Twenty Second National Radio Science Conference (NRSC 2005)
March 15-17, 2005 Cairo - Egypt

Direction-Of-Arrival Methods (DOA) and Time Difference of Arrival (TDOA) Position Location Technique

HASSAN ELKAMCHOUCHI — IEEE SENIOR MEMBER

MOHAMED ABD ELSALAM MOFEED — IEEE MEMBER

Abstract

This paper provides a detailed overview of the various methods for estimation of Direction-Of-Arrival [1]-[3] (also called Angle-Of-Arrival) of a radio signal using an antenna array. The array-based Directional-Of-Arrival (DOA) estimation techniques considered here are broadly divided into two different types: conventional techniques, and subspace based techniques. Conventional methods are based on classical beamforming techniques and require a large number of elements to achieve high resolution. Subspace based methods are high resolution sub-optimal techniques which exploit the eigen structure of the input data matrix.

This paper also uses the Time-Difference-Of-Arrival (TDOA) technique [4]-[6] for estimating the time difference between the received signals to base stations of a certain subscriber using the Generalized Cross Correlation method. The Fang's method and Chan's method are used in solving the resulting hyperbolic equations for finding the Position Location (PL) of a certain subscriber. The Fang's method and Chan's method give accurate and same result. This means that, an accurate position location estimation of a certain subscriber requires an efficient hyperbolic position location estimation method. The paper also clarifies that a more data samples give a more accurate result.

1. Direction of Arrival Methods

1.1 Conventional Methods for DOA Estimation

Conventional methods for Direction-Of-Arrival estimation [1]-[3] are based on the concepts of beamforming and null-steering and do not exploit the nature of the model of the received signal vector $\mathbf{u}(k)$ or the statistical model of the signals and noise. The conventional methods discussed here are the delay-and-sum method (classical beamformer) and Capon's minimum variance method.

1.1.1 Delay-and-Sum Method

Figure 1 shows the classical narrowband beamformer structure, where the output signal $y(k)$ is

$$y(k)=\mathbf{w}^{H}\mathbf{u}(k) \tag{1}$$

The output power at the classical beamformer as a function of the Angle-Of-Arrival is given by

$$P_{cbf}=\mathbf{w}^{H}\mathbf{R}_{uu}\mathbf{w}=\mathbf{a}^{H}(\phi)\mathbf{R}_{uu}\mathbf{a}(\phi) \tag{2}$$

1.1.2 Capon's Minimum Variance Method

Capon's minimum variance technique attempts to overcome the poor resolution problems associated with the delay-and-sum method. This technique minimizes the contribution of the undesired interferences by minimizing the output power while maintaining the gain along the look direction to be constant, usually unity. That is,

$$\min_{\mathbf{w}} E\left[|y(k)|^{2}\right]=\min_{\mathbf{w}}\mathbf{w}^{H}\mathbf{R}_{uu}\mathbf{w} \qquad \text{subject to } \mathbf{w}^{H}\mathbf{a}(\phi_0)=1 \tag{3}$$

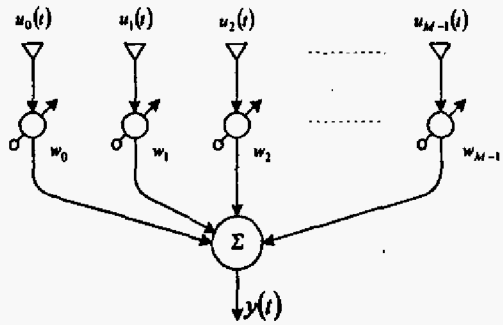


Fig. 1 Illustration of the classical beamforming structure

Now the output power of the array as a function of the Angle-Of-Arrival, using Capon's beamforming method, is given by Capon's spatial spectrum.

$$P_{Capon}(\phi) = \frac{1}{\mathbf{a}^H(\phi)\mathbf{R}_{uu}^{-1}\mathbf{a}(\phi)} \tag{4}$$

1.2 Subspace Methods for DOA Estimation

The Capon's minimum variance method are often successful and are widely used, this method has some fundamental limitations in resolution. Schmidt [1]-[3] derived a complete geometric solution to the DOA estimation problem in the absence of noise and in the presence of noise. The technique proposed by Schmidt is called the MUltiple SIgnal Classification (MUSIC) algorithm.

