Core Cosmology Library: Precision Cosmological Predictions for LSST

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The Core Cosmology Library (CCL) provides routines to compute basic cosmological observables with validated numerical accuracy. These routines have been validated to an accuracy level, documented here, against the results of the Code Comparison Project. In the current version, predictions are provided for distances and background quantities, angular auto- and cross-spectra of cosmic shear and clustering, and the halo mass function. Fiducial specifications for the expected LSST galaxy distributions and clustering bias are also included, together with the capability of computing redshift distributions for a user-defined photometric redshift model. CCL is written in C, with a Python interface. In this note, we explain the functionality of the publicly released (CCL v0.2) library.

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1. **Introduction**

In preparation for constraining cosmology with the Large Synoptic Survey Telescope (LSST), it is necessary to be able to produce rigorous theoretical predictions for the cosmological quantities that will be measured. The Core Cosmology Library[1] (CCL) aims to provide, in one library, a way of making predictions that are validated to a well-documented numerical accuracy, for the purpose of constraining cosmology with LSST. By constructing a cosmology library specifically with LSST in mind, it is possible to ensure that it is flexible, adaptable, and validated for all cases of interest, as well as user-friendly and appropriate for the needs of all working groups.

The Core Cosmology Library is written in C and incorporates the CLASS code (Blas et al. 2011) to provide predictions for the matter power spectrum.[2] A Python wrapper is also provided for ease of use.

This note describes how to install CCL (Section 2), how CCL is documented (Section 2.7), its functionality (Section 3), the relevant unit tests (Section 4), directions for finding a CCL example (Section 5), the Python wrapper (Section 6), future plans (Section 7), means to contact the developers (Section 8), instructions on how to cite CCL (and CLASS, Section 9), and the license under which CCL is released (Section 10).

2. **Installation**

2.1. **Dependencies**

- GNU Scientific Library GSL,[3] GSL-2.1 or higher.
- The SWIG[4] Python wrapper generator is not needed to run CCL, but must be installed if you intend to modify CCL in any way.
- FFTW3[5] is required for computation of correlation functions.
- FFTlog[6] is provided within CCL, with minor modifications.
- The C library associated to the CLASS code. The installation of this library is described below.

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[1] https://github.com/LSSTDESC/CCL
[2] Future versions of the library will incorporate other power-spectrum libraries and methods.
[3] https://www.gnu.org/software/gsl/
[4] http://www.swig.org/
[5] http://www.fftw.org
[6] http://casa.colorado.edu/~ajsh/FFTLog/ and https://github.com/slosar/FFTLog

2.2. **Installing CLASS**

CCL uses CLASS as one of the possible ways of computing the matter power spectrum. In order to communicate with CLASS, CCL must be linked to its library. Before installing CCL proper you must therefore install this library first. Since this process is not necessarily straightforward, we provide a python script `class_install.py` that automatically downloads and install the latest tagged stable version of CLASS. You should run this script (i.e. $ `python class_install.py`) before carrying out the next steps. By default, the script assumes that your main C compiler is `gcc`. If that's not the case, pass the name of your C compiler to the script via the command-line argument `--c_comp` (e.g. $ `python class_install.py --c_comp=icc`). Type `python class_install.py -h` for further details.

This procedure has one final caveat: if you already have a working installation of CCL, `class_install.py` may fail the first time you run it. This can be fixed by either simply running `class_install.py` a second time, or by starting from scratch (i.e. downloading or cloning CCL).

Note that, if you want to use your own version of CLASS, you should follow the steps described in Section 2.5 below.

2.3. **Installation Procedure**

CCL can be installed through an `autotools`-generated configuration file. UNIX users should be familiar with the process: navigate to the directory containing the library and type

```
$ ./configure
$ make
$ make install
```

(You may need to pre-append `sudo` to the last command, depending on your default privileges.) Users without admin privileges can install the library in a user-defined directory (e.g. `/home/desc_fan/`) by running

```
$ ./configure --prefix=/home/desc_fan
$ make
$ make install
```

This will create two directories (if not present already): `/home/desc_fan/include` and `/home/desc_fan/lib` where the header and shared library files will be placed after running `make install`. CCL has been successfully installed on several different Linux and Mac OS X systems.[7]

After installing the C library, you can make sure that it works correctly by typing `make check`, which will run the unit tests described in Section 4. Assuming that the tests pass, you can then move on to installing the Python wrapper, as described in Section 6 (or see Section 2.4 below).

After pulling a new version of CCL from the `git` repository, you can recompile the library by running:

```
$ make clean; make uninstall
$ make; make install
```

CCL library can be called from C++ code without any additional requirements or modifications. To make sure that there are no problems you can run

```
$ make check-cpp
$ ./tests/ccl_sample_run
```

2.4. **Alternative installation and pip**

It is possible to install both the C library and the Python wrapper together in one step, by running

```
$ python setup.py install --prefix=/home/desc_fan
```

This will automatically perform the previous steps to install the C library, and will also install the Python library in `/home/desc_fan/lib/python2.7/site-packages`. You might need to add this path to the `$PYTHONPATH` environment variable to be able to use it. In the near future, the package will be made available through `pip`.

Once installed you can check the installation status by typing

---

[7] We know of one case with Mac OS where libtools had the "lock" function set to "yes" and this caused the installation to stall. However, this is very rare. If this happens, after the `configure` step, edit libtool to set the "lock" to "no".

```
$ python setup.py test
```

This will run the embedded unit tests. Using this last method to install the Python library allows you to uninstall it simply by running

```
$ python setup.py uninstall
```

2.5. **Compiling against an external version of CLASS**

The default installation procedure for `CCL` implies automatically downloading and installing a tagged version of `CLASS`. Optionally, you can also link `CCL` against an external version of `CLASS`. This section is useful if you want to use a modified version of `CLASS`, or a different or more up-to-date version of the standard `CLASS`. For example, we have successfully compiled and run `CCL` with `hiCLASS` (Zumalacarregui et al. 2016).

To compile `CCL` with an external version of `CLASS`, you must first prepare the external copy so that it can be linked as a shared library. By default, the `CLASS` build tools create a static library. After compiling `CLASS` in the usual way (by running `make`), look for a static library file called `libclass.a` that should have been placed in the root source directory. Then, run the following command from that directory (Linux only):

```
$ gcc -shared -o libclass.so -Wl,--whole-archive libclass.a \
                            -Wl,--no-whole-archive -lgomp
```

This should create a new shared library, `libclass.so`, in the same directory. (N.B. The `-lgomp` flag has to appear at the end of the command; otherwise the linker can fail.) If you are running Mac OS X, use the following command instead:

```
$ gcc -fpic -shared -o libclass.dylib -Wl,-all_load libclass.a \
                            -Wl,-noall_load
```

Next, change to the root `CCL` directory and run `make clean` if you have previously run the compilation process. Then, set the `CLASSDIR` environment variable to point to the directory containing `libclass.so`:

```
export CLASSDIR=/path/to/external/class
```

Then, run `./configure` and compile and install `CCL` as usual. The `CCL` build tools should take care of linking to the external version of `CLASS`.

Once compilation has finished, run `make check` to make sure everything is working correctly. Remember to add the external `CLASS` library directory to your system library path, using either `export LD_LIBRARY_PATH=/path/to/external/class` (Linux) or `export DYLD_FALLBACK_LIBRARY_PATH=/path/to/external/class` (Mac). The system must be able to find both the `CCL` and `CLASS` libraries; it is not enough to only add `CCL` to the library path.

2.6. **Creating a Docker Image**

`CCL` supports the use of Docker, for the deployment of applications inside software containers. The Dockerfile included with `CCL` makes it possible to quickly create an image file that can be used to run virtual machines that require no additional dependencies beyond the Docker software. The Docker website[8] details the installation and start-up process for most common operating systems.

With Docker installed and the Docker Daemon running, creating an image is relatively straightforward:

```
docker build -t ccl .
```

This will begin the process of creating the image file for `CCL` locally. This Dockerfile contains all of the C libraries needed by `CCL`, as well as the Python wrapper. It currently uses `python:2.7` as a base image, and supports both ipython and Jupyter notebook. The virtualization process should have minimum impact on performance.

The resulting Docker image has two main functions. The first is a command that will open a Jupyter notebook, tied to a port on your local machine. This can be accessed by running:

```
docker run -p 8888:8888 ccl
```

---

[8] https://www.docker.com

Table 1. Physical constants in `CCL`.

	G_{Newt}	k_b	σ_{SB}	h	c	eV	$M_\odot$	pc
NIST	-3.0e-05	-1.4e-06	-5.9e-06	1.6e-07	0.0e+00	8.4e-08	–	–
PDG 2013	-6.0e-05	-1.1e-06	-4.8e-06	9.2e-08	0.0e+00	4.9e-08	-2.0e-04	1.1e-09
GSL	-1.9e-04	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	-4.5e-11
CLASS	0.0e+00	0.0e+00	–	0.0e+00	0.0e+00	0.0e+00	–	-6.7e-11

The Jupyter notebook can then be accessed through a browser at the address `localhost:8888`. The second possibility is to run a `bash` terminal, by issuing the command:

```
docker run -it ccl bash
```

This will allow for full access to the virtual machine via a terminal, so you can install additional dependencies etc.

2.7. **Documentation**

`CCL` has basic `doxygen`[9] documentation for its C routines. This can be found in the directory `doc/html` within the `CCL` repository by opening the `index.html` file in your browser. The `python` routines are documented in situ; you can view the documentation for a function by calling `help(function_name)` from within `python`.

3. **Functionality**

3.1. **Physical constants**

We have performed a comparison of the physical constants used in `CCL` and in-cluded dependencies and external sources. See Table 1 for percent differences of the constants between these sources.

3.2. **Supported cosmological models**

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[9] http://www.stack.nl/~dimitri/doxygen/

Ultimately, CCL plans to incorporate theoretical predictions for all cosmological models of interest to LSST. Currently, the following families of models are supported:

- Flat ΛCDM

- wCDM and the CPL model ($w_0 + w_a$, Chevallier & Polarski 2001 and Linder 2003)

- Non-zero curvature (K)

- All of the above, plus an arbitrary, user-defined modified growth function (see description in Section 3.6)

- Massive neutrinos, in combination with any of the above (except the user-defined modified growth function).

Not all features of CCL are available for all models. For a guide to which predictions are available for each model, see Table 2. Note that if users install their own version of CLASS, CCL can then make predictions for a more extended set of cosmologies. Users should take care to understand the validity of the CCL assumptions for their own models.

In its default configuration, CCL adopts the nonlinear matter power spectrum from CLASS through the Halofit implementation and the Tinker mass function for cluster number counts.

3.3. **Creating a cosmology**

To use CCL, the first step is to create a ccl_cosmology structure, containing all of the information required to compute cosmological observables. A ccl_cosmology structure can be generated using the information from a ccl_parameters object and a ccl_configuration object.

ccl_parameters objects contain information about the cosmological parameters of a given model, and are initialized using one of the following routines (the full syntax for each function can be found in the header file ccl_core.h):

- ccl_parameters_create(double Omega_c, double Omega_b, double Omega_k, double N_nu_rel, double N_nu_mass, double *mnu, double w0, double wa, double h, double norm_pk, double n_s,int nz_mgrowth,double *zarr_mgrowth,double *dfarr_mgrowth, int

Table 2. Cosmologies implemented in CCL.

Observable/Model	flat ΛCDM	ΛCDM+K	ΛCDM + m_ν	wCDM	$w_0 + w_a$	MG
Distances	✓	✓	✓	✓	✓	X
Growth	✓	✓	X	✓	✓	✓
$P_m(k,z)$	✓	✓	✓	✓	✓	X
Halo Mass Function	✓	✓	X	✓	✓	X
C_l, number counts	✓	✓	X	✓	✓	X
C_l, weak lensing only	✓	✓	✓	✓	✓	X

*status): general `ccl_parameters` constructor supporting all the models described above.

- `ccl_parameters_create_flat_lcdm(...)`: particular constructor for flat ΛCDM cosmologies.

- `ccl_parameters_create_flat_wcdm(...)`: constant w cosmologies.

- `ccl_parameters_create_flat_wacdm(...)`: $w_0 + w_a$.

- `ccl_parameters_create_lcdm(...)`: curved ΛCDM cosmologies.

The argument `norm_pk` can be used to specify the power spectrum normalization in terms of either σ_8 or A_s. The `ccl_parameters_create` functions assume σ_8 normalization if `norm_pk` $> 1.0e - 5$ and A_s normalization otherwise.

To include massive neutrinos, the suffix '`_nu`' may be appended to the last four `ccl_parameters_create` functions above. Using these functions without the `_nu` suffix will set the number of massive neutrinos to 0 and the effective number of massless neutrinos to 3.046. In the case of non-zero massive neutrinos, it may be desirable to set `N_nu_rel` such that $N_{\mathrm{eff}} = 3.046$ in the early universe. In agreement with `CLASS`, when using the default value of T_{NCDM} as described in section 3.4 below, `N_nu_rel` can be set to 2.0328, 1.0196, and 0.00641 for 1, 2 and 3 massive neutrinos respectively to achieve this.

`ccl_configuration` objects contain information about the prescriptions to be used to compute transfer functions, power spectra, mass functions, etc. A default `ccl_configuration` object is made available as `default_config`, which specifies that CLASS will be used to compute transfer functions, HaloFit will be used to calculate the matter power spectrum, there will be no impact of baryons included and the Tinker 2010 prescription will be used to compute the halo mass function.

After initializing instances of `ccl_parameters` and `ccl_configuration`, use the function `ccl_cosmology_create(ccl_parameters, ccl_configuration)` to return a pointer to a `ccl_cosmology` structure. You will need to pass this pointer around to every CCL function.

An example of using CCL is provided in Section 5. The README file has additional extensive documentation for the example run, as well as installation.

3.4. **Distances**

The routines described in this subsection are implemented in `ccl_background.c`.

The Hubble parameter is calculated as

$$
\frac{H(a)}{H_0} = a^{-3/2} \Big(\Omega_{M,0} + \Omega_{\Lambda,0} a^{-3(w_0+w_a)} \exp[3w_a(a-1)] + \Omega_{K,0} a
$$

$$
+ (\Omega_{g,0} + \Omega_{\nu,\mathrm{rel}}) a^{-1} + \Omega_{\nu,\mathrm{m}}(a) a^3 \Big)^{\frac{1}{2}}. \tag{1}
$$

The radial comoving distance is calculated via a numerical integral,

$$
\chi(a) = c \int_a^1 \frac{da'}{a'^2 H(a')}. \tag{2}
$$

The transverse comoving distance is computed in terms of the radial comoving distance as:

$$
r(\chi) = \begin{cases} k^{-1/2} \sin(k^{1/2}\chi) & k > 0 \\ \chi & k = 0 \\ |k|^{-1/2} \sinh(|k|^{1/2}\chi) & k < 0 \end{cases} \tag{3}
$$

The usual angular diameter distance is $d_A = a\, r(a)$, and the luminosity distance is $d_L = r(a)/a$.

CCL can also compute the distance modulus, defined as,

$$
\mu = 5 \log_{10}(d_L/\mathrm{pc}) - 5 \tag{4}
$$

and $a(\chi)$, the inverse of $\chi(a)$.

3.5. **Density parameter functions**

The routines described in this subsection are implemented in `ccl_background.c`.

The density parameter functions $\Omega_X(a)$ can be calculated for six components:

- matter density parameter $\Omega_M(a) = \Omega_{M,0}H_0^2/(a^3H^2(a))$,

- dark energy density parameter $\Omega_\Lambda(a) = \Omega_{\Lambda,0}H_0^2/H^2(a)$,

- radiation density parameter $\Omega_g(a) = \Omega_{g,0}H_0^2/(a^4H^2(a))$,

- curvature density parameter $\Omega_K(a) = \Omega_{K,0}H_0^2/(a^2H^2(a))$,

- massless neutrino density parameter $\Omega_{v,\mathrm{rel}}(a) = \Omega_{v,\mathrm{rel},0}H_0^2/(a^4H^2(a))$,

- massive neutrino density parameter $\Omega_{v,\mathrm{m}}(a)$,

all using the Hubble parameter defined in equation 1.

For massive neutrinos, $\Omega_{v,\mathrm{m}}(a)$ is calculated by calling a set of functions contained in `ccl_neutrinos.c`. We first compute a quantity called the effective temperature:

$$T_v^{\mathrm{eff}} = T_{\mathrm{CMB}}T_{\mathrm{NCDM}}. \tag{5}$$

T_{NCDM} is used to explicitly set the value of m_v/Ω_v^0. Note that despite its name, T_{NCDM} is a dimensionless temperature rescaling rather than an actual temperature, following the nomenclature used by CLASS. We choose a default value of $T_{\mathrm{NCDM}} = 0.71611$, which corresponds to $m_v/\Omega_v^0 = 93.14$ eV, to agree with the default value set by CLASS.

For each species of massive neutrino with mass m_v^i, we define

$$\mu^i = \frac{m_v^i a}{T_v^{\mathrm{eff}}} \tag{6}$$

in units such that μ is dimensionless. We conduct the phase-space integral required to get the neutrino density once, and store the result as a `gsl` spline for subsequent access. Given then the value of this integral, we multiply by appropriate factors to obtain $\Omega_{v,\mathrm{m}}(a)$:

$$\Omega_{v,\mathrm{m}}(a) = \sum_{i=1}^{N_v} \frac{8\pi^2(\pi k_b)^3 k_b}{15(ch_{\mathrm{P}})^3} \frac{8\pi G}{3h^2c^2} \left(\frac{T_v^{\mathrm{eff}}}{a}\right)^4 \left(\frac{7}{8}\int_0^{x_{\mathrm{max}}} dx\, x^2 \frac{\sqrt{x^2 + (\mu^i)^2}}{\exp(x)+1}\right) \tag{7}$$

where h_{P} is Planck's constant and h is $H_0/100$ with H_0 in units of km / s / Mpc. x_{max} is set to 1000. The final bracketed term which includes the phase-space integral can be simplified in the limit where μ is very large or very small: for small μ, it is set to $\frac{7}{8}$, and for large μ, it becomes $\frac{5\zeta(3)}{18\pi^4}\mu \sim 0.2776\mu$.

3.6. **Growth function**

The routines described in this subsection are implemented in `ccl_background.c`. To compute the growth function, $D(a)$, the growth factor of matter perturbations, CCL solves the following differential equation:

$$\frac{d}{da}\left(a^3 H(a)\frac{dD}{da}\right) = \frac{3}{2}\Omega_M(a)aH(a)D, \tag{8}$$

using a Runge-Kutta Cash-Karp algorithm.

In doing this, CCL simultaneously computes the growth rate $f(a)$, defined as:

$$f(a) = \frac{d\ln D}{d\ln a}. \tag{9}$$

CCL provides different functions that return the growth normalized to $D(a = 1) = 1$ and to $D(a \ll 1) \to a$.

Note that the above is strictly valid for a Universe containing only dust-like matter components. A scale-independent growth rate is, for example, ill-defined in the presence of massive neutrinos.

Currently CCL allows for an alternative 'modified gravity' cosmological model defined by a regular background $(w_0 + w_a)$CDM (with arbitrary K) as well as a user-defined $\Delta f(a)$, such that the true growth rate in this model is given by $f(a) = f_0(a) + \Delta f(a)$, where $f_0(a)$ is the growth rate in the background model. Note that this model is only consistently implemented with regards to the computation of the linear growth factor and growth rates (which will also scale the linear power spectrum). All other CCL functions (including the non-linear power spectrum) will ignore these modifications. This model, and the interpretation of the predictions given by CCL, should therefore be used with care.

3.7. **Matter power spectrum**

There are several options for obtaining the matter power spectrum in CCL. The routines described in this subsection are implemented in `ccl_power.c`.

3.7.1. **BBKS**

CCL implements the analytical BBKS approximation to the transfer function ([Bardeen et al. 1986](#)), given by

$$T(q \equiv k/\Gamma h\text{Mpc}^{-1}) = \frac{\ln[1 + 2.34q]}{2.34q}[1 + 3.89q + (16.2q)^2 + (5.47q)^3 + (6.71q)^4]^{-0.25}$$
(10)

where $\Gamma = \Omega_m h$. The power spectrum is related to the transfer function by $\Delta(k) \propto T^2(k)k^{3+n}$ and $\Delta^2(k) \propto k^3 P(k)$. The normalization of the power spectrum is defined at $z = 0$ by setting σ_8 to its value today. The BBKS power spectrum option is primarily used as a precisely-defined input for testing the numerical accuracy of CCL routines (as described in Sect. [4](#)), and it is not recommended for other uses.

3.7.2. **Eisenstein and Hu**

CCL also provides an approximation to the matter power spectrum as implemented by [Eisenstein & Hu](#) ([1998](#)) (E&H; we refer the reader to this paper for a detailed discussion of the fitting formulae).[10]

The Eisenstein & Hu and BBKS approximations are not very accurate (generally only to within a few percent), and so should not be used to derive precise cosmological constraints. They are, however, computationally faster, and therefore useful for code testing and comparison.

3.7.3. **CLASS**

The fiducial configuration calls the CLASS software ([Blas et al. 2011](#)) within CCL to obtain either linear or nonlinear matter power spectra at given redshifts. On initializing the cosmology object, we construct a bi-dimensional spline in k and the scale-factor which is then evaluated by the relevant routines to obtain the matter power spectrum at the desired wavenumber and redshift. The relevant routines can be found within `ccl_power.c`. CLASS currently computes the non-linear power spectrum using the HaloFit prescription of [Takahashi et al.](#) ([2012](#)).

As discussed in Section [2.5](#), the user can compile CCL with an external version of CLASS as well.

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[10] Note that the implementation in CCL modifies Eq. 5 of [Eisenstein & Hu](#) ([1998](#)) using $a^{-1} = 1 + z$ instead of the approximation $a^{-1} \sim z$. The difference in the resulting power spectra is negligible, but larger than 1 part in 10^4 for $k < 10\,h\,\text{Mpc}^{-1}$.

3.7.4. **Impact of baryons**

CCL incorporates the impact of baryons on the total matter power spectrum via the "baryonic correction model" (BCM) of Schneider & Teyssier (2015). This is implemented through a flag in the configuration object, which currently accepts the following options: `ccl_nobaryons` or `ccl_bcm`. When `ccl_bcm` is set, the nonlinear matter power spectrum (whichever the method that provides it) is multiplied by a correction factor $F(k, z)$ which models the impact of baryons.

The main consequences of baryonic processes are: to suppress the power spectrum at intermediate scales ($k \sim$ a few $h/$Mpc) due to the ejection of gas by Active Galactic Nuclei feedaback, and to enhance it at smaller scales due to enhanced cooling. Three effective parameters govern the contribution of baryonic processes to modifying the total matter power spectrum:

- $\log_{10}[M_c/(\mathrm{M}_\odot/h)]$: the mass of the clusters responsible for feedback, which regulates the amount of suppression of the matter power spectrum at intermediate scales;

- η_b: a dimensionless parameter which determines the scale at which suppression peaks;

- and k_s [$h/$Mpc]: the wavenumber that determines the scale of the stellar profile.

If the BCM parameters are not specified and the user sets CCL to compute the power spectrym with baryonic feedback included, CCL will assume the default parameters of Schneider & Teyssier (2015). The user may pass these explicitly if desired.

Other baryonic impact models might be incorporated in the future as they become available.

3.7.5. **Spline parameters & the INI file**

All spline and grid parameters relevant for computing quantities such as distances, growth functions, power spectra, and halo mass functions are defined in the `ccl_params.ini` file in the `include` folder of the repository. This file allows the user to configure the following constants:

- `A_SPLINE_NLOG`: number of points in the logarithmic part of the scale factor spline. The default is `A_SPLINE_NLOG`= 250.

- `A_SPLINE_NA`: number of points in the linear part of the scale factor spline. The default is `A_SPLINE_NA`= 250.

- `A_SPLINE_MINLOG`: minimum value for the scale factor spline. The default is `A_SPLINE_MINLOG`= 0.0001. This only applies to background quantities.

- `A_SPLINE_MIN`: maximum value for the logarithmic part and minimum value for the linear part of the scale factor spline. The default is `A_SPLINE_MIN`= 0.01. This only applies to background quantities.

- `A_SPLINE_MAX`: maximum value for the scale factor spline. The default is `A_SPLINE_MAX`= 1.

- `LOGM_SPLINE_DELTA`: spacing of the mass function spline (logarithm). The default is `LOGM_SPLINE_DELTA`= 0.025.

- `LOGM_SPLINE_NM`: number of points in the mass function spline (logarithm). The default is `LOGM_SPLINE_NA`= 440.

- `LOGM_SPLINE_MIN`: minimum value for the mass function spline (logarithm). The default is `LOGM_SPLINE_MIN`= 6.

- `LOGM_SPLINE_MAX`: maximum value for the mass function spline (logarithm). The default is `LOGM_SPLINE_MAX`= 17.

- `A_SPLINE_NA_PK`: number of bins for the scale factor in the case of two-dimensional splines (where the second variable is the wavenumber). The default is `A_SPLINE_NA_PK`= 50.

- `A_SPLINE_MIN_PK`: minimum value for the scale factor spline. The default is `A_SPLINE_MIN_PK`= 0.1. This only applies for the 2D power spectrum splines.

- `K_MAX_SPLINE`: The maximum value of wavenumber considered in the spline. This is explained in more detail in the coming subsections. The default is `K_MAX_SPLINE`= 50/Mpc.

- `K_MAX`: The maximum value of wavenumber when integrating over k. The default is `K_MAX`= 1000/Mpc.

- `K_MIN_DEFAULT`: The minimum value of wavenumber when integrating over k. The default is `K_MIN_DEFAULT`= 5×10^{-5}/Mpc.

- `N_K`: Number of bins per decade for the wavenumber. The default is `N_K`= 167.

Note that a copy of `ccl_params.ini` is installed along with the library, so changing the version of this file inside the source directory will not have any effect unless you reinstall.

For the matter power spectrum, the spline is performed in two variables: the logarithmically-spaced wavenumber and the linearly-spaced scale factor. Splining the `CLASS` output leads to some precision loss (compared to direct outputs from `CLASS`). We quantify this, along with the impact of extrapolation, in the following subsection.

3.7.6. **Extrapolation for the nonlinear power spectrum**

The computation of the power spectrum from `CLASS` can be significantly sped up by extrapolating in the range $k >$ K_MAX_SPLINE and $k <$ K_MIN. In this section, we describe the implementation of the extrapolation and the accuracy attained. These tests are performed in a flat ΛCDM cosmology with $\Omega_c = 0.25$, $\Omega_b = 0.05$, $A_s = 2.1 \times 10^{-9}$, $h = 0.7$ and $n_s = 0.96$.

We first describe the extrapolation at high wavenumbers. The introduction of the parameter K_MAX_SPLINE allows us to spline the matter power spectrum within the `cosmo` structure up to that value of k (in units of $1/\text{Mpc}$). A separate K_MAX parameter sets the limit for evaluation of the matter power spectrum. The range between K_MAX_SPLINE$< k <$K_MAX is evaluated by performing a second order Taylor expansion in $\ln k$ within the static routine `ccl_power_extrapol_highk`.

First, we compute the first and second derivative of the $\ln P(k, z)$ at $k_0 =$ K_MAX $- 2\Delta \ln k$ by computing the numerical derivatives by finite differences using GSL. The fiducial choice for $\Delta \ln k$ is 10^{-2}. We then apply a second order Taylor expansion to extrapolate the matter power spectrum to $k >$K_MAX_SPLINE. The Taylor expansion gives

$$\ln P(k, z) \simeq \ln P(k_0, z) + \frac{d \ln P}{d \ln k}(\ln k_0, z)(\ln k - \ln k_0) + \frac{1}{2}\frac{d^2 \ln P}{d \ln k^2}(\ln k_0, z)(\ln k - \ln k_0)^2.$$

(11)

The accuracy of this approximation is shown in Figure 1 for redshifts $z = 0$ and $z = 3$. We compare the nonlinear matter power spectrum at $z = 0$ computed with the previously described approximation, to the matter power spectrum obtained by directly evaluating `CLASS` at the desired k value. In figure 1, we vary $\Delta \ln k$ and K_MAX_SPLINE at a time to determine the impact of the choice of parameters. We find that for typical values of $\Delta \ln k = 10^{-2}$ and K_MAX_SPLINE$= 50/\text{Mpc}$, $\ln P$ has
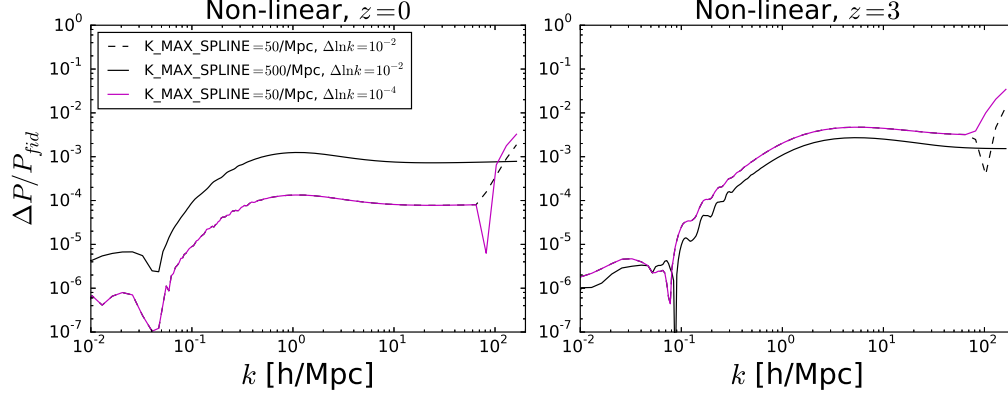
Figure 1. The relative error compared to direct CLASS outputs, P_{fid}, produced by splining the nonlinear matter power spectrum up to `K_MAX_SPLINE` and extrapolating beyond this value with a second order Taylor expansion the natural logarithm of the matter power spectrum. The left panel shows the results at $z = 0$. The right panel shows the results at $z = 3$. The standard `CCL` parameters adopted are those corresponding to the black dashed curve. The solid curve shows the accuracy when using `K_MAX_SPLINE`$= 500$/Mpc. The magenta curve shows the impact of using the higher resolution $\Delta \ln k = 10^{-4}$ in Eq. 11. For comparison, the impact of baryonic physics on the matter power spectrum is $\sim 10\%$ at $k = 1$/Mpc (Schneider & Teyssier 2015).

converged to an accuracy that surpasses the expected impact of baryonic effects on the matter power spectrum at $k > 10$/Mpc. (For an estimate of the impact of baryons on the total matter power spectrum, see Schneider & Teyssier 2015.) The lower `K_MAX_SPLINE` is, the faster `CCL` will run. The optimum choice of `K_MAX_SPLINE` is left to the user for their particular application.

We also extrapolate the power spectrum at small wavenumbers within the static routine `ccl_power_extrapol_lowk`. In this case, the power spectrum below `K_MIN` is obtained by a power-law extrapolation with index n_s:

$$\log P(k < \texttt{K\_MIN}, z) = \log P(\texttt{K\_MIN}, z) + n_s(\log k - \log \texttt{K\_MIN}) \qquad (12)$$

The value adopted for `K_MIN` depends on the choice of power spectrum method. For CLASS and the nonlinear power spectrum, we adopt `K_MIN` that coincides with the smallest wavenumber output by CLASS, `K_MIN`$= 7 \times 10^{-6}$/Mpc.[11] Note that this parameter is different from `K_MIN_DEFAULT`, which sets the minimum k for integrations and which is set by default to `K_MIN_DEFAULT`$= 5 \times 10^{-5}$/Mpc. Hence, in practice, no extrapolation is occurring in this case, unless the user specifically asks for an output power spectra below `K_MIN_DEFAULT` for their own purposes.

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[11] For BBKS, the power spectrum is computed analytically at all k, there is no extrapolation. For the Eisenstein & Hu implementation, the splines of the power spectrum span `K_MIN_DEFAULT`$<$ $k <$`K_MAX_SPLINE`, so there is only extrapolation at high k.
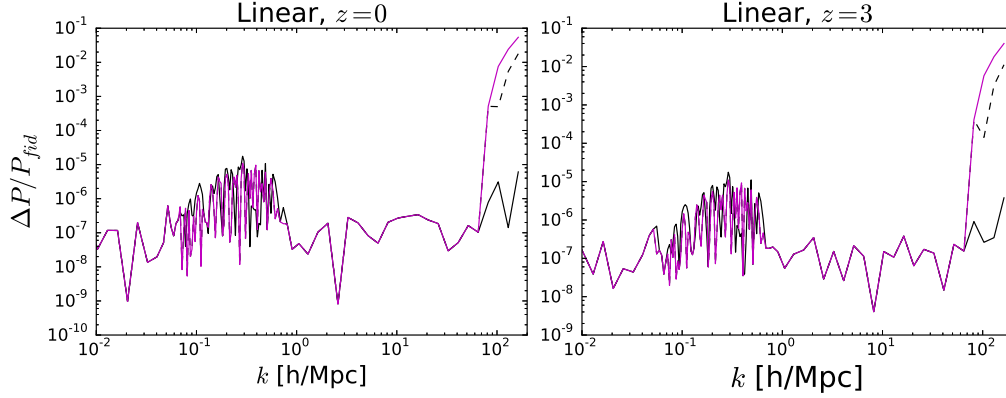
Figure 2. Same as Fig. 1 but for the linear matter power spectrum at $z = 0$ (left) and $z = 3$ (right).

3.7.7. **Extrapolation for the linear power spectrum**

With the implementation described in the previous section, the power spectrum splines are initialized up to `K_MAX_SPLINE`. This is also true for the linear matter power spectrum, which is used within `CCL` in particular to obtain σ_8 (see Eq. 39). We have tested here how the procedure described in the previous section affects the convergence of the linear matter power spectrum. We compare the fiducial `CCL` output to the case where we set `K_MAX_SPLINE`$= 5 \times 10^3$/Mpc. The result is shown in Figure 2. Although there is a significant difference ($\gtrsim 10\%$) between the linear power spectra at large k, we have confirmed that the difference in σ_8 is negligible. Nevertheless, for other applications that use the linear power spectrum, the user might need to increase the value of `K_MAX_SPLINE`.

As in the previous section, the power spectrum at small wavenumber is extrapolated using a power-law. This extrapolation is performed below a fiducial value of `K_MIN_DEFAULT`$= 5 \times 10^{-5}$.

We have found that changing `N_A` to 200, or changing the sampling of the wavenumber to 5000 points, does not change the results presented in Figures 1 and 2 in this section.

3.7.8. **Wishlist for the future**

We plan to implement the following power spectrum methods in the future:

- CAMB,

- Cosmic emulators,

- Halo model/HOD.

3.7.9. **Normalization of the power spectrum**

There are two alternative schemes for normalization of the matter power spectrum. The first one is to specify the value of A_s, the amplitude of the primordial power spectrum, which is passed directly to `CLASS`. This option is available in the case of the linear/nonlinear matter power spectrum implementation. For these, as well as for BBKS and E&H transfer functions, there is the additional option to set the normalization of the matter power spectrum by specifying σ_8, the RMS density contrast averaged over spheres of radius $8h^{-1}$Mpc. The computation of σ_8 is described in Section 3.10.

In practice, there is only one argument that encodes the normalization. This is the argument `norm_pk`, which can be passed the power spectrum normalization parameterized by σ_8 or A_s. As noted above, `ccl_parameters_create` switches to σ_8 normalization if `norm_pk` $> 10^{-5}$, and to A_s normalization otherwise.

In the `python` implementation, `CCL` allows for either $\sigma 8$ or `A_s` to be passed as parameters.

3.8. **Angular power spectra**

In this section we will distinguish between *observables* (quantities observed on the sky, such as number counts in a redshift bin, shear or CMB temperature fluctuations) and *contributions* to the total observed fluctuations of these observables (such as the biased matter density term in number counts, redshift-space distortions, magnification, ISW, etc.). The routines described in this subsection are implemented in `ccl_cls.c`.

3.8.1. **Exact expressions**

The angular power spectrum between two observables a and b can be written as:

$$C_\ell^{ab} = 4\pi \int_0^\infty \frac{dk}{k}\, \mathcal{P}_\Phi(k) \Delta_\ell^a(k) \Delta_\ell^b(k), \tag{13}$$

where $\mathcal{P}_\Phi(k)$ is the dimensionless power spectrum of the primordial curvature perturbations, and Δ^a and Δ^b are, using the terminology of CLASS, the transfer functions corresponding to these observables. Each transfer function will receive contributions from different terms. Currently CCL supports two observables (also labelled "tracers"), number counts and galaxy shape distortions, with the following contributions:

Number counts. —The transfer function for number counts can be decomposed into three terms: $\Delta^{\mathrm{NC}} = \Delta^{\mathrm{D}} + \Delta^{\mathrm{RSD}} + \Delta^{\mathrm{M}}$, where

- Δ^{D} is the standard density term proportional to the matter density:

$$\Delta_\ell^{\mathrm{D}}(k) = \int dz\, p_z(z)\, b(z)\, T_\delta(k,z)\, j_\ell(k\chi(z)), \tag{14}$$

  where T_δ is the matter transfer function. Note that CCL currently does not support non-linear or scale-dependent bias. Here, $p_z(z)$ is the normalized distribution of sources in redshift (selection function). Thus CCL understands each individual redshift bin as a separate "observable".

- Δ^{RSD} is the linear contribution from redshift-space distortions:

$$\Delta_\ell^{\mathrm{RSD}}(k) = \int dz\, p_z(z)\frac{(1+z)p_z(z)}{H(z)}T_\theta(k,z)j_\ell''(k\chi(z)), \tag{15}$$

  where $T_\theta(k,z)$ is the transfer function of θ, the divergence of the comoving velocity field. $T_\theta(k,z)$ depends on the growth, which CCL does not compute for massive neutrino cosmologies. C_ℓ is instead computed assuming a linear-theory relation between the matter overdensity and peculiar velocity fields. While this should not be problematic for wide photometric redshift bins, users should exercise care when interpreting results for narrow window functions.

- Δ^{M} is the contribution from magnification lensing:

$$\Delta_\ell^{\mathrm{M}}(k) = -\ell(\ell+1) \int \frac{dz}{H(z)}W^{\mathrm{M}}(z)T_{\phi+\psi}(k,z)j_\ell(k\chi(z)), \tag{16}$$

  where $T_{\phi+\psi}$ is the transfer function for the Newtonian-gauge scalar metric perturbations, and W^{M} is the magnification window function:

$$W^{\mathrm{M}}(z) \equiv \int_z^\infty dz'\, p_z(z')\frac{2 - 5s(z')}{2}\frac{r(\chi(z') - \chi(z))}{r(\chi(z'))}. \tag{17}$$

  Here $s(z)$ is the magnification bias, given as the logarithmic derivative of the number of sources with magnitude limit, and $r(\chi)$ is the angular comoving distance (see Eq. 3).

Note that `CCL` currently does not compute relativistic corrections to number counts Challinor & Lewis (2011); Bonvin & Durrer (2011). Although these should be included in the future, their contribution to the total fluctuation is largely subdominant (see Alonso et al. (2015) and the two references above), and therefore it is safe to work without them in most cases.

Galaxy shape distortions. —The transfer function for shape distortions is currently decomposed into two terms: $\Delta^{\rm SH} = \Delta^{\rm WL} + \Delta^{\rm IA}$, where

- $\Delta^{\rm L}$ is the standard lensing contribution:

$$\Delta_\ell^{\rm L}(k) = -\frac{1}{2}\sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \int \frac{dz}{H(z)} W^{\rm L}(z) T_{\phi+\psi}(k,z) j_\ell(k\chi(z)), \qquad (18)$$

where $W^{\rm L}$ is the lensing kernel, given by

$$W^L(z) \equiv \int_z^\infty dz' p_z(z') \frac{r(\chi(z') - \chi(z))}{r(\chi(z'))}. \qquad (19)$$

- $\Delta^{\rm IA}$ is the transfer function for intrinsic galaxy alignments. `CCL` currently supports the so-called "non-linear alignment model", according to which the galaxy inertia tensor is proportional the local tidal tensor Hirata & Seljak (2004); Hirata et al. (2007).

$$\Delta_\ell^{\rm IA}(k) = \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \int dz\, p_z(z)\, b_{\rm IA}(z)\, f_{\rm red}(z)\, T_\delta(k,z) \frac{j_\ell(k\chi(z))}{(k\chi(z))^2}. \qquad (20)$$

It is worth noting that the equations above should be modified for non-flat cosmologies by replacing the spherical Bessel functions j_ℓ with their hyperspherical counterparts Kamionkowski & Spergel (1994). Since the library currently only uses the Limber approximation (documented below), this is not currently an issue. It will be revisited in future versions of `CCL`.

CMB lensing. —The transfer function lensing convergence from a source at redshift $z_{\rm S}$ is given by:

$$\Delta_\ell^{\rm C}(k) = -\frac{\ell(\ell+1)}{2} \int_0^{\chi_{\rm S}} d\chi \frac{r(\chi_{\rm S}) - r(\chi)}{r(\chi)r(\chi_{\rm S})} T_{\phi+\psi}(k,\chi) j_\ell(k\chi), \qquad (21)$$

where $\chi_{\rm S} \equiv \chi(z_{\rm S})$.

3.8.2. **The Limber approximation**

As shown above, computing each transfer function involves a radial projection (i.e. an integral over redshift or χ), and thus computing full power spectrum consists of a triple integral for each ℓ. This can be computationally intensive, but can be significantly simplified in certain regimes by using the Limber approximation, given by:

$$j_\ell(x) \simeq \sqrt{\frac{\pi}{2\ell+1}} \, \delta\left(\ell + \frac{1}{2} - x\right). \tag{22}$$

Thus for each k and ℓ we can define a radial distance $\chi_\ell \equiv (\ell + 1/2)/k$, with corresponding redshift z_ℓ. This approximation works best for wide radial kernels and high multipoles.

Substituting this in the expressions above, the simplified versions become:

$$C_\ell^{ab} = \frac{2}{2\ell+1} \int_0^\infty dk \, P_\delta\left(k, z_\ell\right) \tilde{\Delta}_\ell^a(k) \tilde{\Delta}_\ell^b(k). \tag{23}$$

where

$$\tilde{\Delta}_\ell^{\mathrm{D}}(k) = p_z(z_\ell) \, b(z_\ell) \, H(z_\ell) \tag{24}$$

$$\tilde{\Delta}_\ell^{\mathrm{RSD}}(k) = \frac{1+8\ell}{(2\ell+1)^2} \, p_z(z_\ell) \, f(z_\ell) \, H(z_\ell) - \tag{25}$$

$$\frac{4}{2\ell+3}\sqrt{\frac{2\ell+1}{2\ell+3}} p_z(z_{\ell+1}) \, f(z_{\ell+1}) \, H(z_{\ell+1}) \tag{26}$$

$$\tilde{\Delta}_\ell^{\mathrm{M}}(k) = 3\Omega_{M,0} H_0^2 \frac{\ell(\ell+1)}{k^2} \frac{(1+z_\ell)}{r(\chi_\ell)} W^{\mathrm{M}}(z_\ell) \tag{27}$$

$$\tilde{\Delta}_\ell^{\mathrm{L}}(k) = \frac{3}{2}\Omega_{M,0} H_0^2 \sqrt{\frac{(\ell+2)!}{(\ell-2)}} \frac{1}{k^2} \frac{1+z_\ell}{r(\chi_\ell)} W^{\mathrm{L}}(z_\ell) \tag{28}$$

$$\tilde{\Delta}_\ell^{\mathrm{IA}}(k) = \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \frac{p_z(z_\ell) \, b_{\mathrm{IA}}(z_\ell) f_{\mathrm{red}}(z_\ell) H(z_\ell)}{(\ell+1/2)^2} \tag{29}$$

$$\tilde{\Delta}_\ell^{\mathrm{C}}(k) = \frac{3}{2}\Omega_{M,0} H_0^2 \, \ell(\ell+1)\frac{1+z_\ell}{k^2} \frac{r(\chi_{\mathrm{S}}) - r(\chi_\ell)}{r(\chi_\ell)r(\chi_{\mathrm{S}})} \Theta(\chi_\ell; 0, \chi_{\mathrm{S}}). \tag{30}$$

Here $(\Theta(x; x_i, x_f))$ is a top-hat function (1 for $x \in [x_i, x_f]$ and 0 otherwise).

3.9. **Correlation functions**

The following expressions relating the angular power spectra and correlation functions are valid in the flat-sky approximation[12]. In all cases, $f_K(\chi)$ is comoving angular diameter distance, which differs from the radial comoving distance χ only in the case of cosmologies with non-zero curvature.

Galaxy-galaxy. The angular correlation function between two number-count tracers (labeled a and b here) is given by

$$\xi^{ab}(\theta) = \int d\ell \frac{\ell}{2\pi} C_\ell^{ab} J_0(\ell\theta), \tag{31}$$

where C_{ab} is the angular power spectrum between both tracers.

Lensing-lensing. The lensing correlation functions are [13]

$$\xi_+^{ab}(\theta) = \int_0^\infty d\ell \frac{\ell}{2\pi} J_0(\ell\theta) C_\ell^{ab}, \tag{32}$$

$$\xi_-^{ab}(\theta) = \int_0^\infty d\ell \frac{\ell}{2\pi} J_4(\ell\theta) C_\ell^{ab}, \tag{33}$$

where the angular lensing convergence power spectrum C_ℓ^{ab} is given above (see Equations 18 and 23).

Galaxy-lensing. The correlation between a number count tracer a and a shear tracer b is given by

$$\xi^{ab}(\theta) = \int d\ell \frac{\ell}{2\pi} C_\ell^{ab} J_2(\ell\theta), \tag{34}$$

Note that, in the above, "Galaxy" and "Lensing" can be replaced by any spin-0 and spin-2 fields on the sphere respectively (e.g. the CMB lensing convergence would play the same role as the galaxy overdensity field in all the formulas above).

For numerical integration of the correlation functions, we make use of the public code `FFTlog`[14](Hamilton 2000; Talman 2009). In brief, `FFTlog` works on functions periodic in log space, by writing the Hankel Transform as a convolution between Bessel functions and the function of interest (in this case C_ℓ). A version of this code is included in `CCL` with minor modifications.

3.10. **Halo mass & halo bias functions**

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[12] See the weak lensing review by Bartelmann & Schneider (2001), page 44 and Joachimi & Bridle (2010).

[13] from Schneider 2002 and Bartelmann & Schneider section 6.4.1

[14] http://casa.colorado.edu/~ajsh/FFTLog/

The routines described in this subsection are implemented in `ccl_massfunc.c`.

The halo mass function is implemented using several different definitions from the literature: Tinker et al. (2008), Tinker et al. (2010), Angulo et al. (2012), and Watson et al. (2013). All four models are tuned to simulation data and tested against observational results. In addition, each of these fits has been implemented using the common halo definition of $\Delta = 200$, where a halo is defined with:

$$\bar{\rho}(r_\Delta) = \Delta \times \rho_\mathrm{m}, \tag{35}$$

where a halo with size r_Δ has an average density $\bar{\rho}$ equal to the overdensity parameter Δ times the mean background density of the universe, ρ_m. Note that another common definition utilizes the critical density of the universe, ρ_c; currently `CCL` requires that an external conversion by the end user between values of Δ with respect to the critical density to values of Δ as defined with respect to the mean density. In the future we plan to allow for self-consistent handling of critical density based definitions, though it is not implemented as of this build.

In addition to the usage of the most common definition, we have implemented an extension for two of the models. The Tinker 2010 model allows for a value of Δ to be given between the values of 200 and 3200 and interpolates the fitting parameters within this range in a space of $\log \Delta$ using splines. We also have implemented interpolation in the same range of Tinker 2008 Δ values. For both Tinker 2008 and Tinker 2010 models we have utilized spline interpolation through `GSL` routines in order to guarantee a match to specified fitting parameters at exact values of Δ. This fitting has slight deviation from the fit as expressed in Tinker 2010.

With the exception of the Tinker 2010 model, we attempt to keep a common form to the multiplicity function whenever possible for ease of extension:

$$f(\sigma) = A\left[\left(\frac{\sigma}{b}\right)^{-a} + 1\right]e^{-c/\sigma^2}, \tag{36}$$

where A, a, b, and c are fitting parameters that have additional redshift scaling and σ is the RMS variance of the density field smoothed on some scale M at some scale factor a. This basic form is modified for the Angulo et al. (2012) formulation. The resulting form is

$$f(\sigma) = A\left[\left(\frac{b}{\sigma} + 1\right)^{-a}\right]e^{-c/\sigma^2}, \tag{37}$$

where the only change is in the formulation of the second term. Note that the fitting parameters in the Angulo et al. (2012) formulation do not contain any redshift dependence and the use of it is primarily for testing and benchmark purposes.

Each call to the halo mass function requires an assumed model (defined within the `ccl_configuration` structure contained in `ccl_cosmology`), in addi-

tion to a value of the halo mass and scale factor for which to evaluate the halo mass function. The currently implemented models can be called with the tags `config.mass_function_method = ccl_tinker`, `ccl_tinker10`, `ccl_angulo`, or `ccl_watson`. It returns the number density of halos in logarithmic mass bins, in the form $dn/d\log_{10} M$, where n is the number density of halos of a given mass and M is the input halo mass.

The halo mass M is related to σ by first computing the radius R that would enclose a mass M in a homogeneous Universe at $z = 0$:

$$M = \frac{H_0^2}{2G} R^3 \to \frac{M}{M_\odot} = 1.162 \times 10^{12} \Omega_M h^2 \left(\frac{R}{1\,\text{Mpc}}\right)^3. \tag{38}$$

The rms density contrast in spheres of radius R can then be computed as

$$\sigma_R^2 = \frac{1}{2\pi^2} \int dk\, k^2\, P_k\, \tilde{W}_R^2(k) \tag{39}$$

where P_k is the matter power spectrum and $\tilde{W}(kR)$ is the Fourier transform of a spherical top hat window function,

$$\tilde{W}_R(k) = \frac{3}{(kR)^3}[\sin(kR) - kR\cos(kR)] \tag{40}$$

This function is directly implemented in `CCL` as well as a specific σ_8 function.

The Tinker et al. (2010) model parameterizes both the halo mass function and the halo bias in terms of the peak height, $\nu = \delta_c/\sigma(M)$, where δ_c is the critical density for collapse and is chosen to be 1.686 for this particular parameterization. We can then parameterize the halo function and halo bias as

$$b(\nu) = 1 - A\frac{\nu^a}{\nu^a + \delta_c{}^a} + B\nu^b + C\nu^c, f(\nu) = \alpha[1 + (\beta\nu)^{-2\phi}]\nu^{2\eta}e(-\gamma\nu^2/2). \tag{41}$$

The currently implemented model in `CCL` allows for an arbitrary overdensity Δ to be chosen, using the fitting functions provided in Tinker et al. (2010). Other halo model definitions are not included in the halo bias calculation, though this remains an area of active work to improve upon.

3.11. **Photo-z implementation**

The functionality described in this section is implemented in `ccl_lsst_specs.c`.

LSST galaxy redshifts will be obtained using photometry. A model is therefore required for the probability of measuring a photometric redshift z_{ph} for an object

with hypothetical spectroscopic redshift z_s. CCL allows the user to flexibly provide their own photometric redshift model.

To take advantage of this functionality, the user writes a function which accepts as input a photometric redshift, a spectroscopic redshift, and a void pointer to a structure containing any further parameters of the photo-z model. This function will return the probability of measuring the input photometric redshift given the input spectroscopic redshift. Explicitly, it should take the form:

`user_pz_probability(double z_ph, double z_s, void * user_par){...}`

This function is responsible for extracting the photo-z model parameters from the void pointer `user_par` itself.

This model can be used when computing $\frac{dN}{dz}^i$ in photometric redshift bin i, as given by equation 44 below. An example of how the user can construct the required functions and structure can be found in `ccl_sample_run.c`. An example that uses a built-in Gaussian photo-z model is also provided.

3.12. **LSST Specifications**

CCL includes LSST specifications for the expected galaxy distributions of the full galaxy clustering sample and the lensing source galaxy sample. These enable the user to easily make forecasts for LSST. The functionality described in this section is implemented in `ccl_lsst_specs.c`.

The functional forms of the expected $\frac{dN}{dz}$ for clustering galaxies and lensing source galaxies are provided. Here, $\frac{dN}{dz}$ is the number density of galaxies as a function of true redshift.

In the case of lensing source galaxies, these forms are given in Chang et al. (2013), wherein three different cases are considered: fiducial, optimistic, and conservative. All three are included in CCL, and are indicated via a label of DNDZ_WL_OPT, DNDZ_WL_FID, and DNDZ_WL_CONS as appropriate. The functional form of $\frac{dN}{dz}$ for lensing source galaxies is given as:

$$\frac{dN}{dz} \propto z^\alpha \exp\left(-\frac{z}{z_0}^\beta\right). \tag{42}$$

The parameters, in the fiducial case, are given as $\alpha = 1.24$, $\beta = 1.01$, and $z_0 = 0.51$. In the optimistic case, this becomes $\alpha = 1.23$, $\beta = 1.05$, and $z_0 = 0.59$. The conservative case is given by $\alpha = 1.28$, $\beta = 0.97$, and $z_0 = 0.41$.

For the case of the clustering galaxy sample, the functional form is given by (LSST Science Collaboration 2009):

$$\frac{dN}{dz} \propto \frac{1}{2z_0} \left(\frac{z}{z_0}\right)^2 \exp\left(-\frac{z}{z_0}\right) \qquad (43)$$

with $z_0 = 0.3$.

In order to be incorporated into forecasts or predictions, the above expressions for $\frac{dN}{dz}$ must be normalized, and the value of $\frac{dN}{dz}$ must be provided in a given photometric redshift bin. Support is provided for the user to input a flexible photometric redshift model, as described in Section 3.11. This takes the form of a function which returns the probability $p(z, z')$ of measuring a particular photometric redshift z, given a spectroscopic redshift z' and other relevant parameters. Also provided are functions to return σ_z at a given redshift for both lensing sources and clustering galaxies, for the case in which the user-defined function is a Gaussian photo-z model.

With this, $\frac{dN^i}{dz}$ of lensing or clustering galaxies in a particular photometric redshift bin i is given by:

$$\frac{dN^i}{dz} = \frac{\frac{dN}{dz} \int_{z_i}^{z_{i+1}} dz' p(z, z')}{\int_{z_{\min}}^{z_{\max}} dz \frac{dN}{dz} \int_{z_i}^{z_{i+1}} dz' p(z, z')} \qquad (44)$$

where z_i and z_{i+1} are the photo-z edges of the bin in question.

Finally, the expected (linear, scale-independent) bias of galaxies in the clustering sample is also provided. It is given by (LSST Science Collaboration 2009):

$$b(z) = \frac{0.95}{D(z)} \qquad (45)$$

where $D(z)$ in the linear growth rate of structure normalized to unity at z=0.

4. **Tests and validation**

Our goal is for outputs of CCL to be validated against the results of a code-comparison project run within LSST-DESC down to a 10^{-4} or pre-established accuracy level if possible. In some cases, this level of accuracy is not necessary, as other systematics which have not been considered in this version of CCL yet are expected to have a larger fractional impact. In the cases where this applies, we make it clear below.

A code comparison project was carried out among members of TJP where the following outputs of cosmological forecast codes were compared and validated:

1. growth factor at $z = 0, 1, 2, 3, 4, 5$,

2. comoving radial distance $[\text{Mpc}/h]$ at the same redshifts, as well as the corresponding distance moduli,

3. linear matter power spectrum, $P(k)$, from BBKS (Bardeen et al. 1986) in units of $(\text{Mpc}/h)^3$ at $z = 0, 2$ in the range $10^{-3} \leq k \leq 10h/\text{Mpc}$ with 10 bins per decade,

4. Eisenstein & Hu matter power spectrum in units of $(\text{Mpc}/h)^3$ at $z = 0$ in the range $10^{-3} \leq k \leq 10h/\text{Mpc}$ with 10 bins per decade, and

5. the mass variance at $z = 0$, $\sigma(M, z = 0)$ for $M = \{10^6, 10^8, 10^{10}, 10^{12}, 10^{14}, 10^{16}\}\text{M}_\odot/h$.

These predictions were produced and compared for different cosmologies, which are listed in the table below. The results agree to better than 0.1% relative accuracy for comoving distance and growth factor among all submissions, and for $P(k)$ and $\sigma(M)$ among codes which use the same BBKS conventions.

Cosmological models for code comparison project								
Model	Ω_m	Ω_b	Ω_Λ	h_0	σ_8	n_s	w_0	w_a
flat LCDM	0.3	0.05	0.7	0.7	0.8	0.96	-1	0
w_0 LCDM	0.3	0.05	0.7	0.7	0.8	0.96	-0.9	0
w_a LCDM	0.3	0.05	0.7	0.7	0.8	0.96	-0.9	0.1
open w_a LCDM	0.3	0.05	0.65	0.7	0.8	0.96	-0.9	0.1
closed w_a LCDM	0.3	0.05	0.75	0.7	0.8	0.96	-0.9	0.1

We noticed that there are 2 typos for the BBKS transfer function in "Modern Cosmology" (Dodelson 2004) compared to the original BBKS paper. The quadratic term should be $(16.1q)^2$ and the cubic term should be $(5.46q)^3$. The BBKS equation is correct in Peacock (1999). Using the wrong equation can give differences in the results above the 10^{-4} level.

From the comparison, we were also able to identify some typical issues which affect convergence at the desired level:

- For achieving 10^{-4} precision in $\sigma(M)$ and the normalisation of the power spectrum, one should check that the integral of σ_8 and $\sigma(M)$ has con-

verged for the chosen values of $\{k_{\mathrm{min}}, k_{\mathrm{max}}\}$. After checking convergence, we achieved the desired precision.

- Also note that for $\sigma(M)$, it is important to set the desired precision level correctly for the numerical integrator. The integral usually yields $\sigma^2(M)$, and not $\sigma(M)$. Hence, one has to set the desired precision taking the exponent into account.

- The value of the gravitational constant, G, enters into the critical density. We found that failure to define G with sufficient precision would result in lack of convergence at the 10^{-4} level between the different submissions. Importantly, note that CAMB barely has 10^{-4} precision in G (and similarly, there might be other constants within CAMB/CLASS for which one should check the precision level). For CCL, we are using the value from the Particle Physics Handbook.

- Including/excluding radiation in the computation of the comoving distances and the growth function can easily make a difference of 10^{-4} at the redshifts required in this comparison.

In a second stage, we used the BBKS linear matter power spectrum from the previous step to compare two-point statistics for two redshift bins, resulting in three tomography combinations, $(1-1),(1-2),(2-2)$. We computed the following quantities:

- projected galaxy clustering tomography power spectra: density term only (no magnification, RSD, etc.) with non-evolving linear bias $b(z) = 1$, in the range $10 < \ell < 10000$, using 5 bins per decade,

- angular convergence tomography power spectra: leading order convergence term only (no magnification), in the same range and with the same resolution as the case above,

- angular galaxy clustering tomography correlation function, in the range $0.01 \, \mathrm{deg} < \theta < 5 \, \mathrm{deg}$, using 5 bins per decade, and

- angular shear tomography correlation functions (ξ_+,ξ_-), similarly to above.

We adopted the following analytic redshift distributions: a Gaussian with $\sigma = 0.15$, centered at $z_1 = 1$; and another Gaussian with the same dispersion but centered at $z_2 = 1.5$. We repeated the exercise for two redshift distribution histograms shown in Figure 3.
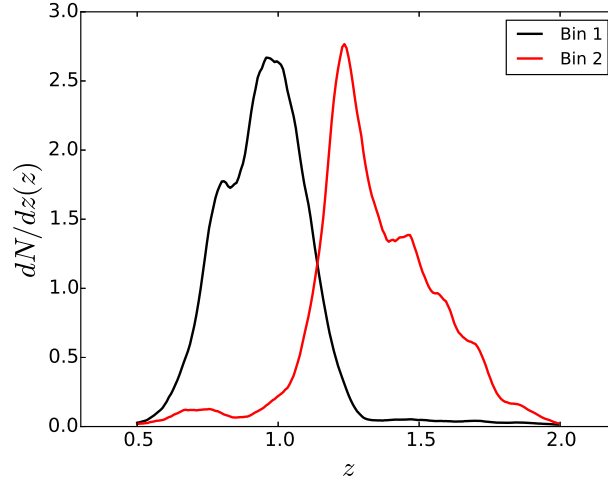
Figure 3. Binned redshift distributions used for code comparison project.

In this second step, only 2 codes have been compared so far. More outputs are needed to guarantee convergence. Preliminarily, from these outputs, we have concluded that:

- The cross-correlation between bins is particularly sensitive to the number of points where the kernels have been sampled.

- The accuracy of the correlation function is sensitive to $\ell_{\max}$. We had to use up to $\ell_{\max} = 3 \times 10^4$ for convergence (and we could not achieve 0.01% convergence).

- The large scales of the correlation function are sensitive to $\ell_{\min}$. The use of the flat-sky approximation is also relevant on these scales.

- For sufficiently high precision, the correlation functions are sensitive to how the power spectrum is sampled and interpolated.

For C_ℓ computations, we required the relative difference between CCL and the benchmarks to be $< 10^{-3}$. We performed the test both for analytic redshift distributions and histograms.

To obtain realistic targets for the convergence of correlation function computations for LSST analyses, we calculate the expected statistical uncertainty of the clustering and lensing correlation functions of the LSST gold sample (c.f. Sect. 3.12), assuming an effective source galaxy density of $n_{\rm eff} = 26\,{\rm gal/sq\,arcmin}$ for galaxy shape distortions, and galaxy density of $n_{\rm gold} = 45\,{\rm gal/sq\,arcmin}$ for number counts. Specifically, we calculate the Gaussian covariance of angular correlation functions following the formalism of Joachimi et al. (2008), and note that leaving out

the non-Gaussian covariance terms makes this convergence criterion more conservative. We split the galaxy samples into 10 tomography bins, defined to contain equal numbers of galaxies. The accuracy test then proceeds as follows. We compared the difference between CCL calculated lensing and clustering correlations and the benchmarks for the analytic redshift distributions and for auto-correlations of redshift bins only. To pass the benchmark test, we required that this difference be smaller than half of the value of the errorbar derived from the covariance for each correlation function computed. Specifically, we take the value of the covariance in the bins centered at $z = 1$ and $z = 1.5$ to compare to the benchmarks.

Additionally, independent codes were utilized to test the accuracy of halo mass function predictions. For the halo mass function, we compare the value of σ, $\log(\sigma^{-1})$, and the value of the halo mass function in the form used in Tinker et al. (2008),

$$\log[(M^2/\bar{\rho}_m)dn/dM]. \tag{46}$$

We note that while we maintain the 10^{-4} for our evaluations of σ, the accuracy degrades to a value of 5×10^{-3} for the halo mass function evaluation, primarily at the high halo mass and high redshift domains. We find that this increased error is acceptable, as the level of precision is significantly better than the accuracy of current halo mass function models.

Finally, for cosmologies involving massive neutrinos, we have included tests which compare comoving distance and distance modulus predictions from CCL from those produced by the astropy (Astropy Collaboration et al. 2013) library. In this case, we consider these comparison tests to pass if the agreement between the two predictions is 0.001 or better; this is due to the fact that astropy uses a fitting function for the neutrino phase-space integral while we compute the integral directly. The fitting function used by astropy is expected to match the true integral at approximately this level. As a sanity check, we have also manually checked that the power spectrum from CLASS as called via CCL matches that as called directly from CLASS in five test cosmologies with massive neutrinos.

CCL has a suite of test routines which, upon compilation, compare its outputs to the benchmarks from code comparison. These are run with `make check`.

5. **Examples for C implementation**

Examples of how to run CCL are provided in the `tests` sub-directory of the library. The first resource for a new user should be the `ccl_sample_run.c` file. This starts by setting up the CCL default configuration. Then, it creates the "cosmo" struc-

ture, which contains distances and power spectra splines, for example. There are example calls for routines that output comoving radial distances, the scale factor, the growth factor and σ_8. Toy models are created for the redshift distributions of galaxies in the clustering and lensing samples, and for the bias of the clustering sample ($b(z) = 1 + z$). These are used for constructing the "tracer" structures via `CCL_Cltracer`, which can then be called to obtain the angular power spectra for clustering, cosmic shear and galaxy lensing.

6. **Python wrapper**

A Python wrapper for `CCL` is provided through a module called `pyccl`. The whole `CCL` interface can be accessed through regular Python functions and classes, with all of the computation happening in the background through the C code. The functions all support `numpy` arrays as inputs and outputs, with any loops being performed in the C code for speed.

