MATLAB SIMULATION FOR COMPUTING PROBABILITY OF DETECTION

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Abstract **– This paper presents a MATLAB simulation that computes the probability of detection for a radar. It allows the user to input a wide range of radar characteristics. The target detection method has the capacity for handling fluctuating targets. This is important because actual targets are observed to scintillate and the probability of detection decreases as the signal-to-noise ratio decreases.**

This paper also considers characteristics of the operational mode of the radar system, such as its PRF (Pulse Repetition Frequency), number of pulses processed, and the necessary range resolving ratio. The simulation utilizes formulas for detection probability associated with a target's range and uses the Swerling cases to model the target's fluctuating cross section. The goal is to build a MATLAB simulation that can display radar observations in a format that facilitates analysis in range versus azimuth.

I. INTRODUCTION

This paper investigates methods for detecting fluctuating and non-fluctuating targets. Specifically, it looks at methods that utilize the Swerling I through V models. These models take into account the different surface types that targets may have and atmospheric attenuation and how a target's cross section varies as a function of time.

This simulation focuses on Medium PRF radar detection. MPRF Operation is ambiguous in both range and Doppler frequency. In typical X-band Medium-PRF Pulse Doppler Radar, the PRFs are between 10 and 30 kHz and the duty factor is about 0.05. The ability of MPRF radars to detect low speed targets as well as high speed targets, as compared to High PRF radars that cannot detect low speed targets very well, make it the preferred system for fighter radar applications, when only a single system is allowable [1]. It should be noted that this simulation can be used to investigate other PRF ranges.

The simulation also takes into account characteristics of the radar and the mode in which the radar is operating for example its average power, the size of its antenna main lobe and its PRF sequence. Thus, the user will not only be able to simulate the appearance and disappearance of a target as it gets closer to or farther away from the radar but will be able to determine the maximum detection range of many types of radar for a target of a selected cross-section, within certain mode specifications.

II. SWERLING MODELS

A. *Swerling I*

The Swerling I case models a target that consists of many independent scattering elements, such as a large complex target. In Swerling I, the RCS (radar cross section) samples measured from a target on any one scan have constant amplitude (i.e. they are correlated throughout an entire scan, but are not correlated from scan to scan). Fig.1 is a plot of results for Swerling case I. Swerling I is also known as the *Chi-Square of Degree 2* and it is sometime called *slow fluctuations*. "Expressed in statistical terms, the normalized autocorrelation function of target cross section is approximately one for the time in which the beam is on target during a single scan, and is approximately zero for a time as long as the interval between scans and this type of fluctuation will henceforth be referred to a scan-to-scan fluctuation" [2]. In other words, Chi-square distribution applies to a wide range of targets where the probability density function is:

$$f(\sigma) = (\exp(-np*\sigma/\sigma_{av}))*\alpha \tag{1}$$

where $\alpha = [np / (\Gamma(np)*\sigma_{av})]*[(np*\sigma)/(\sigma_{av})]^{np-1}$

where
$$\Gamma(np) = \int_0^\infty t^{(np-1)}*e^{(-t)}\,dt \tag{2}$$

and σ_{av} is the average value of RCS, np is the number of pulses, and t is the threshold. This probability density function mathematically imitates the many scattering elements of a target with similarly sized surface areas (Skolnik 66). Also the target model in Swerling I is sometimes called a *Rayleigh Scatterer*. The equation leading to the PDF for Swerling I is given by:

$$f(\sigma) = (1/\sigma_{av})*\exp(-\sigma/\sigma_{av}) \qquad \text{for } \sigma \geq 0 \tag{3}$$

and the probability of detection is given by:

for $n_p = 1$
$$P_D = e^{-V_T/(1+SNR)} \tag{4}$$

for $n_p \geq 2$
$$P_D = 1 - \Gamma_I(V_T, n_p - 1) + \left(1 + \frac{1}{n_p \cdot SNR}\right) + \theta \cdot e^{V_T/(1+n_p \cdot SNR)} \tag{5}$$

where
$$\theta = \Gamma\left(\frac{V_T}{1+\dfrac{1}{n_p \cdot SNR}}, n_p - 1\right)$$