1.2.1 MUSIC algorithm

The MUSIC algorithm proposed by Schmidt in 1979 is a high resolution multiple signal classification technique based on exploiting the eigenstructure of the input covariance matrix. If there are D signals incident on the array, the received input data vector at an M element array can be expressed as a linear combination of the D incident waveforms and noise. That is,

$$\mathbf{u}(t) = \sum_{l=0}^{D-1}\mathbf{a}(\phi_l)s_l(t) + \mathbf{n}(t) \tag{5}$$

where $\mathbf{s}^T(t) = [s_0(t)\ s_1(t)\ \ldots\ s_{D-1}(t)]$ is the vector of incident signals, $\mathbf{a}(\phi) = [\mathbf{a}(\phi_0)\ \mathbf{a}(\phi_1)\ \ldots\ \mathbf{a}(\phi_{D-1})]$ is the array steering vector corresponding to the Direction-Of-Arrival of the l^{th} signal, and $\mathbf{n}^T(t) = [n_0(t)\ n_1(t)\ \ldots\ n_{D-1}(t)]$ is the noise vector. Then the DOAs of the multiple incident signals can be estimated by locating the peaks of a MUSIC spatial spectrum given by

$$P_{MUSIC}(\phi) = \frac{1}{\mathbf{a}^H(\phi)\mathbf{V}_n\mathbf{V}_n^H\mathbf{a}(\phi)} \tag{6}$$

where $\mathbf{a}(\phi)$ is the steering vector, and $\mathbf{V}_n$ is the matrix of noise eigenvectors. Orthogonality between $\mathbf{a}(\phi)$ and $\mathbf{V}_n$ will minimize the denominator and hence will give rise to peaks in the MUSIC spectrum defined in equation (6).

1.2.2 Spatial Smoothing Techniques

This method improves the MUSIC algorithm and make it work in the presence of coherent signals. Let a linear uniform array with M identical sensors be divided into overlapping forward subarrays of size p, such that the sensor elements $\{0, \ldots, p-1\}$ form the first forward subarray and sensors $\{1, \ldots, p\}$ form the second forward subarray, etc. Let $\mathbf{u}_k(t)$ denote the vector of the received signals at the k^{th} forward subarray. Based on the notation of equation (5) we can model the signals received at each subarray as

$$\mathbf{u}_k^f(t) = \mathbf{A}\mathbf{F}^{(k-1)}s(t) + \mathbf{n}_k(t) \tag{7}$$

where $\mathbf{F}^{(k)}$ denotes the k^{th} power of the diagonal matrix

$$\mathbf{F} = \mathrm{diag}\left\{ e^{-j\beta\cos\phi_0} \quad . \quad . \quad . \quad e^{-j\beta\cos\phi_{D-1}} \right\} \tag{8}$$

Pillar and Kwon proved that by making use of a set of forward and conjugate backward subarrays simultaneously, it is possible to detect up to $2M/3$ coherent signals. In this scheme, in addition to splitting the array into overlapping forward subarrays, it is also split into overlapping backward arrays such that the first backward subarray is formed using elements $\{M, M\text{-}1, \ldots, M\text{-}p\text{+}1\}$, the second subarray is formed using elements $\{M\text{-}1, M\text{-}2, \ldots, M\text{-}p\}$, and so on.
Similar to equation (7), the complex conjugate of the received signal vector at the k^{th} backward subarray can be expressed as

$$\begin{aligned} \mathbf{u}_k^b &= \left[u_{M-k+1}^* \quad u_{M-k}^* \quad . \quad . \quad . \quad u_{p-k+1}^* \right]^T \\ &= \mathbf{A}\mathbf{F}^{k-1}\left(\mathbf{F}^{M-1}s\right)^* + \mathbf{n}_k^* \quad 0 \le k \le L\text{-}1 \end{aligned} \tag{9}$$

where $\mathbf{F}$ is defined in equation (8). Now the forward/conjugate backward smoothed covariance matrix $\hat{\mathbf{R}}$ is defined as the mean of $\mathbf{R}^f$ and $\mathbf{R}^b$, i.e,

