MICROWAVE EMITTER POSITION LOCATION: PRESENT AND FUTURE

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Abstract: Position location (determination) of enemy microwave emitters (radars, jammers) is one of the most important tasks of electronic warfare (EW) systems, particularly for electronic intelligence (ELINT) and electronic support measures (ESM) systems. Correct performed location of those emitters yields data allowing to deduce an allocation of enemy radar-controlled weapon systems, to detect associated threats and makes possible to eliminate the emitters from a battlefield by jamming or missiles (weapon delivery). Effectiveness of emitter location is significant condition of successful military action. Since contemporary battlefield is more and more saturated by electronic systems, thus the significance of location systems using sophisticated emitter location techniques and algorithms (ELT&A) will be permanent increase.The paper is a comprehensive review (tutorial) presenting current state of knowledge concerning ELT&A including last obtained results. This paper undertakes the most important technological and theoretical aspects of the emitter location considered by many significant contributors in this field, including some results obtained by the author.

1. INTRODUCTION

1.1. Emitter location philosophy

To locate various sources of enemy signals (emitters) is one of the most important tasks of electronic intelligence/electronic warfare ELINT/EW systems [1-5].
Emitter position location provides:

- determining electronic order of battle
- weapon sensor location
- precision target aim/designation
- emitter differentiation/separation.

Coordinates of the emitter may be estimated by processing measurement results (data):

- obtained from a stationary observation system consisting of spatially distributed measuring points or
- collected by a single moving observation system.

In the observation system are received (intercepted) signals from the emitter and are measured parameters of the signals.

Possible signal parameters useful to locate emitters are:
(1) amplitude
(2) phase shift
(3) time delay (time difference of arrival-TDOA)
(4) frequency shift (frequency difference of arrival-FDOA).
A result of the signal parameter measurement called here the primary observation is the concrete value of measured physical quantity.

Each mentioned quantity (signal parameter) is functionally related with appropriate geometrical quantity or its derivative.
Geometrical quantities suitable for emitter position location:
(1) angle of arrival (AOA) or direction of arrival (DOA) of the signal from the emitter to the measuring point
(2) range difference from the emitter to two measuring points
(3) derivative of specific range difference (velocity of emitter relative to the observer or vice versa).
AOA (DOA) is determined using amplitude or phase shift.
Range difference is determined by TDOA measurements.
Range difference derivative of specific range difference is obtained from FDOA measurements.

The concrete value of given geometrical quantity will be called the secondary observation.
Such an observation determines:
- line of position (LOP) on the plane or
- surface of position (SOP) in three-dimensional space.

The emitter lies on the LOP or on the SOP if no errors exist. In such a case position of the emitter can be determined as a point of intersection of two LOPs or three SOPs.
Unfortunately, it is pure theoretical case.

Properties of secondary observation set depend on:
(1) kind of measured signal parameters
(2) used measurement techniques
(3) procedures of data collecting.
These elements determine concrete emitter location technique (ELT), see Fig.1.

A type of ELT determines a type of applied emitter location algorithm (ELA).
Each concrete ELA is defined by:
(1) assumed observation model
(2) estimation method
(3) procedure(s) of numerical computations.
Observation model:
- describes kinds of errors taken into consideration
- determines functional relationship between given secondary observation, emitter coordinates and observation errors.

Estimation method:
- is resulting from introduced criterion of estimation quality
- yields an estimator of emitter position.

Note: Complexity of the estimator depends on: observation model, existing limitations, a priori knowledge.
Procedures of numerical computations influence on:
- processing speed and range
- numerical stability.

Note : Since these factors are mostly contrary , trade-off sometimes is necessary.

Each concrete emitter location method (ELM) can be considered as a fusion of ELT with appropriate ELA, as it is illustrated in Fig.1.

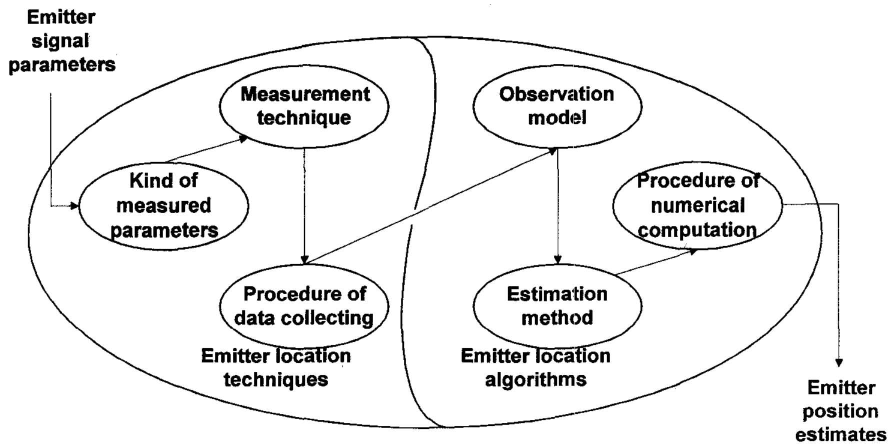

Figure 1: Elements determining emitter location methods

1.2. Brief history of emitter location methods development

Milestones/ Significant contributors/ Solved questions

- Earliest described ELM were applied to location of ground emitters using land-based stationary measuring systems.
  The first work in this field has been published by Stansfield [6]. Method presented there is a fusion of typical direction finding (DF) with some original emitter location algorithm. This method is widely used in COMINT and ELINT and it is known as the triangulation method [3,5,7-13]. The algorithm was referred later in its original form as so called Stansfield's algorithm [13-15] and was presented also in several modified forms [7,15-18]. New results concerning triangulation were recently published by Gavish and Weiss [19].

- Stansfield's idea has been later utilized by Daniels [20] and Marchand [21] to time-difference-of-arrival (TDOA) measurements (see also Lee [22] and Torrieri [15]).
  ELA using TDOA measurements is called the multilateration algorithm [22] or range-difference (hyperbolic) algorithm.

- Novel algorithm based on TDOA measurements and called LOCA (location on the conic axis) has been done by Schmidt [57]. It can be considered the dual one to typical (ordinary) algorithm utilizing TDOA (hyperbolic).

