Optimized Algorithm for Solving Phase Interferometer Ambiguity

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Abstract—**Direction finding (DF) system using phase interferometer of long baselines gives high accuracy of angle of arrival (AOA), however, it also involves phase ambiguity and phase error. The paper presents a measurement of phase interferometer system, which was configured with three and four antenna elements, providing information of phase error. Moreover, a method including rotation of measured phase differences, labeling the phase lines and rounding value of phase difference was applied for determination of unambiguous AOA. The algorithm of AOA calculation was fast and optimized. The method was verified by experimental measurement combining with signal processing in MATLAB environment. The RMS error of phase measurement was 5.4° and RMS error of AOA determination was around 0.09°.**

Index Terms—**Direction finding; phase interferometer; angle of arrival; three-element array; phase ambiguity; phase error**

I. INTRODUCTION

There are three main techniques used in passive direction finding (DF): time difference of arrival (TDOA) [1-3], amplitude comparison [4] and phase interferometer, in which phase interferometer is known to be a high accuracy and effective method. The wider antenna spacing, the higher accuracy of the interferometer system is, however increment of antenna spacing also leads to ambiguity of AOA calculation. Phase measurement in interferometer system moves only in range of $(-\pi, +\pi)$. Ambiguities begin to occur when the antenna spacing is longer than haft of signal wavelength. In 2004, Lim *et al*. introduced a direction finding device using combination of phase interferometer and amplitude comparison to determine AOA. The device was designed as a circular array using 8 antennas. This device had 0.7 degree RMS error in azimuth and 0.6 degree RMS error in elevation angle [5].

DF ambiguity was solved by the rotation interferometer [6], which has high successful rate, but not instantaneously. A good direction finding system, which operated in a wide band of frequencies, was performed by using hybrid multi-mode interferometer [7]. The hybrid interferometer needed only three elements and no short baseline to solve the phase ambiguity. However, it must use both phase and amplitude comparison for estimation of AOA.

A phase interferometer system with five elements was demonstrated in [8]. The system was configured by one small baseline ($<\lambda/2$) and three larger baselines ($>\lambda/2$). Kalman prediction was used to correct errors in solving phase ambiguity of multi baseline phase interferometer in other way [9]. The proposed algorithm in [9] took Kalman predictive values as criteria to verify, if resolved ambiguities were right or not and then corrected the error positions.

Ly *et al.* demonstrated an effective algorithm SODA (Second Order Difference Array) which obtained unambiguous AOA for a single narrowband signal using system configured with three antennas spaced as $d_{13} = 2\cdot d_{12}+d_{\Delta}$, where $d_{\Delta} \le \lambda_{min}/2$ [10, 11, 12]. Then, the unambiguous AOA was computed following d_{Δ} and virtual phase difference $\Delta\varphi_{\Delta} = \mathrm{mod}(\Delta\varphi_{13} - 2\cdot\Delta\varphi_{12}, 2\pi)$. This algorithm, however, leads to a decrease in accuracy of AOA. This disadvantage could be improved by using SODA-Based Inference (SBI), still it gets more sensitive to noise errors.

In this work, a method using rotation of phase differences, labeling the phase lines and rounding value of phase difference was performed in order to determine unambiguous AOA. The phase interferometer system was configured with three and four antenna elements and was verified by experimental measurement combining with signal processing in MATLAB environment, providing information of phase error.

II. FORMULATION OF LONG BASELINE PHASE INTERFEROMETER

Classical phase interferometer DF measures phase difference of two antennas for estimation of AOA. Angle θ created by incident signal with antenna array is calculated in relationship with phase difference $\Delta\phi$ and spacing d of two antennas:

$$\theta = \arcsin\left(\frac{\Delta\phi\cdot\lambda}{2\pi\cdot d}\right). \tag{1}$$

The equation (1) brings a problem of unlimited solution of θ because it is periodic. Unique AOA is obtained when antenna spacing is smaller than a half of wavelength of receiving signal. Phase measurement is unambiguous in case that $\Delta\varphi$ is in interval of $(0, 2\pi)$ or $(-\pi, +\pi)$ for direction of arrival signal in range of (-90°, +90°). The condition for unambiguous measurement is $d \le \lambda/2$. But in reality of microwave spectrum, this condition is hard to perform because it is so hard

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to produce two antennas which are spaced in short distance. The second reason is that space between two antennas is smaller than half of wavelength so that leads to big error. This error of AOA measurement at interferometer method depends on value of antenna spacing:

$$\sigma_\theta = \sigma_{\Delta\varphi}\left(\frac{\lambda}{2\pi d\cos\theta}\right). \qquad (2)$$

The longer baseline, the smaller error of AOA calculation. To employ benefit of long baseline, the problem of ambiguity must be solved by addition of the third antenna or more combining with suitable algorithm.