$$\hat{\mathbf{R}} = \frac{\mathbf{R}^f + \mathbf{R}^b}{2} \tag{10}$$

where

$$\mathbf{R}^f = \frac{1}{L}\sum_{k=0}^{L-1}\mathbf{R}_k^f, \quad \mathbf{R}^b = \frac{1}{L}\sum_{k=0}^{L-1}\mathbf{R}_k^b = \mathbf{A}\mathbf{R}_{ss}^b\mathbf{A}^H + \sigma_n^2\mathbf{I} \tag{11}$$

is the forward averaged spatially smoothed covariance matrix $\mathbf{R}^f$, and the backward spatially smoothed covariance matrix $\mathbf{R}^b$. Using an M element array, applying MUSIC on $\hat{\mathbf{R}}$, it is possible to detect up to $2M/3$ coherent signals.

2. TDOA Estimation Techniques

Hyperbolic position location estimation is accomplished in two stages. The first stage involves estimation of the TDOA between base station receivers through the use of time delay estimation techniques. The estimated TDOAs are then transformed into range-difference measurements between base stations, resulting in a set of nonlinear hyperbolic range-difference equations. The second stage utilizes efficient algorithms to produce an unambiguous solution to these nonlinear hyperbolic equations. The solution produced by these algorithms results in the estimated position location of the source.

2.1 General Model for TDOA Estimation

For signal $s(t)$ radiating from a remote source through a channel with interference and noise, a model for the time delay estimation between received signals at two base stations, $x_1(t)$ and $x_2(t)$, is given by

$$\begin{aligned} x_1(t) &= A_1 s(t - d_1) + n_1(t) \\ x_2(t) &= A_2 s(t - d_2) + n_2(t) \end{aligned} \tag{12}$$

where A_1 and A_2 scale the amplitude of each signal, $n_1(t)$ and $n_2(t)$ consist of noise and interfering signals, and d_1 and d_2 are the signal delay times, or arrival times. This model assumes that $s(t)$, $n_1(t)$ and $n_2(t)$ are real and jointly stationary, zero mean (time average) random processes and that $s(t)$ is uncorrelated with noise $n_1(t)$ and $n_2(t)$. referring the delay time and scaling amplitudes to the receiver with the shortest TOA, assuming $d_1 < d_2$, the model of equation (12) can be rewritten as

$$\begin{aligned} x_1(t) &= s(t) + n_1(t) \\ x_2(t) &= As(t - D) + n_2(t) \end{aligned} \tag{13}$$

where A is the amplitude ratio between the two versions of $s(t)$, and $D = d_2\text{-}d_1$. It is desired to estimate D, the TDOA of $s(t)$ between the two receivers. It may also be desirable to estimate the scaling amplitude A, to augment TDOA estimates with path loss considerations.

2.2 Generalized Cross Correlation Method

For accurate estimation of D that requires the use of estimation techniques that provide resistance to noise and interference and the ability to resolve multipath signal components we use generalized cross correlation (GCC) method. GCC method cross correlate pre-filtered versions of the received signals at two receiving stations, then estimate the TDOA between them as the location of the peak of the cross correlation estimate. Pre-filtering is intended to accentuate frequencies for which signal-to-noise (SNR) is highest and to attenuate the noise before the signal is passed to the correlator.

When $x_1(t)$ and $x_2(t)$ are filtered, the cross power spectrum between the filtered outputs is given by

$$G_{y_2y_1}(f)=H_1(f)H_2^*(f)G_{x_2x_1}(f) \tag{14}$$

where * denotes the complex conjugate. Therefore, the generalized cross correlation, specified by superscript G, between $x_1(t)$ and $x_2(t)$ is

$$R^G_{y_2y_1}(\tau)=\int_{-\infty}^{\infty}\Psi_G(f)G_{x_2x_1}(f)e^{j\pi f\tau}df \tag{15}$$

where

$$\Psi_G(f)=H_1(f)H_2^*(f) \tag{16}$$

and denotes the general frequency weighting, or filter function. Because only an estimate of $R^G_{y_2y_1}(\tau)$ can be obtained, equation (15) is rewritten as

$$\hat{R}^G_{y_2y_1}(\tau)=\int\Psi_G(f)\hat{G}_{x_2x_1}(f)e^{j\pi f\tau}df \tag{17}$$

which is used to estimate D. The GCC methods use filter functions $\Psi_G(f)$ to minimize the effect of noise and interference.

