

# Gnowee Benchmark Results

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## ABSTRACT

This is a comprehensive listing of all of the benchmark results in tabular format. The associated plots can be found at <https://github.com/SlaybaughLab/Gnowee/tree/development/Benchmarks/plots>. The results are organized by benchmark type (continuous, mixed-integer, and combinatorial). The continuous benchmarks are further subdivided into constrained and unconstrained problems. In each section, the parameters used are listed first, followed by the individual problem detailed results, and rounded off with function evaluation and FOM summary tables. Each algorithm was run for 100 iterations against each benchmark shown to compile sufficient statistics. A select subset of these problems can be found in: "Gnowee: A Metaheuristic Optimization Algorithm for Constrained, Black Box, Combinatorial Mixed-Integer Engineering Design Problems."

## Gnowee Optimized Settings

From the hyper-optimization results, available at <https://github.com/SlaybaughLab/Gnowee/tree/development/Benchmarks/plots/HyperOptimization>, these were found to be the best settings across the range of problem types considered. They may not be the best for any given type, but they are the most likely to be best given a generic problem that combines multiple variable types.

**Table 1.** Gnowee algorithm settings.

Parameter	Gnowee
$P$	25
$S_i$	LHC
$\alpha$	0.5
$\gamma$	1
$\beta$	10
$f_1$	1.0
$f_d$	0.2
$f_e$	0.2

## Continuous Design Benchmarks

The convergence criteria used for the continuous benchmarks is shown in Table 2. Additionally, a 1% of optimal fitness convergence criteria was used for all problems and algorithms.

**Table 2.** Control settings for the continuous optimization algorithms.

Algorithm	$F_{max}$	$F_{stall}$	$P$
GA[1]	200,000	10,000	50
SA[1]	200,000	10,000	1
PSO[1]	200,000	10,000	100
CS[2]	200,000	10,000	25
MCS[3]	200,000	10,000	20
MEIGO[4]	200,000	10,000	50
Gnowee	200,000	10,000	25

# Unconstrained Functions

Functions considered:

1. 3D Ackley
2. 4D De Jong
3. 2D Easom
4. 6D Greiwank
5. 5D Rastrigin
6. 5D Rosenbrock

**Table 3.** Three dimensional Ackley optimization detailed results. Bold results indicate fitness greater than 1% convergence criteria. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 0.0.

	GA[1]	SA[1]	PSO[1]	CS [2]	MCS[3]	MEIGO[4]	Gnowee
$x_1^{avg}$	<b>-0.000142 ± 0.13784</b>	-0.000005 ± 0.00210	0.000189 ± 0.00171	0.000168 ± 0.00188	<b>-0.001334 ± 0.01217</b>	<u>0.000154 ± 0.00114</u>	-0.000202 ± 0.00201
$x_2^{avg}$	<b>-0.010039 ± 0.25649</b>	-0.000065 ± 0.00204	0.000301 ± 0.00186	-0.000011 ± 0.00186	<b>0.000669 ± 0.01164</b>	<u>0.000054 ± 0.00119</u>	-0.000069 ± 0.00175
$x_3^{avg}$	<b>0.018987 ± 0.19196</b>	-0.000009 ± 0.00272	0.000098 ± 0.00171	0.000077 ± 0.00192	<b>-0.000824 ± 0.01333</b>	<u>-0.000050 ± 0.00111</u>	0.000093 ± 0.00175
$f_{avg}(\vec{x})$	<b>0.178450 ± 0.72239</b>	0.008015 ± 0.00587	0.006896 ± 0.00220	0.007394 ± 0.00227	<b>0.052226 ± 0.03022</b>	<u>0.006551 ± 0.00232</u>	0.007264 ± 0.00200
$N_{f(\vec{x})}^{avg}$	<b>2411 ± 2310</b>	3692 ± 3332	2945 ± 648	8445 ± 1056	<b>19053 ± 6129</b>	<u>618 ± 431</u>	1898 ± 269
$FOM_{avg}$	<b>1666.8</b>	109.7	33.7	85.9	<b>1955.5</b>	<u>12.5</u>	19.7
$x_1^{best}$	-0.000118	0.000246	0.000057	-0.000484	0.000142	<u>0.000286</u>	-0.000581
$x_2^{best}$	-0.000175	-0.000043	0.000130	0.000041	-0.001033	<u>-0.000179</u>	-0.000203
$x_3^{best}$	-0.000119	-0.000874	0.000772	-0.000038	-0.000798	<u>-0.000079</u>	-0.000268
$f_{best}(\vec{x})$	0.000561	0.002113	0.001824	0.001129	0.003063	<u>0.001134</u>	0.001558
$N_{f(\vec{x})}^{best}$	2000	3587	2700	8175	10845	<u>566</u>	1680
$FOM_{best}$	1.1	7.6	4.9	9.2	33.2	<u>0.6</u>	2.6

**Table 4.** Four dimensional DeJong optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 0.0.

	GA[1]	SA[1]	PSO[1]	CS [2]	MCS[3]	MEIGO[4]	Gnowee
$x_1^{avg}$	0.007030 $\pm$ 0.03815	0.001011 $\pm$ 0.04421	-0.001355 $\pm$ 0.03941	-0.001043 $\pm$ 0.04236	0.008158 $\pm$ 0.03965	-0.005668 $\pm$ 0.03503	-0.000338 $\pm$ 0.04201
$x_2^{avg}$	0.003892 $\pm$ 0.04043	0.001925 $\pm$ 0.04607	-0.005754 $\pm$ 0.04030	0.002235 $\pm$ 0.04028	0.005167 $\pm$ 0.03935	-0.007166 $\pm$ 0.03545	-0.001091 $\pm$ 0.03982
$x_3^{avg}$	0.004782 $\pm$ 0.03706	-0.000971 $\pm$ 0.04227	0.002082 $\pm$ 0.03578	-0.004713 $\pm$ 0.03947	-0.002679 $\pm$ 0.04141	0.001778 $\pm$ 0.03410	0.000343 $\pm$ 0.03779
$x_4^{avg}$	0.004358 $\pm$ 0.04272	0.005013 $\pm$ 0.04613	-0.000697 $\pm$ 0.03325	-0.008977 $\pm$ 0.04188	-0.003806 $\pm$ 0.03883	-0.000570 $\pm$ 0.02998	-0.000592 $\pm$ 0.03963
$f_{avg}(\vec{x})$	0.006332 $\pm$ 0.00240	0.007942 $\pm$ 0.00165	0.005547 $\pm$ 0.00238	0.006769 $\pm$ 0.00225	0.006394 $\pm$ 0.00245	0.005705 $\pm$ 0.00265	0.006287 $\pm$ 0.00239
$N_{f(\vec{x})}^{avg}$	946 $\pm$ 248	1714 $\pm$ 1683	1833 $\pm$ 300	3033 $\pm$ 488	1100 $\pm$ 291	396 $\pm$ 70	892 $\pm$ 173
$FOM_{avg}$	10.7	53.7	15.2	30.5	12.6	3.5	8.9
$x_1^{best}$	-0.001593	-0.019614	-0.007134	0.017529	-0.006102	-0.010734	0.006149
$x_2^{best}$	-0.005017	0.017816	-0.003572	0.012948	0.006524	0.005440	-0.016879
$x_3^{best}$	0.014303	0.005006	-0.014717	-0.004303	-0.004873	-0.014811	-0.008211
$x_4^{best}$	0.005449	-0.030062	0.006252	0.015140	-0.019086	-0.001821	-0.021813
$f_{best}(\vec{x})$	0.000262	0.001631	0.000319	0.000723	0.000468	0.000368	0.000866
$N_{f(\vec{x})}^{best}$	1200	142	2300	3025	1364	345	840
$FOM_{best}$	0.3	0.2	0.7	2.2	0.6	0.1	0.7

