

Problem Set 1

1 Grid snapping

Task: For geometric algorithms, it is often helpful to discretize the input points. In this task, we consider a standard grid in \mathbb{R}^2 . Given the grid width and a set of points, compute the lower left corner of the grid cell that each point falls into.

Input: The first line contains the number of points n , which is at most 100. The second line contains the grid width with two digits after the decimal point. The coordinates are between -10000 and 10000 . The following n lines each contain a point, consisting of two floating point numbers with two digits after the decimal point, separated by a space.

Output: A line for each input point, containing the lower left corner of the grid cell that it falls into, with two digits after the decimal point.

Sample Input:

```
3
0.75
0.00 0.00
-0.75 0.00
1.00 2.15
```

Sample Output:

```
0.00 0.00
-0.75 0.00
0.75 1.50
```

2 k -means I

Task: Given a set of points P in \mathbb{R}^2 with integer coordinates, compute the sum of the squared lengths of all points in P , i.e., $\sum_{x \in P} \|x\|^2$. The length of a point $x = (x_1, x_2) \in \mathbb{R}^2$ is defined as $\sqrt{x_1^2 + x_2^2}$.

Input: The first line contains the number of points n . The next n lines contain one point per line. Each point is specified by two integers, separated by a space, and the integers lie between -10000 and 10000 . The cardinality of P is at most 10000.

Output: A line containing $\sum_{x \in P} \|x\|^2$, with 30 decimal places and without rounding error in these decimal places.

Sample Input:

```
4
1 1
1 -1
-1 1
-1 -1
```

Sample Output:

```
8.000000000000000000000000000000
```

3 Area of a Polygon

Task: You are given the n vertices p_0, \dots, p_{n-1} of a non-self-intersecting polygon P . Compute the area A of P .

Input: The first line of the input contains the number n of vertices. Then, n lines follow describing the vertices in clockwise order along the boundary of the polygon. Each line contains the x and y coordinate of one vertex. You may assume that all coordinates are integers, and that we have $-1000 \leq x, y \leq 1000$.

Output: Output the area A of the described polygon.

Sample Input:

```
5
-1 0
0 1
2 0
1 -1
0 -2
```

Sample Output:

```
4.5
```

4 Matrix Multiplication

Task: Let $n \in \mathbb{N}$. You are given an $n \times n$ -matrix A , and a vector v of size n . Most of the entries in A and v will be equal to zero. Compute the vector Av .

Input: The first line contains the dimension n of the matrix A . You may assume that $1 \leq n \leq 100000$. The second line contains the number m of non-zero entries in A . Then, m lines follow: Each of the following lines contain three integers i, j , and a_{ij} , which means that in row i and column j the matrix holds the value a_{ij} . You may assume that $0 \leq i, j \leq n-1$, and that $-5 \leq a_{ij} \leq 5$. The values of the matrix are given in increasing lexicographical order.

After the m lines, a line follows which contains the number b of non-zero entries in v . Then, each of the following b lines contains a number i with $0 \leq i \leq n-1$, and the value v_i .

Output: Output the values of Av that are different from zero. That is, if the i -th value of Av is non-zero, then print a line containing at first i and then the corresponding value of Av .

Sample Input:

```
3
4
0 0 1
0 1 -5
0 2 -4
1 0 -3
2
0 3
2 3
```

Sample Output:

```
0 -9
1 -9
```