33MeV and ETA Activation Analysis

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Energy Calibration

To make the energy calibration and efficiency curves, ^{241}Am, ^{133}Ba, ^{60}Co, ^{137}Cs, ^{152}Eu, and ^{88}Y sources were measured at 1cm and 18cm from the detector.

At 1cm, the energy calibration was

$$E_\gamma = (0.39716)CN - 0.34861$$

Where E_γ is the energy of a gamma, and CN is the channel number. This line has a standard slope error of 0.00008440535516 and a standard y-intercept error of 0.1847826124

At 18cm, the energy calibration was

$$E_\gamma = (0.39720)CN - 0.43063$$

with a standard slope error of 0.00009099026011 and a standard y-intercept error of 0.1991944395

Efficiency Calculation

The efficiency function was calculated using a least squares method. On physical grounds, the efficiency curves asymptotically approach a E_γ^{-1} relationship, so we do a linear least squares regression with the basis vectors x, 1, $\frac{1}{\mathrm{x}}$, $\frac{1}{\mathrm{x}^2}$ to fit the inverse efficiencies.

$$\mathscr{E}_{18}(E_\gamma) = \frac{{E_\gamma}^2}{(0.7115){E_\gamma}^3 + (147.8817){E_\gamma}^2 - (1169.8)E_\gamma + 63775}$$

We used this function for the efficiency instead of the function in James' iPython notbeook because it was discovered that there was a small peak at 1090keV with a branching ratio of 1.73% hiding in the 1086keV line for ^{152}Eu so the area of that peak was being visibly over-counted. Since James' peak-fitting program finds the peaks itself and fits them and then fits an efficiency curve based on the measured peak areas, it is not clear to us if we can point it to that extra peak so it doesn't include those counts when making the efficiency curve. Because of this complication, we used Ninad's efficiency curve which was calculated using the adjusted peak area for ^{152}Eu at 1086keV.