Interferometer system with three antennas is usually used for detecting AOA in field of passive surveillance application. This system is configured as description in Fig. 1.

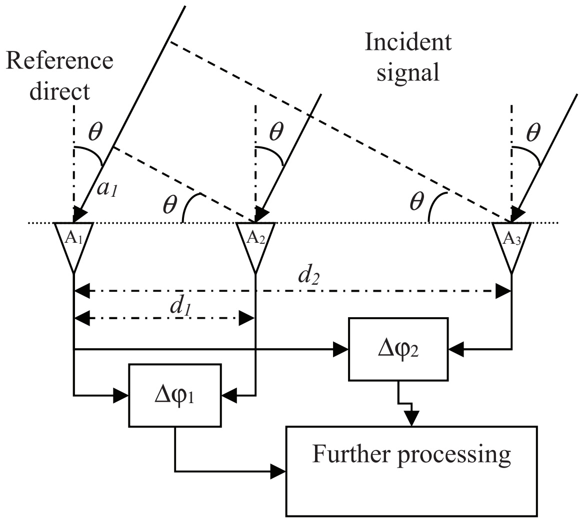

Figure 1. Configuration of phase interferometer system with 3 antenna elements.

Incident signals are received by three antennas and their phase differences are detected by phase comparators. Then the phase differences of each antenna pair are processed to determine AOA. In reality, phase differences are measured only in range of (- π, +π) or (0, 2π), but true phase differences are:

$$\Delta\phi_i = \Delta\varphi_i + 2\pi n_i = \frac{2\pi d_i}{\lambda}\sin(\theta)\ , \qquad (3)$$

where $\Delta\varphi_i$ is measured phase differences, $n_i = fix\{d_i\sin(\theta)/\lambda\}$, $i = 1, 2$, function $fix(x)$ returns integer part of number x.

Relationship of the phase differences is written as:

$$A = \frac{\Delta\phi_i}{\Delta\phi_j} = \frac{d_i}{d_j} = \frac{M}{N}\ , i, j = 1, 2,... \qquad (4)$$

Number A is named as spacing ratio. Fig. 2a shows relationship of phase differences. We need to choose suitable spacing ratio A for corresponding error of phase measurements. We can calculate distance of two nearby phase lines as following:

$$V = \frac{2\pi}{\sqrt{M^2 + N^2}}\ , \qquad (5)$$

Phase error accepted in measurement is smaller than $V/3$ (Fig. 2b).

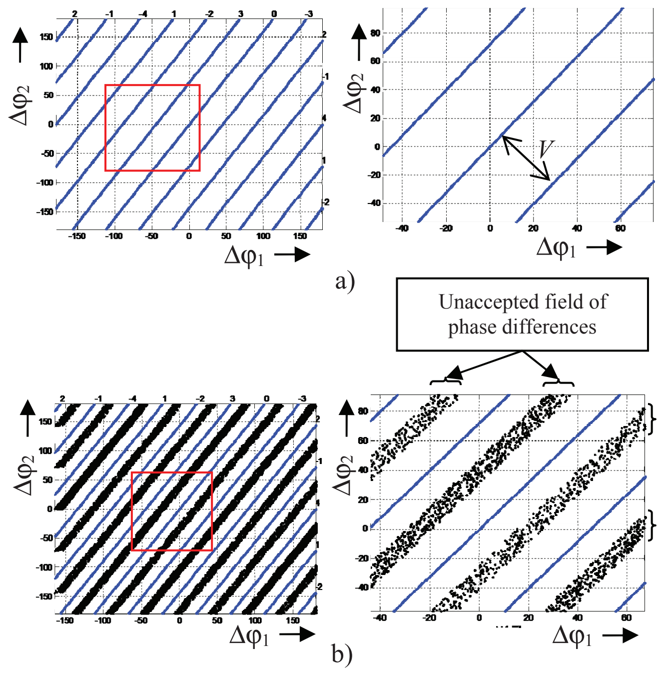

Figure 2. Relationship of phase differences corresponding with parameters $d_1 = 5\ \lambda/2$, $d_2 = 8\ \lambda/2$.

