# Modelica Library of Chemical Processes

(first draft)

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## Basic definitions

There are two base components, which should be included in almost all chemical models. The chemical solution and the chemical substance. Chemical solution join together all substances inside and provide to each chemical substance the current state of temperature “T” and pressure “p”. These physical quantities are the characterization of the solution. They are the result of the fundamental relations of thermodynamics:

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|  | 1. Pressure-Volume relationship |
|  | 1. Temperature relationship |

Where “U” is the free internal energy as the integration of heat energies added the solution “∂Q” and mechanical power of the solution “∂W” as equation (3); “G” is a free Gibbs energy defined as the sum of all chemical potential “μj” multiplied by amount of the j-th substance “nj” in the solution as equation (4); “H” is the enthalpy of the solution, which is also extensive property and can be calculated from molar enthalpies of the substances “Hm” as equation (5); “V” is a volume of the solution, which is also extensive property as the sum of all molar volumes “Vm” of the substances in the solution as equation (6); and amount of all substances “n” is the sum of amount of each substance in the solution as equation (7).

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|  | 1. Free Internal energy |
|  | 1. Free Gibbs energy |
|  | 1. Free enthalpy |
|  | 1. Volume |
|  | 1. Amount of solution |

Some mechanical aspects must be added to make a work with the solution, for example as piston in the motor with a fixed area “A” equaled to πr2, where r is an internal radius of the cylinder. The solution pressure can be converted to force by equation (9) and the force can be accumulated by displacement of the piston to the work of the solution “W” using equation (8) and (10). Change of this mechanical energy becomes from internal energy of the solution, so derivation of the work “∂W” is part of the change of free internal energy of the solution as mentioned in equation (3).

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|  | 1. Work of the chemical solution |
|  | 1. Pressure-Force relation |
|  | 1. Displacement-Volume relation |

Having these equations for solution it is well defined the relation between added heat “∂Q” and temperature “T” and the relation between force “F” and the position “s”. It means that the solution has two standard additional connectors – the thermal connector of Modelica Standard Library 3.2 (MSL) as Modelica.Thermal.HeatTransfer.Interfaces.HeatPort and mechanical connector from MSL as Modelica.Mechanics.Translational.Interfaces.Flange\_a. And Modelica compilers will handle the causality for each couple of these physical quantities.

Expect of this thermodynamics can have the chemical solution also the electrical properties as electrical potential “ϕ” and electrical current “i”. The non-zero electric potential “ϕ” has a direct impact to each substance with non-zero charge number. And the change of the charges has the meaning of electric current as in equation (11) , where “nj” is the molar amount of j-th substance, “zj” is the number of charge of the j-th substance (e.g. -1 for electron, +2 for Ca2+), and “F” is the Faraday’s constant.

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|  | 1. Electric current of the solution |

Having internal relation between electric potential “ϕ” and electric current “i” there is presenter also standard electrical port for the solution as Modelica.Electrical.Analog.Interfaces.Pin. However typical electroneutral solution should have zero electric potential, so the typical usage of this port is to connect it in the electrical ground. Setting some non-zero voltage source can cause the electrochemical processes, which are dependent on electric potential of the solution.

Each extensive property of the chemical solution, which can be calculated from properties of the chemical substances inside, is connected to the substances of the solution via solutionPort. The solution port redefine the sums above into the integration equations, whose have the same meaning as the original sum equations. This mathematical trick is used only to simplify the usage to user, which can connect all substances into one port of the chemical solution. The summation of each change is done by Kirchhoff’s law of connected flow variables. And in the side of the chemical solution is all changes restored to total integrated property, which can be presented in connector as non-flow variable having the same value for each substance.

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| flow variable of the substance | Kirchhoff's junction rule | Integrated properties of the solution | non-flow variable of the solution |
|  |  |  | – amount of solution |
|  |  |  | – mass |
|  |  |  | – free Gibbs energy |
|  |  |  | – temperature |
|  |  |  | – pressure |
|  |  |  | - electric potential |

The properties of the substance as molar mass “MM”, charge number of ion “z”, molar volume “Vm” or molar enthalpy “Hm” can be expressed in the substance definition, which will be shown at the end of this article.

The model of substance is accumulating the molar flow of the substance “∂n­j” into the amount of substance “nj” using equation (12). Having the amount of substance “nj” and the amount of solution “n” there is possible to present the mole fraction of the substance in solution “xj” as equation (13), which is typically the same as activity of the substance “a­j” defined by equation (14). However there exist some special cases, when the activity is different. This can be corrected by activity coefficient “γj” different as 1.

