LSST DESC Notes

Core Cosmology Library: Precision Cosmological Predictions for LSST

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The Core Cosmology Library (CCL) provides routines to compute basic cosmological observables with validated numerical accuracy. These routines have been validated to an accuracy level, documented here, against the results of the Code Comparison Project. In the current version, predictions are provided for distances and background quantities, angular auto- and cross-spectra of cosmic shear and clustering, and the halo mass function. Fiducial specifications for the expected LSST galaxy distributions and clustering bias are also included, together with the capability of computing redshift distributions for a user-defined photometric redshift model. CCL is written in C, with a Python interface. In this note, we explain the functionality of the publicly released (CCL v0.2) library.

Contents

1. Introduction 5

2. Installation 5

2.1. Dependencies 5

2.2. Installing CLASS 6

2.3. Installing Angpow 6

2.4. Installation Procedure 7

2.5. Alternative installation and pip 8

2.6. Compiling against an external version of CLASS 8

2.7. Creating a Docker Image 9

2.8. Documentation 10

3. Functionality 11

3.1. Physical constants 11

3.2. Supported cosmological models 11

3.3. Creating a cosmology 12

3.4. Distances 13

3.5. Density parameter functions 14

3.6. Growth function 15

3.7. Matter power spectrum 16

3.7.1. BBKS 16

3.7.2. Eisenstein and Hu 16
3.7.3. CLASS 17
3.7.4. Cosmic emulator 17
3.7.5. Impact of baryons 19
3.7.6. Spline parameters & the INI file 20
3.7.7. Extrapolation for the nonlinear power spectrum 21
3.7.8. Extrapolation for the linear power spectrum 23
3.7.9. Wishlist for the future 23
3.7.10. Normalization of the power spectrum 24
3.8. Angular power spectra 24
3.8.1. Exact expressions 24
3.8.2. The Limber approximation 27
3.8.3. Beyond limber 27
3.9. Correlation functions 30
3.10. Halo mass & halo bias functions 31
3.11. Photo-z implementation 33
3.12. LSST Specifications 34
4. Tests and validation 35
5. Examples for C implementation 40
6. Python wrapper 40

6.1. Python installation 40

6.2. Python example 42

6.3. Technical notes on how the Python wrapper is implemented 43

7. Future functionality to be included 45

8. Feedback 45

9. Citing CCL 45

10. License 46

1. Introduction

In preparation for constraining cosmology with the Large Synoptic Survey Telescope (LSST), it is necessary to be able to produce rigorous theoretical predictions for the cosmological quantities that will be measured. The Core Cosmology Library[1] (CCL) aims to provide, in one library, a way of making predictions that are validated to a well-documented numerical accuracy, for the purpose of constraining cosmology with LSST. By constructing a cosmology library specifically with LSST in mind, it is possible to ensure that it is flexible, adaptable, and validated for all cases of interest, as well as user-friendly and appropriate for the needs of all working groups.

The Core Cosmology Library is written in C and incorporates the CLASS code (Blas et al. 2011) to provide predictions for the matter power spectrum. It also incorporates emulated power spectra from the cosmic emulator of Lawrence et al. (2017).[2] A Python wrapper is also provided for ease of use.

This note describes how to install CCL (Section 2), how CCL is documented (Section 2.8), its functionality (Section 3), the relevant unit tests (Section 4), directions for finding a CCL example (Section 5), the Python wrapper (Section 6), future plans (Section 7), means to contact the developers (Section 8), instructions on how to cite CCL (and CLASS, Section 9), and the license under which CCL is released (Section 10).

2. Installation

2.1. Dependencies

- GNU Scientific Library GSL,[3] GSL-2.1 or higher.
- The SWIG[4] Python wrapper generator is not needed to run CCL, but must be installed if you intend to modify CCL in any way.
- FFTW3[5] is required for computation of correlation functions.
- FFTlog[6] is provided within CCL, with minor modifications.

[1] https://github.com/LSSTDESC/CCL
[2] Future versions of the library will incorporate other power-spectrum libraries and methods.
[3] https://www.gnu.org/software/gsl/
[4] http://www.swig.org/
[5] http://www.fftw.org
[6] http://casa.colorado.edu/~ajsh/FFTLog/ and https://github.com/slosar/FFTLog

- The C library associated to the CLASS code. The installation of this library is described below.

2.2. Installing CLASS

CCL uses CLASS as one of the possible ways of computing the matter power spectrum. In order to communicate with CLASS, CCL must be linked to its library. Before installing CCL proper you must therefore install this library first. Since this process is not necessarily straightforward, we provide a python script `class_install.py` that automatically downloads and install the latest tagged stable version of CLASS. You should run this script (i.e. `$ python class_install.py`) before carrying out the next steps. By default, the script assumes that your main C compiler is `gcc`. If that's not the case, pass the name of your C compiler to the script via the command-line argument `--c_comp` (e.g. `$ python class_install.py --c_comp=icc`). Type `python class_install.py -h` for further details.

This procedure has one final caveat: if you already have a working installation of CCL, `class_install.py` may fail the first time you run it. This can be fixed by either simply running `class_install.py` a second time, or by starting from scratch (i.e. downloading or cloning CCL).

Note that, if you want to use your own version of CLASS, you should follow the steps described in Section 2.6 below.

2.3. Installing Angpow

CCL provides an optional link to the Angpow library that enables fast and accurate computations of the angular power spectra without using the Limber approximation (written in C++). We provide a python script to automatically install Angpow and make the link with CCL. You should first run this script (`$ python angpow_install.py`) before carrying out the next steps. The installation downloads the last release of Angpow adapted to CCL from a dedicated DESC github repository[7] and looks for a C++ compiler compatible with OpenMP. It provides a dedicated file `./angpow/src/angpow_ccl.cc`‘ that makes the interface between the CCL structures and the Angpow classes.

[7] https://github.com/LSSTDESC/Angpow4CCL

To remove Angpow, run $ python angpow_install.py --clean and install CCL again.

2.4. Installation Procedure

CCL can be installed through an autotools-generated configuration file. UNIX users should be familiar with the process: navigate to the directory containing the library and type

```
$ ./configure
$ make
$ make install
```

(You may need to pre-append sudo to the last command, depending on your default privileges.) Users without admin privileges can install the library in a user-defined directory (e.g. /home/desc_fan/) by running

```
$ ./configure --prefix=/home/desc_fan
$ make
$ make install
```

This will create two directories (if not present already): /home/desc_fan/include and /home/desc_fan/lib where the header and shared library files will be placed after running make install. CCL has been successfully installed on several different Linux and Mac OS X systems.[8]

After installing the C library, you can make sure that it works correctly by typing make check, which will run the unit tests described in Section 4. Assuming that the tests pass, you can then move on to installing the Python wrapper, as described in Section 6 (or see Section 2.5 below).

After pulling a new version of CCL from the git repository, you can recompile the library by running:

```
$ make clean; make uninstall
$ make; make install
```

[8] We know of one case with Mac OS where libtools had the "lock" function set to "yes" and this caused the installation to stall. However, this is very rare. If this happens, after the configure step, edit libtool to set the "lock" to "no".

CCL library can be called from C++ code without any additional requirements or modifications. To make sure that there are no problems you can run

```
$ make check-cpp
$ ./examples/ccl_sample_run
```

2.5. Alternative installation and pip

It is possible to install both the C library and the Python wrapper together in one step, by running

```
$ python setup.py install --prefix=/home/desc_fan
```

This will automatically perform the previous steps to install the C library, and will also install the Python library in /home/desc_fan/lib/python2.7/site-packages. You might need to add this path to the $PYTHONPATH environment variable to be able to use it. In the near future, the package will be made available through pip.

Once installed you can check the installation status by typing

```
$ python setup.py test
```

This will run the embedded unit tests. Using this last method to install the Python library allows you to uninstall it simply by running

```
$ python setup.py uninstall
```

2.6. Compiling against an external version of CLASS

The default installation procedure for CCL implies automatically downloading and installing a tagged version of CLASS. Optionally, you can also link CCL against an external version of CLASS. This section is useful if you want to use a modified version of CLASS, or a different or more up-to-date version of the standard CLASS. For example, we have successfully compiled and run CCL with hiCLASS (Zumalacarregui et al. 2016).

To compile CCL with an external version of CLASS, you must first prepare the external copy so that it can be linked as a shared library. By default, the CLASS build tools create a static library. After compiling CLASS in the usual way (by running make), look for a static library file called libclass.a that should have been placed in the root source directory. Then, run the following command from that directory (Linux only):

```
$ gcc -shared -o libclass.so -Wl,--whole-archive libclass.a \
                             -Wl,--no-whole-archive -lgomp
```

This should create a new shared library, libclass.so, in the same directory. (N.B. The -lgomp flag has to appear at the end of the command; otherwise the linker can fail.) If you are running Mac OS X, use the following command instead:

```
$ gcc -fpic -shared -o libclass.dylib -Wl,-all_load libclass.a \
                             -Wl,-noall_load
```

Next, change to the root CCL directory and run make clean if you have previously run the compilation process. Then, set the CLASSDIR environment variable to point to the directory containing libclass.so:

```
export CLASSDIR=/path/to/external/class
```

Then, run ./configure and compile and install CCL as usual. The CCL build tools should take care of linking to the external version of CLASS.

Once compilation has finished, run make check to make sure everything is working correctly. Remember to add the external CLASS library directory to your system library path, using either export LD_LIBRARY_PATH=/path/to/external/class (Linux) or export DYLD_FALLBACK_LIBRARY_PATH=/path/to/external/class (Mac). The system must be able to find both the CCL and CLASS libraries; it is not enough to only add CCL to the library path.

2.7. **Creating a Docker Image**

CCL supports the use of Docker, for the deployment of applications inside software containers. The Dockerfile included with CCL makes it possible to quickly create an image file that can be used to run virtual machines that require no additional

dependencies beyond the Docker software. The Docker website[9] details the installation and start-up process for most common operating systems.

With Docker installed and the Docker Daemon running, creating an image is relatively straightforward:

```
docker build -t ccl .
```

This will begin the process of creating the image file for CCL locally. This Dockerfile contains all of the C libraries needed by CCL, as well as the Python wrapper. It currently uses `python:2.7` as a base image, and supports both ipython and Jupyter notebook. The virtualization process should have minimum impact on performance.

The resulting Docker image has two main functions. The first is a command that will open a Jupyter notebook, tied to a port on your local machine. This can be accessed by running:

```
docker run -p 8888:8888 ccl
```

The Jupyter notebook can then be accessed through a browser at the address `localhost:8888`. The second possibility is to run a `bash` terminal, by issuing the command:

```
docker run -it ccl bash
```

This will allow for full access to the virtual machine via a terminal, so you can install additional dependencies etc.

2.8. **Documentation**

CCL has basic `doxygen`[10] documentation for its C routines. This can be found in the directory `doc/html` within the CCL repository by opening the `index.html` file in your browser. The `python` routines are documented in situ; you can view the documentation for a function by calling `help(function_name)` from within `python`.

[9] https://www.docker.com
[10] http://www.stack.nl/~dimitri/doxygen/

Table 1. Physical constants in CCL.

