Gnowee Benchmark Results

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ABSTRACT

This is a comprehensive listing of all of the benchmark results in tabular format. The associated plots can be found at `https://github.com/SlaybaughLab/Gnowee/tree/development/Benchmarks/plots`. The results are organized by benchmark type (continuous, mixed-integer, and combinatorial). The continuous benchmarks are further subdivided into constrained and unconstrained problems. In each section, the parameters used are listed first, followed by the individual problem detailed results, and rounded off with function evaluation and FOM summary tables. Each algorithm was run for 100 iterations against each benchmark shown to compile sufficient statistics. A select subset of these problems can be found in: "Gnowee: A Metaheuristic Optimization Algorithm for Constrained, Black Box, Combinatorial Mixed-Integer Engineering Design Problems."

Gnowee Optimized Settings

From the hyper-optimization results, available at `https://github.com/SlaybaughLab/Gnowee/tree/development/Benchmarks/plots/HyperOptimization`, these were found to be the best settings across the range of problem types considered. They may not be the best for any given type, but they are the most likely to be best given a generic problem that combines multiple variable types.

Table 1. Gnowee algorithm settings.

Parameter	Gnowee
P	25
S_i	LHC
α	0.5
γ	1
β	10
f_1	1.0
f_d	0.2
f_e	0.2

Continuous Design Benchmarks

The convergence criteria used for the continuous benchmarks is shown in Table 2. Additionally, a 1% of optimal fitness convergence criteria was used for all problems and algorithms.

Table 2. Control settings for the continuous optimization algorithms.

Algorithm	F_{max}	F_{stall}	P
GAMathworks2015	200,000	10,000	50
SAMathworks2015	200,000	10,000	1
PSOMathworks2015	200,000	10,000	100
CSYang2014	200,000	10,000	25
MCSWalton2011	200,000	10,000	20
MEIGOEgea2014	200,000	10,000	50
Gnowee	200,000	10,000	25

Unconstrained Functions

Functions considered:

1. 3D Ackley
2. 4D De Jong
3. 2D Easom
4. 6D Greiwank
5. 5D Rastrigin
6. 5D Rosenbrock

Table 3. Three dimensional Ackley optimization detailed results. Bold results indicate fitness greater than 1% convergence criteria. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 0.0.

	GAMathworks2015	SAMathworks2015	PSOMathworks2015	CS Yang2014	MCSWalton2011	MEIGOEgea2014	Gnowee
x_1^{avg}	**-0.000142 ± 0.13784**	-0.000005 ± 0.00210	0.000189 ± 0.00171	0.000168 ± 0.00188	**-0.001334 ± 0.01217**	0.000154 ± 0.00114	-0.000202 ± 0.00201
x_2^{avg}	**-0.010039 ± 0.25649**	-0.000065 ± 0.00204	0.000301 ± 0.00186	-0.000011 ± 0.00186	**0.000669 ± 0.01164**	0.000054 ± 0.00119	-0.000069 ± 0.00175
x_3^{avg}	**0.018987 ± 0.19196**	-0.000009 ± 0.00272	0.000098 ± 0.00171	0.000077 ± 0.00192	**-0.000824 ± 0.01333**	-0.000050 ± 0.00111	0.000093 ± 0.00175
$f_{avg}(\vec{x})$	**0.178450 ± 0.72239**	0.008015 ± 0.00587	0.006896 ± 0.00220	0.007394 ± 0.00227	**0.052226 ± 0.03022**	0.006551 ± 0.00232	0.007264 ± 0.00200
$N_{f(\vec{x})}^{avg}$	**2411 ± 2310**	3692 ± 3332	2945 ± 648	8445 ± 1056	**19053 ± 6129**	618 ± 431	1898 ± 269
FOM_{avg}	**1666.8**	109.7	33.7	85.9	**1955.5**	12.5	19.7
x_1^{best}	-0.000118	0.000246	0.000057	-0.000484	0.000142	0.000286	-0.000581
x_2^{best}	-0.000175	-0.000043	0.000130	0.000041	-0.001033	-0.000179	-0.000203
x_3^{best}	-0.000119	-0.000874	0.000772	-0.000038	-0.000798	-0.000079	-0.000268
$f_{best}(\vec{x})$	0.000561	0.002113	0.001824	0.001129	0.003063	0.001134	0.001558
$N_{f(\vec{x})}^{best}$	2000	3587	2700	8175	10845	566	1680
FOM_{best}	1.1	7.6	4.9	9.2	33.2	0.6	2.6

Table 4. Four dimensional DeJong optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 0.0.

