西安電子科技大學
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人工智能学院

智能数据挖掘课程作业报告

基于 kNN 的高光谱图像分类

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目录

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1 数据集简介

使用 Indian Pines 数据集，该数据集是由 AVIRIS 传感器在印第安纳州西北部的 Indian Pines 试验场上空采集的。图像大小为，共有 224 个光谱带。其空间分辨率为每像素 20 米。一般在实验中通过去除 4 个空带和 20 个损坏带，保留了 200 个光谱通道。该数据将共有 10249 个像素包含了真值信息，它们分别属于 16 个不同的类。

数据集地址: (数据集为.mat 格式) `http://www.ehu.eus/ccwintco/index.php/Hyperspectral_Remote_Sensing_Scenes`(下载 corrected Indian Pines 和 Indian Pines groundtruth)

2 原理分析

2.1 kNN 方法

k 近邻 (k-Nearest Neighbor, kNN) 学习是一种常用的监督学习方法，其工作原理：给定测试样本，基于某种距离度量找出训练集中与其最靠近的 k 个训练样本，然后基于这 k 个“邻居”的信息来进行预测。通常，在分类任务中可使用“投票法”，即选择这 k 个样本中出现最多的类别标记作为预测结果；在回归任务中可使用“平均法”，即将这 k 个样本的实值输出标记的平均值作为预测结果；还可基于距离远近进行加权平均或加权投票，距离越近的样本权重越大。分类中 kNN 的伪代码如算法 1 所示。

Algorithm 1 kNN 方法

Input: X: 训练数据, Y: X 的标签类, x: 未知数据;
Output: x 的预测类别;
for $x^{'} \in$ X **do**
    计算距离 $d(x^{'}, x)$;
end for
计算包含 k 个最短距离 $d(X_i, x)$ 的下标集合 I;
return 在 I 中最多的标签类别 Y_i;

2.2 评价指标

为了能够公正客观评价分类方法的表现，需要选取相应的评价指标。有四种常用的高光谱图像分类评价指标，分别为：总体分类精度 (Overall Accuracy，OA)、类别分类精度 (Class Accuracy，CA)、平均分类精度 (Average Accuracy，AA) 和 Kappa 系数。

假设 N 为所有测试样本的总个数，通过将获得的分类结果图和地面真实图像相同位置进行对比可计算出混淆矩阵 $\mathrm{K} \in \mathbb{R}^{C\times C}$，其中 C 表示样本的总类别数，且 $N = \sum_{i=1}^{C}\sum_{j=1}^{C}\mathrm{K}_{ij}$，$\mathrm{K}_{ij}$ 的值代表第 i 类样本被划分成第 j 类的样本数量。OA 代表分类结果与参考数据相吻合的概率，其计算公式为：

$$\mathrm{OA} = \frac{\sum\limits_{i=1}^{C}\mathrm{K}_{ii}}{N} \tag{1}$$

CA_i 代表第 i 类样本被正确分类的概率，其计算公式为：

$$\mathrm{CA}_i = \frac{\mathrm{K}_{ii}}{\sum\limits_{j=1}^{C}\mathrm{K}_{ij}} \tag{2}$$

AA 代表各个类别分类精度的平均值，其计算公式为：

$$\mathrm{AA} = \frac{\sum\limits_{i=1}^{C}\mathrm{CA}_i}{C} \tag{3}$$

Kappa 系数是一种分类一致性评价指标，它在评价分类精度时还考虑了不确定性对分类结果造成的影响，其计算公式为：

$$\mathrm{Kappa} = \frac{N\sum\limits_{i=1}^{C}\mathrm{K}_{ii} - \sum\limits_{i=1}^{C}\left(\sum\limits_{j=1}^{C}\mathrm{K}_{ij}\sum\limits_{j=1}^{C}\mathrm{K}_{ji}\right)}{N^2 - \sum\limits_{i=1}^{C}\left(\sum\limits_{j=1}^{C}\mathrm{K}_{ij}\sum\limits_{j=1}^{C}\mathrm{K}_{ji}\right)} \tag{4}$$

3　实验过程

3.1　数据预处理

通过 Python 中的 `scipy.io.loadmat` 导入 Indian Pines 高光谱数据集并查看相关信息得到，数据集的形状为 $(145, 145, 200)$，ground truth 的形状是 $(145, 145)$，其中标签类别及数目如表 1 所示，查看不同光谱下的图像示例如图 1 所示。

表 1: 标签类别及数目

0	11	2	14	10	3	6	12	5	8	15	4	13	16	1	7	9
10776	2455	1428	1265	972	830	730	593	483	478	386	237	205	93	46	28	20

所有不同分类方法均采用 MIN-MAX 归一化预处理数据,数据集分割代码 `train_test_split (pca, y, test_size=0.95, random_state=42, stratify=y)` 中的 `stratify=y` 表示按每类比例分割，得到训练集标签类别及数目如表 2 所示。

