Phase Interferometry for Approach Radars

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Abstract **– This paper discusses the design of an interferometric system that the role of a passive precision approach radar should perform. The brief overview of requirements for precision approach radar parameters is presented. The proposal of an interferometric system for a multi-base phase interferometer is described. An evaluation of the accuracy of determining the angle of arrival of the incident signal is described. Models of standard deviations of a determine the angle of arrival of the incident signal, depending on the individual system parameters are determined. The impact of ground reflection on the signal is discussed. Finally, the results of these models are discussed and a proposal for minimum requirements for the deployment of individual phase interferometer receivers is made.**

Keywords **– Phase Interferometry; Precision Approach Radar; Radar; Reflection from Terrain**

I. Introduction

This paper discusses the design of an interferometric system that the role of a passive precision approach radar should perform. The requirements for precision approach radar are described in detail in the standard ICAO [1]. In this paper, the most interesting parameters where are important for setting of a phase interferometer are described. PAR (Precision approach radar) must be able to provide and determine the aircraft position with the reflective surface of 15 m^2 or more. This aircraft shall be located in a sector at an angle of 20 ° in the azimuth plane and in a vertical plane by a sector 7 ° wide by a distance of at least 16.7 km from the radar antenna. The azimuth resolution must be at least 1.2 °. The maximum permissible error due to deviations from the approach axis must be less than 0.34 °. The elevation resolution must be at least 0.6 °. The maximum permissible error due to deviations from the approach axis must be less than 0.23 °. The full display must be repeated at least once per second.

The system consists of two interferometers. One is used to determine the azimuth of the target and the second is used to determine the elevation of the target. The system design can be done for both interferometers simultaneously, since the requirements for their parameters are almost identical.

II. System Based on a Phase Interferometry

The phase interferometer [6] is used to determine the angle at which the plane wave falls on the plane fitted by the receivers. Two or more base interferometers are used to increase accuracy - see Fig. 1

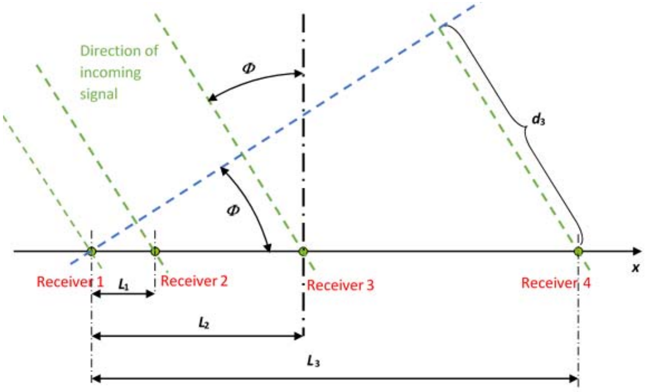

Figure 1. Phase interferometer with three bases (L_1, L_2, L_3)

Where:

Φ - The angle of arrival of the signal (ie the angle of plane wave impact on the plane of the antenna)

d_m - Extending the length of the beam path to the *m*-th antenna element

L_m - The length of the *m* base

The phase delay of the wave impinging on the *m* antenna element ψ_m can be calculated according to relation 1.

$$\psi_m = 2\pi \frac{d_m}{\lambda} = 2\pi \frac{L_m \sin\Phi}{\lambda} \tag{1}$$

Where λ is the wavelength of the received signal.

The total voltage (complex signal envelope) received on the m-element is calculated according to the relation 2.

$$u_m = U \exp(-j\psi_m) + n_m; \tag{2}$$

Where *U* is the amplitude of the signal received on the *m*-th antenna element and n_m is the noise voltage (complex envelope) on the *m*-th antenna element.

