Time-Delay Direction Finding Based on Canonical Correlation Analysis

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Abstract—**Direction finding is an important component of Electronic Warfare (EW) systems. Direction finding precision may directly affect signal sorting, recognition, location, and jamming decisions, etc. Thus, it is an urgent task to improve direction finding precision. To solve the problems of time-delay direction finding's sensitivity to interference, this paper proposes a novel technique based on Canonical Correlation Analysis (CCA) to improve time-delay direction finding performance and completely analyzes the algorithm. The implementation of this time-delay direction finding method is simple and the computational complexity is low. The precision of direction finding by the new method is much better than that by conventional way. Simulations show that applying CCA to time-delay direction finding is efficient and feasible.**

I. INTRODUCTION

In the modern EW signal environments, the signals are dense and the types of which are also complicated. This makes the task of signal processing in electronic reconnaissance very hard. As the position of emitters is relatively fixed or the variety of speed relative to signals is very low, direction finding become a key technology in current electronic reconnaissance. At the same time, there are more demands for direction finding such as high precision, high resolving ability, multi signal simultaneous processing and real-time direction finding, etc. An effective novel method should been acquired to solve these problems in direction finding, which may be very helpful to signal sorting and recognition in electronic reconnaissance, especially to passive location [1].

Time-delay direction finding is an important method featured the high sensitivity, high precision and excellent real-time. Time-delay direction finding method can obtain the arrival of azimuth (AOA) of electric wave by the measurement of the difference of arrival time to each direction finding antenna, which is the time delay. As the bad anti-jamming property and the carrier wave must have determinate modulate, time-delay direction finding has not been broadly applied at present.

Canonical Correlation Analysis (CCA) is an important method of multivariate statistical since it was proposed by H.Hotelling in 1936 [2], which can find the basic vectors from two sets of variables. There are detail descriptions of CCA in [3][4]. CCA has been applied in some preliminary work [5][6][7] in recent years. Uncorrelated components can be obtained by CCA, in addition, have maximum spatial or temporal correlation within each component. Then we can apply CCA to seek the correlate components of the data from double receiving antennas. After general correlation of the two canonical components, we can obtain the time delay and complete the computation of azimuth. The structure of this paper is as follows: In the Section 2, we will analyze the issue of time-delay direction finding and pose the problem. In the Section 3, the direction finding algorithm will be analyzed. Then some experiments of the algorithm applied in this paper are conducted in the Section 4. Finally a conclusion is given.

II. PROBLEM FORMULATION

The model of time delay between the signals received by two groups of separate antennas is shown as Fig.1.

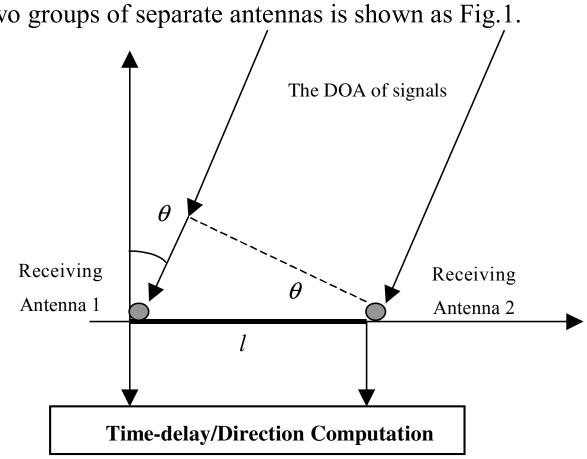

Figure. 1. Signal receiving model

Supposing that receiving station has m receiving channels, the mutual distance is l. If have $n(n \le m)$ narrow band signals, $\boldsymbol{r}(\theta_i)$ is the antenna respond to narrow band signal on the direction θ_i, then $\boldsymbol{r}(\theta_i)=\left[1, e^{j\phi_i}, ..., e^{j(m-1)\phi_i}\right]^T$, where $\phi_i = 2\pi l \sin\theta_i / \lambda$, λ is the wave length. Receiving signal model could be described as:

$$\boldsymbol{x}(t) = \boldsymbol{H} \cdot \sum_{i=1}^{n} \boldsymbol{r}(\theta_i) s_i(t) + \boldsymbol{n}(t) = \boldsymbol{A} \cdot \boldsymbol{s}(t) + \boldsymbol{n}(t) \qquad (1)$$

where $\boldsymbol{x}(t)$ is the receiving signals of antenna. $\boldsymbol{A}$ is a $m \times n$ mixing matrix, which is the product of antenna responding function $\boldsymbol{R} = \left[\boldsymbol{r}(\theta_1), ..., \boldsymbol{r}(\theta_n)\right]$ and the mixing matrix $\boldsymbol{H}$ during the signal transmitting process. $\boldsymbol{s}(t)$ are the source signals including radar signals, interference signals etc. $\boldsymbol{n}(t)$ are $m \times 1$ dimension noise signals, which include exterior noise, inner noise and electricity noise etc. The signals are assumed as mutually independent and independent with noise in the following analysis. The key problem of direction finding based time delay is how to eliminate or reduce the affect of noise, interference and mixing signals to time delay computation by correlation, which is the main problem that this paper want to solve.