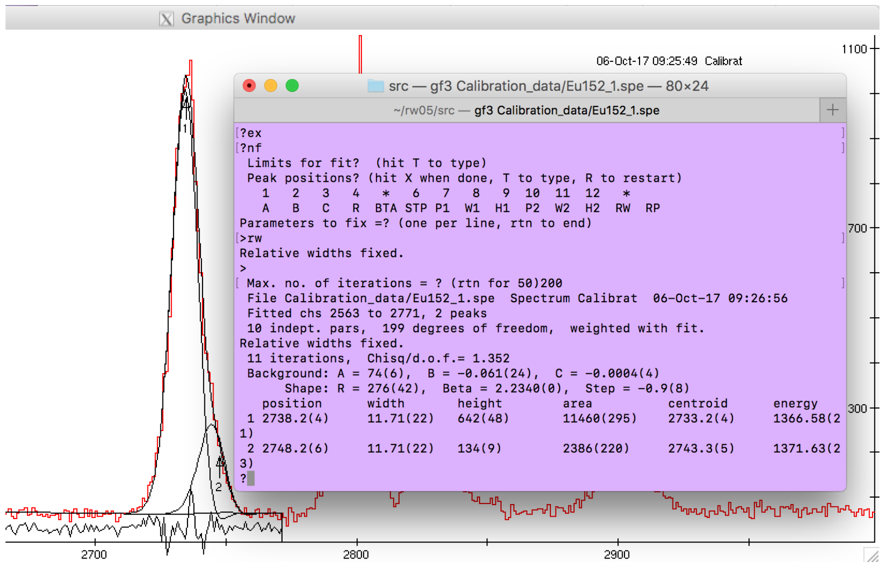

Figure 1: Corrected peak fit for Eu152 with extra peak at 1090keV

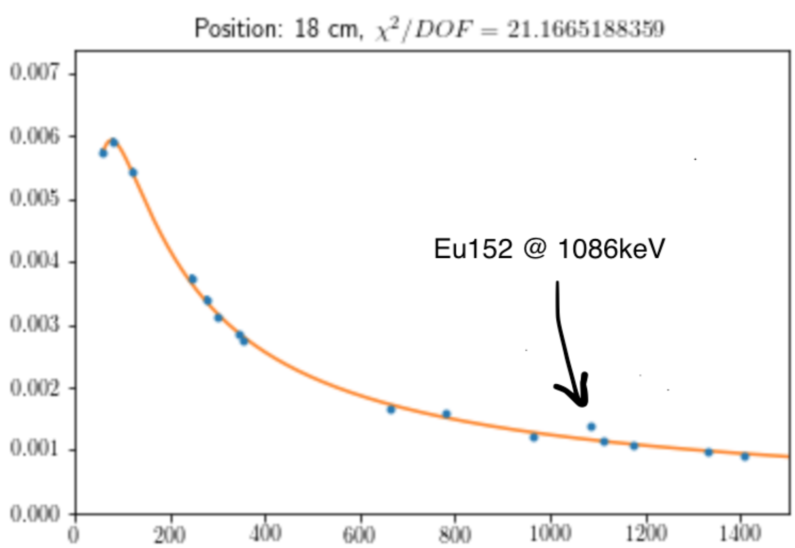

Figure 2: Efficiency curve before correction

Initial Activity Formula using Efficiency Ratios

The initial activity calculation is based on the following equation from Darren's Equipment Damage paper, adjusted for decay between irradiation and measurement.

$$A_0 = \frac{C\lambda e^{\lambda \Delta t_j}}{(1 - e^{-\lambda \Delta t_c})\epsilon(E_\gamma) f_l I_\gamma}$$

A_0 = the initial activity of the source at creation
C = measured counts
I = intensity of that gamma
f_l = the detector live-time fraction
λ = the decay constant associated with that gamma
t_0 = the irradiation stop time
t_1 = the counting start time
t_2 = the counting stop time
$\Delta t_c = t_2 - t_1$ = the total count time
$\Delta t_j = t_1 - t_0$ = the time between the end of irradiation and the start of the counting
$\epsilon(E_\gamma)$ = the efficiency of the detector for a given line

Since data sets 2 and 3 are both at 1cm, we want the efficiency at 1cm, ϵ_1. The efficiency factor can be split into three parts that matter for our purposes:

$F_{cm}(Nuclide)$, is the peak summing correction factor for a given characteristic gamma ray of a given nuclide at that distance.,

G_{cm}, the geometric correction factor to the efficiency,

$\mathscr{E}_{cm}(keV)$, the intrinsic energy dependent efficiency of the detector at that distance for single gamma sources.

So the total efficiency for a nuclide at a distance d is

$$\epsilon_d(E_\gamma) = F_d(Nuclide) G_d \mathscr{E}_d(E_\gamma)$$

If we want the corrected activity of the E_γ reaction for a given nuclide, n, at a distance of 1cm, then

$$A_0 = \frac{C_1 \lambda e^{\lambda \Delta t_{j(1cm)}}}{(1 - e^{-\lambda \Delta t_{c(1cm)}}) F_1(n) G_1 \mathscr{E}_1(E_\gamma) f_l I_\gamma}$$

$$A_0 = \frac{C_{18} \lambda e^{\lambda \Delta t_{j(18cm)}}}{(1 - e^{-\lambda \Delta t_{c(18cm)}}) F_{18}(n) G_{18} \mathscr{E}_{18}(E_\gamma) f_l I_\gamma}$$

$$\frac{C_1 \lambda e^{\lambda \Delta t_{j(1cm)}}}{(1 - e^{-\lambda \Delta t_{c(1cm)}}) F_1(n) G_1 \mathscr{E}_1(E_\gamma) f_l I_\gamma} = \frac{C_{18} \lambda e^{\lambda \Delta t_{j(18cm)}}}{(1 - e^{-\lambda \Delta t_{c(18cm)}}) F_{18}(n) G_{18} \mathscr{E}_{18}(E_\gamma) f_l I_\gamma}$$