- Application of the Stansfield's method to locate of ground emitters using measurement results collected by airborne systems have been presented at first by Ancker [14], Blachman [17], Sparagna et al [7]. Later there have been introduced airborne methods: azimuth/elevation presented by Baron et al [8], circulation (angle-difference) presented by Poirot and Arbid [12], constant heading angle methods proposed by Mahapatra [23] (so called spiral method) and Mangel [9] (three bearing method) respectively.

- Above methods were developed under simplifying assumptions of flat Earth surface and straightforward propagation.
  However, for large distances occur significant errors of the environment model (Houston and Nelson [24]). In this case such a simplification may be unsatisfactory.
  Some ELM without flat Earth surface approximation has been presented by Wangsness [25].

Note: All listed here methods were elaborated to locate emitters on the Earth surface. Methods for emitter location in 3-D space have been introduced later.

- Problem of position estimation of a target in 3-D space has been formulated at first by Cooper and Laite [26,27] for range-range systems. However, such systems are disable to locate emitters.

- A method of emitter location in 3-D space using range-difference location system was described by Lee [22]. An original algorithm to locate the emitter 3-D using two-dimensional angle measurements has been presented by author [28,29].

Note: Although methods to locate emitters on the plane using DF systems were described earliest, such methods for 3-D applications have been introduced relatively latest. Common feature of early period of ELM development was that all works have presented only homogeneous methods of emitter location. The homogeneous method utilizes only one type of determined geometrical quantities (e.g. AOA only or TDOA only), whereas the non-homogeneous method utilizes various types of these quantities (e.g. AOA together with TDOA).

- Formulation of emitter location problem for non- homogeneous (mixed) observations has been presented by Foy [16] to locate emitters on the plane.

- General method of emitter location in three-dimensional space observations has been presented by Wax [30] and the author [31] (observer position uncertainty i.e. measuring platform position errors have been included).

- Latest introduced emitter location method is called frequency-difference-of-arrival (FDOA) method or differential Doppler (DD) [3,16] utilizing measurements of Doppler shift of the emitter frequency caused by relative motion between the observer and the emitter. Some results of investigations concerning the method have been done by Chestnut [32], Chan and Towers [33], Becker [34].

- ELA most often used in mentioned ELM are principally based on the maximum likelihood (ML) estimator or on the least squares (LS) estimator. The ML estimator may be used if probability density function (pdf) of measurement errors is known, otherwise the LS estimator is used.

- Alternative estimation approach is based on Kalman filtering [10,15,16] but it requires more a priori (initial) knowledge (Spingarn [35]).

Since approach to emitter position estimation problem depends on an emitter uncertainity model (prior knowledge concerning the emitter position), two major models of the emitter uncertainity can be specified:
(1) fisherian model, if lack of any a priori knowledge concerning the emitter
(2) bayesian model, if this knowledge is available.

An application of the fisherian model leads to fisherian estimation algorithm whereas application of the bayesian model leads to the bayesian algorithm. In practice, fisherian algorithms are absolutely predominant, although bayesian emitter algorithms occur too, e.g. Butterly [36] or Marple et al [37].

2. EMITTER LOCATION TECHNIQUES

2.1. Classification of emitter location techniques

Each concrete emitter location technique depends on:
- kind of measured parameters of emitted signal
- used measurement technique
- procedure of collecting measurement results.

Taking into account these elements, following ELT can be mentioned (Fig.2):
(1) triangulation
(2) azimuth/elevation
(3) circulation
(4) range-difference (hyperbolic)
(5) differential Doppler.
Moreover, combinations of above techniques may be utilized and they are called mixed mode location techniques[16].

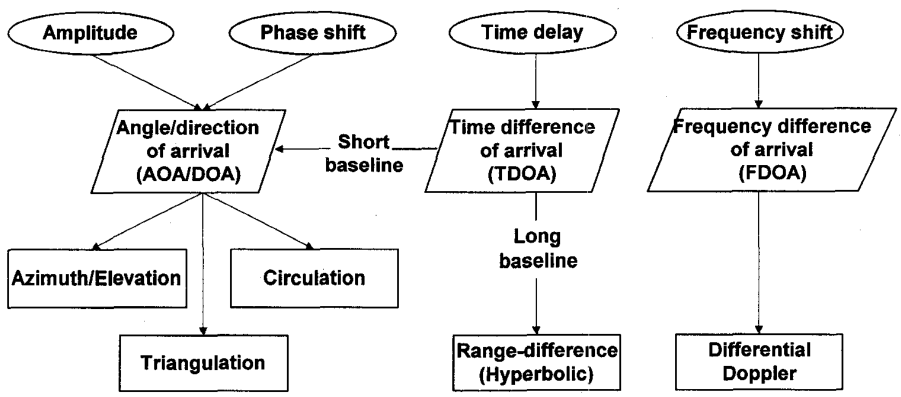

Figure 2: Classification of emitter location techniques resulting from kind of measured parameters, measurement techniques and procedure of data collecting

Triangulation. Location technique involving only angle measurements taken from more than one observation site (most often called triangulation [3,5,7-13] eventually back resection [38]).
This technique generally utilizes intersections of LOPs on the plane or of SOPs in 3-D space.
Triangulation is used in:
- moving observation systems consisting of single measuring platform or
- stationary systems consisting of two or more measuring platforms.

This technique is very flexible in use. It may be utilized to locate emitters operating either with continuous wave (CW) signals and pulsed signals.

Triangulation allows to minimize effect of random data measurement errors on estimate of emitter position but is vulnerable to DF systematic errors (bias). It is well described in many publications, [3, 5, 7, 8, 13, 15, 17, 38].

Azimuth/Elevation. This technique is capable to provide single pulse instantaneous emitter location. It may be applied to location of stationary ground-based emitters from airborne measuring platforms. The technique provides emitter location by simultaneous measuring an azimuth and elevation (depression) angle of the emitter. Angle measurements are made from the airborne platform flying at known altitude relative to the emitter.
This technique requires only one pair of angles (azimuth and elevation angle) and allows locate emitters lying on the Earth surface in front of the aircraft nose.
For fixed elevation angle this technique provides better accuracy at high altitudes [3,8].
It may be used to locate all types of emitters (pulsed and CW).