	GAMathworks2015	**SAMathworks2015**	**PSOMathworks2015**	**CS Yang2014**	**MCSWalton2011**	**MEIGOEgea2014**	**Gnowee**
x_1^{avg}	0.007030 $\pm$ 0.03815	0.001011 $\pm$ 0.04421	-0.001355 $\pm$ 0.03941	-0.001043 $\pm$ 0.04236	0.008158 $\pm$ 0.03965	-0.005668 $\pm$ 0.03503	-0.000338 $\pm$ 0.04201
x_2^{avg}	0.003892 $\pm$ 0.04043	0.001925 $\pm$ 0.04607	-0.005754 $\pm$ 0.04030	0.002235 $\pm$ 0.04028	0.005167 $\pm$ 0.03935	-0.007166 $\pm$ 0.03545	-0.001091 $\pm$ 0.03982
x_3^{avg}	0.004782 $\pm$ 0.03706	-0.000971 $\pm$ 0.04227	0.002082 $\pm$ 0.03578	-0.004713 $\pm$ 0.03947	-0.002679 $\pm$ 0.04141	0.001778 $\pm$ 0.03410	0.000343 $\pm$ 0.03779
x_4^{avg}	0.004358 $\pm$ 0.04272	0.005013 $\pm$ 0.04613	-0.000697 $\pm$ 0.03325	-0.008977 $\pm$ 0.04188	-0.003806 $\pm$ 0.03883	-0.000570 $\pm$ 0.02998	-0.000592 $\pm$ 0.03963
$f_{avg}(\vec{x})$	0.006332 $\pm$ 0.00240	0.007942 $\pm$ 0.00165	0.005547 $\pm$ 0.00238	0.006769 $\pm$ 0.00225	0.006394 $\pm$ 0.00245	0.005705 $\pm$ 0.00265	0.006287 $\pm$ 0.00239
$N_{f(\vec{x})}^{avg}$	946 $\pm$ 248	1714 $\pm$ 1683	1833 $\pm$ 300	3033 $\pm$ 488	1100 $\pm$ 291	396 $\pm$ 70	892 $\pm$ 173
FOM_{avg}	10.7	53.7	15.2	30.5	12.6	3.5	8.9
x_1^{best}	-0.001593	-0.019614	-0.007134	0.017529	-0.006102	-0.010734	0.006149
x_2^{best}	-0.005017	0.017816	-0.003572	0.012948	0.006524	0.005440	-0.016879
x_3^{best}	0.014303	0.005006	-0.014717	-0.004303	-0.004873	-0.014811	-0.008211
x_4^{best}	0.005449	-0.030062	0.006252	0.015140	-0.019086	-0.001821	-0.021813
$f_{best}(\vec{x})$	0.000262	0.001631	0.000319	0.000723	0.000468	0.000368	0.000866
$N_{f(\vec{x})}^{best}$	1200	142	2300	3025	1364	345	840
FOM_{best}	0.3	0.2	0.7	2.2	0.6	0.1	0.7

Table 5. Two dimensional Easom optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = -1.0.

	GAMathworks2015	SAMathworks2015	PSOMathworks2015	CS Yang2014	MCSWalton2011	MEIGOEgea2014	Gnowee
x_1^{avg}	**2.869975 ± 2.20867**	**3.224050 ± 2.1422**	3.145620 ± 0.03864	3.146342 ± 0.04087	3.138960 ± 0.04199	3.141590 ± 0.00009	3.141648 ± 0.03949
x_2^{avg}	**2.865983 ± 1.77447**	**3.057748 ± 2.29850**	3.145847 ± 0.03396	3.141920 ± 0.04075	3.143228 ± 0.03854	3.141591 ± 0.00008	3.140957 ± 0.03939
$f_{avg}(\vec{x})$	**-0.617519 ± 0.48584**	**-0.228645 ± 0.31523**	-0.996029 ± 0.00270	-0.995035 ± 0.00269	-0.995176 ± 0.00297	-0.996433 ± 0.00294	-0.995393 ± 0.00289
$N_{f(\vec{x})}^{avg}$	**4459 ± 4448**	**13831 ± 5076**	2088 ± 307	7040 ± 1601	3747 ± 2510	464 ± 140	828 ± 193
FOM_{avg}	**6809.3**	**22415.2**	11.9	58.8	54.4	3.2	6.5
x_1^{best}	3.131134	3.135441	3.138883	3.148398	3.146351	3.141674	3.144702
x_2^{best}	3.137317	3.144053	3.150349	3.147757	3.146351	3.141387	3.145677
$f_{best}(\vec{x})$	-0.999809	-0.999934	-0.999874	-0.999874	-0.999932	-0.999996	-0.999960
$N_{f(\vec{x})}^{best}$	1050	18461	2200	7575	185	560	1140
FOM_{best}	0.2	1.2	0.0	1.0	0.3	0.0	0.0

Table 6. Six dimensional Griewank optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 0.0.

	GAMathworks2015	SAMathworks2015	PSOMathworks2015	CS Yang2014	MCSWalton2011	MEIGOEgea2014	Gnowee
x_1^{avg}	0.201159 ± 1.88417	**1.398034 ± 14.43923**	**1.378999 ± 5.22680**	**-0.565628 ± 8.15575**	**1.493224 ± 10.15150**	**0.253461 ± 2.66910**	**-0.249655 ± 3.90880**
x_2^{avg}	0.366524 ± 2.49624	**1.586266 ± 14.05641**	**0.033644 ± 4.95374**	**-0.349229 ± 7.21675**	**1.848447 ± 11.77585**	**0.088039 ± 3.27772**	**0.309768 ± 3.04246**
x_3^{avg}	0.116993 ± 0.76640	**0.409451 ± 15.88555**	**0.216585 ± 6.41644**	**0.499912 ± 7.80187**	**1.991870 ± 10.94855**	**0.153182 ± 3.03737**	**-0.268980 ± 3.12493**
x_4^{avg}	-0.054621 ± 0.63482	**0.599289 ± 17.37281**	**-0.138224 ± 5.97189**	**0.464491 ± 8.57921**	**1.168596 ± 12.17359**	**-0.185988 ± 3.01119**	**0.508842 ± 4.42702**
x_5^{avg}	-0.003029 ± 0.10780	**0.784290 ± 14.00737**	**0.580790 ± 5.53637**	**-1.234143 ± 7.51511**	**0.365900 ± 11.69798**	**-0.633978 ± 3.31817**	**-0.206882 ± 4.04043**
x_6^{avg}	-0.046531 ± 0.78031	**-1.892031 ± 16.97844**	**-0.831285 ± 6.70667**	**-0.487097 ± 8.80328**	**0.723274 ± 10.68837**	**0.237986 ± 3.17676**	**0.612019 ± 4.83796**
$f_{avg}(\vec{x})$	0.009721 ± 0.01310	**0.687428 ± 0.18887**	**0.055998 ± 0.04546**	**0.171407 ± 0.06900**	**0.380347 ± 0.09967**	**0.016864 ± 0.00962**	**0.022621 ± 0.01468**
$N_{f(\vec{x})}^{avg}$	1951 ± 2507	**20419 ± 6603**	**29043 ± 12138**	**37820 ± 14534**	**21559 ± 7489**	**68392 ± 28540**	**31535 ± 12324**
FOM_{avg}	92.1	**27654.5**	**3665.4**	**13956.8**	**16746.4**	**2597.3**	**1549.7**
x_1^{best}	-0.014629	**9.267509**	0.002698	**3.123689**	**2.856185**	-3.140023	0.073978
x_2^{best}	0.027934	**-17.667665**	-0.018476	**4.312563**	**4.696731**	-4.438444	-0.008706
x_3^{best}	-0.010507	**-0.338062**	0.081153	**-5.272732**	**-0.143986**	-0.000000	-0.043893
x_4^{best}	-0.027760	**-5.927008**	-0.167562	**5.921846**	**-18.690699**	0.000000	0.044161
x_5^{best}	0.015453	**6.907948**	-0.007364	**-0.013710**	**0.111012**	0.000000	0.004432
x_6^{best}	0.126265	**-7.423495**	-0.001446	**-0.074905**	**-7.710662**	-0.000000	0.017419
$f_{best}(\vec{x})$	0.001773	**0.190155**	0.004704	**0.048529**	**0.173032**	0.008251	0.003347
$N_{f(\vec{x})}^{best}$	1200	**14484**	18100	**46725**	**19395**	19419	23901
FOM_{best}	2.1	**2754.2**	85.1	**2267.5**	**3356.0**	160.2	80.0

