CMPU1018_CA1

Click on a question number to see how your answers were marked and, where available, full solutions.

Question Number	Score	Review
Question 1	5/5	
Question 2	2/2	
Question 3	4/4	
Question 4	3/3	
Question 5	3/3	
Question 6	1/1	
Question 7	4/4	
Total	22/22	(100%)

Performance Summary

Exam Name:	CMPU1018_CA1
Session ID:	12552393888
Student's Name:	Kliutko, Yaroslav (90006)
Exam Start:	Mon Oct 14 2024 08:57:50
Exam Stop:	Mon Oct 14 2024 09:33:18
Time Spent:	0:35:27

Question 1

This question will test your use of the Division Algorithm and the Extended Euclidean Algorithm.

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a)

Use the division algorithm to find the quotient q and the remainder r when dividing 83 by 7.

q = 11 ✔ Expected answer: 11 , r =

6 ✔ Expected answer: 6

Score: 2/2 ✔

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b)

The gcd of 25 and 54 is 1 ✔ Expected answer: 1 .

Score: 1/1 ✔

▶ Show feedback

c)

$gcd(25, 54) =$ 13 ✔ Expected answer: 13 x$25+$

-6 ✔ Expected answer: -6 x54

Score: 2/2 ✔

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Question 2

Euclidean Algorithm

Apply the Euclidean algorithm to find the greatest common denominator between 1204 and 280. Show the steps.

Part a) Algorithm

Please enter all the steps of the Euclidean algorithm below (please enter the number of steps you need in the "Rows:" box.)

Rows: 3

1204	=	4	×	280	+	84	✔
280	=	3	×	84	+	28	
84	=	3	×	28	+	0	

Expected answer:

1204	=	4	×	280	+	84
280	=	3	×	84	+	28
84	=	3	×	28	+	0

Score: 1/1 ✔

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Part b) gcd

gcd(1204,280)=

28 ✔

Expected answer: 28

Score: 1/1 ✔

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Advice

In order to apply the Euclidean algorithm to find the gcd, you need to apply integer division repeatedly, until the remainder is 0.

Once you know the gcd between two numbers p and q you can find their lcm using the following formula:

$$\operatorname{lcm}(p, q) = \frac{p \times q}{\gcd(p, q)}$$

Question 3

Multiplication in modular arithmetic looks different to regular multiplication. Here is an example for multiplication in modulo 13.

$\times$	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	1	3	5	7	9	11
3	0	3	6	9	12	2	5	8	11	1	4	7	10
4	0	4	8	12	3	7	11	2	6	10	1	5	9
5	0	5	10	2	7	12	4	9	1	6	11	3	8
6	0	6	12	5	11	4	10	3	9	2	8	1	7
7	0	7	1	8	2	9	3	10	4	11	5	12	6
8	0	8	3	11	6	1	9	4	12	7	2	10	5
9	0	9	5	1	10	6	2	11	7	3	12	8	4
10	0	10	7	4	1	11	8	5	2	12	9	6	3
11	0	11	9	7	5	3	1	12	10	8	6	4	2
12	0	12	11	10	9	8	7	6	5	4	3	2	1

Use this multiplication table to answer following questions.

a)

The number x such that $7x \equiv 1 \pmod{13}$ is called the **inverse** of 7 in $\mathbb{Z}_{13}$. Find the inverse of 7 in $\mathbb{Z}_{13}$.

2 ✔ Expected answer: 2

Score: 1/1 ✔

▶ Show feedback

b)

Solve $7x \equiv 8 \pmod{13}$ for x.

3 ✔ Expected answer: 3

Score: 1/1 ✔

▶ Show feedback

c)

Solve $7x \equiv 10 \pmod{13}$ for x.

7 ✔ Expected answer: 7

Score: 1/1 ✔

▶ Show feedback

d)

Solve $7x + 12 \equiv 2 \pmod{13}$ for x.

6 ✔ Expected answer: 6

Score: 1/1 ✔

▶ Show feedback

Advice

This question introduces the concept of multiplication in modular arithmetic. All the answers can be found by looking up the appropriate row and column in the given multiplication table.

Aside: It is easy to find the inverse of a when working in $\mathbb{Z}_p$ for some prime number p. By Fermat's Little Theorem $a^p \equiv a \pmod{p}$, so if a is not a multiple of p then $a^{-1} \equiv a^{p-2} \pmod{p}$.

Question 4

Given integers $a, b \in \mathbb{Z}$ we can write a in the form

$$a = qb + r$$

for some $q, r \in \mathbb{Z}$ where $0 \leq r < b$. The numbers q and r are called the **quotient** and **remainder** when a is divided by b:

$$\frac{a}{b} = q + \frac{r}{b}.$$

In **modular arithmetic** (in this case, modulo b) our interest is focused on the remainder. The operation $a \bmod b = r$.

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a)

Which numbers have a remainder of 4 when divided by 6? These numbers would be considered equal in modulo 6.

- [ ] 11
- [x] -2
- [x] 22
- [x] -8
- [ ] -3
- [x] 10
- [ ] -7

✔

Expected answer:

- [ ] 11
- [x] -2
- [x] 22
- [x] -8
- [ ] -3
- [x] 10
- [ ] -7

Score: 1/1 ✔

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b)

Which numbers are equal to 2 in modulo 4?

13	-3	7	2	-2	-6	6
☐	☐	☐	☑	☑	☑	☑

✔

Expected answer:

13	-3	7	2	-2	-6	6
☐	☐	☐	☑	☑	☑	☑

Score: 1/1 ✔

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c)

A number a is a **multiple** of b precisely when $a = qb$ for some number q. In terms of modular arithmetic we would say $a \bmod b = 0$ because a has no remainder when divided by b.

Enter a non-zero number which is equivalent to $0 \pmod 6$ and also equivalent to $0 \pmod 4$.

24 ✔

Expected answer: 24

Score: 1/1 ✔

▶ Show feedback

Advice

The first two questions introduce the idea that different numbers in $\mathbb{Z}$ can be the same in modulo b. For example in modulo 6 the numbers

$$6, 12, 18, \cdots$$

are equivalent to zero. While the numbers

$$5, 9, 13, \cdots$$

are equivalent to $1 \pmod 4$.

The next question introduce a simple divisibility test: $a = qb = qb + 0$ exactly when $a \equiv 0 \pmod b$. This is just another way to say that b divides a, or that a is a multiple of b. In particular

$$24 = 6 \times 4 \text{ is divisible by } 6 \text{ and } 4.$$

Question 5

In each of these questions, write the given number as a product of powers of primes in ascending order.

Give your answer in list form, where

the 1st element represents the power of 2

the 2nd element represents the power of 3 the 3rd element represents the power of 5

the 4th element represents the power of 7

For example

$210 = 2 \times 3 \times 5 \times 7$ we would give our answer as $[1, 1, 1, 1]$

$63 = 3^2 \times 7$ we would give our answer as $[0, 2, 0, 1]$

$750 = 2 \times 3 \times 5^3$ we would give our answer as $[1, 1, 3, 0]$

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a)

$2268 =$

[2,4,0,1] $[2, 4, 0, 1]$ ✔

Expected answer: [2, 4, 0, 1] $[2, 4, 0, 1]$

Score: 1/1 ✔

▶ Show feedback

b)

$84 =$

[2,1,0,1] $[2, 1, 0, 1]$ ✔

Expected answer: [2, 1, 0, 1] $[2, 1, 0, 1]$

Score: 1/1 ✔

▶ Show feedback

c)

$15750 =$

`[1,2,3,1]` $[1, 2, 3, 1]$ ✔

Expected answer: `[ 1, 2, 3, 1 ]` $[1, 2, 3, 1]$

Score: 1/1 ✔

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Advice

To find the prime factorisation of a number, keep breaking that number down into pairs of factors until all number written down are primes.

e.g. To find the prime factorisation of 2450 we could note that

$$2450 = 245 \times 10$$

$$2450 = 245 \times \underline{5} \times \underline{2}$$

$$2450 = \underline{5} \times 49 \times \underline{5} \times \underline{2}$$

$$2450 = \underline{5} \times \underline{7} \times \underline{7} \times \underline{5} \times \underline{2}$$

where we have underlined the primes as we go along.

Rewriting this in ascending order of primes, we obtain $2450 = 2 \times 5 \times 5 \times 7 \times 7$

which we can also write as $2450 = 2 \times 5^2 \times 7^2$

(a) Applying a similar method to the given questions, we can obtain:

We can write the given numbers as products of prime factors as follows:

$2268 = 2^2 \times 3^4 \times 7$

$84 = 2^2 \times 3 \times 7$

$15750 = 2 \times 3^2 \times 5^3 \times 7$

Question 6

Find α and β.

Using the rules of logs:

$$\log_a(x^b) = b\log(x)$$
$$\log_a(x) + \log_a(y) = \log_a(xy),$$
$$\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right).$$

Express the following in terms of $\log_a(x)$ and $\log_a(y)$

$$\log_a(x^{-6}y^{15}) = \alpha\log_a(x) + \beta\log_a(y)$$

$\alpha =$ -6 ✔ Expected answer: -6 , $\beta =$ 15 ✔ Expected answer: 15

Score: 1/1 ✔

▶ Show feedback

Advice

The rules for combining logs are

$$\log_a(bc) = \log_a(b) + \log_a(c)$$

$$\log_a\left(\frac{b}{c}\right) = \log_a(b) - \log_a(c)$$

$$\log_a(b^r) = r\log_a(b)$$

a)
Using these rules gives:

$$\begin{aligned}\log_a(x^{-6}y^{15}) &= \log_a(x^{-6}) + \log_a(y^{15}) \\ &= -6\log_a(x) + 15\log_a(y)\end{aligned}$$

b)

$$\begin{aligned}\frac{17}{4}\log_a(x) + \log_a(9x+8) - \log_a(x^{\frac{1}{4}}) &= \log_a(x^{\frac{17}{4}}) + \log_a(9x+8) - \log_a(x^{\frac{1}{4}}) \\ &= \log_a\left(x^4\ (9x+8)\right) \\ &= \log_a\left(x^4\ (9x+8)\right)\end{aligned}$$

Question 7

a)

Express the following as a single power of 9:

$$9^4 \times 9^{11}$$

Enter the power 15 15 ✔ Expected answer: 15 15

Score: 1/1 ✔

▶ Show feedback

b)

Express the following as a single power of 2:

$$(2^2)^5$$

Enter the power 10 10 ✔ Expected answer: 10 10

Score: 1/1 ✔

▶ Show feedback

c)

Express the following as a power of a single number. The power should be a prime.

$$16^{11} \times 23^{11}$$

Enter the number 368 368 ✔ Expected answer: 368 368 and the power 11 11 ✔ Expected answer: 11 11

Score: 2/2 ✔

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Advice

(a) Add the powers: $4 + 11 = 15$ (first index law).

(b) Multiply the powers $2 \times 5 = 10$ (second index law).

(c) Multiply 16 and 23 to get 368, with the power unchanged at 11. [This is the only way for the power to be a prime.]