Circulation. The technique is used to locate emitters on the Earth surface from airborne platforms and utilizes intersecting circles instead of intersecting lines. It has been suggested by Poirot and Arbid [12] to eliminate certain types of systematic errors.
In the circulation are utilized angle measurements called relative bearings or heading angles (measured from the aircraft nose clockwise or counterclockwise to the reference line passing through actual aircraft position and position of the emitter). Systematic measurement error of the same relative bearing is always constant although unknown. This attribute gives a possibility of systematic errors elimination using appropriate processing of collected angle measurements.
The circulation has some disadvantages. It is more sensitive to random DF errors as the triangulation. Ambiguous intersections of LOPs require additional procedure for resolving. This ambiguity can be resolved by use of typical bearings. However, when systematic errors occur, the circulation yields significantly better results as the triangulation.
There are known two slightly another methods eliminating systematic errors. They can be considered as versions of the triangulation because of the same general idea.
The first one, proposed by Mahapatra [23], can be called constant heading angle method or spiral trajectory method. Principal requirement is that actual relative bearing must be held constant during whole observation process. A trajectory of the observer is a logarithmic spiral. Basic feature of this method: actual value of bearing is not used in the emitter position estimation. Major advantage of this method: complete absence of ambiguities occuring in circulation. Main disadvantage is that required trajectory is strong constrained by a requirement of constant relative bearings [23].
The second method called three bearings method proposed by Mangel [9], can be considered as a particular case of the constant heading angle method. Number of measured bearings necessary to determine emitter position is limited to three what makes this method operationally simple.

Range difference. The range difference (hyperbolic) ELM is based on TDOA measurements only. TDOA approach requires three or more measuring platforms that cooperate in emitter position determining. TDOA technique utilizes measurements of the TOA of pulsed signals at three or more spatially separated receivers. TDOA measurement value determines on the plane an isodelay curve (hyperbola).
TDOA measurement error influences on the emitter location error. For pulsed signals, the error in TDOA measuring strongly depends on the bandwidth of the measurement channel. If the bandwidth wider then the error smaller. Therefore for very accurate TDOA measurements very short pulses are preferred [3].

Differential Doppler. A technique called DD or FDOA is based upon measurements of the frequency of the received signal at two spatially separated moving receivers. The concrete frequency shift (difference) resulting due to difference of radial velocity of receivers relative to the emitters generates an iso-Doppler curve [3].
The technique utilizes receiver motion to information extraction. If the receiver speed greater then more accurate results. Without relative motion is any measurement possible.
Doppler measurement accuracy increases if the pulse width of the received signal increases. In practice, the DD technique can be most useful to locate CW emitters. It is the significant limitation for application of the DD technique to EW tasks.

2.2 Comparison of emitter location techniques

Main features. All AOA measurement techniques can be used to position determination of emitters generating all types of signals.

The hyperbolic location technique based upon TDOA measurements can be used to locate emitters of pulsed signals only.

Differential Doppler technique based on FDOA measurements can be applied to locate emitters generating CW signals mainly. DD technique may be used only in moving measurement systems.

All AOA location techniques and the hyperbolic location technique can be used in both types of measurement systems, i.e. stationary and moving.

AOA location techniques don't require simultaneous operating of measuring platforms. a single measuring platform may be sufficient.

In the case of use hyperbolic or DD techniques the simultaneous operating of measuring platforms is necessary. Minimum two (for the DD technique) or three (for the hyperbolic technique) measuring platforms are needed here.

Consideration concerning all AOA location techniques leads to following conclusions:

- Azimuth/elevation technique is useful especially to locate stationary, ground-based emitters from airborne measuring platforms.
  This technique requires a single aircraft, therefore is flexible operationally yielding an instantaneous location.
  However, the technique is sufficiently effective (accurate) only at relative high altitudes of flight.
- Triangulation is effective to locate ground emitters by moving or stationary measuring platforms in presence of pure random measurement errors.
  The methods is very sensitive to systematic measurement errors.
- Application of constant relative bearing methods (spiral trajectory or three bearing method) allow to eliminate the effect of systematic errors.
  The methods can be used only in moving measurement systems operating with strongly restricted own trajectory.
- Circulation eliminates an influence of systematic measurement errors but it is sensitive to random measurement errors.
  The method introduces an ambiguity caused by intersecting circles.

Some technological aspects. Main limitations related to emitter position location accuracy are resulting from physical restrictions and technological difficulties which exist in AOA, TDOA and FDOA measurements.

Practically used techniques of AOA determination:

- amplitude maximum
- amplitude comparison
- phase comparison (interferometry).

The first technique is the cheapest one, quick directional search to provide high probability of signal intercept is needed, so called spinning antenna systems may be used.
Main advantage is constructional simplicity (single receiving channel) but it is sensitive to amplitude fluctuations.
Amplitude comparison is not sensitive to the fluctuations but it is more complex (two or more channels).
Both methods are sensitive to the multipath effect.
Phase comparison provides very high accuracy of AOA measurements and is relatively insensitive to multipath. However, it is the most expensive method exploiting multiple baseline theory to eliminate AOA ambiguity [8], complicated receivers to accurate determining of signal frequency and using sophisticated methods of digital signal processing at microwave frequencies [3, 39-41].

The significantly cheaper alternative to phase interferometry offers some AOA determination technique using TDOA measurements.
The measurements are realized using a short baseline (metres) instead the long baseline (kilometres). Main advantage of TDOA direction-finding technique over phase interferometry is that AOA calculation is independent of signal frequency. Moreover, phase ambiguities don't occur here, there is no need to use auxiliary resolving channels. In addition, because of very short time interval measurements, effect of multipath reflections is eliminated (Cusdin and Mallinson [42]).
This technique is sometimes called differential time of arrival (DTOA). It requires extremely accurate time delay measurements.

Main advantage of the range-difference (hyperbolic) ELM based upon TDOA measurements, realized in the system with long baselines, is that the most accurate and rapid emitter location may be provided because of relatively easy and accurate measurements of time delay.
This technique requires at least three simultaneously operating measuring platforms, high quality receivers, wideband data link between the platforms, very high performance central processor and very high reliability of cooperating subsystems.
It is very useful to locate emitters generating pulsed signals, but it can not be used to locate CW-type emitters.

DD technique has several practical limitations. Usefulness is restricted mainly to locate emitters generating CW signals with constant frequency. The technique is poor to locate of pulsed emitters, particularly in EW application.
Measuring receivers must be precisely synchronized in time and they must have extreme stability.
Location of receivers and changes of their position during the measurement time interval must be estimated with high accuracy [32].

