Principal Component Analysis

import library

In [ ]:

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.colors as colors
from matplotlib import cm
```

load data

In [ ]:

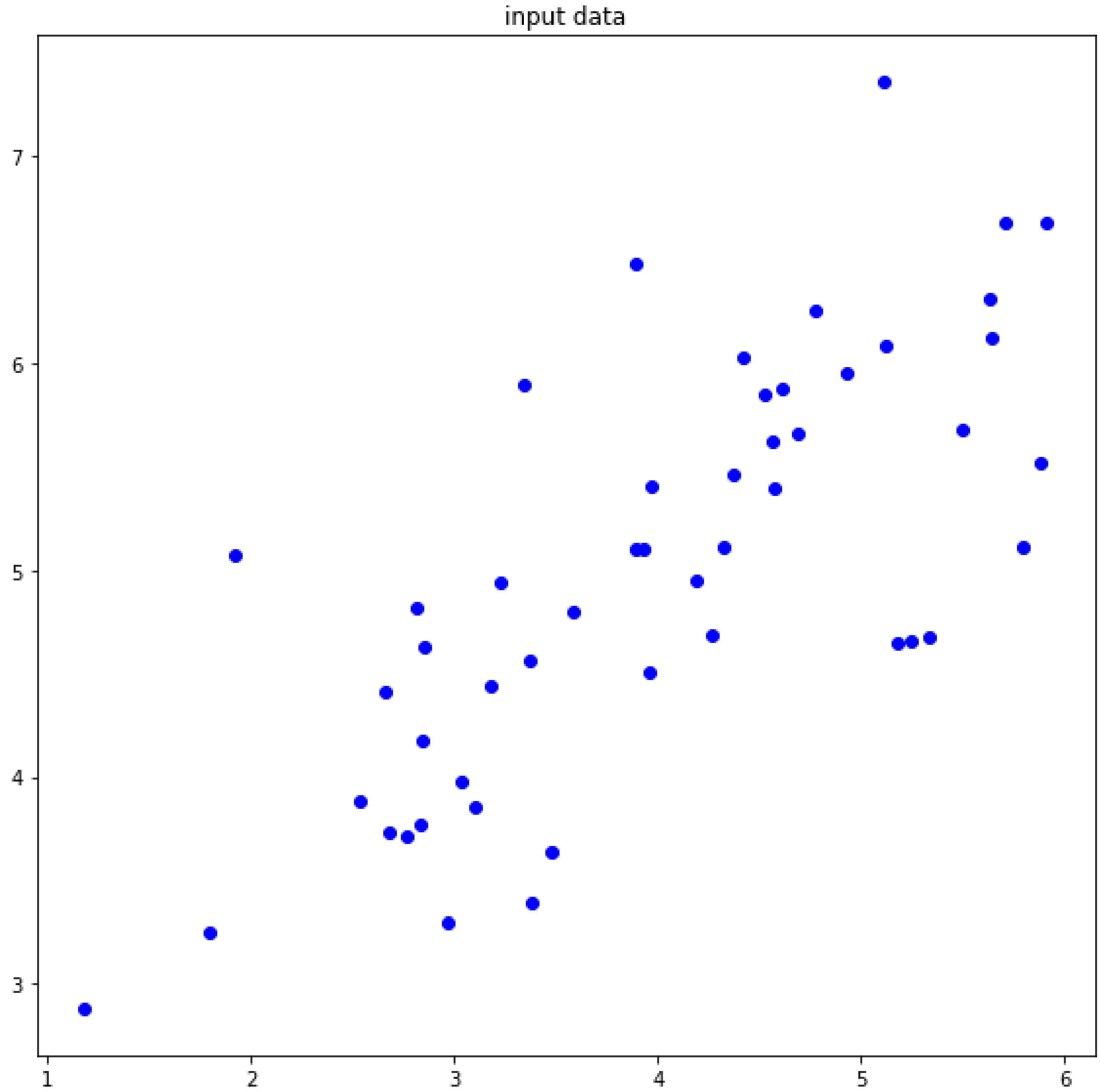
```
fname_data  = 'assignment_12_data.txt'
feature0    = np.genfromtxt(fname_data, delimiter=',')

number_data     = np.size(feature0, 0)
number_feature  = np.size(feature0, 1)

print('number of data : {}'.format(number_data))
print('number of feature : {}'.format(number_feature))
```

```
number of data : 50
number of feature : 2
```

plot the input data

In [ ]:

```
plt.figure(figsize=(8,8))
plt.title('input data')

x0 = feature0[:,0]
y0 = feature0[:,1]

plt.scatter(x0, y0, color='blue')

plt.tight_layout()
plt.show()
```



Normalization (Z-scoring)

- shape of feature = $n \times m$ where n is the number of data and m is the dimension of features

In [ ]:
```
def normalize(feature):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    x = feature[:,0]
    y = feature[:,1]
    x_mean = x.mean()
    y_mean = y.mean()

    x = (x - x_mean)/x.std()
    y = (y - y_mean)/y.std()

    feature_normalize = np.zeros((feature[:,0].size,2))
    feature_normalize[:,0] = x
    feature_normalize[:,1] = y

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++
```

```
    return feature_normalize
```

In [ ]:

```
feature = normalize(feature0)

x   = feature[:, 0]
y   = feature[:, 1]

min_x = np.min(x)
min_y = np.min(y)

max_x = np.max(x)
max_y = np.max(y)
```

compute covariance matrix

- shape of feature = $n \times m$ where n is the number of data and m is the dimension of features

In [ ]:

```
def compute_covariance(feature):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    number_data = np.size(feature, 0)
    Sigma = np.matmul(feature.T, feature) / number_data

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return Sigma
```

compute principal components

- `np.linalg.eig`
- `argsort()`
- return the eigenvalues and the eigenvectors in a decreasing order according to the eigenvalues

In [ ]:

```
def compute_principal_component(feature):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    Sigma = compute_covariance(feature)
    S, U = np.linalg.eig(Sigma)  #  Eigenvalue: S, Eigenvector: U
    sort_value = S.argsort()
    S = S[sort_value[::-1]]


    principal_component_1 = S[0] * U[:, 0]
    principal_component_2 = S[1] * U[:, 1]

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++
```

```
    return (principal_component_1, principal_component_2)
```

compute the projection of point onto the axis

- `np.matmul`
- `np.dot`
- shape of feature = $n \times m$ where n is the number of data and m is the dimension of features
- shape of vector = $m \times 1$ where m is the dimension of features

In [ ]:

```
def compute_projection_onto_line(feature, vector):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    temp = (np.dot(feature, vector) / np.dot(vector,vector)).reshape(50,1)
    projection = np.matmul(temp, vector.reshape(1,2))

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    return projection
```

compute the principal components and the projection of feature

In [ ]:

```
(principal_component_1, principal_component_2) = compute_principal_component(feature)

projection1 = compute_projection_onto_line(feature, principal_component_1)
projection2 = compute_projection_onto_line(feature, principal_component_2)
```

---

functions for presenting the results

In [ ]:

```
def function_result_01():

    plt.figure(figsize=(8,8))
    plt.title('data normalized by z-scoring')
    plt.scatter(x, y, color='blue')

    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)

    plt.tight_layout()
    plt.show()
```

In [ ]:

```
def function_result_02():

    plt.figure(figsize=(8,8))
```

```
    plt.title('principal components')

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    plt.scatter(x, y, marker= 'o', color='blue')

    plt.arrow(0,0,principal_component_1[0],principal_component_1[1], width= 0.02, heac
    plt.arrow(0,0,principal_component_2[0],principal_component_2[1], width= 0.02, heac

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)

    plt.tight_layout()
    plt.show()
```

In [ ]:

```
def function_result_03():

    plt.figure(figsize=(8,8))
    plt.title('first principle axis')

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    Sigma = compute_covariance(feature)
    S, U = np.linalg.eig(Sigma)

    plt.scatter(x, y, marker= 'o', color='blue')
    plt.plot([-5*U[0][0], 5*U[0][0]], [-5*U[1][0], 5*U[1][0]], color='red')

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)

    plt.tight_layout()
    plt.show()
```

In [ ]:

```
def function_result_04():

    plt.figure(figsize=(8,8))
    plt.title('second principle axis')

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    Sigma = compute_covariance(feature)
    S, U = np.linalg.eig(Sigma)

    plt.scatter(x, y, marker= 'o', color='blue')
    plt.plot([-5*U[0][1], 5*U[0][1]], [-5*U[1][1], 5*U[1][1]], color='red')


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++
```

```
    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)

    plt.tight_layout()
    plt.show()
```

In [ ]:

```
def function_result_05():

    plt.figure(figsize=(8,8))
    plt.title('projection onto the first principle axis')

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    Sigma = compute_covariance(feature)
    S, U = np.linalg.eig(Sigma)

    plt.scatter(x, y, marker= 'o', color='blue')
    plt.plot([-5*U[0][0], 5*U[0][0]], [-5*U[1][0], 5*U[1][0]], color='red')


    plt.scatter(projection1[:,0], projection1[:,1], marker= 'o', c= "g")

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)

    plt.tight_layout()
    plt.show()
```

In [ ]:

```
def function_result_06():

    plt.figure(figsize=(8,8))
    plt.title('projection onto the second principle axis')

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    Sigma = compute_covariance(feature)
    S, U = np.linalg.eig(Sigma)

    plt.scatter(x, y, marker= 'o', color='blue')
    plt.plot([-5*U[0][1], 5*U[0][1]], [-5*U[1][1], 5*U[1][1]], color='red')

    plt.scatter(projection2[:,0], projection2[:,1], marker='o', c= "g")



    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)

    plt.tight_layout()
    plt.show()
```

In [ ]:

```
def function_result_07():
```

```
    plt.figure(figsize=(8,8))
    plt.title('projection onto the first principle axis')

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    Sigma = compute_covariance(feature)
    S, U = np.linalg.eig(Sigma)

    plt.scatter(x, y, marker= 'o', color='blue')
    plt.plot([-5*U[0][0], 5*U[0][0]], [-5*U[1][0], 5*U[1][0]], color='red')

    plt.scatter(projection1[:,0], projection1[:,1], marker = 'o', c= "g")

    for i in range(number_data):
        plt.plot([feature[i,0], projection1[i,0]], [feature[i,1], projection1[i,1]],'-

