

DUT – RU International School of Information Science & Engineering

Topic # 5-6

Light and Optics,

Oscillations and Waves

Contents

- 1. Reflection, Refraction, and Ray Approximation
- 2. Flat Mirrors and Thin Lenses
- 3. Simple Harmonic Motion
- 4. Waves and Their Types

The Subject of Optics

The ability to manipulate **light** has greatly enhanced our capacity to investigate and understand the nature of the universe.

One goal of physics is to discover the basic laws governing light, such as the law of refraction. A broader goal is to put those laws to use, and perhaps the most important use is the production of **images**.



Reflection, Refraction, and Ray Approximation

When light travelling in one medium encounters a boundary leading into a second

medium, the processes of reflection and refraction can occur.

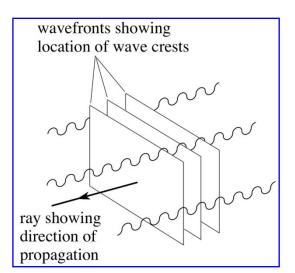


in **reflection**, part of the light encountering the second medium **bounces** off that medium



in **refraction**, the light passing into the second medium **bends** through an angle with respect to the normal to the boundary

Note: Often, both processes occur at the same time, with part of the light being reflected and part refracted.

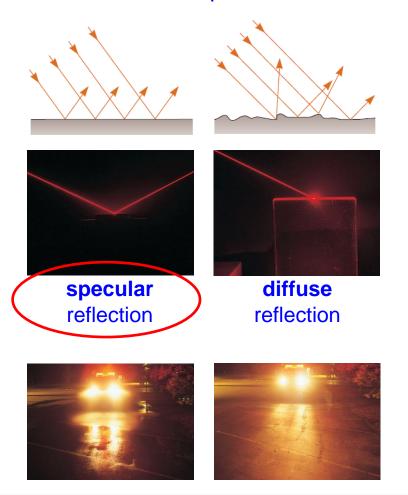


In order to study both phenomena, we use the **ray** approximation for light:

light travels in a straight-line path in a homogeneous medium, until it encounters a boundary between two different materials

Reflection of Light

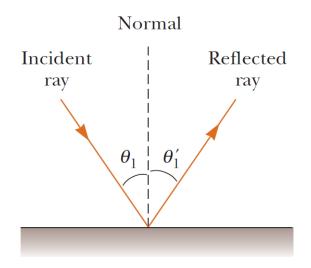
When a light ray traveling in a transparent medium encounters a boundary leading into a second medium, part of the incident ray is reflected back into the first medium.



The **law of reflection**: (deduced experimentally)

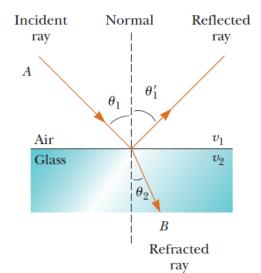
$$\theta_1' = \theta_1$$

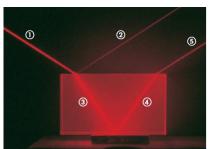
the angle of reflection equals the angle of incidence



Refraction of Light

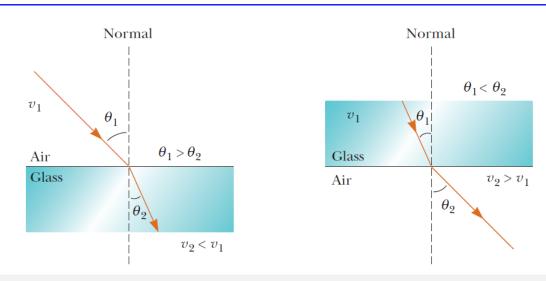
When a ray of light traveling through a transparent medium encounters a boundary leading into another transparent medium, part of the ray is reflected, and part enters the second medium. The ray that enters the second medium is bent at the boundary and is said to be refracted.





The incident ray, the reflected ray, the refracted ray, and the normal at the point of incidence all lie in the same plane

The path of a light ray through a refracting surface is reversible



Refraction of Light

When light passes from one transparent medium to another, it's refracted because the speed of light is different in the two media.

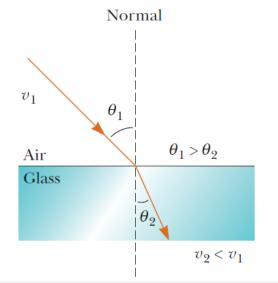
The index of refraction



The law of refraction ■



$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} = \text{constant}$$



$$n \equiv \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v}$$

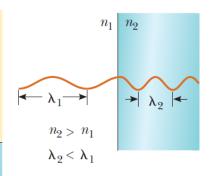




 λ_2

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Snell's law of refraction



index of refraction and wave aspect of light

$$v_1 = f\lambda_1$$

$$v_2 = f\lambda_2$$

$$\lambda_1 n_1 = \lambda_2 n_2$$

Note: the frequency of the wave does **NOT** change as the wave passes from one medium to another