	G_{Newt}	k_b	σ_{SB}	h	c	eV	$M_\odot$	pc
NIST	-3.0e-05	-1.4e-06	-5.9e-06	1.6e-07	0.0e+00	8.4e-08	–	–
PDG 2013	-6.0e-05	-1.1e-06	-4.8e-06	9.2e-08	0.0e+00	4.9e-08	-2.0e-04	1.1e-09
GSL	-1.9e-04	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	-4.5e-11
CLASS	0.0e+00	0.0e+00	–	0.0e+00	0.0e+00	0.0e+00	–	-6.7e-11

3. Functionality

3.1. Physical constants

We have performed a comparison of the physical constants used in CCL and included dependencies and external sources. See Table 1 for percent differences of the constants between these sources.

3.2. Supported cosmological models

Ultimately, CCL plans to incorporate theoretical predictions for all cosmological models of interest to LSST. Currently, the following families of models are supported:

- Flat ΛCDM
- wCDM and the CPL model ($w_0 + w_a$, Chevallier & Polarski 2001 and Linder 2003)
- Non-zero curvature (K)
- All of the above, plus an arbitrary, user-defined modified growth function (see description in Section 3.6)
- A single massive neutrino species or multiple equal-mass massive neutrinos, in combination with any of the above (except the user-defined modified growth function).

Not all features of CCL are available for all models. For a guide to which predictions are available for each model, see Table 2. Note that if users install their own version of CLASS, CCL can then make predictions for a more extended set of cosmologies.

Table 2. Cosmologies implemented in CCL.

Observable/Model	flat ΛCDM	ΛCDM+K	ΛCDM + m_ν	wCDM	$w_0 + w_a$	MG
Distances	✓	✓	✓	✓	✓	X
Growth	✓	✓	X	✓	✓	✓
$P_m(k,z)$	✓	✓	✓	✓	✓	X
Halo Mass Function	✓	✓	X	✓	✓	X
C_l, number counts	✓	✓	X	✓	✓	X
C_l, weak lensing only	✓	✓	✓	✓	✓	X

Users should take care to understand the validity of the CCL assumptions for their own models.

In its default configuration, CCL adopts the nonlinear matter power spectrum from CLASS through the Halofit implementation and the Tinker mass function for cluster number counts.

3.3. **Creating a cosmology**

To use CCL, the first step is to create a `ccl_cosmology` structure, containing all of the information required to compute cosmological observables. A `ccl_cosmology` structure can be generated using the information from a `ccl_parameters` object and a `ccl_configuration` object.

`ccl_parameters` objects contain information about the cosmological parameters of a given model, and are initialized using one of the following routines (the full syntax for each function can be found in the header file `ccl_core.h`):

- `ccl_parameters_create(double Omega_c, double Omega_b, double Omega_k, double N_nu_rel, double N_nu_mass, double mnu, double w0, double wa, double h, double norm_pk, double n_s,int nz_mgrowth,double *zarr_mgrowth,double *dfarr_mgrowth, int *status)`: general `ccl_parameters` constructor supporting all the models described above.
- `ccl_parameters_create_flat_lcdm(...)`: particular constructor for flat ΛCDM cosmologies.
- `ccl_parameters_create_flat_wcdm(...)`: constant w cosmologies.
- `ccl_parameters_create_flat_wacdm(...)`: $w_0 + w_a$.

- `ccl_parameters_create_lcdm(...)`: curved ΛCDM cosmologies.

The argument `norm_pk` can be used to specify the power spectrum normalization in terms of either σ_8 or A_s. The `ccl_parameters_create` functions assume σ_8 normalization if `norm_pk` $>$ `1.0e` $-$ `5` and A_s normalization otherwise.

To include massive neutrinos, the suffix '`_nu`' may be appended to the last four `ccl_parameters_create` functions above. Using these functions without the `_nu` suffix will set the number of massive neutrinos to 0 and the effective number of massless neutrinos to 3.046. In the case of non-zero massive neutrinos, it may be desirable to set `N_nu_rel` such that $N_{\rm eff} = 3.046$ in the early universe. In agreement with `CLASS`, when using the default value of $T_{\rm NCDM}$ as described in section 3.5 below, `N_nu_rel` can be set to 2.0328, 1.0196, and 0.00641 for 1, 2 and 3 massive neutrinos respectively to achieve this.

`ccl_configuration` objects contain information about the prescriptions to be used to compute transfer functions, power spectra, mass functions, etc. A default `ccl_configuration` object is made available as `default_config`, which specifies that CLASS will be used to compute transfer functions, HaloFit will be used to calculate the matter power spectrum, there will be no impact of baryons included and the Tinker 2010 prescription will be used to compute the halo mass function.

After initializing instances of `ccl_parameters` and `ccl_configuration`, use the function `ccl_cosmology_create(ccl_parameters, ccl_configuration)` to return a pointer to a `ccl_cosmology` structure. You will need to pass this pointer around to every CCL function.

An example of using `CCL` is provided in Section 5. The `README` file has additional extensive documentation for the example run, as well as installation.

3.4. **Distances**

The routines described in this subsection are implemented in `ccl_background.c`.

The Hubble parameter is calculated as

$$\frac{H(a)}{H_0} = a^{-3/2}\Big(\Omega_{M,0} + \Omega_{\Lambda,0}a^{-3(w_0+w_a)}\exp[3w_a(a-1)] + \Omega_{K,0}a + (\Omega_{g,0} + \Omega_{\nu,\rm rel})a^{-1} + \Omega_{\nu,\rm m}(a)a^3\Big)^{\frac{1}{2}}. \quad (1)$$

The radial comoving distance is calculated via a numerical integral,

$$\chi(a) = c \int_a^1 \frac{da'}{a'^2 H(a')}. \tag{2}$$

The transverse comoving distance is computed in terms of the radial comoving distance as:

$$r(\chi) = \begin{cases} k^{-1/2} \sin(k^{1/2}\chi) & k > 0 \\ \chi & k = 0 \\ |k|^{-1/2} \sinh(|k|^{1/2}\chi) & k < 0 \end{cases} \tag{3}$$

The usual angular diameter distance is $d_A = a\, r(a)$, and the luminosity distance is $d_L = r(a)/a$.

CCL can also compute the distance modulus, defined as,

$$\mu = 5 \log_{10}(d_L/\mathrm{pc}) - 5 \tag{4}$$

and $a(\chi)$, the inverse of $\chi(a)$.

3.5. **Density parameter functions**

The routines described in this subsection are implemented in `ccl_background.c`.

The density parameter functions $\Omega_X(a)$ can be calculated for six components:

- matter density parameter $\Omega_M(a) = \Omega_{M,0} H_0^2/(a^3 H^2(a))$,
- dark energy density parameter $\Omega_\Lambda(a) = \Omega_{\Lambda,0} H_0^2/H^2(a)$,
- radiation density parameter $\Omega_g(a) = \Omega_{g,0} H_0^2/(a^4 H^2(a))$,
- curvature density parameter $\Omega_K(a) = \Omega_{K,0} H_0^2/(a^2 H^2(a))$,
- massless neutrino density parameter $\Omega_{\nu,\mathrm{rel}}(a) = \Omega_{\nu,\mathrm{rel},0} H_0^2/(a^4 H^2(a))$,
- massive neutrino density parameter $\Omega_{\nu,\mathrm{m}}(a)$,

all using the Hubble parameter defined in equation 1.

For massive neutrinos, $\Omega_{\nu,\mathrm{m}}(a)$ is calculated by calling a set of functions contained in `ccl_neutrinos.c`. First, we assume that the mass of one neutrino is equal

to $m_\nu^{\rm tot}/N_{\rm m}^\nu$ (recalling that we assume a single massive neutrino or equal-mass neutrinos). We then compute a quantity called the effective temperature:

$$T_\nu^{\rm eff} = T_{\rm CMB} T_{\rm NCDM}. \tag{5}$$

Here, $T_{\rm NCDM}$ is used to explicitly set the value of m_ν/Ω_ν^0. Note that despite its name, $T_{\rm NCDM}$ is a dimensionless temperature rescaling rather than an actual temperature, following the nomenclature used by `CLASS`. We choose a default value of $T_{\rm NCDM} = 0.71611$, which corresponds to $m_\nu/\Omega_\nu^0 = 93.14$ eV, to agree with the default value set by `CLASS`. We define

$$\mu = \frac{m_\nu^{\rm tot} a}{N_{\rm m}^\nu T_\nu^{\rm eff}} \tag{6}$$

in units such that μ is dimensionless. We then conduct the phase-space integral required to get the neutrino density, and multiply by appropriate factors to obtain $\Omega_{\nu,{\rm m}}(a)$:

$$\Omega_{\nu,{\rm m}}(a) = N_{\rm m}^\nu \frac{8\pi^2(\pi k_b)^3 k_b}{15(ch_{\rm P})^3} \frac{8\pi G}{3h^2c^2} \left(\frac{T_\nu^{\rm eff}}{a}\right)^4 \left(\frac{7}{8}\int_0^{x_{\rm max}} dx\, x^2 \frac{\sqrt{x^2+\mu^2}}{\exp(x)+1}\right) \tag{7}$$

where $h_{\rm P}$ is Planck's constant and h is $H_0/100$ with H_0 in units of km / s / Mpc. $x_{\rm max}$ is set to 1000. The final bracketed term which includes the phase-space integral can be simplified in the limit where μ is very large or very small: for small μ, it is set to $\frac{7}{8}$, and for large μ, it becomes $\frac{5\zeta(3)}{18\pi^4}\mu \sim 0.2776\mu$.

3.6. **Growth function**

The routines described in this subsection are implemented in `ccl_background.c`. To compute the growth function, $D(a)$, the growth factor of matter perturbations, `CCL` solves the following differential equation:

$$\frac{d}{da}\left(a^3 H(a)\frac{dD}{da}\right) = \frac{3}{2}\Omega_M(a) a H(a) D, \tag{8}$$

using a Runge-Kutta Cash-Karp algorithm.

In doing this, `CCL` simultaneously computes the growth rate $f(a)$, defined as:

$$f(a) = \frac{d\ln D}{d\ln a}. \tag{9}$$

`CCL` provides different functions that return the growth normalized to $D(a=1)=1$ and to $D(a \ll 1) \to a$.

Note that the above is strictly valid for a Universe containing only dust-like matter components. A scale-independent growth rate is, for example, ill-defined in the presence of massive neutrinos.

Currently CCL allows for an alternative 'modified gravity' cosmological model defined by a regular background $(w_0 + w_a)$CDM (with arbitrary K) as well as a user-defined $\Delta f(a)$, such that the true growth rate in this model is given by $f(a) = f_0(a) + \Delta f(a)$, where $f_0(a)$ is the growth rate in the background model. Note that this model is only consistently implemented with regards to the computation of the linear growth factor and growth rates (which will also scale the linear power spectrum). All other CCL functions (including the non-linear power spectrum) will ignore these modifications. This model, and the interpretation of the predictions given by CCL, should therefore be used with care.

3.7. Matter power spectrum

There are several options for obtaining the matter power spectrum in CCL. The routines described in this subsection are implemented in `ccl_power.c`.