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|  | 1. Amount of the substance |
|  | 1. Mole fraction of the substance |
|  | 1. Activity of the substance |

The main equation of the substance is the definition of electro-chemical potential as the fundamental equation of physical chemistry – equation (15), where “μo” is the electro-chemical potential of the pure substance (aj=1) at zero electric potential (ϕ=0), and R is gas constant.

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|  | 1. **Electro-chemical potential** of the substance |

The definition of electrochemical potential of the pure substance μo is the main property of the substance. It is temperature and pressure dependent and its derivation for ideal gas substance and for incompressible substance is in the end of this article.

Using this electro-chemical potential “μ” together with molar flow “q” of the substance it is defined the substance connector.

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| flow variable | non-flow variable |
| – molar flow of the substance | – electro-chemical potential of the substance |

## Chemical processes

The most, if not all, chemical processes are equilibration of the electro-chemical potentials. The chemical equilibrium is state, when the chemical process stops and it does not generate any molar changes of any substance. And at this time the sum electro-chemical potentials of reactants is the same as the sum of electrochemical potentials of products. Each equilibration of chemical potentials is described by the equation of chemical kinetics (16), where “kC” is a kinetics coefficient making the rate of the chemical process, and “vj” is stoichiometry coefficient negative for reactants and positive for products (e.g. transport of the substance Ain <-> Aout has stoichiometry vin=-1, vout=1; chemical reaction 3 A1 <-> 2 A2 + 4 A3 has stoichiometry v1=-3, v2=2, v3=4).

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|  | 1. Chemical kinetics |
|  | 1. Molar change of the substance |

Where at equilibrium all molar flows “∂n­j” and “∂n­p” is zero, so the equilibrium is independent of the size of the kinetics coefficient “kC”.

Each chemical reaction in solution without electric potential (ϕ=0) reaches the standard equilibrium coefficient “K” as equation (18), which is also the direct result of molar Gibbs energy of the reaction calculated using Hess’ law from chemical potentials of pure substances as usually calculated in physical chemistry. If it is zero flow (∂n­p=0) in the chemical kinetics equation (16) then for each value of kC is algebraically derivable the equilibrium of equation (18) using the definition of electro-chemical potential (15).

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|  | 1. Equilibrium (dissociation) coefficient of the chemical reaction |

The equilibrium of the diffusion process of the substance ends with the same electro-chemical potentials “μ“ of the substance in each place of the solution. The same “μ” means the same activity “a” of the substance and the same “a” means the same mole fraction “x” of the substance. So the ideal diffusion ends with homogenous mixture, where the concentration of the substance is the same at each place.

The osmosis is the diffusion of electroneutral (zj=0) substances through semipermeable membrane. It is a membrane, through which some substances can freely diffuse and other cannot. For example the cellular membrane in biology is very selective and even a water must have a membrane channel to be enabled the transport through it. The water cross the membrane to reach the same mole fraction on both sides if there are the same pressure on both side of the membrane. If not, and the pressures are different, then the pressure dependence at equilibrium is reaching osmotic pressure “Π” of the substance with molar volume “Vm” as in equation (18) derived from the equilibrium of (16).

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|  | 1. Osmotic pressure |

The other situation is on semipermeable membrane for electrolytes (zj≠0), the substances which has an electric charge. Because at steady state the Donnan’s equilibrium is reached as expected, the concentrations of the free diffusible ion can be different on both side of the membrane during this equilibrium. However the electro-chemical potentials of this ion is the same on both side of the membrane. This phenomena generate the direct relation between Donnan’s coefficient as ration of ion concentration and membrane potential as difference of electric potentials of the solutions. This equation is known as Nernst potential of the permeable ion and it is the direct result of the equilibration equation (16), where the stoichiometry of the same ion is vin=-1, vout=1 and electrochemical potentials of the same ion are μin and μout with the same μo, temperature T and activity coefficient γ on both side of the membrane. And the membrane potential is the difference of the electric voltages of the solutions as in equation (19) derived from the equilibrium of (16).

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|  | 1. Membrane potential |

Gas dissolution in liquids equilibrates the electro-chemical potentials between the gaseous and the dissolved substance. This process is called Henry’s law and the ratio between gaseous and dissolved concentration is called Henry’s coefficient “kH”. From the steady state of equation (16) is derived the relation between the tabulated Henry’s coefficients as equation (1), μog as electro-chemical potential of the pure gaseous substanceand μod as extrapolated electro-chemical potential of the pure dissolved substance. The same principle is the vaporization of the solvent called Raoult’s law (2), where the fraction between liquid dissolved activity ad and gaseous form ag=pvap/pair is also determined by μod of pure liquid and μog of pure vapor of the substance. The last relation from gas dissolution series is dissolution in solid substances called as Sievert’s law (3), which is equilibrating a little longer, but the situation at equilibrium is similar.