Refraction of Light

Indices of Refraction for Various Substances, Measured with Light of Vacuum Wavelength $\lambda_0 = 589 \text{ mn}$

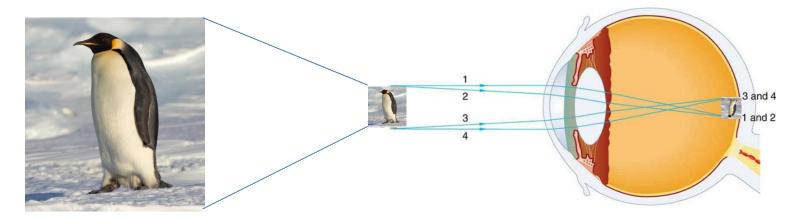
Substance	Index of Refraction	Substance	Index of Refraction
Solids at 20°C		Liquids at 20°C	
Diamond (C)	2.419	Benzene	1.501
Fluorite (CaF ₂)	1.434	Carbon disulfide	1.628
Fused quartz (SiO ₂)	1.458	Carbon tetrachloride	1.461
Glass, crown	1.52	Ethyl alcohol	1.361
Glass, flint	1.66	Glycerine	1.473
Ice (H_2O) (at $0^{\circ}C)$	1.309	Water	1.333
Polystyrene	1.49		almost one
Sodium chloride (NaCl)	1.544	Gases at 0°C, 1 atm	aimost one
Zircon	1.923	Air	1.000 293
		Carbon dioxide	$1.000\ 45$

Mirrors and Lenses

The development of the technology of **mirrors** and **lenses** led to a revolution in the progress of science. These devices, relatively simple to construct from cheap materials, led to microscopes and telescopes, extending human sight and opening up new pathways to knowledge, from microbes to distant planets.



Images can be formed by **reflection** from mirrors or by **refraction** through lenses.





Your visual system goes through this **processing** and recognition even if the light rays **do not come directly** from the penguin, but instead reflect toward you from a mirror or refract through the lenses in a pair of binoculars.

light actually passes through the image point

REAL IMAGE

VIRTUAL IMAGE

light diverges from the image point

Flat Mirrors

A mirror is a surface that can reflect a beam of light in one direction instead of either scattering it widely in many directions or absorbing it. Flat mirrors are the mirrors

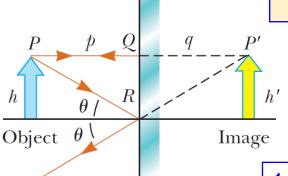
having the flat reflecting surface.

Images are formed at the point where rays of light actually intersect or where they appear to originate



To find out where an image is formed, it's necessary to follow at least two rays of light as they reflect from the mirror



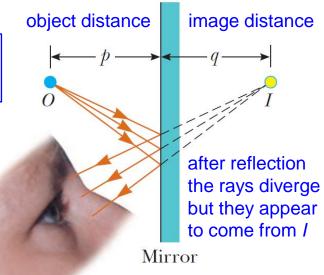


$$M \equiv \frac{\text{image height}}{\text{object height}} = \frac{h'}{h}$$

Flat mirror: M = 1

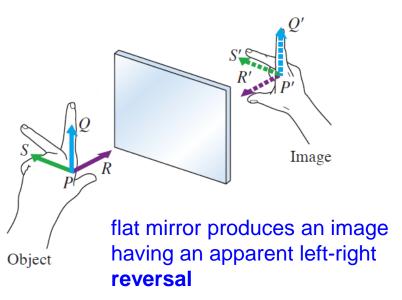
Note: word magnification, as used in optics, doesn't always mean enlargement, because the image could be smaller than the object.

- The image is as far behind the mirror as the object is in front.
- The image is unmagnified, virtual, and upright.



Flat Mirrors

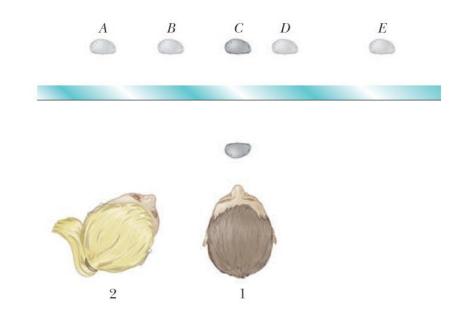




QUIZ

Check your understanding:

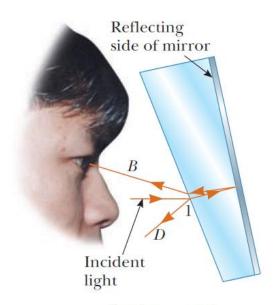
In the overhead view, the image of the stone seen by observer 1 is at C.
Where does observer 2 see the image – at A, at B, at C, at E, or not at all?



Flat Mirrors

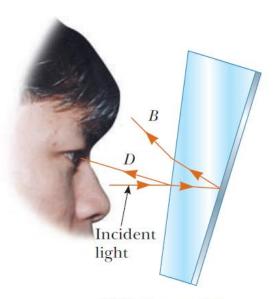
Most **rearview** mirrors in cars have a day setting and a night setting. The night setting greatly diminishes the intensity of the image so that lights from trailing cars will not

blind the driver.