III. Direction Finding Algorithm

Direction finding algorithm includes three steps: canonical correlation analysis, time delay estimation, and azimuth computation. Before all these steps, the receiving signals should be normalized to avoid the influence to following processing.

A. Canonical Correlation Analysis

The main difference between CCA and the other three methods is that CCA is closely related to mutual information [5]. Hence CCA can be easily motivated in information based on tasks and our natural selection.

Consider two sets of input data $x_1, x_2, \cdots, x_p$ and $y_1, y_2, \cdots, y_q$, $p \le q$, we attempt to find the coefficient $\boldsymbol{a} = (a_{i1}, a_{i2}, \cdots, a_{ip})$ and $\boldsymbol{b} = (b_{i1}, b_{i2}, \cdots, b_{iq})$ by the idea of principle components. The two sets of data can be written as combination of some pairs of variables ξ_i and η_i, which can be described as:

$$\begin{cases} \xi_1 = a_{11}x_1' + \cdots + a_{1p}x' & \eta_1 = b_{11}y_1' + \cdots + b_{1q}y_q' \\ \quad \cdots & \\ \xi_p = a_{p1}x_1' + \cdots + a_{pp}x_p' & \eta_p = b_{p1}y_1' + \cdots + b_{pq}y_q' \end{cases} \qquad (2)$$

where x', y' are the standardization value of x, y respectively. ξ_1, η_1 are the first pair of canonical variable, the correlation coefficient can be describe as $r_{\xi_1\eta_1}$, brief written as r_1. ξ_2, η_2 are the second pair of canonical variable, the correlation coefficient can be describe as $r_{\xi_2\eta_2}$, brief written as r_2. Then p pairs of canonical variables and p canonical correlation coefficients can be obtained.

Mutual independent variables can be obtained by the method of canonical correlation. Here we introduce a theorem [4] (which will be proven in appendix A).

Theorem *Let* $\boldsymbol{x} = (x_1, x_2, \cdots, x_p)^T$, $\boldsymbol{y} = (y_1, y_2, \cdots, y_q)^T$ *are two sets of random variables,* $\operatorname{cov}(\boldsymbol{x}) = \boldsymbol{\Sigma}_{xx}, \operatorname{cov}(\boldsymbol{y}) = \boldsymbol{\Sigma}_{yy}$, $\operatorname{cov}(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{\Sigma}_{xy}, \operatorname{cov}(\boldsymbol{y}, \boldsymbol{x}) = \boldsymbol{\Sigma}_{yx}$, $\boldsymbol{\Sigma}_{xx}, \boldsymbol{\Sigma}_{yy}$ *are positive definite, then* $\boldsymbol{\Sigma}_{xx}^{-1}\boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}_{yy}^{-1}\boldsymbol{\Sigma}_{yx}$ *and* $\boldsymbol{\Sigma}_{yy}^{-1}\boldsymbol{\Sigma}_{yx}\boldsymbol{\Sigma}_{xx}^{-1}\boldsymbol{\Sigma}_{xy}$ *have the same non-zero latent roots* $\lambda_1^2 \ge \lambda_2^2 \ge \cdots \ge \lambda_r^2 > 0$. *If their mutual orthogonal identity eigenvectors are* $\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \cdots, \boldsymbol{\alpha}_r$ *and* $\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \cdots, \boldsymbol{\beta}_r$ *respectively, then* $\boldsymbol{a}_i^* = \boldsymbol{\Sigma}_{xx}^{-1/2}\boldsymbol{\alpha}_i$, $\boldsymbol{b}_i^* = \boldsymbol{\Sigma}_{yy}^{-1/2}\boldsymbol{\beta}_i$ $(i = 1, 2, ..., r)$ *are the ith pair of canonical correlation variables,* λ_i *is the ith canonical correlation coefficient.*

By the theorem, canonical correlation variable and coefficients can be obtained by the follow steps:

Step1: computing the correlations of the two sets of variables as:

$$\boldsymbol{\Sigma} = \begin{bmatrix} r_{x_1x_1} & \cdots & r_{x_1x_p} & r_{x_1y_1} & \cdots & r_{x_1y_q} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ r_{x_px_1} & \cdots & r_{x_1x_p} & r_{x_py_1} & \cdots & r_{x_py_q} \\ r_{y_1x_1} & \cdots & r_{y_1x_p} & r_{y_1y_1} & \cdots & r_{y_1y_q} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ r_{y_qx_1} & \cdots & r_{y_qx_p} & r_{y_qy_1} & \cdots & r_{y_py_q} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Sigma}_{xx} & \boldsymbol{\Sigma}_{xy} \\ \boldsymbol{\Sigma}_{yx} & \boldsymbol{\Sigma}_{yy} \end{bmatrix} \qquad (3)$$

The right matrix in (3) is a partitioned matrix.

Setp2: Computing the canonical correlation coefficients r_i. Firstly, we compute two matrices $\boldsymbol{L}$ and $\boldsymbol{M}$, where $\boldsymbol{L} = \boldsymbol{\Sigma}_{xx}^{-1}\boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}_{yy}^{-1}\boldsymbol{\Sigma}_{yx}, \boldsymbol{M} = \boldsymbol{\Sigma}_{yy}^{-1}\boldsymbol{\Sigma}_{yx}\boldsymbol{\Sigma}_{xx}^{-1}\boldsymbol{\Sigma}_{xy}$, secondly, we compute the eigenvalue λ_i of matrix $\boldsymbol{L}$ and $\boldsymbol{M}$, we can obtain $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_i \ge \cdots \ge \lambda_p$, then $r_i = \sqrt{\lambda_i}$ can be obtained.

Step3: Computing the canonical variables ξ_i and η_i. First computing the eigenvectors of matrix $\boldsymbol{L}$ about λ_i, we can obtain the coefficient matrix $\boldsymbol{A}$. Second computing the eigenvectors of matrix $\boldsymbol{M}$ about λ_i, we can obtain the coefficient matrix $\boldsymbol{B}$. Thirdly we introduce standardization variables x_j' and y_j', then the v canonical variables ξ_i and η_i can be obtained.

Then canonical variables ξ_i and η_i can be obtained in turns, which is the basis of time delay estimation.

B. Time delay Estimation

After the canonical variables ξ_i and η_i having been obtained, the following step is to estimate the time delay $\Delta\tau$ of the two independent receiving signals, viz. the canonical variables ξ_i and η_i. To a same emitter, the receiving time is different as the as the different receiving position. In this case, we must consider the resemble property of the two signals must be considered during the time varying. Delaying $\Delta\tau$ to $s_1(n)$ and change it to $s_1(n-\tau)$, then the coefficient $R_{s_1s_2}(\tau)$ can be written as:

$$R_{s_1s_2}(\tau)=\sum_{n=-\infty}^{\infty}s_1(n)s_2(n-\tau) \tag{4}$$

When τ varying from $-\infty$ to $+\infty$, $R_{s_1s_2}(\tau)$ is a function of τ, call $R_{s_1s_2}(\tau)$ as the coefficient of $s_1(n)$ and $s_2(n-\tau)$, τ is the time delay of $s_2(n-\tau)$. When $|R_{s_1s_2}(\tau)|$ reach the maximal value at τ_0, then τ_0 is the time difference $\Delta\tau$ of the two signals.

C. Azimuth Computation

After $\Delta\tau$ has been obtained, the computation of azimuth is relatively simple. From Fig.1, the following equations can be obtained: $\Delta l = l\sin\theta$, $\Delta\tau c=\Delta l = l\sin\theta$, where $c=3\times10^8 m/s$ is the electric wave transmitting speed, the DOA can be obtained as:

$$\theta=\arccos\left((c\cdot\Delta\tau)/l\right) \tag{5}$$

From (5) we can see that the precision of DOA decided by l and the measure precision $\Delta\tau$, where $\Delta\tau=(l\sin\theta)/c$. Partial derivative of $\Delta\tau$ can be written as: $d(\Delta\tau)=\frac{\partial(\Delta\tau)}{\partial\theta}d\theta+\frac{\partial(\Delta\tau)}{\partial l}dl+\frac{\partial(\Delta\tau)}{\partial c}dc$, where l, c are all constant, so the result of partial derivative is $d(\Delta\tau)=\frac{l}{c}\cos\theta d\theta$, then:

$$d\theta=\left(c\cdot d(\Delta\tau)\right)/(l\cos\theta) \tag{6}$$

From (6) we can know that the precision of direction finding is limited by the base line length l and the measure precision $\Delta\tau$, it also relate to the measure angle. In order to improve the performance of direction finding, usually 4 groups of antenna are applied to cover the whole azimuth.