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)

    plt.tight_layout()
    plt.show()
```

In [ ]:

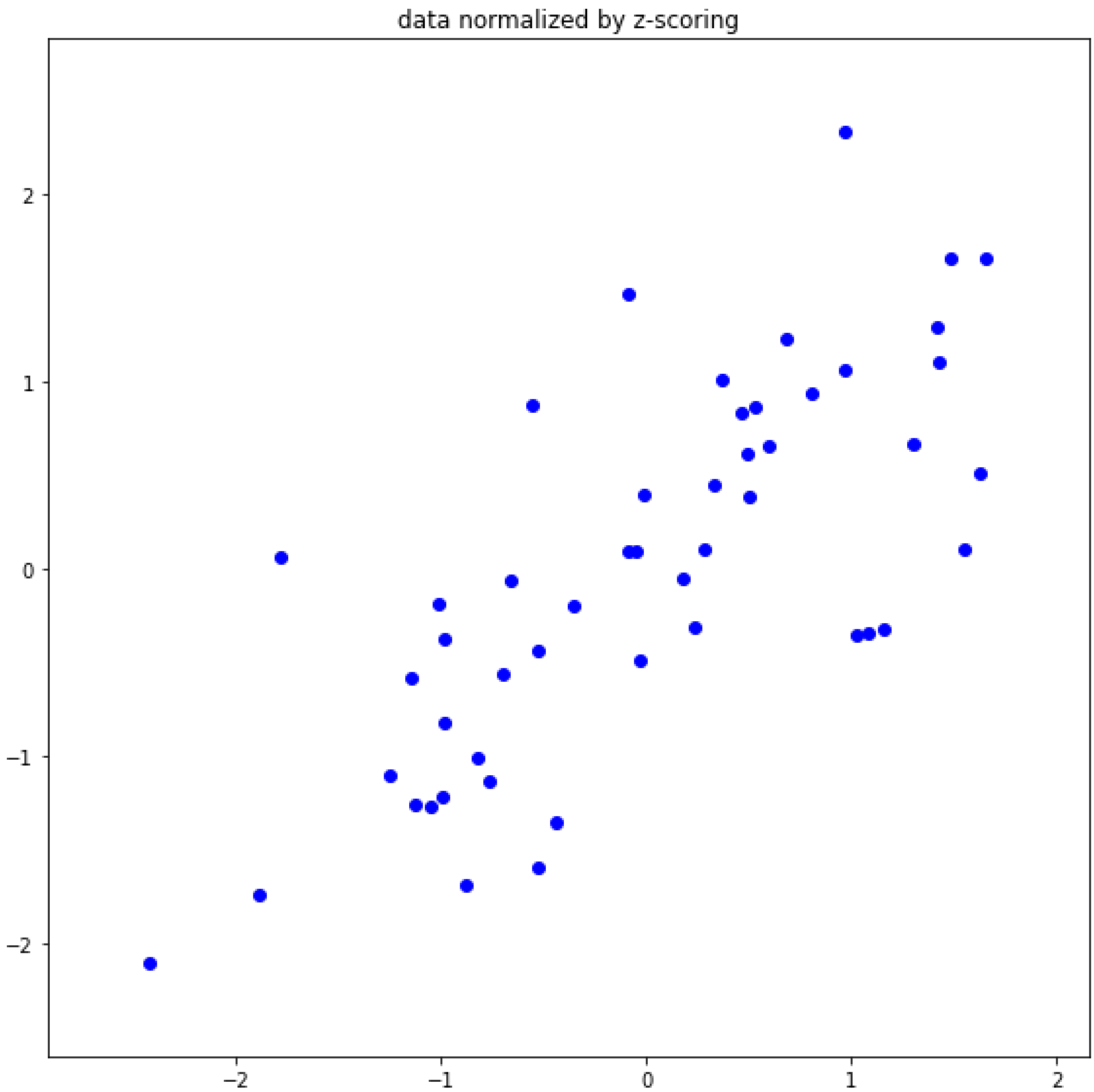
```
def function_result_08():

    plt.figure(figsize=(8,8))
    plt.title('projection to the second principle axis')

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    Sigma = compute_covariance(feature)
    S, U = np.linalg.eig(Sigma)

    plt.scatter(x, y, marker= 'o', color='blue')
    plt.plot([-5*U[0][1], 5*U[0][1]], [-5*U[1][1], 5*U[1][1]], color='red')

    plt.scatter(projection2[:,0], projection2[:,1], marker = 'o', c= "g")

    for i in range(number_data):
        plt.plot([feature[i,0], projection2[i,0]], [feature[i,1], projection2[i,1]],'-


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)