III. Phase Difference and AOA Computation

Unambiguous AOA is determined by interferometer system using two long baselines whose ratio must be rational number. If length of baseline is $d_i = n\ \lambda/2$ (where n = 1, 2, 3, …), with one value of phase difference, n values of AOA are calculated. Fig. 3 shows relationship of phase differences of baselines and AOA. With configuration of $d_1 = 8\ \lambda/2$ and $d_2 = 11\ \lambda/2$, it is clear that one value of phase difference between antenna 1 and 2 can calculate 8 values of AOA (Fig. 3a). Similarly, one value of phase difference between antenna 1 and 3 can calculate 11 values of AOA (Fig. 3b). Among 8 AOA values of baseline d_1 there is one value, which is involved in 11 AOA values of baseline d_2.

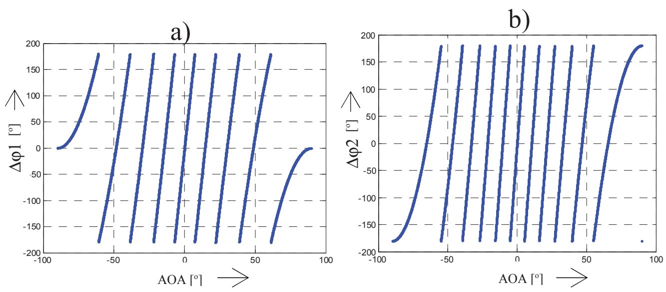

Figure 3. Relationship of phase differences and AOA

A. *Three-antenna system*

Unambiguous phase differences of two baselines are computed by using phase plane and its rotation and then unambiguous AOA is determined. Principle for obtaining unambiguous phase differences and AOA is described in steps below.

First, phase plane (relationship of phase differences between two baselines) is created by known parameters such as d_1, d_2. The lines of phase plane are labeled with corresponding numbers. Relationship between true phase differences and ambiguous phase differences (for baseline d_2) are written as $\Delta\phi_2 = \Delta\varphi_2 + 2\pi n_2$, where $n_2 = 0, \pm1, \pm2, \pm3, \pm4, \ldots$ Phase lines are labeled corresponding to values of n_2, for instance, if $n_2 = 0$, relationship of phase differences $\Delta\varphi_1$ and $\Delta\varphi_2$ is presented by line labeled "0" (Fig. 4a).

Second, phase plane and measured phase point are rotated about angle α and then horizontal axis is modified by following equations (Fig. 4b):

$$\begin{bmatrix} \Delta\varphi_{xn} \\ \Delta\varphi_{yn} \end{bmatrix} = \begin{bmatrix} \cos(\alpha)/V & -\sin(\alpha)/V \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \times \begin{bmatrix} \Delta\varphi_1 \\ \Delta\varphi_2 \end{bmatrix}, \tag{6}$$

for phase lines and

$$\begin{bmatrix} \Delta\varphi_{xn\_m} \\ \Delta\varphi_{yn\_m} \end{bmatrix} = \begin{bmatrix} \cos(\alpha)/V & -\sin(\alpha)/V \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \times \begin{bmatrix} \Delta\varphi_{1\_m} \\ \Delta\varphi_{2\_m} \end{bmatrix}, \tag{7}$$

for measured point, where angle $\alpha = tg^{-1}(d_1/d_2)$. (8)

In the third step, value of $\Delta\varphi_{xn\_m}$ is rounded. After rounding this value, the measured point should lie on the phase line (Fig. 4c).

$$\Delta\varphi_{xnr\_m} = round(\Delta\varphi_{xn1\_m}), \tag{10}$$

Next, new phase lines and new measured point are re-rotated to original.

$$\begin{bmatrix} \Delta\varphi_{1\_m\_new} \\ \Delta\varphi_{2\_m\_new} \end{bmatrix} = \begin{bmatrix} \cos(-\alpha)\cdot V & -\sin(-\alpha) \\ \sin(-\alpha)\cdot V & \cos(-\alpha) \end{bmatrix} \times \begin{bmatrix} \Delta\varphi_{xnr\_m} \\ \Delta\varphi_{yn\_m} \end{bmatrix} \tag{11}$$

At this point, the measured point which is the correct one keeps lying on the phase line (Fig. 4d).

