, b) \vdash (q_8, \epsilon)$
 $\delta(q_8, a, a) \vdash (q_9, \epsilon)$
 $\delta(q_9, \epsilon, Z_0) \vdash (q_{10}, \epsilon)$

State diagram:

