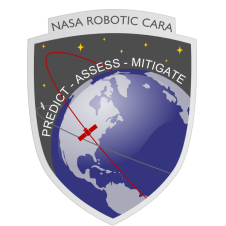


Software Development Kit: Monte Carlo Probability of Collision (Pc) from TCA (in equinoctial elements)

CONJUNCTION ASSESSMENT AND RISK ANALYSIS (CARA) PROGRAM



*Astrorum, Inc*

*Waco, Tx*

*Omitron, Inc*

*Colorado Springs, CO*

|  |
| --- |
|  |

National Aeronautics and

Space Administration

Goddard Space Flight Center,

Greenbelt, MD 20771

**December 2019Preface**

This document outlines the Monte Carlo Probability of Collision Calculation submitted as part of the Software Development Kit (SDK). The SDK is intended to provide both industry and government customers with a code base with which to perform standard calculations inherent to the Collision Avoidance (CA) problem and as outlined in the CA Standard.

# Table of Contents

[1.0 Introduction 4](#_Toc27649152)

[1.1 Required Software 4](#_Toc27649153)

[2.0 Risk Assessment Algorithms 5](#_Toc27649154)

[2.1 Monte Carlo Assessment of Probability of Collision at Time of Close Approach 5](#_Toc27649155)

[3.0 Acronyms 11](#_Toc27649156)

[4.0 References 12](#_Toc27649157)

**List of Tables**

[Table 1: Monte Carlo Routine Input Parameters 7](#_Toc27649158)

[Table 2: Monte Carlo Routine Output Parameters 8](#_Toc27649159)

[Table 3: Monte Carlo Unit Test Case Descriptions 10](#_Toc27649160)

# Introduction

The CARA Software Development Kit (SDK) contains entries and artifacts for each major algorithm needed to perform the required Collision Avoidance (CA) calculations outlined in the CA Standard. For a typical algorithm, the SDK will include a version of the algorithm, a driver program to take information from a text format CDM and execute the algorithm, producing the needed calculation or output, and a series of test cases that exercise the algorithm and produce validated results.

This document describes a specific algorithm, its associated inputs and outputs, the methodology used within the algorithm and examples of usage.

## Required Software

The following list is of software and hardware requirements for use of this SDK:

* Matlab 2016b
* Matlab Parallel Processing Toolbox

# Risk Assessment Algorithms

## Monte Carlo Assessment of Probability of Collision at Time of Close Approach

A method that may be employed as a method of determining the probability of collision is to perform a Monte Carlo simulation of both the primary and secondary object states at the time of close approach and statistically determine the probability of collision based on the number of trials which violate a predetermined proximity threshold.

A Monte Carlo simulation is a computational technique that allows a probabilistic process to be modelled using random sampling from a known multivariate distribution. As the process of orbit determination yields both a best estimate of a satellite state and its associated uncertainty, the problem of collision probability lends itself well to using Monte Carlo sampling methodology.

### Monte Carlo Sampling Methodology

#### General Monte Carlo Process for Estimating Probability of Collision

Using a best estimate of a satellite state, ***,*** and a corresponding measure of the uncertainty of that state using the covariance matrix,***C***, a series of Monte Carlo samples may be generated using the following methodology using the Eigen values, ***λn,*** and Eigen vectors, ***vn,*** of the covariance matrix:

This sampling assumes that uncertainty on the object states follows a normal distribution, and using this technique, any number of sample states may be generated. In this process, a sample state for both the primary and secondary object are generated independently and using identical processes. This set of a sampled primary and secondary object state are then taken as a single trial for the purposes of assessing collision probability.

The primary and secondary object states then have their time and geometry of close approach geometry estimated using linear interpolation between tightly spaced ephemeris nodes that are generated for both the primary and secondary satellites using a two body propagation. This close approach geometry is then compared to the given hard body radius or “keep-out” region and this trial is considered a collision if the minimum distance between the two objects is less than the hard body radius. This technique mirrors representing the exclusion zone of the two objects as a circle when calculating the two-dimensional probability of collision. With a sufficient number of trials, the probability of collision may be inferred by comparing the number of trial collisions to the total number of trials performed.

#### Variances in Approach to Monte Carlo Methodology

There are a few variations in the manner in which the Monte Carlo Approach may be employed, with different benefits and detriments inherent to each approach. First, there are two different methods of sampling the satellite states at the time of close approach (TCA).

1. Sampling using Cartesian state and covariance
2. Sampling using Equinoctial state and covariance

This implementation of Monte Carlo methodology performs sampling in the equinoctial frame, and if object states are provided in Cartesian frames, relevant state transformations are applied. Sampling in the Equinoctial frame preserves the non-linear nature of the satellite propagation, but requires additional state transformations to transform the state and covariance to Equinoctial elements for sampling, and then back to Cartesian elements for ease of determining each samples’ time and distance of close approach. The concern with non-linearities arises most prominently when either one of the objects’s covariance matrix is exceeding large, or the encounter time is extended due to low relative velocities between the two objects.

Second, there are two methods of determining the time and distance of close approach for each sample using given satellite state vectors:

1. Calculating close approach using a rectilinear approximation
2. Calculating close approach using two body Keplerian dynamics

The first approach is significantly faster as it can leverage vector dot products to rapidly find an answer to the time and geometry of close approach, but may incur significant errors when the non-linear nature of orbit propagation becomes an issue. Calculating the close approaches using Keplerian dynamics is more accurate, but incurs a significant calculation time increase due to propagation and interpolation.

### Monte Carlo – Source Code Description

The primary function contained within the SDK used for estimating the Probability of Collision (Pc) of a close approach event is the:

Pc\_MC\_Kep2Body\_parallel.m

routine, which estimates the time and geometry of the individual close approaches from the Monte Carlo trials using Keplerian 2-body motion.