3.7.1. BBKS

CCL implements the analytical BBKS approximation to the transfer function (Bardeen et al. 1986), given by

$$T(q \equiv k/\Gamma h\mathrm{Mpc}^{-1}) = \frac{\ln[1+2.34q]}{2.34q}[1+3.89q+(16.2q)^2+(5.47q)^3+(6.71q)^4]^{-0.25} \tag{10}$$

where $\Gamma = \Omega_m h$. The power spectrum is related to the transfer function by $\Delta(k) \propto T^2(k)k^{3+n}$ and $\Delta^2(k) \propto k^3 P(k)$. The normalization of the power spectrum is defined at $z = 0$ by setting σ_8 to its value today. The BBKS power spectrum option is primarily used as a precisely-defined input for testing the numerical accuracy of CCL routines (as described in Sect. 4), and it is not recommended for other uses.

3.7.2. Eisenstein and Hu

CCL also provides an approximation to the matter power spectrum as implemented by Eisenstein & Hu (1998) (E&H; we refer the reader to this paper for a detailed discussion of the fitting formulae).[11]

The Eisenstein & Hu and BBKS approximations are not very accurate (generally only to within a few percent), and so should not be used to derive precise cosmological constraints. They are, however, computationally faster, and therefore useful for code testing and comparison.

3.7.3. **CLASS**

The fiducial configuration calls the CLASS software (Blas et al. 2011) within CCL to obtain either linear or nonlinear matter power spectra at given redshifts. On initializing the cosmology object, we construct a bi-dimensional spline in k and the scale-factor which is then evaluated by the relevant routines to obtain the matter power spectrum at the desired wavenumber and redshift. The relevant routines can be found within `ccl_power.c`. CLASS currently computes the non-linear power spectrum using the HaloFit prescription of Takahashi et al. (2012).

As discussed in Section 2.6, the user can compile CCL with an external version of CLASS as well.

3.7.4. **Cosmic emulator**

The cosmic emulator of Lawrence et al. (2017) is integrated into CCL and it is available as one of the matter power spectrum and transfer function methods (both have to be specified). The cosmic emulator provides predictions for the matter power spectrum based on an interpolation over simulation outputs spanning a defined set of cosmological parameter values.

The emulator provides accurate predictions for the nonlinear matter power spectrum, at the 1% level, for $z \leq 2$ and in the wavenumber range $k = [10^{-3}, 5]$ Mpc^{-1}. If a redshift above $z = 2$ is passed, the emulator will quit and return an error message. For k values below and above the previously specified range, we extrapolate in the manner specified in the following section.

[11] Note that the implementation in CCL modifies Eq. 5 of Eisenstein & Hu (1998) using $a^{-1} = 1 + z$ instead of the approximation $a^{-1} \sim z$. The difference in the resulting power spectra is negligible, but larger than 1 part in 10^4 for $k < 10\,h\,\mathrm{Mpc}^{-1}$.

The allowed range of cosmological parameters is as follows:

$$\begin{aligned} 0.12 \leq \Omega_m h^2 \leq 0.155, \\ 0.0215 \leq \Omega_b h^2 \leq 0.0235, \\ 0.7 \leq \sigma_8 \leq 0.9, \\ 0.55 \leq h \leq 0.85, \\ 0.85 \leq n_s \leq 1.05, \\ -1.3 \leq w_0 \leq -0.7, \\ -1.73 \leq w_a \leq -0.7, \\ 0.0 \leq \Omega_\nu h^2 \leq 0.01. \end{aligned} \tag{11}$$

Actually, w_a and w_0 are constrained jointly to be $0.3 \leq (-w_0 - w_a)^{1/4}$. Note that CCL only allows a subregion within this parameter space. For models in which w(z) crosses -1 at some given redshift, CLASS will crash because this value corresponds to a true cosmological constant, which by definition should have no perturbations.[12] The linear matter power spectrum is still obtained from CLASS as described in the section above.

Neutrino species—The emulator is set up to consider an effective number of relativistic species of 3.04 and the user can provide $\Omega_\nu h^2$ to set the neutrino mass. This is different from the standard CCL configuration, which takes as input the parameters the number of relativistic species, $N_{\nu,\mathrm{rel}}$, the number of massive neutrinos, $N_{\nu,\mathrm{mass}}$ and their mass in eV, m_ν. For best compatibility with the emulator, the user can decide to pass the following set of parameters to CCL:

- $\{N_{\nu,\mathrm{rel}}, N_{\nu,\mathrm{mass}}, m_\nu\} = \{0.00641, 3, m_\nu\}$ for massive neutrinos or
- $\{N_{\nu,\mathrm{rel}}, N_{\nu,\mathrm{mass}}, m_\nu\} = \{3.04, 0, 0\}$ otherwise.

If other combinations of parameters are passed in emulator mode, CCL will quit and issue an error.

Notice that ideally we would like pass a non-integer number of massive neutrino species for best compatibility with the emulator set-up. However, since this is not possible within CCL, we have opted to verify that neither the growth function nor the comoving radial distance computation are affected by this approximation to more than 10^{-4} in the range $0.01 < a < 1$, where a is the scale factor. We have verified this by comparing this prediction for a fiducial cosmology

[12] We thank Emilio Bellini and Miguel Zumalacárregui for clarifying this for us.

$\{\Omega_c = 0.27, \Omega_b = 0.049, h = 0.67, \sigma_8 = 0.8, n_s = 0.96\}$ with the following neutrino parameters: $\{N_{\nu,\mathrm{rel}}, N_{\nu,\mathrm{mass}}, m_\nu\} = \{0.00641, 3, 0.06\mathrm{eV}\}$, $\{N_{\nu,\mathrm{rel}}, N_{\nu,\mathrm{mass}}, m_\nu\} = \{0, 3.04, 0.06\mathrm{eV}\}$ and $\{N_{\nu,\mathrm{rel}}, N_{\nu,\mathrm{mass}}, m_\nu\} = \{0, 3.046, 0.06\mathrm{eV}\}$. The discrepancy between distances and growth results between these choices of neutrino parameters can raise above 10^{-4} at $a < 0.01$ and the user should be mindful of this for their particular application.

3.7.5. **Impact of baryons**

`CCL` incorporates the impact of baryons on the total matter power spectrum via the "baryonic correction model" (BCM) of Schneider & Teyssier (2015). This is implemented through a flag in the configuration object, which currently accepts the following options: `ccl_nobaryons` or `ccl_bcm`. When `ccl_bcm` is set, the nonlinear matter power spectrum (whichever the method that provides it) is multiplied by a correction factor $F(k, z)$ which models the impact of baryons.

The main consequences of baryonic processes are: to suppress the power spectrum at intermediate scales ($k \sim$ a few $h/$Mpc) due to the ejection of gas by Active Galactic Nuclei feedaback, and to enhance it at smaller scales due to enhanced cooling. Three effective parameters govern the contribution of baryonic processes to modifying the total matter power spectrum:

- $\log_{10}[M_c/(\mathrm{M}_\odot/h)]$: the mass of the clusters responsible for feedback, which regulates the amount of suppression of the matter power spectrum at intermediate scales;
- η_b: a dimensionless parameter which determines the scale at which suppression peaks;
- and k_s [$h/$Mpc]: the wavenumber that determines the scale of the stellar profile.

If the BCM parameters are not specified and the user sets `CCL` to compute the power spectrym with baryonic feedback included, `CCL` will assume the default parameters of Schneider & Teyssier (2015). The user may pass these explicitly if desired.

Other baryonic impact models might be incorporated in the future as they become available.

3.7.6. **Spline parameters & the INI file**

All spline and grid parameters relevant for computing quantities such as distances, growth functions, power spectra, and halo mass functions are defined in the `ccl_params.ini` file in the `include` folder of the repository. This file allows the user to configure the following constants:

- `A_SPLINE_NLOG`: number of points in the logarithmic part of the scale factor spline. The default is `A_SPLINE_NLOG`$= 250$.
- `A_SPLINE_NA`: number of points in the linear part of the scale factor spline. The default is `A_SPLINE_NA`$= 250$.
- `A_SPLINE_MINLOG`: minimum value for the scale factor spline. The default is `A_SPLINE_MINLOG`$= 0.0001$. This only applies to background quantities.
- `A_SPLINE_MIN`: maximum value for the logarithmic part and minimum value for the linear part of the scale factor spline. The default is `A_SPLINE_MIN`$= 0.01$. This only applies to background quantities.
- `A_SPLINE_MAX`: maximum value for the scale factor spline. The default is `A_SPLINE_MAX`$= 1$.
- `LOGM_SPLINE_DELTA`: spacing of the mass function spline (logarithm). The default is `LOGM_SPLINE_DELTA`$= 0.025$.
- `LOGM_SPLINE_NM`: number of points in the mass function spline (logarithm). The default is `LOGM_SPLINE_NA`$= 440$.
- `LOGM_SPLINE_MIN`: minimum value for the mass function spline (logarithm). The default is `LOGM_SPLINE_MIN`$= 6$.
- `LOGM_SPLINE_MAX`: maximum value for the mass function spline (logarithm). The default is `LOGM_SPLINE_MAX`$= 17$.
- `A_SPLINE_NLOG_PK`: number of bins for the logarithmic part of the scale factor in the case of two-dimensional splines (where the second variable is the wavenumber). The default is `A_SPLINE_NLOG_PK`$= 11$.
- `A_SPLINE_NA_PK`: number of bins for the linear part of the scale factor in the case of two-dimensional splines (where the second variable is the wavenumber). The default is `A_SPLINE_NA_PK`$= 40$. In the case of the emulator's non-linear power spectrum, the maximum wavenumber is `K_MAX_SPLINE`$= 5/$Mpc, the interpolation is only linear and we use `A_SPLINE_NA_PK`$= 40$ as default.

- A_SPLINE_MINLOG_PK: minimum value for the scale factor spline. The default is A_SPLINE_MINLOG_PK$= 0.01$. This only applies for the 2D power spectrum splines. For the cosmic emulator and in the case of the nonlinear matter power spectrum, we use a linear spline only, adopting A_SPLINE_MIN_PK$= 1/3$.
- A_SPLINE_MIN_PK: maximum value for the logarithmic part and minimum value for the linear part of the scale factor spline. The default is A_SPLINE_MIN_PK$= 0.1$. This only applies for the 2D power spectrum splines.
- K_MAX_SPLINE: The maximum value of wavenumber considered in the spline. This is explained in more detail in the coming subsections. The default is K_MAX_SPLINE$= 50/$Mpc. In the case of the emulator's nonlinear power spectrum, the maximum wavenumber is K_MAX_SPLINE$= 5/$Mpc.
- K_MAX: The maximum value of wavenumber when integrating over k. The default is K_MAX$= 1000/$Mpc.
- K_MIN_DEFAULT: The minimum value of wavenumber when integrating over k. The default is K_MIN_DEFAULT$= 5 \times 10^{-5}/$Mpc.
- N_K: Number of bins per decade for the wavenumber. The default is N_K$= 167$.

Note that a copy of ccl_params.ini is installed along with the library, so changing the version of this file inside the source directory will not have any effect unless you reinstall.

For the matter power spectrum, the spline is performed in two variables: the logarithmically-spaced wavenumber and the linearly-spaced scale factor. Splining the CLASS output leads to some precision loss (compared to direct outputs from CLASS). We quantify this, along with the impact of extrapolation, in the following subsection.

3.7.7. **Extrapolation for the nonlinear power spectrum**

The computation of the power spectrum from CLASS can be significantly sped up by extrapolating in the range $k >$ K_MAX_SPLINE and $k <$ K_MIN. In this section, we describe the implementation of the extrapolation and the accuracy attained. These tests are performed in a flat ΛCDM cosmology with $\Omega_c = 0.25$, $\Omega_b = 0.05$, $A_s = 2.1 \times 10^{-9}$, $h = 0.7$ and $n_s = 0.96$.

We first describe the extrapolation at high wavenumbers. The introduction of the parameter K_MAX_SPLINE allows us to spline the matter power spectrum within the

cosmo structure up to that value of k (in units of $1/\mathrm{Mpc}$). A separate K_MAX parameter sets the limit for evaluation of the matter power spectrum. The range between K_MAX_SPLINE$< k <$K_MAX is evaluated by performing a second order Taylor expansion in $\ln k$ within the static routine ccl_power_extrapol_highk.

First, we compute the first and second derivative of the $\ln P(k,z)$ at $k_0 =$ K_MAX $- 2\Delta \ln k$ by computing the numerical derivatives by finite differences using GSL. The fiducial choice for $\Delta \ln k$ is 10^{-2}. We then apply a second order Taylor expansion to extrapolate the matter power spectrum to $k >$K_MAX_SPLINE. The Taylor expansion gives

$$\ln P(k,z) \simeq \ln P(k_0,z) + \frac{d\ln P}{d\ln k}(\ln k_0, z)(\ln k - \ln k_0) + \frac{1}{2}\frac{d^2 \ln P}{d\ln k^2}(\ln k_0, z)(\ln k - \ln k_0)^2. \tag{12}$$

The accuracy of this approximation is shown in Figure 1 for redshifts $z = 0$, $z = 3$ and $z = 20$. We compare the nonlinear matter power spectrum at these redshifts computed with the previously described approximation, to the matter power spectrum obtained by setting the power spectrum splines to high values. We find that for typical values of $\Delta \ln k = 10^{-2}$ and K_MAX_SPLINE$= 50/\mathrm{Mpc}$, $\ln P$ has converged to an accuracy that surpasses the expected impact of baryonic effects on the matter power spectrum at $k > 10/\mathrm{Mpc}$. (For an estimate of the impact of baryons on the total matter power spectrum, see Schneider & Teyssier 2015.) The lower K_MAX_SPLINE is, the faster CCL will run. The optimum choice of K_MAX_SPLINE is left to the user for their particular application.

We also extrapolate the power spectrum at small wavenumbers within the static routine ccl_power_extrapol_lowk. In this case, the power spectrum below K_MIN is obtained by a power-law extrapolation with index n_s:

$$\log P(k < \mathrm{K\_MIN}, z) = \log P(\mathrm{K\_MIN}, z) + n_s(\log k - \log \mathrm{K\_MIN}) \tag{13}$$

The value adopted for K_MIN depends on the choice of power spectrum method. For CLASS and the nonlinear power spectrum, we adopt K_MIN that coincides with the smallest wavenumber output by CLASS, K_MIN$= 7 \times 10^{-6}/\mathrm{Mpc}$.[13] Note that this parameter is different from K_MIN_DEFAULT, which sets the minimum k for integrations and which is set by default to K_MIN_DEFAULT$= 5 \times 10^{-5}/\mathrm{Mpc}$. Hence, in practice, no extrapolation is occurring in this case, unless the user specifically asks for an output power spectra below K_MIN_DEFAULT for their own purposes.

[13] For BBKS, the power spectrum is computed analytically at all k, there is no extrapolation. For the Eisenstein & Hu implementation, the splines of the power spectrum span K_MIN_DEFAULT$<$ $k <$K_MAX_SPLINE, so there is only extrapolation at high k.

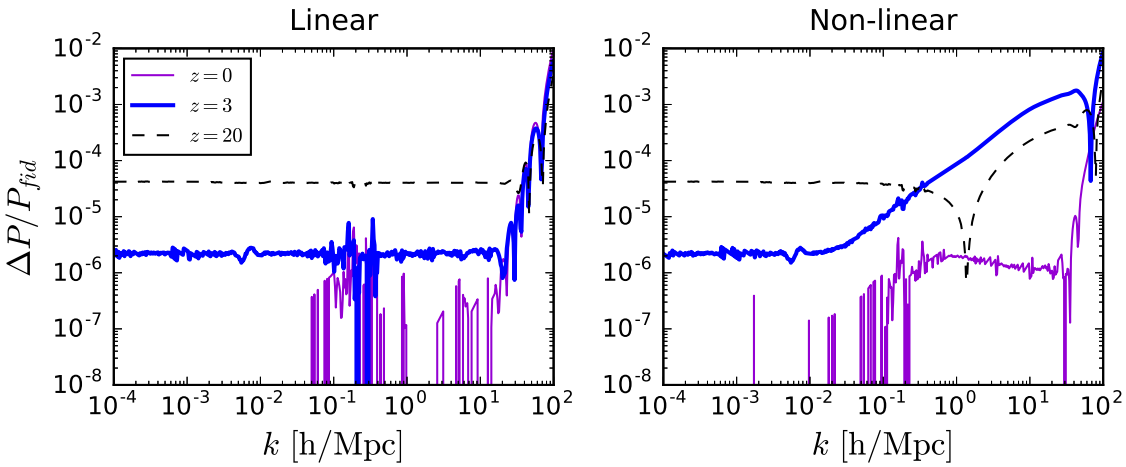

Figure 1. The relative error compared to power spectra produced with high values of the power spectrum splines, P_{fid}, produced by splining the matter power spectrum up to `K_MAX_SPLINE` and extrapolating beyond this value with a second order Taylor expansion the natural logarithm of the matter power spectrum. The left panel shows the relative errors for the linear matter power spectrum at $z = 0$, $z = 3$ and $z = 20$. The right panel shows the results for the non-linear matter power spectrum at the same redshifts. The standard `CCL` parameters adopted are those corresponding to the black dashed curve. For comparison, the impact of baryonic physics on the matter power spectrum is $\sim 10\%$ at $k = 1$/Mpc (Schneider & Teyssier 2015).

3.7.8. **Extrapolation for the linear power spectrum**

With the implementation described in the previous section, the power spectrum splines are initialized up to `K_MAX_SPLINE`. This is also true for the linear matter power spectrum, which is used within `CCL` in particular to obtain σ_8 (see Eq. 45). We have tested here how the procedure described in the previous section affects the convergence of the linear matter power spectrum. The result is shown in Figure 1. For some applications that use the linear power spectrum, the user might need to increase the value of `K_MAX_SPLINE`.

As in the previous section, the power spectrum at small wavenumber is extrapolated using a power-law. This extrapolation is performed below a fiducial value of `K_MIN_DEFAULT`$= 5 \times 10^{-5}$.

We have found that changing `N_A` to 200, or changing the sampling of the wavenumber to 5000 points, does not change the results presented in Figures 1 in this section.

3.7.9. **Wishlist for the future**

We plan to implement the following power spectrum methods in the future:

- CAMB,
- other emulators,
- Halo model/HOD.

3.7.10. **Normalization of the power spectrum**

There are two alternative schemes for normalization of the matter power spectrum. The first one is to specify the value of A_s, the amplitude of the primordial power spectrum, which is passed directly to `CLASS`. This option is available in the case of the linear/nonlinear matter power spectrum implementation. For these, as well as for BBKS and E&H transfer functions, there is the additional option to set the normalization of the matter power spectrum by specifying σ_8, the RMS density contrast averaged over spheres of radius $8h^{-1}$Mpc. The computation of σ_8 is described in Section 3.10.

In practice, there is only one argument that encodes the normalization. This is the argument `norm_pk`, which can be passed the power spectrum normalization parameterized by σ_8 or A_s. As noted above, `ccl_parameters_create` switches to σ_8 normalization if `norm_pk` $> 10^{-5}$, and to A_s normalization otherwise.

In the `python` implementation, `CCL` allows for either $\sigma 8$ or `A_s` to be passed as parameters.

3.8. **Angular power spectra**

In this section we will distinguish between *observables* (quantities observed on the sky, such as number counts in a redshift bin, shear or CMB temperature fluctuations) and *contributions* to the total observed fluctuations of these observables (such as the biased matter density term in number counts, redshift-space distortions, magnification, ISW, etc.). The routines described in this subsection are implemented in `ccl_cls.cc`.

3.8.1. **Exact expressions**

The angular power spectrum between two observables a and b can be written as:

$$C_\ell^{ab} = 4\pi \int_0^\infty \frac{dk}{k}\, \mathcal{P}_\Phi(k) \Delta_\ell^a(k) \Delta_\ell^b(k), \tag{14}$$

where $\mathcal{P}_\Phi(k)$ is the dimensionless power spectrum of the primordial curvature perturbations, and Δ^a and Δ^b are, using the terminology of CLASS, the transfer functions corresponding to these observables. Each transfer function will receive contributions from different terms. Currently `CCL` supports two observables (also labelled "tracers"), number counts and galaxy shape distortions, with the following contributions:

Number counts. —The transfer function for number counts can be decomposed into three terms: $\Delta^{\rm NC} = \Delta^{\rm D} + \Delta^{\rm RSD} + \Delta^{\rm M}$, where

- $\Delta^{\rm D}$ is the standard density term proportional to the matter density:

$$\Delta_\ell^{\rm D}(k) = \int dz\, p_z(z)\, b(z)\, T_\delta(k,z)\, j_\ell(k\chi(z)), \tag{15}$$

  where T_δ is the matter transfer function. Note that `CCL` currently does not support non-linear or scale-dependent bias. Here, $p_z(z)$ is the normalized distribution of sources in redshift (selection function). Thus `CCL` understands each individual redshift bin as a separate "observable".

- $\Delta^{\rm RSD}$ is the linear contribution from redshift-space distortions:

$$\Delta_\ell^{\rm RSD}(k) = \int dz\, p_z(z) \frac{(1+z)p_z(z)}{H(z)} T_\theta(k,z) j_\ell''(k\chi(z)), \tag{16}$$

  where $T_\theta(k,z)$ is the transfer function of θ, the divergence of the comoving velocity field. $T_\theta(k,z)$ depends on the growth, which `CCL` does not compute for massive neutrino cosmologies. C_ℓ is instead computed assuming a linear-theory relation between the matter overdensity and peculiar velocity fields. While this should not be problematic for wide photometric redshift bins, users should exercise care when interpreting results for narrow window functions.

- $\Delta^{\rm M}$ is the contribution from magnification lensing:

$$\Delta_\ell^{\rm M}(k) = -\ell(\ell+1) \int \frac{dz}{H(z)} W^{\rm M}(z) T_{\phi+\psi}(k,z) j_\ell(k\chi(z)), \tag{17}$$

  where $T_{\phi+\psi}$ is the transfer function for the Newtonian-gauge scalar metric perturbations, and $W^{\rm M}$ is the magnification window function:

$$W^{\rm M}(z) \equiv \int_z^\infty dz' p_z(z') \frac{2-5s(z')}{2} \frac{r(\chi(z')-\chi(z))}{r(\chi(z'))}. \tag{18}$$

Here $s(z)$ is the magnification bias, given as the logarithmic derivative of the number of sources with magnitude limit, and $r(\chi)$ is the angular comoving distance (see Eq. 3).

Note that CCL currently does not compute relativistic corrections to number counts Challinor & Lewis (2011); Bonvin & Durrer (2011). Although these should be included in the future, their contribution to the total fluctuation is largely subdominant (see Alonso et al. (2015) and the two references above), and therefore it is safe to work without them in most cases.

Galaxy shape distortions. —The transfer function for shape distortions is currently decomposed into two terms: $\Delta^{\rm SH} = \Delta^{\rm WL} + \Delta^{\rm IA}$, where

- $\Delta^{\rm L}$ is the standard lensing contribution:

$$\Delta^{\rm L}_\ell(k) = -\frac{1}{2}\sqrt{\frac{(\ell+2)!}{(\ell-2)!}}\int \frac{dz}{H(z)} W^{\rm L}(z) T_{\phi+\psi}(k,z)\, j_\ell(k\chi(z)), \tag{19}$$

  where $W^{\rm L}$ is the lensing kernel, given by

$$W^L(z) \equiv \int_z^\infty dz' p_z(z') \frac{r(\chi(z')-\chi(z))}{r(\chi(z'))}. \tag{20}$$

- $\Delta^{\rm IA}$ is the transfer function for intrinsic galaxy alignments. CCL currently supports the so-called "non-linear alignment model", according to which the galaxy inertia tensor is proportional the local tidal tensor Hirata & Seljak (2004); Hirata et al. (2007).

$$\Delta^{\rm IA}_\ell(k) = \sqrt{\frac{(\ell+2)!}{(\ell-2)!}}\int dz\, p_z(z)\, b_{\rm IA}(z)\, f_{\rm red}(z)\, T_\delta(k,z)\, \frac{j_\ell(k\chi(z))}{(k\chi(z))^2}. \tag{21}$$

It is worth noting that the equations above should be modified for non-flat cosmologies by replacing the spherical Bessel functions j_ℓ with their hyperspherical counterparts Kamionkowski & Spergel (1994). Since the library currently only uses the Limber approximation (documented below), this is not currently an issue. It will be revisited in future versions of CCL.

CMB lensing. —The transfer function lensing convergence from a source at redshift $z_{\rm S}$ is given by:

$$\Delta^{\rm C}_\ell(k) = -\frac{\ell(\ell+1)}{2}\int_0^{\chi_{\rm S}} d\chi \frac{r(\chi_{\rm S})-r(\chi)}{r(\chi)r(\chi_{\rm S})} T_{\phi+\psi}(k,\chi)\, j_\ell(k\chi), \tag{22}$$

where $\chi_{\rm S} \equiv \chi(z_{\rm S})$.

3.8.2. **The Limber approximation**

As shown above, computing each transfer function involves a radial projection (i.e. an integral over redshift or χ), and thus computing full power spectrum consists of a triple integral for each ℓ. This can be computationally intensive, but can be significantly simplified in certain regimes by using the Limber approximation, given by:

$$j_\ell(x) \simeq \sqrt{\frac{\pi}{2\ell+1}}\,\delta\left(\ell+\frac{1}{2}-x\right). \tag{23}$$

Thus for each k and ℓ we can define a radial distance $\chi_\ell \equiv (\ell+1/2)/k$, with corresponding redshift z_ℓ. This approximation works best for wide radial kernels and high multipoles.

Substituting this in the expressions above, the simplified versions become:

$$C_\ell^{ab} = \frac{2}{2\ell+1}\int_0^\infty dk\, P_\delta\,(k, z_\ell)\,\tilde{\Delta}_\ell^a(k)\tilde{\Delta}_\ell^b(k). \tag{24}$$

where

$$\tilde{\Delta}_\ell^{\rm D}(k) = p_z(z_\ell)\, b(z_\ell)\, H(z_\ell) \tag{25}$$

$$\tilde{\Delta}_\ell^{\rm RSD}(k) = \frac{1+8\ell}{(2\ell+1)^2}\, p_z(z_\ell)\, f(z_\ell)\, H(z_\ell) - \tag{26}$$

$$\frac{4}{2\ell+3}\sqrt{\frac{2\ell+1}{2\ell+3}} p_z(z_{\ell+1})\, f(z_{\ell+1})\, H(z_{\ell+1}) \tag{27}$$

$$\tilde{\Delta}_\ell^{\rm M}(k) = 3\Omega_{M,0}H_0^2\frac{\ell(\ell+1)}{k^2}\,\frac{(1+z_\ell)}{r(\chi_\ell)} W^{\rm M}(z_\ell) \tag{28}$$

$$\tilde{\Delta}_\ell^{\rm L}(k) = \frac{3}{2}\Omega_{M,0}H_0^2\sqrt{\frac{(\ell+2)!}{(\ell-2)}}\frac{1}{k^2}\,\frac{1+z_\ell}{r(\chi_\ell)} W^{\rm L}(z_\ell) \tag{29}$$

$$\tilde{\Delta}_\ell^{\rm IA}(k) = \sqrt{\frac{(\ell+2)!}{(\ell-2)!}}\frac{p_z(z_\ell)\, b_{\rm IA}(z_\ell) f_{\rm red}(z_\ell) H(z_\ell)}{(\ell+1/2)^2} \tag{30}$$

$$\tilde{\Delta}_\ell^{\rm C}(k) = \frac{3}{2}\Omega_{M,0}H_0^2\,\ell(\ell+1)\frac{1+z_\ell}{k^2}\,\frac{r(\chi_{\rm S})-r(\chi_\ell)}{r(\chi_\ell)r(\chi_{\rm S})}\,\Theta(\chi_\ell; 0, \chi_{\rm S}). \tag{31}$$

Here ($\Theta(x; x_i, x_f)$) is a top-hat function (1 for $x \in [x_i, x_f]$ and 0 otherwise).

3.8.3. **Beyond limber**

Native CCL computation.

CCL incorporates routines to compute the C_ℓ^{ab} angular power spectra as described above but without the Limber approximation. The algorithm performs first the integrals over z for both tracers, and ends with the k integral. This computation is much slower than using the Limber approximation, but it ends up with precise angular power spectra at low ℓ, and correct cross-correlations between tracers (the Limber approximation fails at reproducing the interference terms $j_\ell(x) \times j_\ell(x')$).

Some parameters are needed to define this integral. First, to fasten the computation the C_ℓ^{ab} function is computed for a selection of ℓ values (linearly spaced at low-ℓ and logarithmicly spaced at high-ℓ) and then the function is splined. Then the integration step of the redshift integrals must be specified in terms of a comoving distance. It is recommanded to start with a low value ($< 3\,$Mpc) for a high precision integration, and then to release this parameter to achieve the user needs in terms of precision and rapidity. A logarithmic step in k must also be provided: it does not correspond to the integration step but to a computation step before splining the transfer functions $\Delta_\ell^a(k)$. Last parameter, a minimum redshift can be given for the redshift integral bound so that the transfer functions are not computed near $z \approx 0$ where they can be numerically undefined.

The integration bounds in redshift are estimated automatically given the a redshift window so that the integration is preformed where the redshift window is relevant within a given precision (set by the CCL_FRAC_RELEVANT parameter). The integration bounds for the k integrals are defined automatically given the ℓ multipole and the comoving distances at play.

It is worth noting that the user can define a threshold multipole ℓ from which the C_ℓ^{ab} computation switches to the Limber approximation which is faster and generally relevant at high ℓ values.

Angpow.

The aim of the Angpow software (Campagne et al. 2017) is to compute the angular power spectra C_ℓ^{ab} without any Limber numerical approximation. CCL has been linked to the Angpow code, which is briefly described here.

The angular power spectrum for two shells C_ℓ^{ab} is computed in Angpow according to the following expression

$$C_\ell^{ab} = \iint_0^\infty \mathrm{d}z\mathrm{d}z' p_{z_1}(z_1) p_{z_2}(z') \times \int_0^\infty \mathrm{d}k\, f_\ell(z,k) f_\ell(z',k). \tag{32}$$

The auxiliary function $f_\ell(z,k)$ can be defined without loss of generality as

$$f_\ell(z,k) \equiv \sqrt{\frac{2}{\pi}}\, k\sqrt{P(k,z)}\, \widetilde{\Delta}_\ell(z,k) \tag{33}$$

with

- $P(k,z)$: the matter power spectrum at redshift z
- $\widetilde{\Delta}_\ell(z,k)$: a function describing the physical processes such as matter density fluctuations, redshift-space distortions as described for instance in references Durrer (2008); Yoo et al. (2009); Yoo (2010); Challinor & Lewis (2011); Bonvin & Durrer (2011). Currently, the `Angpow` version delivered with `CCL` only can deal with galaxy clustering tracers (no lensing) and this without the magnification lensing term (equation 17). The incorporation of those transfer functions is left for future work, but in principle `Angpow` has already the capability to treat them. For now, for galaxy clustering tracers we defined $\widetilde{\Delta}_\ell(z,k)$ as
$$\widetilde{\Delta}_\ell(z,k) \approx b j_\ell(k\chi(z)) - f(z) j''_\ell(k\chi(z)) \tag{34}$$
with $j_\ell(x)$ and $j''_\ell(x)$ the spherical Bessel function of order ℓ and its second derivative, and $f(a)$ the growth rate as defined subsection 3.6 (derivative of the growth function with respect to the scale factor a).

To proceed to a numerical evaluation of equation 32, `Angpow` first conducts inside the rectangle $[z_{1\min}, z_{1\max}] \times [z_{2\min}, z_{2\max}]$ given by the $p_z(z)$ selection functions a Cartesian product of one-dimensional (1D) quadrature defined by the set of sample nodes z_i and weights w_i. In practice, the Clenshaw-Curtis quadrature is used. The corresponding sampling points (z_{1i}, z_{2j}) are weighted by the product $w_i w_j$ using the 1D quadrature sample points and weights on both redshift regions with $i = 0, \ldots, N_{z_1} - 1$ and $j = 0, \ldots, N_{z_2} - 1$. Then, one gets the following approximation:

$$C_\ell^{ab} \approx \sum_{i=0}^{N_{z_1}-1} \sum_{j=0}^{N_{z_2}-1} w_i w_j p_{z_1}(z_i) p_{z_2}(z_j) \widehat{P}_\ell(\chi_i, \chi_j) \tag{35}$$

with the notations $z_i = z_{1i}$, $z_j = z_{2j}$ and $\chi_i = \chi(z_{1i})$, $\chi_j = \chi(z_{2j})$ and

$$\widehat{P}_\ell(z_i, z_j) = \int_0^\infty dk\, f_\ell(z_i,k) f_\ell(z_j,k), \tag{36}$$

To conduct the computation of such integral of highly oscillating functions we use the 3C-algorithm described in details in reference (Campagne et al. 2017). In brief this algorithm proceeds the following way:

1. the total integration k interval (eg. $[k_{\rm min}, k_{\rm max}]$) in equation (36) is cut on several k-sub-intervals;

2. on each sub-interval the functions $f_{i\,\ell}(k) = f_\ell(z_i, k)$ and $f_{j\,\ell}(k) = f_\ell(z_j, k)$ are projected onto Chebyshev series of order 2^N;

3. the product of the two Chebyshev series is performed with a 2^{2N} Chebyshev series;

4. then, the integral on the sub-interval is computed thanks to the Clenshaw-Curtis quadrature.

All the Chebyshev expansions and the Clenshaw-Curtis quadrature are performed via the DCT-I fast transform of FFTW.

Thanks to the 3C-algorithm, the `Angpow` is able to compute the C_ℓ^{ab} in a fast and accurate way. It was tested against `CLASS` and the native `CCL` computation and can performed the computation an order of magnitude faster, which is more suitable for an extensive exploration of the cosmological parameter space ($\mathcal{O}(1s)$). Note that `Angpow` is written in C++ with OpenMP, to distribute the computation of a single C_ℓ on a single thread. The computation can also be switch by the user to the faster Limber approximation setting a threshold multipole ℓ.

Precision tests.

The code has been compared with `CLASS` and the native `CCL` computation and all three softwares agrees perfectly if precision parameters are pushed to high levels.

A few parameters must be provided to set the precision of this computation. First the order of the Chebyshev polynomials is set to 2^{10} by default, and the number of k sub-intervals to 200, and we checked this is enough for the current uses. Then the redshift quadrature stepping is set automatically given the redshift windows to recover the native `CCL` computation boosted with high precision parameters: its precision is optimised so that the relative numerical error compared with the native method is two orders of magnitude below the relative cosmic variance $\sqrt{2/(2\ell+1)}$, from $\ell = 2$ to $\ell = 1000$. The $k_{\rm min}$ and $k_{\rm max}$ bounds are also automatically set given the current multipole ℓ and the comoving distance χ involved in the inner integral.

3.9. **Correlation functions**

The following expressions relating the angular power spectra and correlation functions are valid in the flat-sky approximation[14]. In all cases, $f_K(\chi)$ is comoving angular diameter distance, which differs from the radial comoving distance χ only in the case of cosmologies with non-zero curvature.

Galaxy-galaxy. The angular correlation function between two number-count tracers (labeled a and b here) is given by

$$\xi^{ab}(\theta) = \int d\ell \frac{\ell}{2\pi} C_\ell^{ab} J_0(\ell\theta), \tag{37}$$

where C_{ab} is the angular power spectrum between both tracers.

Lensing-lensing. The lensing correlation functions are [15]

$$\xi_+^{ab}(\theta) = \int_0^\infty d\ell \frac{\ell}{2\pi} J_0(\ell\theta) C_\ell^{ab}, \tag{38}$$

$$\xi_-^{ab}(\theta) = \int_0^\infty d\ell \frac{\ell}{2\pi} J_4(\ell\theta) C_\ell^{ab}, \tag{39}$$

where the angular lensing convergence power spectrum C_ℓ^{ab} is given above (see Equations 19 and 24).

Galaxy-lensing. The correlation between a number count tracer a and a shear tracer b is given by

$$\xi^{ab}(\theta) = \int d\ell \frac{\ell}{2\pi} C_\ell^{ab} J_2(\ell\theta), \tag{40}$$

Note that, in the above, "Galaxy" and "Lensing" can be replaced by any spin-0 and spin-2 fields on the sphere respectively (e.g. the CMB lensing convergence would play the same role as the galaxy overdensity field in all the formulas above).

For numerical integration of the correlation functions, we make use of the public code `FFTlog`[16](Hamilton 2000; Talman 2009). In brief, `FFTlog` works on functions periodic in log space, by writing the Hankel Transform as a convolution between Bessel functions and the function of interest (in this case C_ℓ). A version of this code is included in `CCL` with minor modifications.

3.10. Halo mass & halo bias functions

[14] See the weak lensing review by Bartelmann & Schneider (2001), page 44 and Joachimi & Bridle (2010).

[15] from Schneider 2002 and Bartelmann & Schneider section 6.4.1

[16] http://casa.colorado.edu/~ajsh/FFTLog/

The routines described in this subsection are implemented in `ccl_massfunc.c`.

The halo mass function is implemented using several different definitions from the literature: Tinker et al. (2008), Tinker et al. (2010), Angulo et al. (2012), and Watson et al. (2013). All four models are tuned to simulation data and tested against observational results. In addition, each of these fits has been implemented using the common halo definition of $\Delta = 200$, where a halo is defined with:

$$\bar{\rho}(r_\Delta) = \Delta \times \rho_{\rm m}, \tag{41}$$

where a halo with size r_Δ has an average density $\bar{\rho}$ equal to the overdensity parameter Δ times the mean background density of the universe, $\rho_{\rm m}$. Note that another common definition utilizes the critical density of the universe, $\rho_{\rm c}$; currently `CCL` requires that an external conversion by the end user between values of Δ with respect to the critical density to values of Δ as defined with respect to the mean density. In the future we plan to allow for self-consistent handling of critical density based definitions, though it is not implemented as of this build.

In addition to the usage of the most common definition, we have implemented an extension for two of the models. The Tinker 2010 model allows for a value of Δ to be given between the values of 200 and 3200 and interpolates the fitting parameters within this range in a space of $\log \Delta$ using splines. We also have implemented interpolation in the same range of Tinker 2008 Δ values. For both Tinker 2008 and Tinker 2010 models we have utilized spline interpolation through `GSL` routines in order to guarantee a match to specified fitting parameters at exact values of Δ. This fitting has slight deviation from the fit as expressed in Tinker 2010.

With the exception of the Tinker 2010 model, we attempt to keep a common form to the multiplicity function whenever possible for ease of extension:

$$f(\sigma) = A\left[\left(\frac{\sigma}{b}\right)^{-a} + 1\right] e^{-c/\sigma^2}, \tag{42}$$

where A, a, b, and c are fitting parameters that have additional redshift scaling and σ is the RMS variance of the density field smoothed on some scale M at some scale factor a. This basic form is modified for the Angulo et al. (2012) formulation. The resulting form is

$$f(\sigma) = A\left[\left(\frac{b}{\sigma} + 1\right)^{-a}\right] e^{-c/\sigma^2}, \tag{43}$$

where the only change is in the formulation of the second term. Note that the fitting parameters in the Angulo et al. (2012) formulation do not contain any redshift dependence and the use of it is primarily for testing and benchmark purposes.

Each call to the halo mass function requires an assumed model (defined within the `ccl_configuration` structure contained in `ccl_cosmology`), in addi-

tion to a value of the halo mass and scale factor for which to evaluate the halo mass function. The currently implemented models can be called with the tags `config.mass_function_method = ccl_tinker`, `ccl_tinker10`, `ccl_angulo`, or `ccl_watson`. It returns the number density of halos in logarithmic mass bins, in the form $dn/d\log_{10} M$, where n is the number density of halos of a given mass and M is the input halo mass.

The halo mass M is related to σ by first computing the radius R that would enclose a mass M in a homogeneous Universe at $z = 0$:

$$M = \frac{H_0^2}{2G} R^3 \rightarrow \frac{M}{M_\odot} = 1.162 \times 10^{12} \Omega_M h^2 \left(\frac{R}{1\,\mathrm{Mpc}}\right)^3 . \tag{44}$$

The rms density contrast in spheres of radius R can then be computed as

$$\sigma_R^2 = \frac{1}{2\pi^2} \int dk\, k^2\, P_k\, \tilde{W}_R^2(k) \tag{45}$$

where P_k is the matter power spectrum and $\tilde{W}(kR)$ is the Fourier transform of a spherical top hat window function,

$$\tilde{W}_R(k) = \frac{3}{(kR)^3}[\sin(kR) - kR\cos(kR)] \tag{46}$$

This function is directly implemented in `CCL` as well as a specific σ_8 function.

The Tinker et al. (2010) model parameterizes both the halo mass function and the halo bias in terms of the peak height, $\nu = \delta_c/\sigma(M)$, where δ_c is the critical density for collapse and is chosen to be 1.686 for this particular parameterization. We can then parameterize the halo function and halo bias as

$$b(\nu) = 1 - A\frac{\nu^a}{\nu^a + {\delta_c}^a} + B\nu^b + C\nu^c, f(\nu) = \alpha[1 + (\beta\nu)^{-2\phi}]\nu^{2\eta} e(-\gamma\nu^2/2). \tag{47}$$

The currently implemented model in `CCL` allows for an arbitrary overdensity Δ to be chosen, using the fitting functions provided in Tinker et al. (2010). Other halo model definitions are not included in the halo bias calculation, though this remains an area of active work to improve upon.

3.11. Photo-z implementation

The functionality described in this section is implemented in `ccl_lsst_specs.c`.

LSST galaxy redshifts will be obtained using photometry. A model is therefore required for the probability of measuring a photometric redshift $z_{\rm ph}$ for an object

with hypothetical spectroscopic redshift z_s. CCL allows the user to flexibly provide their own photometric redshift model.

To take advantage of this functionality, the user writes a function which accepts as input a photometric redshift, a spectroscopic redshift, and a void pointer to a structure containing any further parameters of the photo-z model. This function will return the probability of measuring the input photometric redshift given the input spectroscopic redshift. Explicitly, it should take the form:

```
user_pz_probability(double z_ph, double z_s, void * user_par){...}
```

This function is responsible for extracting the photo-z model parameters from the void pointer `user_par` itself.

This model can be used when computing $\frac{dN}{dz}^i$ in photometric redshift bin i, as given by equation 50 below. An example of how the user can construct the required functions and structure can be found in `ccl_sample_run.c`. An example that uses a built-in Gaussian photo-z model is also provided.

3.12. LSST Specifications

CCL includes LSST specifications for the expected galaxy distributions of the full galaxy clustering sample and the lensing source galaxy sample. These enable the user to easily make forecasts for LSST. The functionality described in this section is implemented in `ccl_lsst_specs.c`.

The functional forms of the expected $\frac{dN}{dz}$ for clustering galaxies and lensing source galaxies are provided. Here, $\frac{dN}{dz}$ is the number density of galaxies as a function of true redshift.

In the case of lensing source galaxies, these forms are given in Chang et al. (2013), wherein three different cases are considered: fiducial, optimistic, and conservative. All three are included in CCL, and are indicated via a label of `DNDZ_WL_OPT`, `DNDZ_WL_FID`, and `DNDZ_WL_CONS` as appropriate. The functional form of $\frac{dN}{dz}$ for lensing source galaxies is given as:

$$\frac{dN}{dz} \propto z^{\alpha} \exp\left(-\frac{z}{z_0}^{\beta}\right). \tag{48}$$

The parameters, in the fiducial case, are given as $\alpha = 1.24$, $\beta = 1.01$, and $z_0 = 0.51$. In the optimistic case, this becomes $\alpha = 1.23$, $\beta = 1.05$, and $z_0 = 0.59$. The conservative case is given by $\alpha = 1.28$, $\beta = 0.97$, and $z_0 = 0.41$.

For the case of the clustering galaxy sample, the functional form is given by (LSST Science Collaboration 2009):

$$\frac{dN}{dz} \propto \frac{1}{2z_0}\left(\frac{z}{z_0}\right)^2 \exp\left(-\frac{z}{z_0}\right) \tag{49}$$

with $z_0 = 0.3$.

In order to be incorporated into forecasts or predictions, the above expressions for $\frac{dN}{dz}$ must be normalized, and the value of $\frac{dN}{dz}$ must be provided in a given photometric redshift bin. Support is provided for the user to input a flexible photometric redshift model, as described in Section 3.11. This takes the form of a function which returns the probability $p(z, z')$ of measuring a particular photometric redshift z, given a spectroscopic redshift z' and other relevant parameters. Also provided are functions to return σ_z at a given redshift for both lensing sources and clustering galaxies, for the case in which the user-defined function is a Gaussian photo-z model.

