

## Homework 2: Q 1-2

$$\forall n \in \mathbb{N}, \sum_{k=1}^n k(k+1) = \frac{n(n+1)(n+2)}{3}$$

1) Base Case ( $n=1$ ):  $\sum_{k=1}^1 k(k+1) = 1(1+1) = 2$   $\frac{1(1+1)(1+2)}{3} = 2$  ✓

2) Inductive Hypothesis: ( $n=k$ )

$$\Rightarrow \sum_{k=1}^k k(k+1) = \frac{k(k+1)(k+2)}{3}$$

(inductive assumption)

3) Inductive step ( $n=k+1$ )

Prove:  $\Rightarrow \sum_{k=1}^{k+1} k(k+1) = \frac{(k+1)(k+2)(k+3)}{3}$

Let

$$\sum_{k=1}^{k+1} k(k+1) = \left( \sum_{k=1}^k k(k+1) \right) + (k+1)(k+2)$$

Inductive hypothesis:

$$\frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$

$$\Rightarrow (k+1)(k+2) \left( \frac{k}{3} + 1 \right) \Rightarrow (k+1)(k+2) \cdot \frac{k+3}{3} = \frac{(k+1)(k+2)(k+3)}{3} \quad \checkmark$$

5) conclude:

$$\sum_{k=1}^n k(k+1) = \frac{n(n+1)(n+2)}{3}, \quad \forall n \in \mathbb{N}$$

QED