6.1. **Python installation**

Before you can build the Python wrapper, you must have compiled and installed the C version of `CCL`, as `pyccl` will be dynamically linked to it. The Python wrapper's build tools currently assume that your C compiler is `gcc`, and that you have a working Python 2.x or 3.x installation with `numpy` and `distutils` with `swig` (the latter is not necessary for using `CCL`, only for development). If you have installed `CCL` in your default library path, you can build and install the `pyccl` module by going to the root `CCL` directory and choosing one of the following options:

- To build and install the wrapper for the current user only, run
  `$ python setup.py install --user`

- To build and install the wrapper for all users, run
  `$ sudo python setup.py install`

- To build the wrapper in-place in the source directory (for testing), run
  `$ python setup.py build_ext --inplace`

If you choose either of the first two options, the `pyccl` module will be installed into a sensible location in your `PYTHONPATH`, and so should be automatically picked up by your Python interpreter. You can then simply import the module using `import pyccl`. If you use the last option, however, you must either start your in-

terpreter from the root `CCL` directory, or manually add the root `CCL` directory to your `PYTHONPATH`.

These options assume that the C library (`libccl`) has been installed somewhere in the default library path. If this isn't the case, you will need to tell the Python build tools where to find the library. This can be achieved by running the following command first, before any of the commands above:

```
python setup.py build_ext --library-dirs=/path/to/lib/
--rpath=/path/to/lib/
```

Here, `/path/to/lib/` should point to the directory where you installed the C library. For example, if you ran `./configure --prefix=/my/path/` before you compiled the C library, the correct path would be `/my/path/lib/`. The command above will build the Python wrapper in-place; you can then run one of the `install` commands, as listed above, to actually install the wrapper. Note that the `rpath` switch makes sure that the `CCL` C library can be found at runtime, even if it is not in the default library path. If you use this option, there should therefore be no need to modify the library path yourself.

On some systems, building or installing the Python wrapper fails with a message similar to:

```
fatal error: 'gsl/gsl_interp2d.h' file not found.
```

This happens when the build tools fail to find the directory containing the GSL header files, e.g. when they have been installed in a non-standard directory. To work around this problem, use the `--include-dirs` option when running the `setup.py build_ext` step above, i.e. if the GSL header files are in the directory `/path/to/include/`, you would run

```
python setup.py build_ext --library-dirs=/path/to/install/lib/
--rpath=/path/to/install/lib/ --include-dirs=/path/to/include/
```

and then run one of the `setup.py install` commands listed above. (Note: As an alternative to the `--include-dirs` option, you can use `-I/path/to/include` instead.)

You can quickly check whether `pyccl` has been installed correctly by running `python -c "import pyccl"` and checking that no errors are returned. For a more in-depth test to make sure everything is working, change to the `tests/` sub-

directory and run `python run_tests.py`. These tests will take a few minutes. Notice that these are not the same tests that are run via `make check`. In the case of the `python` tests, the library will only check for finite outputs of the routines called from `pyccl`. There is no benchmark comparison in this case.

6.2. **Python example**

The Python module has essentially the same functions as the C library, just presented in a more standard Python-like way. You can inspect the available functions and their arguments by using the built-in Python `help()` function, as with any Python module.

Below is a simple example Python script that creates a new `Cosmology` object, and then uses it to calculate the C_ℓ's for a simple lensing cross-correlation. It should take a few seconds on a typical laptop.

```
import pyccl as ccl
import numpy as np

# Create new Cosmology object with a given set of parameters. This keeps track
# of previously-computed cosmological functions
cosmo = ccl.Cosmology(Omega_c=0.27, Omega_b=0.045, h=0.67, A_s=2e-9, n_s=0.96)

# Define a simple binned galaxy number density curve as a function of redshift
z_n = np.linspace(0., 1., 200)
n = np.ones(z_n.shape)

# Create objects to represent tracers of the weak lensing signal with this
# number density (with has_intrinsic_alignment=False)
lens1 = ccl.ClTracerLensing(cosmo, False, n=(z_n, n))
lens2 = ccl.ClTracerLensing(cosmo, False, n=(z_n, n))

# Calculate the angular cross-spectrum of the two tracers as a function of ell
ell = np.arange(2, 10)
cls = ccl.angular_cl(cosmo, lens1, lens2, ell)
print cls
```

Further examples are collected in several Jupyter notebooks available in the `tests/` directory. These are:

```
Photo-z example.ipynb,
```

```
Power spectrum example.ipynb,
```

```
Distance Calculations Example.ipynb,
```

```
HMFexample.ipynb,
```

```
Lensing angular power spectrum.ipynb,
```

```
Correlation.ipynb,
```

```
MCMC Likelihood Analysis.ipynb
```

Note that the likelihood analysis in the last notebook is not intended to be realistic, but it gives an operational example of how `CCL` can be integrated into such an analysis. In particular, the notebook only considers cosmic shear over one wide redshift bin and the covariance matrix adopted solely includes a contribution from cosmic variance. The "data vector" is simply a simulated using `CCL` theoretical predictions. For speed, theoretical predictions use the BBKS power spectrum plementation.

6.3. **Technical notes on how the Python wrapper is implemented**

The Python wrapper is built using the `swig` tool, which automatically scans the `CCL` C headers and builds a matching interface in Python. The default autogenerated `swig` interface can be accessed through the `pyccl.lib` module if necessary. A more user-friendly wrapper has been written on top of this to provide more structure to the module, allow `numpy` vectorization, and provide more natural Python objects to use (instead of opaque `swig`-generated objects).

The key parts of the wrapper are as follows:

`setup.py` —This instructs `swig` and other build tools on how to find the right source files and set compile-time variables correctly. Most of this information is provided by header files and SWIG interface files that are included through the `pyccl/ccl.i` interface file.

Note that certain compiler flags, like `-fopenmp`, are also set in `setup.py`. If you are not using `gcc`, you may need to modify these flags (see the `extra_compile_args` argument of the `setup()` function).

Interface (`.i`) files—These are kept in the `pyccl/` directory, and tell `swig` which functions to extract from the C headers. There are also commands in these files to generate basic function argument documentation, and remove the `ccl_` prefix from function names.

The interface files also contain code that tells `swig` how to convert C array arguments to `numpy` arrays. For certain functions, this code may also contain a simple loop to effectively vectorize the function.

The main interface file is `pyccl/ccl.i`, which imports all of the other interface files. Most of the CCL source files (e.g. `core.c`) have their own interface file too. For other files, mostly containing support/utility functions, `swig` only needs the C header (`.h`) file to be specified in the main `ccl.i` file, however. (The C source file must also be added to the list in `setup.py` for it to be compiled successfully.)

Python module files—The structure of the Python module, as seen by the user, is organized through the `pyccl/__init__.py` file, which imports only the parts of the `swig` wrapper that are useful to the user. The complete autogenerated `swig` interface can be accessed through the `pyccl.lib` sub-module if necessary.

Individual sub-modules from CCL are wrapped in their own Python scripts (e.g. `power.py`), which typically provide a nicer "Pythonic" interface to the underlying CCL functions and objects. This includes automatically choosing whether to use the vectorized C function or not, as well as some conversions from Python objects to the autogenerated `swig` objects. Most of the core Python objects, like `Parameters` and `Cosmology`, are defined in `core.py`. These objects also do some basic memory management, like calling the corresponding `ccl_free_*` C function when the Python object is destroyed.

Auto-generated wrapper files—The `swig` command is triggered when you run `setup.py`, and automatically generates a number of C and Python wrapper files in the `pyccl/` directory. These typically have names like `ccl_*.c` and `ccl_*.py`, and should not be edited directly, as `swig` will overwrite them when it next runs.

`pyccl/pyutils.py` —This file contains several generic helper functions for passing `numpy` arrays in and out of Python functions in a convenient way, and for performing error checking and some type conversions.

The build process will also create a `pyccl/ccllib.py` file, which is the raw autogenerated Python interface, and `_ccllib.so`, which is a C library containing all of the C functions and their Python bindings. A `build/` directory and `pyccl.egg-info/` directory will also be created in the same directory as `setup.py` when you compile `pyccl`. These (plus the `pyccl/_ccllib.so` file) should be removed if you want to do a clean recompilation. Running `python setup.py clean --all` will remove some, but not all, of the generated files.

7. **Future functionality to be included**

In the future, we hope that CCL will include other functionalities. Functionalities which are currently under development:

- a link to `angpow` (Campagne et al. 2017) for going beyond the Limber approximation,

- a link to `FAST-PT` (McEwen et al. 2016) for efficient implementation of nonlinear perturbation theory,

- and more power spectrum methods (see 3.7.8).

8. **Feedback**

If you would like to contribute to CCL or contact the developers, please do so through the CCL github repository located at https://github.com/LSSTDESC/CCL.

9. **Citing** CCL

If you use CCL in your work, please provide a link to the repository and cite it as LSST DESC (in preparation). For free use of the CLASS library, the CLASS developers require that the CLASS paper be cited: *CLASS II: Approximation schemes*, D. Blas, J. Lesgourgues, T. Tram, arXiv:1104.2933, JCAP 1107 (2011) 034. The CLASS repository can be found in http://class-code.net.

10. **License**

feedback received from other working groups of DESC, including Strong Lensing, Supernovae and Photometric Redshifts.

Anže Slosar: Wrote and reviewed code.
Antonio Villarreal: Contributed to initial benchmarking, halo mass function code, and general code and issues review.
Michal Vrastil: Wrote documentation and example code, reviewed code.
Joe Zuntz: Wrote initial infrastructure, C testing setup, and reviewed code.

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