FLASH: A Framework for Programming Distributed Graph Processing Algorithms

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Abstract—**As a result of decades of studies, a broad spectrum of graph algorithms have been developed for graph analytics, including assortativity, clustering, centrality, traversal, matching, mining, etc. However, the majority of recent graph processing frameworks only focus on a handful of fix-point graph algorithms such as breadth-first search, PageRank, shortest path, etc. It leaves the distributed computation of a large variety of graph algorithms suffering from low efficiency, limited expressiveness, or high implementation complexity with existing frameworks.**

In this paper, we propose FLASH, a framework for programming distributed graph processing algorithms, which achieves good expressiveness, productivity and efficiency at the same time. Thanks to its high-level interface, FLASH allows users to implement complex distributed graph algorithms with high performance with only a few lines of code. We have implemented 72 graph algorithms for 49 different problems in FLASH. In further evaluations, we found that FLASH beats other state-of-the-art graph processing frameworks with the speedups of up to 2 orders of magnitudes while takes up to 92% less lines of code.

Index Terms—**Graph computing, Programming model, Distributed computing, Code generation**

I. INTRODUCTION

Graph algorithms serve as the essential building blocks for a diverse variety of real-world applications such as social network analytics [2], data mining [3], network routing [4], and scientific computing [5]. As the graph data becomes increasingly huge, there is an urgent need of scaling graph algorithms in the distributed context, and many distributed graph frameworks have emerged to fill in the gap, such as Pregel [6], Giraph [7], GraphLab [8], PowerGraph [9], GraphX [10], Gemini [11] and others [12], [13], [14], [15], [16]. However, as pointed out by [17], all these graph processing frameworks are only evaluated via a handful of specific graph algorithms that are similar in computation patterns. Such evaluation is far from sufficient lacking in the coverage of diversity and usability required from real-world applications. To comprehensively evaluate a distributed graph framework, we carefully consider three metrics that we call **EPE**, namely *expressiveness*, *productivity* and *efficiency*. Specifically,

- Expressiveness represents the capability of the programming interface to express different kinds of graph algorithms. Expressiveness is arguably the most important metric that must be fulfilled in order to support the diverse graph applications in practice.
- Productivity shows the convenience of the interface or the required effort for users to implement their algorithms, which is also important given that otherwise graph computation will be a privilege to a few expertise [16].
- Efficiency refers to the performance factor of executing graph algorithms, which should be concerned because real-world applications on large-scale graphs are often time-consuming and/or memory-intensive.

After thoroughly analyzing existing graph processing frameworks, we find out that none of them has arrived at the proper tradeoff for all the EPE metrics. To pursuit productivity, an abstraction called “thinking like a vertex” (or vertex-centric) was proposed in Pregel [6], and shared among many existing graph processing frameworks. Under this philosophy, graphs are divided into partitions for scalability and updates are bound to the granularity of vertices for parallelization. The vertex-centric implementation of a graph algorithm follows a common iterative, single-phased and value-propagation-based (short of *ISVP*) pattern [17]: the algorithm runs iteratively until convergence, and in each iteration, all vertices receive messages from their neighbors to update their own states, then they send the updated states as messages to the neighbors for the next iteration. Due to the productivity of the vertex-centric model, a lot of graph frameworks follow the philosophy for their abstraction, including the GraphLab variant [8], the Scatter-Gather model [18], and the *GAS* model [9].

Such high-level abstraction brings productivity to some extent to users, however, at the sacrifice of expressiveness and efficiency. Regarding expressiveness, we argue that the abstraction, while designed specifically for the ISVP algorithms, is almost infeasible to be applied to a large variety of algorithms that are not of the kind, such as K-clique Counting, Rectangle Counting, Clustering Coefficient and Bridge Detection, to just name a few. Actually, there are typically tens of algorithms available to representative graph processing frameworks. As a comparison, NetworkX [19], a Python graph library, supports over 328 graph algorithms. At the same time, modern graph scenarios bring in the needs of more advanced and complex graph algorithms, which poses a big challenge for existing graph processing frameworks. Regarding efficiency, it's not surprising that some hardcoded algorithms can run much faster than the high-level alternative implementation on a framework. The general graph frameworks, especially distributed frameworks lose efficiency because of communication/synchronization overhead, load balance issues and higher software complexities [11]. As we evaluated, a hardcoded algorithm for Connected Components [20] can perform orders of magnitude faster than the vertex-centric counterparts. To overturn the efficiency issue, Gemini [11] has been developed that significantly lower the latency of executing certain algorithms. However, the user is often required to program more codes compared to the vertex-centric alternatives, which to some extent harms the productivity. Moreover, in order to exploit the extreme efficiency, some constraints are enforced by Gemini on implementing an algorithm that can deteriorate the expressiveness. For example, Gemini requires an algorithm to communicate only with the neighbors in each iteration, and the core computation must satisfy the *associative* and *commutative* properties. Obviously, a graph framework based on the low-level interfaces such as MPI can provide the optimal result for expressiveness and efficiency, but it may not be appealing to the users in terms of productivity. Another work Ligra [21] and its following work [22] provide a programming model which is more succinct for expressing a class of parallel graph algorithms. However, these frameworks are based on the shared memory machines, thus cannot scale to the same size as distributed ones. And they are still limited in expressiveness for implementing all kinds of non-ISVP algorithms.

Motivated by this, we propose a new distributed graph processing framework called FLASH in this paper, and claim that FLASH finally approaches a sweet spot for all EPE metrics. We still follow the vertex-centric philosophy due to its productivity, but we move a step further by basing the FLASH programming interfaces on Ligra for more succinct and user-friendly coding. Note that Ligra is a single-machine parallel library, and our extension of Ligra to the distributed context is non-trivial, for which we must carefully handle communication, synchronization, data races and task scheduling. To do so, we propose a middleware called FLASHWARE that hides all the above details for distribution, and provides the capability to apply multiple system optimizations automatically and adaptively at the runtime. As for expressiveness, we thoroughly study the issues such as control flow and communication pattern of the vertex-centric-based frameworks on some non-ISVP algorithms, including Betweenness Centrality, Graph Coloring and the optimized Connected Components algorithm [20]. By providing flexible control flow, beyond-neighborhood communication and the operations on arbitrary vertex sets, FLASH significantly improves the expressiveness of distributed graph algorithms. In fact, we have implemented 72 graph algorithms in FLASH for 49 different commonly used applications. Among them, some algorithms are extremely difficult or nearly impossible to implement on existing graph processing frameworks, including the optimized algorithm to compute minimum spanning tree [20], faster betweenness centrality algorithm [23], k-core decomposition [24], graph coloring [25], k-clique counting [26], to just name a few.

TABLE I: Comparison of different models.

Algo. [19]	**Pregel**	**PowerG.**	**Gemini**	**Ligra**	**FLASH**
Expressiveness & Productivity [LLoCs [27], lower is better]					
CC-basic	● 30	● 36	● 50	● 26	● 12
CC-opt	◒ 63	○	○	○	● 56
BFS	● 22	● 25	● 56	● 20	● 13
BC	◒ 49	◒ 162	◒ 139	● 75	● 33
MIS	● 48	● 53	● 112	● 37	● 23
MM-basic	● 57	● 66	● 98	● 59	● 20
MM-opt	◒ 84	○	○	○	● 27
KC	● 35	● 32	○	● 45	● 20
TC	● 31	◒ 181	○	● 38	● 22
GC	● 48	● 58	○	○	● 24
SCC	◒ 275	○	○	○	● 74
BCC	◒ 1057	○	○	○	● 77
LPA	● 51	● 46	○	○	● 26
MSF	◒ 208	○	○	○	● 24
RC	○	○	○	○	● 23
CL	○	○	○	○	● 33

The algorithms are demonstrated in the full version [1]. ● means that it is well-supported; ○ means that we failed to express it; ◒ means that it could be implemented in a non-intuitively way at the cost of performance.

