Find the Laplace Transforms of the following.

(xv) $2e^{4t} + 3e^{-2t}$ (done)

(1) $L\{e^{-t}\cos 2t\}$ =

Now $L\{\cos 2t\} = \frac{s}{s^2+4}$

By first shifting property

$L\{e^{-t}\cos 2t\} = \frac{s+1}{(s+1)^2+4}$

(2) $L\{t^2 e^{-3t}\}$

Now $L\{t^2\} = \frac{2!}{s^{2+1}} = \frac{2}{s^3}$

By I^{st} Shifting Property

$L\{e^{-3t}t^2\} = \frac{2}{(s+3)^3}$ //

(3) $L\{e^{4t}\sin 2t\cos t\}$.

Now $L\{\sin 2t\cos t\} = ?$

$\boxed{\text{NOW } 2\sin A\cos B = \sin(A+B) + \sin(A-B)}$

$$\therefore L\{\sin 2t \cos t\} = \frac{1}{2} L[\sin 3t + \sin t]$$

$$= \frac{1}{2}\left\{\frac{3}{s^2+9} + \frac{1}{s^2+1}\right\}.$$

Using the I^{st} shifting property.

$$L\{e^{4t} \sin 2t \cos t\} = \frac{1}{2}\left\{\frac{3}{(s-4)^2+9} + \frac{1}{(s-4)^2+1}\right\}$$

$$= \frac{1}{2}\left\{\frac{3}{s^2-8s+25} + \frac{1}{s^2-8s+17}\right\}$$

~~Trigonometric~~ Results

$$\left\{\begin{aligned} \sin x &= \frac{e^{ix} - e^{-ix}}{2i} \\ \cos x &= \frac{e^{ix} + e^{-ix}}{2} \\ \sinh(x) &= \frac{e^{x} - e^{-x}}{2} \\ \cosh(x) &= \frac{e^{x} + e^{-x}}{2} \\ \cos x + i \sin x &= e^{ix} \end{aligned}\right\}$$

Result If $L\{f(t)\} = \bar{f}(s)$ show that

$$L[(\sinh at) f(t)] = \frac{1}{2}\left\{\bar{f}(s-a) - \bar{f}(s+a)\right\}$$

$$L[(\cosh at) f(t)] = \frac{1}{2}\left\{\bar{f}(s-a) + \bar{f}(s+a)\right\}$$

and hence evaluate ① $\sinh 2t \sin 3t$

② $\cosh 3t \cos 2t$

Now $L[(\cosh at) f(t)] = L\left\{\left[\frac{1}{2}(e^{at}+e^{-at})\right] f(t)\right\}$

$= \frac{1}{2}\left[L\{e^{at} f(t)\} + L\{e^{-at} f(t)\}\right]$

$= \frac{1}{2}\left[\bar{f}(s-a) + \bar{f}(s+a)\right]$

By first shifting property

$L[(\sinh at) f(t)] = L\left\{\left[\frac{1}{2}(e^{at}-e^{-at})\right] f(t)\right\}$

$= \frac{1}{2}\left[L\{e^{at} f(t)\} - L\{e^{-at} f(t)\}\right]$

$= \frac{1}{2}\left[\bar{f}(s-a) - \bar{f}(s+a)\right]$

By I^{st} shifting property.

① $L\{\sinh 2t \sin 3t\}$?

Here $L\{\sinh at \underline{f(t)}\} = L\{\sinh 2t \underline{\sin 3t}\}$

Now $L\{\sin 3t\} = \frac{3}{s^2+9} \rightarrow \bar{f}(s)$

$\therefore L\{\sinh 2t \sin 3t\} = \frac{1}{2}\left\{\bar{f}(s-2) - \bar{f}(s+2)\right\}$

$$\therefore\ L\{\sinh 2t \sin 3t\} = \frac{1}{2}\left[\frac{3}{(S-2)^2+9} - \frac{3}{(S+2)^2+9}\right]$$

$$= \frac{12S}{S^4+10S^2+169} \quad \text{(Simplifying)}$$

(2) $L\{\cosh 3t \cos 2t\} = ?$

This is of the form $L\{\cosh at\, f(t)\}$

and we know $\underline{L\{\cosh at\, f(t)\}} = \frac{1}{2}\left[\bar{f}(S-a) + \bar{f}(S+a)\right]$

Now $L\{\cosh 3t \underbrace{\cos 2t}_{f(t)}\}$

$$\therefore\ L\{\cos 2t\} = \frac{S}{S^2+4} = \bar{f}(S)$$

Now $L\{\cosh \underbrace{3}_{a}t \cos 2t\} = \frac{1}{2}\left[\bar{f}(S-3) + \bar{f}(S+3)\right]$

$$= \frac{1}{2}\left[\frac{S-3}{(S-3)^2+4} + \frac{S+3}{(S+3)^2+4}\right]$$

Simplifying $L\{\cosh 3t \cos 2t\} = \frac{2S(S^2-5)}{S^4-10S^2+169}$

Find Laplace Transform of

(1) $e^{-3t}(2\cos 5t - 3\sin 5t)$

(2) $e^{3t}\sin^2 t$

(3) $(t+2)^2 e^t$