**Table 5.** Two dimensional Easom optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = -1.0.

	GA[1]	SA[1]	PSO[1]	CS [2]	MCS[3]	MEIGO[4]	Gnowee
$x_1^{avg}$	<b>2.869975</b> $\pm$ <b>2.20867</b>	<b>3.224050</b> $\pm$ <b>2.1422</b>	3.145620 $\pm$ 0.03864	3.146342 $\pm$ 0.04087	3.138960 $\pm$ 0.04199	<u>3.141590</u> $\pm$ 0.00009	3.141648 $\pm$ 0.03949
$x_2^{avg}$	<b>2.865983</b> $\pm$ <b>1.77447</b>	<b>3.057748</b> $\pm$ <b>2.29850</b>	3.145847 $\pm$ 0.03396	3.141920 $\pm$ 0.04075	3.143228 $\pm$ 0.03854	<u>3.141591</u> $\pm$ 0.00008	3.140957 $\pm$ 0.03939
$f_{avg}(\vec{x})$	<b>-0.617519</b> $\pm$ <b>0.48584</b>	<b>-0.228645</b> $\pm$ <b>0.31523</b>	-0.996029 $\pm$ 0.00270	-0.995035 $\pm$ 0.00269	-0.995176 $\pm$ 0.00297	<u>-0.996433</u> $\pm$ 0.00294	-0.995393 $\pm$ 0.00289
$N_{f(\vec{x})}^{avg}$	<b>4459</b> $\pm$ <b>4448</b>	<b>13831</b> $\pm$ <b>5076</b>	2088 $\pm$ 307	7040 $\pm$ 1601	3747 $\pm$ 2510	<u>464</u> $\pm$ <u>140</u>	828 $\pm$ 193
$FOM_{avg}$	<b>6809.3</b>	<b>22415.2</b>	11.9	58.8	54.4	<u>3.2</u>	6.5
$x_1^{best}$	3.131134	3.135441	<u>3.138883</u>	3.148398	3.146351	<u>3.141674</u>	<u>3.144702</u>
$x_2^{best}$	3.137317	3.144053	<u>3.150349</u>	3.147757	3.146351	<u>3.141387</u>	<u>3.145677</u>
$f_{best}(\vec{x})$	-0.999809	-0.999934	<u>-0.999874</u>	-0.999874	-0.999932	<u>-0.999996</u>	<u>-0.999960</u>
$N_{f(\vec{x})}^{best}$	1050	18461	<u>2200</u>	7575	185	<u>560</u>	<u>1140</u>
$FOM_{best}$	0.2	1.2	<u>0.0</u>	1.0	0.3	<u>0.0</u>	<u>0.0</u>

**Table 6.** Six dimensional Griewank optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 0.0.

	GA[1]	SA[1]	PSO[1]	CS [2]	MCS[3]	MEIGO[4]	Gnowee
$x_1^{avg}$	<u>0.201159 ± 1.88417</u>	<b>1.398034 ± 14.43923</b>	<b>1.378999 ± 5.22680</b>	<b>-0.565628 ± 8.15575</b>	<b>1.493224 ± 10.15150</b>	<b>0.253461 ± 2.66910</b>	<b>-0.249655 ± 3.90880</b>
$x_2^{avg}$	<u>0.366524 ± 2.49624</u>	<b>1.586266 ± 14.05641</b>	<b>0.033644 ± 4.95374</b>	<b>-0.349229 ± 7.21675</b>	<b>1.848447 ± 11.77585</b>	<b>0.088039 ± 3.27772</b>	<b>0.309768 ± 3.04246</b>
$x_3^{avg}$	<u>0.116993 ± 0.76640</u>	<b>0.409451 ± 15.88555</b>	<b>0.216585 ± 6.41644</b>	<b>0.499912 ± 7.80187</b>	<b>1.991870 ± 10.94855</b>	<b>0.153182 ± 3.03737</b>	<b>-0.268980 ± 3.12493</b>
$x_4^{avg}$	<u>-0.054621 ± 0.63482</u>	<b>0.599289 ± 17.37281</b>	<b>-0.138224 ± 5.97189</b>	<b>0.464491 ± 8.57921</b>	<b>1.168596 ± 12.17359</b>	<b>-0.185988 ± 3.01119</b>	<b>0.508842 ± 4.42702</b>
$x_5^{avg}$	<u>-0.003029 ± 0.10780</u>	<b>0.784290 ± 14.00737</b>	<b>0.580790 ± 5.53637</b>	<b>-1.234143 ± 7.51511</b>	<b>0.365900 ± 11.69798</b>	<b>-0.633978 ± 3.31817</b>	<b>-0.206882 ± 4.04043</b>
$x_6^{avg}$	<u>-0.046531 ± 0.78031</u>	<b>-1.892031 ± 16.97844</b>	<b>-0.831285 ± 6.70667</b>	<b>-0.487097 ± 8.80328</b>	<b>0.723274 ± 10.68837</b>	<b>0.237986 ± 3.17676</b>	<b>0.612019 ± 4.83796</b>
$f_{avg}(\vec{x})$	<u>0.009721 ± 0.01310</u>	<b>0.687428 ± 0.18887</b>	<b>0.055998 ± 0.04546</b>	<b>0.171407 ± 0.06900</b>	<b>0.380347 ± 0.09967</b>	<b>0.016864 ± 0.00962</b>	<b>0.022621 ± 0.01468</b>
$N_f^{avg}(\vec{x})$	<u>1951 ± 2507</u>	<b>20419 ± 6603</b>	<b>29043 ± 12138</b>	<b>37820 ± 14534</b>	<b>21559 ± 7489</b>	<b>68392 ± 28540</b>	<b>31535 ± 12324</b>
$FOM_{avg}$	<u>92.1</u>	<b>27654.5</b>	<b>3665.4</b>	<b>13956.8</b>	<b>16746.4</b>	<b>2597.3</b>	<b>1549.7</b>
$x_1^{best}$	<u>-0.014629</u>	<b>9.267509</b>	0.002698	<b>3.123689</b>	<b>2.856185</b>	-3.140023	0.073978
$x_2^{best}$	<u>0.027934</u>	<b>-17.667665</b>	-0.018476	<b>4.312563</b>	<b>4.696731</b>	-4.438444	-0.008706
$x_3^{best}$	<u>-0.010507</u>	<b>-0.338062</b>	0.081153	<b>-5.272732</b>	<b>-0.143986</b>	-0.000000	-0.043893
$x_4^{best}$	<u>-0.027760</u>	<b>-5.927008</b>	-0.167562	<b>5.921846</b>	<b>-18.690699</b>	0.000000	0.044161
$x_5^{best}$	<u>0.015453</u>	<b>6.907948</b>	-0.007364	<b>-0.013710</b>	<b>0.111012</b>	0.000000	0.004432
$x_6^{best}$	<u>0.126265</u>	<b>-7.423495</b>	-0.001446	<b>-0.074905</b>	<b>-7.710662</b>	-0.000000	0.017419
$f_{best}(\vec{x})$	<u>0.001773</u>	<b>0.190155</b>	0.004704	<b>0.048529</b>	<b>0.173032</b>	0.008251	0.003347
$N_f^{best}(\vec{x})$	<u>1200</u>	<b>14484</b>	18100	<b>46725</b>	<b>19395</b>	19419	23901
$FOM_{best}$	<u>2.1</u>	<b>2754.2</b>	85.1	<b>2267.5</b>	<b>3356.0</b>	160.2	80.0