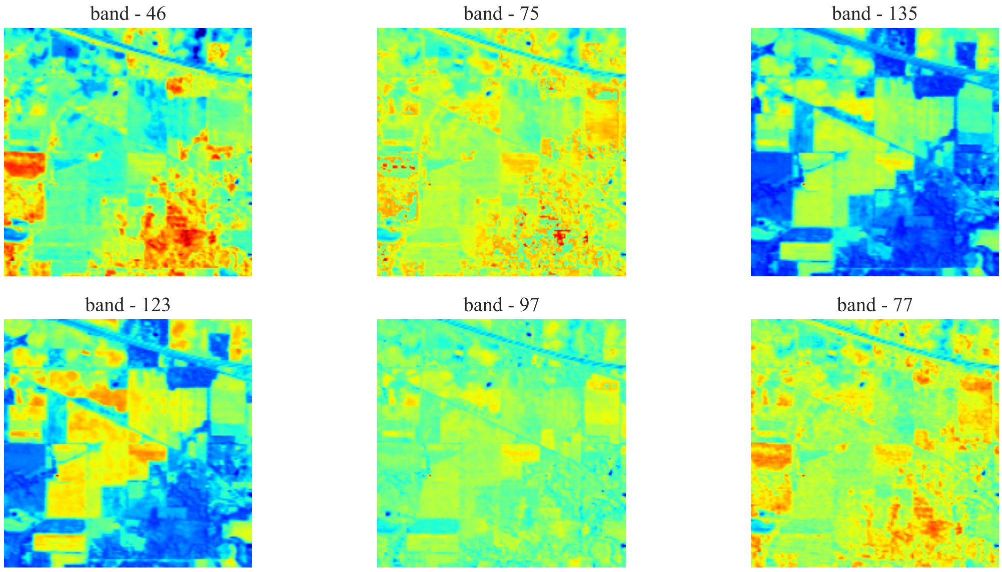


图 1: 不同光谱下的图像示例

表 2: 训练集标签类别及数目

0	11	2	14	10	3	6	12	5	8	15	4	13	16	1	7	9
539	123	71	63	49	41	37	30	24	24	19	12	10	5	2	1	1

3.2 kNN 进行分类

首先用 PCA 降成 $2\sim16$ 维等 15 个不同特征维度，每个不同特征维度 kNN 中 k 取值为 $1\sim20$，得到不同特征维度的最好准确率如表 3 所示，以及不同 k 值下的准确率对比如图 2 所示，并且可以求出在特征维度为 8 且 $k=7$ 时有最大准确率 0.645。

表 3: kNN 分类不同特征维度最好准确率

维度	2	3	4	5	6	7	8
准确率	0.588	0.629	0.627	0.635	0.642	0.642	0.643
9	10	11	12	13	14	15	16
0.639	0.637	0.637	0.638	0.640	0.640	0.640	0.640

以最好的参数实验可以得到分类结果为 $\mathrm{OA}=0.645$，$\mathrm{AA}=0.352, \mathrm{kappa}=0.466$，$k$NN 分类结果图和 Ground Truth 分别如图 3 所示和图 4 所示。进行 10 折交叉验证可以得到准确率为 0.670。

kNN 分类得到的混淆矩阵和标准化后的混淆矩阵见附录。

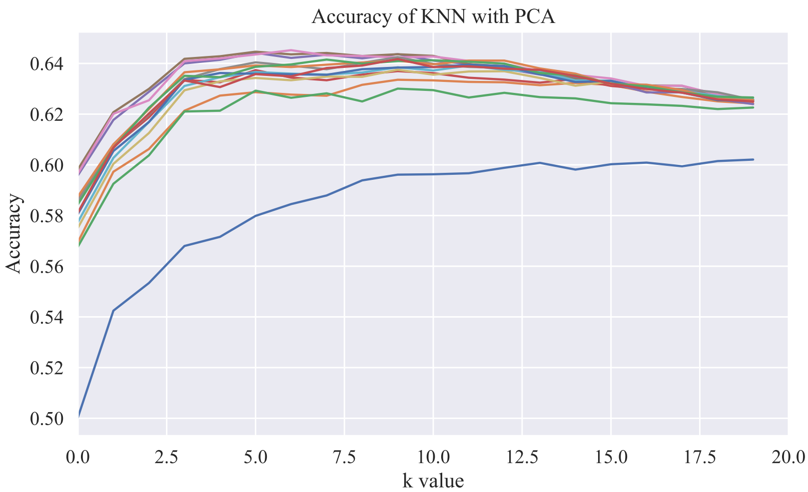


图 2: kNN 方法不同 k 值下的准确率

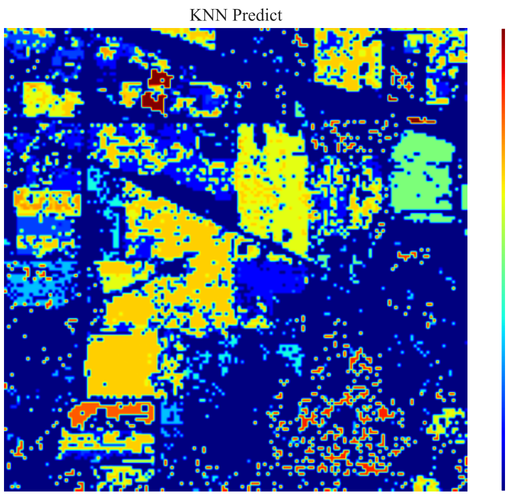


图 3: kNN 分类结果图

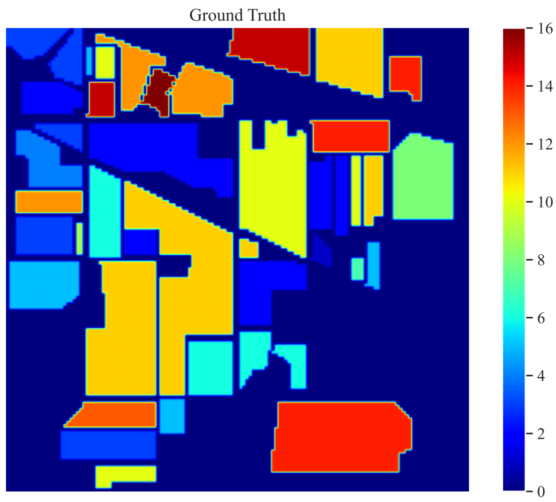


图 4: Ground Truth

4 附录

In [1]:
```
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
from scipy.io import loadmat
from sklearn.model_selection import train_test_split
from sklearn.neighbors import KNeighborsClassifier
from sklearn import metrics
from sklearn.decomposition import PCA
from sklearn.metrics import cohen_kappa_score, confusion_matrix
from operator import truediv
import time
%matplotlib inline
```

In [2]:
```
sns.set_style("darkgrid", {"grid.color": ".6", "grid.linestyle": ":"})
sns.set_theme(font='Times New Roman', font_scale=1.2)
plt.rc("figure", autolayout=True)
# Chinese support
plt.rcParams['font.sans-serif'] = ['SimHei']
plt.rcParams['axes.unicode_minus'] = False
```