Daytime setting

the silvered back surface of the mirror reflects a bright ray B into the driver's eyes.



Nighttime setting

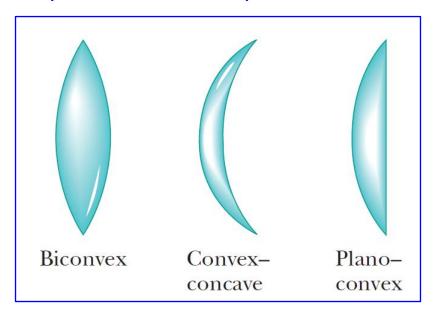
the glass of the unsilvered front surface of the mirror reflects a dim ray D into the driver's eyes.

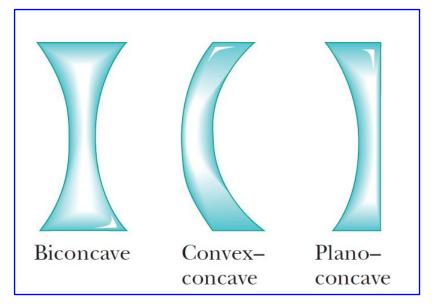






A typical **thin lens** consists of a piece of glass or plastic, ground so that each of its two refracting surfaces is a segment of either a sphere or a plane. Lenses are commonly used to form images by refraction in optical instruments, such as cameras, telescopes, and microscopes.





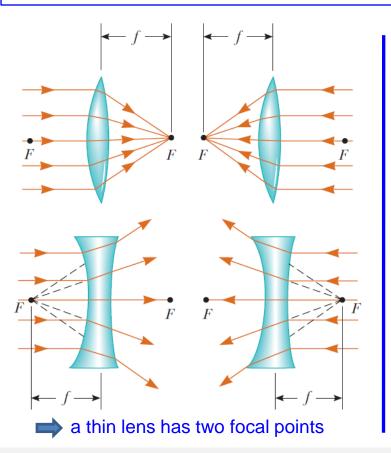
converging lenses

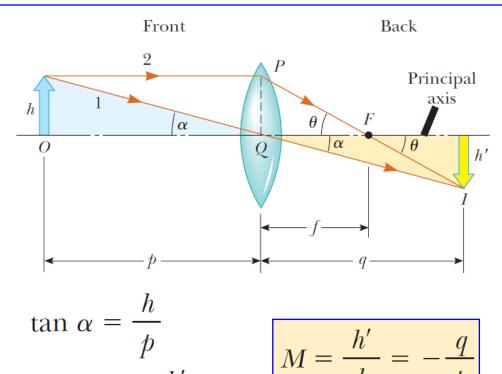
diverging lenses

Note: Converging lenses are thicker at the center than at the rim, whereas diverging lenses are thinner at the center than at the rim.

It is convenient to define a point called the **focal point** F for a lens. The distance from the focal point to the lens is called the **focal length** f.

The focal length is the image distance that corresponds to an **infinite** object distance





It is convenient to define a point called the **focal point** *F* for a lens. The distance from the focal point to the lens is called the **focal length** *f*.

The focal length is the image distance that corresponds to an **infinite** object distance

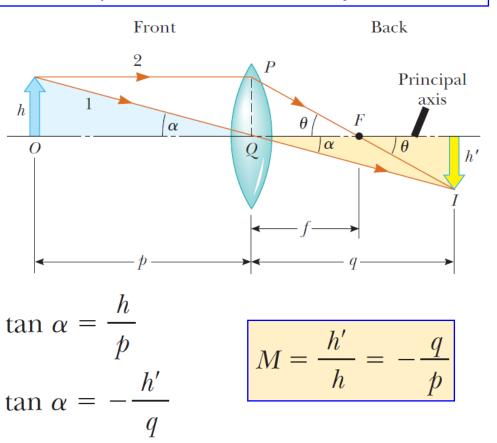
$$\tan \theta = \frac{PQ}{f} \quad \tan \theta = -\frac{h'}{q - f}$$

$$\frac{h}{f} = -\frac{h'}{q - f} \quad \frac{h'}{h} = -\frac{q - f}{f}$$

$$\frac{q}{p} = \frac{q - f}{f}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

thin-lens equation



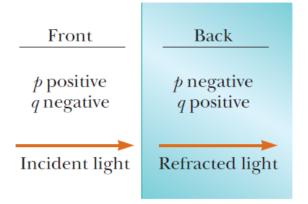
Thin-lens equation can be used with both converging and diverging lenses if we adhere to a set of sign conventions.

Sign Conventions for Thin Lenses							
Quantity	Symbol	In Front	In Back	Convergent	Divergent		
Object location	þ	+	_				
Image location	q	_	+				
Lens Radii	R_1, R_2	_	+				
Focal Length	f			+	_		

Note: a converging lens has a positive focal length under this convention and a diverging lens has a negative focal length.