**Table 7.** Five dimensional Rastrigin optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 0.0.

	GA[I]	SA[I]	PSO[I]	CS [2]	MCS[3]	MCS[4]	Gnowee
$x_1^{avg}$	<b>0.059971 ± 0.34112</b>	<b>0.128445 ± 0.73054</b>	<b>-0.059811 ± 0.39477</b>	<b>0.009766 ± 0.54304</b>	<b>0.129483 ± 0.67286</b>	<b>0.010135 ± 0.17291</b>	<b>-0.000037 ± 0.00239</b>
$x_2^{avg}$	<b>0.010147 ± 0.38716</b>	<b>0.109002 ± 0.74688</b>	<b>-0.029804 ± 0.45724</b>	<b>-0.087902 ± 0.51953</b>	<b>-0.028690 ± 0.72844</b>	<b>0.000636 ± 0.14297</b>	<b>-0.000118 ± 0.14144</b>
$x_3^{avg}$	<b>0.050058 ± 0.29575</b>	<b>0.039757 ± 0.76083</b>	<b>-0.019501 ± 0.31564</b>	<b>-0.092107 ± 0.50942</b>	<b>0.099634 ± 0.65522</b>	<b>-0.020182 ± 0.19898</b>	<b>-0.010414 ± 0.09948</b>
$x_4^{avg}$	<b>0.049819 ± 0.45551</b>	<b>-0.059536 ± 0.75964</b>	<b>0.010146 ± 0.36051</b>	<b>-0.017504 ± 0.52925</b>	<b>0.059926 ± 0.70538</b>	<b>-0.030188 ± 0.26285</b>	<b>0.000230 ± 0.00256</b>
$x_5^{avg}$	<b>0.070279 ± 0.35357</b>	<b>0.010283 ± 0.80608</b>	<b>-0.041048 ± 0.31722</b>	<b>0.077687 ± 0.56729</b>	<b>0.098970 ± 0.71294</b>	<b>-0.019731 ± 0.24429</b>	<b>-0.009626 ± 0.09958</b>
$f_{avg}(\vec{x})$	<b>0.700755 ± 1.25056</b>	<b>2.938346 ± 1.24507</b>	<b>0.727492 ± 1.16489</b>	<b>2.003227 ± 1.88113</b>	<b>2.475966 ± 1.50516</b>	<b>0.233308 ± 0.43974</b>	<b>0.046677 ± 0.19456</b>
$N_{f(\vec{x})}^{avg}$	<b>6869 ± 5247</b>	<b>18730 ± 7638</b>	<b>16709 ± 7918</b>	<b>54486 ± 27535</b>	<b>30959 ± 10655</b>	<b>18272 ± 10361</b>	<b>17455 ± 6700</b>
$FOM_{avg}$	<b>15843</b>	<b>122362</b>	<b>29436</b>	<b>274627</b>	<b>155804</b>	<b>11514.8</b>	<b>1753.0</b>
$x_1^{best}$	<u>0.001626</u>	<b>0.000272</b>	-0.001102	0.001539	0.001879	-0.000334	-0.000047
$x_2^{best}$	<u>-0.000293</u>	<b>-0.000114</b>	0.000834	0.000695	-0.002179	-0.000307	0.000830
$x_3^{best}$	<u>0.000268</u>	<b>-0.000060</b>	0.001411	0.000208	0.000568	-0.000172	-0.000916
$x_4^{best}$	<u>0.001571</u>	<b>-0.994495</b>	0.001792	-0.001619	0.004230	-0.000634	0.000049
$x_5^{best}$	<u>0.000950</u>	<b>-0.000333</b>	0.000570	-0.002458	0.003963	-0.000023	-0.000199
$f_{best}(\vec{x})$	<u>0.001225</u>	<b>0.995042</b>	0.001476	0.002293	0.008371	0.000744	0.000312
$N_{f(\vec{x})}^{best}$	<u>2750</u>	<b>12850</b>	6600	86125	41017	9604	18180
$FOM_{best}$	<u>3.4</u>	<b>12786.3</b>	9.7	198	343	7.1	5.7



**Table 8.** Five dimensional Rosenbrock optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 0.0.

	GA[1]	SA[1]	PSO[1]	CS [2]	MCS[3]	MEIGO[4]	Gnowee
$x_1^{avg}$	<b>0.664163 <math>\pm</math> 0.74193</b>	<b>0.913465 <math>\pm</math> 0.24200</b>	<b>0.930016 <math>\pm</math> 0.33490</b>	<b>0.958113 <math>\pm</math> 0.03493</b>	<b>0.670798 <math>\pm</math> 0.74108</b>	0.999510 $\pm$ 0.00063	<u>0.996755 <math>\pm</math> 0.00587</u>
$x_2^{avg}$	<b>0.987771 <math>\pm</math> 0.05248</b>	<b>0.892751 <math>\pm</math> 0.18282</b>	<b>0.976758 <math>\pm</math> 0.03037</b>	<b>0.919281 <math>\pm</math> 0.06734</b>	<b>0.995704 <math>\pm</math> 0.13957</b>	0.999007 $\pm$ 0.00126	<u>0.993776 <math>\pm</math> 0.01133</u>
$x_3^{avg}$	<b>0.979232 <math>\pm</math> 0.10844</b>	<b>0.830949 <math>\pm</math> 0.22824</b>	<b>0.955287 <math>\pm</math> 0.05926</b>	<b>0.852081 <math>\pm</math> 0.12937</b>	<b>1.011265 <math>\pm</math> 0.28502</b>	0.998055 $\pm$ 0.00246	<u>0.977659 <math>\pm</math> 0.02125</u>
$x_4^{avg}$	<b>0.970839 <math>\pm</math> 0.23840</b>	<b>0.742486 <math>\pm</math> 0.28736</b>	<b>0.916683 <math>\pm</math> 0.12106</b>	<b>0.743928 <math>\pm</math> 0.23910</b>	<b>1.104951 <math>\pm</math> 0.62274</b>	0.996133 $\pm$ 0.00488	<u>0.975512 <math>\pm</math> 0.04473</u>
$x_5^{avg}$	<b>0.998742 <math>\pm</math> 0.61122</b>	<b>0.632370 <math>\pm</math> 0.35765</b>	<b>0.854708 <math>\pm</math> 0.27400</b>	<b>0.609654 <math>\pm</math> 0.46952</b>	<b>1.604654 <math>\pm</math> 1.71378</b>	0.992275 $\pm$ 0.00970	<u>0.953105 <math>\pm</math> 0.08908</u>
$f_{avg}(\vec{x})$	<b>0.732415 <math>\pm</math> 1.48960</b>	<b>0.341593 <math>\pm</math> 0.77527</b>	<b>0.148894 <math>\pm</math> 0.67298</b>	<b>0.228330 <math>\pm</math> 0.19906</b>	<b>1.176830 <math>\pm</math> 1.70477</b>	0.008955 $\pm$ 0.00121	<u>0.007585 <math>\pm</math> 0.00210</u>
$N_{f(\vec{x})}^{avg}$	<b>92171 <math>\pm</math> 51761</b>	<b>15837 <math>\pm</math> 6841</b>	<b>76826 <math>\pm</math> 32339</b>	<b>69941 <math>\pm</math> 45990</b>	<b>41264 <math>\pm</math> 26915</b>	11743 $\pm$ 4440	<u>11560 <math>\pm</math> 2834</u>
$FOM_{avg}$	<b>181239</b>	<b>12420</b>	<b>25884</b>	<b>47472</b>	<b>143587</b>	224.4	<u>152.2</u>
$x_1^{best}$	0.995179	<u>0.994959</u>	0.993468	0.996795	<b>0.993286</b>	0.999108	0.996602
$x_2^{best}$	0.988557	<u>0.988849</u>	0.987842	0.997276	<b>0.987204</b>	0.998287	0.994582
$x_3^{best}$	0.981532	<u>0.980139</u>	0.970684	0.993203	<b>0.968473</b>	0.997278	0.990845
$x_4^{best}$	0.962683	<u>0.959798</u>	0.942644	0.991269	<b>0.936557</b>	0.994757	0.979425
$x_5^{best}$	0.926854	<u>0.918535</u>	0.885976	0.988646	<b>0.870901</b>	0.989218	0.959626
$f_{best}(\vec{x})$	0.004111	<u>0.003610</u>	0.007758	0.007634	<b>0.013065</b>	0.003514	0.001572
$N_{f(\vec{x})}^{best}$	3950	<u>187</u>	72300	34975	<b>33664</b>	11296	12369
$FOM_{best}$	16	<u>0.7</u>	561	1399	<b>440</b>	39.7	19.4