Table 7. Five dimensional Rastrigin optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 0.0.

	GAMathworks2015	SAMathworks2015	PSOMathworks2015	CS Yang2014	MCSWalton2011	MCSEgea2014	Gnowee
x_1^{avg}	**0.059971 ± 0.34112**	**0.128445 ± 0.73054**	**-0.059811 ± 0.39477**	**0.009766 ± 0.54304**	**0.129483 ± 0.67286**	**0.010135 ± 0.17291**	**-0.000037 ± 0.00239**
x_2^{avg}	**0.010147 ± 0.38716**	**0.109002 ± 0.74688**	**-0.029804 ± 0.45724**	**-0.087902 ± 0.51953**	**-0.028690 ± 0.72844**	**0.000636 ± 0.14297**	**-0.000118 ± 0.14144**
x_3^{avg}	**0.050058 ± 0.29575**	**0.039757 ± 0.76083**	**-0.019501 ± 0.31564**	**-0.092107 ± 0.50942**	**0.099634 ± 0.65522**	**-0.020182 ± 0.19898**	**-0.010414 ± 0.09948**
x_4^{avg}	**0.049819 ± 0.45551**	**-0.059536 ± 0.75964**	**0.010146 ± 0.36051**	**-0.017504 ± 0.52925**	**0.059926 ± 0.70538**	**-0.030188 ± 0.26285**	**0.000230 ± 0.00256**
x_5^{avg}	**0.070279 ± 0.35357**	**0.010283 ± 0.80608**	**-0.041048 ± 0.31722**	**0.077687 ± 0.56729**	**0.098970 ± 0.71294**	**-0.019731 ± 0.24429**	**-0.009626 ± 0.09958**
$f_{avg}(\vec{x})$	**0.700755 ± 1.25056**	**2.938346 ± 1.24507**	**0.727492 ± 1.16489**	**2.003227 ± 1.88113**	**2.475966 ± 1.50516**	**0.233308 ± 0.43974**	**0.046677 ± 0.19456**
$N_{f(\vec{x})}^{avg}$	**6869 ± 5247**	**18730 ± 7638**	**16709 ± 7918**	**54486 ± 27535**	**30959 ± 10655**	**18272 ± 10361**	**17455 ± 6700**
FOM_{avg}	**15843**	**122362**	**29436**	**274627**	**155804**	**11514.8**	**1753.0**
x_1^{best}	0.001626	**0.000272**	-0.001102	0.001539	0.001879	-0.000334	-0.000047
x_2^{best}	-0.000293	**-0.000114**	0.000834	0.000695	-0.002179	-0.000307	0.000830
x_3^{best}	0.000268	**-0.000060**	0.001411	0.000208	0.000568	-0.000172	-0.000916
x_4^{best}	0.001571	**-0.994495**	0.001792	-0.001619	0.004230	-0.000634	0.000049
x_5^{best}	0.000950	**-0.000333**	0.000570	-0.002458	0.003963	-0.000023	-0.000199
$f_{best}(\vec{x})$	0.001225	**0.995042**	0.001476	0.002293	0.008371	0.000744	0.000312
$N_{f(\vec{x})}^{best}$	2750	**12850**	6600	86125	41017	9604	18180
FOM_{best}	3.4	**12786.3**	9.7	198	343	7.1	5.7

Table 8. Five dimensional Rosenbrock optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 0.0.