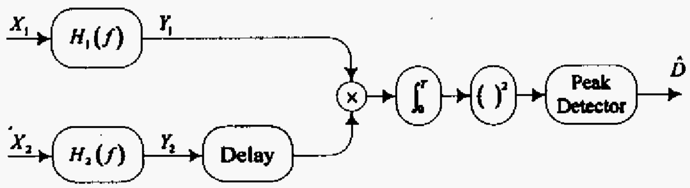


Figure 2 Generalized Cross Correlation Method for TDOA Estimation

2.3 Methods for Hyperbolic Position Location Estimation Technologies

Accurate position location estimation of a source requires an efficient hyperbolic position location estimation algorithm. Fang's method and Chan's method have been proposed for position location estimation based on TDOA estimation.

2.3.1 Fang's Method

For a 2D hyperbolic PL system using three base stations to estimate the source location (x, y), the coordinates of the first base station are (X_1, Y_1), the coordinates of the second base station are (X_2, Y_2), and the coordinates of the third base station are (X_3, Y_3). The distance between the source and the i^{th} base station is

$$\begin{aligned}R_i(x,y)&=\sqrt{(X_i-x)^2+(Y_i-y)^2}\\&=\sqrt{X_i^2+Y_i^2-2X_ix-2Y_iy+x^2+y^2}\end{aligned} \tag{18}$$

Let

$$\begin{aligned}X_{i,1}&=X_i-X_1\\Y_{i,1}&=Y_i-Y_1\end{aligned} \tag{19}$$

The range difference between base stations with respect to the base station where the signal arrives first is

$$R_{i,1} = c\tau_{i,1} = R_i - R_1 = \sqrt{(X_i - x)^2 + (Y_i - y)^2} - \sqrt{(X_1 - x)^2 + (Y_1 - y)^2} \tag{20}$$

first transform the set of nonlinear equations in equation (20) into another set of equations. Rearranging the form of these equations to get

$$dx^2 + ex + f = 0 \tag{21}$$

where

$$d = (2X_{2,1} + 2Y_{2,1}g)^2 - 4R_{2,1}^2(1+g^2)$$
$$e = -2(2X_{2,1} + 2Y_{2,1}g)\left[(X_2^2 - X_1^2) + (Y_2^2 - Y_1^2) - R_{2,1}^2 - 2Y_{2,1}h\right] + 4R_{2,1}^2\left[2X_1 + 2g(Y_1 - h)\right]$$
$$f = \left[(X_2^2 - X_1^2) + (Y_2^2 - Y_1^2) - R_{2,1}^2 - 2Y_{2,1}h\right]^2 - 4R_{2,1}^2\left[X_1^2 + (Y_1 - h)^2\right]$$

Solving the quadratic equation (21), we get two values for x. It has been found that one of the roots of equation (21) which is $\left(-e + \sqrt{e^2 - 4df}\right)/2d$ always results in x values which give mobile's position estimation that is well beyond the cell coverage area.

2.3.2 Chan's Method

A non iterative solution to the hyperbolic position estimation problem which is capable of achieving optimum performance for arbitrarily placed sensors was proposed by Chan.

For a three base stations system, producing two TDOA's, x and y can be solved in terms of R_1. The solution is in the form of

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\begin{bmatrix} X_{2,1} & Y_{2,1} \\ X_{3,1} & Y_{3,1} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} R_{2,1} \\ R_{3,1} \end{bmatrix} R_1 + \frac{1}{2} \begin{bmatrix} R_{2,1}^2 - K_2 + K_1 \\ R_{3,1}^2 - K_3 + K_1 \end{bmatrix} \right\} \tag{22}$$

where

$$K_1 = X_1^2 + Y_1^2$$
$$K_2 = X_2^2 + Y_2^2$$
$$K_3 = X_3^2 + Y_3^2$$

when equation (22) is substituted in equation (18) with $i = 1$, a quadratic equation in terms of R_1 is

$$aR_1^2 + bR_1 + c = 0 \tag{23}$$

only the following root should be considered for cellular PL

$$R_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{24}$$

2.4 Measures of Position Location Accuracy

A set of benchmarks is required to evaluate the accuracy of the hyperbolic position location technique. A commonly used measure of PL accuracy is the Root Mean Square (RMS).

