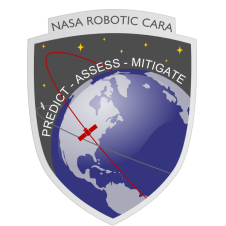


Software Development Kit: Three-Dimensional Probability of Collision (Pc) Calculations

CONJUNCTION ASSESSMENT AND RISK ANALYSIS (CARA) PROGRAM



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**Preface**

This document outlines the 3D Probability of Collision Calculation submitted as part of the Software Development Kit (SDK). The SDK is intended to provide both industry and government customers with a code base with which to perform standard calculations inherent to the Collision Avoidance (CA) problem and as outlined in the CA Standard.

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# Introduction

The CARA Software Development Kit (SDK) contains entries and artifacts for each major algorithm needed to perform the required Collision Avoidance (CA) calculations outlined in the CA Standard. For a typical algorithm, the SDK will include a version of the algorithm, a driver program to take information from a text format CDM and execute the algorithm, producing the needed calculation or output, and a series of test cases that exercise the algorithm and produce validated results.

This document describes a specific algorithm, its associated inputs and outputs, the methodology used within the algorithm and examples of usage.

## Required Software

The following list is of software and hardware requirements for use of this SDK:

* Matlab 2016b

# Risk Assessment Algorithm

## 3D Probability of Collision Calculation

One method that may be employed to estimate the probability of collision (Pc) is the “3D-Nc” algorithm formulated by Hall (2021)[[1]](#endnote-1) which estimates the statistically expected number of collisions (Nc) for satellite encounters. When applied to a single, high relative velocity satellite conjunction, the 3D-Nc method provides a direct estimate of the collision probability (i.e., Pc = Nc for temporally-isolated conjunctions). When applied to a low relative velocity conjunction and/or to a multi-encounter interaction between two satellites, the 3D-Nc method provides an upper limit estimate of the collision probability (i.e., Pc ≤ Nc for extended/blended conjunctions).

### 3D-Pc for an Isolated Conjunction – Mathematical Formulas

For an isolated conjunction, the three-dimensional probability of collision (3D-Pc) approximation is calculated by integrating the statistically expected collision rate over time:

This is the number of collisions statistically expected to occur at some time during the interval . For high relative velocity conjunctions, this time interval typically only needs to span a very short duration near the time of closest approach (TCA).

The statistically expected collision rate is calculated by integrating the product of two functions over the unit sphere:

This collision rate typically peaks very sharply in time at or near TCA for high relative velocity conjunctions. The radius represents the combined circumscribing, hard-body radii of the two satellites. The two required integrand functions and their arguments are calculated from the TCA orbital states and covariance matrices of the two satellites involved in the conjunction, as described in detail in the Hall (2021) 3D-Nc method paper.

The required integrals above are both evaluated numerically, with the 1D time integration using trapezoidal quadrature, and the 2D unit-sphere integration using Lebedev quadrature. Calculating the overall Pc estimate therefore entails 3D numerical integration.

### 3D-Pc for an Isolated Conjunction – Source Code Description

The primary function contained within the SDK used for estimating the 3D Probability of Collision of an isolated close approach event is:

Pc3D\_Hall.m

This function estimates the probability of collision using the formula above.

As inputs, the function accepts the following:

Table 1: 3D Probability of Collision Function Input Parameters

|  |  |
| --- | --- |
| Input Variable | Definition |
| r1 | [3X1] TCA ECI Position Vector of the Primary Object (meters) |
| v1 | [3X1] TCA ECI Velocity Vector of the Primary Object (meters/second) |
| cov1 | [6X6] TCA Primary State covariance matrix corresponding to input primary object reference frame |
| r2 | [3X1] TCA ECI Position Vector of the Secondary Object (meters) |
| v2 | [3X1] TCA ECI Velocity Vector of the Secondary Object (meters/second) |
| cov2 | [6X6] TCA Secondary State covariance matrix corresponding to input primary object reference frame |
| HBR | Combined hard body radius or exclusion zone of the two objects (m) |
| params | A Matlab structure holding parameters required for the calculation (e.g., Lebedev unit-sphere quadrature coefficients). The function default\_params\_Pc3D\_Hall.m initiates these parameters to their default values. |

The 3D Probability of Collision function outputs the following:

Table 2: 3D Probability of Collision Function Output Parameters

|  |  |
| --- | --- |
| Output Variable | Definition |
| Pc | Probability of Collision calculated using the 3D-Nc method approximation as applied to a single conjunction. |
| out | A Matlab structure holding a large number of auxiliary output variables, such as arrays holding the integration time points, , and the associated estimated collision rates, . See the header of the Pc3D\_Hall.m function for a detailed description of these auxiliary outputs. For users that only need conjunction Pc estimates, this output can be neglected. |

Validation cases for the algorithm are contained within the unit test suite for the SDK at:

..\SDK\ThreeDimensionalPc\UnitTest\Pc3D\_Hall\_UnitTest.m

These test cases were developed using previously defined stressing cases developed by Alfano (2009)[[2]](#endnote-2), supplemented with test cases developed by Omitron to test specific issues related to the 3D-Pc calculation.

Table 3: 3D Probability of Collision Function Unit Test Cases

|  |  |
| --- | --- |
| Test ID | Description |
| test01 | Alfano test case 1 |
| test02 | Alfano test case 2 |
| test03 | Alfano test case 3 |
| test04 | Alfano test case 4 |
| test05 | Alfano test case 5 |
| test06 | Alfano test case 6 |
| test07 | Alfano test case 7 |
| test08 | Alfano test case 8 |
| test09 | Alfano test case 9 |
| test10 | Alfano test case 10 |
| test12 | Omitron test case designed to test circular cross-sectional area calculation of probability of collision |
| test13 | Omitron test case designed to test non-positive definite (NPD) covariance matrix processing |
| test14 | Omitron test case designed to test a high-velocity conjunction for which the 3D-Pc estimate has been verified by Monte Carlo simulation Pc analysis, and for which 2D-Pc estimate is erroneously ~15 orders of magnitude too small. |

# Acronyms

|  |  |
| --- | --- |
| 2D | Two Dimensional |
| 3D | Three Dimensional |
| CARA | Conjunction Assessment Risk Analysis |

|  |  |
| --- | --- |
| CDM | Conjunction Data Message |

|  |  |
| --- | --- |
| ECI | Earth Centered Inertial |
| HBR | Hard Body Radius |
| NPD | Non-Positive Definite |
| Pc | Probability of Collision |
| SDK | Software Development Kit |
| TCA | Time of Closest Approach |

# References

1. Hall, D. T. *Expected Collision Rates for Tracked Satellites*. Journal of Spacecraft and Rockets, In Press, Jan 2021. [↑](#endnote-ref-1)
2. Alfano, S. *Satellite Conjunction Monte Carlo Analysis.* AAS 09-233. 2009. [↑](#endnote-ref-2)