On the individual receivers, we measure the u_m signals, based on this signals we can estimate $\hat{\Phi}$ the real direction of Φ and estimate $\hat{U}$ the amplitude *U*. Where voltages are known and estimates $\hat{\Phi}$, $\hat{U}$ and the noise n_m is not known. We must find the most credible values $\hat{\Phi}$ and $\hat{U}$, when we know that n_m is uncorrelated noise (3), with normal amplitude distribution (4).

$$P[n_0, n_1, .. n_N] = \prod_{m=0}^{N} [w(n_m)] \approx \exp\left(-\frac{\sum_{m=0}^{N} n_m^2}{2\sigma^2}\right) \tag{3}$$

$$w(n_m) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{n_m^2}{2\sigma^2}\right) \tag{4}$$

We assume that if the n_m noise did not work, the voltage u_n would be equal (5).

$$u_{m0} = U \exp\left[-j2\pi \frac{L_m \sin(\Phi)}{\lambda}\right] \tag{5}$$

The presence of noise causes that the estimates of the variables U and Φ will differ from the actual values by a small phase errors $\delta\Phi = \hat{\Phi} - \Phi$ and amplitudes $\delta U = \hat{U} - U$. The equation (5) can be linearized against $\delta\Phi$ and δU and in matrix form we get the equation (6).

$$\mathbf{A}(\Phi, U).\mathbf{\Delta}_{\mathbf{\Phi,U}} = \mathbf{N} \tag{6}$$

Where:

$\mathbf{A}(\Phi, U)$ - Matrix of the left side of the system (6) of size 2x(N+1)

$\mathbf{\Delta}_{\mathbf{\Phi,U}}$ - Column error vector $\delta\Phi$ of determining the angle of arrival Φ and error δU of amplitude U of incidental waves

$\mathbf{N}$ - Column vector of noise voltages n_i (N+1 elements)

$$\mathbf{A} = \|A_{ik}\|; \quad \mathbf{\Delta}_{\mathbf{\Phi,U}} = \begin{vmatrix} \delta\Phi \\ \delta U \end{vmatrix}; \quad \mathbf{N} = \begin{vmatrix} n_0 \\ n_1 \\ ::: \\ n_N \end{vmatrix};$$
$$A_{i1} = -\exp\left[-j2\pi \frac{L_i \sin(\Phi)}{\lambda}\right]; \quad A_{i2} = 2\pi U_0 \frac{L_i \cos(\Phi)}{\lambda} \exp\left[-j2\pi \frac{L_i \sin(\Phi)}{\lambda}\right] \tag{7}$$

By solving equation (6)) we get a relation for a column error vector (8).

$$\mathbf{\Delta}_{\mathbf{\Phi,U}} = \left(\mathbf{A}^{\mathbf{H}}.\mathbf{A}\right)^{-1}.\mathbf{A}^{\mathbf{H}}.\mathbf{N} \tag{8}$$

Equation (8) represents the linear dependence of error estimates of the arrival angle and the amplitude of the incident signal on the amplitudes of the noise signals. The noise is uncorrelated and therefore the variance of all noise voltages n_i is the same – equal σ_n^2. The covariant matrix of estimation errors can be expressed as (9).

$$\mathbf{cov}[\mathbf{\Delta}_{\mathbf{\Phi,U}}] = \sigma_n^2.\left(\mathbf{A}^{\mathbf{H}}.\mathbf{A}\right)^{-1} \tag{9}$$

The Covariant error matrix $\mathbf{cov}[\mathbf{\Delta}_{\mathbf{\Phi,U}}]$ is a square matrix 2x2 with elements S_{ik}. The elements $S_{11} = \sigma_U^2$ a $S_{22} = \sigma_\Phi^2$ are equal to a variance of amplitude and angle of arrival estimates.

III. Results of Simulation of the Mean Quadratic Deviation of the Determination of the Angle of Arrival of the Signal

Based on the equations described above, a computer model of an interferometer, using four receivers, was assembled. The system therefore has three bases (the distance between the first receiver and the others). The first base is selected fixedly λ/2. The remaining bases can be changed in the model. The size of the second and especially the third (the longest) of the base has a direct and significant effect on the accuracy of determining the angle of arrival of a signal. The signal-to-noise ratio can be set in the model. This parameter has a major effect on the accuracy of the signal angle of arrival evaluation. The following graph (Fig. 2) shows the dependence of the mean quadratic deviation of the determination of the angle of arrival of the signal in relation to the angle of the received signal against the axis of the interferometer system and the noise signal ratio, at the 0.5 * λ, 8 * λ and 20 * λ bases.