The Root Mean Square (RMS) position location error is as follows

$$RMS = \sqrt{\varepsilon} = \sqrt{E\left[(x - x_v)^2 + (y - y_v)^2\right]} \tag{25}$$

3. Simulation Results

Figure 3 compares the resolution performance of the Delay-and-Sum, Capon's Minimum Variance, MUSIC, and MUSIC with Spatial Smoothing methods. Using six signals of equal power at an SNR of 10 dB arrive at a 10-element uniformly spaced array with an interelement spacing equal to 0.5λ, and the Direction-Of-Arrivals are 40°, 60°, 80°, 100°, 120°, 140°. From the figure we can conclude that the four methods succeed to differentiate between the six signals.

Figure 4 compares the resolution performance of Delay-and-Sum, and Capon's Minimum Variance methods. Using eight signals of equal power at an SNR of 10 dB arrive at a 10-element uniformly spaced array with an interelement spacing equal to 0.5λ, and the Direction-Of-Arrivals are 50°, 60°, 70°, 80°, 100°, 110°, 120°, 130°. From the figure we can conclude that Capon's method is able to distinguish between the eight signals, while the Delay-and-Sum method fails to differentiate between the eight signals.

Figure 5 compares the resolution performance of Capon's Minimum Variance and MUSIC methods. Using eight signals of equal power at an SNR of 20 dB arrive at a 10-element uniformly spaced array with an interelement spacing equal to 0.5λ, and the Direction-Of-
Arrivals are 71°, 74°, 77°, 80°, 100°, 103°, 106°, 109°. From the figure we can conclude that MUSIC can detect closely spaced signals, while Capon's method fails to differentiate between them.

Figure 6 compares the resolution performance of MUSIC and MUSIC with Spatial Smoothing methods. Using four coherent signals of equal power at an SNR of 20 dB arrive at a 6-element uniformly spaced array with an interelement spacing equal to 0.5λ, and the Direction-Of-Arrivals are 55°, 75°, 105°, 125°. From the figure we can conclude that MUSIC with Spatial Smoothing can detect clearly the coherent signals, while MUSIC fails completely to detect them.

Using the simulation module in figure 7, which includes three base stations and a mobile source. The radius of each cell is equal to 5 km. The coordinates of BS 1 is (X_1,Y_1) = (0, 8660), BS 2 is (X_2,Y_2) = (0, 0), and BS 3 is (X_3, Y_3) = (7500, 4330). Figure 8 shows the Generalized Cross Correlation output using Roth Impulse Response for estimating the TDOA between the arrived signals at BS 2 relative to BS 1, and when investigating the data we find two closely peaks so we take the average between them to get the value of estimated τ at $-2.75 \cdot 10^{-5}$. Also, figure 9 shows the Generalized Cross Correlation output using Roth Impulse Response for estimating the TDOA between the arrived signals at BS 3 relative to BS 1, and when investigating the data we find two closely peaks so we take the average between them to get the value of estimated τ at $-1.85 \cdot 10^{-5}$. The total number of taken samples is 12400. After that, we get the position of the mobile source by using the three methods, Chan's method, Fang's method, and the Mathcad built in function method. All of these methods give the same result. The calculated coordinates from the previous methods is (x, y) = (2002, 11840), but the actual one is (2000, 12250); this means that the error is 0.1%, and 3.3% which is in acceptable range. Figure 10 gives the behavior between Root Mean Square error against the number of iterations. The error is calculated using different number of iterations from 1000 samples till 12000 samples.