3. EMITTER LOCATION ALGORITHMS

3.1. Classification of emitter location algorithms.

Geometrical quantities most useful to locate emitters may be as follows:
(1) angles (bearings)
  (a) horizontal bearing (azimuth) Θ
  (b) vertical bearing (elevation angle) ε
(2) range difference Δr
(3) angle difference $\Delta\Theta$
Set of considered geometrical quantities can be described by

$$U = \{\Theta,\ \varepsilon,\ \Delta r,\ \Delta\Theta\}$$

If a location system can measure the Θ and ε simultaneously, then can be used single integrated quantity called generalized bearing or slant azimuth $\alpha = f(\Theta, \varepsilon)$ [28,29].
Since α is not a measured quantity, it is called the pseudo-observation.

In emitter location systems may be utilized geometrical quantities of the same type or of various types. It characterizes an input data structure Using this criterion ELA can be classified into two groups:
(1) homogeneous algorithms which use the same type of geometrical quantities;
(2) non-homogeneous (heterogeneous) algorithms using geometrical quantities of various types.

Considering type of used geometrical quantities ELA can be classified as follows (Fig.3):
(1) angular algorithms using Θ and ε or α
(2) hyperbolic algorithms using Δr
(3) circulation algorithms using $\Delta\Theta$.

The classification with respect to type of utilized geometrical quantities can be considered as the secondary one with respect to the input data structure.

Considering uncertainty model concerning emitter the ELA can be classified as follows (Fig.4).
1) fisherian algorithms, in which a priori knowledge concerning the emitter is not taken into account; the ML or LS estimators can be used here
2) bayesian algorithms, in which a priori knowledge about the emitter is included; minimum risk (MR) or maximum a posteriori probability (MAP) estimators can be used here
3) pseudo-bayesian algorithms, in which join particular features of both above-mentioned algorithms;
  there can be distinguish:
  a) bayesian-fisherian algorithms, in which apriori information is obtained using the fisherian estimator and whereafter the bayesian estimator is used; fisherian estimator is here formally extended to a form of the bayesian estimator
  b) fisherian-bayesian estimator, in which a priori information is obtained but it is very uncertain; initially bayesian estimator reduces itself to the fisherian form.

In consideration of input data collecting strategy, ELA can be classified as follows (Fig.5):
(1) algorithms for simultaneous observations taken from many observation sites at the same time
(2) algorithms for asynchronous observations taken from many sites at various time or from a

single observation point.

Considering of input data processing strategy, such ELA can be distinguished (Fig.5):
(1) algorithms utilizing parallel (bath) data processing, where all input data are commonly (together) processed at the same time
(2) algorithms utilizing serial (stepwise) data processing, where successively obtained input data are processed step by step.

3.2. Emitter location algorithms synthesis

Observation model. Let $\boldsymbol{x}=\left[x,y,z\right]^T$ be the emitter unknown position.

Vector $\boldsymbol{x}$ will be estimated from set $\left\{u_i;\ i=1,2,\ldots,n\right\}$ obtained from spatially separated elementary measuring subsystems (EMS).

EMS may be:

- measuring installation located at single site (Θ or ε) or
- two measuring installations located at two spatially separated sites (Δr or $\Delta\Theta$).

Position of the EMS will be described by:

$$\boldsymbol{X}_i=\begin{cases}\begin{bmatrix}\boldsymbol{X}_i'\\ \boldsymbol{0}\end{bmatrix} & \text{for} \quad u_i\in\left\{\Theta,\varepsilon\right\}\\ \begin{bmatrix}\boldsymbol{X}_i'\\ \boldsymbol{X}_i''\end{bmatrix} & \text{for} \quad u_i\in\left\{\Delta r,\Delta\Theta\right\}\end{cases}$$

where: $\boldsymbol{X}_i'=\left[x_i',y_i',z_i'\right]^T$, $\boldsymbol{X}_i''=\left[x_i'',y_i'',z_i''\right]^T$, $\boldsymbol{0}=\left[0,0,0\right]^T$

$\boldsymbol{X}_i'$, $\boldsymbol{X}_i''$ - EMS coordinates vector.

In the presence of additive, random measurement errors, single observation (measurement equation) can be expressed by [31,43,44]

$$u_i=f_i\left(\boldsymbol{x},\boldsymbol{X}_i\right)+\zeta_i, \qquad i=1,2,\ldots,n$$

Introducing integrated vector $\boldsymbol{X}=\left[\boldsymbol{x}^T,\boldsymbol{X}_i^T\right]^T$, measurement equation can be presented as

$$u_i=f_i\left(\boldsymbol{X}\right)+\zeta_i$$

Linearization of prediction function $f_i\left(\boldsymbol{X}\right)$ about initial reference point $\boldsymbol{X}_0$ leads to

$$u_i=f_i\left(\boldsymbol{X}_0\right)+\frac{\partial f_i\left(\boldsymbol{X}\right)}{\partial \boldsymbol{X}}_{|X-X_0}\left(\boldsymbol{X}-\boldsymbol{X}_0\right)+\zeta_i$$

where: $X_0=\left[\boldsymbol{x}_0^T,\widetilde{\boldsymbol{X}}_i^T\right]^T$

x_0 - some initial guess of the emitter true position

$\widetilde{\boldsymbol{X}}_i$ - vector of EMS coordinates at the moment of u_i observation.