    plt.tight_layout()
    plt.show()
```

---

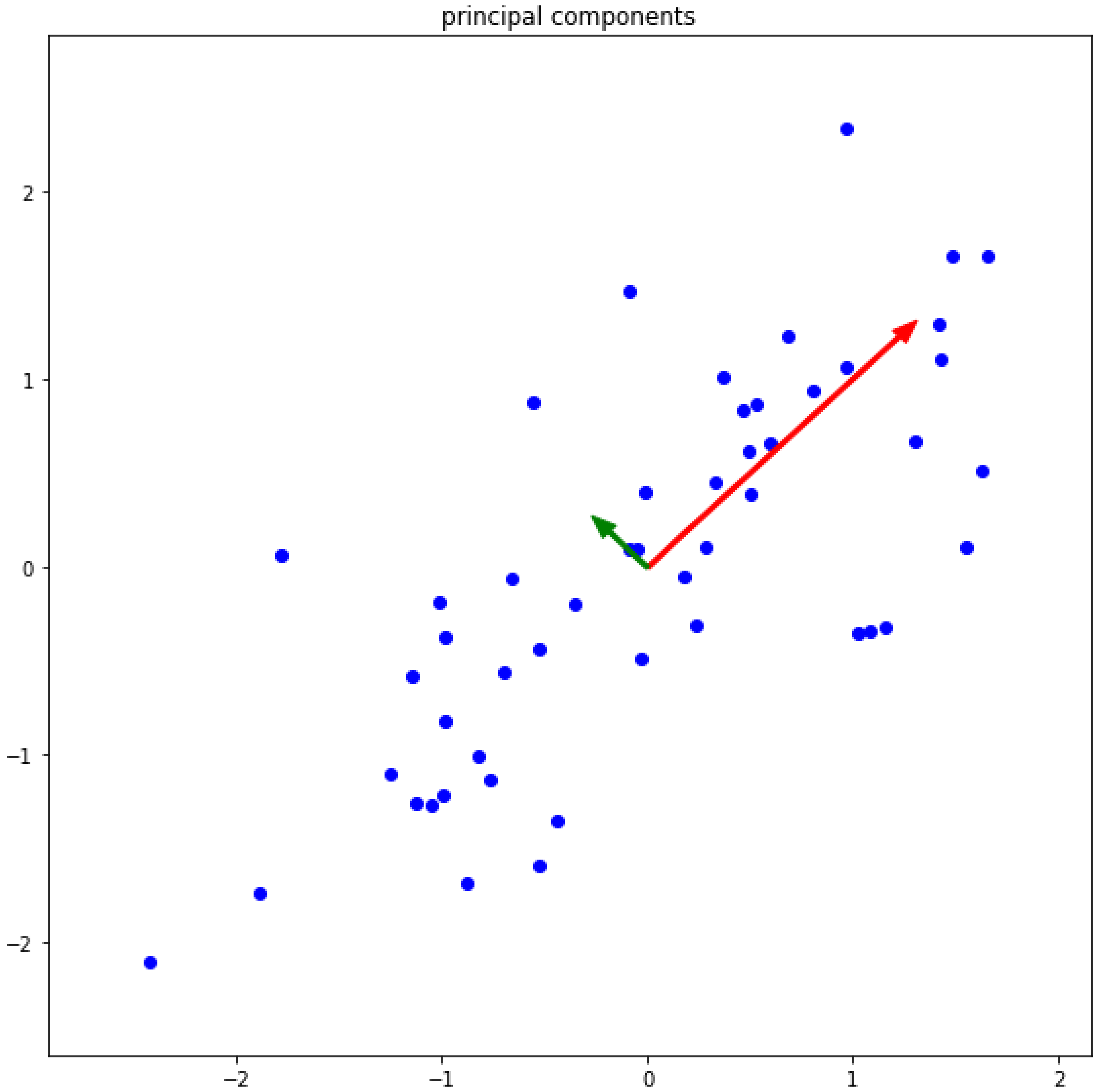
results

In [ ]:
```
number_result = 8

for i in range(number_result):
    title = '## [RESULT {:02d}]'.format(i+1)
    name_function = 'function_result_{:02d}()'.format(i+1)