IV. SIMULATIONS

In order to verify the validity of this direction finding algorithm proposed in this paper, here a series of experiments have been conducted of direct correlation, GCC and CCA method. Background of the experiments assumed as: there are two groups of receiving antennas and each includes two antenna units. Distance between the antennas is $30m$, the relative direction of DOA to really north way is 30º. From the receiving model established in Fig.1 that the time delay is $50ns$, if the sampling interval is $1ns$, then the delay is 50 sampling intervals. Source signals include one FM noisy jamming and which can be generated using the corresponding steps from [8]. The other signal is a FM signal, described as: $s_{FM}(t)=\cos(2\pi f_0 t+\phi(t))$, where f_0 is the carrier frequency, $\phi(t)$ is a modulating component. Simulation steps as: first taking a direct correlation to the filtered receiving signals in the interference background (shown as in Fig.2). From the result as Fig.3, we can see that there are easy to have two extremums, so as to GCC method.

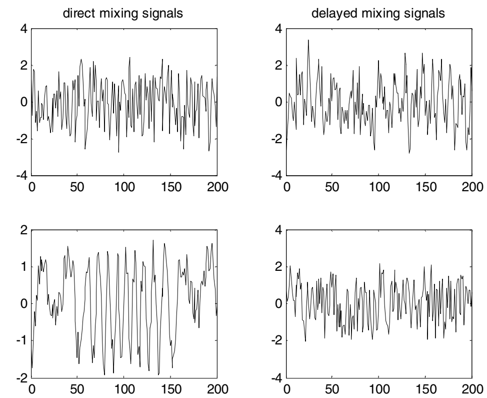

Figure 2. Mixing signals

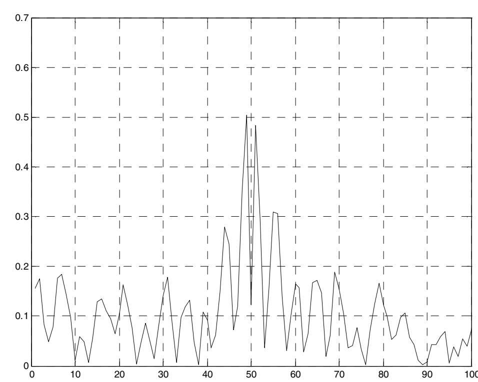

Figure 3. Direct correlation results

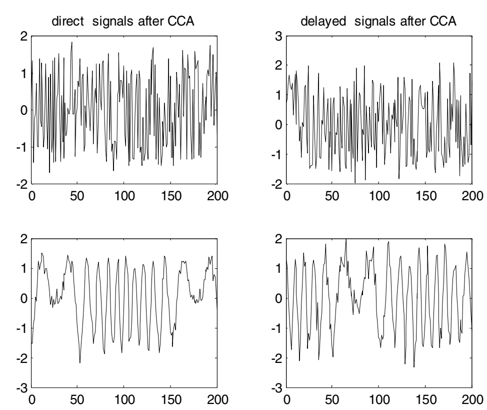

Figure 4. CCA results

Then conduct correlation processing to the canonical variable results (shown as Fig.4) by CCA. The correlation result as Fig.5, the time delay is just 50 sampling intervals, which is accuracy and stable.

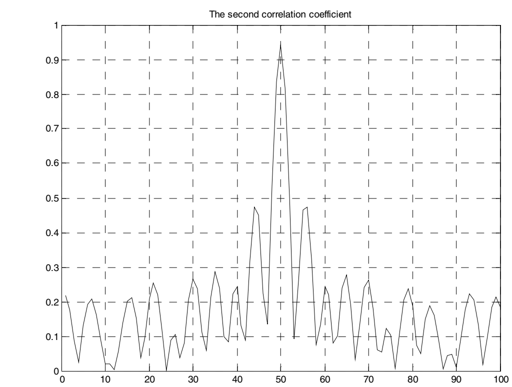

Figure 5. Canonical variables correlation results

A lot of experiments have been conducted by the changing the impact of additive noise. Results as in Fig.6 show that the estimation of time delay after CCA processing is more accuracy and stabilization than direct correlation and GCC. The anti-jamming property is also excellent.