**Table 9.** Summary of function evaluation results for continuous unconstrained optimization benchmarks. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run.

	Ackley (3-D)	De Jong (4-D)	Easom (2-D)	Griewank (6-D)	Rastrigin (5-D)	Rosenbrock (5-D)
GA[1]	<b>2411</b> $\pm$ <b>2310</b>	968 $\pm$ 248	<b>4459</b> $\pm$ <b>4448</b>	<u>1951</u> $\pm$ <u>2507</u>	<b>6869</b> $\pm$ <b>5247</b>	<b>92171</b> $\pm$ <b>51761</b>
SA[1]	3692 $\pm$ 3332	1714 $\pm$ 1683	<b>13831</b> $\pm$ <b>5076</b>	<b>20419</b> $\pm$ <b>6603</b>	<b>18730</b> $\pm$ <b>7638</b>	<b>15837</b> $\pm$ <b>6841</b>
PSO[1]	2945 $\pm$ 648	1833 $\pm$ 300	2088 $\pm$ 307	<b>29043</b> $\pm$ <b>12138</b>	<b>16709</b> $\pm$ <b>7918</b>	<b>76826</b> $\pm$ <b>3.2339</b>
CS[2]	8445 $\pm$ 1056	3033 $\pm$ 488	7040 $\pm$ 1601	<b>37820</b> $\pm$ <b>14534</b>	<b>54486</b> $\pm$ <b>27535</b>	<b>69941</b> $\pm$ <b>45990</b>
MCS[3]	<b>19053</b> $\pm$ <b>6129</b>	1100 $\pm$ 291	3747 $\pm$ 2510	<b>21559</b> $\pm$ <b>7489</b>	<b>30959</b> $\pm$ <b>10655</b>	<b>41264</b> $\pm$ <b>26915</b>
MEIGO[4]	<u>618</u> $\pm$ <u>431</u>	<u>369</u> $\pm$ <u>70</u>	<u>464</u> $\pm$ <u>140</u>	<b>68392</b> $\pm$ <b>28540</b>	<b>18272</b> $\pm$ <b>10361</b>	11743 $\pm$ 4440
Gnowee	1898 $\pm$ 269	892 $\pm$ 173	828 $\pm$ 193	<b>31535</b> $\pm$ <b>12324</b>	<u><b>17455</b></u> $\pm$ <u><b>6700</b></u>	<u>11560</u> $\pm$ <u>2834</u>

**Table 10.** Summary of FOM results for continuous unconstrained optimization benchmarks. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run.

	Ackley (3-D)	De Jong (4-D)	Easom (2-D)	Griewank (6-D)	Rastrigin (5-D)	Rosenbrock (5-D)
GA[1]	<b>1666.8</b>	10.7	<b>6809.3</b>	<u>92.1</u>	<b>15843</b>	<b>181239</b>
SA[1]	109.7	53.7	<b>22415.2</b>	<b>27654.5</b>	<b>122362</b>	<b>12420</b>
PSO[1]	33.7	15.2	11.9	<b>3665.4</b>	<b>29436</b>	<b>25884</b>
CS[2]	85.9	30.5	58.8	<b>13956.8</b>	<b>274627</b>	<b>47472</b>
MCS[3]	<b>1955.5</b>	12.6	54.4	<b>16746.4</b>	<b>155804</b>	<b>143587</b>
MEIGO[4]	<u>12.5</u>	<u>3.5</u>	<u>3.2</u>	<b>2597.3</b>	<b>11514.8</b>	224.4
Gnowee	19.7	8.9	6.5	<b>1549.7</b>	<u><b>1753.0</b></u>	<u>152.2</u>

## Constrained Functions

Functions considered:

1. Welded Beam [5]
2. Pressure Vessel [5]
3. Speed Reducer [5]
4. Spring [5]

The Pressure Vessel problem used was generally the same between the MI and continuous case. For the continuous case, the thickness of shell and head were made to be continuous variables. Everything else was left the same between the two cases. The optimal found was 5885.33285347.