4. Conclusion

Each of the given methods has a degree of resolution depending on the angle of separation between the signals, according to their figures, their ranks from lower to higher resolution is delay-and-sum, Capon's minimum variance, and MUSIC. Also, this paper clarifies that when the spatial smoothing technique applied on MUSIC algorithm, it improves its performance in coherent signals environment, and can detect $2M/3$ coherent signals, where M is the number of array elements.

Also, this paper clarifies that the Generalized Cross Correlation method is an effective method for Time-Difference-Of-Arrival estimation technique. The Roth Impulse Response is used in our model, which gives an estimated result nearly to the actual result. The Fang's method and Chan's method are used in solving the resulting hyperbolic equations for finding the Position Location (PL) of a certain subscriber. Fang's method and Chan's method give an accurate and efficient result. This means that, an accurate position location estimation of a certain subscriber requires an efficient hyperbolic position location estimation method. And when many samples are taken a more accurate result is accomplished.

5. References

[1] Z. Xu and M. Tsatsanis, "Blind adaptive algorithms for minimum variance CDMA receivers", IEEE Transactions on Communications, Vol. 49, pp. 180-194, January 2001

[2] J. Liberti and T. Rappaport, Smart Antennas for Wireless Communications, Prentice Hall PTR, New Jersey, 1999

[3] D. Linebarger, R. DeGroat, and E. Dowling, "Efficient Direction-Finding Methods Employing Forward/Backward Averaging", IEEE Trans. On Signal Processing, Vol. 42, pp. 2136-2145, August 1994.

[4] P. Aarabi and G. Shi, "Phase-Based Dual-Microphone Robust Speech Enhancement", IEEE Trans. on Systems, Man, and Cybernetics, Part B: Cybernetics, Vol. 34, No. 4, August 2004.

[5] C. Marro, Y. Mahieux, and K. Simmer, "Analysis of Noise Reduction and Dereverberation Techniques Based on Microphone Arrays with Post-filteering", IEEE Trans. on Speech Audio Processing, Vol. 6, pp. 240-259, May 1998.

[6] W. Gardner and C. Chen, "Signal-Selective Time-Difference-of-Arrival Estimation for Passive Location of Man-Made Signal Sources in Highly Corruptive Environments, Part I: Theory and Method", IEEE Trans. on Signal Processing, Vol. 40, No. 5, pp. 1168-1184, May 1992.