    print('**************************************************')
    print(title)
    print('**************************************************')
    eval(name_function)
```

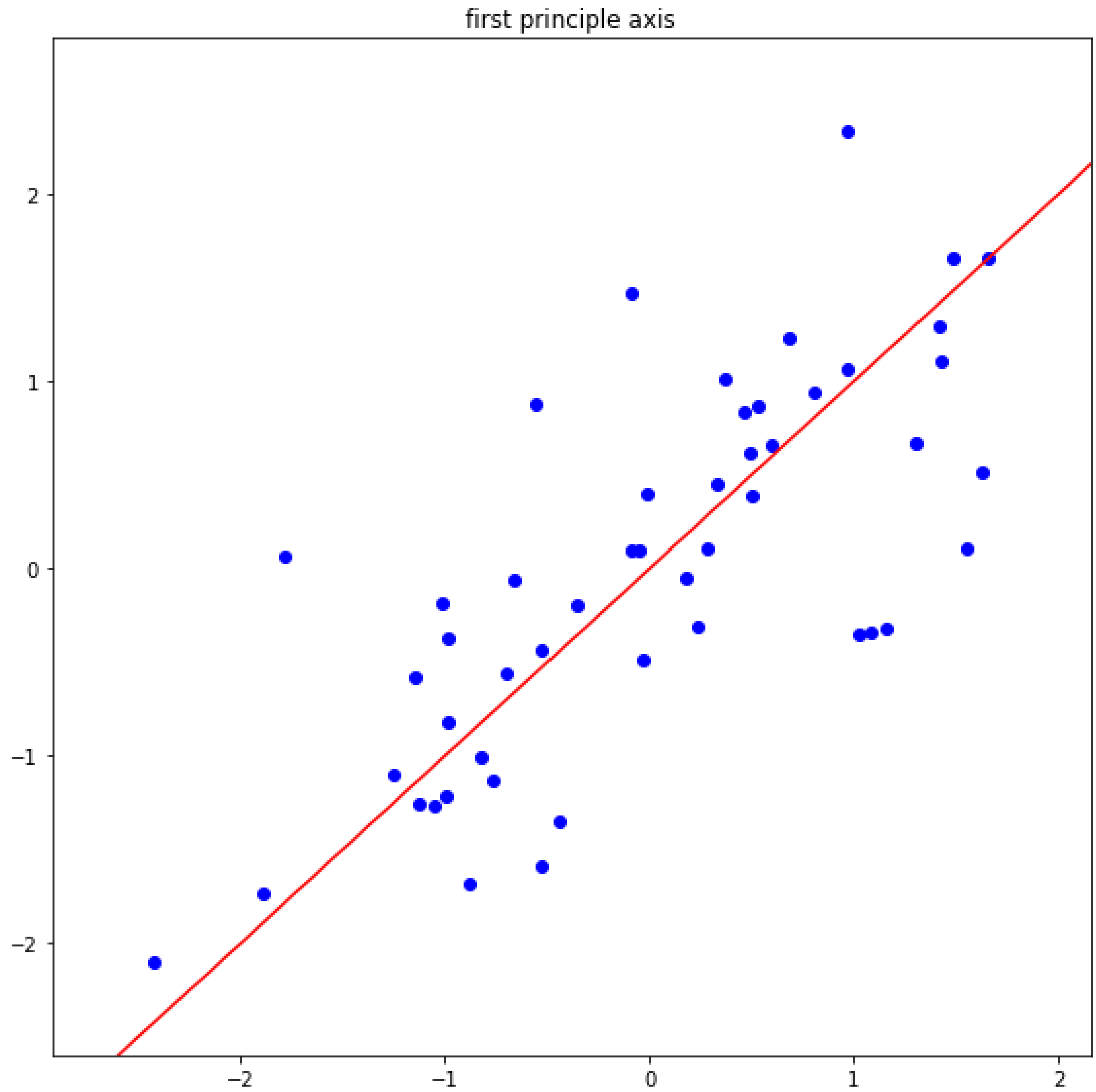
```
**************************************************
## [RESULT 01]
**************************************************
```



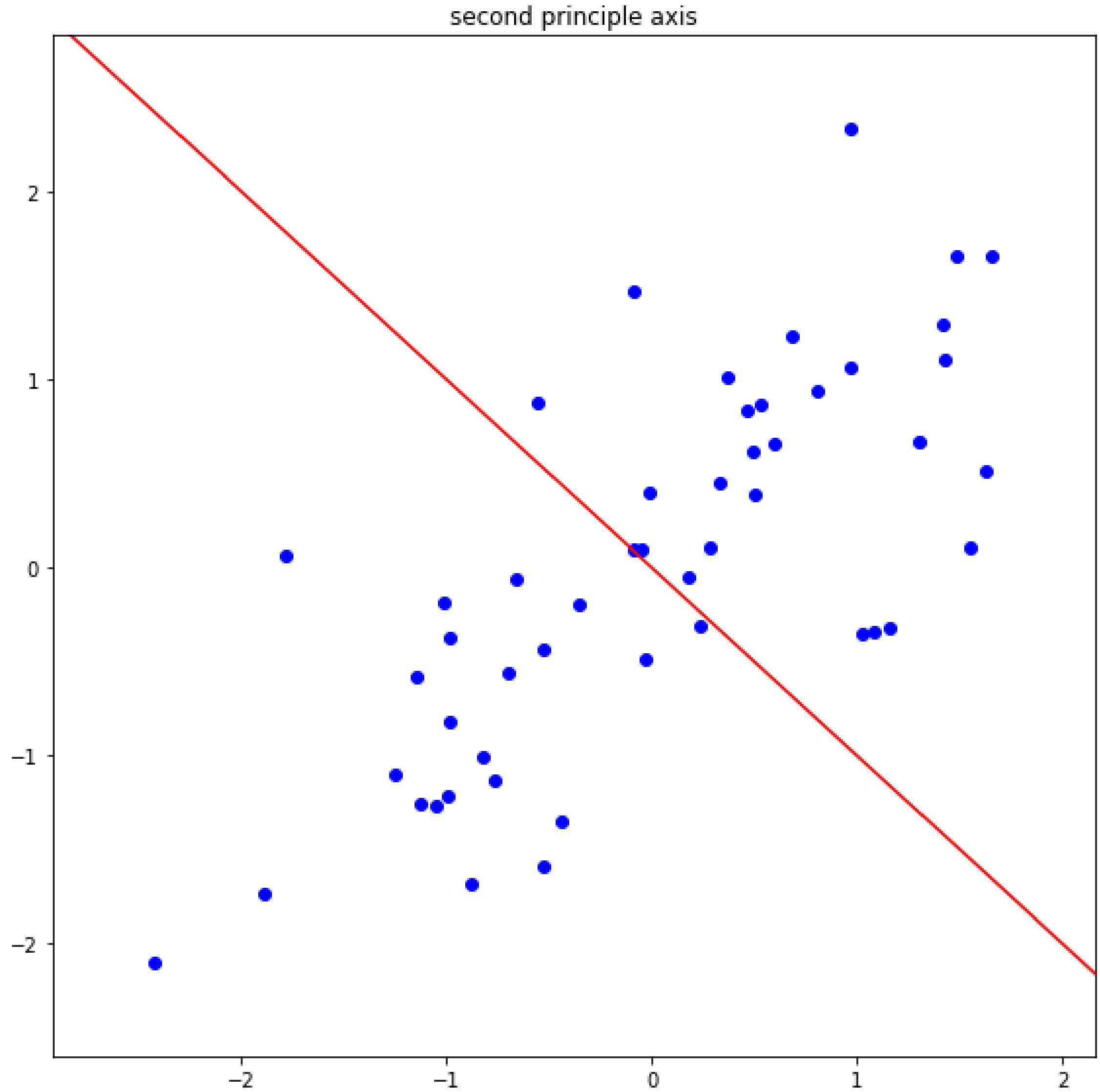