**Table 11.** Welded Beam optimization results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 1.724852 [5].

	GA[1]	SA[1]	PSO[1]	CS [2]	MCS[3]	MEIGO[4]	Gnowee
$x_1^{avg}$	<b>0.535380</b> $\pm$ <b>0.27754</b>	<b>0.159921</b> $\pm$ <b>0.03097</b>	0.203742 $\pm$ 0.00440	0.203259 $\pm$ 0.00302	<b>0.206113</b> $\pm$ <b>0.02361</b>	<u>0.205698</u> $\pm$ <u>0.00004</u>	0.203344 $\pm$ 0.00255
$x_2^{avg}$	<b>2.430234</b> $\pm$ <b>1.31992</b>	<b>5.454721</b> $\pm$ <b>1.42779</b>	3.533882 $\pm$ 0.08177	3.551189 $\pm$ 0.06770	<b>3.602632</b> $\pm$ <b>0.57210</b>	<u>3.471443</u> $\pm$ <u>0.00082</u>	3.539995 $\pm$ 0.05569
$x_3^{avg}$	<b>5.244808</b> $\pm$ <b>1.20508</b>	<b>9.064175</b> $\pm$ <b>0.37189</b>	9.040988 $\pm$ 0.07305	9.053524 $\pm$ 0.03750	<b>8.913393</b> $\pm$ <b>0.31918</b>	<u>9.036835</u> $\pm$ <u>0.00053</u>	9.049939 $\pm$ 0.04067
$x_4^{avg}$	<b>0.707833</b> $\pm$ <b>0.28695</b>	<b>0.211915</b> $\pm$ <b>0.01399</b>	0.206968 $\pm$ 0.00282	0.206465 $\pm$ 0.00080	0.213877 $\pm$ 0.01599	<u>0.205741</u> $\pm$ <u>0.00001</u>	0.206546 $\pm$ 0.00090
$f_{avg}(\vec{x})$	<b>3.268729</b> $\pm$ <b>0.75854</b>	<b>1.938610</b> $\pm$ <b>0.12968</b>	1.740241 $\pm$ 0.00813	1.740365 $\pm$ 0.00420	<b>1.775602</b> $\pm$ <b>0.06015</b>	<u>1.738767</u> $\pm$ <u>0.00253</u>	1.738984 $\pm$ 0.00255
$N_{f(\vec{x})}^{avg}$	<b>21786</b> $\pm$ <b>32462</b>	<b>18648</b> $\pm$ <b>7571</b>	7895 $\pm$ 9676	29307 $\pm$ 6633	<b>35775</b> $\pm$ <b>21311</b>	<u>1803</u> $\pm$ <u>844</u>	5910 $\pm$ 1311
$FOM_{avg}$	<b>106667.6</b>	<b>5125.8</b>	329.4	442.6	<b>2933.8</b>	<u>35.0</u>	80.6
$x_1^{best}$	<b>0.210064</b>	0.205379	0.205379	0.205390	0.204258	<u>0.205700</u>	0.205479
$x_2^{best}$	<b>3.477943</b>	3.513764	3.479932	3.487902	3.500719	<u>3.470942</u>	3.533783
$x_3^{best}$	<b>8.782271</b>	9.063732	9.065610	9.072675	9.049050	<u>9.037137</u>	9.042888
$x_4^{best}$	<b>0.217819</b>	0.206069	0.205654	0.205737	0.205855	<u>0.205731</u>	0.205732
$f_{best}(\vec{x})$	<b>1.778071</b>	1.737478	1.730026	1.732982	1.729743	<u>1.731085</u>	1.730380
$N_{f(\vec{x})}^{best}$	<b>11850</b>	6074	3600	23825	26083	<u>1074</u>	8440
$FOM_{best}$	<b>365.6</b>	44.5	10.8	112.3	74.0	<u>3.9</u>	27.1

**Table 12.** Pressure Vessel optimization results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 5885.332800.

	GA[1]	SA[1]	PSO[1]	CS [2]	MCS[3]	MEIGO[4]	Gnowee
$x_1^{avg}$	0.864415 ± 0.04369	0.940023 ± 0.02975	0.864334 ± 0.06945	0.787252 ± 0.00918	0.858054 ± 0.05374	<b>0.799934 ± 0.02224</b>	<u>0.786637 ± 0.00659</u>
$x_2^{avg}$	0.427522 ± 0.02125	0.464654 ± 0.01471	0.428286 ± 0.03319	0.393513 ± 0.00557	0.425774 ± 0.02601	<b>0.395689 ± 0.01113</b>	<u>0.393452 ± 0.00392</u>
$x_3^{avg}$	44.774889 ± 2.28465	48.705831 ± 1.54145	44.208110 ± 3.52936	40.689886 ± 0.49253	44.167221 ± 2.73328	<b>41.438960 ± 1.15286</b>	<u>40.673817 ± 0.35776</u>
$x_4^{avg}$	147.679491 ± 24.28765	112.457234 ± 14.98499	150.432099 ± 38.31871	195.452603 ± 6.49264	153.122188 ± 29.85924	<b>185.551070 ± 14.24299</b>	<u>195.630753 ± 4.94062</u>
$f_{avg}(\vec{x})$	<b>6056.931682 ± 89.12492</b>	<b>6318.467444 ± 201.83904</b>	<b>6071.894010 ± 138.45163</b>	5937.014124 ± 13.89697	<b>6068.109699 ± 110.82856</b>	<b>5950.171681 ± 29.31413</b>	<u>5934.575903 ± 7.93851</u>
$N_{f(\vec{x})}^{avg}$	<b>12900 ± 4412</b>	<b>38069 ± 25758</b>	<b>14420 ± 7912</b>	24348 ± 11980	<b>211091 ± 8775</b>	<b>34464 ± 35935</b>	<u>5132 ± 2608</u>
$FOM_{avg}$	<b>762.0</b>	<b>8488.6</b>	<b>1209.5</b>	529.4	<b>1472.6</b>	<b>1567.4</b>	<u>108.4</u>
$x_1^{best}$	0.791185	0.819423	0.782836	0.780368	0.780094	<u>0.780903</u>	0.787062
$x_2^{best}$	0.392003	0.405041	0.390073	0.388495	0.388176	<u>0.386011</u>	0.389085
$x_3^{best}$	40.899704	42.457133	40.542649	40.426503	40.395892	<u>40.459183</u>	40.760369
$x_4^{best}$	192.096828	172.316973	197.503497	198.768809	199.407274	<u>198.080156</u>	194.157149
$f_{best}(\vec{x})$	5923.533202	5961.436530	5917.675018	198.768809	5909.301443	<u>5907.774153</u>	5907.938547
$N_{f(\vec{x})}^{best}$	3300	157259	3300	16500	9899	<u>827</u>	3270
$FOM_{best}$	21.4	2033.5	18.1	50.9	40.3	<u>3.2</u>	14.3

**Table 13.** Speed Reducer optimization results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 2996.348165 [5].

	GA[I]	SA[I]	PSO[I]	CS [2]	MCS[3]	MEIGO[4]	Gnowee
$x_1^{avg}$	3.523479 ± 0.01115	<b>3.505564 ± 0.00781</b>	3.524163 ± 0.01931	3.515357 ± 0.00987	3.512536 ± 0.00950	<u>3.500023 ± 0.00002</u>	3.516265 ± 0.01256
$x_2^{avg}$	0.701443 ± 0.00096	<b>0.700006 ± 0.00003</b>	0.700172 ± 0.00031	0.700121 ± 0.00034	0.700315 ± 0.00047	<u>0.700000 ± 0.00000</u>	0.700154 ± 0.00029
$x_3^{avg}$	17.014020 ± 0.01545	<b>17.093946 ± 0.08501</b>	17.001355 ± 0.00383	17.001275 ± 0.00620	17.006690 ± 0.01137	<u>17.000019 ± 0.00003</u>	17.006133 ± 0.01393
$x_4^{avg}$	7.388391 ± 0.12907	<b>7.820906 ± 0.26983</b>	7.739903 ± 0.35150	7.670151 ± 0.34593	7.835118 ± 0.37921	<u>7.300106 ± 0.00030</u>	7.706041 ± 0.34363
$x_5^{avg}$	7.836127 ± 0.04244	<b>8.030792 ± 0.12827</b>	7.975181 ± 0.16944	7.982819 ± 0.15050	8.067321 ± 0.17831	<u>7.800107 ± 0.00031</u>	7.977768 ± 0.15026
$x_6^{avg}$	3.362148 ± 0.01162	<b>3.358619 ± 0.01043</b>	3.365029 ± 0.01253	3.371809 ± 0.01829	3.367024 ± 0.01393	<u>3.350250 ± 0.00004</u>	3.369766 ± 0.01646
$x_7^{avg}$	5.293224 ± 0.00673	<b>5.289101 ± 0.00359</b>	5.292234 ± 0.00513	5.295841 ± 0.00725	5.293021 ± 0.00596	<u>5.286697 ± 0.00001</u>	5.294405 ± 0.00572
$f_{avg}(\vec{x})$	3023.396151 ± 2.76750	<b>3028.255983 ± 11.74571</b>	3021.959120 ± 4.06056	3021.876766 ± 4.17776	3022.858369 ± 2.93269	<u>3018.674196 ± 5.01003</u>	3021.984575 ± 3.34277
$N_{avg}^{avg}$	3323 ± 977	<b>38725 ± 32075</b>	2255 ± 1673	3538 ± 826	1439 ± 501	<u>830 ± 247</u>	2113 ± 454
$FOM_{avg}$	56.5	<b>1437.1</b>	62.2	51.3	26.0	<u>11.7</u>	29.7
$x_1^{best}$	3.507376	3.500246	3.502503	3.501448	3.505454	<u>3.500002</u>	3.513615
$x_2^{best}$	0.700978	0.700000	0.700002	0.700000	0.700000	<u>0.700000</u>	0.700365
$x_3^{best}$	17.010740	17.058220	17.000000	17.000000	17.000000	<u>17.000024</u>	17.022727
$x_4^{best}$	7.318865	8.120390	7.559614	7.996768	7.300000	<u>7.301175</u>	7.333132
$x_5^{best}$	7.854669	7.845934	7.818193	7.800000	7.988013	<u>7.801708</u>	7.831905
$x_6^{best}$	3.369326	3.358398	3.361828	3.358396	3.366356	<u>3.350309</u>	3.361983
$x_7^{best}$	5.291011	5.289320	5.291013	5.294357	5.297392	<u>5.286691</u>	5.290604
$f_{best}(\vec{x})$	3014.563118	3018.564074	3005.772033	3010.066122	3013.589560	<u>3005.867843</u>	3013.796361
$N_{f(\vec{x})}^{best}$	3200	21205	2400	4525	1687	<u>763</u>	1980
$FOM_{best}$	19.5	157.2	7.5	20.7	9.7	<u>2.4</u>	11.5