In [3]:
```
data = loadmat('./Indian_pines_corrected.mat')['indian_pines_corrected']
labels = loadmat('./Indian_pines_gt.mat')['indian_pines_gt']
print(f"Dataset: {data.shape}\nGround Truth: {labels.shape}") # 打印形状
print(np.unique(labels))
pd.DataFrame(pd.Series(labels.reshape(-1,)).value_counts()).T
```

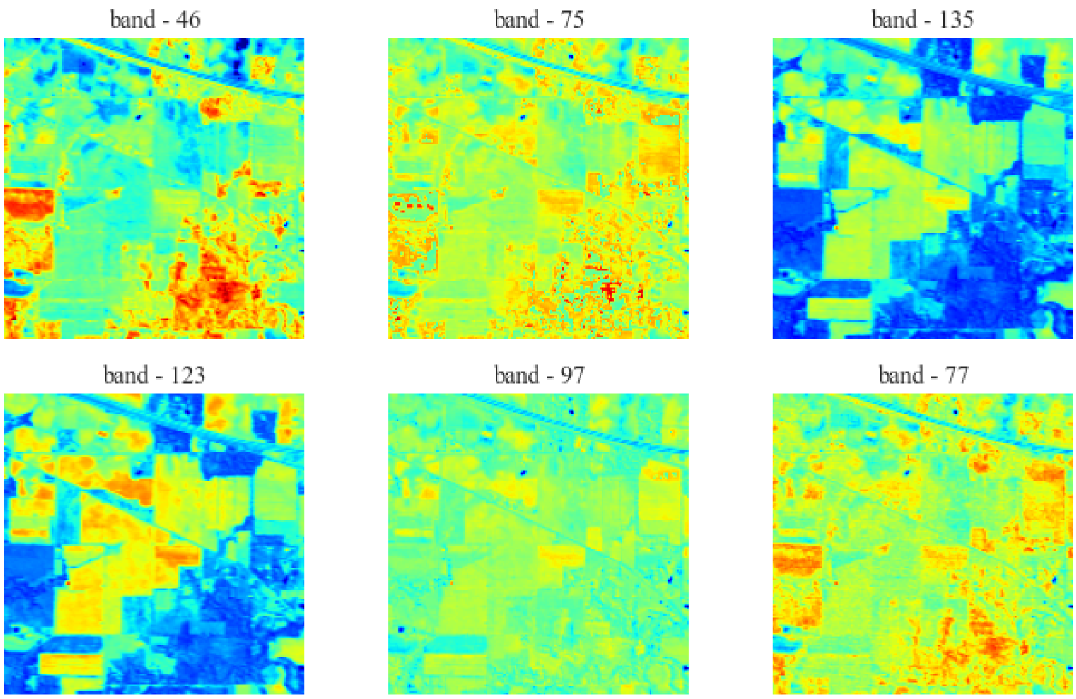
```
Dataset: (145, 145, 200)
Ground Truth: (145, 145)
[ 0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16]
```

Out[3]:

	0	11	2	14	10	3	6	12	5	8	15	4	13	16	\
0	10776	2455	1428	1265	972	830	730	593	483	478	386	237	205	93	

	1	7	9
0	46	28	20

In [4]:
```
sns.axes_style('whitegrid')
fig = plt.figure(figsize=(12, 6))

# 查看不同光谱下的图像示例
for i in range(1, 1+6):
    fig.add_subplot(2, 3, i)
    band = np.random.randint(data.shape[2])
    plt.imshow(data[:, :, band], cmap='jet')
    plt.axis('off')
    plt.title(f'band - {band}')

plt.savefig('./document/figure/diffband6.pdf')
plt.show()
```



In [5]:
```
def extract_pixels(X, y, save_name='indian_pines_all'):
    q = X.reshape(-1, X.shape[2])
    df = pd.DataFrame(q)
    df = pd.concat([df, pd.DataFrame(y.ravel())], axis=1)
```

```
        df.columns= [f'band{i}' for i in range(1, 1+X.shape[2])]+['class']
        #df.to_csv(f'data/{save_name}.csv')
        return df

    df = extract_pixels(data, labels, save_name='indian_pines_all')
    df.head()
```

Out[5]:

```
   band1  band2  band3  band4  band5  band6  band7  band8  band9  band10  ...  \
0   3172   4142   4506   4279   4782   5048   5213   5106   5053    4750  ...
1   2580   4266   4502   4426   4853   5249   5352   5353   5347    5065  ...
2   3687   4266   4421   4498   5019   5293   5438   5427   5383    5132  ...
3   2749   4258   4603   4493   4958   5234   5417   5355   5349    5096  ...
4   2746   4018   4675   4417   4886   5117   5215   5096   5098    4834  ...

   band192  band193  band194  band195  band196  band197  band198  band199  \
0     1094     1090     1112     1090     1062     1069     1057     1020
1     1108     1104     1117     1091     1079     1085     1064     1029
2     1111     1114     1114     1100     1065     1092     1061     1030
3     1122     1108     1109     1109     1071     1088     1060     1030
4     1110     1107     1112     1094     1072     1087     1052     1034

   band200  class
0     1020      3
1     1020      3
2     1016      3
3     1006      3
4     1019      3

[5 rows x 201 columns]
```

In [6]:

```
# x = df[df['class'] != 0] # 标签 0 没有关键的类别，去除
x = df
X = x.iloc[:, :-1].values
y = x.loc[:, 'class'].values
X.shape
```

Out[6]: (21025, 200)

In [7]:

```
from sklearn import preprocessing
```

```
def norm(X):
    min_max_scaler = preprocessing.MinMaxScaler()
    X = min_max_scaler.fit_transform(X)
    return X

def standartize(X): # 数据标准化
    scaler = preprocessing.StandardScaler().fit(X)
    X = scaler.transform(X)
    return X

X = norm(X)
```