2

Γ_I is the incomplete gamma function
V_T is the threshold
SNR is the Signal-to-Noise Ratio
np is the number of integrated pulses

B. *Swerling II*

The Swerling II case has the same probability density function as Swerling I, but the fluctuations are pulse to pulse related and not scan to scan related. Fig. 2 is a plot of Swerling case II. The probability of detection is defined as:

for $n_p < 50$

$$P_D = 1 - \Gamma_I\left(\frac{V_T}{1+SNR}, n_p\right) \tag{6}$$

for $n_p > 50$ use Gram-Charlier series (see equation 17) with its coefficients C_3, C_4, and C_6 and ω:

$$C_3 = -\frac{1}{3\sqrt{n_p}} \tag{7}$$

$$C_4 = \frac{1}{4n_p} \tag{8}$$

$$C_6 = \frac{C_3^2}{2} \tag{9}$$

$$\varpi = \sqrt{n_p}(1+SNR) \tag{10}$$

C. *Swerling III*

In the Swerling III case, the RCS is assumed to be constant and the fluctuations are scan to scan related. The Swerling III case models a target that is made up of one main scattering element, such as a simple shaped target. Fig. 3 is a plot of Swerling case III. Swerling III is also known as the *Chi-Square of Degree 4* and the probability density function is:

$$f(\sigma) = ((4*\sigma)/\sigma^2_{av})*\exp(-(2*\sigma)/\sigma_{av}) \quad \text{for } \sigma \geq 0 \tag{11}$$

The probability of detection is:

for $n_p = 1,2$

$$P_D = \exp\left(\frac{-V_T}{1+n_p SNR/2}\right)\left(1+\frac{2}{n_p SNR}\right)^{n_p-2} \times \gamma = K_0 \tag{12}$$

where $\gamma = 1 + \frac{V_T}{1+n_p SNR/2} - \frac{2}{n_p SNR}(n_p - 2)$

for $n_p > 2$

$$P_D = \delta + 1 - \Gamma_I(V_T, n_p - 1) + K_0 \times \Gamma_I\left(\frac{V_T^{n_p-1}e^{-V_T}}{1+2/n_p SNR}, n_p - 1\right) \tag{13}$$

where $\delta = \frac{V_T^{n_p-1}e^{-V_T}}{(1+n_p SNR/2)(n_p-2)!}$

D. *Swerling IV*

The Swerling IV case has the same probability density function as Swerling III, but the fluctuations are only pulse to pulse related. Fig.4 is a plot of Swerling case IV. The probability of detection for Swerling IV target is:

for $n_p < 50$

$$P_D = 1 - \varphi \times \left(1 + \frac{SNR}{2}\right)^{-n_p} \tag{14}$$

where $\varphi = \gamma_0 + \left(\frac{SNR}{2}\right) n_p \gamma_1 + \left(\frac{SNR}{2}\right)^2 \frac{n_p(n_p-1)}{2!}\gamma_2 + \ldots + \left(\frac{SNR}{2}\right)^{n_p}\gamma_{n_p}$

and $\gamma_i = \Gamma_I \frac{(V_T)}{1+(SNR)/2}, n_p + i$

by using the recursive formula

$$\gamma_i = \gamma_{i-1} - A_i \quad \text{for } i > 0 \tag{15}$$

$$A_i = \frac{V_T/(1+(SNR)/2)}{n_p + i - 1} \quad \text{for } i > 1$$

$$A_1 = \frac{(V_T/(1+(SNR)/2))^{n_p}}{n_p!\exp(V_T/(1+(SNR)/2))}$$

$$\gamma_0 = \Gamma_I\left(\frac{V_T}{(1+(SNR)/2)}, n_p\right)$$

for $n_p > 50$

The Gram-Charlier series is used as follows (see equation 17)