```
**************************************************
## [RESULT 02]
**************************************************
```



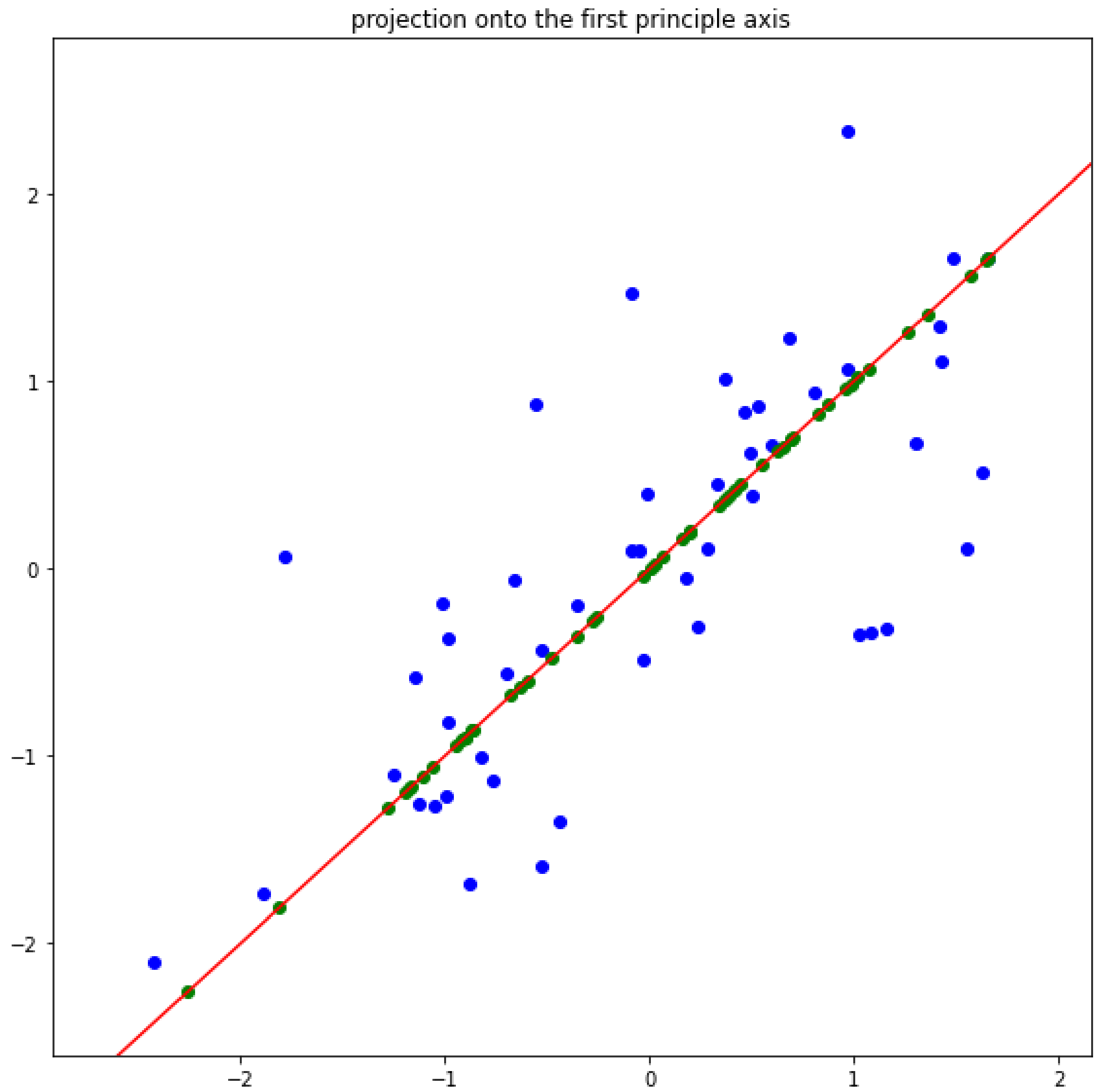