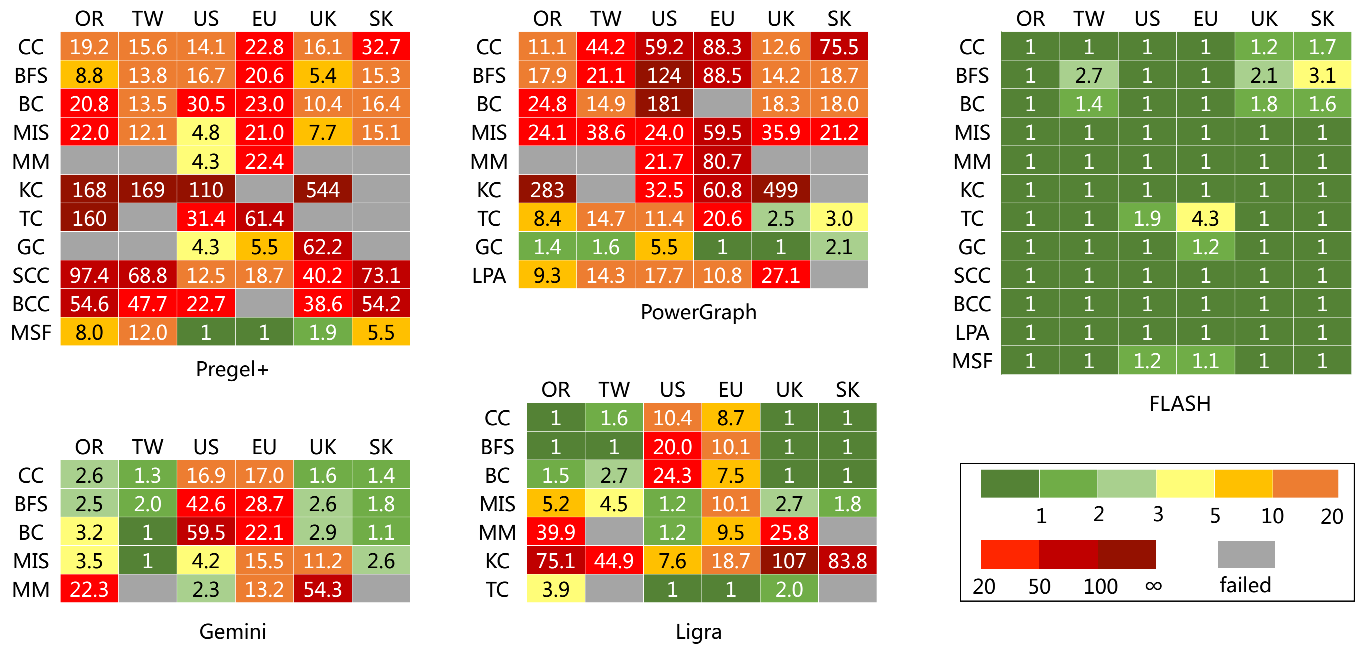
Moreover, thanks to FLASHWARE, we can now program much more succinct codes using the FLASH programming interfaces, which also helps productivity. Table I summarizes the comparison in expressiveness and productivity of FLASH and the representative graph frameworks, while Figure 1 presents the comparison in efficiency. No a single framework can beat FLASH in all metrics, while FLASH does outperform Pregel+ and PowerGraph by a large margin in all aspects.

In summary, we make the following contributions.

(1). We propose the FLASH model, a novel high-level abstraction for programming distributed graph algorithms (Section III). We define its succinct interface after presenting some preliminaries (Section II), and show its expressiveness through some representative examples. With the ability of simulating the vertex-centric models, FLASH can be used to implement existing vertex-centric algorithms. We also make a comparison with other models to highlight FLASH's advantages.

(2). We provide an efficient implementation of the FLASH model (Section IV). It is based on a novel design of the system architecture containing several main components, including a middleware named FLASHWARE for completing communication and updates for the distributed runtime, and a compiler for code generation from the high-level APIs. Moreover, we explore adaptive runtime choices to further enhance the performance, including the density-aware dual-mode propagation scheme, overlapping communication with computation, synchronizing only critical properties, and communicating with only necessary mirrors.

(3). We provide a thorough experimental evaluation of our framework from expressiveness, productivity and efficiency (Section V). The results of different applications on various input graphs demonstrate FLASH's capability of expressing many advanced graph algorithms, while providing a satisfac-



	OR	TW	US	EU	UK	SK
CC	19.2	15.6	14.1	22.8	16.1	32.7
BFS	8.8	13.8	16.7	20.6	5.4	15.3
BC	20.8	13.5	30.5	23.0	10.4	16.4
MIS	22.0	12.1	4.8	21.0	7.7	15.1
MM			4.3	22.4		
KC	168	169	110		544	
TC	160		31.4	61.4		
GC			4.3	5.5	62.2	
SCC	97.4	68.8	12.5	18.7	40.2	73.1
BCC	54.6	47.7	22.7		38.6	54.2
MSF	8.0	12.0	1	1	1.9	5.5

Pregel+

	OR	TW	US	EU	UK	SK
CC	11.1	44.2	59.2	88.3	12.6	75.5
BFS	17.9	21.1	124	88.5	14.2	18.7
BC	24.8	14.9	181		18.3	18.0
MIS	24.1	38.6	24.0	59.5	35.9	21.2
MM			21.7	80.7		
KC	283		32.5	60.8	499	
TC	8.4	14.7	11.4	20.6	2.5	3.0
GC	1.4	1.6	5.5	1	1	2.1
LPA	9.3	14.3	17.7	10.8	27.1	

PowerGraph

	OR	TW	US	EU	UK	SK
CC	1	1	1	1	1.2	1.7
BFS	1	2.7	1	1	2.1	3.1
BC	1	1.4	1	1	1.8	1.6
MIS	1	1	1	1	1	1
MM	1	1	1	1	1	1
KC	1	1	1	1	1	1
TC	1	1	1.9	4.3	1	1
GC	1	1	1	1.2	1	1
SCC	1	1	1	1	1	1
BCC	1	1	1	1	1	1
LPA	1	1	1	1	1	1
MSF	1	1	1.2	1.1	1	1

FLASH

	OR	TW	US	EU	UK	SK
CC	2.6	1.3	16.9	17.0	1.6	1.4
BFS	2.5	2.0	42.6	28.7	2.6	1.8
BC	3.2	1	59.5	22.1	2.9	1.1
MIS	3.5	1	4.2	15.5	11.2	2.6
MM	22.3		2.3	13.2	54.3	

Gemini

	OR	TW	US	EU	UK	SK
CC	1	1.6	10.4	8.7	1	1
BFS	1	1	20.0	10.1	1	1
BC	1.5	2.7	24.3	7.5	1	1
MIS	5.2	4.5	1.2	10.1	2.7	1.8
MM	39.9		1.2	9.5	25.8	
KC	75.1	44.9	7.6	18.7	107	83.8
TC	3.9		1	1	2.0	

Ligra

Fig. 1: A heat map of slowdowns of various frameworks compared to the fastest framework for 12 representative applications on 6 real-world graphs. "failed" means that the execution did not terminate within 5000s or failed due to exhausted memory.

tory performance at the same time. It beats other state-of-the-art graph processing frameworks in 84.5% cases, and may achieve significant speedups of up to 2 orders of magnitudes while takes up to 92% less lines of code.

II. Preliminaries and Related Work

We first define some basic graph notations and discuss the mainstream programming models of graph processing frameworks.

Functions. FLASH uses the functional programming paradigm. A variable with a type is denoted as $var : type$. A function $f(x : X) \mapsto Y$ means that for each input $x \in X$, it has an output $y \in Y$ such that $f(x) = y$.

Graphs. FLASH is defined over the property graph, which can be represented as $G = (V, E)$. V represents a finite set of vertices, and each $v \in V$ has a unique identifier ($v.id \in \mathbb{N}$) and some associated properties. With the cartesian product of sets A and B denoted by $A \times B$ where $A \times B = \{(a, b) : a \in A \wedge b \in B\}$, $E \subseteq V \times V$ represents a set of (directed) edges. For each edge (s, d), the first vertex s is the source of it and the second vertex d is the target. The number of vertices in a graph is denoted by $|V|$ and the number of edges is denoted by $|E|$. We denote a weighted graph by $G = (V, E, w)$, where w is a function which maps an edge to a real value, thus each edge $e \in E$ is associated with the weight $w(e)$.

Graphs partitions. In a distributed cluster containing m workers, the input graph will be partitioned into m partitions (also called subgraphs), with each worker holding one of the partitions ($P_i = (V_i, E_i)$ for worker i). A m-way partition scheme for the graph $G(V, E)$ should guarantee that $V = \cup_{i=1}^{m} V_i$ and $E = \cup_{i=1}^{m} E_i$. There are two main schemes for partitioning the graph, namely *edge-cut* and *vertex-cut*, and we use *vertex-cut* which is more common in existing works. To be specific, each vertex belongs to only one partition, so that $V_i \cap V_j = \emptyset$, for $i \neq j$. Two endpoints of an edge may belong to different partitions. To this end, FLASH uses the master-mirror notion as PowerGraph [9] proposed: each vertex is assigned to and owned by one partition, where it is a *master*, as its primary copy. While in other partitions, there may also be replicas for this vertex, called *mirrors*, which are used for update propagation and data synchronization. We will explain this more specifically in Section IV.

Graph algorithms. A graph algorithm takes a graph G as input, does processing and analytics on it to solve real-world problems. As assumed in many parallel graph processing frameworks, a graph algorithm updates the information stored in vertices, while edges are viewed as immutable objects. Generally, A graph algorithm propagates updates along the edges (original edges of G, or virtual edges which are generated dynamically during the execution) iteratively, until the convergence condition is met or a given number of iterations are completed. Vertices with ongoing updates are called *active vertices* (or *frontiers*), with outgoing edges from them called *active edges*. This paper considers the Bulk Synchronous Parallel (BSP) model [28], where an algorithm consists of a series of supersteps. In each superstep, computation and communication (message passing) take place on active vertices. The supersteps are executed synchronously, so that messages sent during one superstep are guaranteed to be delivered in the beginning of the next superstep.

Pregel. There are many parallel programming abstractions for processing large graphs [29], [30], [17]. Pregel [6] proposed a vertex-centric abstraction which is designed based on the

BSP model. By distributing vertices and associated adjacent edges to each worker, the Pregel framework has fixed routines for all the algorithms. In each superstep, a Pregel program calls the `compute()` on each vertex, which performs the predefined user-specific computation. During the `compute()`, each vertex processes the incoming messages from the previous superstep, then sends messages to other vertices. The communication is based on message-passing, with messages can be aggregated if the user provides a `combine()` function. The early-aggregation of messages can reduce the number of messages to be materialized, thus improves the performance and reduces the memory usage. The Pregel model greatly simplifies the parallel graph algorithms. Therefore, it almost becomes a standard of large graph processing, followed by many graph processing frameworks [7], [13], [12], [10], [11].

GAS. GraphLab [8] and PowerGraph [9] are also based on the vertex-centric abstraction. They proposed the "Gather-Apply-Scatter" (GAS) model to further simplify the graph algorithms, but at the cost of limiting the data-exchange to only happen between adjacent vertices. GAS hides the communication details from programmers, and the users only have the view of each vertex and its neighbors, which means that the control flow of a graph algorithm is highly rigid. Thus, GAS is less expressive but more effortless than Pregel.

As we have described above, traditional vertex-centric models (such as Pregel and GAS) do not achieve enough expressiveness and productivity to program for many advanced non-ISVP graph algorithms. They do not support the users to define the custom control flow, which is important for multi-phased algorithms. Besides, these models force the programmers to *think like a vertex*, thus loss the global perspective, for example, to maintain a group of specific vertices. The Betweenness Centrality algorithm [23] requires both of the flexible control flow and recording the frontiers in each step, consequently, it cannot be easily expressed in these models. Actually, the implementation of this algorithm provided by [11] needs more than 400 lines of code in total.

Ligra. Ligra [21] proposed a new model that supports a ***vertexSubset*** type. It represents a subset of vertices $U \subseteq V$ of the graph G. Based on this data structure, it is easy to apply the update logics on any subset of vertices every time. It exposes the control flow to the users, supporting arbitrary combination of these operations during the execution. Ligra is proved to be useful when programming a wide variety of parallel graph algorithms in the shared-memory environment.

Nevertheless, Ligra is still limited in some aspects. Since the communication is through the EDGEMAP interface, only the messages along the edges of G are possible. However, communication beyond neighborhood or along a set of specific edges is an important feature in some advanced graph algorithms [20]. Moreover, Ligra is designed for shared-memory systems, thus lacks the distributed semantics. It requires the programmers to use atomic instructions (e.g., the compare-and-swap instruction), which are too low-level and not fitting in the distributed scenario.

III. Programming Model

As depicted in Section II, previous frameworks fail to achieve the **EPE** metrics due to fixed control flow, neighborhood-exchange limitation, the limitation of the associative and commutative aggregation and the dependence on the shared-memory architecture. To address the challenges, we propose the FLASH programming model by making a sufficient extension to Ligra's programming model. For supporting communication beyond neighborhood, FLASH allows the communication possible to happen on arbitrary vertex pairs. In this section, we first show the interface of FLASH, and how to flexibly express graph algorithms with the given primitives. Then we show that our FLASH model can be fully compatible with the well-known vertex-centric models. Finally, we highlight some characteristics of FLASH to show its advantages over the classical programming models.

A. Interface

FLASH is a functional programming model specific for distributed graph processing. It follows the Bulk Synchronous Parallel (BSP) computing paradigm [28], with each of the primary functions (SIZE, VERTEXMAP and EDGEMAP) constitutes a single superstep. We made the interface of FLASH much similar to Ligra's, therefore, it is easy to port a program written in Ligra to our model. The *vertexSubset* type represents a set of vertices of the graph G, which only contains a set of integers, representing the vertex id for each vertex in this set. The associated properties of vertices are maintained only once for a graph, shared by all *vertexSubset*s. The following describes the APIs of FLASH based on this type.

(1). SIZE$(U : vertexSubset) \mapsto \mathbb{N}$

This function returns the size of a *vertexSubset*, i.e., $|U|$.

(2). VERTEXMAP$(U : vertexSubset,$
$F(v : vertex) \mapsto bool,$
$M(v : vertex) \mapsto vertex) \mapsto vertexSubset$

The VERTEXMAP interface applies the map function M to each vertex in U that passes the condition checking function F. The ids of the output vertices form the resulting *vertexSubset*. That is to say, we have:

$Out = \{v.id | v.id \in U \wedge F(v) = true\}$
$v^{new} = M(v),\ v.id \in U \wedge F(v) = true$

Since in the scope of M, only data of the vertex itself is available, this function is used to conduct local updates. Specially, the M function could be omitted, which represents that the vertex data attached will not be changed. This is common when implementing the *filter* semantics, where F executes the condition checking. The execution of VERTEXMAP on each vertex is independent and without any communication, thus this function can run in parallel naturally, as shown in Algorithm 1.

(3). EDGEMAP$(U : vertexSubset,$
$H : edgeSet,$
$F(s : vertex, d : vertex) \mapsto bool,$
$M(s : vertex, d : vertex) \mapsto vertex,$
$C(v : vertex) \mapsto bool,$
$R(t : veretex, d : vertex) \mapsto vertex) \mapsto vertexSubset$

Algorithm 1 VERTEXMAP

```
function VERTEXMAP(U, F, M)
    $Out = \{\}$
    parfor $u.id \in U$ do
        if $(F(u) == true)$ then
            $u^{new} = M(u)$
            Add $u.id$ to $Out$
    return $Out$
```

The EDGEMAP interface in Ligra is used to transfer messages between neighbor vertices. We extend and redefine this interface for stronger expressiveness and the support for distributed semantics. For a graph $G = (V, E)$, EDGEMAP applies the update logic to the specific edges with source vertex in U and target vertex satisfying C. H represents the edge set to conduct updates, which is E in common cases. We allow the users to define arbitrary edge sets they want dynamically at runtime, even virtual edges generated during the algorithm's execution. The edge set can be defined through defining a function which maps a source vertex id to a set of ids of the targets. We also provide some pre-defined operators for convenience, such as reverse edges (`reverse(E)`), two-hop neighbors (`join(E, E)`), or edges with targets in U (`join(E, U)`). This extension makes the communication beyond the neighborhood-exchange limitation.

If a chosen edge passes the condition checking (F), the map function M is applied on it. The output of the function M represents a temporary new value of the target vertex. This new value is applied immediately and sequentially if it is in the *pull* mode, while in the *push* mode, another parameter R is required to apply all the temporary new values on a specific vertex to get its final value. It is unnecessary in the Ligra's API because Ligra is a shared-memory framework, which uses atomic operations to ensure consistency. On the contrary, the FLASH model is designed for distributed systems. Therefore, we use a reduce function R, which takes an old value and a new value for a single vertex, and reduces them to output the updated state. The updated target vertices form the output set of EDGEMAP. The reduce function R is required to be associative and commutative to ensure correctness. Or it is not required to provide R for sequentially applying the function M, i.e., to run EDGEMAP always in the *pull* mode, as we will explained in Section III-C.

More precisely, the active edge set is defined as:

$$E_a = \{(s, d) \in H | s.id \in U \land C(d) = true\}.$$

Then, F and M is applied to each element in E_a. If it is in the *pull* mode:

$$d^{new} = M(s, d^{new}), (s, d) \in E_a \land F(s, d^{new}) = true.$$

Or, in the *push* mode:

$$T = \{M(s, d) | (s, d) \in E_a \land F(s, d) = true\},$$

$$d^{new} = R(..., R(T_2^d, R(T_1^d, d))), T_i^d \in T \land T_i^d.id = d.id.$$

And the ids of the updated targets form output set:

$$Out = \{d.id | (s, d) \in E_a \land F(s, d) = true\}.$$

The function C is useful in algorithms where a value associated with a vertex only needs to be updated once. We retain the default function used by Ligra (CTURE) which always returns true, since the user does not need this functionality sometimes. Similarly, the F function of EDGEMAP and VERTEXMAP can also be supplied using CTURE, if it is unnecessary.

The auxiliary operators. Other auxiliary APIs are provided by FLASH for conveniently conducting set operations, including UNION, MINUS, INTERSACT, ADD, CONTAIN and so on.

Expressing other programming models. FLASH has the ability to simulate the traditional vertex-centric programming models. As a consequence, it is possible to port existing vertex-centric programs in our model. Below we outline a proof (see the full version [1] for details).

The typical execution workflow of the vertex-centric model proceeds in synchronized iterations (or *supersteps*). In each superstep, all active vertices (called *frontiers*) execute the same user-defined vertex function in parallel, which receives a set of messages as input (*inbox*) and can produce one or more messages as output (*outbox*) . At the end of a superstep, the runtime receives the messages from the *outbox* of each vertex and computes the set of active vertices for the next superstep. The execution terminates when there are no active vertices or when some convergence condition is met. The local computation in each superstep can be implemented in FLASH through a VERTEXMAP function, which processes the *inbox* and produces the updated value as well as the *outbox* for each vertex, and a following EDGEMAP function adds a message to the *inbox* of the target.

B. Examples

To demonstrate the usage of FLASH and display its ability to express different algorithms, we show several representative examples in this section. Please refer to the full version [1] for more examples.

Breadth First Search (BFS). As with standard parallel BFS algorithms [31], [32], we implement a BFS algorithm in FLASH, as shown in Algorithm 2. For each vertex, a property named dis is created and initialized to represent the distance from the root to this vertex. On each iteration/superstep, we maintain a frontier. More specifically, for superstep i (starting from 0), the frontier U_i contains all vertices reachable from the root in i hops (i.e., $v.dis = i$, $\forall v \in U_i$). At the beginning of the algorithm, a *vertexSubset* that only contains the root is created, representing the frontier. To use a global variable such as r in a local function, we provide a *bind* operator to supply additional input parameters (line 11). In each of the following supersteps, the EDGEMAP function is applied on outgoing edges of the frontier, to check if any neighbor d of an active vertex s is visited. If the neighbor d has not been visited, updates it and then adds this vertex to the next frontier. The COND function tells EDGEMAP to only consider the neighbors that have not been visited. Although it could be replaced by the default condition function CTRUE, we provide this function for efficiency. As dis for a vertex d is ensured to be same no matter it is updated by which neighbor in the same superstep, we can simply remain any new value for it in the REDUCE function. The iterative execution will terminate when there are no vertices in the frontier, means that all reachable vertices from the root have been visited.

Algorithm 2 Breadth-First Search

```
function INIT($v, root$):
    $v.dis = (v.id == root ? 0 : INF)$
    return v
function FILTER($v, root$): return $v.id == root$
function UPDATE($s, d$):
    $d.dis = s.dis + 1$
    return $d$
function COND($v$): return $v.dis == INF$
function REDUCE($t, d$): return $t$

$U$ = VERTEXMAP($V$, CTRUE, INIT.bind(0))                    ▷ initialize
$U$ = VERTEXMAP($V$, FILTER.bind(0))                          ▷ root=0
while SIZE($U$) $\neq 0$ do
    $U$ = EDGEMAP($U, E$, CTRUE, UPDATE, COND, REDUCE)
```

Algorithm 3 Betweenness Centrality

```
function INIT($v, r$):
    if ($v.id == r$) then $v.level = 0$, $v.num = 1$, $v.b = 0$
    else $v.level = -1$, $v.num = 0$, $v.b = 0$
    return $v$
function FILTER($v, r$): return $v.id == r$

function UPDATE1($s, d$):
    $d.num = d.num + s.num$
    return $d$
function COND1($v$): return $v.level == -1$
function R1($t, d$):
    $d.num = d.num + t.num$
    return $d$

function LOCAL($v, curLevel$):
    $v.level = curLevel$
    return $v$

function F2($s, d$):
    return $d.level == s.level - 1$
function UPDATE2($s, d$):
    $d.b = d.b + \frac{d.num}{s.num} * (1 + s.b)$
    return $d$
function R2($t, d$):
    $d.b = d.b + t.b$
    return $d$

function BC($S$, $curLevel$)
    if (SIZE($S$) == 0) then return
    $A$ = EDGEMAP ($S, E$, CTRUE, UPDATE1, COND1, R1)
    $A$ = VERTEXMAP ($A$, CTRUE, LOCAL.bind($curLevel$))
    BC($A, curLevel + 1$)
    $A$ = EDGEMAP ($S$, reverse($E$), F2, UPDATE2, CTRUE, R2)

$r = 0$                                                   ▷ choose a BFS root
$U$ = VERTEXMAP($V$, CTRUE, INIT.bind($r$))                 ▷ initialize
$U$ = VERTEXMAP($V$, FILTER.bind($r$))                      ▷ create the frontier
BC($U$, 1)                    ▷ calculate the betweenness centrality scores
```

Betweenness Centrality (BC). The betweenness centrality index [33] is useful in social network analysis and has been well studied. Precisely, the betweenness centrality of a vertex v, denoted by $C_B(v)$, is equal to $\sum_{s \neq v \neq t \in V} \delta_{st}(v)$. The pair-dependency $\delta_{st}(v)$ is calculated by $\frac{\sigma_{st}(v)}{\sigma_{st}}$, where σ_{st} is the number of shortest paths from s to t, and $\sigma_{st}(v)$ is the number of shortest paths from s to t that pass through v. To compute the betweenness centrality scores, Brandes [23] presents an algorithm which decrease the number of operations from $O(|V|^3)$ to $O(|V||E| + |V|^2)$ for unweighted graphs.

This algorithm works in two phases, the first phase uses a BFS-like procedure to calculate the number of shortest paths form r to each vertex, and the second phase computes the dependency scores through a backward propagation. Since the frontiers visited in every step of the first phase need to be tracked, it is difficult to directly implement this algorithm in a traditional vertex-centric model which does not supply a *vertexSubset* structure. In contrast, its implementation in FLASH is intuitively, as Algorithm 3 shows. In the second phase, the edges are need to point in the reverse direction, so we define the edge set for the EDGEMAP to be `reverse(E)`.

Connected Components (CC). A weakly connected component is a maximal subgraph of a graph such that for every pair of vertices in it, there is an undirected path connecting them. In existing vertex-centric models, the standard method for calculating CC is label propagation. In this algorithm, each vertex is attached with a property which represents its component label, being its own vertex id initially. In the subsequent supersteps, a vertex will update its label if it receives a smaller id and then it propagates this id to all its neighbors. This algorithm is both simple and scalable, but not necessarily efficient. As the label is propagated only one hop at a time, it may require many iterations to converge, especially for graphs that have large diameters.

An optimized CC algorithm [20] is proposed to overcome this problem. It utilizes a parent pointer $p(v)$ for each vertex to maintain a tree (forest) structure, with each rooted tree represents a connected component of the graph. In each iteration, the algorithm uses $StarDetection$ to identify stars (tree of depth one), in which every vertex points to one common rooted vertex (the rooted vertex is self-pointing); then it merges stars that are connected via some edges using two $StarHooking$ operations; finally it applies $PointerJumping$ to assign $p(v) = p(p(v))$. When the algorithm terminates, there must be isolated stars, each standing for one connected component with the rooted vertex as its id. In this algorithm, virtual edges are generated to maintain the tree structure. Since the messages are not always along the original edges, it could not be implemented in the models that do not support communication beyond neighborhood. While in FLASH, this algorithm is expressed without much effort.

Maximal Matching (MM). In graph theory, a matching in an undirected graph is a set of edges without common vertices. And a maximal matching is a matching of a graph G that is not a subset of any other matching. This problem could be solved in a greedy algorithm which always tries to build a match for each unmatched vertex (called temporary match) in the first phase. And then, if two vertices are matched each other, they will be added to the result in the second phase, means the temporary match is successful. This algorithm executes iteratively until no more vertices could be added to the result. Also, we can implement an optimized version for this algorithm (named MM-opt) by conducting computation for an unmatched vertex only if its temporary matched vertex is matched successful in the last iteration. This is done by executing the EDGEMAPSPARSE operation from successfully matched vertices targeting their specific neighbors (through defining the edge set of EDGEMAPSPARSE based on the temporary matching results).

C. Advantages of FLASH

Dual update propagation model. During graph processing, the type of an active set may be *dense* or *sparse*, typically determined by the size of active vertices and associated outgoing edges. Ligra dispatches different computation kernels for different types of the active set. More specifically, it adopts the *push* mode for the sparse active sets and the *pull* mode for the dense active sets. FLASH follows this design and extend it to fit the distributed scenario, using an adaptive switching between these two modes according to the size of active sets.

As shown in Algorithm 4, the EDGEMAP function calls one of EDGEMAPDENSE (Algorithm 5) and EDGEMAPSPARSE (Algorithm 6) according to the density, which implements the *pull* mode and the *push* mode, respectively. The users can set the threshold to decide if it is dense. EDGEMAPDENSE loops all vertices in the graph in parallel and for each vertex v, it sequentially applies the F function and the M function for its neighbors that are in U and the edges are in H, until C returns false. After that, its id will be added to the result. Since all updates are applied immediately in this procedure, the R function is omitted in EDGEMAPDENSE.

Algorithm 4 EDGEMAP

```
function EDGEMAP(U, H, F, M, C, R)
    if (the density of $U >$ threshold) then
        return EDGEMAPDENSE (U, H, F, M, C)
    else
        return EDGEMAPSPARSE(U, H, F, M, C, R)
```

Algorithm 5 EDGEMAPDENSE

```
function EDGEMAPDENSE(U, H, F, M, C)
    $Out = \{\}$
    parfor $v.id \in V$ do
        $v^{new} = v$
        for $(ngh, v) \in H$ do
            if $(C(v^{new}) == 0)$ then break
            if $(ngh.id \in U$ and $F(ngh, v^{new}) == 1)$ then
                $v^{new} = M(ngh, v^{new})$
                Add $v.id$ to $Out$
    return $Out$
```

Algorithm 6 EDGEMAPSPARSE

```
function EDGEMAPSPARSE(U, H, F, M, C, R)
    $Out = \{\}, Tmp = \{\}$
    parfor $u.id \in U$ do
        parfor $(u, ngh) \in H$ do
            if $(C(ngh) == 1$ and $F(u, ngh) == 1)$ then
                Add $(M(u, ngh), ngh.id)$ to $Tmp$
                Add $ngh.id$ to $Out$
    parfor $v.id \in Out$ do
        $v^{new} = v$
        for $(t, v.id) \in Tmp$ do
            $v^{new} = R(t, v^{new})$
    return $Out$
```

On the contrary, EDGEMAPSPARSE loops all vertices in the active set U in parallel and for each vertex $u \in U$, it executes $F(u, ngh)$ and $M(u, ngh)$ in parallel to update its qualified neighbors. If a neighbor is updated, it will be added to Out. As a vertex may be updated by different neighbors at the same time in a single EDGEMAPSPARSE function, all the new values will be applied on the target vertex through the R function.

This auto-switch scheme is proved to be useful for real-world graphs and is also adopted by some other works [34], [35], [11]. Also, FLASH 's dual mode processing is optional: users may choose to execute in only one of the two modes through calling EDGEMAPDENSE or EDGEMAPSPARSE, instead of calling EDGEMAP.

The *vertexSubset* type. Besides expressing existing algorithms that follow the vertex-centric models, FLASH provides the possibility of expressing more advanced algorithms. FLASH is the first distributed graph processing model to provide the *vertexSubset* type, which is a global-perspective data structure supplementing the perspective of a single vertex. By maintaining a set of vertices, it can conduct updates on arbitrary vertices. Moreover, multiple vertex sets can be maintained at the same time in a program, they can even be defined in a recursive function. On the contrary, when other vertex-centric frameworks implement such algorithms, they have to start from the whole graph every time and pick up specific vertices.

Flexible control flow. Another important difference between FLASH and other vertex-centric models is that we allow the users to define the arbitrary control flow by combining the primitives, thus FLASH can naturally support multi-phased algorithms. In traditional vertex-centric models, these algorithms are supported in an awkward way since previous frameworks only allow to provide a single user-defined function.

To make matters worse, in some works, global variables are not supported (e.g., the number of iterations). Therefore, such global states have to be duplicated on each vertex, which is not efficient and user friendly. FLASH enables the users to program multi-phased algorithms by chaining SIZE, EDGEMAP and VERTEXMAP, as well as supports to use global variables.

Communication beyond neighborhood. FLASH makes an extension to Ligra by allowing the users to provide the arbitrary edge set they want to transfer messages, even when the edges do not exist in the original graph. While in most vertex-centric models (e.g., GAS), communication is only allowed between immediate neighbors in the graph. Therefore, these models fail to express some advanced graph algorithms. By defining the edge set in EDGEMAP, algorithms that contain communication beyond neighborhood can be expressed intuitively, such as the optimized CC algorithm [20].

IV. SYSTEM DESIGN AND IMPLEMENTATION

To realize the programming model above, we design and implement a new distributed framework. In this section, we first describe the system architecture. The implementation details of the main components will also be presented. And then, we will introduce some system optimizations we have explored.

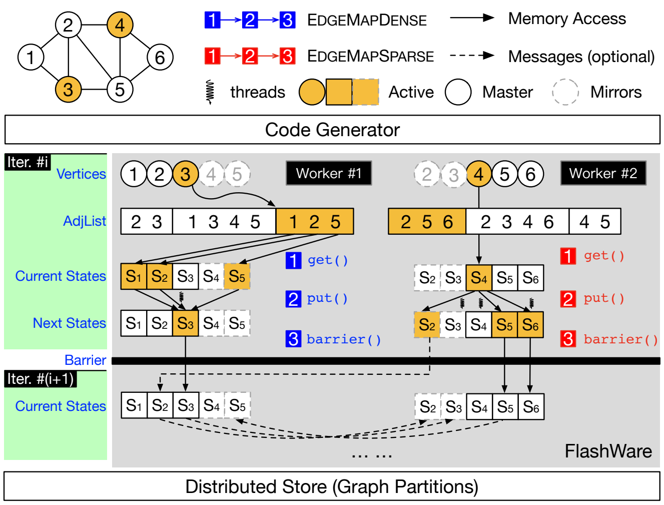


Fig. 2: System architecture and the design of FLASHWARE.

There are several main components in the framework, as shown in Figure 2. The first is a code generator which takes the high-level FLASH APIs as input, and generates execution code to be run on the second component named FLASHWARE, which is a middleware designed and optimized for the distributed graph processing. The FLASHWARE executes the code produced by the code generator on the distributed runtime, which contains multiple CPUs of the cluster machines. Each process acts as an individual worker and could contain multiple working threads. Each worker processes a fragment of the distributed computation, and the communication between different workers is implemented through MPI.

A. FLASHWARE

FLASHWARE is designed as a middle layer that completes intra-node updating and inter-node communication (message passing). FLASHWARE enables FLASH's interface to hide the details of communication and data distribution, as well as provides the capability to apply multiple system optimizations automatically and adaptively at the runtime.

Data layout. The graph is partitioned using an *edge-cut* scheme, as we described in Section II. Every worker is assigned with a set of disjoint vertices (and the associated edges). For a worker, the vertex data is held in its memory. While for the edges, it depends: if there are enough memory capacity, the edges are cached in memory all the time; otherwise, they will be maintained in the disks, only loaded when necessary.

Suppose an m-worker cluster, the given graph $G = (V, E)$ will be partitioned into m partitions. V_i is the vertex subset held by the i-th worker, with each vertex in V_i owns a *master* on this worker. There are also mirrors created for remote vertices (i.e., vertices assigned to other workers). Figure 2 gives an example, where the first three vertices locate at worker #1, and the other three vertices belong to worker #2. For each *vertexSubset*, a worker simply maintains a set of vertex *ids*, representing the master vertices in the set that locate on it.

FLASH's execution is based on the BSP model, hence FLASHWARE distinguishes the *current states* and the *next states*. The current states of a vertex are ensured to be consistent on all workers who access it in the current superstep. On the other hand, the updated values are written in the next states, which is not visible to other vertices in the current superstep and may be different between workers. To save the memory, the next states are only created when necessary.

Interface. FLASHWARE exposes several APIs to the upstream:

(1). $\boldsymbol{get}(id : \mathbb{N}) \mapsto vertex$

The first function is used to access a specific vertex with id through reading the current states. Since this kind of data is consistent, a worker can access arbitrary vertices safely, no matter masters or mirrors.

(2). $\boldsymbol{put}(id : \mathbb{N},$
$\quad v : vertex,$
$\quad R(new : vertex, old : vertex) \mapsto vertex) \mapsto None$

Another function is used to update the vertex with id using a new value v. It is completed through writing the next states. Since concurrency updates may happen (in EDGEMAPSPARSE), a reduce function R may be required. In this case, the new value will be aggregated with the old value to get the updated result.

(3). $\boldsymbol{barrier}() \mapsto None$

The final function $barrier()$ acts as a block similar to $MPI\_Barrier$. It forces the workers to wait until all workers have completed the processing for the current superstep. When it finishes, all updates in this superstep become visible in the current states for starting a new superstep.

Usages. To demonstrate the usages of FLASHWARE, we discuss the three primary operations of our programming model. VERTEXMAP is used to conduct local computation, it $gets$ data of a master vertex, and updates that vertex by put, no need of the reduce function. EDGEMAPDENSE is similar, except that it also reads the current states of other vertices, instead of the central vertex only, as Figure 2 shows. Both of VERTEXMAP and EDGEMAPDENSE make updates for the masters, thus messages are generated adaptively to synchronize these updates to related mirrors. The EDGEMAPSPARSE function is more complex. A working thread reads the current states for master vertices and may update arbitrary vertices. Therefore, the reduce function R is required for aggregating multiple updates. This procedure is three-phased: a mirror first merges updates to form a temporary new value for this vertex; and then mirrors send messages to the master which processes these messages to decide the final state; a master broadcasts its final state to necessary mirrors. As a result, there are two rounds of message-passing in EDGEMAPSPARSE.

B. Code Generation

The code generator in FLASH is responsible for generating code to be executed by FLASHWARE, from the high-level APIs which the users provide. It also enables some optimizations through static analysis of the program, for example, to decide the critical attributes, as we will discuss specifically in the next section. Take the VERTEXMAP as an illustration, we now explain the logics of the code generation. The code generation

TABLE II: The rules to decide critical properties.

	VERTEXMAP	EDGEMAPDENSE		EDGEMAPSPARSE	
		source	target	source	target
get	×	✓	×	×	✓
put	×	–	×	–	✓

"✓" means that the property is decided to be critical; "×" means that cannot decide yet; "–" means that this kind of operation never happens.

for EDGEMAP is similar but requires more function calls of get and put. A VERTEXMAP contains calling $get()$ and $put()$ on the master vertices in parallel inside a worker. Once a master vertex is updated, messages are generated adaptively to synchronize the new state to its mirrors. The $barrier()$ is called at last to ensure that local computation is completed on each worker, and all messages are transferred and processed. After that, the superstep for this VERTEXMAP finishes.

C. Optimizations

Overlap communication with computation. For a worker with c cores, it maintains a thread pool containing c threads. Among them, one thread is responsible for inter-worker message sending and one is for receiving via MPI. The other threads perform parallel vertex-centric processing. As separate threads are created to execute message passing, the communication is overlapped with computation, leading to the performance improved.

Take the VERTEXMAP function as an example, the local computation is conducted on each vertex in parallel. Once a vertex has been processed and updated (i.e., applying the function M on its master), the new value will be immediately added to the buffer which contains messages to be sent. The sending thread continually picks up messages from the sending buffer, and send the batched messages to another worker. At the same time, the receiving thread checks if there are messages from each of the other workers in turn. The messages will be processed by updating target mirrors. In this way, the computation and communication tasks are co-scheduled.

Synchronize critical properties only. In some cases, there are multiple vertex properties, but not all of them are *critical*. We say a property critical only if it is accessed by other vertices, thus the update to the master need to be broadcasted to its mirrors. On the contrary, if a property is only read by the master, means that it is only useful in local computation, it is not critical. For such properties, they are stored locally with the master vertex inside a partition, while the mirrors do not hold such data. This optimization leads to a reduction on both the network traffic and the memory consumed.

More precisely, given a vertex property, we can decide if it is critical through static analysis of the program before the execution. Table II gives the rules based on a classification of the operations on a property. If and only if it is got as the property of the source vertex in an EDGEMAPDENSE function, or it is got/putted as the property of the target vertex in an EDGEMAPSPARSE function, it is critical.

Communicate with necessary mirrors only. Another intuitive way to eliminate redundant messages is to communicate with only the necessary mirrors. For normal graph applications, the messages are transferred along the edges. Therefore, the master of a vertex should only communicate with its mirrors that located in the partitions that containing at least one neighbor of this vertex. Only in the cases that the programmers define virtual edges for EDGEMAP, which beyond the scope of the edge set E, FLASHWARE synchronizes the update on a master to the mirrors in all partitions.

V. EVALUATION

In this section, we present the evaluation results to show the superiority of FLASH over previous works in terms of expressiveness, productivity, and efficiency. (1) For expressiveness, we choose 14 representative graph applications from [19] and implement them in FLASH. Each of them represents a general category of graph algorithms and is frequently used in real-world applications, which demonstrates that FLASH has a large range of applicability. Some applications are rarely provided in previous works, indicating FLASH's strong expressiveness for programming complex algorithms. (2) For productivity, we try our best to implement algorithms for these applications in state-of-the-art graph frameworks, and show the *Logical Line of Codes* (LLoCs) [27] for implementing them. Due to the limitations of the expressiveness in previous frameworks, we can only implement the sub-optimal version for some algorithms, or even failed to implement the naivest version in them. (3) For efficiency, we compare the execution time of FLASH with other graph processing frameworks. The experimental results show that FLASH achieves the best overall performance. It beats other frameworks with the speedups of up to 2 orders of magnitudes. Besides, we conduct some micro benchmarks to reveal why FLASH runs faster in most of cases.

A. Experimental Setup

We compare FLASH with state-of-the-art graph processing frameworks using following setups:

Platform. All our experiments are conducted on a 4-node Intel(R) Xeon(R) CPU E5-2682 v4 based system. Each node has 32 cores running at 2.50 GHz and 512GB of main memory. All nodes are connected with a 10Gb ethernet. In order to perform the comparison, the latest version of Pregel+, PowerGraph, Gemini and Ligra are installed on the cluster. We leverage OpenMP to manage threads and Open MPI to manage message passing between nodes.

Datasets. We use a collection of real-world graphs, including social networks (SN), web graphs (WG), and road networks (RN). The basic characteristics of each dataset are illustrated in Table III. For the social network, it is characterized by a very skew distribution of edges, usually with some "hot" vertices having an extremely high degree. For the road network, it has a very long diameter and fewer associated edges per vertex. And the web graph lies somewhere in middle. For unweighted graphs, random weights are added to each of the edges if necessary.

TABLE III: A collection of real-world graphs.

Abbr.	Dataset	|V|	|E|	Diameter	Domain
OR [36]	soc-orkut	3.07M	117M	9	SN
TW [37]	soc-twitter	41.7M	1.47B	15	SN
US [38]	road-USA	23.9M	28.9M	1452	RN
EU [38]	europe-osm	50.9M	54.1M	2037	RN
UK [38]	uk-2002	18.5M	298M	25	WG
SK [38]	sk-2005	50.6M	1.95B	23	WG

TABLE IV: A collection of representative graph applications.

Abbr.	Application	Abbr.	Application
CC	connected components	BFS	breadth-first search
BC	betweenness centrality	MIS	maximal independent set
MM	maximal matching	KC	k-core decomposition
TC	triangle counting	GC	graph coloring
SCC	strongly CC	BCC	biconnected components
LPA	label propagation	MSF	minimum spanning forest
RC	rectangle counting	CL	k-clique counting

Applications. We choose 14 representative graph applications to demonstrate FLASH's ability for programming distributed graph algorithms effortlessly and effectively, as shown in Table IV. CC and BFS are well supported by all of the tested frameworks, because the ISVP algorithms perform well enough for most cases, and there is no need of high expressiveness. We also implemented an optimized CC algorithm [20] in FLASH since it performs better on large-diameter graphs. For BC, MIS and MM, every system we evaluated is able to express a basic algorithm correctly, but they failed to express the advanced versions of these algorithms (e.g., for MM), suffering either poor performance or complicated programs. FLASH is not only able to implement the basic version with less effort, but also make the advanced version possible. As for KC, TC and GC, even the naivest algorithms are not feasible to implement in some frameworks, while FLASH is able to implement them with less effort, because of the support for non-ISVP algorithms. As for SCC, BCC, LPA, MSF, RC and CL, they are rarely implemented and tested in existing graph frameworks, and it is often failed to implement them. While it is easy for FLASH to do so.