With this, $\frac{dN^i}{dz}$ of lensing or clustering galaxies in a particular photometric redshift bin i is given by:

$$\frac{dN^i}{dz} = \frac{\frac{dN}{dz}\int_{z_i}^{z_{i+1}} dz' p(z, z')}{\int_{z_{\rm min}}^{z_{\rm max}} dz \frac{dN}{dz}\int_{z_i}^{z_{i+1}} dz' p(z, z')} \tag{50}$$

where z_i and z_{i+1} are the photo-z edges of the bin in question.

Finally, the expected (linear, scale-independent) bias of galaxies in the clustering sample is also provided. It is given by (LSST Science Collaboration 2009):

$$b(z) = \frac{0.95}{D(z)} \tag{51}$$

where $D(z)$ in the linear growth rate of structure normalized to unity at z=0.

4. Tests and validation

Our goal is for outputs of `CCL` to be validated against the results of a code-comparison project run within LSST-DESC down to a 10^{-4} or pre-established accuracy level if possible. In some cases, this level of accuracy is not necessary, as other systematics which have not been considered in this version of `CCL` yet are expected to have a larger fractional impact. In the cases where this applies, we make it clear below.

A code comparison project was carried out among members of TJP where the following outputs of cosmological forecast codes were compared and validated:

1. growth factor at $z = 0, 1, 2, 3, 4, 5$,
2. comoving radial distance $[\mathrm{Mpc}/h]$ at the same redshifts, as well as the corresponding distance moduli,
3. linear matter power spectrum, $P(k)$, from BBKS (Bardeen et al. 1986) in units of $(\mathrm{Mpc}/h)^3$ at $z = 0, 2$ in the range $10^{-3} \leq k \leq 10h/\mathrm{Mpc}$ with 10 bins per decade,
4. Eisenstein & Hu matter power spectrum in units of $(\mathrm{Mpc}/h)^3$ at $z = 0$ in the range $10^{-3} \leq k \leq 10h/\mathrm{Mpc}$ with 10 bins per decade, and
5. the mass variance at $z = 0$, $\sigma(M, z = 0)$ for $M = \{10^6, 10^8, 10^{10}, 10^{12}, 10^{14}, 10^{16}\}\mathrm{M}_\odot/h$.

These predictions were produced and compared for different cosmologies, which are listed in the table below. The results agree to better than 0.1% relative accuracy for comoving distance and growth factor among all submissions, and for $P(k)$ and $\sigma(M)$ among codes which use the same BBKS conventions.

Cosmological models for code comparison project								
Model	Ω_m	Ω_b	Ω_Λ	h_0	σ_8	n_s	w_0	w_a
flat LCDM	0.3	0.05	0.7	0.7	0.8	0.96	-1	0
w_0 LCDM	0.3	0.05	0.7	0.7	0.8	0.96	-0.9	0
w_a LCDM	0.3	0.05	0.7	0.7	0.8	0.96	-0.9	0.1
open w_a LCDM	0.3	0.05	0.65	0.7	0.8	0.96	-0.9	0.1
closed w_a LCDM	0.3	0.05	0.75	0.7	0.8	0.96	-0.9	0.1

We noticed that there are 2 typos for the BBKS transfer function in "Modern Cosmology" (Dodelson 2004) compared to the original BBKS paper. The quadratic term should be $(16.1q)^2$ and the cubic term should be $(5.46q)^3$. The BBKS equation is correct in Peacock (1999). Using the wrong equation can give differences in the results above the 10^{-4} level.

From the comparison, we were also able to identify some typical issues which affect convergence at the desired level:

- For achieving 10^{-4} precision in $\sigma(M)$ and the normalisation of the power spectrum, one should check that the integral of σ_8 and $\sigma(M)$ has con-

verged for the chosen values of $\{k_{\rm min}, k_{\rm max}\}$. After checking convergence, we achieved the desired precision.

- Also note that for $\sigma(M)$, it is important to set the desired precision level correctly for the numerical integrator. The integral usually yields $\sigma^2(M)$, and not $\sigma(M)$. Hence, one has to set the desired precision taking the exponent into account.

- The value of the gravitational constant, G, enters into the critical density. We found that failure to define G with sufficient precision would result in lack of convergence at the 10^{-4} level between the different submissions. Importantly, note that CAMB barely has 10^{-4} precision in G (and similarly, there might be other constants within CAMB/CLASS for which one should check the precision level). For `CCL`, we are using the value from the Particle Physics Handbook.

- Including/excluding radiation in the computation of the comoving distances and the growth function can easily make a difference of 10^{-4} at the redshifts required in this comparison.

In a second stage, we used the BBKS linear matter power spectrum from the previous step to compare two-point statistics for two redshift bins, resulting in three tomography combinations, $(1-1)$,$(1-2)$,$(2-2)$. We computed the following quantities:

- projected galaxy clustering tomography power spectra: density term only (no magnification, RSD, etc.) with non-evolving linear bias $b(z) = 1$, in the range $10 < \ell < 10000$, using 5 bins per decade,

- angular convergence tomography power spectra: leading order convergence term only (no magnification), in the same range and with the same resolution as the case above,

- angular galaxy clustering tomography correlation function, in the range $0.01\,{\rm deg} < \theta < 5\,{\rm deg}$, using 5 bins per decade, and

- angular shear tomography correlation functions (ξ_+,ξ_-), similarly to above.

We adopted the following analytic redshift distributions: a Gaussian with $\sigma = 0.15$, centered at $z_1 = 1$; and another Gaussian with the same dispersion but centered at $z_2 = 1.5$. We repeated the exercise for two redshift distribution histograms shown in Figure 2.

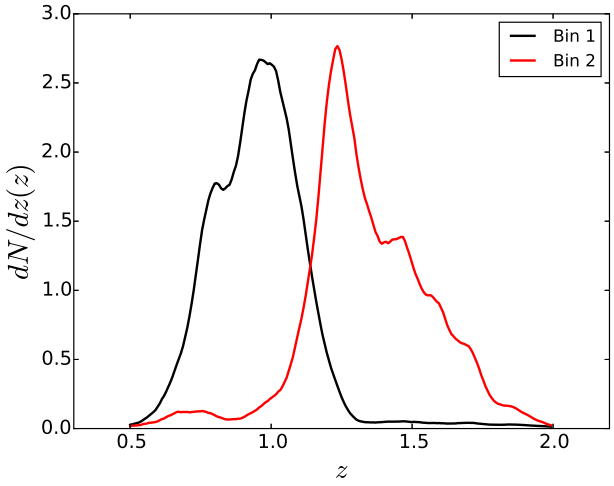

Figure 2. Binned redshift distributions used for code comparison project.

In this second step, only 2 codes have been compared so far. More outputs are needed to guarantee convergence. Preliminarily, from these outputs, we have concluded that:

- The cross-correlation between bins is particularly sensitive to the number of points where the kernels have been sampled.
- The accuracy of the correlation function is sensitive to $\ell_{\rm max}$. We had to use up to $\ell_{\rm max} = 3 \times 10^4$ for convergence (and we could not achieve 0.01% convergence).
- The large scales of the correlation function are sensitive to $\ell_{\rm min}$. The use of the flat-sky approximation is also relevant on these scales.
- For sufficiently high precision, the correlation functions are sensitive to how the power spectrum is sampled and interpolated.

For C_ℓ computations, we required the relative difference between `CCL` and the benchmarks to be $< 10^{-3}$. We performed the test both for analytic redshift distributions and histograms.

To obtain realistic targets for the convergence of correlation function computations for LSST analyses, we calculate the expected statistical uncertainty of the clustering and lensing correlation functions of the LSST gold sample (c.f. Sect. 3.12), assuming an effective source galaxy density of $n_{\rm eff} = 26 {\rm gal/sq\,arcmin}$ for galaxy shape distortions, and galaxy density of $n_{\rm gold} = 45 {\rm gal/sq\,arcmin}$ for number counts. Specifically, we calculate the Gaussian covariance of angular correlation functions following the formalism of Joachimi et al. (2008), and note that leaving out

the non-Gaussian covariance terms makes this convergence criterion more conservative. We split the galaxy samples into 10 tomography bins, defined to contain equal numbers of galaxies. The accuracy test then proceeds as follows. We compared the difference between CCL calculated lensing and clustering correlations and the benchmarks for the analytic redshift distributions and for auto-correlations of redshift bins only. To pass the benchmark test, we required that this difference be smaller than half of the value of the errorbar derived from the covariance for each correlation function computed. Specifically, we take the value of the covariance in the bins centered at $z = 1$ and $z = 1.5$ to compare to the benchmarks.

Additionally, independent codes were utilized to test the accuracy of halo mass function predictions. For the halo mass function, we compare the value of σ, $\log(\sigma^{-1})$, and the value of the halo mass function in the form used in Tinker et al. (2008),

$$\log[(M^2/\bar{\rho}_m)dn/dM]. \tag{52}$$

We note that while we maintain the 10^{-4} for our evaluations of σ, the accuracy degrades to a value of 5×10^{-3} for the halo mass function evaluation, primarily at the high halo mass and high redshift domains. We find that this increased error is acceptable, as the level of precision is significantly better than the accuracy of current halo mass function models.

The implementation of the matter power spectrum emulation code from Lawrence et al. (2017) in CCL has been validated at first instance by comparing the direct output of the original code to CCL for the best fit cosmology (M000) in the range of wavenumbers spanned by the emulator predictions and confirming that the agreement is within the expectation from numerical errors (fractional difference of $< 10^{-5}$). We have also compared the CCL emulator outputs for certain cosmologies to smoothed power spectra from the simulations presented in Lawrence et al. (2017).[17] The check ensures that the CCL outputs when calling the emulator are within 3% of the simulation benchmarks at $z = 0$ and for the k range of validity of the emulator. This essentially corresponds to a verification of the results presented in Figure 5 and Figure 6 of Lawrence et al. (2017).

Finally, we note that formal tests for predictions in cosmologies with neutrinos are not yet included. The support for neutrino cosmologies in the current release is therefore not yet formally validated, although outputs have been informally compared against other codes where available. Formal validation of this functionality is ongoing.

[17] Courtesy of Earl Lawrence and Katrin Heitmann.

CCL has a suite of test routines which, upon compilation, compare its outputs to the benchmarks from code comparison. These are run with `make check`.

5. Examples for C implementation

Examples of how to run CCL are provided in the `tests` sub-directory of the library. The first resource for a new user should be the `ccl_sample_run.c` file. This starts by setting up the CCL default configuration. Then, it creates the "cosmo" structure, which contains distances and power spectra splines, for example. There are example calls for routines that output comoving radial distances, the scale factor, the growth factor and σ_8. Toy models are created for the redshift distributions of galaxies in the clustering and lensing samples, and for the bias of the clustering sample ($b(z) = 1 + z$). These are used for constructing the "tracer" structures via `CCL_Cltracer`, which can then be called to obtain the angular power spectra for clustering, cosmic shear and galaxy lensing.

6. Python wrapper

A Python wrapper for CCL is provided through a module called `pyccl`. The whole CCL interface can be accessed through regular Python functions and classes, with all of the computation happening in the background through the C code. The functions all support `numpy` arrays as inputs and outputs, with any loops being performed in the C code for speed.