	GAMathworks2015	SAMathworks2015	PSOMathworks2015	CS Yang2014	MCSWalton2011	MEIGOEgea2014	Gnowee
x_1^{avg}	**0.664163 ± 0.74193**	**0.913465 ± 0.24200**	**0.930016 ± 0.33490**	**0.958113 ± 0.03493**	**0.670798 ± 0.74108**	0.999510 ± 0.00063	0.996755 ± 0.00587
x_2^{avg}	**0.987771 ± 0.05248**	**0.892751 ± 0.18282**	**0.976758 ± 0.03037**	**0.919281 ± 0.06734**	**0.995704 ± 0.13957**	0.999007 ± 0.00126	0.993776 ± 0.01133
x_3^{avg}	**0.979232 ± 0.10844**	**0.830949 ± 0.22824**	**0.955287 ± 0.05926**	**0.852081 ± 0.12937**	**1.011265 ± 0.28502**	0.998055 ± 0.00246	0.977659 ± 0.02125
x_4^{avg}	**0.970839 ± 0.23840**	**0.742486 ± 0.28736**	**0.916683 ± 0.12106**	**0.743928 ± 0.23910**	**1.104951 ± 0.62274**	0.996133 ± 0.00488	0.975512 ± 0.04473
x_5^{avg}	**0.998742 ± 0.61122**	**0.632370 ± 0.35765**	**0.854708 ± 0.27400**	**0.609654 ± 0.46952**	**1.604654 ± 1.71378**	0.992275 ± 0.00970	0.953105 ± 0.08908
$f_{avg}(\vec{x})$	**0.732415 ± 1.48960**	**0.341593 ± 0.77527**	**0.148894 ± 0.67298**	**0.228330 ± 0.19906**	**1.176830 ± 1.70477**	0.008955 ± 0.00121	0.007585 ± 0.00210
$N_{f(\vec{x})}^{avg}$	**92171 ± 51761**	**15837 ± 6841**	**76826 ± 32339**	**69941 ± 45990**	**41264 ± 26915**	11743 ± 4440	11560 ± 2834
FOM_{avg}	**181239**	**12420**	**25884**	**47472**	**143587**	224.4	152.2
x_1^{best}	0.995179	0.994959	0.993468	0.996795	**0.993286**	0.999108	0.996602
x_2^{best}	0.988557	0.988849	0.987842	0.997276	**0.987204**	0.998287	0.994582
x_3^{best}	0.981532	0.980139	0.970684	0.993203	**0.968473**	0.997278	0.990845
x_4^{best}	0.962683	0.959798	0.942644	0.991269	**0.936557**	0.994757	0.979425
x_5^{best}	0.926854	0.918535	0.885976	0.988646	**0.870901**	0.989218	0.959626
$f_{best}(\vec{x})$	0.004111	0.003610	0.007758	0.007634	**0.013065**	0.003514	0.001572
$N_{f(\vec{x})}^{best}$	3950	187	72300	34975	**33664**	11296	12369
FOM_{best}	16	0.7	561	1399	**440**	39.7	19.4

Table 9. Summary of function evaluation results for continuous unconstrained optimization benchmarks. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run.

	Ackley (3-D)	De Jong (4-D)	Easom (2-D)	Griewank (6-D)	Rastrigin (5-D)	Rosenbrock (5-D)
GAMathworks2014a	**2451 ± 2310**	968 ± 248	**4459 ± 4448**	1951 ± 2507	**6869 ± 5247**	**92171 ± 51761**
SAMathworks2014a	**3592 ± 3332**	1714 ± 1683	**13831 ± 5076**	**20419 ± 6603**	**18730 ± 7638**	**15837 ± 6841**
PSOMathworks2014a	**2045 ± 648**	1833 ± 300	2088 ± 307	**29043 ± 12138**	**16709 ± 7918**	**76826 ± 3.2339**
CSYang2014	8445 ± 1056	3033 ± 488	7040 ± 1601	**37820 ± 14534**	**54486 ± 27535**	**69941 ± 45990**
MCSWalton2011	**19053 ± 6129**	1100 ± 291	3747 ± 2510	**21559 ± 7489**	**30959 ± 10655**	**41264 ± 26915**
MEIGOEgea2014	618 ± 431	369 ± 70	464 ± 140	**68392 ± 28540**	**18272 ± 10361**	11743 ± 4440
Gnowee	1898 ± 269	892 ± 173	828 ± 193	**31535 ± 12324**	**17455 ± 6700**	11560 ± 2834

Table 10. Summary of FOM results for continuous unconstrained optimization benchmarks. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run.

	Ackley (3-D)	De Jong (4-D)	Easom (2-D)	Griewank (6-D)	Rastrigin (5-D)	Rosenbrock (5-D)
GAMathworks2015	**1666.8**	10.7	**6809.3**	92.1	**15843**	**181239**
SAMathworks2015	109.7	53.7	**22415.2**	**27654.5**	**122362**	**12420**
PSOMathworks2015	33.7	15.2	11.9	**3665.4**	**29436**	**25884**
CSYang2014	85.9	30.5	58.8	**13956.8**	**274627**	**47472**
MCSWalton2011	**1955.5**	12.6	54.4	**16746.4**	**155804**	**143587**
MEIGOEgea2014	12.5	3.5	3.2	**2597.3**	**11514.8**	224.4
Gnowee	19.7	8.9	6.5	**1549.7**	**1753.0**	152.2

Constrained Functions

Functions considered:

1. Welded Beam **Cagnina2008**
2. Pressure Vessel **Cagnina2008**
3. Speed Reducer **Cagnina2008**
4. Spring **Cagnina2008**

The Pressure Vessel problem used was generally the same between the MI and continuous case. For the continuous case, the thickness of shell and head were made to be continuous variables. Everything else was left the same between the two cases. The optimal found was 5885.33285347.

Table 11. Welded Beam optimization results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 1.724852 **Cagnina2008**

	GAMathworks2015	SAMathworks2015	PSOMathworks2015	CS Yang2014	MCSWalton2011	MEIGOEgea2014	Gnowee
x_1^{avg}	**0.535380 ± 0.27754**	**0.159921 ± 0.03097**	0.203742 ± 0.00440	0.203259 ± 0.00302	**0.206113 ± 0.02361**	0.205698 ± 0.00004	0.203344 ± 0.00255
x_2^{avg}	**2.430234 ± 1.31992**	**5.454721 ± 1.42779**	3.533882 ± 0.08177	3.551189 ± 0.06770	**3.602632 ± 0.57210**	3.471443 ± 0.00082	3.539995 ± 0.05569
x_3^{avg}	**5.244808 ± 1.20508**	**9.064175 ± 0.37189**	9.040988 ± 0.07305	9.053524 ± 0.03750	**8.913393 ± 0.31918**	9.036835 ± 0.00053	9.049939 ± 0.04067
x_4^{avg}	**0.707833 ± 0.28695**	**0.211915 ± 0.01399**	0.206968 ± 0.00282	0.206465 ± 0.00080	0.213877 ± 0.01599	0.205741 ± 0.00001	0.206546 ± 0.00090
$f_{avg}(\vec{x})$	**3.268729 ± 0.75854**	**1.938610 ± 0.12968**	1.740241 ± 0.00813	1.740365 ± 0.00420	**1.775602 ± 0.06015**	1.738767 ± 0.00253	1.738984 ± 0.00255
$N_{f(\vec{x})}^{avg}$	**21786 ± 32462**	**18648 ± 7571**	7895 ± 9676	29307 ± 6633	**35775 ± 21311**	1803 ± 844	5910 ± 1311
FOM_{avg}	**106667.6**	**5125.8**	329.4	442.6	**2933.8**	35.0	80.6
x_1^{best}	**0.210064**	0.205379	0.205379	0.205390	0.204258	0.205700	0.205479
x_2^{best}	**3.477943**	3.513764	3.479932	3.487902	3.500719	3.470942	3.533783
x_3^{best}	**8.782271**	9.063732	9.065610	9.072675	9.049050	9.037137	9.042888
x_4^{best}	**0.217819**	0.206069	0.205654	0.205737	0.205855	0.205731	0.205732
$f_{best}(\vec{x})$	**1.778071**	1.737478	1.730026	1.732982	1.729743	1.731085	1.730380
$N_{f(\vec{x})}^{best}$	**11850**	6074	3600	23825	26083	1074	8440
FOM_{best}	**365.6**	44.5	10.8	112.3	74.0	3.9	27.1