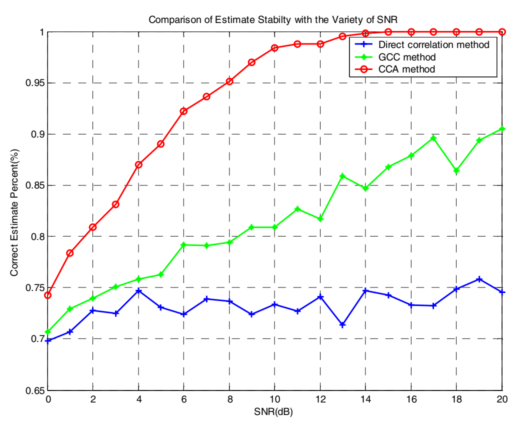

Figure 6. Performance comparison

V. Conclusion

This paper proposes the application of CCA method to time-delay direction finding, which can not only effectively overcome the sensitivity to interference of this direction finding method, but also avoid the localization of time-delay direction finding and improve the precision of the direction finding method, so as to enhance the location precision. The important contribution of this direction finding method proposed by this paper can improve the applicability of time-delay direction finding, which will play an important role in military and civilian affairs.

Acknowledgment

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Appendix A: Proof of theorem

Computing the variable is to choose $\boldsymbol{a},\boldsymbol{b}$ in the condition of $\boldsymbol{a}^T\boldsymbol{\Sigma}_{xx}\boldsymbol{a}=1,\boldsymbol{b}^T\boldsymbol{\Sigma}_{yy}\boldsymbol{b}=1$, then we will obtain the maximum value of $\boldsymbol{a}^T\boldsymbol{\Sigma}_{xy}\boldsymbol{b}$. Taking Lagrangean function as the assistant function as :

$$\Phi(\boldsymbol{a},\boldsymbol{b},s,t)=\boldsymbol{a}^T\boldsymbol{\Sigma}_{xy}\boldsymbol{b}+s\left(\boldsymbol{a}^T\boldsymbol{\Sigma}_{xx}\boldsymbol{a}-1\right)+t\left(\boldsymbol{b}^T\boldsymbol{\Sigma}_{yy}\boldsymbol{b}-1\right) \quad \text{(A1)}$$

In order to maximize Φ, Φ must content with following condition:

$$\begin{cases}\partial\Phi/\partial\boldsymbol{a}=\boldsymbol{\Sigma}_{xy}\boldsymbol{b}+2s\boldsymbol{\Sigma}_{xx}\boldsymbol{a}=0\\ \partial\Phi/\partial\boldsymbol{b}=\boldsymbol{\Sigma}_{yx}\boldsymbol{a}+2t\boldsymbol{\Sigma}_{yy}\boldsymbol{b}=0\end{cases} \quad \text{(A2)}$$

Then we can obtain:

$$2s\boldsymbol{a}=-\boldsymbol{\Sigma}_{xx}^{-1}\boldsymbol{\Sigma}_{xy}\boldsymbol{b} \quad \text{(A3)}$$

$$2t\boldsymbol{b}=-\boldsymbol{\Sigma}_{yy}^{-1}\boldsymbol{\Sigma}_{yx}\boldsymbol{a} \quad \text{(A4)}$$

Equation (A3) left multiplies $\boldsymbol{a}^T\boldsymbol{\Sigma}_{xx}$, then $2s=-\boldsymbol{a}^T\boldsymbol{\Sigma}_{xy}\boldsymbol{b}$. Equation (A4) left multiplies $\boldsymbol{b}^T\boldsymbol{\Sigma}_{yy}$, then $2t=-\boldsymbol{b}^T\boldsymbol{\Sigma}_{yx}\boldsymbol{a}$, so $s=t$. $\boldsymbol{a},\boldsymbol{b}$ can be written as:

$$\begin{cases}\boldsymbol{a}=\boldsymbol{\Sigma}_{xx}^{-1}\boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}_{yy}^{-1}\boldsymbol{\Sigma}_{yx}\boldsymbol{a}/4s^2\\ \boldsymbol{b}=\boldsymbol{\Sigma}_{yy}^{-1}\boldsymbol{\Sigma}_{yx}\boldsymbol{\Sigma}_{xx}^{-1}\boldsymbol{\Sigma}_{xy}\boldsymbol{b}/4s^2\end{cases} \quad \text{(A5)}$$

By (A5), we can know that $\boldsymbol{a},\boldsymbol{b}$ are the eigenvectors of $\boldsymbol{\Sigma}_{xx}^{-1}\boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}_{yy}^{-1}\boldsymbol{\Sigma}_{yx}$, $\boldsymbol{\Sigma}_{yy}^{-1}\boldsymbol{\Sigma}_{yx}\boldsymbol{\Sigma}_{xx}^{-1}\boldsymbol{\Sigma}_{xy}$ and they have the same eigenvalue.