4.1 KNN classification

```
In [8]: acc = np.zeros((15,20))

        start = time.time()
        for hell in range(2,17):
          start1 = time.time()
          model = PCA(n_components=hell)
          pca = model.fit_transform(X)
          X_train, X_test, y_train, y_test = train_test_split(pca,
                                                              y,
                                                              test_size=0.95,
                                                              random_state=42,
                                                              stratify=y)
          print("For band =", hell)
          for i in range(1,21):
            knn = KNeighborsClassifier(n_neighbors=i)
            knn.fit(X_train, y_train)
            y_pred=knn.predict(X_test)
            acc[hell-2][i-1]=metrics.accuracy_score(y_test, y_pred)
          end=time.time()
          print("Time taken for band", hell," is =",end-start1,"sec")
          max=np.argmax(acc[hell-2])
          print("Max accuracy =", acc[hell-2][max])
```

```
        end=time.time()
        print("Total Time Taken =",end-start,"sec")
```

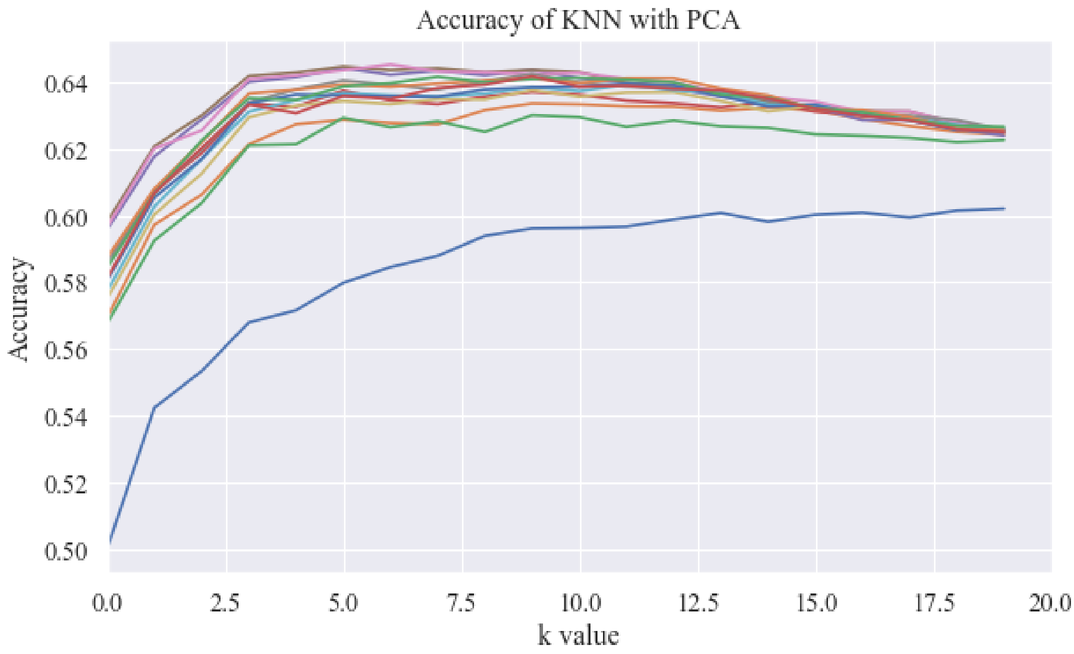
```
For band = 2
Time taken for band 2  is = 1.5301227569580078 sec
Max accuracy = 0.6020827075197757
For band = 3
Time taken for band 3  is = 1.7832973003387451 sec
Max accuracy = 0.6335736457394613
For band = 4
Time taken for band 4  is = 1.9694814682006836 sec
Max accuracy = 0.6300690898167618
For band = 5
Time taken for band 5  is = 2.1276233196258545 sec
Max accuracy = 0.6373785921698207
For band = 6
Time taken for band 6  is = 2.4373369216918945 sec
Max accuracy = 0.6441874436767798
For band = 7
Time taken for band 7  is = 2.6667490005493164 sec
Max accuracy = 0.6446380294382698
For band = 8
Time taken for band 8  is = 2.8386828899383545 sec
Max accuracy = 0.6452388104535897
For band = 9
Time taken for band 9  is = 3.0289556980133057 sec
Max accuracy = 0.64248523080004
For band = 10
Time taken for band 10  is = 3.4666008949279785 sec
Max accuracy = 0.6375287874236507
For band = 11
Time taken for band 11  is = 3.4986419677734375 sec
Max accuracy = 0.6391809352157806
For band = 12
Time taken for band 12  is = 3.7522194385528564 sec
Max accuracy = 0.6394813257234405
For band = 13
```

```
Time taken for band 13  is = 3.7696914672851562 sec
Max accuracy = 0.6411334735155703
For band = 14
Time taken for band 14  is = 3.9831297397613525 sec
Max accuracy = 0.6415339941924502
For band = 15
Time taken for band 15  is = 4.22843599319458 sec
Max accuracy = 0.6417342545308902
For band = 16
Time taken for band 16  is = 10.74459195137024 sec
Max accuracy = 0.6419845799539401
Total Time Taken = 51.828561305999756 sec
```

In [9]:
```
acc.shape
```

Out[9]: (15, 20)

In [10]:
```
plt.figure(figsize=(8, 5), dpi=80)
for i in range(0,14):
  plt.plot(acc[i])
plt.xlim(0,20)
plt.title('Accuracy of KNN with PCA')
plt.xlabel('k value')
plt.ylabel('Accuracy')
plt.savefig('./document/figure/KNN_with_PCA.pdf')
plt.show()
```