Table 12. Pressure Vessel optimization results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 5885.332800.

	GAMathworks2015	SAMathworks2015	PSOMathworks2015	CS Yang2014	MCSWalton2011	MEIGOEgea2014	Gnowee
x_1^{avg}	**0.864415 ± 0.04369**	**0.940023 ± 0.02975**	**0.864334 ± 0.06945**	0.787252 ± 0.00918	**0.858054 ± 0.05374**	**0.799934 ± 0.02224**	0.786637 ± 0.00659
x_2^{avg}	**0.427522 ± 0.02125**	**0.464654 ± 0.01471**	**0.428286 ± 0.03319**	0.393513 ± 0.00557	**0.425774 ± 0.02601**	**0.395689 ± 0.01113**	0.393452 ± 0.00392
x_3^{avg}	**44.774889 ± 2.28465**	**48.705831 ± 1.54145**	**44.208110 ± 3.52936**	40.689886 ± 0.49253	**44.167221 ± 2.73328**	**41.438960 ± 1.15286**	40.673817 ± 0.35776
x_4^{avg}	**147.679491 ± 24.28765**	**112.457234 ± 14.98499**	**150.432099 ± 38.31871**	195.452603 ± 6.49264	**153.122188 ± 29.85924**	**185.551070 ± 14.24299**	195.630753 ± 4.94062
$f_{avg}(\vec{x})$	**6056.931682 ± 89.12492**	**6318.467444 ± 201.83904**	**6071.894010 ± 138.45163**	5937.014124 ± 13.89697	**6068.109699 ± 110.82856**	**5950.171681 ± 29.31413**	5934.575903 ± 7.93851
$N_{f(\vec{x})}^{avg}$	**12900 ± 4412**	**38069 ± 25758**	**14420 ± 7912**	24348 ± 11980	**211091 ± 8775**	**34464 ± 35935**	5132 ± 2608
FOM_{avg}	**762.0**	**8488.6**	**1209.5**	529.4	**1472.6**	**1567.4**	108.4
x_1^{best}	0.791185	0.819423	0.782836	0.780368	0.780094	0.780903	0.787062
x_2^{best}	0.392003	0.405041	0.390073	0.388495	0.388176	0.386011	0.389085
x_3^{best}	40.899704	42.457133	40.542649	40.426503	40.395892	40.459183	40.760369
x_4^{best}	192.096828	172.316973	197.503497	198.768809	199.407274	198.080156	194.157149
$f_{best}(\vec{x})$	5923.533202	5961.436530	5917.675018	198.768809	5909.301443	5907.774153	5907.938547
$N_{f(\vec{x})}^{best}$	3300	157259	3300	16500	9899	827	3270
FOM_{best}	21.4	2033.5	18.1	50.9	40.3	3.2	14.3

Table 13. Speed Reducer optimization results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 2996.348165 **Cagnina2008**

	GAMathworks2015	**SAMathworks2015**	**PSOMathworks2015**	**CS Yang2014**	**MCSWalton2011**	**MEIGOEgea2014**	**Gnowee**
x_1^{avg}	3.523479 ± 0.01115	**3.505564 ± 0.00781**	3.524163 ± 0.01931	3.515357 ± 0.00987	3.512536 ± 0.00950	3.500023 ± 0.00002	3.516265 ± 0.01256
x_2^{avg}	0.701443 ± 0.00096	**0.700006 ± 0.00003**	0.700172 ± 0.00031	0.700121 ± 0.00034	0.700315 ± 0.00047	0.700000 ± 0.00000	0.700154 ± 0.00029
x_3^{avg}	17.014020 ± 0.01545	**17.093946 ± 0.08501**	17.001355 ± 0.00383	17.001275 ± 0.00620	17.006690 ± 0.01137	17.000019 ± 0.00003	17.006133 ± 0.01393
x_4^{avg}	7.388391 ± 0.12907	**7.820906 ± 0.26983**	7.739903 ± 0.35150	7.670151 ± 0.34593	7.835118 ± 0.37921	7.300106 ± 0.00030	7.706041 ± 0.34363
x_5^{avg}	7.836127 ± 0.04244	**8.030792 ± 0.12827**	7.975181 ± 0.16944	7.982819 ± 0.15050	8.067321 ± 0.17831	7.800107 ± 0.00031	7.977768 ± 0.15026
x_6^{avg}	3.362148 ± 0.01162	**3.358619 ± 0.01043**	3.365029 ± 0.01253	3.371809 ± 0.01829	3.367024 ± 0.01393	3.350250 ± 0.00004	3.369766 ± 0.01646
x_7^{avg}	5.293224 ± 0.00673	**5.289101 ± 0.00359**	5.292234 ± 0.00513	5.295841 ± 0.00725	5.293021 ± 0.00596	5.286697 ± 0.00001	5.294405 ± 0.00572
$f_{avg}(\vec{x})$	3023.396151 ± 2.76750	**3028.255983 ± 11.74571**	3021.959120 ± 4.06056	3021.876766 ± 4.17776	3022.858369 ± 2.93269	3018.674196 ± 5.01003	3021.984575 ± 3.34277
$N_{f(\vec{x})}^{avg}$	3323 ± 977	**38725 ± 32075**	2255 ± 1673	3538 ± 826	1439 ± 501	830 ± 247	2113 ± 454
FOM_{avg}	56.5	**1437.1**	62.2	51.3	26.0	11.7	29.7
x_1^{best}	3.507376	3.500246	3.502503	3.501448	3.505454	3.500002	3.513615
x_2^{best}	0.700978	0.700000	0.700002	0.700000	0.700000	0.700000	0.700365
x_3^{best}	17.010740	17.058220	17.000000	17.000000	17.000000	17.000024	17.022727
x_4^{best}	7.318865	8.120390	7.559614	7.996768	7.300000	7.301175	7.333132
x_5^{best}	7.854669	7.845934	7.818193	7.800000	7.988013	7.801708	7.831905
x_6^{best}	3.369326	3.358398	3.361828	3.358396	3.366356	3.350309	3.361983
x_7^{best}	5.291011	5.289320	5.291013	5.294357	5.297392	5.286691	5.290604
$f_{best}(\vec{x})$	3014.563118	3018.564074	3005.772033	3010.066122	3013.589560	3005.867843	3013.796361
$N_{f(\vec{x})}^{best}$	3200	21205	2400	4525	1687	763	1980
FOM_{best}	19.5	157.2	7.5	20.7	9.7	2.4	11.5