$$C_3 = \frac{1}{3\sqrt{n_p}}\frac{2B^3}{(2B^2-1)^{1.5}}$$

$$C_4 = \frac{1}{4n_p}\frac{2B^4 - 1}{(2B^2 - 1)^2}$$

$$C_6 = \frac{C_3^2}{2}$$

$$\varpi = \sqrt{n_p(2 \bullet B^{2-1})}$$

$$B = 1 + \frac{SNR}{2}$$

E. *Swerling V*

The Swerling V case models a target whose RCS remains constant and has mainly one large surface. Fig. 5 is a plot of Swerling case V. The equation is defined by Marcum probability of false alarm for the case when np > 1 as

$$P_{fa} = 1 - (P_{50})^{np/nfa} \approx \ln(2)(n_p/n_{fa}) \quad (16)$$

where $P_{50} = 0.5$

The Gram-Charlier series is used to compute the probability of detection and this equation is shown below:

$$P_D \cong \frac{erfc(V/\sqrt{2})}{2} - \frac{e^{-V^2/2}}{\sqrt{2\pi}}[C_3(V^2-1) + C_4V(3-V^2) - C_6V(V^4 - 10V^2 + 15)] \quad (17)$$

Where V is

$$V = \frac{V_T - n_p(1 + SNR)}{\varpi}$$

And C3, C4, and C6 are the coefficients from the Gram-Charlier series

$$C_3 = -\frac{SNR + 1/3}{\sqrt{n_p}(2 \cdot SNR + 1)^{1.5}}$$

$$C_4 = \frac{SNR + 1/4}{n_p(2 \cdot SNR + 1)^2}$$

$$C_6 = C_3^2/2$$

$$\varpi = \sqrt{n_p(2 \cdot SNR + 1)}$$

II. Range and SNR Relations

Other than the Swerling model equations, the simulation utilizes three main equations in order to compute a given target's probability of detection. The first equation, the Radar Range Equation, allows one to find the radar detection range for a given probability of detection:

$$R_{Pd} = \left[\frac{P_{avg}(Ae)^2\sigma \cdot tot}{(4\pi)^2 SNRreq \cdot kTs \cdot \lambda^2}\right]^{1/4} \quad (18)$$

P_{avg} = average power of the radar
Ae = effective array size
σ = target cross section
tot = the time on target
SNRreq = the required signal to noise ratio for a given probability of detection
k = Boltzmann's constant
Ts = the absolute temperature of the system with internal and external noise
λ = wavelength.

With the exception of the constants, the user of the simulation inputs the characteristics of both the radar and the target. Generally this equations allows one to find the range that correlates to a given probability of detection. Using SNRreq equal to one gives the reference range, Ro, of the radar (and considering losses in the receiver and the rest of the system):

$$R_0 = \left[\frac{P_{avg}(Ae)^2\sigma \cdot tot}{(4\pi)^2 \cdot kTo \cdot F_n \cdot \lambda^2 \cdot L}\right]^{1/4} \quad (19)$$

To = Normal noise temperature (290 degree Kelvin, used to compute kTs)
F_n = Receiver noise Figure
L= Total losses (not used in general case)
(note: $kTo = 4x10^{-21}$)

Then the equation:

$$SNR = 40 \bullet \log10(Ro/R) \quad (20)$$

is used to find the signal to noise ratio at a given range, R. The simulation proceeds to find the probability of detection for the given range using the selected Swerling model equation. Once the probability of detecting a target at a given range is computed it is used to display the target in a simulated radar display. Because the simulation will have adjustable inputs, the user will be able to experiment with different types of radars. Early simulations show the normal correlations one would expect: the probability of detection increases when the size and power of the radar are increased and decreases if the time on target or target's cross section are decreased, etc.