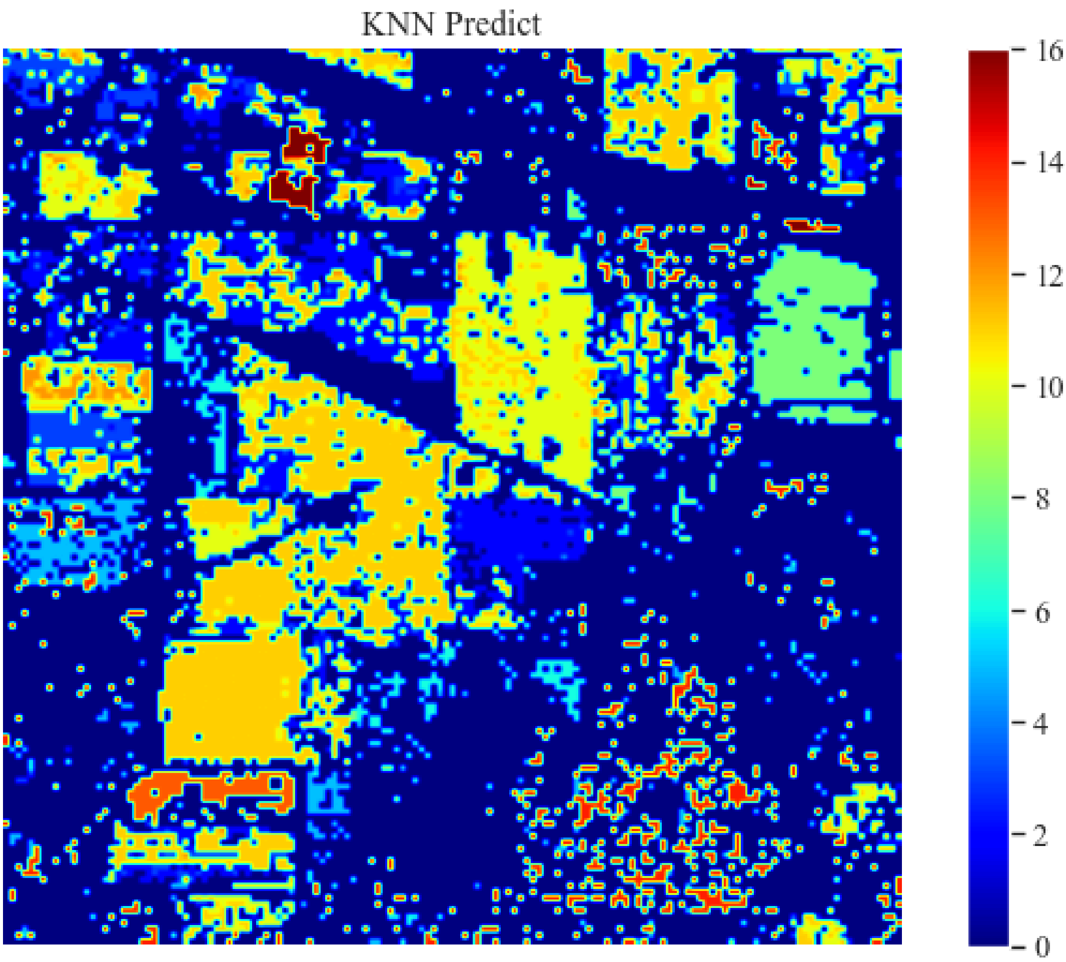

```
In [11]: temp = band = k = 0
         for i in range(0,14):
           for j in range(0,20):
             if temp < acc[i][j]:
               temp = acc[i][j]
               band = i + 2
               k = j + 1
         print("BAND =", band, "K-Value =",k, "MAX ACCURACY =", acc[band-2][k-1])
```

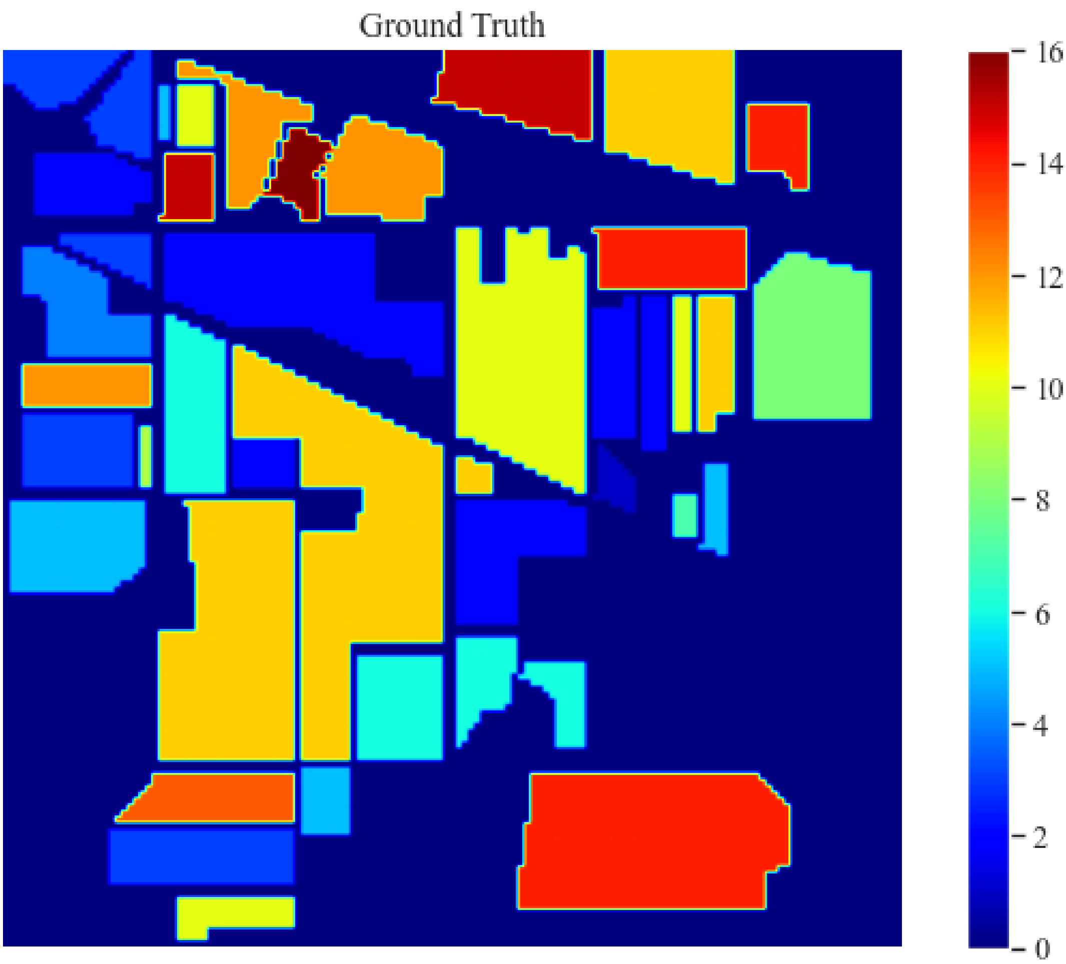
```
BAND = 8 K-Value = 7 MAX ACCURACY = 0.6452388104535897
```

```
In [12]: def AA_andEachClassAccuracy(confusion_matrix):
             counter = confusion_matrix.shape[0]
             list_diag = np.diag(confusion_matrix)
             list_raw_sum = np.sum(confusion_matrix, axis=1)
             each_acc = np.nan_to_num(truediv(list_diag, list_raw_sum))
             average_acc = np.mean(each_acc)
             return each_acc, average_acc
```