Table 14. Spring optimization results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 0.012665 **Cagnina2008**

	GAMathworks2015	SAMathworks2015	PSOMathworks2015	CS Yang2014	MCSWalton2011	MEIGOEgea2014	Gnowee
x_1^{avg}	**0.060063 ± 0.00558**	0.050810 ± 0.00090	**0.052298 ± 0.00314**	0.051703 ± 0.00127	**0.058462 ± 0.00592**	0.052817 ± 0.00133	0.051620 ± 0.00119
x_2^{avg}	**0.610416 ± 0.18392**	0.335680 ± 0.02133	**0.375903 ± 0.08398**	0.357026 ± 0.03064	**0.560899 ± 0.18946**	0.385194 ± 0.03386	0.355059 ± 0.02874
x_3^{avg}	**5.461663 ± 3.19803**	12.840958 ± 1.32526	**11.363353 ± 3.39514**	11.560822 ± 1.83294	**6.573904 ± 3.75091**	9.997362 ± 1.53760	11.648626 ± 1.72111
$f_{avg}(\vec{x})$	**0.014398 ± 0.00161**	0.012778 ± 0.00002	**0.012890 ± 0.00030**	0.012771 ± 0.00004	**0.014068 ± 0.00156**	0.012785 ± 0.00006	0.012763 ± 0.00002
$N_{f(\vec{x})}^{avg}$	**16947 ± 6379**	5820 ± 5085	**7959 ± 10523**	16948 ± 7174	**19013 ± 13684**	10279 ± 9835	4738 ± 1836
FOM_{avg}	**4938**	188.3	**702.9**	323.2	**6654**	376.7	79.0
x_1^{best}	0.051341	0.050318	0.052021	0.051230	0.051768	0.052121	0.051092
x_2^{best}	0.348240	0.324492	0.364483	0.345779	0.358488	0.367169	0.342205
x_3^{best}	11.878832	13.476806	10.886333	11.987921	11.224128	10.703094	12.210091
$f_{best}(\vec{x})$	0.012740	0.012715	0.012711	0.012694	0.012705	0.012691	0.012694
$N_{f(\vec{x})}^{best}$	7900	835	27900	5225	4328	1090	3813
FOM_{best}	47	3.3	100.3	12.1	13.5	2.2	8.7

Table 15. Summary of FOM results for continuous constrained optimization benchmarks. Bold values indicate average fitness greater than 1% from the global optimum. Underlined values indicate the best average performance.

	Welded Beam Cagnina2008	Pressure Vessel Cagnina2008	Speed Reducer Cagnina2008	Spring Cagnina2008
GAMathworks2015	**1046.6**	**762.0**	56.5	**4938**
SAMathworks2015	**5132.8**	**8488.6**	**1437.1**	188.3
PSOMathworks2015	328.2	**1209.5**	62.2	**702.9**
CSYang2014	42.6	529.4	51.3	323.2
MCSWalton2011	**2283.8**	**1472.6**	26.0	**6654**
MEIGOEgea2014	320.1	**1567.4**	11.7	355.3
Gnowee	80.6	108.4	29.7	79.0

Table 16. Summary of function evaluation results for continuous constrained optimization benchmarks. Bold values indicate average fitness greater than 1% from the global optimum. Underlined values indicate the best average performance.

	Welded Beam Cagnina2008	Pressure Vessel Cagnina2008	Speed Reducer Cagnina2008	Spring Cagnina2008
GAMathworks2015	**21786** $\pm$ **3456**	**12900** $\pm$ **4412**	3323 $\pm$ 977	**16947** $\pm$ **6379**
SAMathworks2015	**18648** $\pm$ **2757**	**38069** $\pm$ **25758**	**38725** $\pm$ **32075**	5820 $\pm$ 5085
PSOMathworks2015	**7895** $\pm$ **2016**	**14420** $\pm$ **7912**	2255 $\pm$ 1673	**7959** $\pm$ **10523**
CSYang2014	**29047** $\pm$ 6633	24348 $\pm$ 11980	3538 $\pm$ 826	16948 $\pm$ 7174
MCSWalton2011	**35775** $\pm$ **11311**	**211091** $\pm$ **8775**	1439 $\pm$ 501	**19013** $\pm$ **13684**
MEIGOEgea2014	**18032** $\pm$ **1844**	**34464** $\pm$ **35935**	830 $\pm$ 247	11424 $\pm$ 10230
Gnowee	5910 $\pm$ 1311	5132 $\pm$ 2608	2113 $\pm$ 454	4738 $\pm$ 1836

Mixed-Integer Design Benchmarks

Functions considered:

1. Mixed-Integer Pressure Vessel **Cagnina2008**
2. Mixed-Integer Spring **Lampinen1999**
3. Mixed-Integer Chemical Process **Yiqing2007**

Table 17. Control settings for the mixed-integer optimization algorithms.