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

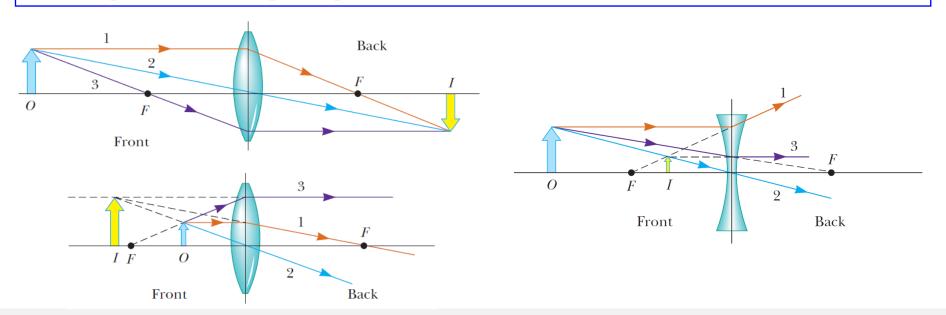
thin-lens equation



Ray Diagrams for Thin Lenses

To locate the image formed by a thin lens, the following three rays are drawn from the top of the objects:

- 1. The first ray is drawn parallel to the principal axis. After being refracted by the lens, this ray passes through (or appears to come from) one of the focal points.
- 2. The second ray is drawn through the center of the lens. This ray continues in a straight line.
- **3.** The third ray is drawn through the other focal point and emerges from the lens parallel to the principal axis.



EXERCISE

Task #1: A converging lens of focal length 10.0 cm forms images of an object situated at various distances. (a) If the object is placed 30.0 cm from the lens, locate the image, state whether it's real or virtual, and find its magnification. (b) Repeat the problem when the object is at 10.0 cm and (c) again when the object is 5.00 cm from the lens.

Solution:

(a)
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{30.0 \text{ cm}} + \frac{1}{q} = \frac{1}{10.0 \text{ cm}}$$

$$q = \frac{1}{15.0 \text{ cm}}$$

$$M = -\frac{q}{p} = -\frac{15.0 \text{ cm}}{30.0 \text{ cm}} = -0.500$$

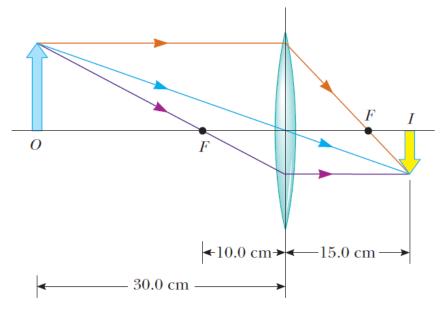


image real

(inverted and smaller than the object)

EXERCISE

Task #1: A converging lens of focal length 10.0 cm forms images of an object situated at various distances. (a) If the object is placed 30.0 cm from the lens, locate the image, state whether it's real or virtual, and find its magnification. (b) Repeat the problem when the object is at 10.0 cm and (c) again when the object is 5.00 cm from the lens.

Solution:

(b)
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{10.0 \text{ cm}} + \frac{1}{q} = \frac{1}{10.0 \text{ cm}} \rightarrow \frac{1}{q} = 0$$

$$q \rightarrow \infty$$

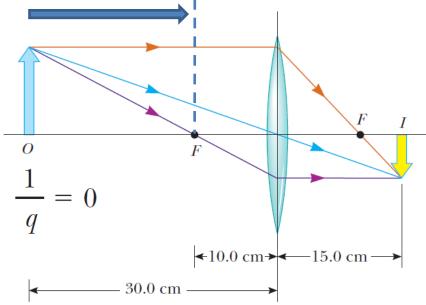


image at infinity

EXERCISE

Task #1: A converging lens of focal length 10.0 cm forms images of an object situated at various distances. (a) If the object is placed 30.0 cm from the lens, locate the image, state whether it's real or virtual, and find its magnification. (b) Repeat the problem when the object is at 10.0 cm and (c) again when the object is 5.00 cm from the lens.

Solution:

image virtual

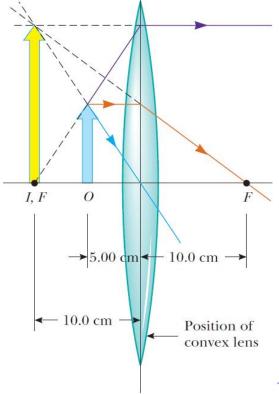
(c)
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$+\frac{1}{q} = \frac{1}{f}$$
 (upright and double the object size)

$$\frac{1}{5.00 \text{ cm}} + \frac{1}{q} = \frac{1}{10.0 \text{ cm}}$$

$$q = -10.0 \text{ cm}$$

$$M = -\frac{q}{p} = -\left(\frac{-10.0 \text{ cm}}{5.00 \text{ cm}}\right) = +2.00$$



EXERCISE

Task #2: A diverging lens of focal length 10.0 cm forms images of an object situated at various distances. (a) If the object is placed 30.0 cm from the lens, locate the image, state whether it's real or virtual, and find its magnification. (b) Repeat the problem when the object is at 10.0 cm and (c) again when the object is 5.00 cm from the lens.

Solution:

(a)
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{30.0 \text{ cm}} + \frac{1}{q} = -\frac{1}{10.0 \text{ cm}}$$

$$q = -7.50 \text{ cm}$$

$$M = -\frac{q}{p} = -\left(\frac{-7.50 \text{ cm}}{30.0 \text{ cm}}\right) = +0.250$$

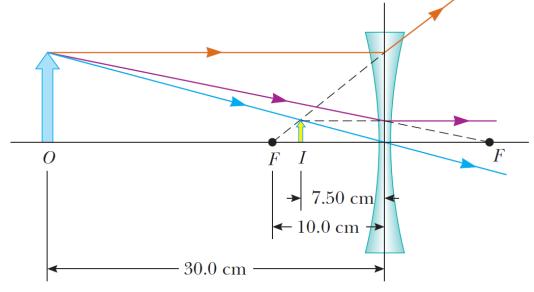


image virtual

(upright and smaller than the object)

EXERCISE

Task #2: A diverging lens of focal length 10.0 cm forms images of an object situated at various distances. (a) If the object is placed 30.0 cm from the lens, locate the image, state whether it's real or virtual, and find its magnification. (b) Repeat the problem when the object is at 10.0 cm and (c) again when the object is 5.00 cm from the lens.

Solution:

(b)
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

 $\frac{1}{10.0 \text{ cm}} + \frac{1}{q} = -\frac{1}{10.0 \text{ cm}}$

$$q = -5.00 \text{ cm}$$

$$M = -\frac{q}{p} = -\left(\frac{-5.00 \text{ cm}}{10.0 \text{ cm}}\right) = +0.500$$

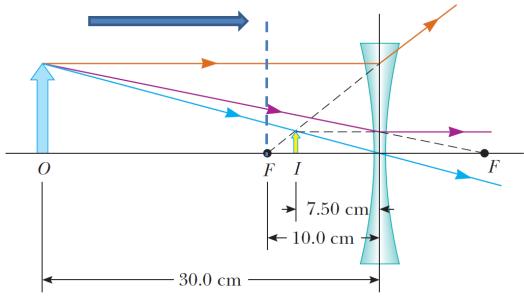


image virtual

(upright and smaller than the object)

EXERCISE

Task #2: A diverging lens of focal length 10.0 cm forms images of an object situated at various distances. (a) If the object is placed 30.0 cm from the lens, locate the image, state whether it's real or virtual, and find its magnification. (b) Repeat the problem when the object is at 10.0 cm and (c) again when the object is 5.00 cm from the lens.

Solution:

(c)
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{5.00 \text{ cm}} + \frac{1}{q} = -\frac{1}{10.0 \text{ cm}}$$

$$q = -3.33 \text{ cm}$$

$$M = -\left(\frac{-3.33 \text{ cm}}{5.00 \text{ cm}}\right) = +0.666$$

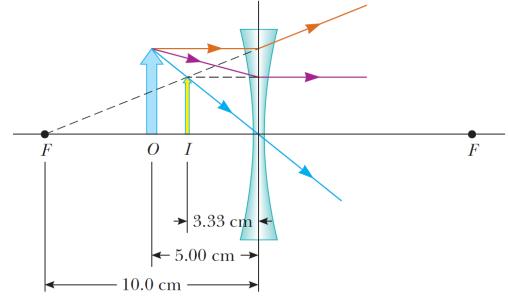


image virtual

(upright and smaller than the object)

The Subject of Vibrations and Waves

Our world is filled with **oscillations** in which objects move back and forth repeatedly. Many oscillations are merely amusing or annoying, but many others are dangerous or financially important. The study and control of oscillations are two of the primary goals of both physics and engineering.

Periodic vibrations can cause disturbances that move through a medium in the form of **waves**. Many kinds of waves occur in nature, such as sound waves, water waves, seismic waves, and electromagnetic waves. However, these very different physical phenomena can be described by common terms and concepts.



wings oscillate in the turbulence

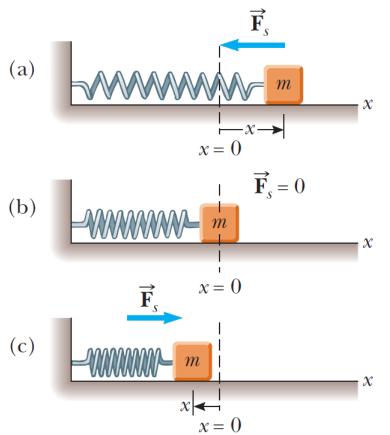


earthquakes



oscillations of power lines due to wind

One of the simplest types of vibrational motion is that of an object attached to a spring. We assume that the object moves on a frictionless horizontal surface.



Hooke's law: (deduced experimentally)

$$F_s = -kx$$

Note: the force is always directed opposite to the displacement of an object, i.e. it pushes or pulls it towards the equilibrium position.

Projecting the equation of motion onto the *x*-axis:

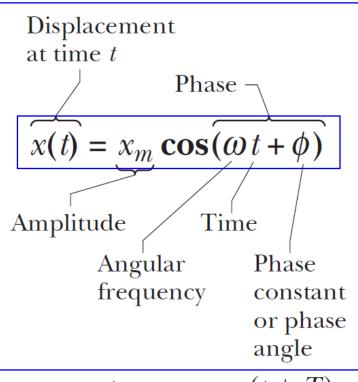
$$m\ddot{x} = -kx$$
 or $m\ddot{x} + kx = 0$

Solving this ODE results in: EXERCISE

$$x(t) = x_m \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

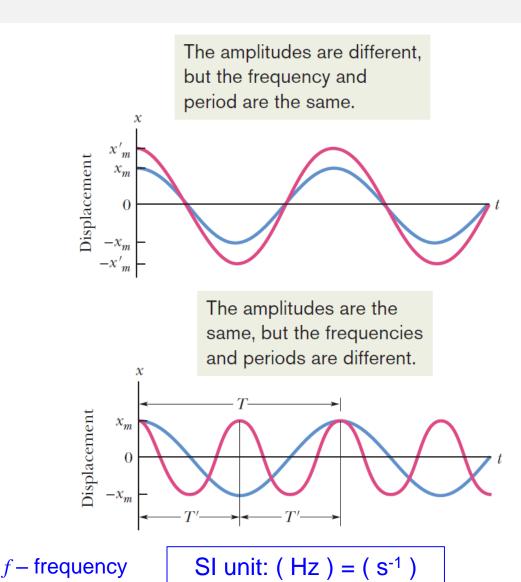
Note: it is also possible to use "sin" in the solution. simple harmonic motion (SHM)

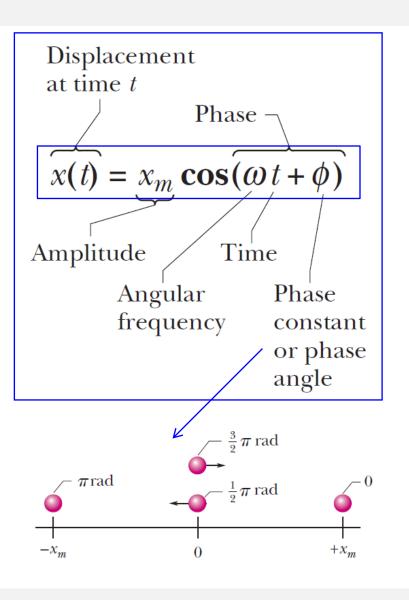


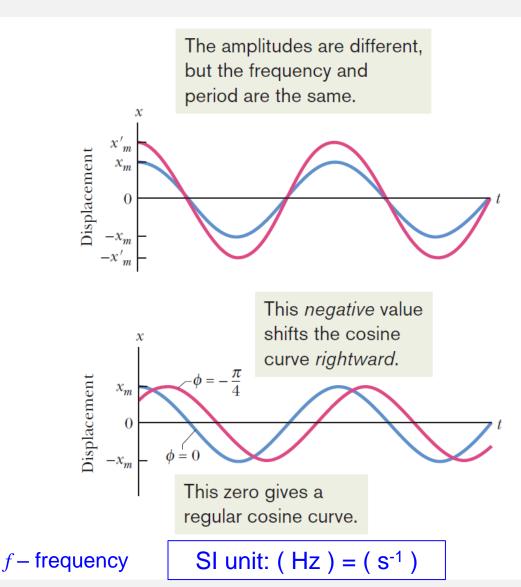
$$x_m \cos \omega t = x_m \cos \omega (t + T)$$

$$\omega(t+T) = \omega t + 2\pi$$

$$T = \frac{2\pi}{\omega} \quad \text{or} \quad T = \frac{1}{f}$$







Velocity and Acceleration of SHM

Let us derive the expressions for both **velocity** and **acceleration** (based on their general definitions) of an object during the SHM.

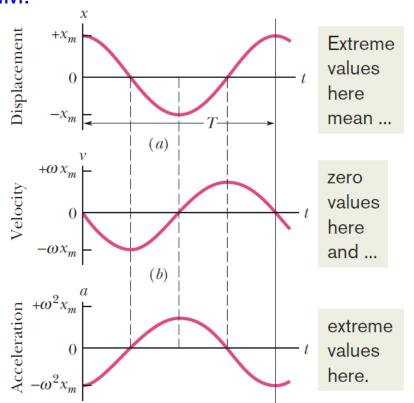
$$v(t) = \frac{dx(t)}{dt} \qquad a(t) = \frac{dv(t)}{dt}$$

$$x(t) = x_m \cos(\omega t + \phi)$$

$$v(t) = -\omega x_m \sin(\omega t + \phi)$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

$$a(t) = -\omega^2 x(t)$$



Note: If you ever see **such a relationship** in an oscillating situation (such as with, say, the current in an electrical circuit, or the rise and fall of water in a tidal bay), you can immediately say that the motion is **SHM** and immediately identify the angular frequency ω of the motion.

QUIZ

Check your understanding:

A particle undergoing simple harmonic oscillation of period T is at $-x_m$ at time t = 0. Is it at $-x_m$, at $+x_m$, at 0, between $-x_m$ and 0, or between 0 and $+x_m$ when (a) t = 2.00T, (b) t = 3.50T, and (c) t = 5.25T?

Which of the following relationships between a particle's acceleration a and its position x indicates simple harmonic oscillation: (a) $a = 3x^2$, (b) a = 5x, (c) a = -4x, (d) a = -2/x? For the SHM, what is the angular frequency (assume the unit of rad/s)?

Energy in Simple Harmonic Motion

If **no friction** is present, the energy transfers back and forth between the kinetic energy and potential energy, while the sum of the two – the mechanical energy *E* of

the oscillator – remains constant.

$$U(t) = \frac{1}{2}kx^{2} = \frac{1}{2}kx_{m}^{2}\cos^{2}(\omega t + \phi)$$

$$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}kx_m^2 \sin^2(\omega t + \phi)$$

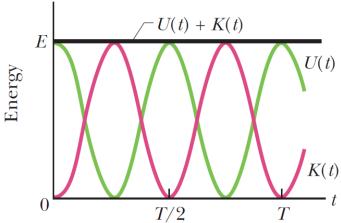
$$E = U + K$$

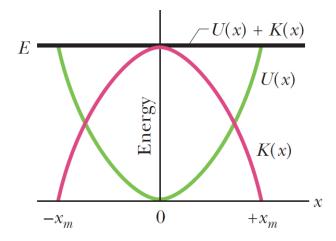
$$= \frac{1}{2}kx_m^2 \cos^2(\omega t + \phi) + \frac{1}{2}kx_m^2 \sin^2(\omega t + \phi)$$

$$= \frac{1}{2}kx_m^2 \left[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right]$$



$$E = U + K = \frac{1}{2}kx_m^2$$





Waves

The world is full of **waves**: sound waves, waves on a string, seismic waves, and electromagnetic waves, such as visible light, radio waves, television signals, and x-rays. All of these waves have as their source a vibrating object, so we can apply the concepts of simple harmonic motion in describing them.

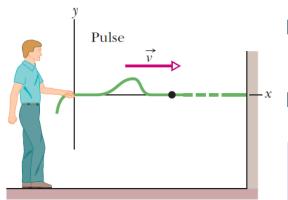


Types of waves

- mechanical waves: all these waves have two central features: (i) they are governed by Newton's laws, and (ii) they can exist only within a material medium, such as water, air, and rock (examples: water waves, sound waves, and seismic waves).
 - electromagnetic waves: these waves require no material medium to exist. Light waves from stars, for example, travel through the vacuum of space to reach us. All electromagnetic waves travel through a vacuum at the same speed 299 792 458 m/s.
 - matter waves: these waves are associated with electrons, protons, and other fundamental particles, and even atoms and molecules. Because we commonly think of these particles as constituting matter, such waves are called matter waves.

Transverse and Longitudinal Waves

One of the simplest ways to demonstrate wave motion is to flip one end of a long rope that is under tension and has its opposite end fixed.





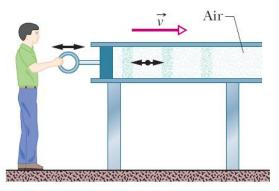
The bump (called a **pulse**) travels to the right with a definite speed. A disturbance of this type is called a **traveling wave**.



Each segment of the rope that is disturbed moves in a direction **perpendicular** to the wave motion.

A traveling wave in which the particles of the disturbed medium move in a direction **perpendicular** to the wave velocity is called a **transverse** wave.

Let us consider how sound waves can be produced. If you suddenly move the piston rightward and then leftward, you can send a pulse of sound along the pipe.



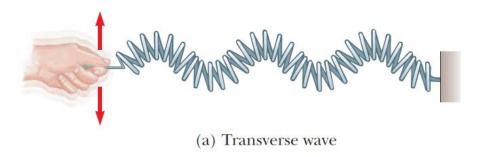


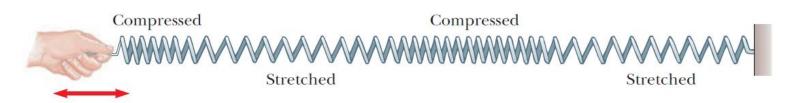
The motion of the elements of air is **parallel** to the direction of the wave's travel.

A traveling wave in which the particles of the disturbed medium undergo displacements **parallel** to the direction of wave motion are called **longitudinal waves**.

Transverse and Longitudinal Waves

Both types of waves can be produced by means of a **spring**.