**Table 14.** Spring optimization results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 0.012665 [5].

	GA[1]	SA[1]	PSO[1]	CS [2]	MCS[3]	MEIGO[4]	Gnowee
$x_1^{avg}$	<b>0.060063</b> $\pm$ <b>0.00558</b>	0.050810 $\pm$ 0.00090	<b>0.052298</b> $\pm$ <b>0.00314</b>	0.051703 $\pm$ 0.00127	<b>0.058462</b> $\pm$ <b>0.00592</b>	0.052817 $\pm$ 0.00133	<u>0.051620</u> $\pm$ <u>0.00119</u>
$x_2^{avg}$	<b>0.610416</b> $\pm$ <b>0.18392</b>	0.335680 $\pm$ 0.02133	<b>0.375903</b> $\pm$ <b>0.08398</b>	0.357026 $\pm$ 0.03064	<b>0.560899</b> $\pm$ <b>0.18946</b>	0.385194 $\pm$ 0.03386	<u>0.355059</u> $\pm$ <u>0.02874</u>
$x_3^{avg}$	<b>5.461663</b> $\pm$ <b>3.19803</b>	12.840958 $\pm$ 1.32526	<b>11.363353</b> $\pm$ <b>3.39514</b>	11.560822 $\pm$ 1.83294	<b>6.573904</b> $\pm$ <b>3.75091</b>	9.997362 $\pm$ 1.53760	<u>11.648626</u> $\pm$ <u>1.72111</u>
$f_{avg}(\vec{x})$	<b>0.014398</b> $\pm$ <b>0.00161</b>	0.012778 $\pm$ 0.00002	<b>0.012890</b> $\pm$ <b>0.00030</b>	0.012771 $\pm$ 0.00004	<b>0.014068</b> $\pm$ <b>0.00156</b>	0.012785 $\pm$ 0.00006	<u>0.012763</u> $\pm$ <u>0.00002</u>
$N_{f(\vec{x})}^{avg}$	<b>16947</b> $\pm$ <b>6379</b>	5820 $\pm$ 5085	<b>7959</b> $\pm$ <b>10523</b>	16948 $\pm$ 7174	<b>19013</b> $\pm$ <b>13684</b>	10279 $\pm$ 9835	<u>4738</u> $\pm$ <u>1836</u>
$FOM_{avg}$	<b>4938</b>	188.3	<b>702.9</b>	323.2	<b>6654</b>	376.7	<u>79.0</u>
$x_1^{best}$	0.051341	0.050318	0.052021	0.051230	0.051768	<u>0.052121</u>	0.051092
$x_2^{best}$	0.348240	0.324492	0.364483	0.345779	0.358488	<u>0.367169</u>	0.342205
$x_3^{best}$	11.878832	13.476806	10.886333	11.987921	11.224128	<u>10.703094</u>	12.210091
$f_{best}(\vec{x})$	0.012740	0.012715	0.012711	0.012694	0.012705	<u>0.012691</u>	0.012694
$N_{f(\vec{x})}^{best}$	7900	835	27900	5225	4328	<u>1090</u>	3813
$FOM_{best}$	47	3.3	100.3	12.1	13.5	<u>2.2</u>	8.7



**Table 15.** Summary of FOM results for continuous constrained optimization benchmarks. Bold values indicate average fitness greater than 1% from the global optimum. Underlined values indicate the best average performance.

	<b>Welded Beam [5]</b>	<b>Pressure Vessel [5]</b>	<b>Speed Reducer [5]</b>	<b>Spring [5]</b>
GA[1]	<b>106667.6</b>	<b>762.0</b>	56.5	<b>4938</b>
SA[1]	<b>5125.8</b>	<b>8488.6</b>	<b>1437.1</b>	188.3
PSO[1]	329.4	<b>1209.5</b>	62.2	<b>702.9</b>
CS[2]	442.6	529.4	51.3	323.2
MCS[3]	<b>2933.8</b>	<b>1472.6</b>	26.0	<b>6654</b>
MEIGO[4]	<u>35.0</u>	<b>1567.4</b>	<u>11.7</u>	355.3
Gnowee	80.6	<u>108.4</u>	29.7	<u>79.0</u>

**Table 16.** Summary of function evaluation results for continuous constrained optimization benchmarks. Bold values indicate average fitness greater than 1% from the global optimum. Underlined values indicate the best average performance.

	<b>Welded Beam [5]</b>	<b>Pressure Vessel [5]</b>	<b>Speed Reducer [5]</b>	<b>Spring [5]</b>
GA[1]	<b>21786 ± 32462</b>	<b>12900 ± 4412</b>	3323 ± 977	<b>16947 ± 6379</b>
SA[1]	<b>18648 ± 7571</b>	<b>38069 ± 25758</b>	<b>38725 ± 32075</b>	5820 ± 5085
PSO[1]	7895 ± 9676	<b>14420 ± 7912</b>	2255 ± 1673	<b>7959 ± 10523</b>
CS[2]	29307 ± 6633	24348 ± 11980	3538 ± 826	16948 ± 7174
MCS[3]	<b>35775 ± 21311</b>	<b>211091 ± 8775</b>	1439 ± 501	<b>19013 ± 13684</b>
MEIGO[4]	<u>1803 ± 844</u>	<b>34464 ± 35935</b>	<u>830 ± 247</u>	11424 ± 10230
Gnowee	5910 ± 1311	<u>5132 ± 2608</u>	2113 ± 454	<u>4738 ± 1836</u>

## Mixed-Integer Design Benchmarks

Functions considered:

1. Mixed-Integer Pressure Vessel [5]
2. Mixed-Integer Spring [6]
3. Mixed-Integer Chemical Process [7]

**Table 17.** Control settings for the mixed-integer optimization algorithms.

Algorithm	$F_{max}$	$F_{stall}$	$P$
GA[1]	200,000	10,000	50
MEIGO[4]	200,000	10,000	50
Gnowee	200,000	10,000	25

**Table 18.** Mixed-integer pressure vessel optimization results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 6059.714335 [5].