In [13]:
```
model = PCA(n_components=band)
pca = model.fit_transform(X)
X_train, X_test, y_train, y_test = train_test_split(pca,
                                                    y,
                                                    test_size =0.95,
                                                    random_state=42,
                                                    stratify=y)
knn = KNeighborsClassifier(n_neighbors = k)
knn.fit(X_train,y_train)
y_pred = knn.predict(X_test)

confusion = confusion_matrix(y_test, y_pred)
each_acc, aa = AA_andEachClassAccuracy(confusion)

print("Accuracy:", metrics.accuracy_score(y_test, y_pred))
print("Average Accuracy:", aa)
print("Kappa Coefficient:", metrics.cohen_kappa_score(y_test, y_pred))
print("Average Time:",(end-start)/320)

output = knn.predict(pca)
output = np.reshape(output,(145,145))
gt = np.reshape(y, (145,145))

plt.figure(figsize=(8, 6), dpi=80)
plt.imshow(output, cmap='jet')
plt.colorbar()
plt.axis('off')
plt.title('KNN Predict')
plt.savefig('./document/figure/KNN_Predict.pdf')

plt.figure(figsize=(8, 6), dpi=80)
plt.imshow(gt, cmap='jet')
plt.colorbar()
plt.axis('off')
plt.title('Ground Truth')
plt.savefig('./document/figure/GT.pdf')
plt.show()
```

```
Accuracy: 0.6452388104535897
Average Accuracy: 0.3522210175641218
Kappa Coefficient: 0.4661555315619679
Average Time: 0.16196425408124923
```





In [14]:
```
from sklearn.model_selection import cross_val_score
knn = KNeighborsClassifier(n_neighbors = k)
scores = cross_val_score(knn, pca, y, cv=10, scoring='accuracy')
print(scores)
print(scores.mean())
```

```
[0.52829291 0.68283405 0.65382786 0.65430338 0.73941988 0.75261656
 0.72549952 0.65889629 0.70599429 0.59705043]
0.669873516742201
```

4.2 KNN report

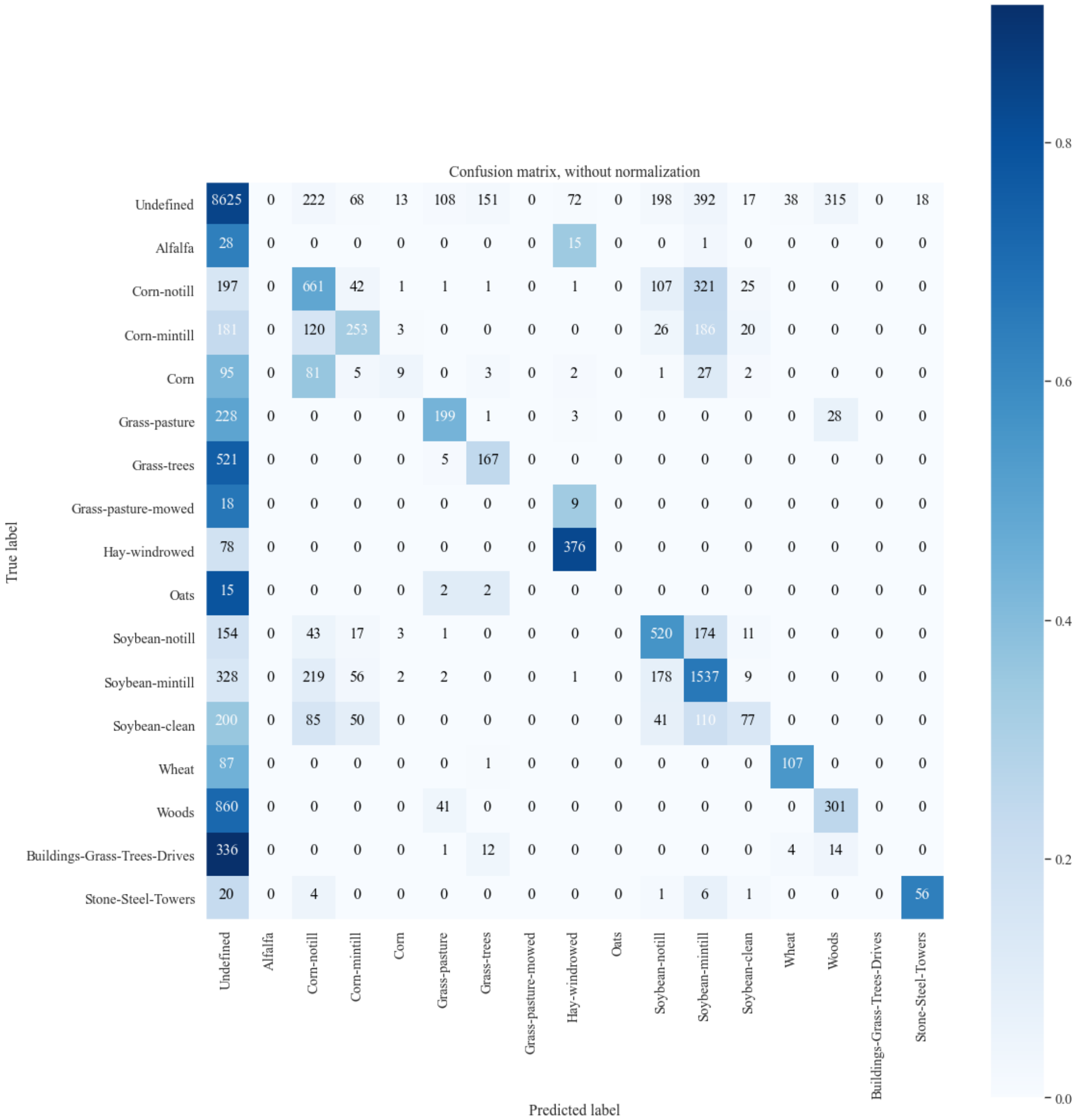
In [15]:
```
model = PCA(n_components=7)
pca = model.fit_transform(X)
X_train, X_test, y_train, y_test = train_test_split(pca,
                                                    y,
                                                    test_size=0.95,
                                                    random_state=42,
                                                    stratify=y)
model = KNeighborsClassifier(n_neighbors=14)
model.fit(X_train, y_train)
y_pred = model.predict(X_test)
```