So

$$\frac{C_1}{C_{18}} = \frac{F_1(n) G_1 \mathscr{E}_1(E_\gamma)}{F_{18}(n) G_{18} \mathscr{E}_{18}(E_\gamma)} \kappa$$

$$\kappa = \frac{e^{\lambda \Delta t_{j(18cm)}} (1 - e^{-\lambda \Delta t_{c(1cm)}})}{e^{\lambda \Delta t_{j(1cm)}} (1 - e^{-\lambda \Delta t_{c(18cm)}})} \frac{f_{l(1cm)}}{f_{l(18cm)}} = e^{\lambda (\Delta t_{j(18cm)} - \Delta t_{j(1cm)})} \frac{(1 - e^{-\lambda \Delta t_{c(1cm)}})}{(1 - e^{-\lambda \Delta t_{c(18cm)}})} \frac{f_{l(1cm)}}{f_{l(18cm)}}$$

$G_{18} \approx 1$ since it acts like a point source very far away, although a more exact value can also be calculated from MCNP.

$F_{18} \approx 1$ as well.

We can get $\frac{F_1(n)G_1\mathscr{E}_1}{F_{18}(n)G_{18}\mathscr{E}_{18}}$ by taking the ratios of the counts for the nuclide from data set 1.

$$F_1(n)G_1\mathscr{E}_1(E_\gamma) = \frac{1}{\kappa}\frac{C_1}{C_{18}}F_{18}(n)G_{18}\mathscr{E}_{18}(E_\gamma)$$

Plugging in, we have our final equation:

$$A_0(\text{Data Set x}) = \frac{C_1(\text{Data Set x})\lambda e^{\lambda\Delta t_j}}{(1-e^{-\lambda\Delta t_c})F_{18}(n)G_{18}\mathscr{E}_{18}(E_\gamma)f_l I_\gamma}\frac{C_{18}(\text{Data Set 1})}{C_1(\text{Data Set 1})}\kappa$$

$$A_0(\text{Data Set x}) = \frac{C_1(\text{Data Set x})\lambda e^{\lambda\Delta t_j}}{(1-e^{-\lambda\Delta t_c})\mathscr{E}_{18}(E_\gamma)f_l I_\gamma}\frac{C_{18}(\text{Data Set 1})}{C_1(\text{Data Set 1})}\kappa$$

where

$$\kappa = e^{\lambda(\Delta t_{j(18cm)}-\Delta t_{j(1cm)})}\frac{(1-e^{-\lambda\Delta t_{c(1cm)}})}{(1-e^{-\lambda\Delta t_{c(18cm)}})}\frac{f_{l(1cm)}}{f_{l(18cm)}}$$

Things to Note

The efficiency ratios in the equation above were calculated using the Correction Factor Data (Data Set 1). For ^{27}Al, Data Set 1 included counts for "Al1a" and "Al1b" as well as a second count of Al1a at 18cm. The Al1a had an elemental purity of 99.999 and a circumference of about 50mm just like the other foils in the Correction Factor Data. Al1b was shop aluminum with an unknown elemental purity and a circumference of 13.5mm.

By the time ^{27}Al and ^{115}In were measured at 1cm, the $843.76keV$ energy for the Al1a foil and the $1293.56keV$ energy for the In1 foil had decayed away. Since Data Sets 2 and 3 needed efficiency ratio data for ^{27}Al(n,p)^{27}Mg reaction, two methods were used to approximate the initial activity. First, the counts from Al1b were used to calculate the activity. This may provide an order-of-magnitude idea of what the activation for this reaction channel was, but this aluminum is impure and does not have the same dimensions as the aluminum for which we want the activation.