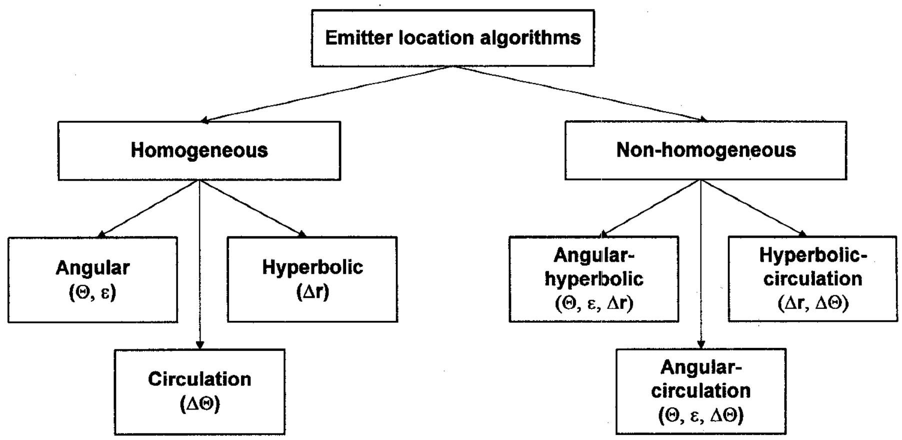

Figure 3: Emitter location algorithms classification with respect to input data structure and type of used geometrical quantities

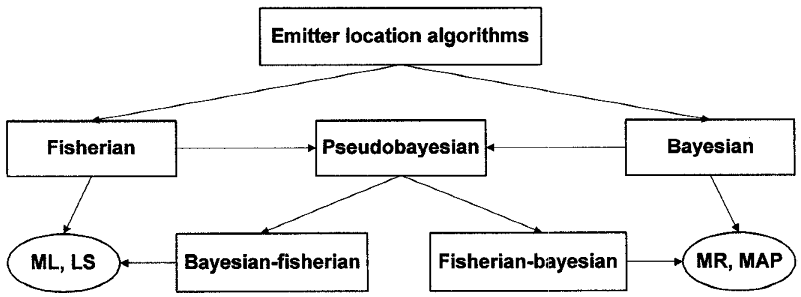

Figure 4: Emitter location algorithms classification with respect to observation object uncertainty model

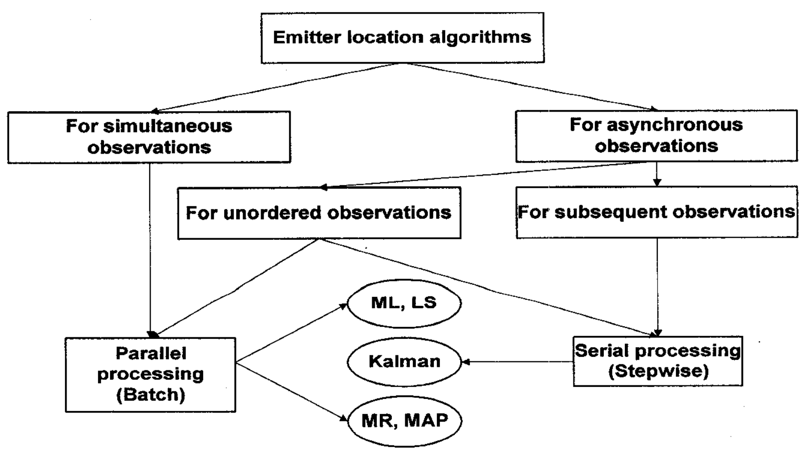

Figure 5: Emitter location algorithms classification with respect to input data collecting and processing strategy

The gradient vector will be decomposed onto two vectors

$$\frac{\partial f_i(X)}{\partial X}\Big|_{(\cdot)} = \left[\frac{\partial f_i(\boldsymbol{x}, \boldsymbol{X}_i)}{\partial \boldsymbol{x}}\Big|_{(\cdot)}, \frac{\partial f_i(\boldsymbol{x}, \boldsymbol{X}_i)}{\partial \boldsymbol{X}_i}\Big|_{(\cdot)}\right] = \left[\boldsymbol{g}_{1i}^T, \boldsymbol{g}_{2i}^T\right] = \boldsymbol{g}_i^T$$

where $(\cdot)$ denotes the point $X_0 = \left(\boldsymbol{x}_0^T, \tilde{\boldsymbol{X}}_i^T\right)$.

Denoting

$$f_{0i} = f_i(X_0) \text{ and } \eta_i = \boldsymbol{X}_i - \tilde{\boldsymbol{X}}_i$$

and assuming that

$$E(\eta_i) = \boldsymbol{0} \text{ and } E\left(\eta_i \eta_i^T\right) = \boldsymbol{R}_{h_i}$$

where $\boldsymbol{R}_{h_i}$ denotes covariance matrix of *i*-th EMS position errors, observation equation may be expressed in modified form

$$w_i = \boldsymbol{g}_{1i}^T \boldsymbol{x} + e_i$$

where: $w_i = u_i + \boldsymbol{g}_{1i}^T \boldsymbol{x}_0 - f_{0i}$

$e_i = \boldsymbol{g}_{2i}^T \eta_i + \zeta_i, \; E(e_i) = 0$

For n observations the vector equation can be presented in form

$$\boldsymbol{w} = \boldsymbol{G}_1 \boldsymbol{x} + \boldsymbol{e}$$

where: $\boldsymbol{w} = \boldsymbol{u} + \boldsymbol{G}_1 \boldsymbol{x}_0 - \boldsymbol{f}_0$

$\boldsymbol{e} = \boldsymbol{G}_2 \eta + \zeta$

$\boldsymbol{G}_1$ - $n \times 3$ matrix of gradients $\boldsymbol{g}_{1i}^T$

$\boldsymbol{G}_2$ - $n \times 6n$ block-diagonal matrix of gradients $\boldsymbol{g}_{2i}^T$.

Since error vectors ζ and η are described by

$$E(\zeta) = \boldsymbol{0}, \; E\left(\zeta\zeta^T\right) = \boldsymbol{R}_z, \; E(\eta) = \boldsymbol{0}, \; E\left(\eta\eta^T\right) = \boldsymbol{R}_h$$

then vector of the joint error $\boldsymbol{e}$ will be described as follows

$$E(\boldsymbol{e}) = \boldsymbol{0}, \; E\left(\boldsymbol{e}\boldsymbol{e}^T\right) = \boldsymbol{R}, \; \boldsymbol{R} = \boldsymbol{G}_2 \boldsymbol{R}_h \boldsymbol{G}_2^T + \boldsymbol{R}_h$$

Fisherian algorithms. These algorithms are realized using ML or LS estimators. ML estimator may be applied if the probability density function (pdf) of $\boldsymbol{e}$ is completely known. Otherwise the LS estimator may be used.