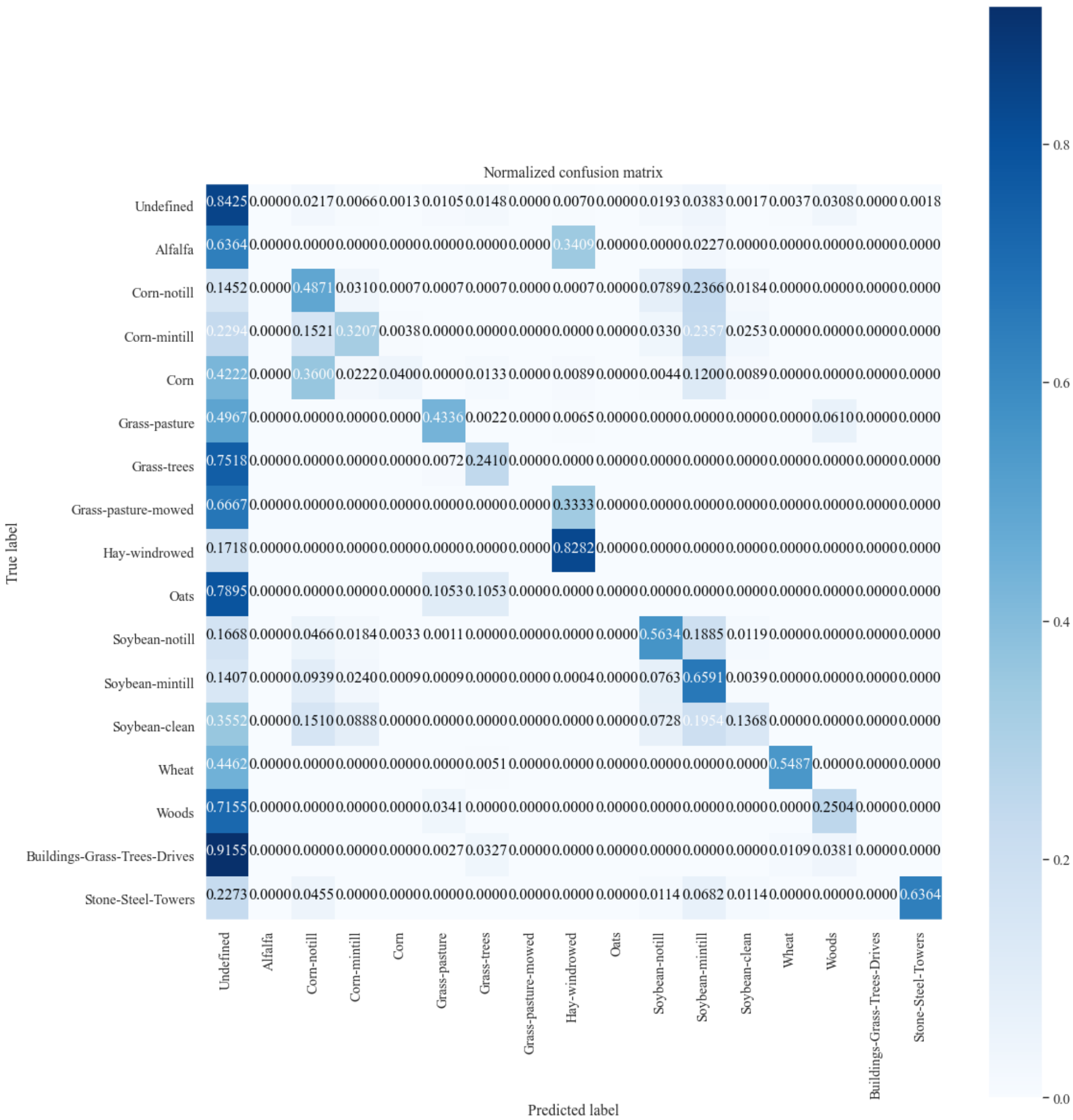
In [16]:
```
target_names = ['Undefined','Alfalfa', 'Corn-notill', 'Corn-mintill', 'Corn'
               ,'Grass-pasture', 'Grass-trees', 'Grass-pasture-mowed',
                'Hay-windrowed', 'Oats', 'Soybean-notill', 'Soybean-mintill',
               'Soybean-clean', 'Wheat', 'Woods', 'Buildings-Grass-Trees-Drives',
               'Stone-Steel-Towers']
```