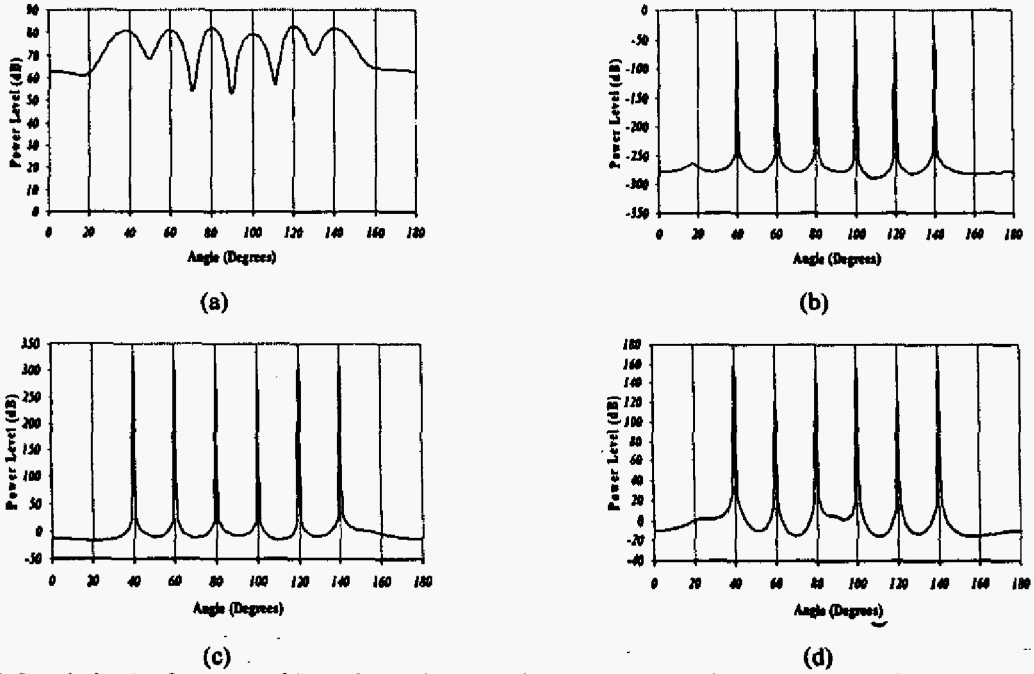


Fig 3. Resolution Performance of (a) Delay-and-sum method (b) Capon's Minimum Variance (c) MUSIC and (d) MUSIC with Spatial Smoothing (6 signals, SNR = 10 dB for all, 10-elements, interelement spacing = 0.5λ, DOAs = 40°, 60°, 80°, 100°, 120°, 140°)

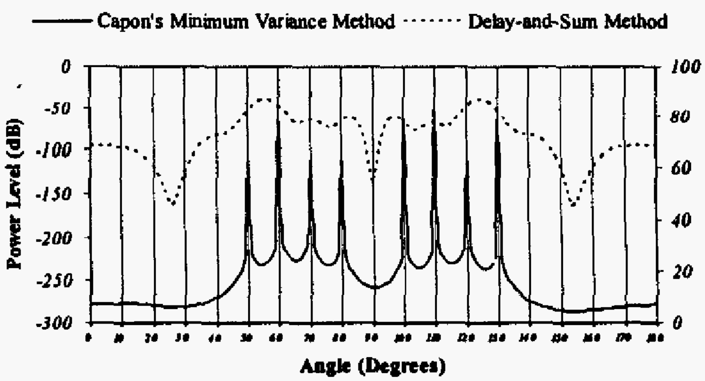


Fig 4. Comparison of resolution performance of Delay-and-Sum Method and Capon's Minimum Variance Method (8 signals, SNR = 10 dB for all, 10-elements, interelement spacing = 0.5λ, DOAs = 50°, 60°, 70°, 80°, 100°, 110°, 120°, 130°)

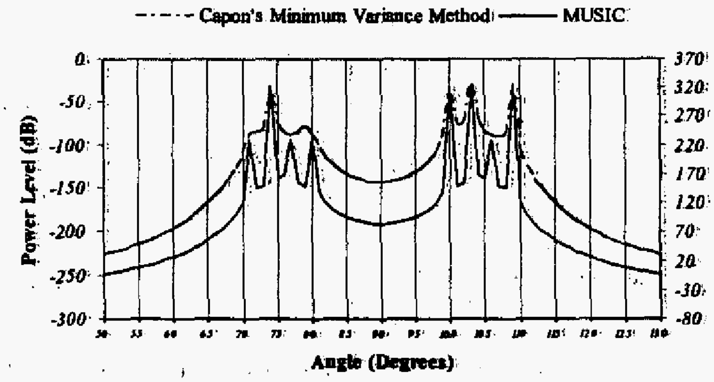


Fig 5. Comparison of resolution performance of MUSIC Method and Capon's Minimum Variance Method (8 signals, SNR = 20 dB for all, 10-elements, interelement spacing = 0.5λ, DOAs = 71°, 74°, 77°, 80°, 100°, 103°, 106°, 109°)

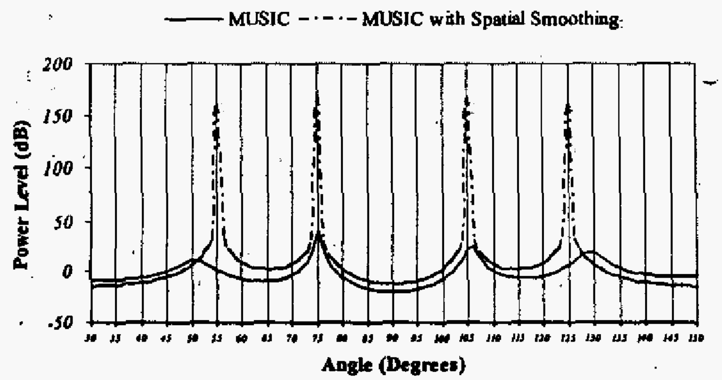


Fig 6. Comparison of resolution performance of MUSIC with Spatial Smoothing Method and MUSIC Method (4 coherent signals, SNR = 20 dB for all, 6-elements, interelement spacing = 0.5λ, DOAs = 55°, 75°, 105°, 125°)