Algorithm	F_{max}	F_{stall}	P
GAMathworks2015	200,000	10,000	50
MEIGOEgea2014	200,000	10,000	50
Gnowee	200,000	10,000	25

Table 18. Mixed-integer pressure vessel optimization results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 6059.714335 **Cagnina2008**

	GAMathworks2015	**MEIGOEgea2014**	**Gnowee**
x_1^{avg}	**45.405737 ± 2.07552**	**43.917185 ± 2.02390**	42.307067 ± 1.08568
x_2^{avg}	**140.963636 ± 21.46900**	**156.687582 ± 21.86685**	175.252438 ± 12.48958
x_3^{avg}	**0.88063 ± 0.03982**	**0.848750 ± 0.03890**	0.820000 ± 0.02041
x_4^{avg}	**0.451250 ± 0.02602**	**0.442500 ± 0.01704**	0.437500 ± 0.00000
$f_{avg}(\vec{x})$	**6177.813265 ± 118.36124**	**6130.653499 ± 73.23186**	6103.390699 ± 13.10013
$N_{f(\vec{x})}^{avg}$	**14131 ± 10646**	**4896 ± 8570**	3385 ± 739
FOM_{avg}	**897.9**	**358.3**	40.4
x_1^{best}	41.817039	42.054521	42.028546
x_2^{best}	180.299557	177.184811	177.507680
x_3^{best}	0.812500	0.812500	0.812500
x_4^{best}	0.437500	0.437500	0.437500
$f_{best}(\vec{x})$	6097.653041	6070.873710	6068.342344
$N_{f(\vec{x})}^{best}$	3401	1473	3875
FOM_{best}	21.3	2.7	5.5

Table 19. Mixed-integer spring optimization results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 2.65872 **Lampinen1999**

	GAMathworks2015	**MEIGOEgea2014**	**Gnowee**
x_1^{avg}	**1.879815 ± 0.35586**	1.445495 ± 0.21823	1.365593 ± 0.20183
x_2^{avg}	**4.370000 ± 1.58691**	6.960000 ± 2.00967	7.720000 ± 1.87530
x_3^{avg}	**0.317080 ± 0.01642**	0.295240 ± 0.01206	0.290680 ± 0.01125
$f_{avg}(\vec{x})$	**2.844345 ± 0.22113**	2.683470 ± 0.01694	2.678752 ± 0.01505
$N_{f(\vec{x})}^{avg}$	**13330 ± 3656**	13504 ± 7319	7821 ± 7015
FOM_{avg}	**1697.9**	330.1	219.2
x_1^{best}	1.226427	1.223043	1.223160
x_2^{best}	9.000000	9.000000	9.000000
x_3^{best}	0.283000	0.283000	0.283000
$f_{best}(\vec{x})$	2.665920	2.658563	2.658818
$N_{f(\vec{x})}^{best}$	101	5826	1172
FOM_{best}	0.3	0.3	0.2

Table 20. Mixed-integer chemical process optimization results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 4.579582 **Yiqing2007**

	GAMathworks2015	**MEIGOEgea2014**	**Gnowee**
x_1^{avg}	**0.531959 ± 0.35574**	0.199985 ± 0.00001	0.191423 ± 0.00586
x_2^{avg}	**0.898655 ± 0.24141**	0.799988 ± 0.00001	0.795269 ± 0.00408
x_3^{avg}	**1.703999 ± 0.21540**	1.907865 ± 0.00001	1.902400 ± 0.00430
x_4^{avg}	**0.350000 ± 0.47937**	1.000000 ± 0.00000	1.000000 ± 0.00000
x_5^{avg}	**0.720000 ± 0.45126**	1.000000 ± 0.00000	1.000000 ± 0.00000
x_6^{avg}	**0.510000 ± 0.50242**	0.000000 ± 0.00000	0.000000 ± 0.00000
x_7^{avg}	**0.340000 ± 0.47610**	1.000000 ± 0.00000	1.000000 ± 0.00000
$f_{avg}(\vec{x})$	**6.084888 ± 0.88504**	4.610416 ± 0.01027	4.616823 ± 0.00737
$N_{f(\vec{x})}^{avg}$	**20088 ± 12461**	3436 ± 3566	6269 ± 1684
FOM_{avg}	**18890.9**	95.2	92.1
x_1^{best}	0.190610	0.199997	0.199530
x_2^{best}	0.799945	0.799997	0.796747
x_3^{best}	1.902600	1.907841	1.907393
x_4^{best}	1.000000	1.000000	1.000000
x_5^{best}	1.000000	1.000000	1.000000
x_6^{best}	0.000000	0.000000	0.000000
x_7^{best}	1.000000	1.000000	1.000000
$f_{best}(\vec{x})$	4.606384	4.584077	4.589212
$N_{f(\vec{x})}^{best}$	10651	5999	7875
FOM_{best}	62.3	5.9	16.6

Table 21. Summary of FOM results for constrained mixed-integer optimization benchmarks. Bold values indicate average fitness greater than 1% from the global optimum. Underlined values indicate the best average performance.

	Pressure Vessel Cagnina2008	**Spring Lampinen1999**	**Chemical Process Yiqing2007**
GAMathworks2015	**8852.9**	**1697.9**	**18890.1**
MEIGOEgea2014	**328.34**	330.1	95.2
Gnowee	40.4	219.2	92.1

Table 22. Summary of function evaluation results for constrained mixed-integer optimization benchmarks. Bold values indicate average fitness greater than 1% from the global optimum. Underlined values indicate the best average performance.