Finally, unambiguous phase difference for baseline d_2 corresponding to this measured point is calculated by:

$$\Delta\phi_{2\_m\_new} = \Delta\varphi_{2\_m\_new} + 2\pi n_{2\_\text{line}} \tag{12}$$

and AOA is calculated by:

$$\theta_m = \arcsin\left(\frac{\Delta\phi_{2\_m\_new}\cdot\lambda}{2\pi\cdot d_2}\right) \tag{13}$$

An interferometer system is considered with configuration of two baselines $d_1 = 3\lambda/2$ and $d_2 = 13\lambda/2$. Assumingly AOA is $\theta = -31°$, the true phase differences for two baselines are calculated: $\Delta\phi_1 = -278.12°$ and $\Delta\phi_2 = -1205.20°$. Measured phase differences without error will be: $\Delta\varphi_1 = 81.88°$ and $\Delta\varphi_2 = -125.19°$. In fact, with influence of noise, phase differences for two baselines are: $\Delta\varphi_{1\_m} = 88.06°$ and $\Delta\varphi_{2\_m} = -139.20°$. Phase error is 14°. AOA calculated by the algorithm is $-31.34°$. The error is $0.34°$.

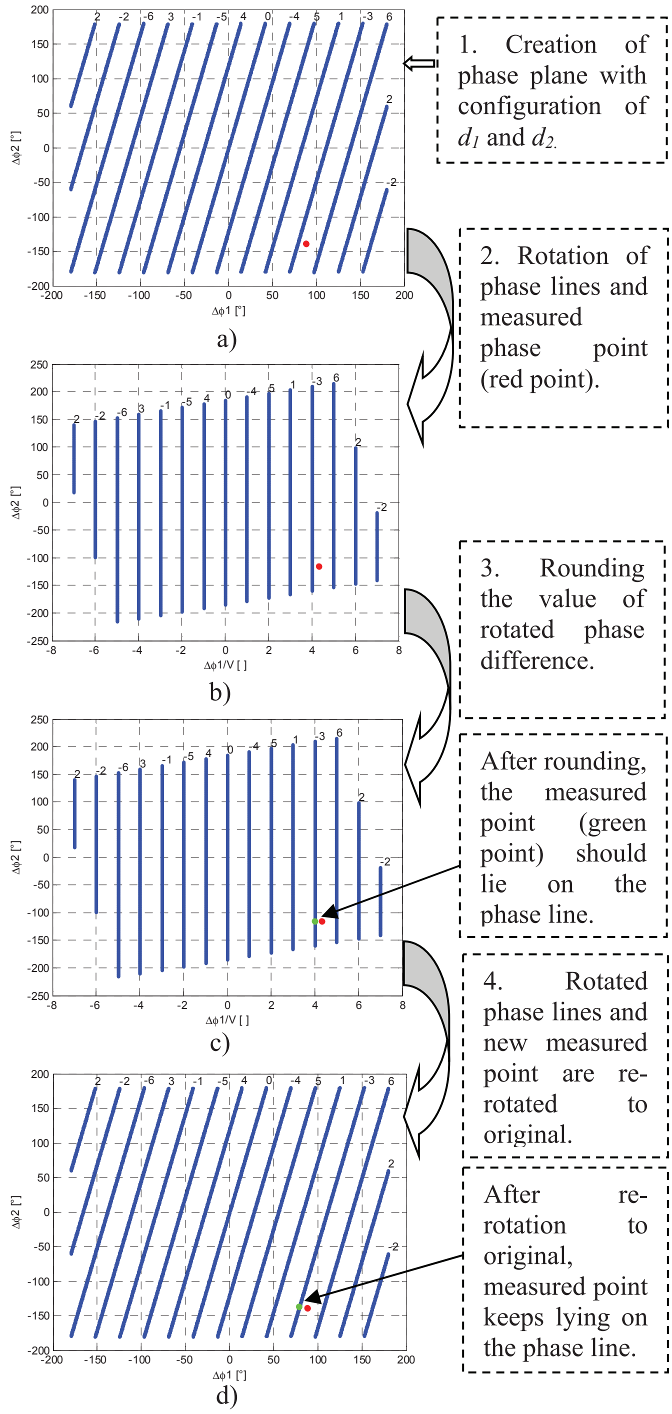

Figure 4. Computing procedure of unambiguous phase differences

B. *Four-antenna system*

For best configuration of interferometric system three baselines, created by four antennas, are chosen with spacing ratio of $A = M^2{:}M{\cdot}N{:}N^2$. In which M<N, they are positive integer numbers and their greatest common divisor is "1".