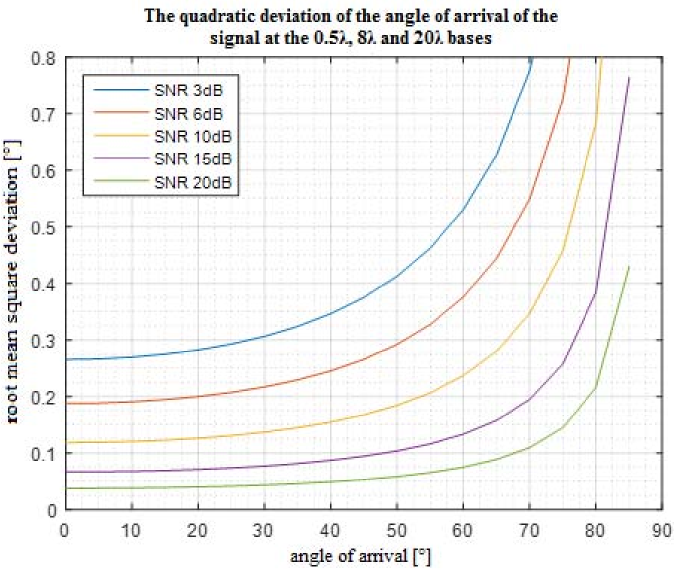

Figure 2. The root mean square deviation of determining the angle of arrival of a signal

Figure 2 shows that the system meets the required parameters for the accuracy of determining the direction of signal arrival at the signal-to-noise ratio 10 dB.

IV. Effect of Reflections from Terrain

In this case, we will consider the effect of reflections of the signal from the plane terrain (Earth). We will consider only the effect of these reflections on the antenna providing vertical coverage (interferometer measuring elevation), where the impact of these reflections cannot be neglected. The schematic arrangement of the vertical interferometer antenna is shown in Figure 3. As can be seen, on each antenna element is incident to the direct signal and the signal reflected from the ground. On the individual antenna elements, these signals are added together. Reflected signals significantly decrease the accuracy of determining the elevation of the target.

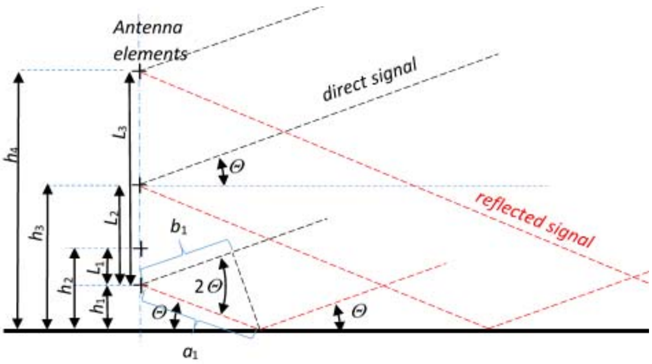

Figure 3. Scheme of mirror reflections from terrain in elevation

In addition, a mathematical apparatus for determining the influence from terrain will be described. The difference in the paths of direct and reflected signals is given by Equation (10).

$$c_i = a_i - b_i = \frac{h_i}{\sin\Theta}(1-\cos 2\Theta) = 2h_i \sin\Theta \tag{10}$$

Then the difference between the phases of direct and reflected beams is (11).

$$\Delta\varphi_i = 2\pi\frac{c_i}{\lambda} = 4\pi\frac{h_i}{\lambda}\sin\Theta \tag{11}$$

When calculating the system errors, u_i is replaced by the expression in relation to the voltage of the received signal (12).