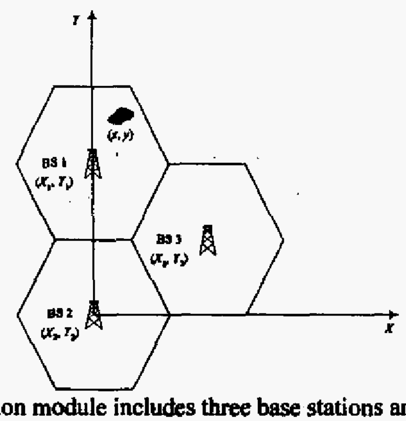
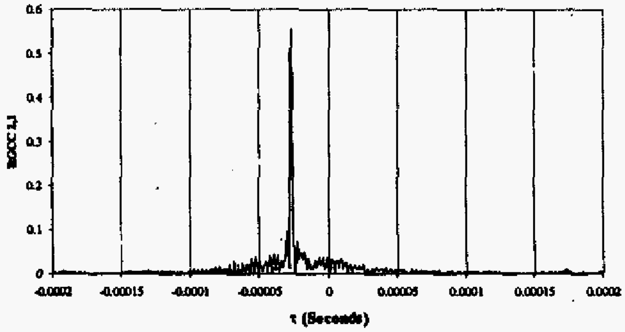
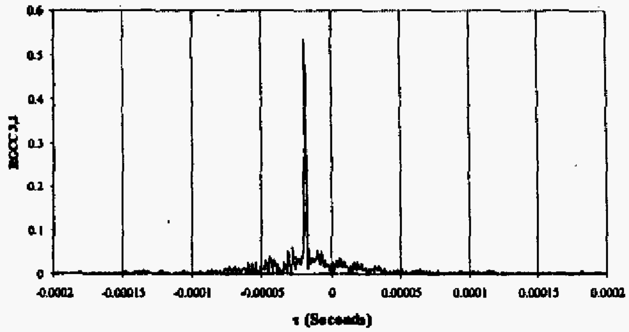


Fig 7 A simulation module includes three base stations and a mobile source



τ	$R_{GCC\,2,1}$
-2.90E-05	0.01705
-2.80E-05	0.3192
-2.70E-05	0.5517
-2.60E-05	0.129
-2.50E-05	0.009725

Fig 8.The Generalized Cross Correlation output using Roth Impulse Response in case of estimating the TDOA between the arrived signals at BS 2 relative to BS 1 (table attached showing the estimated τ at $-2.75 \cdot 10^{-5}$)



τ	$R_{GCC\,3,1}$
-2.10E-05	0.03143
-2.00E-05	0.07122
-1.90E-05	0.5275
-1.80E-05	0.3551
-1.70E-05	0.01596

Fig 9.The Generalized Cross Correlation output using Roth Impulse Response in case of estimating the TDOA between the arrived signals at BS 3 relative to BS 1 (table attached showing the estimated τ at $-1.85 \cdot 10^{-5}$)

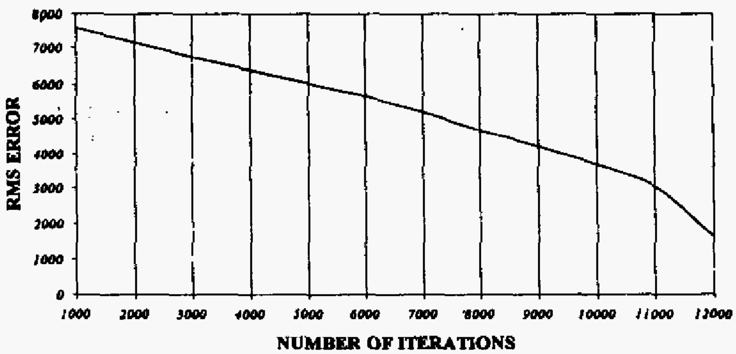


Fig 10 Root Mean Square Error against the Number of Iterations