	GA[1]	MEIGO[4]	Gnowee
$x_1^{avg}$	<b>45.405737 <math>\pm</math> 2.07552</b>	<b>43.917185 <math>\pm</math> 2.02390</b>	<u>42.307067 <math>\pm</math> 1.08568</u>
$x_2^{avg}$	<b>140.963636 <math>\pm</math> 21.46900</b>	<b>156.687582 <math>\pm</math> 21.86685</b>	<u>175.252438 <math>\pm</math> 12.48958</u>
$x_3^{avg}$	<b>0.88063 <math>\pm</math> 0.03982</b>	<b>0.848750 <math>\pm</math> 0.03890</b>	<u>0.820000 <math>\pm</math> 0.02041</u>
$x_4^{avg}$	<b>0.451250 <math>\pm</math> 0.02602</b>	<b>0.442500 <math>\pm</math> 0.01704</b>	<u>0.437500 <math>\pm</math> 0.00000</u>
$f_{avg}(\vec{x})$	<b>6177.813265 <math>\pm</math> 118.36124</b>	<b>6130.653499 <math>\pm</math> 73.23186</b>	<u>6103.390699 <math>\pm</math> 13.10013</u>
$N_{f(\vec{x})}^{avg}$	<b>14131 <math>\pm</math> 10646</b>	<b>4896 <math>\pm</math> 8570</b>	<u>3385 <math>\pm</math> 739</u>
$FOM_{avg}$	<b>897.9</b>	<b>358.3</b>	<u>40.4</u>
$x_1^{best}$	41.817039	<u>42.054521</u>	42.028546
$x_2^{best}$	180.299557	<u>177.184811</u>	177.507680
$x_3^{best}$	0.812500	<u>0.812500</u>	0.812500
$x_4^{best}$	0.437500	<u>0.437500</u>	0.437500
$f_{best}(\vec{x})$	6097.653041	<u>6070.873710</u>	6068.342344
$N_{f(\vec{x})}^{best}$	3401	<u>1473</u>	3875
$FOM_{best}$	21.3	<u>2.7</u>	5.5

**Table 19.** Mixed-integer spring optimization results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 2.65872 [6].

	GA[1]	MEIGO[4]	Gnowee
$x_1^{avg}$	<b>1.879815 <math>\pm</math> 0.35586</b>	1.445495 $\pm$ 0.21823	<u>1.365593 <math>\pm</math> 0.20183</u>
$x_2^{avg}$	<b>4.370000 <math>\pm</math> 1.58691</b>	6.960000 $\pm$ 2.00967	<u>7.720000 <math>\pm</math> 1.87530</u>
$x_3^{avg}$	<b>0.317080 <math>\pm</math> 0.01642</b>	0.295240 $\pm$ 0.01206	<u>0.290680 <math>\pm</math> 0.01125</u>
$f_{avg}(\vec{x})$	<b>2.844345 <math>\pm</math> 0.22113</b>	2.683470 $\pm$ 0.01694	<u>2.678752 <math>\pm</math> 0.01505</u>
$N_{f(\vec{x})}^{avg}$	<b>13330 <math>\pm</math> 3656</b>	13504 $\pm$ 7319	<u>7821 <math>\pm</math> 7015</u>
$FOM_{avg}$	<b>1697.9</b>	330.1	<u>219.2</u>
$x_1^{best}$	<u>1.226427</u>	<u>1.223043</u>	<u>1.223160</u>
$x_2^{best}$	<u>9.000000</u>	<u>9.000000</u>	<u>9.000000</u>
$x_3^{best}$	<u>0.283000</u>	<u>0.283000</u>	<u>0.283000</u>
$f_{best}(\vec{x})$	<u>2.665920</u>	<u>2.658563</u>	<u>2.658818</u>
$N_{f(\vec{x})}^{best}$	<u>101</u>	<u>5826</u>	<u>1172</u>
$FOM_{best}$	<u>0.3</u>	<u>0.3</u>	<u>0.2</u>

**Table 20.** Mixed-integer chemical process optimization results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 4.579582 [7].

	GA[1]	MEIGO[4]	Gnowee
$x_1^{avg}$	<b>0.531959</b> $\pm$ <b>0.35574</b>	0.199985 $\pm$ 0.00001	<u>0.191423</u> $\pm$ 0.00586
$x_2^{avg}$	<b>0.898655</b> $\pm$ <b>0.24141</b>	0.799988 $\pm$ 0.00001	<u>0.795269</u> $\pm$ 0.00408
$x_3^{avg}$	<b>1.703999</b> $\pm$ <b>0.21540</b>	1.907865 $\pm$ 0.00001	<u>1.902400</u> $\pm$ 0.00430
$x_4^{avg}$	<b>0.350000</b> $\pm$ <b>0.47937</b>	1.000000 $\pm$ 0.00000	<u>1.000000</u> $\pm$ 0.00000
$x_5^{avg}$	<b>0.720000</b> $\pm$ <b>0.45126</b>	1.000000 $\pm$ 0.00000	<u>1.000000</u> $\pm$ 0.00000
$x_6^{avg}$	<b>0.510000</b> $\pm$ <b>0.50242</b>	0.000000 $\pm$ 0.00000	<u>0.000000</u> $\pm$ 0.00000
$x_7^{avg}$	<b>0.340000</b> $\pm$ <b>0.47610</b>	1.000000 $\pm$ 0.00000	<u>1.000000</u> $\pm$ 0.00000
$f_{avg}(\vec{x})$	<b>6.084888</b> $\pm$ <b>0.88504</b>	4.610416 $\pm$ 0.01027	<u>4.616823</u> $\pm$ 0.00737
$N_{f(\vec{x})}^{avg}$	<b>20088</b> $\pm$ <b>12461</b>	3436 $\pm$ 3566	<u>6269</u> $\pm$ 1684
$FOM_{avg}$	<b>18890.9</b>	95.2	92.1
$x_1^{best}$	0.190610	<u>0.199997</u>	0.199530
$x_2^{best}$	0.799945	<u>0.799997</u>	0.796747
$x_3^{best}$	1.902600	<u>1.907841</u>	1.907393
$x_4^{best}$	1.000000	<u>1.000000</u>	1.000000
$x_5^{best}$	1.000000	<u>1.000000</u>	1.000000
$x_6^{best}$	0.000000	<u>0.000000</u>	0.000000
$x_7^{best}$	1.000000	<u>1.000000</u>	1.000000
$f_{best}(\vec{x})$	4.606384	<u>4.584077</u>	4.589212
$N_{f(\vec{x})}^{best}$	10651	<u>5999</u>	7875
$FOM_{best}$	62.3	<u>5.9</u>	16.6

**Table 21.** Summary of FOM results for constrained mixed-integer optimization benchmarks. Bold values indicate average fitness greater than 1% from the global optimum. Underlined values indicate the best average performance.