As inputs, the routine accepts the following:

Table 1: Monte Carlo Routine Input Parameters

|  |  |
| --- | --- |
| Input Variable | Definition |
| Nsample\_total | Number of Monte Carlo samples to generate |
| tfc\_bins | Time bin definitions for outputting time of first contact between the primary and secondary objects |
| r1 | [3X1] ECI Position Vector of the Primary Object (meters)  - OR –  [6X1] Equinoctial State of the Primary Object |
| v1 | [3X1] ECI Velocity Vector of the Primary Object (meters/second)  - OR –  [ ] Empty Entry if primary state is defined in equinoctial elements |
| P1 | [6X6] Primary State covariance matrix corresponding to input primary object reference frame |
| r2 | [3X1] ECI Position Vector of the Secondary Object (meters)  - OR –  [6X1] Equinoctial State of the Secondary Object |
| v2 | [3X1] ECI Velocity Vector of the Secondary Object (meters/second)  - OR –  [ ] Empty Entry if secondary state is defined in equinoctial elements |
| P2 | [6X6] Secondary State covariance matrix corresponding to input primary object reference frame |
| HBR | Combined hard body radius or exclusion zone of the two objects (m) |
| GM | Central body gravitational constant (m3/s2)  For Earth, this is: 3.986004418e14 m3/s2 |
| motion\_mode | Dynamical model: 'k2bpri', or 'k2balt' |
| conf\_level | (optional) Desired Monte Carlo fractional confidence level (intended to bound estimates)  Example: a 95% confidence interval would be input as 0.95  If no value is input, defaults to 1-sigma confidence interval (68.27%) |
| Nsample\_batch | (optional) Number of Monte Carlo samples to run in individual batches, this is done to manage the parallel processing in Matlab.  If no value is input, a default value of 1000 samples per batch is used. |
| plot\_ca\_dist\_path | (optional) Path name to which to save the output Close Approach (CA) distribution plot  If no path is input, no plots will be generated |

The Monte Carlo analysis routine outputs the following:

Table 2: Monte Carlo Routine Output Parameters

|  |  |
| --- | --- |
| Output Variable | Definition |
| Pc\_all | Probability of Collision using all time of first contact counts |
| Uc\_all | Uncertainty bounds on the reported probability of collision using confidence bounds as determined by the “conf\_level” input variable |
| Nc\_tfc\_all | Number of all collision counts for times of first contact |
| Pc\_att | Attenuated Probability of Collision using only the first time of first contact counts. |
| Uc\_att | Uncertainty bounds on the reported attenuated probability of collision using confidence bounds as determined by the “conf\_level” input variable |
| Nc\_tfc\_att | Attenuated number of all collision counts |
| Pc\_0 | Probability of Collision at the first time included in the “tfc\_bins”, this reflects the probability of collision at all times prior to this bin[[1]](#endnote-1) and indicates that the time range input was insufficient.  For rapid conjunctions, this set of outputs may be neglected. |
| Uc\_0 | Uncertainty bounds on the reported probability of collision using confidence bounds as determined by the “conf\_level” input variable |
| Nc\_tfc\_0 | Number of all collision counts at the first time included in the “tfc\_bins” |

Validation cases for this algorithm are contained within the unit test suite for the SDK at:

..\MonteCarloPcCalcEQN\UnitTest\MonteCarloCode\Pc\_MC\_Kep2body\_parallel\_UnitTest.m

These test cases were developed using both operational data from a subset of previous events exceeding a probability of collision of 1E-4 and previously defined stressing cases developed by Alfano 2009[[2]](#endnote-2). Alfano test cases 7 through 12 were omitted due to either a desire to limit testing time to a reasonable span of time or due to the encounter time being poorly defined. As these are Monte Carlo processes, the Unit Test Cases may occasionally fail due to the random nature of Monte Carlo sampling. If this occurs please examine the failure mode first, and attempt to re-run the unit test cases.

Table 3: Monte Carlo Unit Test Case Descriptions

|  |  |
| --- | --- |
| Test ID | Description |
| test01 | Operational close approach event with maximum estimated probability of collision from selected subset of events using original hard body radius of 20 meters. |
| test02 | Operational close approach event with maximum secondary object radial position uncertainty from selected subset of events using a modified hard body radius of 100 meters for more rapid testing. |
| test03 | Operational close approach event with maximum secondary object intrack position uncertainty from selected subset of events using a modified hard body radius of 100 meters for more rapid testing. |
| test05 | Operational close approach event with minimum miss distance from selected subset of events using a modified hard body radius of 20 meters for more rapid testing. |
| test06 | Operational close approach event with minimum relative velocity from selected subset of events using original hard body radius of 20 meters. |
| test07 | Alfano test case 1 |
| test08 | Alfano test case 3 |
| test09 | Alfano test case 4 |
| test10 | Alfano test case 5 |
| test11 | Alfano test case 6 |

# Acronyms

|  |  |
| --- | --- |
| CA | Close Approach |
| CARA | Conjunction Assessment Risk Analysis |
| CDM | Conjunction Data Message |
| ECI | Earth Centered Inertial |
| HBR | Hard Body Radius |
| Pc | Probability of Collision |
| SDK | Software Development Kit |

# References

1. Coppola, Vincent T. *Evaluating the Short Encounter Assumption of the Probability of Collision Formula.* AAS 12-248. 2012. [↑](#endnote-ref-1)
2. Alfano, S. *Satellite Conjunction Monte Carlo Analysis.* AAS 09-233. 2009. [↑](#endnote-ref-2)