$$u_i \cdot \left(1 + F\left(\Theta, h_i\right)\right); \tag{12}$$

Where $F(\Theta,h_i)$ it is called "formfactor" due to multipath propagation and is given by the following equation (13)

$$F\left(\Theta, h_i\right) = f\left(\Theta\right) + \rho \cdot f\left(-\Theta\right)\exp\left(\mathrm{j}\Delta\phi_i\right); \tag{13}$$

Where:

$\rho = \rho_0\rho_s\rho_v D$ - The total complex coefficient of reflection from the terrain

$f(\Theta)$ - Voltage radiation pattern of the antenna element (in the vertical plane)

ρ_0 - Fresnel's (complex) mirror reflection coefficient from the plane interface

ρ_S - Correction of the reflection coefficient due to terrain roughness (Specular roughness factor)

ρ_v - Correction of reflection coefficient due to vegetation cover (Vegetation factor for land surface)

D - Divergence factor due to the curvature of the Earth (Spherical earth divergency factor)

The individual members of the above equation for the total complex mirror reflection coefficient from the terrain will be described in detail below.

The size of the Fresnel coefficient of specular reflection depends on the selected polarization. For vertical polarization, the following relationships are applied (14) [2], [3].

$$\rho_{0\mathrm{V}} = \frac{Y^2\sin\Theta - \sqrt{Y^2 - \cos^2\Theta}}{Y^2\sin\Theta + \sqrt{Y^2 - \cos^2\Theta}} \quad Y = \sqrt{\frac{\varepsilon_{rc}}{\mu_{rc}}} \cong \sqrt{\varepsilon_r + \frac{\sigma}{\mathrm{j}\omega}} \tag{14}$$

For horizontal polarization, the following relationships are applying (15) [2], [3].

$$\rho_{0\mathrm{H}} = \frac{\sin\Theta - \sqrt{Y^2 - \cos^2\Theta}}{\sin\Theta + \sqrt{Y^2 - \cos^2\Theta}} \quad Y = \sqrt{\frac{\varepsilon_{rc}}{\mu_{rc}}} \cong \sqrt{\varepsilon_r + \frac{\sigma}{\mathrm{j}\omega}} \tag{15}$$

Where:

Θ - The angle of impact of the signal in elevation (as shown in Fig. 3)

ε_r - Relative permittivity of the terrain

σ - Conductivity of the terrain

ω - The angular frequency of the received signal

The following table1 [4] shows typical magnitudes of relative permittivity and conductivity for different terrains.

TABLE I. PARAMETERS OF DIFFERENT TERRAINS

Surface type	ε_r	σ	σ_H
Moving grass	10	0.001	0.01
Tall grass	10	0.001	0.1
Gravel	4	0.001	0.02
Asphalt	6	0.001	0.0004
Brush	4	0.001	0.5
Snow	2.5	0.001	0.003
Desert	2.5	0.001	0.003
Trees	1.5	0.001	1.5
Seawater	80	4	-
Clean water	67	0.1	-

The size of the coefficient ρ_S, which corrects the reflection coefficient of the terrain due to terrain roughness, can be mathematically described using the following equation (16).

$$\rho_S = \exp\left[-\frac{1}{2}\left(4\pi\frac{\sigma_H}{\lambda}\sin\Theta\right)^2\right] \tag{16}$$

Where σ_H is the mean quadratic value of surface roughness. Its typical size for the different surfaces are in Table 1.

The effect of vegetation must be included in calculating the surface-reflected energy. The absorption is modeled as a vegetation factor ρ_v that multiplies ρ_0 in calculating the reflection coefficient. This vegetation factor can be calculated via equation (17) [3].

$$\rho_v = \left(1 - \sqrt{a\lambda}\right)\exp\left(-\frac{b\sin\psi}{\lambda}\right) + \sqrt{a\lambda} \le 1.0 \tag{17}$$

where the coefficients a and b are:

$a = 3.2$, $b = 1$ for thin grass;

$a = 0.32$, $b = 3$ for brush or dense weeds;

$a = 0.032$, $b = 5$ for dense trees.