In [17]:
```
from sklearn.metrics import classification_report
import itertools

def plot_confusion_matrix(cm, classes,
                          normalize=False,
                          title='Confusion matrix',
                          cmap=plt.get_cmap("Blues")):
    Normalized = cm.astype('float') / cm.sum(axis=1)[:, np.newaxis]
    if normalize:
        cm = Normalized
        print("Normalized confusion matrix")
    else:
        print('Confusion matrix, without normalization')

    plt.imshow(Normalized, interpolation='nearest', cmap=cmap)
    plt.colorbar()
    plt.title(title)
    tick_marks = np.arange(len(classes))
    plt.xticks(tick_marks, classes, rotation=90)
```

```
        plt.yticks(tick_marks, classes)

        fmt = '.4f' if normalize else 'd'
        for i, j in itertools.product(range(cm.shape[0]), range(cm.shape[1])):
            thresh = cm[i].max() / 2.
            plt.text(j, i, format(cm[i, j], fmt),
                     horizontalalignment="center",
                     color="white" if cm[i, j] > thresh else "black")

        plt.tight_layout()
        plt.ylabel('True label')
        plt.xlabel('Predicted label')

    plt.figure(figsize=(15,15),dpi=80)
    plt.grid(False)
    plot_confusion_matrix(confusion, classes=target_names, normalize=False,
                          title='Confusion matrix, without normalization')
    plt.savefig('./document/figure/confusion_mat.pdf')

    plt.figure(figsize=(15,15),dpi=80)
    plt.grid(False)
    plot_confusion_matrix(confusion, classes=target_names, normalize=True,
                          title='Normalized confusion matrix')
    plt.savefig('./document/figure/confusion_mat_norm.pdf')
    plt.show()
```

```
Confusion matrix, without normalization
Normalized confusion matrix
```


Confusion matrix, without normalization
True label
Predicted label
Undefined 8625 0 222 68 13 108 151 0 72 0 198 392 17 38 315 0 18
Alfalfa 28 0 0 0 0 0 0 0 15 0 0 1 0 0 0 0 0
Corn-notill 197 0 661 42 1 1 1 0 1 0 107 321 25 0 0 0 0
Corn-mintill 181 0 120 253 3 0 0 0 0 0 26 186 20 0 0 0 0
Corn 95 0 81 5 9 0 3 0 2 0 1 27 2 0 0 0 0
Grass-pasture 228 0 0 0 0 199 1 0 3 0 0 0 0 0 28 0 0
Grass-trees 521 0 0 0 0 5 167 0 0 0 0 0 0 0 0 0 0
Grass-pasture-mowed 18 0 0 0 0 0 0 0 9 0 0 0 0 0 0 0 0
Hay-windrowed 78 0 0 0 0 0 0 0 376 0 0 0 0 0 0 0 0
Oats 15 0 0 0 0 2 2 0 0 0 0 0 0 0 0 0 0
Soybean-notill 154 0 43 17 3 1 0 0 0 0 520 174 11 0 0 0 0
Soybean-mintill 328 0 219 56 2 2 0 0 1 0 178 1537 9 0 0 0 0
Soybean-clean 200 0 85 50 0 0 0 0 0 0 41 110 77 0 0 0 0
Wheat 87 0 0 0 0 0 1 0 0 0 0 0 0 107 0 0 0
Woods 860 0 0 0 0 41 0 0 0 0 0 0 0 0 301 0 0
Buildings-Grass-Trees-Drives 336 0 0 0 0 1 12 0 0 0 0 0 0 4 14 0 0
Stone-Steel-Towers 20 0 4 0 0 0 0 0 0 0 1 6 1 0 0 0 56
Undefined
Alfalfa
Corn-notill
Corn-mintill
Corn
Grass-pasture
Grass-trees
Grass-pasture-mowed
Hay-windrowed
Oats
Soybean-notill
Soybean-mintill
Soybean-clean
Wheat
Woods
Buildings-Grass-Trees-Drives
Stone-Steel-Towers
0.8
0.6
0.4
0.2
0.0


Normalized confusion matrix
True label
Predicted label
Undefined
Alfalfa
Corn-notill
Corn-mintill
Corn
Grass-pasture
Grass-trees
Grass-pasture-mowed
Hay-windrowed
Oats
Soybean-notill
Soybean-mintill
Soybean-clean
Wheat
Woods
Buildings-Grass-Trees-Drives
Stone-Steel-Towers
0.8425 0.0000 0.0217 0.0066 0.0013 0.0105 0.0148 0.0000 0.0070 0.0000 0.0193 0.0383 0.0017 0.0037 0.0308 0.0000 0.0018
0.6364 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3409 0.0000 0.0000 0.0227 0.0000 0.0000 0.0000 0.0000 0.0000
0.1452 0.0000 0.4871 0.0310 0.0007 0.0007 0.0007 0.0000 0.0007 0.0000 0.0789 0.2366 0.0184 0.0000 0.0000 0.0000 0.0000
0.2294 0.0000 0.1521 0.3207 0.0038 0.0000 0.0000 0.0000 0.0000 0.0000 0.0330 0.2357 0.0253 0.0000 0.0000 0.0000 0.0000
0.4222 0.0000 0.3600 0.0222 0.0400 0.0000 0.0133 0.0000 0.0089 0.0000 0.0044 0.1200 0.0089 0.0000 0.0000 0.0000 0.0000
0.4967 0.0000 0.0000 0.0000 0.0000 0.4336 0.0022 0.0000 0.0065 0.0000 0.0000 0.0000 0.0000 0.0000 0.0610 0.0000 0.0000
0.7518 0.0000 0.0000 0.0000 0.0000 0.0072 0.2410 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.6667 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3333 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.1718 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.8282 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.7895 0.0000 0.0000 0.0000 0.0000 0.1053 0.1053 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.1668 0.0000 0.0466 0.0184 0.0033 0.0011 0.0000 0.0000 0.0000 0.0000 0.5634 0.1885 0.0119 0.0000 0.0000 0.0000 0.0000
0.1407 0.0000 0.0939 0.0240 0.0009 0.0009 0.0000 0.0000 0.0004 0.0000 0.0763 0.6591 0.0039 0.0000 0.0000 0.0000 0.0000
0.3552 0.0000 0.1510 0.0888 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0728 0.1954 0.1368 0.0000 0.0000 0.0000 0.0000
0.4462 0.0000 0.0000 0.0000 0.0000 0.0000 0.0051 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.5487 0.0000 0.0000 0.0000
0.7155 0.0000 0.0000 0.0000 0.0000 0.0341 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.2504 0.0000 0.0000
0.9155 0.0000 0.0000 0.0000 0.0000 0.0027 0.0327 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0109 0.0381 0.0000 0.0000
0.2273 0.0000 0.0455 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0114 0.0682 0.0114 0.0000 0.0000 0.0000 0.6364
0.0
0.2
0.4
0.6
0.8