JULES

June 19, 2019

1 Equation Set

The equation set is written in terms of five prognostic conservative variables: three momenta, mass density, and specific entropy. While deriving these equations, it will prove convenient to define a 4-vector $u^\alpha = (1, u, v, w)$ for $\alpha = (t, x, y, z)$. This allows us to write a conservation law for a variable ϕ as

$$\partial_t \rho\phi + \partial_i \rho u_i \phi = \partial_\alpha \rho u^\alpha \phi = \text{ sources and sinks.}$$

Because mass conservation written in this form is just

$$\partial_\alpha \rho u^\alpha = 0,$$

this immediately provides some useful properties, namely that

$$\partial_\alpha \rho u^\alpha \phi = \rho u^\alpha \partial_a \phi$$

and

$$u^\alpha \partial_\alpha \rho = \rho \partial_\alpha u^\alpha.$$