CPSC-354 Report

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Abstract

This report follows the course template and starts with Week 1. I go over the MU Puzzle from Chapter 1, explain why **MU** cannot be made from **MI**, and add a few notes and a question for discussion.

Contents

1 Introduction

This report will grow during the semester. For Week 1 I looked at the MU Puzzle to practice writing about a simple formal system: an axiom, rules of inference, derivations, and the idea of working *inside* versus *outside* the system. This part is based on Chapter 1 of Hofstadter [HEB].

2 Week by Week

2.1 Week 1

2.1.1 Notes and Exploration

- A *formal system* here consists of strings over $\{M, I, U\}$, an axiom MI, and four rules. A *theorem* is any string reachable from the axiom by finitely many rule applications.
- Working *inside* the system = generate strings by the rules; working *outside* the system = reason about all possible derivations (e.g., invariants).

2.1.2 Homework

The MU Puzzle is explained in Chapter 1 [HEB]. Here are the rules in my own words:

Rules.

(R1) $xI \to xIU$	(append U if the string ends in I)
(R2) $Mx \to Mxx$	(duplicate the part after M)
(R3) replace III by U	(wherever it occurs)
(R4) delete UU	(wherever it occurs).

Claim. The string **MU** cannot be made from MI by these rules.

Reasoning about the number of I's. We can track just how many I's there are in a string:

- Rule 1 adds a U, so the I's stay the same.
- Rule 2 doubles the part after M, so the number of I's doubles.
- Rule 3 removes three I's at once.
- Rule 4 only touches U's, so the I's stay the same.

We start with MI, which has 1 I. Doubling moves us between 1 and 2, taking away 3 doesn't change that cycle, and the other rules don't affect the I's. So the number of I's will always be either 1 or 2. It will never become 0. Since MU has 0 I's, it's impossible to reach it from MI.

Tiny example trail.

$$MI \xrightarrow{\text{R1}} MIU \xrightarrow{\text{R2}} MIUIU$$

Here the number of I's goes $1 \to 1 \to 2$ and never becomes a multiple of 3.

2.1.3 Questions

Q1. Besides counting I's, are there other simple invariants for the MIU-system (e.g., about the parity or positions of U's) that separate additional non-theorems from theorems?

3 Essay

(Reserved for the synthesis essay later in the term.)

4 Evidence of Participation

Joined Discord; created GitHub repo with `report.tex` and compiled `report.pdf`.

5 Conclusions

References

[HEB] Douglas R. Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid*, Basic Books, 1979. Chapma University