Principle for obtaining unambiguous AOA of four-antenna system is described as following. Phase space (relationship of phase differences of three baselines) is created by known parameters such as M and N. Relationship between true phase differences and ambiguous phase differences (for baseline d_3) is written as $\Delta\phi_3 = \Delta\varphi_3 + 2\pi n_3$, where $n_3 = 0, \pm1, \pm2, \pm3, \pm4, \ldots$. Phase lines are labeled corresponding to values of n_3 (Fig. 5).

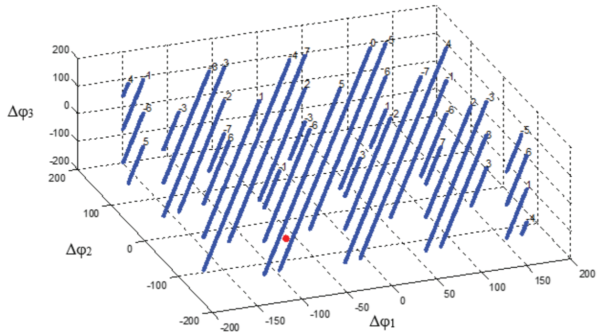

Figure 5. Relationship of phase differences of three baselines $d_1 = 9\lambda/2$, $d_2 = 12\lambda/2$ and $d_3 = 16\lambda/2$. Measured phase differences is represented by red point.

Phase space and measured point are rotated by using matrix **R**.

$$\begin{bmatrix}\Delta\varphi_x\\ \Delta\varphi_y\\ \Delta\varphi_z\end{bmatrix} = \mathbf{R}\times\begin{bmatrix}\Delta\varphi_1\\ \Delta\varphi_2\\ \Delta\varphi_3\end{bmatrix} \text{ for phase lines} \quad (14)$$

$$\text{and } \begin{bmatrix}\Delta\varphi_{x\_m}\\ \Delta\varphi_{y\_m}\\ \Delta\varphi_{z\_m}\end{bmatrix} = \mathbf{R}\times\begin{bmatrix}\Delta\varphi_{1\_m}\\ \Delta\varphi_{2\_m}\\ \Delta\varphi_{3\_m}\end{bmatrix} \text{ for measured point,} \quad (15)$$

where, $$\mathbf{R} = \begin{bmatrix}\cos(\alpha) & \sin(\alpha) & 0\\ -\sin(\alpha)\cos(\beta) & \cos(\alpha)\cos(\beta) & \sin(\beta)\\ \sin(\alpha)\sin(\beta) & -\cos(\alpha)\sin(\beta) & \cos(\beta)\end{bmatrix}$$

is rotating matrix, angles α and β are defined as

$$\alpha = \arctan(\frac{M}{N}) \text{ and } \beta = \arctan(\frac{M\cdot\sqrt{M^2+N^2}}{N^2}) \quad (16)$$

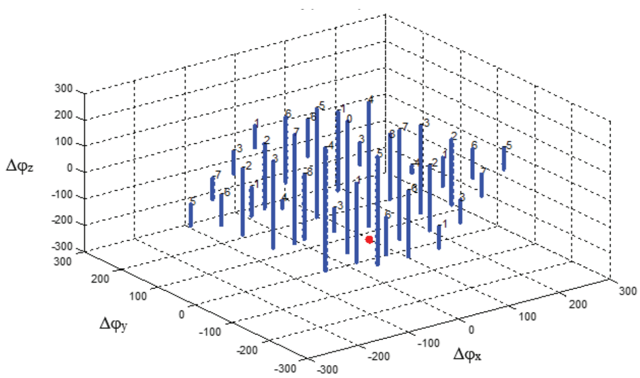

Figure 6. Relationship of phase differences after rotation

Distance between phase lines is calculated as:

$$V = \sqrt{V_x^2 + V_y^2} \quad (17)$$

$$\text{where } V_x = 360/\sqrt{M^2+N^2} \text{ and } Vy = V_x\cdot\sin(\beta)\cos(\alpha) \quad (18)$$

