

Decomposition Using Functional Dependencies

legal instance

There are usually a variety of constraints (rules) on the data in the real world. An instance of a relation that satisfies all such real-world constraints is called a legal instance of the relation

superkey

a set of one or more attributes that, taken collectively, allows us to identify uniquely a tuple in the relation. Given $r(R)$, a subset K of R is a superkey of $r(R)$ if, in any legal instance of $r(R)$, for all pairs t_1 and t_2 of tuples in the instance of r if $t_1 \neq t_2$, then $t_1[K] \neq t_2[K]$. That is, no two tuples in any legal instance of relation $r(R)$ may have the same value on attribute set K . 3 If no two tuples in r have the same value on K , then a K -value uniquely identifies a tuple in r .

functional dependency

- Given an instance of $r(R)$, we say that the instance **satisfies** the **functional dependency** $\alpha \rightarrow \beta$ if for all pairs of tuples t_1 and t_2 in the instance such that $t_1[\alpha] = t_2[\alpha]$, it is also the case that $t_1[\beta] = t_2[\beta]$.
- We say that the functional dependency $\alpha \rightarrow \beta$ **holds** on schema $r(R)$ if, every legal instance of $r(R)$ satisfies the functional dependency.

We shall use functional dependencies in two ways:

1. To test instances of relations to see whether they *satisfy* a given set F of functional dependencies.
2. To specify constraints on the set of legal relations. We shall thus concern ourselves with *only* those relation instances that satisfy a given set of functional dependencies. If we wish to constrain ourselves to relations on schema $r(R)$ that satisfy a set F of functional dependencies, we say that F **holds** on $r(R)$.

A	B	C	D
a_1	b_1	c_1	d_1
a_1	b_2	c_1	d_2
a_2	b_2	c_2	d_2
a_2	b_3	c_2	d_3
a_3	b_3	c_2	d_4

Observe that $A \rightarrow C$ is satisfied

The functional dependency $C \rightarrow A$ is not satisfied,

trivial Functional Dependency

In general, a functional dependency of the form $\alpha \rightarrow \beta$ is trivial if $\beta \subseteq \alpha$.

1) $A \rightarrow A$

2) $AB \rightarrow A$

3) $AB \twoheadrightarrow B$

ALL ARE CALLED TRIVIAL

1) $A \twoheadrightarrow C$

C IS NOT A SUBSET OF A, SO THIS IS NONTRIVIAL

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It is important to realize that an instance of a relation may satisfy some functional dependencies that are **not required to hold** on the relation's schema. Eg **room number \rightarrow capacity**

<i>building</i>	<i>room_number</i>	<i>capacity</i>
Packard	101	500
Painter	514	10
Taylor	3128	70
Watson	100	30
Watson	120	50

However, we would expect the functional dependency **building, room number \rightarrow capacity** to hold on the classroom schema

inferences in FD

$r(A, B, C)$

if **$A \rightarrow B$** and **$B \rightarrow C$** hold on r

we can infer the functional dependency **$A \rightarrow C$** must also hold on r

closure F^+

Given $F = \{A \rightarrow B, B \rightarrow C, C \rightarrow D\}$, which of the following represents F^+ ?

- ☐ $\{A \rightarrow B, B \rightarrow C, C \rightarrow D, A \rightarrow BCD\}$
- ☐ $\{A \rightarrow B, B \rightarrow C, C \rightarrow D, A \rightarrow C, B \rightarrow D, A \rightarrow D, B \rightarrow CD, A \rightarrow BCD\}$
- ☐ $\{A \rightarrow C, B \rightarrow D, A \rightarrow D\}$
- ☐ $\{A \rightarrow C, B \rightarrow D, A \rightarrow D, A \rightarrow BCD\}$

the set of all functional dependencies that can be inferred given the set F

Lossless Decomposition and Functional Dependencies/to check whether functional dependencies are lossless with the help of functional dependencies

use functional dependencies to show when certain decompositions are lossless

R_1 and R_2 form a lossless decomposition of R if at least one of the following functional dependencies is in F^+ :

- $R_1 \cap R_2 \rightarrow R_1$
- $R_1 \cap R_2 \rightarrow R_2$

if $R_1 \cap R_2$ forms a superkey for either R_1 or R_2

in dep(ID , name, salary, dept name, building, budget)

instructor (ID , name, dept name, salary)
department (dept name, building, budget)

the intersection of these two schemas, which is dept name

dept name \rightarrow dept name, building, budget, the lossless-decomposition rule is satisfied.

relation schema $r(R)$ into $r_1(R_1)$ and $r_2(R_2)$, where $R_1 \cap R_2 \rightarrow R_1$

following SQL constraints must be imposed on the decomposed schema

- $R_1 \cap R_2$ is the primary key of r_1 .
This constraint enforces the functional dependency.
- $R_1 \cap R_2$ is a foreign key from r_2 referencing r_1 .
This constraint ensures that each tuple in r_2 has a matching tuple in r_1 , without which it would not appear in the natural join of r_1 and r_2 .