Find the Inverse Laplace Transform of

① $\dfrac{3(s^2-1)^2}{2s^5}$ ② $\dfrac{3s+5\sqrt{2}}{s^2+8}$ ③ $\dfrac{4s+15}{16s^2-25}$

① $\dfrac{3(s^2-1)^2}{2s^5} = \dfrac{3(s^4-2s^2+1)}{2s^5} = \dfrac{3s^4-6s^2+3}{2s^5}$

Now $L^{-1}\left\{\dfrac{3(s^2-1)^2}{2s^5}\right\} = L^{-1}\left\{\dfrac{3s^4}{2s^5} - L^{-1}\dfrac{6s^2}{2s^5} + L^{-1}\dfrac{3}{2s^5}\right\}$

$= L^{-1}\left\{\dfrac{3}{2s} - L^{-1}\dfrac{6}{2s^3} + L^{-1}\dfrac{3}{2s^5}\right\}$

$= \dfrac{3}{2}L^{-1}\left(\dfrac{1}{s}\right) - 3L^{-1}\left(\dfrac{1}{s^3}\right) + \dfrac{3}{2}L^{-1}\left(\dfrac{1}{s^5}\right)$

$= \underline{\underline{\dfrac{3}{2} - \dfrac{3t^2}{2} + \dfrac{3}{2}\left[\dfrac{t^4}{4!}\right]}}$

Formula $L^{-1}\left(\dfrac{n!}{s^{n+1}}\right) = t^n$

Now $-3L^{-1}\left\{\dfrac{1}{s^3}\right\} = -3L^{-1}\left\{\dfrac{2!}{s^3}\right\} = t^2 \Rightarrow \dfrac{-3}{} L^{-1}\left\{\dfrac{1}{s^3}\right\} = \dfrac{t^2}{2!}$

ie $-3L^{-1}\left\{\dfrac{1}{s^3}\right\} = \dfrac{-3t^2}{2!}$

Also $\dfrac{3}{2}L^{-1}\left(\dfrac{1}{s^5}\right) = \dfrac{3}{2}L^{-1}\left(\dfrac{4!}{s^5}\right) = t^4 \Rightarrow \dfrac{3}{2}L^{-1}\left(\dfrac{1}{s^5}\right) = \dfrac{t^4}{4!}$

$\therefore \dfrac{3}{2}L^{-1}\left(\dfrac{1}{s^5}\right) = \dfrac{3}{2}\cdot\dfrac{t^4}{4!}$

(2) $L^{-1}\left(\frac{3s+5\sqrt{2}}{s^2+8}\right) = ?$

Now $\frac{3s+5\sqrt{2}}{s^2+8} = \frac{3s+5\sqrt{2}}{s^2+(2\sqrt{2})^2} \Rightarrow$

$$\frac{3s}{s^2+(2\sqrt{2})^2} + \frac{5\sqrt{2}}{s^2+(2\sqrt{2})^2}$$

$$\therefore L^{-1}\left(\frac{3s+5\sqrt{2}}{s^2+8}\right) = L^{-1}\left\{\frac{3s}{s^2+(2\sqrt{2})^2}\right\} + L^{-1}\left\{\frac{5\sqrt{2}}{s^2+(2\sqrt{2})^2}\right\}$$

$$= 3\cos(2\sqrt{2}t) + 5\sqrt{2}\, L^{-1}\left(\frac{1}{s^2+(2\sqrt{2})^2}\right)$$

$$= 3\cos(2\sqrt{2}t) + 5\sqrt{2}\cdot\frac{1}{2\sqrt{2}}\sin(2\sqrt{2}t)$$

$$\therefore L^{-1}\left(\frac{3s+5\sqrt{2}}{s^2+8}\right) = \underline{\underline{3\cos(2\sqrt{2}t) + \frac{5}{2}\sin(2\sqrt{2}t)}}$$

(3) $L^{-1}\left\{\frac{4s+15}{16s^2-25}\right\} = ?$

Now $\frac{4s+15}{16s^2-25} = \frac{4s}{16s^2-25} + \frac{15}{16s^2-25}$

$$= \frac{4s}{(4s)^2-5^2} + \frac{15}{(4s)^2-5^2}$$

Now $$\frac{4s+15}{16s^2-25} = \frac{4s}{(4s)^2-5^2} + \frac{15}{(4s)^2-5^2}$$

$$= \frac{4s}{4^2\left[s^2-\frac{5^2}{4^2}\right]} + \frac{15}{4^2\left[s^2-\frac{5^2}{4^2}\right]}$$

$$= \frac{4s}{16\left[s^2-\left(\frac{5}{4}\right)^2\right]} + \frac{15}{16\left[s^2-\left(\frac{5}{4}\right)^2\right]}$$

$$L^{-1}\left[\frac{4s+15}{16s^2-25}\right] = L^{-1}\frac{4s}{16\left[s^2-\left(\frac{5}{4}\right)^2\right]} + L^{-1}\frac{15}{16\left[s^2-\left(\frac{5}{4}\right)^2\right]}$$

$$= \frac{1}{4}\cosh\left(\frac{5}{4}t\right) + \frac{15}{16}\cdot\frac{1}{\frac{5}{4}}\sinh\left(\frac{5}{4}t\right)$$

$$= \frac{1}{4}\cosh\left(\frac{5}{4}t\right) + \frac{15}{16}\cdot\frac{4}{5}\sinh\left(\frac{5}{4}t\right)$$

$$= \frac{1}{4}\cosh\left(\frac{5}{4}t\right) + \frac{3}{4}\sinh\left(\frac{5}{4}t\right)$$

(4) Find the invese Laplace transform of

$$\frac{2s^2-4}{(s+1)(s-2)(s-3)}$$