The second method was to use the $1014.52keV$ gamma peak for this reaction, but to use data from the ^{115}In(n,n')^{115m}In to calculate the correction factor $\frac{C_{18}(\text{Data Set 1})}{C_1(\text{Data Set 1})}\kappa$ for this peak since neither this $1014.52keV$ from ^{27}Mg nor the $336.24keV$ peak from ^{115m}In experience significant coincidence summing. The high purity aluminum was used to calculate the initial activity for the ^{27}Al$(n,a)^{24}$Na reaction so that activity should be fine, but note that the second count for Al1a (Row 5, Data Set 1) at 18cm was used to calculate this. Since there was no alternative foil for ^{115}In, the initial activity could not be calculated for the ^{115}In $(n,\gamma)^{116m}$In reaction.

Results

33MeV Beam Only					
Foil	Reaction	Gamma (keV)	Initial Activity (Bq)	Sim Init Act (Bq)	#Atoms Activated
^{27}Al	^{27}Al(n,p)^{27}Mg	843.76	1809^*	1098^1	1481000^*
^{27}Al	^{27}Al(n,p)^{27}Mg	1014.52	$1673^\dagger$	1098^1	$1370000^\dagger$
^{27}Al	^{27}Al(n,a)^{24}Na	1368.63	222.06 ± 2.69	218	17134000 ± 209000
^{197}Au	^{197}Au(n,2n)^{196}Au	355.7	30.978 ± 0.385	23.9	23863000 ± 297000
^{197}Au	^{197}Au(n,γ)^{198}Au	411.8	19.411 ± 0.325	0.7^2	6518600 ± 109100
^{115}In	^{115}In(n,n')^{115m}In	336.241	1695.7 ± 9.2	622^2	39508000 ± 216000
^{115}In	^{115}In(n,γ)^{116m}In	1293.56	unknown	209^2	unknown
^{58}Ni	^{58}Ni(n,2n)^{57}Ni	1377.63	17.11 ± 0.26	17.7	3164000 ± 48000
^{58}Ni	^{58}Ni(n,p)^{58}Co	810.76	9.1003 ± 0.0990	15.3^3	80379000 ± 874000
^{90}Zr	^{90}Zr(n,2n)^{89}Zr	909.15	54.14 ± 0.60	64.7^3	22045000 ± 246000

Activations in ETA				
Foil	Reaction	Gamma (keV)	Initial Activity (Bq)	#Atoms Activated
^{27}Al	^{27}Al(n,p)^{27}Mg	843.76	454.4^*	372000^*
^{27}Al	^{27}Al(n,p)^{27}Mg	1014.52	$397.5^\dagger$	$325400^\dagger$
^{27}Al	^{27}Al(n,a)^{24}Na	1368.63	382.11 ± 3.46	29689000 ± 269000
^{197}Au	^{197}Au(n,2n)^{196}Au	355.7	88.197 ± 1.140	67941000 ± 878000
^{197}Au	^{197}Au(n,γ)^{198}Au	411.8	148.57 ± 2.14	49892000 ± 720000
^{115}In	^{115}In(n,n')^{115m}In	336.241	2778.4 ± 20.1	64733000 ± 468000
^{115}In	^{115}In(n,γ)^{116m}In	1293.56	unknown	unknown
^{58}Ni	^{58}Ni(n,2n)^{57}Ni	1377.63	42.00 ± 0.52	7766000 ± 97000
^{58}Ni	^{58}Ni(n,p)^{58}Co	810.76	27.501 ± 0.266	242910000 ± 2350000
^{90}Zr	^{90}Zr(n,2n)^{89}Zr	909.15	139.9 ± 1.5	5698000 ± 620000

*These values determined using irradiated shop aluminum as a reference as discussed above.

†These values determined using data from ^{115}In(n,n')^{115m}In reaction as a reference.

^{1} This value should be modeled well, so the discrepancy should be investigated.