Another factor affecting the reflected rays is the divergence factor D, expressed approximately by equation (18), [5].

$$D = \sqrt{\frac{1}{3}\left(1+\frac{2\zeta}{\sqrt{\zeta^2+3}}\right)} \qquad \zeta = \sqrt{\frac{a_{eff}}{2\ h_{radar}}\tan\theta_d} \qquad (18)$$

Where:

a_{eff} – The effective earth's radius

h_{radar} – The radar height above the ground

Θ – The elevation angle

Considering all the above-mentioned coefficients and fitting them into the equation for the total complex coefficient of reflection from the terrain, we obtain the following courses for different terrain in vertical polarization – Figure 4.

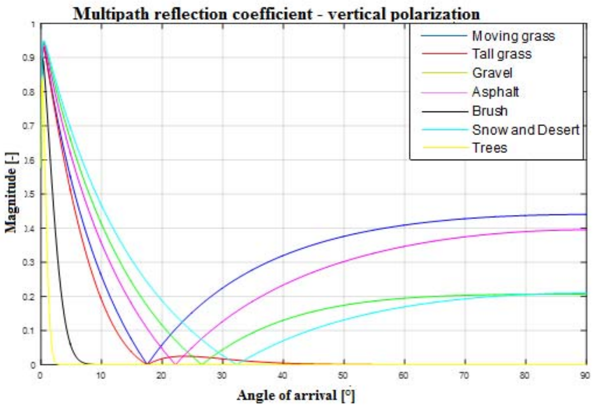

Figure 4. Reflection coefficient of multipath propagation for different terrain and vertical polarization of the signal

Subsequently we express the signals received on the individual antenna elements of the interferometer, we use the equation (12). In our case, we consider the use of patch antenna elements, the $f(\Theta)$ factor can be neglected, because it can be assumed that the radiation pattern of such antenna elements is almost constant in the elevation range. Applying the above equations, we obtain the following results for the values of the mean quadratic deviation of the angle of arrival of the signal, depending on the different terrains in the near vicinity of the interferometer antenna.

For calculation, we consider the required signal to noise ratio of 10 dB and the individual base distances 0.5λ, 12λ and 30λ. The height of the basic base antenna element above the terrain is 3 m. Under these conditions and vertical polarization, we obtain the following dependencies for different terrain types - Figure 5.

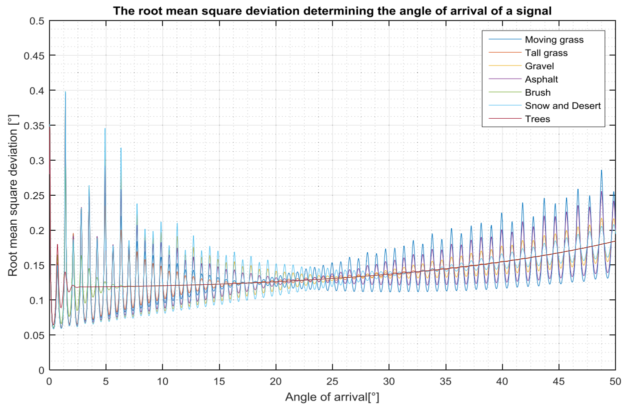

Figure 5. Root mean square deviation determining the angle of arrival of a signal for different terrains and for vertical polarization

V. Conclusion

From the simulated results, it is obvious that in the case of vertical polarization of the incident signal, there is no significant or very significant influence on the mean quadratic deviation of the determination of direction of angle of arrival of signal, when we considering all simulated terrains. When using horizontal polarization, the magnitude of the mean quadratic deviation of the determination of direction of angle of arrival of signal will increase significantly. Therefore, it is appropriate to use the vertical polarization and the antenna elements with a high polarization purity.

It is also apparent from the simulation results that the magnitude of the mean quadratic deviation of the determination of direction of arrival of the signal depends on the height of the first (base) antenna element of the interferometric system. It is therefore necessary to choose the appropriate height of the system above the terrain for the given type of terrain.

Acknowledgment

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