Fisherian estimator $\hat{\boldsymbol{x}}_F$ of the $\boldsymbol{x}$ may be expressed by [15,31,43]

$$\hat{\boldsymbol{x}}_F = \left(\boldsymbol{G}_1^T \boldsymbol{R}^{-1} \boldsymbol{G}_1\right)^{-1} \boldsymbol{G}_1^T \boldsymbol{R}^{-1} \boldsymbol{w}$$

$$\boldsymbol{R}_F = \left(\boldsymbol{G}_1^T \boldsymbol{R}^{-1} \boldsymbol{G}_1\right)^{-1}$$

Position estimation errors are described by covariance matrix $\boldsymbol{R}_F$ or by parameters of error ellipsoid [15,16,30,31,43].

Bayesian algorithms. These algorithms may be created using MR or MAP estimators:

- MR estimator be used if the a priori probability distribution of the $\boldsymbol{x}$ and the loss function concerning estimation errors of the $\boldsymbol{x}$ are known.
- MAP estimator can be used if loss functions is unknown.

Bayesian estimator can be expressed by [43,44]

$$\hat{\boldsymbol{x}}_B = \boldsymbol{R}_B\left(\boldsymbol{G}_1^T \boldsymbol{R}^{-1}\boldsymbol{w} + \boldsymbol{R}_{ap}^{-1}\hat{\boldsymbol{x}}_{ap}\right)$$

$$\boldsymbol{R}_B = \left(\boldsymbol{G}_1^T \boldsymbol{R}^{-1}\boldsymbol{G}_1 + \boldsymbol{R}_{ap}^{-1}\right)^{-1}$$

where: $\hat{\boldsymbol{x}}_{ap}$ - expected value (mean) of random vector

$\boldsymbol{R}_{ap}$ - covariance matrix of this vector

$\boldsymbol{R}_B$ - covariance matrix of estimator $\hat{\boldsymbol{x}}_B$ errors.

Another form of $\hat{\boldsymbol{x}}_B$:

$$\hat{\boldsymbol{x}}_B = \left(\boldsymbol{R}_F^{-1} + \boldsymbol{R}_{ap}^{-1}\right)^{-1}\left(\boldsymbol{R}_F^{-1}\hat{\boldsymbol{x}}_F + \boldsymbol{R}_{ap}^{-1}\hat{\boldsymbol{x}}_{ap}\right)$$

is a weighted sum of the fisherian and the a priori estimators.
Two boundary cases can be specified:

1) if $\boldsymbol{R}_F^{-1} \to \boldsymbol{0}$ (maximum measurements uncertainty) then $\hat{\boldsymbol{x}}_B \to \hat{\boldsymbol{x}}_{ap}$,

2) if $\boldsymbol{R}_{ap}^{-1} \to \boldsymbol{0}$ (maximum a priori uncertainty) then $\hat{\boldsymbol{x}}_B \to \hat{\boldsymbol{x}}_F$,

Next alternative form of $\hat{\boldsymbol{x}}_B$:

$$\hat{\boldsymbol{x}}_B = \hat{\boldsymbol{x}}_{ap} + \boldsymbol{K}\left(\boldsymbol{w} - \boldsymbol{G}_1\hat{\boldsymbol{x}}_{ap}\right)$$

$$\boldsymbol{R}_B = \left(\boldsymbol{I} - \boldsymbol{K}\boldsymbol{G}_1\right)\boldsymbol{R}_{ap}$$

$$\boldsymbol{K} = \boldsymbol{R}_{ap}\boldsymbol{G}_1^T\left(\boldsymbol{G}_1\boldsymbol{R}_{ap}\boldsymbol{G}_1^T + \boldsymbol{R}\right)^{-1}$$

where: $\boldsymbol{K}$ - $3 \times n$ gain matrix
$\boldsymbol{I}$ - 3×3 identity matrix.

Pseudo-bayesian algorithms. Two cases can be specified :

- bayesian-fisherian (B-F) or
- fisherian-bayesian (F-B).

The B-F algorithm is created by extending the fisherian estimator to the bayesian form.
If vector $\boldsymbol{w}_*$ of observations obtained earlier as the vector $\boldsymbol{w}$ is accessible (available), then the estimator $\hat{\boldsymbol{x}}_*$ and matrix $\boldsymbol{R}_*$ may be determined using expressions for $\hat{\boldsymbol{x}}_F$ and $\boldsymbol{R}_F$.
Treating results $\hat{\boldsymbol{x}}_*$ and $\boldsymbol{R}_*$ as the specific a priori knowledge, the B-F estimator will be

$$\hat{\boldsymbol{x}}_{B-F} = \boldsymbol{R}_{B-F}\left(\boldsymbol{G}_1^T\boldsymbol{R}^{-1}\boldsymbol{w} + \boldsymbol{R}_*^{-1}\hat{\boldsymbol{x}}_*\right)$$

$$\boldsymbol{R}_{B-F} = \left(\boldsymbol{G}_1^T\boldsymbol{R}^{-1}\boldsymbol{G}_1 + \boldsymbol{R}_*^{-1}\right)^{-1}$$

Vector $\boldsymbol{x}$ is here interpreted as deterministic one, but unknown.

The F-B algorithm is created by reduction of the bayesian estimator to a form of the fisherian one.

If a priori knowledge will be limited to $\boldsymbol{R}_{ap}$ only, so called bayesian LS estimator can be obtained [37]

$$\hat{\boldsymbol{x}}_{BLS} = \boldsymbol{R}_B \boldsymbol{G}_1^T \boldsymbol{R}^{-1} \boldsymbol{w}$$

If a priori knowledge will be decrease more and more then $\boldsymbol{R}_{ap}^{-1} \to \boldsymbol{0}$, the F-B estimator will be in pure fisherian form.

$$\hat{\boldsymbol{x}}_{F-B} = \hat{\boldsymbol{x}}_F$$

However, vector $\boldsymbol{x}$ has a random character here.

Kalmanian algorithms. These algorithms are created using Kalman filter [45].
As a result of k-th observation the observation vector $w(k)$ is obtained. This vector consists of m parameters determined simultaneously at the k-th moment. Modified observation equation is

$$\boldsymbol{w}(k) = \boldsymbol{G}_1(k)\boldsymbol{x}(k) + \boldsymbol{e}(k)$$

where: $\boldsymbol{G}_1(k)$ - $m \times 3$ measurement matrix

$\boldsymbol{x}(k)$ - 3×1 state vector (emitter position coordinates)

$\boldsymbol{e}(k)$ - $m \times 1$ vector of measurement errors.