^{2} These channels will be underpredicted in the model due to use of Meulder's as a starting source (no neutrons < 2 MeV).

^{3} These channels are also fed from other reactions on other isotopes in natural Ni/Zr. Therefore, we'd expect the model to underpredict these channels, but the opposite happened.

1 Geometric Correction for Shop Aluminum

As before, use efficiency ratios to determine the efficiency of the detection for this line:

$$\frac{C_1}{C_{18}} = \frac{F_1(n)G_1\mathscr{E}_1(E_\gamma)}{F_{18}(n)G_{18}\mathscr{E}_{18}(E_\gamma)}\kappa$$

But now there is an additional geometric correction factor in G_d so the equation becomes

$$\frac{C_1}{C_{18}} = \frac{F_1(n)G_{1(\text{corrected})}\mathscr{E}_1(E_\gamma)}{F_{18}(n)G_{18(\text{corrected})}\mathscr{E}_{18}(E_\gamma)}\kappa$$

But we took $G_{18} \approx 1$ since it acts like a point source very far away. This is even more so true now and $F_{18} \approx 1$ still so we have

$$\frac{C_1}{C_{18}} = \frac{F_1(n)G_{1(\text{corrected})}\mathscr{E}_1(E_\gamma)}{\mathscr{E}_{18}(E_\gamma)}\kappa = \frac{F_1(n)G_1R_{corr}\mathscr{E}_1(E_\gamma)}{\mathscr{E}_{18}(E_\gamma)}\kappa$$

where R_{corr} = the geometric correction ratio for foils with circumferences other than 50mm.

The area of a disk with circumference c is

$$a = \frac{c^2}{4\pi}$$

The area subtended by the solid angle from the detector to the source is changed from the $50mm$ sources by a factor of $R_{corr} = \frac{a_{13.5mm}}{a_{50mm}} = \frac{(13.5)^2}{(50)^2}$

So the efficiency at $1cm$ using a $13.5mm$ foil for the efficiency ratio is given by

$$\frac{C_1}{C_{18}} \frac{\mathscr{E}_{18}(E_\gamma)}{R_{corr}} \frac{1}{\kappa} = F_1(n) G_1 \mathscr{E}_1(E_\gamma)$$

So

$$A_0(\text{Al-27}) = \frac{C_1(\text{Al-27})\lambda e^{\lambda \Delta t_j} R_{corr}}{(1 - e^{-\lambda \Delta t_c})\mathscr{E}_{18}(E_\gamma) f_l I_\gamma} \frac{C_{18}(\text{Al1b})}{C_1(\text{Al1b})} \kappa$$

The calculation of the initial activity without this correction factor was only 8 percent off from the initial activity calculated using In115(n,n') to calculate the efficiency ratios. Applying the correction factor this way gives an initial activity of $132Bq$ which is off from the initial activity 92 percent which is 92 percent off from the initial activity calculated using In115(n,n').

Uncertainty Propagation

An approximate uncertainty in the initial activities was calculated using propagation of uncertainty in the number of counts in each gamma peak. The uncertainty is given by

$$\Delta_{A_0(\text{Data Set x})} \approx \kappa \frac{\lambda e^{\lambda \Delta t_j}}{(1 - e^{-\lambda \Delta t_c})\mathscr{E}_{18}(E_\gamma) f_l I_\gamma} \Delta_{counts}$$

where

$$\Delta_{counts} = \sqrt{\left(\frac{C_{18}(\text{Data 1})}{C_1(\text{Data 1})} \Delta_{C_1(\text{Data x})}\right)^2 + \Big(C_1(\text{Data x})\Big)^2 \left(\left(\frac{\Delta_{C_{18}(\text{Data 1})}}{C_1(\text{Data 1})}\right)^2 + \left(\frac{C_{18}(\text{Data 1})\Delta_{C_1(\text{Data 1})}}{(C_1(\text{Data 1}))^2}\right)^2\right)}$$