III. Probability Of Detection In Terms Of Range

From the supplied inputs, the simulation will output the target on the radar display based on its probability of detection. The user inputs the radar's average power, radar's antenna size, target's cross section, time on target, SNR requirement, wavelength, total noise loss in power ratio terms, receiver noise in power ratio terms, maximum range of interest, and the number of pulses within each PRF segment. A curve showing Pd versus range is produced. This curve is called the Pd Range Curve, which is the probability of detecting a given target at a given range on any one scan across the target. The higher the probability specified, the shorter the range will be [3]. The following are
Pd Range Curves for the different Swerling cases where each curve different represents the number of pulses processed:

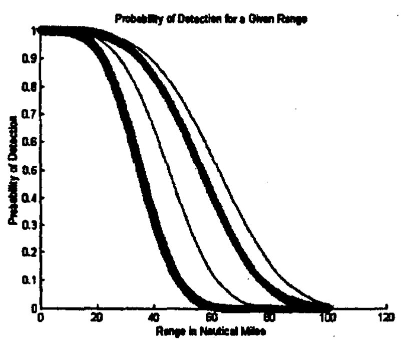

Fig. 1. Swerling 1 Case with np = 4, 16, 64, 128 from left to right. (note: assume Pfa = 10^{-9}).

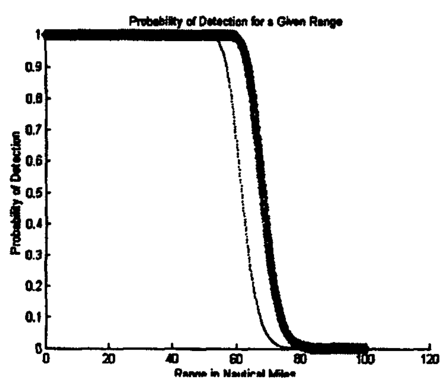

Fig. 2. Swerling 2 Case with np = 64, 128 left and right respectively. (note: assume Pfa = 10^{-9}).

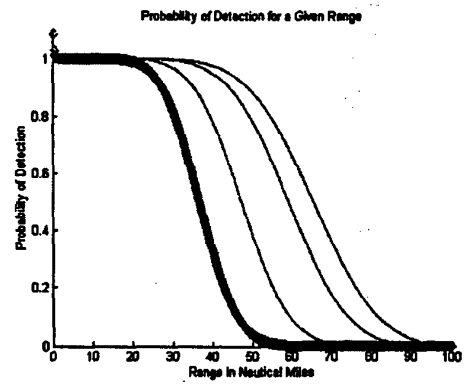

Fig. 3. Swerling 3 Case with np = 4, 16, 64, 128 from left to right. (note: assume Pfa = 10^{-9}).

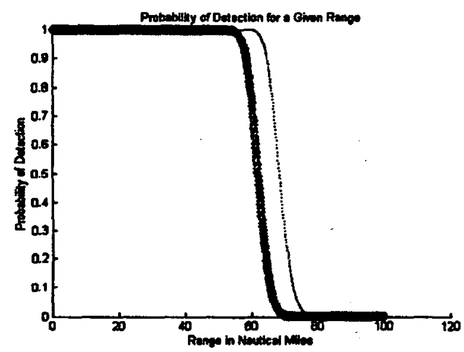

Fig. 4. Swerling 4 Case with np = 64, 128 from thick left to right. (note: this figure illustrates the conditional quality of Swerling 5 centered on np = 50 and assume Pfa = 10^{-9}).

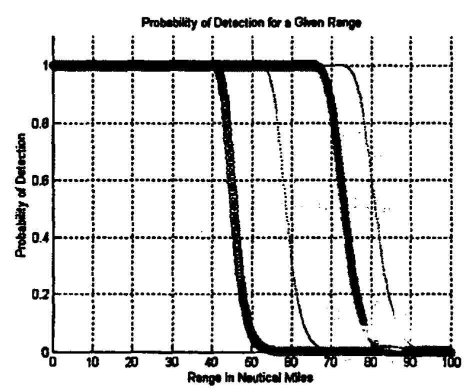

Fig. 5. Swerling 5 Case with np = 4, 16, 64, 128 from left to right. (note: assume Pfa = 10^{-9}).