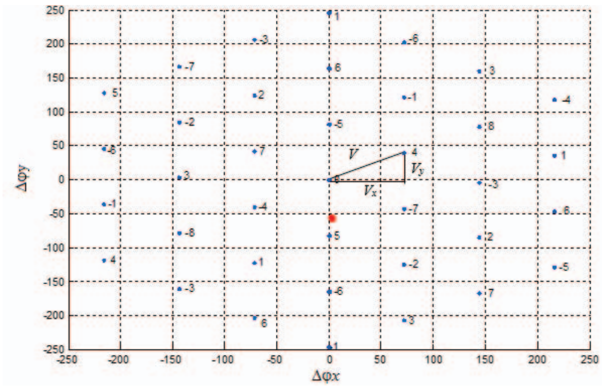

Figure 7. View from top to bottom of phase space after rotation

The phase space is changed as shown in Fig. 8.
For phase lines:

$$\Delta\varphi_{xn} = \frac{\Delta\varphi_x}{V_x} \text{ and } \Delta\varphi_{yn} = \frac{\Delta\varphi_y - V_y\cdot\Delta\varphi_{xn}}{V} \quad (19)$$

$$\text{For measured point: } \Delta\varphi_{xn\_m} = \frac{\Delta\varphi_{x\_m}}{V_x} \quad (20)$$

$$\Delta\varphi_{xnr\_m} = round(\Delta\varphi_{xn\_m}) \quad (21)$$

$$\Delta\varphi_{yn\_m} = \frac{\Delta\varphi_{y\_m} - V_y\cdot\Delta\varphi_{xnr\_m}}{V}) \quad (22)$$

$$\Delta\varphi_{ynr\_m} = round(\Delta\varphi_{yn\_m}) \quad (23)$$

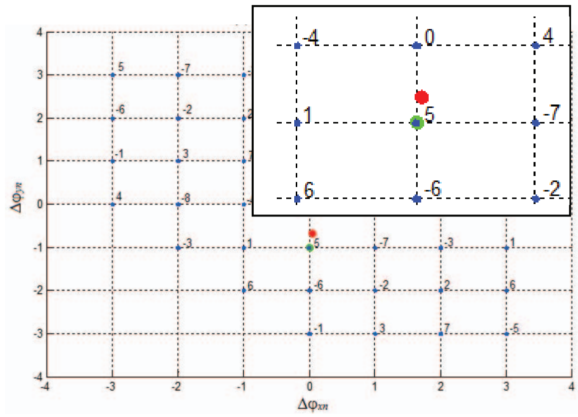

Figure 8. View from top to bottom of phase space after rotation and modification

Phase lines and new measured point are re-rotated to original (Fig. 9).

$$\begin{bmatrix}\Delta\varphi_1\\ \Delta\varphi_2\\ \Delta\varphi_3\end{bmatrix} = \mathbf{R}^{-1}\times\begin{bmatrix}\Delta\varphi_{xn}\cdot V_x\\ \Delta\varphi_{yn}\cdot V + V_y\cdot\Delta\varphi_{xn}\\ \Delta\varphi_z\end{bmatrix} \text{ for phase lines,} \quad (24)$$

$$\begin{bmatrix} \Delta\varphi_{1\_m\_new} \\ \Delta\varphi_{2\_m\_new} \\ \Delta\varphi_{3\_m\_new} \end{bmatrix} = \mathbf{R}^{-1} \times \begin{bmatrix} \Delta\varphi_{xnr\_m} \cdot V_x \\ \Delta\varphi_{ynr\_m} \cdot V + V_y \cdot \Delta\varphi_{xnr\_m} \\ \Delta\varphi_{z\_m} \end{bmatrix} \qquad (25)$$

for measured point, where

$$\mathbf{R}^{-1} = \begin{bmatrix} \cos(-\alpha) & -\cos(-\beta)\sin(-\alpha) & \sin(-\beta)\sin(-\alpha) \\ \sin(-\alpha) & \cos(-\beta)\cos(-\alpha) & -\sin(-\beta)\cos(-\alpha) \\ 0 & \sin(-\beta) & \cos(-\beta) \end{bmatrix}$$

is a re-rotating matrix for return.

Finally, unambiguous phase difference for baseline d_3 corresponding to this measured point is calculated by:

$$\Delta\phi_{3\_m\_new} = \Delta\varphi_{3\_m\_new} + 2\pi n_{3\_\mathrm{line}} \qquad (26)$$

and AOA is calculated by:

$$\theta_m = \arcsin\left(\frac{\Delta\phi_{3\_m\_new} \cdot \lambda}{2\pi \cdot d_3}\right) \qquad (27)$$

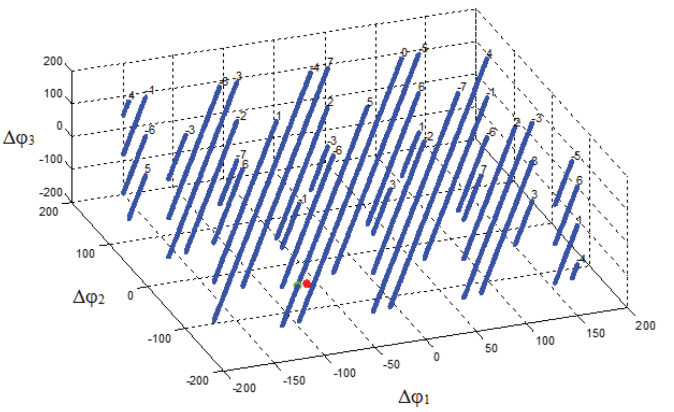

Figure 9. The new measured point (green) lies in a phase line (number “5”).

Method for determination of number n_{3_line} is described as following. Each spacing ratio of interferometric system creates unique characteristic matrix **C.** Elements of matrix **C** are numbers which are labeled for phase lines. The matrix **C** has form of:

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1m} \\ C_{21} & C_{22} & \cdots & C_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ C_{m1} & C_{m2} & \cdots & C_{mm} \end{bmatrix} \qquad (28)$$

Indexes of element C_{ij} are computed as:

$$i = \Delta\varphi_{xnr\_m} + fix\left(\frac{M+N-1}{2}\right) + 1 \qquad (29)$$

$$j = \Delta\varphi_{ynr\_m} + fix\left(\frac{M+N-1}{2}\right) + 1 \qquad (30)$$

Then, we have $n_{3\_\mathrm{line}} = C_{\mathrm{ij}}$.

To verify the proposed algorithm we consider an interferometer system with four baselines d_1=9λ/2, d_2 = 12λ/2 and d_3 = 16λ/2 . Assumingly AOA is θ = 31 °, the true phase differences for three baselines are calculated: $\Delta\phi_1$ = 834.36 °, $\Delta\phi_2$ = 1112.5 ° and $\Delta\phi_3$ = 1483.3 °. Phase differences without error will be: $\Delta\varphi_1$ = 114.36 °, $\Delta\varphi_2$ = 32.48 ° and $\Delta\varphi_3$ = 43.31 °. In fact, phase differences with influence of noise are: $\Delta\varphi_{1\_meas}$ = 129.36 °, $\Delta\varphi_{2\_meas}$ = 47.48 ° and $\Delta\varphi_{3\_meas}$ = 58.31 °. Phase error is 15 °. AOA calculated by the algorithm is 31.43 °. The error is 0.44 °.

IV. Experimental Measurement and Results

To validate proposed method, a phase interferometer system for measurement consisting of four passive receiving antennas, high resolution oscilloscope and transmitter of signal with frequency of 1090 MHz was established. Fig. 10 shows photography of the four-antenna system. Fig. 11 shows block schema of the measurement system.

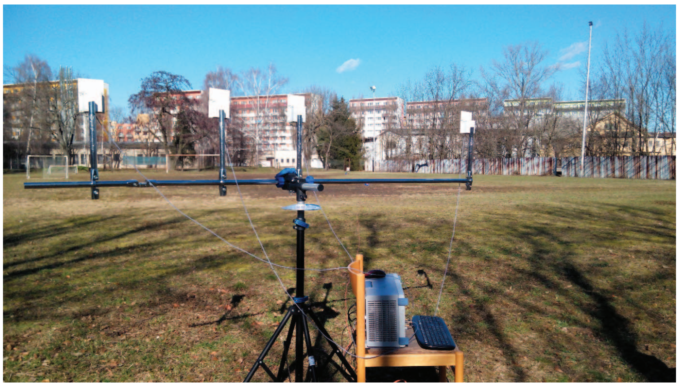

Figure 10. Photography of four-antenna system.