Properties of the state vector are described by the state equation [44, 45]

$$\boldsymbol{x}(k+1) = \boldsymbol{\Phi}(k+1,k)\boldsymbol{x}(k) + \boldsymbol{\Gamma}(k+1,k)\boldsymbol{v}(k)$$

where: $\boldsymbol{\Phi}(k+1,k)$ - 3×3 transition matrix

$\boldsymbol{\Gamma}(k+1,k)$ - $3 \times r$ trajectory noise gain matrix

$\boldsymbol{v}(k)$ - $r \times 1$ trajectory noise vector.

Under initial conditions $\boldsymbol{P}(0) = E\left[\boldsymbol{x}(k)\boldsymbol{x}^T(k)\right]$, $\hat{\boldsymbol{x}}(0) = \boldsymbol{x}_0$ optimal state estimate for filtering at the time k is given by the recursive equation [44]

$$\hat{\boldsymbol{x}}(k) = \hat{\boldsymbol{x}}(k/k-1) + \boldsymbol{K}(k)\left[\boldsymbol{w}(k) - \boldsymbol{G}_1(k)\,\hat{\boldsymbol{x}}(k/k-1)\right]$$

where: $\hat{\boldsymbol{x}}(k) \equiv \hat{\boldsymbol{x}}(k/k)$ - state vector estimate at the k-th moment for filtering;

$\hat{\boldsymbol{x}}(k/k-1)$ - state vector estimate on k-th moment for prediction based upon estimate $\hat{\boldsymbol{x}}(k-1) \equiv \hat{\boldsymbol{x}}(k-1/k-1)$

$\boldsymbol{K}(k)$ - $3 \times m$ gain matrix (called also Kalman gain).

Characteristics of position estimation errors. Vector of emitter position estimation errors is defined as difference between $\hat{\boldsymbol{x}}$ and $\boldsymbol{x}$.
Covariance matrix of vector $\boldsymbol{x}$ estimation errors can be expressed using following matrices:

1) $\boldsymbol{R}_F$ in fisherian algorithm
2) $\boldsymbol{R}_B$ in bayesian algorithm
3) $\boldsymbol{R}_{B-F}$, $\boldsymbol{R}_B$ or $\boldsymbol{R}_F$ in pseudo-bayesian algorithms
4) $\boldsymbol{P}(k)$ in kalmanian algorithm.

Error region for the estimate of $\boldsymbol{x}$ can be described by an ellipsoid called error ellipsoid [15,46,47].
Error ellipsoid mathematically exact but very inconvenient for any comparing.
Some monoparametric measure is more convenient in practical use. The Grubbs's analytical approximation for the spherical error probable (SEP) is proposed here [47].

3.3. Comparison of emitter location algorithms

Main features. In consideration of a manner of input data collecting, kalmanian algorithms essentially differ from the other algorithms.
On the other hand, all described algorithms are similar with respect to estimators form formal structure, and estimation philosophy (logic structure). It concern especially bayesian and pseudo-bayesian estimators.

Occurring differences concern the manner of input data collecting. It leads to different data processing organisation.

Emitter position estimation using fisherian, bayesian and pseudo-bayesian algorithms is realised after collecting all accessible measurement data. The data create usually large data files, that must be stored on the whole. It is parallel (batch) data processing realized once for given data file.
Following features of this data processing can be distinguished here:

(1) to initialize the data processing an initial estimate $\hat{x}(0) = x_0$ is needed
(2) nonlinear prediction function is linearized about the nominal estimate from previous iteration
(3) .the largest matrix being inverted is of $n \times n$ dimensionality, therefore it may be large
(4) for small number of observations n variance position estimation error can strongly depend on variance of a priori estimate;
(5) bias of emitter position estimate does not occur
(6) in an idealized case, i.e. in absence of measurement errors, these algorithms are convergent to true value of $\boldsymbol{x}$.

Emitter position estimation using kalmanian algorithms is realized sequentially (successively) as a step by step process after each obtained observation. A file (set) of stored data is limited to two observations: the current one and the previous one. It is serial (stepwise) data processing.
Following features can be specified here:

(1) to initialize the data processing the initial estimate $\hat{x}(0)$ and the matirx $\boldsymbol{P}(0)$ are needed
(2) nonlinear prediction function is linearized about the predicted current state estimate
(3) the largest matrix being inverted is of $m \times m$ dimensionality, if $m \leq 3$ then the matrix will be small
(4) variance of estimation error decreases stepwise and may be smaller than in previously considered algorithms
(5) bias in position estimate occurs
(6) in the idealized case these algorithms don't exactly converge to true value of $\boldsymbol{x}$.

Some practical remarks. The of batch-processing-type algorithms may be useful in situations when many measurements from many spatially distributed measuring systems are obtained simultaneously or at various moments of the time.
These algorithms are preferable for land-based stationary or mobile EW systems (Air Defense/ Army).

The of stepwise-processing-type algorithms are particularly useful in situations when a little number of measurements is successively obtained from single measuring system (platform). These algorithms are preferable for moving (airborne or shipborne) EW systems (Air Force/Navy).

4. DEVELOPMENT TRENDS OF EMITTER LOCATION METHODS

4.1. Trends of emitter location techniques development

In practice, the most often used ELT are:
- angular(triangulation, azimuth/elevation)
- range-difference.

For both angular ELT the most perspectivical techniques seems to be phase comparison (phase interferometry) and DTOA. DTOA is the AOA determination technique time-difference-of-arrival measurements realized using short baseline.

The most sophisticated ELT are based upon AOA determination principle by phase comparison using theory of multiple baseline interferometry [8] and very expensive interferometer structures. The interferometers are built as complex receiving systems providing digital signal processing at microwave frequencies [39-41]. Advanced interferometer technology provides already very high accuracy of AOA determination. Main problems are still related to elimination of AOA determination ambiguity and to necessity of accurate measurements of signal frequency before AOA determination.
This technique is preferable to precise location of emitters and it is currently applied in sophisticated land-based, airborne and shipborne ELINT systems.