```
**************************************************
## [RESULT 03]
**************************************************
```



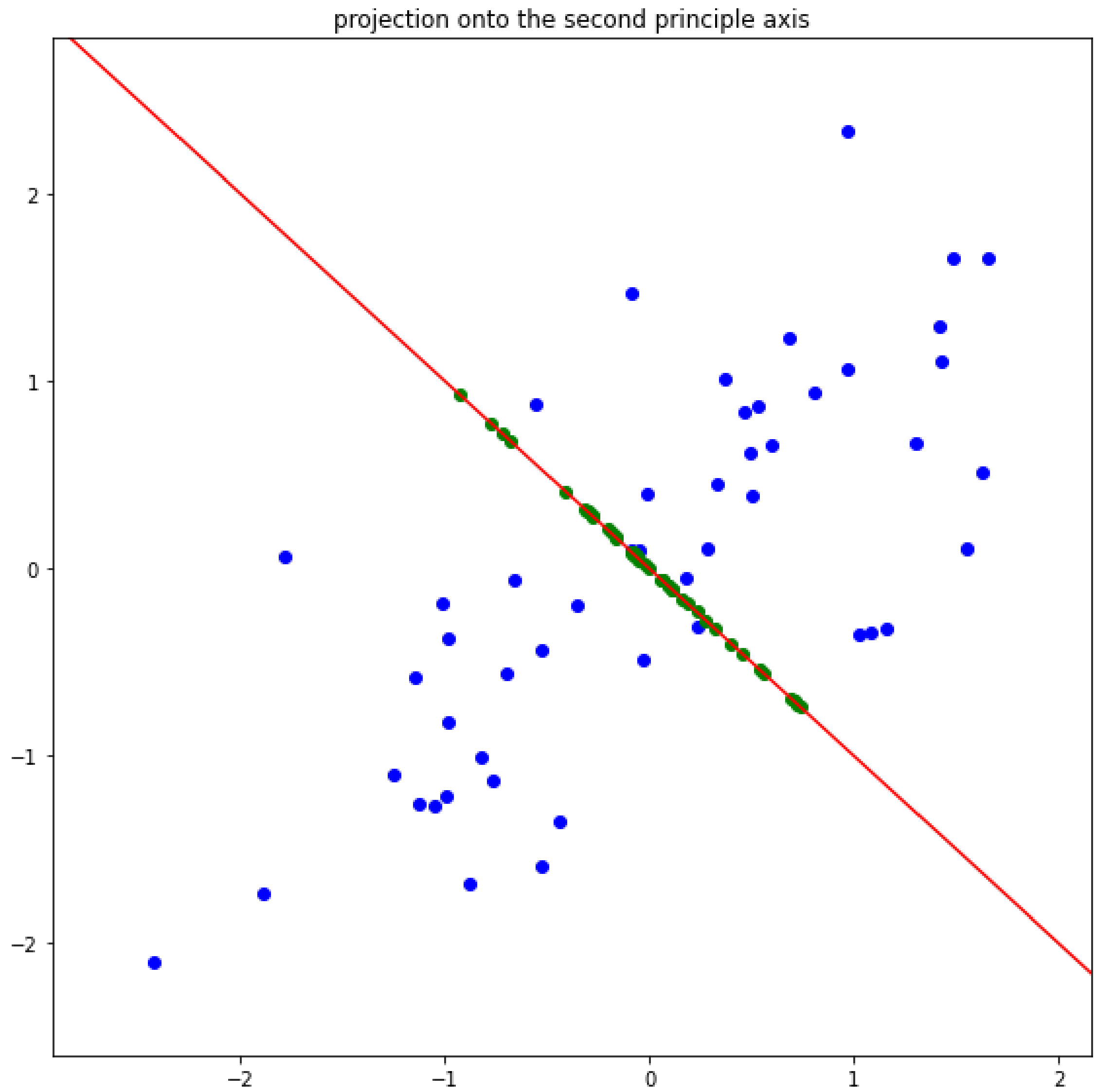

```
*************************************************
## [RESULT 04]
*************************************************
```



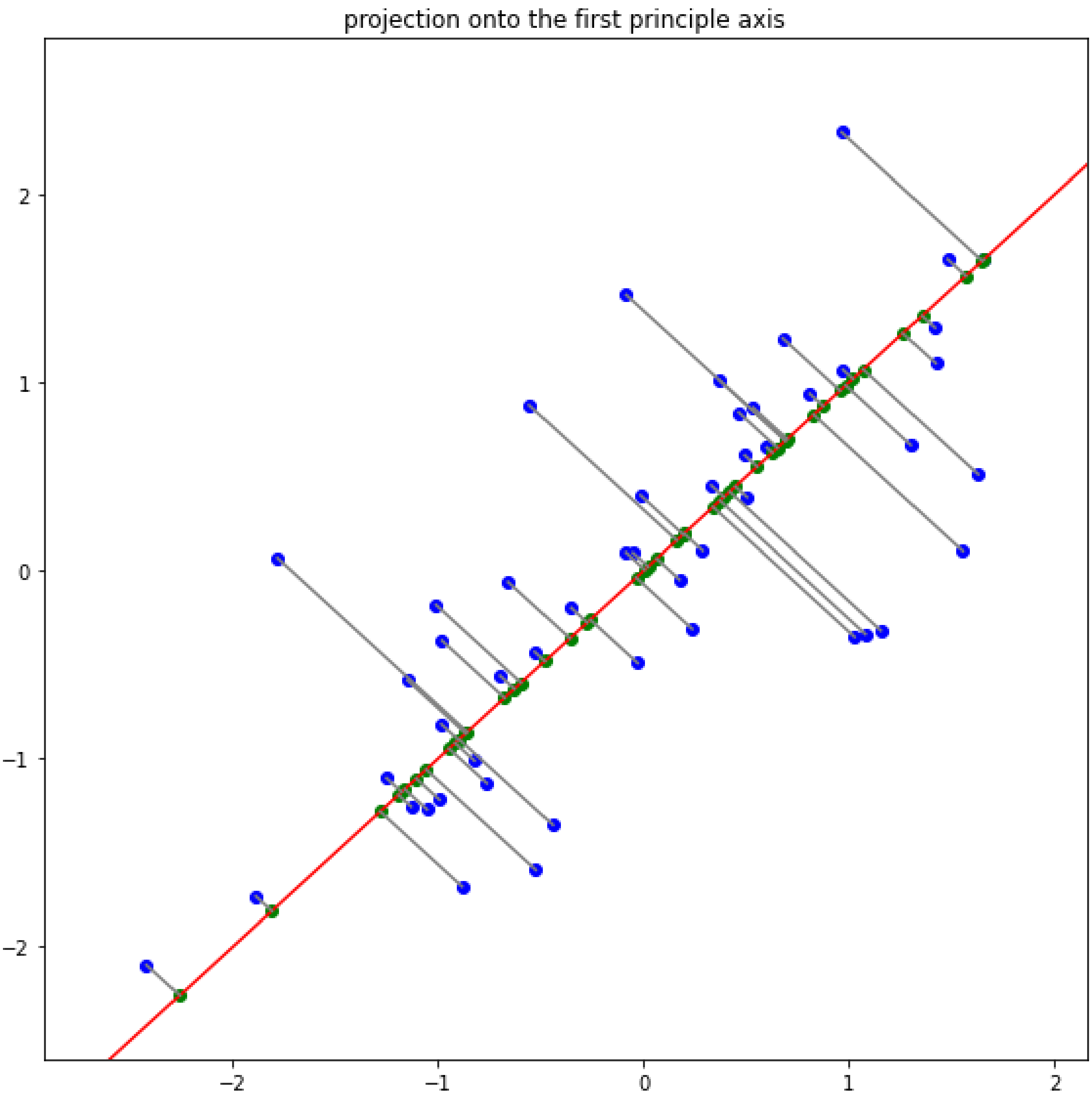