IV. M OUT OF N RANGE RESOLVING RATIO

In many modes of radar, more than one PRF is used in a scan. Using PRF sequences helps for both de-ghosting and range resolution. This simulation outputs the combined probability of detection for a certain M/N ratio and displays the target accordingly. The M/N range resolving ratio is the number of PRFs with threshold crossings, M, needed out of the total number of PRFs in the sequence, N to confidently detect the target. The probability of detection from each PRF used in the M/N computation is accessed from Pd tables saved in MATLAB files are shown in Fig. 6.

table12000.mat

Fig. 6. Pd tables are saved in the MATLAB files.

The tables are created using the above equations. Each Pd table is accessed via PRF, range, and number of pulses, np. The method used to compute the combined probability associated with a given M/N ratio, (i.e. P(m,n), the probability of getting <u>at least</u> M hits out of N PRFs) is given in the equation below, where Pn is the probability of getting a hit for a specific PRF nominally numbered n.

$$P(m,n) = Pn*P(m-1,n-1)+(1-Pn)*P(m,n-1) \quad (34)$$

Proof for M/N = 3/8:

$\nexists$ m>n $\therefore$ if m>n P(m,n) = 0
P(0,n) = 1

$P(3,8) = P_8*P(2,7)+(1-P_8)*P(3,7)$
$P(3,7) = P_7*P(2,6)+(1-P_7)*P(3,6)$
$P(3,6) = P_6*P(2,5)+(1-P_6)*P(3,5)$
$P(3,5) = P_5*P(2,4)+(1-P_5)*P(3,4)$
$P(3,4) = P_4*P(2,3)+(1-P_4)*P(3,3)$

$P(2,7) = P_7*P(1,6)+(1-P_7)*P(2,6)$
$P(2,6) = P_6*P(1,5)+(1-P_6)*P(2,5)$
$P(2,5) = P_5*P(1,4)+(1-P_5)*P(2,4)$
$P(2,4) = P_4*P(1,3)+(1-P_4)*P(2,3)$
$P(2,3) = P_3*P(1,2)+(1-P_4)*P(2,2)$

$P(1,6) = P_6*P(0,5)+(1-P_6)*P(1,5)$
$P(1,5) = P_5*P(0,4)+(1-P_5)*P(1,4)$
$P(1,4) = P_4*P(0,3)+(1-P_4)*P(1,2)$
$P(1,3) = P_3*P(0,2)+(1-P_3)*P(1,1)$
$P(1,2) = P_2*P(0,1)+(1-P_2)*P(1,0)$
$P(1,n) = Pd_{PRFn}$

Computational example with M/N = 2/4:

Given: $P_1 = 0.8$, $P_2 = 0.7$, $P_3 = 0.6$, $P_4 = 0.5$

[1] $P(2,4) = P_4*P(1,3)+(1-P_4)*P(2,3)$
$= 0.5*P(1,3) + 0.5*P(2,3)$
[2] $P(2,3) = P_3*P(1,2)+(1-P_3)*P(2,2)$
$= 0.6*P(1,2)+0.4*P(2,2)$
[3] $P(2,2) = P_2*P(1,1)+(1-P_2)*P(2,1)$
$= 0.7*P(1,1)+(0.3)*(0)$
$= (0.7)*(0.8) = 0.56$

[2] Continue
$P(1,2) = P_2*P(0,1)+(1-P_2)*P(1,1)$
$= (0.7)*(1)+(0.3)(0.8) = 0.94$
$P(2,3) = P_3*P(1,2)+(1-P_3)*P(2,2)$
$= (0.6)(1)+(0.4)(0.941) = 0.7880$

[1] Continue
$P(1,3) = P_3*P(0,2)+(1-P_3)*P(1,2)$
$= (0.6)(1)+(0.4)(0.941) = 0.976$
$P(2,4) = P_4*P(1,3)+(1-P_4)*P(2,3)$
$=(0.5)(0.976)+(0.5)(0.7880) = 0.882$ (final answer)

V. DISPLAY

The simulation interface allows the user to place a target on the screen by clicking on the desired area with a mouse. Then the probabilities is automatically computed from the range (the arch indicates the maximum range of the simulation, see fig. 7). To simulate the random nature of detection, the simulation generates a random number between 0 and 1 and if the cumulative probability of detection is less than the random number the target is displayed. The simulation cycles through this process causing the target to flicker according to its probability of detection. It is also displayed with the target's elevation and the number of pulses used in each PRF of the sequence. The radar display is shown in Fig. 7. Table I is the probability of detection table at a given PRF sequence of 10 KHz and the results are simulated in the MATLAB program. The values in Table II are the probabilities of detection at a given PRF sequence of 20 KHz. The output of the program is shown below.

