COVID model

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1 Introduction

2 Model

Qualitatively we model the epidemic dynamics with the following subpopulations:

- Susceptible individuals $[S]$ are exposed to the virus by contact with an infected individual.
- Exposed individuals $[E]$ progress towards a symptomatic state on average time t_l
- Infected individuals $[I]$ infect an average of R_0 secondary infections. On a time-scale of t_i, infected individuals either recover or progress towards hospitalization.
- Hospitalized individuals $[H]$ either recover or worsen towards a critical state on a time-scale of t_h.
- Critical individuals $[C]$ model ICU usage. They either return to the hospital state or die $[D]$ on a time-scale of t_c
- Recovered individuals $[R]$ can not be infected again.

Subpopulations are delineated by age classes, indexed by a, to allow for variable transition rates dependent upon age. Quantitatively, we solve the following system of equations to estimate hospital usage:

$$\frac{dS_a(t)}{dt} = -\beta_a(t)S_a(t)\sum_b I_b(t) \quad (1)$$

$$\frac{dE_a(t)}{dt} = \beta_a(t)S_a(t)\sum_b I_b(t) - E_a(t)/t_l \quad (2)$$

$$\frac{dI_a(t)}{dt} = E_a(t)/t_l - I_a(t)/t_i \quad (3)$$

$$\frac{dH_a(t)}{dt} = (1-m_a)I_a(t)/t_i + (1-f_a)C_a(t)/t_c - H_a(t)/t_h \quad (4)$$

$$\frac{dC_a(t)}{dt} = c_a H_a(t)/t_h - C_a(t)/t_c \quad (5)$$

$$\frac{dR_a(t)}{dt} = m_a I_a(t)/t_i + (1-c_a)H_a(t) \quad (6)$$

$$\frac{dD_a(t)}{dt} = f_a C_a(t)/t_c \quad (7)$$

The parameters of this model fall into three categories: a time dependent infection rate $\beta(t)$ time scales of transition to a different subpopulation t_l, t_i, t_h, t_c, and age specific parameters m_a, c_a and f_a that determine relative rates of different outcomes. The latency time from infection to infectiousness i's t_l, the time an individual is infectious after which he/she either recovers or falls severely ill is t_i, the time a sick person recovers or deteriorates into a critical state is t_h, and the time a person remains critical before dying or stabilizing is t_c. The fraction of infectious that are asymptomatic or mild is m_a, the fraction of severe cases that turn critical is c_a, and the fraction of critical cases that are fatal is f_a.

The transmission rate $\beta_a(t)$ is given by

$$\beta_a(t) = R_0 I_a M(t)(1 + \varepsilon \cos(2\pi(t - t_{max})))/t_i \quad (8)$$

where I_a is the degree to which particular age groups are isolated from the rest of the population, $M(t)$ is the time course of mitigation measures, ε is the amplitude of seasonal variation in transmissibility, and t_{max} is the time of the year of peak transmission.

Estimates for these parameters are taken from the following publications:

- R_0:
- Serial interval:
- Severity: China CDC
- Clinical: