Solutions to Bain and Engelhardt's Introduction to Probability and Mathematical Statistics

06.01 This is where the first solution will go.
06.02 This is where the second solution will go.
06.15 This is a simplified version of example 6.4.5.

$X_1, X_2 \sim POI(\lambda)$ so the MGF of both is $e^{\lambda(e^t-1)}$. Thus by theorem 6.4.4

$$M_Y(t) = e^{\lambda(e^t-1)}e^{\lambda(e^t-1)} = e^{2\lambda(e^t-1)} \sim POI(2\lambda)$$

The pdf then of Y is $\frac{e^{-2\lambda}2\lambda^x}{x!}$
06.16 Note: the pdf of $f_{x_1,x_2} = \frac{1}{x_1^2}\frac{1}{x_2^2}$
a) We need to find $f_{u,v} = f_{x_1,x_2}(x_1(u,v), x_2(u,v))|J|$ where J is our jacobian. First we let $u = x_1x_2$ and $v = x_1$ thus $x_1 = v$ and $x_2 = \frac{u}{v}$, now we can find J.

$$J = \begin{vmatrix} 0 & 1 \\ \frac{1}{v} & 0 \end{vmatrix} = \frac{1}{v}$$

Finally, our pdf is:

$$\begin{aligned} f_{U,V}(u,v) &= f_{x_1,x_2}(v, \frac{u}{v}) \left|\frac{1}{v}\right| \\ &= \frac{1}{v^2}\frac{1}{(\frac{u}{v})^2}\left|\frac{1}{v}\right| \\ &= \frac{1}{u^2 v}, 1 < v < u < \infty \end{aligned}$$