6.1. Python installation

Before you can build the Python wrapper, you must have compiled and installed the C version of CCL, as `pyccl` will be dynamically linked to it. The Python wrapper's build tools currently assume that your C compiler is `gcc`, and that you have a working Python 2.x or 3.x installation with `numpy` and `distutils` with `swig` (the latter is not necessary for using CCL, only for development). If you have installed CCL in your default library path, you can build and install the `pyccl` module by going to the root CCL directory and choosing one of the following options:

- To build and install the wrapper for the current user only, run
  `$ python setup.py install --user`

- To build and install the wrapper for all users, run
  `$ sudo python setup.py install`

- To build the wrapper in-place in the source directory (for testing), run
  `$ python setup.py build_ext --inplace`

If you choose either of the first two options, the `pyccl` module will be installed into a sensible location in your `PYTHONPATH`, and so should be automatically picked up by your Python interpreter. You can then simply import the module using `import pyccl`. If you use the last option, however, you must either start your interpreter from the root `CCL` directory, or manually add the root `CCL` directory to your `PYTHONPATH`.

These options assume that the C library (`libccl`) has been installed somewhere in the default library path. If this isn't the case, you will need to tell the Python build tools where to find the library. This can be achieved by running the following command first, before any of the commands above:

```
python setup.py build_ext --library-dirs=/path/to/lib/
--rpath=/path/to/lib/
```

Here, `/path/to/lib/` should point to the directory where you installed the C library. For example, if you ran `./configure --prefix=/my/path/` before you compiled the C library, the correct path would be `/my/path/lib/`. The command above will build the Python wrapper in-place; you can then run one of the `install` commands, as listed above, to actually install the wrapper. Note that the `rpath` switch makes sure that the `CCL` C library can be found at runtime, even if it is not in the default library path. If you use this option, there should therefore be no need to modify the library path yourself.

On some systems, building or installing the Python wrapper fails with a message similar to:

```
fatal error: 'gsl/gsl_interp2d.h' file not found.
```

This happens when the build tools fail to find the directory containing the GSL header files, e.g. when they have been installed in a non-standard directory. To work around this problem, use the `--include-dirs` option when running the `setup.py build_ext` step above, i.e. if the GSL header files are in the directory `/path/to/include/`, you would run

```
python setup.py build_ext --library-dirs=/path/to/install/lib/
--rpath=/path/to/install/lib/ --include-dirs=/path/to/include/
```

and then run one of the `setup.py install` commands listed above. (Note: As an alternative to the `--include-dirs` option, you can use `-I/path/to/include` instead.)

You can quickly check whether `pyccl` has been installed correctly by running `python -c "import pyccl"` and checking that no errors are returned. For a more in-depth test to make sure everything is working, change to the `tests/` subdirectory and run `python run_tests.py`. These tests will take a few minutes. Notice that these are not the same tests that are run via `make check`. In the case of the `python` tests, the library will only check for finite outputs of the routines called from `pyccl`. There is no benchmark comparison in this case.

6.2. **Python example**

The Python module has essentially the same functions as the C library, just presented in a more standard Python-like way. You can inspect the available functions and their arguments by using the built-in Python `help()` function, as with any Python module.

Below is a simple example Python script that creates a new `Cosmology` object, and then uses it to calculate the C_ℓ's for a simple lensing cross-correlation. It should take a few seconds on a typical laptop.

```
import pyccl as ccl
import numpy as np

# Create new Cosmology object with a given set of parameters. This keeps track
# of previously-computed cosmological functions
cosmo = ccl.Cosmology(Omega_c=0.27, Omega_b=0.045, h=0.67, A_s=2e-9, n_s=0.96)

# Define a simple binned galaxy number density curve as a function of redshift
z_n = np.linspace(0., 1., 200)
n = np.ones(z_n.shape)

# Create objects to represent tracers of the weak lensing signal with this
# number density (with has_intrinsic_alignment=False)
lens1 = ccl.ClTracerLensing(cosmo, False, n=(z_n, n))
```

```
lens2 = ccl.ClTracerLensing(cosmo, False, n=(z_n, n))

# Calculate the angular cross-spectrum of the two tracers as a function of ell
ell = np.arange(2, 10)
cls = ccl.angular_cl(cosmo, lens1, lens2, ell)
print cls
```

Further examples are collected in several Jupyter notebooks available in the `examples/` directory. These are:

`Correlation.ipynb`,

`Distance Calculations Example.ipynb`,

`HMFexample.ipynb`,

`Lensing angular power spectrum.ipynb`,

`MCMC Likelihood Analysis.ipynb`,

`Photo-z example.ipynb`,

`Power spectrum example.ipynb`

Note that the likelihood analysis in the last notebook is not intended to be realistic, but it gives an operational example of how `CCL` can be integrated into such an analysis. In particular, the notebook only considers cosmic shear over one wide redshift bin and the covariance matrix adopted solely includes a contribution from cosmic variance. The "data vector" is simply a simulated using `CCL` theoretical predictions. For speed, theoretical predictions use the BBKS power spectrum implementation.

6.3. **Technical notes on how the Python wrapper is implemented**

The Python wrapper is built using the `swig` tool, which automatically scans the `CCL` C headers and builds a matching interface in Python. The default autogenerated `swig` interface can be accessed through the `pyccl.lib` module if necessary. A more user-friendly wrapper has been written on top of this to provide more structure to the module, allow `numpy` vectorization, and provide more natural Python objects to use (instead of opaque `swig`-generated objects).

The key parts of the wrapper are as follows:

`setup.py` —This instructs `swig` and other build tools on how to find the right source files and set compile-time variables correctly. Most of this information is provided by header files and SWIG interface files that are included through the `pyccl/ccl.i` interface file.

Note that certain compiler flags, like `-fopenmp`, are also set in `setup.py`. If you are not using `gcc`, you may need to modify these flags (see the `extra_compile_args` argument of the `setup()` function).

Interface (`.i`) files —These are kept in the `pyccl/` directory, and tell `swig` which functions to extract from the C headers. There are also commands in these files to generate basic function argument documentation, and remove the `ccl_` prefix from function names.

The interface files also contain code that tells `swig` how to convert C array arguments to `numpy` arrays. For certain functions, this code may also contain a simple loop to effectively vectorize the function.

The main interface file is `pyccl/ccl.i`, which imports all of the other interface files. Most of the `CCL` source files (e.g. `core.c`) have their own interface file too. For other files, mostly containing support/utility functions, `swig` only needs the C header (`.h`) file to be specified in the main `ccl.i` file, however. (The C source file must also be added to the list in `setup.py` for it to be compiled successfully.)

Python module files —The structure of the Python module, as seen by the user, is organized through the `pyccl/__init__.py` file, which imports only the parts of the `swig` wrapper that are useful to the user. The complete autogenerated `swig` interface can be accessed through the `pyccl.lib` sub-module if necessary.

Individual sub-modules from `CCL` are wrapped in their own Python scripts (e.g. `power.py`), which typically provide a nicer “Pythonic” interface to the underlying `CCL` functions and objects. This includes automatically choosing whether to use the vectorized C function or not, as well as some conversions from Python objects to the autogenerated `swig` objects. Most of the core Python objects, like `Parameters` and `Cosmology`, are defined in `core.py`. These objects also do some basic memory management, like calling the corresponding `ccl_free_*` C function when the Python object is destroyed.

Auto-generated wrapper files—The `swig` command is triggered when you run `setup.py`, and automatically generates a number of C and Python wrapper files in the `pyccl/` directory. These typically have names like `ccl_*.c` and `ccl_*.py`, and should not be edited directly, as `swig` will overwrite them when it next runs.

`pyccl/pyutils.py`—This file contains several generic helper functions for passing `numpy` arrays in and out of Python functions in a convenient way, and for performing error checking and some type conversions.

The build process will also create a `pyccl/ccllib.py` file, which is the raw autogenerated Python interface, and `_ccllib.so`, which is a C library containing all of the C functions and their Python bindings. A `build/` directory and `pyccl.egg-info/` directory will also be created in the same directory as `setup.py` when you compile `pyccl`. These (plus the `pyccl/_ccllib.so` file) should be removed if you want to do a clean recompilation. Running `python setup.py clean --all` will remove some, but not all, of the generated files.

7. Future functionality to be included

In the future, we hope that `CCL` will include other functionalities. Functionalities which are currently under development:

- a link to `angpow` (Campagne et al. 2017) for going beyond the Limber approximation,
- a link to `FAST-PT` (McEwen et al. 2016) for efficient implementation of nonlinear perturbation theory,
- support for cosmologies with multiple unequal-mass neutrinos,
- and more power spectrum methods (see 3.7.9).

8. Feedback

If you would like to contribute to `CCL` or contact the developers, please do so through the `CCL` github repository located at `https://github.com/LSSTDESC/CCL`.

9. Citing `CCL`

If you use CCL in your work, please provide a link to the repository and cite it as LSST DESC (in preparation). For free use of the CLASS library, the CLASS developers require that the CLASS paper be cited: *CLASS II: Approximation schemes*, D. Blas, J. Lesgourgues, T. Tram, arXiv:1104.2933, JCAP 1107 (2011) 034. The CLASS repository can be found in http://class-code.net.

10. License

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Author contributions are listed below.
Husni Almoubayyed: wrote an mcmc jupyter notebook example, reviewed code/contributed to issues.
David Alonso: Co-led project; developed structure for angular power spectra; implemented autotools; integrated into LSS pipeline; contributed to: background, power spectrum, mass function, documentation and benchmarks; reviewed code
Jonathan Blazek: Planning capabilities and structure; documentation and testing.
Philip Bull: Implemented the Python wrapper and wrote documentation for it; general bug fixes, maintenance, and code review; enhanced the installer and error handling system.
Jean-Éric Campagne: Angpow builder and contributed to the interface with CCL.
N. Elisa Chisari: Co-led project, coordinated hack projects & communication, contributed to: correlation function & power spectrum implementation, documentation, and comparisons with benchmarks.
Alex Drlica-Wagner: Helped with document preparation.
Tim Eifler: Reviewed/tested code.
Renée Hlozek: Contributed initial code for error handling structures, reviewed other code edits.
Mustapha Ishak: Contributed to planning of code capabilities and structure; reviewed code; identified and fixed bugs.
Matthew Kirby: Performed comparison of physical constants.
David Kirkby: Writing, testing and reviewing code. Asking questions.
Elisabeth Krause: Initiated and co-led project; developed CLASS interface and error handling; contributed to other code; reviewed pull requests.
C. Danielle Leonard: Wrote and tested code for LSST specifications, user-defined photo-z interface, and support of neutrinos; reviewed other code; wrote text for this note.
Christiane S. Lorenz: Contributed to accurate high-redshift cosmological background quantities and benchmarked background splines.
Phil Marshall: Helped with document preparation.
Thomas McClintock: Wrote Python documentation.
Sean McLaughlin: Wrote doxygen documentation and fixed bugs/added function-

ality to distances.
Jérémy Neveu: Contributed to Angpow and built the interface with CCL.
Stéphane Plaszczynski: Contributed to Angpow and contributed to the interface with CCL.
Javier Sanchez: Modified setup.py to allow pip installation and uninstall.
Sukhdeep Singh: Contributed to the correlation functions code.
Anže Slosar: Wrote and reviewed code.
Antonio Villarreal: Contributed to initial benchmarking, halo mass function code, and general code and issues review.
Michal Vrastil: Wrote documentation and example code, reviewed code.
Joe Zuntz: Wrote initial infrastructure, C testing setup, and reviewed code.

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