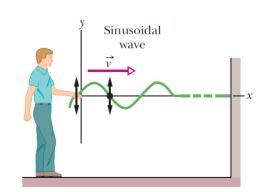


(b) Longitudinal wave

Note: Waves **need not** be purely transverse or purely longitudinal. For example, ocean waves exhibit a **superposition** of **both** types. When an ocean wave encounters a cork, the cork executes a circular motion, going up and down while going forward and back.

Sinusoidal Wave

To completely describe a wave on a string (and the motion of any element along its length), we need a function that gives the shape of the wave.





we need a relation in the form:

$$y = h(x, t)$$

In case of a **sinusoidal** wave:

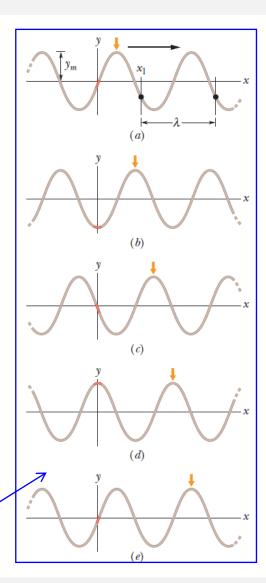
$$y(x,t) = y_m \sin(kx - \omega t)$$

(transverse displacement of any string element)

Note: because this equation is written in terms of position x, it can be used to find the displacements of all the elements of the string as a function of time. Thus, it can tell us the shape of the wave at any given time.

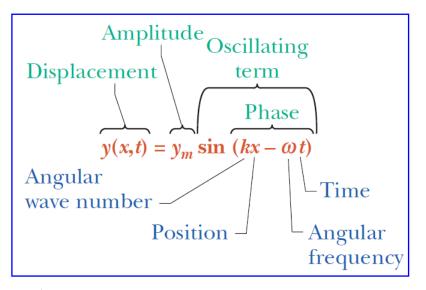
Note: as the wave sweeps through succeeding elements of the string, the elements oscillate parallel to the *y*-axis.

"snapshots" of a string wave travelling in the positive direction of an x-axis



Amplitude, Phase and Wavelength

Let us consider the quantities which define the transversal displacement of the elements of the string caused by the propagation of the sinusoidal wave.





amplitude of the wave is the magnitude of the maximum displacement of the elements from their equilibrium positions as the wave passes through them (always positive).



phase of the wave is the argument of the sine function. As the wave sweeps through a string element at a particular position *x*, the phase changes linearly with time, so that it undergoes harmonic oscillations.



wavelength (λ) of the wave is the distance (parallel to the direction of the wave's travel) between repetitions of the wave shape.

$$y_m \sin kx_1 = y_m \sin k(x_1 + \lambda) = y_m \sin(kx_1 + k\lambda)$$
 \rightarrow $k\lambda = 2\pi$



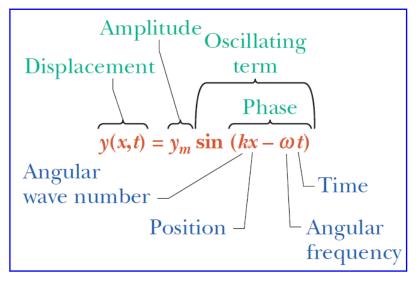
angular wave number:

$$k = \frac{2\pi}{\lambda}$$

SI units: $(rad / m) = (m^{-1})$

Period, Angular Frequency and Frequency

Let us consider the quantities which define the transversal displacement of the elements of the string caused by the propagation of the sinusoidal wave.



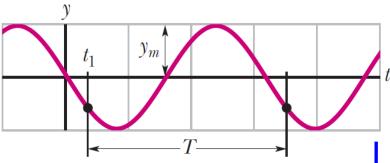


period of oscillation of a wave is the time any string element takes to move through one full oscillation. (x = 0)

$$y(0,t) = y_m \sin(-\omega t) = -y_m \sin \omega t$$

$$-y_m \sin \omega t_1 = -y_m \sin \omega (t_1 + T) =$$

$$= -y_m \sin(\omega t_1 + \omega T) \rightarrow \omega T = 2\pi$$



Note: this is a graph, not a snapshot!



angular frequency:

$$\omega = \frac{2\pi}{T}$$



frequency:

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

(as it was for SHM, this is a number of oscillations per unit time)

Conclusions

- **reflection** and **refraction** are the phenomena which occur when the light travelling in one medium encounters a boundary leading into a second medium. The physics of both is governed by the **law of reflection** and **law of refraction** correspondingly
- the **index of refraction** of a material the ratio of the **speed of light** in the **vacuum** to the speed of light in the **material**
- **images** can be formed by **reflection** from mirrors or by **refraction** through lenses. We considered the basic cases of such mechanisms as **flat mirrors** and **thin lenses**

- **periodic** motion is a very common type of motion in nature. The systems which undergo such types of motion are called **oscillators**
- **simple harmonic motion** occurs when the net force on an object along the direction of motion is **proportional** to the object's **displacement** and in the **opposite** direction
- wave is a motion of some disturbance in space. According to the direction of the displacements of the considered objects (quantities) we distinguish between transverse and longitudinal waves