```
**************************************************
## [RESULT 05]
**************************************************
```



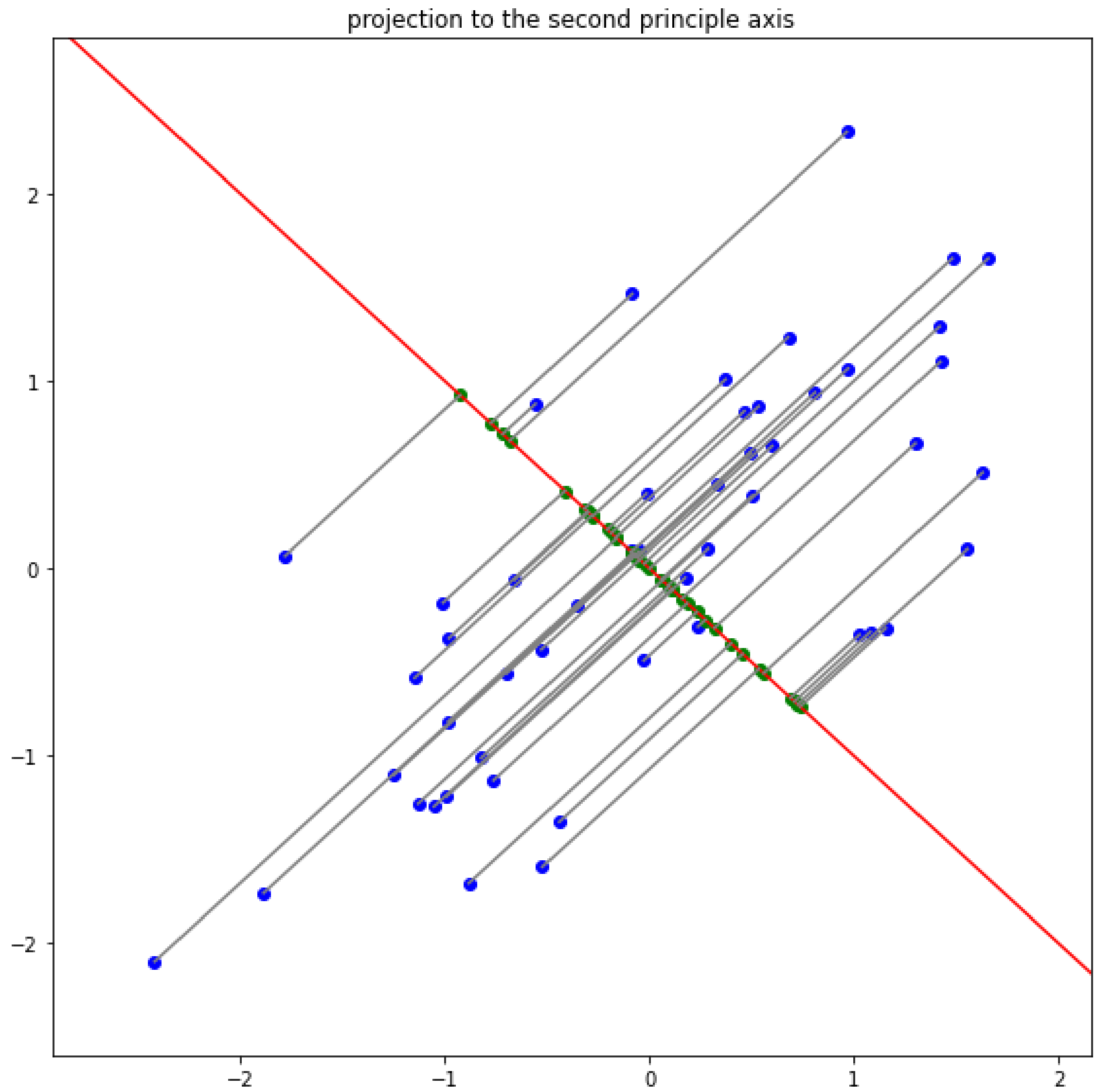

```
**************************************************
## [RESULT 06]
**************************************************
```




```
**************************************************
## [RESULT 07]
**************************************************
```




```
**************************************************
## [RESULT 08]
**************************************************
```


projection to the second principle axis
2
1
0
−1
−2
−2
−1
0
1
2