Please give the PRFs of the PRF series in their used order?
[10000]
pdA =

TABLE I
Probability of Detection Table at 10 KHz

Range	Number Of Pulses				
(nmi)	2	4	8	16	32
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.9997	0.9999	0.9999	1.0000	1.0000
3	0.9987	0.9993	0.9997	0.9998	0.9999

4	0.9958	0.9979	0.9990	0.9995	0.9997
5	0.9899	0.9949	0.9975	0.9987	0.9994
6	0.9791	0.9895	0.9947	0.9974	0.9987
7	0.9617	0.9806	0.9903	0.9951	0.9976
8	0.9356	0.9672	0.9835	0.9917	0.9958
9	0.8989	0.9480	0.9736	0.9867	0.9933
10	0.8503	0.9219	0.9601	0.9798	0.9899

Please give the PRFs of the PRF series in their used order > [20000]

pdB =

TABLE II
Probability of Detection Table at 20 KHz

Range (nmi)	Number Of Pulses				
	2	4	8	16	32
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.9995	0.9997	0.9999	0.9999	1.0000
3	0.9974	0.9987	0.9993	0.9997	0.9998
4	0.9917	0.9958	0.9979	0.9990	0.9995
5	0.9798	0.9899	0.9949	0.9975	0.9987
6	0.9587	0.9791	0.9895	0.9947	0.9974
7	0.9249	0.9617	0.9806	0.9903	0.9951
8	0.8755	0.9356	0.9672	0.9835	0.9917
9	0.8085	0.8989	0.9480	0.9736	0.9867
10	0.7240	0.8503	0.9219	0.9601	0.9798

Please present the prf sequence> [10000 12000 14000 16000 18000 20000 22000 24000]
Please enter the number of pulses in the prf sections> 32
Please enter the m of the m/n ratio> 3
please enter the targets relative elevation in radians> .2

Output from the MATLAB simulation:
range = 25.6546
degree = -0.2264
pd = 0.8201

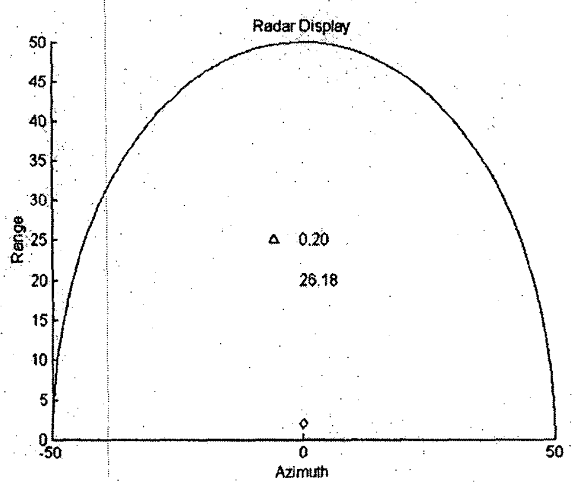

Fig. 7. Radar display.

VI. Conclusion

The simulation illustrates what one would expect to find in actual radar system. The simulation shows that as certain parameters of the radar are increased, such as the array size, the average power, the number of pulses processed, and the target's cross section, the probability of detection increases. Conversely as the parameters decrease the probability of detection also decreases. The simulation not only allows the user to experiment with different radar system configurations but allows comparison of different PRF sequences to maximize the capabilities of a given radar. Furthermore, the user can compare each of the Swerling models to see which best matches the radar system(s) of interest.

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