CPSC-354 Report

Linus Fujisawa
Chapman University

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The MU Puzzle

The MU Puzzle, introduced in Chapter 1, starts with a very simple setup the axiom string **MI** and four rules that let us change or extend the string using the letters M, I, and U. The puzzle asks whether it's possible to reach the string **MU** by applying these rules.

At first glance it seems like it might work if you just keep trying different combinations, but after some trial and error this problem will not have a solution. The key observation is that no matter what rule you apply, the number of I's never becomes a multiple of three. It always stays in a certain pattern.

- The starting axiom MI has one I.

- Rule 1 ($xI \rightarrow xIU$) adds a U at the end, but doesn't change the number of I's.

- Rule 2 ($Mx \rightarrow Mxx$) doubles the part after the M, which changes the number of I's but never makes it a multiple of three.

- Rule 3 ($III \rightarrow U$) removes three I's, so the overall pattern stays the same.

- Rule 4 ($UU \rightarrow$ delete) only affects U's, not I's.

So the system never produces a string where the number of I's is a multiple of three. Since MU has zero I's, it doesn't fit the pattern, which means there's no way to reach it from MI.

For example, starting with MI (which has 1 I), Rule 1 gives MIU. This still has 1 I. If we keep going, $MIU \rightarrow MIUIU$ using Rule 2, the number of I's doubles to 2. At no point does the number of I's ever become a multiple of three, which is why MU can't be reached.

Conclusion

In the end, the MU puzzle has no solution. The rules simply don't allow us to create MU starting from MI. By spotting the pattern with the I's, we can prove once and for all that MU is unreachable.