	Pressure Vessel [5]	Spring [6]	Chemical Process [7]
GA[1]	<b>897.9</b>	<b>1697.9</b>	<b>18890.1</b>
MEIGO[4]	<b>358.3</b>	330.1	95.2
Gnowee	<u>40.4</u>	<u>219.2</u>	<u>92.1</u>

**Table 22.** Summary of function evaluation results for constrained mixed-integer optimization benchmarks. Bold values indicate average fitness greater than 1% from the global optimum. Underlined values indicate the best average performance.

	Pressure Vessel [5]	Spring [6]	Chemical Process [7]
<b>GA[1]</b>	<b>14131 ± 10646</b>	<b>13330 ± 3656</b>	<b>20088 ± 12461</b>
<b>MEIGO[4]</b>	<b>4896 ± 8570</b>	13504 ± 7319	3436 ± 3566
<b>Gnowee</b>	<u>3385 ± 739</u>	<u>7821 ± 7015</u>	<u>6269 ± 1684</u>

## Combinatorial Design Benchmarks

Functions considered:

1. Mixed-Integer Pressure Vessel [5]
2. Mixed-Integer Spring [6]
3. Mixed-Integer Chemical Process [7]

**Table 23.** Control settings for the combinatorial optimization algorithms.

Algorithm	$F_{max}$	$F_{stall}$	$P$
<b>GA[1]</b>	200,000	15,000	100
<b>DCS[8]</b>	200,000	15,000	100
<b>Gnowee</b>	200,000	15,000	25

**Table 24.** Eil51 TSP optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 426.0

	GA[1]	DCS[8]	Gnowee
$f_{avg}(\vec{x})$	<b>452.470708 ± 9.11127</b>	428.890000 ± 1.13614	<b><u>434.640000 ± 3.97345</u></b>
$N_{f(\vec{x})}^{avg}$	<b>103415 ± 24218</b>	104341 ± 39398	<b><u>9294 ± 6033</u></b>
$FOM_{avg}$	<b>10940.4</b>	1509.7	<b><u>555.6</u></b>
$f_{best}(\vec{x})$	<b>432.941007</b>	427.000000	<u>427.000000</u>
$N_{f(\vec{x})}^{best}$	<b>103700</b>	112745	<u>3639</u>
$FOM_{best}$	<b>1689.6</b>	264.7	<u>8.5</u>

**Table 25.** St70 TSP optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 675.0

	GA[1]	DCS[8]	Gnowee
$f_{avg}(\vec{x})$	<b>742.166695 ± 21.86713</b>	<b>683.910000 ± 3.36078</b>	<b><u>695.720000 ± 12.01353</u></b>
$N_{f(\vec{x})}^{avg}$	<b>157455 ± 3.042629e+04</b>	<b>174362 ± 37953</b>	<b><u>16238 ± 9823</u></b>
$FOM_{avg}$	<b>27452.0</b>	<b>3804.6</b>	<b><u>1403.1</u></b>
$f_{best}(\vec{x})$	<b>695.738869</b>	<u>675.000000</u>	676.000000
$N_{f(\vec{x})}^{best}$	<b>200000</b>	<u>126881</u>	4740
$FOM_{best}$	<b>6145.2</b>	<u>0.0</u>	7.0

**Table 26.** Pr107 TSP optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 44303.0

	GA[1]	DCS[8]	Gnowee
$f_{avg}(\vec{x})$	<b>50228.032262 ± 1786.57531</b>	<b>45402.500000 ± 503.76310</b>	<b><u>46275.720000 ± 1320.43734</u></b>
$N_{f(\vec{x})}^{avg}$	<b>159692 ± 3.790020e+04</b>	<b>198006 ± 15497</b>	<b><u>27447 ± 16157</u></b>
$FOM_{avg}$	<b>36563.1</b>	<b>6067.9</b>	<b><u>3380.5</u></b>
$f_{best}(\vec{x})$	<b>46926.395272</b>	44679.000000	<u>44566.000000</u>
$N_{f(\vec{x})}^{best}$	<b>200000</b>	175768	<u>26143</u>
$FOM_{best}$	<b>11842.95</b>	1491.7	<u>155.2</u>

**Table 27.** Bier127 TSP optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 118282.0

	GA[1]	DCS[8]	Gnowee
$f_{avg}(\vec{x})$	<b>144124.296501 ± 4196.67351.</b>	<b>122257.960000 ± 1183.97614</b>	<u><b>123335.690000 ± 1867.54089</b></u>
$N_{f(\vec{x})}^{avg}$	<b>197340 ± 10306</b>	<b>228991 ± 6345</b>	<u><b>37483 ± 18077</b></u>
$FOM_{avg}$	<b>49869.9</b>	<b>8337.3</b>	<u><b>3918.6</b></u>
$f_{best}(\vec{x})$	<b>133399.493454</b>	119359.000000	<u><b>119785.000000</b></u>
$N_{f(\vec{x})}^{best}$	<b>200000</b>	230030	<u><b>38043</b></u>
$FOM_{best}$	<b>25561.8</b>	2094.5	<u><b>483.4</b></u>

**Table 28.** Ch150 TSP optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 6528.0

	GA[1]	DCS[8]	Gnowee
$f_{avg}(\vec{x})$	<b>10128.421320 ± 298.53102</b>	<b>6907.260000 ± 87.74741</b>	<u><b>6807.390000 ± 91.98374</b></u>
$N_{f(\vec{x})}^{avg}$	<b>199603 ± 3276</b>	<b>231786 ± 5338</b>	<u><b>48757 ± 24725</b></u>
$FOM_{avg}$	<b>115495.0</b>	<b>14396.6</b>	<u><b>5261.4</b></u>
$f_{best}(\vec{x})$	<b>6793.691618</b>	<b>6696.000000</b>	<u><b>6663.000000</b></u>
$N_{f(\vec{x})}^{best}$	<b>200000</b>	<b>232480</b>	<u><b>90289</b></u>
$FOM_{best}$	<b>8140</b>	<b>5982.9</b>	<u><b>1867.2</b></u>

**Table 29.** Summary of function evaluation results for TSP optimization benchmarks. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run

	GA[1]	DCS[8]	Gnowee
<b>Eil51</b>	<b>103415 ± 24218</b>	104341 ± 39398	<u><b>9294 ± 6033</b></u>
<b>St70</b>	<b>157455 ± 30426</b>	174362 ± 37953	<u><b>16238 ± 9823</b></u>
<b>Pr107</b>	<b>159692 ± 37900</b>	<b>198006 ± 15497</b>	<u><b>27447 ± 16157</b></u>
<b>Bier127</b>	<b>197340 ± 10306</b>	<b>228991 ± 6345</b>	<u><b>37483 ± 18077</b></u>
<b>Ch150</b>	<b>199603 ± 3276</b>	<b>231786 ± 5338</b>	<u><b>48757 ± 24725</b></u>

**Table 30.** Summary of FOM results for TSP optimization benchmarks. Bold indicates fitness greater than 1% from the global optimum. Underlined indicates the best average performance.

	GA[1]	DCS[8]	Gnowee
<b>Eil51</b>	<b>10940.4</b>	1509.7	<u><b>555.6</b></u>
<b>St70</b>	<b>27452.0</b>	3804.6	<u><b>1403.1</b></u>
<b>Pr107</b>	<b>36563.1</b>	<b>6067.9</b>	<u><b>3380.5</b></u>
<b>Bier127</b>	<b>49869.9</b>	<b>8337.3</b>	<u><b>3918.6</b></u>
<b>Ch150</b>	<b>115495.0</b>	<b>14396.6</b>	<u><b>5261.4</b></u>

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