The second technique providing high accuracy of AOA determination is DTOA (differential time of arrival). It may be considered as a short-baseline version of TDOA technique [42]. To improve accuracy of TDOA measurements they are realized at an intermediate frequency. By very high accuracy of TDOA measurements high accuracy of AOA determination is obtained without necessity of signal frequency determining. Moreover, elimination of AOA determination ambiguity is not needed here.
DTOA technique may be useful particularly for airborne ELINT and ESM platforms, where elements of antenna system must be spatially separated at relatively small distances only.

Range-difference ELT provides high accuracy of emitter position location. The technique is useful for land-based stationary and airborne EW systems to locate emitters very precise. This technique requires wideband data transmission lines and high performance central processor. Such a technique, when utilized in airborne version, provides long detection range and yields emitter position almost instantaneously.
Range-difference airborne emitter location systems can operate at large distances. They are preferable rather for strategic applications.
Worldwide dissemination of NAVSTAR GPS creates some possibility of non-typical use of the system to emitter location using TDOA techniques with long baseline (hyperbolic), but without use of wideband transmission lines. If separate receivers of location system have an acces to precise timing (reference time base of GPS, rubidium time standard provides nanoseconds accuracy), there is no need to transmit synchronising pulses on-line through wideband transmission line to determine necessary time differences. These differences can be determined off-line in everyone receiving point where time of arrival (TOA) of the pulse to other receivers of the system has been delivered using whichever narrowband transmission

line. However, in dense signal environment resolving the same pulses from individual receivers of location system can be extremely difficult or impossible without sophisticated (fine) interpulse and/or intrapulse analysis. Last experiences in ELINT with so called specific emitter identification(SEI) techniques offer some possibilities of solving this problem by pulse differentiation (resolving) using unintentional modulation on pulse (UMOP) measurements [48-52].

Improvement of technical/operational possibilities in existing ELT may be achieved by:
- appropriate combining/merge of various ELT
- reasonable data fusion/validation/correlation

Combining triangulation and circulation (AOA/DAOA) techniques gives a possibility to avoid vulnerability of emitter location system to both types of measurements errors (random and systematic).
Combining TDOA and FDOA techniques gives a possibility of effective use of the location system independently of both types of received signals (pulsed or CW)
Proper data fusion makes emitter location system more effective (precise) in use and flexible operationally.

4.2. Trends of emitter location algorithms development

In today's emitter location systems most often utilized geometrical quantity is the AOA. Since the existing equipment often doesn't provide sufficiently high AOA resolution, measuring equipment is aided by suitable AOA processing to increase AOA resolution.

Several algorithms providing increased AOA resulotion are known as algorithms called:
- MUSIC (multiple signal classification), invented by Schmidt [3,57]
- Min-Norm (minimum-norm), introduced by Kumaresan and Tufts [58]
- SSR (state- space realization), given by Kung, Arun and Bhaskar Rao [59]
- ESPIRIT (estimation of signal parameter via rotational invariance techniques), done by Paulraj, Roy and Kailath [53, 59, 61]
- BEWE (bearing estimation without eigendecomposition), presented by Yeh [60]
- WSF (weighted subspace filtering), developed by Viberg, Ottersten and Kailath [61]
- FINE (first principal vector), proposed by Xu, Buckley and Marks [62].

In works concerning emitter location algorithms it can observe tendencies towards
- improving operation speed
- providing their better numerical stability.

Research efforts are concentrated mainly on fisherian, bayesian-fisherian and kalmanian algorithms.
Numerical stability can be improved by application of the rectangular matrix pseudoinverse [43,54]. At present there are accessible numerically effective procedures for calculation of matrix pseudoinverse based upon so called SVD (singular value decomposition) [41, 55].

The pseudoinverse of a rectangular matrix may be applied to the fisherian estimator as follows [43]

$$\hat{\mathbf{x}}_F = \underline{\mathbf{G}}_1^{+} \underline{\mathbf{w}} \qquad \text{(a)}$$

where $\underline{\mathbf{G}}_1^{+}$ denotes pseudoinverse of the matrix $\underline{\mathbf{G}}_1$ whereas the $\underline{\mathbf{w}}$ denotes transformed observation vector. Matrix $\underline{\mathbf{G}}_1$ and vector $\underline{\mathbf{w}}$ are expressed respectively by

$$\underline{\mathbf{G}}_1 = \mathbf{R}^{-1/2}\mathbf{G}_1 \ \text{ and } \ \underline{\mathbf{w}} = \mathbf{R}^{-1/2}\mathbf{w} \qquad \text{(b, c)}$$

where $\mathbf{R}^{-1/2}$ denotes a square root of the matrix inverse $\mathbf{R}^{-1}$ while $\boldsymbol{R}$ is covariance matrix of observation errors and $\boldsymbol{w}$ is the observation vector, identically as it is given in section 3.2.. Utilizing the formulae (b,c) and properties of a matrix pseudoinverse, the matrix $\underline{\mathbf{G}}_1^+$ can be expressed by

$$\underline{\mathbf{G}}_1^+ = \left(\underline{\mathbf{G}}_1^T \underline{\mathbf{G}}_1\right)^{-1} \underline{\mathbf{G}}_1^T \qquad \text{(d)}$$

Putting expression (b) into (d) and subsequently the (d) and (c) into (a) the fisherian estimator in its typical form (4) is obtained.

Emitter location algorithm using Kalman filters are also intensively examined. Main direction concerns improving estimation effectiveness by decreasing an estimation bias.

All types of ELA should be theoretical considered and numerically examined under assumptions closer to reality. These assumptions concern more complex observation models including various types of errors that can appear in real measurement environment (e.g. unknown systematic errors).

5. CONCLUDING REMARKS

From studies, research and investigations it results, that offensive of new technologies concerning ELT is focused mainly on antenna/receiver complexes, i.e. on microwave parts of EW systems.
Sophisticated techniques are preferable here such as angular techniques using phase interferometry or DTOA and the range-difference technique. Phase interferometry and DTOA are suitable in moving EW systems designed for tactical tasks realized in relative small observation area while the range-difference approach is better in stationary or moving EW systems operating in large observation area.

In further stages of data processing, efforts of designers and producers are concentrated on organization of measurements process and elaboration of flexible and effective software for data processing. Software requires reliable ELA providing effective and quick emitter estimation with high numerical stability.
ELA using parallel (batch) data processing are preferable for multiplatform, stationary EW systems while emitter location algorithms using serial (stepwise) data processing are more suitable for single-platform, moving EW systems.

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