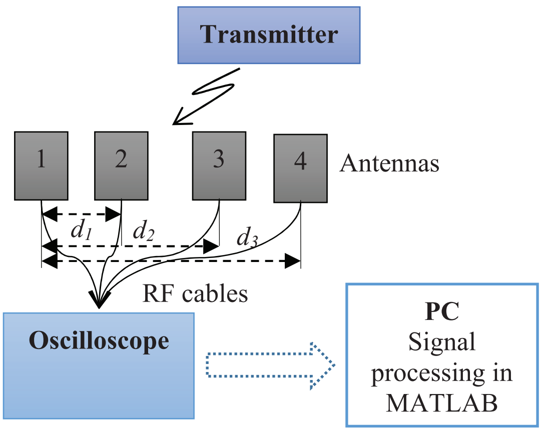

Figure 11. Configuration of measurement system.

Transmitter generated signal with frequency of 1090 MHz. This signal was received by four antennas and measured using oscilloscope. The four antennas were spaced in straight line to create three baselines d_1 = 9λ/2, d_2 = 12λ/2 and d_3 = 16λ/2. All signals displayed on the oscilloscope were saved to files for further signal processing. Results of measured phase differences were shown in Fig. 13. It was acquired that values of mean phase differences were -103.42 ° for baseline d_1, 108.43 ° for baseline d_2 and -119.61 ° for baseline d_3. Maximum errors of phase difference measurement were: 9.08 °, 9.37 ° and 9.03 °. The root mean square (RMS) value of the phase differences were: 5.41 °, 5.04 ° and 5.45 °.

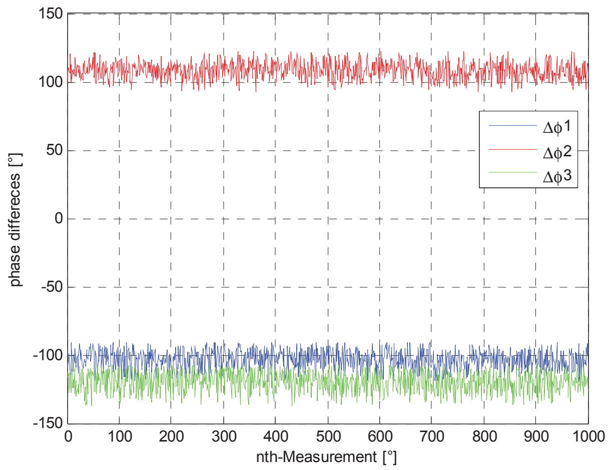

Figure 12. Results of phase difference measurements

Results of AOA determinations were shown in Fig. 13. The values of AOA moved in a limited range of (-17°, -16.5°). Mean value of AOA was -16.77 °. Maximum error of AOA was 0.22 °. The RMSE value of the AOA was 0.09 °.

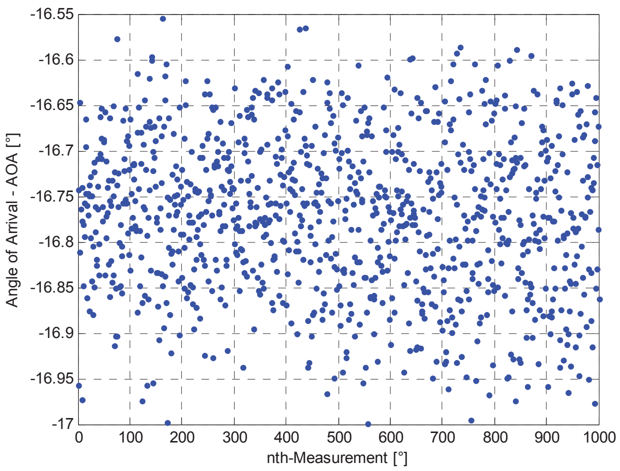

Figure 13. AOA measurements and their errors.

V. Conclusion

The presented method in the paper proposed an approach to solve ambiguity of the phase difference and to determine AOA, as well as provided information of allowable maximum error. Phase error information was determined according to spacing ratio, which provided to configure phase interferometer systems. Phase error of measurement depended on quality of devices and it had to be smaller than accepted error calculated by spacing of neighboring lines. Each spacing ratio gave a corresponding phase relationship, limited error and matrix of numbers labeled on lines. Proposed algorithm for obtaining unambiguous phase differences and AOA was fast and optimized due to use of phase line rotation, labeling and rounding. The phase interferometer measurement achieved very good results. Phase RMS errors of measurement of three baselines were 5.41 °, 5.04 ° and 5.45 °. Maximal error of the AOA determination was 0.22 ° and the RMS value was 0.09 °.

Acknowledgment

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