④ $L(\sin^3 2t) = L\left\{\frac{3\sin 2t - \sin 6t}{4}\right\}$

$$= \frac{3L}{4}\{\sin 2t\} - \frac{1}{4}L\{\sin 6t\}$$

$$= \frac{3}{4}\left(\frac{2}{s^2+4}\right) - \frac{1}{4}\left(\frac{6}{s^2+36}\right)$$

$$= \frac{6}{4}\left(\frac{1}{s^2+4}\right) - \frac{6}{4}\left(\frac{1}{s^2+36}\right)$$

$$= \frac{6}{4}\left[\frac{1}{s^2+4} - \frac{1}{s^2+36}\right]$$

**II$^{nd}$ Property** **First Shifting Property**

If $L\{f(t)\} = \bar{f}(s)$ then $L\{e^{at}f(t)\} = \bar{f}(s-a)$

**Proof** By definition, $L\{e^{at}f(t)\}$

$$= \int_0^\infty e^{-st}e^{at}f(t)\,dt$$

$$= \int_0^\infty e^{-(s-a)t}f(t)\,dt$$

$$= \int_0^\infty e^{-rt}f(t)\,dt \qquad \text{where } r = s-a$$

$$= \bar{f}(r) \qquad \bar{f}(r) = \bar{f}(s-a)$$

$$= \bar{f}(s-a)$$

Thus if we know the transform $\bar{f}(s)$ of $f(t)$ we can write the transform for $e^{at}f(t)$ by simply replacing $s$ by $s-a$ to get $\bar{f}(s-a)$.

Application of this property

① $L(e^{at}) = \frac{1}{s-a}$

② $L(e^{at}t^n) = \frac{n!}{(s-a)^{n+1}}$

③ $L(e^{at}\sin bt) = \frac{b}{(s-a)^2+b^2}$

④ $L(e^{at}\cos bt) = \frac{s-a}{(s-a)^2+b^2}$

⑤ $L(e^{at}\sinh bt) = \frac{b}{(s-a)^2-b^2}$

⑥ $L(e^{at}\cosh bt) = \frac{s-a}{(s-a)^2-b^2}$

in each case $s>0$

Problems to do

① $L(5\sin 3t - 8\cos 2t)$

② $L\{(t^2+1)^2\}$

③ $L\{\sin^2 t\}$