	Pressure Vessel Cagnina2008	Spring Lampinen1999	Chemical Process Yiqing2007
GAMathworks2015	**14131 ± 10646**	**13330 ± 3656**	**20088 ± 12461**
MEIGOMetaheuristics	**12962 ± 8570**	13504 ± 7319	3436 ± 3566
Gnowee	3385 ± 739	7821 ± 7015	6269 ± 1684

Combinatorial Design Benchmarks

Functions considered:

1. Mixed-Integer Pressure Vessel **Cagnina2008**
2. Mixed-Integer Spring **Lampinen1999**
3. Mixed-Integer Chemical Process **Yiqing2007**

Table 23. Control settings for the combinatorial optimization algorithms.

Algorithm	F_{max}	F_{stall}	P
GAMathworks2015	200,000	15,000	100
DCSZhou2014	200,000	15,000	100
Gnowee	200,000	15,000	25

Table 24. Eil51 TSP optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 426.0

	GAMathworks2015	DCSZhou2014	Gnowee
$f_{avg}(\vec{x})$	**452.470708 ± 9.11127**	428.890000 ± 1.13614	**434.640000 ± 3.97345**
$N^{avg}_{f(\vec{x})}$	**103415 ± 24218**	104341 ± 39398	**9294 ± 6033**
FOM_{avg}	**10940.4**	1509.7	**555.6**
$f_{best}(\vec{x})$	**432.941007**	427.000000	427.000000
$N^{best}_{f(\vec{x})}$	**103700**	112745	3639
FOM_{best}	**1689.6**	264.7	8.5

Table 25. St70 TSP optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 675.0

	GAMathworks2015	DCSZhou2014	Gnowee
$f_{avg}(\vec{x})$	**742.166695 ± 21.86713**	**683.910000 ± 3.36078**	**695.720000 ± 12.01353**
$N^{avg}_{f(\vec{x})}$	**157455 ± 3.042629e+04**	**174362 ± 37953**	**16238 ± 9823**
FOM_{avg}	**27452.0**	**3804.6**	**1403.1**
$f_{best}(\vec{x})$	**695.738869**	675.000000	676.000000
$N^{best}_{f(\vec{x})}$	**200000**	126881	4740
FOM_{best}	**6145.2**	0.0	7.0

Table 26. Pr107 TSP optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 44303.0

	GAMathworks2015	DCSZhou2014	Gnowee
$f_{avg}(\vec{x})$	**50228.032262 ± 1786.57531**	**45402.500000 ± 503.76310**	**46275.720000 ± 1320.43734**
$N^{avg}_{f(\vec{x})}$	**159692 ± 3.790020e+04**	**198006 ± 15497**	**27447 ± 16157**
FOM_{avg}	**36563.1**	**6067.9**	**3380.5**
$f_{best}(\vec{x})$	**46926.395272**	44679.000000	44566.000000
$N^{best}_{f(\vec{x})}$	**200000**	175768	26143
FOM_{best}	**11842.95**	1491.7	155.2

Table 27. Bier127 TSP optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 118282.0

	GAMathworks2015	DCSZhou2014	Gnowee
$f_{avg}(\vec{x})$	**144124.296501 ± 4196.67351**.	**122257.960000 ± 1183.97614**	**123335.690000 ± 1867.54089**
$N^{avg}_{f(\vec{x})}$	**197340 ± 10306**	**228991 ± 6345**	**37483 ± 18077**
FOM_{avg}	**49869.9**	**8337.3**	**3918.6**
$f_{best}(\vec{x})$	**133399.493454**	119359.000000	**119785.000000**
$N^{best}_{f(\vec{x})}$	**200000**	230030	**38043**
FOM_{best}	**25561.8**	2094.5	**483.4**

Table 28. Ch150 TSP optimization detailed results. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run. Optimum fitness = 6528.0

	GAMathworks2015	DCSZhou2014	Gnowee
$f_{avg}(\vec{x})$	**10128.421320 ± 298.53102**	**6907.260000 ± 87.74741**	**6807.390000 ± 91.98374**
$N^{avg}_{f(\vec{x})}$	**199603 ± 3276**	**231786 ± 5338**	**48757 ± 24725**
FOM_{avg}	**115495.0**	**14396.6**	**5261.4**
$f_{best}(\vec{x})$	**6793.691618**	**6696.000000**	**6663.000000**
$N^{best}_{f(\vec{x})}$	**200000**	**232480**	**90289**
FOM_{best}	**8140**	**5982.9**	**1867.2**

Table 29. Summary of function evaluation results for TSP optimization benchmarks. Bold results indicate fitness greater than 1% from the global optimum. Underlined results indicate the best performance for the average and the overall best run

	GAMathworks2015	DCSZhou2014	Gnowee
Eil51	**103415 ± 24218**	104341 ± 39398	**9294 ± 6033**
St70	**157455 ± 30426**	174362 ± 37953	**16238 ± 9823**
Pr107	**159692 ± 37900**	**198006 ± 15497**	**27447 ± 16157**
Bier127	**197340 ± 10306**	**228991 ± 6345**	**37483 ± 18077**
Ch150	**199603 ± 3276**	**231786 ± 5338**	**48757 ± 24725**

Table 30. Summary of FOM results for TSP optimization benchmarks. Bold indicates fitness greater than 1% from the global optimum. Underlined indicates the best average performance.

	GAMathworks2015	DCSZhou2014	Gnowee
Eil51	**10940.4**	1509.7	**555.6**
St70	**27452.0**	3804.6	**1403.1**
Pr107	**36563.1**	**6067.9**	**3380.5**
Bier127	**49869.9**	**8337.3**	**3918.6**
Ch150	**115495.0**	**14396.6**	**5261.4**