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|  | 1. Henry’s coefficient |
|  | 1. Raoult’s vapor pressure |
|  | 1. Sieverts’ coefficient |

As it is known the vaporization is highly dependent on pressure and temperature. The vaporization curve is known as the temperature relationship of vaporization pressure at pair = 100 kPa and ad­=1 as equation (2).

## Chemical substance

The most of previous processes are well described at standard temperature of 25°C and standard pressure of 100kPa. In all equations of equilibrium of the chemical processes as (18), (19), (20), (21), (22) and (23) the measurable coefficients have the meaning of differences between μo. This means, that only a relative values for μo is needed to know of equilibriums coefficients. Typically there are selected the substances in their typical phase as reference with μo=0 and the other phases and composite substances are relatively defined to these reference substances. These relative μo values are called free formation molar Gibbs energies of the substances “ΔfGo“ and they are typically tabulated at temperature T0=25°C and p0=100kPa. So it is reasonable for these standard conditions to define μo25C,1bar = ΔfGo and Hm,25C,1bar= ΔfHo, where “ΔfHo“ is the free formation molar enthalpy of the substance as the amount of heat consumed by one mole of the substance during whole formation process from the reference substances. Also the molar entropy ΔfSo at standard temperatures can be expressed from equation (24). This value is usually not tabulated for the substances and must be really calculated by equation (24). Man must not be confused with the standard molar entropy of the substance, which is an absolute quantity usually noted as “So”. However the Hess’ law calculation of process entropies should give the same entropy of the chemical process.

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|  |  | 1. Free molar entropy of formation at T0=298.15K and p0=100kPa |

The enthalpies and the entropies at different pressure and temperature conditions can be easily extended for idealized substances as ideal gas (molar volume Vm=R\*T/p) or incompressible substance, for which the molar volume Vm is constant at each temperature and pressure.

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|  |  | 1. Molar enthalpy of the ideal gaseous substance |
|  |  | 1. Molar enthalpy of the incompressible substance |
|  |  | 1. Molar entropy of the ideal gas substance |
|  |  | 1. Molar entropy of the incompressible substance |
|  |  | 1. Electro-chemical potential of the pure ideal gas substance |
|  |  | 1. Electro-chemical potential of the pure incompressible substance |

Where cp is molar heat capacity of the substance at constant pressure. The meaning of this substance property is the ratio of heat change per change of temperature in one mole of the substance. If the solution does not exchange any other energy with environment then the heat flow from environment is the same as change of internal energy as equation (3). The assumption is that the change of Gibbs energy is equal to change of temperature multiplied by entropy during isobaric heating as equation (31).

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|  | 1. The assumption for change of Gibbs energy during isobaric heating |
|  | 1. Ideal gas isobaric heating |
|  | 1. Incompressible substance isobaric heating |

So the temperature shift of ideal gas is the solution of differential equation ∂Hm=∂T\*(cp+R) for free molar enthalpy and the differential equation T\*∂Sm=∂T\*(cp+R) for free molar entropy. In the case of incompressible substances are the equations ∂Hm=∂T\*cp and T\*∂Sm=∂T\*cp. These relations give the temperature shifts as presented in (25), (26), (27) and (28).

If we look at the relation (19) of osmotic pressure of incompressible substance then we see the pressure shift of (p-p0)\*Vm. By definition this pressure-volume energy is part of the internal energy, but not a part of enthalpy, so it must be included in entropy of the substance as in equation (28).

And if we imagine that the chemical processes of the ideal gas substance are driven only by partial pressure independently of ambient pressure of the whole gaseous solution we need to add a correction shift also to molar entropy of the pure gas, which change the mole fraction of gas to meaning of partial pressure in scale of different pressures of the solutions. Mathematically it is the total pressure of solution “p” is extracted from chemical potential defined by equation (29) at 25°C and 0V as following algebraic operation expressed by equation (34).

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And because this is not the only one possible way how to define the substance consistently with previous theory, there is an option to redefine the substance model with new calculation of the molar enthalpy, molar entropy, free molar Gibbs energy, molar volume and other base substance properties with any possible dependences on pressure, temperature, electric potential and ionic strength of the solution.