Baselines. Four representative state-of-the-art graph processing frameworks are tested as the baselines: Pregel+, PowerGraph, Gemini and Ligra. Pregel+ [13] is the representative framework that uses Pregel's vertex-centric model. It is not just an open-source Pregel implementation, but a substantially improved distributed system with effective message reduction. Compared with other Pregel-like frameworks (e.g., Giraph [7] and GPS [12]), Pregel+ provides simpler interface and higher efficiency. PowerGraph [9] is the representative framework that adopts the GAS programming model. Gemini [11] is a state-of-the-art distributed graph processing system, which is reported to significantly outperform all well-known existing distributed graph processing frameworks, however, its expressiveness is weaker than other vertex-centric frameworks. Ligra [21] provides similar interfaces compared with FLASH's, such as the flexible control flow and the *vertexSubset* type. Since Ligra is a shared-memory system, the experiments for Ligra are conducted on a single node of the cluster.

These frameworks provide some pre-optimized built-in algorithms, but none of them provides implementations for all the applications listed in Table IV, due to their limitations of expressiveness. For fair comparison, we test the built-in algorithms if they are provided, or, we try our best to implement them in these frameworks. Note that some frameworks provide multiple implementations for an application, *e.g.,* PowerGraph provides a hash-table-based algorithm and an edge-iterator-based one for TC. We report the best performance of these variants when conducting tests on them.

B. Overall Performance

Table V reports the results on overall execution time of the first eight applications on six datasets. All these tests are conducted on a 4-node cluster except Ligra, which only uses a single node. For fair comparison, the initial pre-processing time (for reading and partitioning the original data) and post-processing time (e.g., writing the final results) of every framework are not recorded. Specially, Pregel+ may decompose the algorithm (*e.g.,* BC, SCC, and BCC) into several individual sub-algorithms and chain them by taking the output of the previous as the input of the next. In this situation, the data sharing time (*e.g.,* reading and writing the results of sub-algorithms) among sub-algorithms will be recorded. Table VI reports the results of the other six applications on six datasets. Since the algorithms for these applications are difficult or not supported to be implemented in previous works, we only compare the performance of FLASH with the most efficient implementation provided by other frameworks (Pregel+ for SCC, BCC and MSF, and PowerGraph for LPA).

As we can see, FLASH achieves higher performance compared with state-of-the-art works. For 84.5% cases, it is faster than all other compared frameworks; for 95.2% cases, it provides competitive performance compared with the one that performs best (within $2\times$ slowdown). FLASH can achieve significant speedups over existing works, up to 2 orders of magnitude. For example, when calculating MM on the TW dataset, FLASH only takes 25.15 seconds, while all the other frameworks failed to get the results within 5000 seconds. Another example is SCC, which requires complex computation and is only provided by Pregel+ as far as we know. The implementation in Pregel+ is $22.7\times$ to $54.6\times$ slower than FLASH. PowerGraph performs efficiently on GC since it implements an asynchronous algorithm, which converges faster than a BSP-based algorithm. Ligra is faster than FLASH in some cases because it is a shared-memory system, with the communication cost much cheaper than that of distributed systems. When the computation amount is very low and the communication time accounts to most percentages of the total time, Ligra outperforms other works.

C. Productivity

To demonstrate the productivity of FLASH, the LLoCs are counted for all implemented cases, as shown in Table I. Note

TABLE V: Execution time for the first eight applications on six datasets (in seconds).

App.	Data	Pregel+	PowerG.	Gemini	Ligra	Flash	App.	Data	Pregel+	PowerG.	Gemini	Ligra	Flash
CC	OR	9.21	5.31	1.24	0.49	**0.48**	BFS	OR	3.07	6.27	0.87	**0.35**	**0.35**
	TW	99.31	281.93	8.60	10.09	**6.38**		TW	31.47	48.11	4.61	**2.28**	6.16
	US	435.42	1832.2	524.34	323.43	**30.96**		US	202.79	1512.3	519.01	244.01	**12.17**
	EU	1740.0	6749.7	1302.3	663.10	**76.47**		EU	1035.5	4453.4	1445.4	506.72	**50.32**
	UK	33.56	26.33	3.33	**2.09**	2.51		UK	5.94	15.51	2.78	**1.09**	2.26
	SK	132.97	307.30	5.57	**4.07**	7.02		SK	29.33	35.96	3.53	**1.92**	6.02
BC	OR	11.23	13.40	1.73	0.81	**0.54**	MIS	OR	11.22	12.30	1.78	2.66	**0.51**
	TW	110.29	121.71	**8.15**	21.62	11.77		TW	55.62	176.77	4.66	20.61	**4.58**
	US	516.86	3066.8	1007.1	411.25	**16.94**		US	4.55	22.58	3.93	1.10	**0.94**
	EU	2981.1	OT	2861.8	978.21	**129.64**		EU	254.88	722.41	188.22	122.41	**12.14**
	UK	22.61	39.91	6.24	**2.18**	3.87		UK	14.05	65.64	20.46	4.92	**1.83**
	SK	116.13	127.23	7.54	**7.08**	11.49		SK	77.54	108.54	13.37	9.24	**5.13**
MM	OR	OT	OT	497.15	889.61	**22.27**	KC	OR	678.44	1140.6	–	302.65	**4.03**
	TW	OT	OT	OT	OT	**25.15**		TW	4937.4	OT	–	1313.4	**29.26**
	US	13.00	65.66	6.96	3.69	**3.03**		US	232.18	68.80	–	16.11	**2.12**
	EU	428.87	1547.7	253.25	182.36	**19.17**		EU	OT	634.68	–	195.04	**10.44**
	UK	OT	OT	1091.8	518.83	**22.11**		UK	2924.6	2682.4	–	577.72	**5.38**
	SK	OT	OT	OT	OT	**114.76**		SK	OT	OT	–	3702.8	**44.16**
TC	OR	529.61	27.86	–	12.90	**3.32**	GC	OR	OT	13.26	–	–	**9.72**
	TW	OOM	720.01	–	OT	**49.10**		TW	OT	426.37	–	–	**264.44**
	US	17.90	6.48	–	**0.57**	1.09		US	10.29	13.11	–	–	**2.38**
	EU	32.56	10.91	–	**0.53**	2.29		EU	242.59	**43.81**	–	–	54.61
	UK	OOM	17.44	–	14.23	**7.00**		UK	2219.7	36.19	–	–	**35.67**
	SK	OOM	211.67	–	OT	**70.59**		SK	OT	706.21	–	–	**331.72**

"–" means that we fail to implement an available algorithm for this case because of the limitations in expressiveness; "OOM" means that the tested algorithm failed due to exhausted memory. "OT" means that the execution did not terminate within 5000s.

TABLE VI: Execution time for the last six applications on six datasets (in seconds).

App.	Data	Baseline	Flash	App.	Data	Baseline	Flash
SCC	OR	120.76	**1.24**	BCC	OR	303.93	**5.57**
	TW	949.60	**13.80**		TW	3615.0	**75.85**
	US	719.91	**57.84**		US	3844.7	**169.58**
	EU	3021.1	**161.35**		EU	OT	**486.14**
	UK	223.22	**5.55**		UK	879.91	**22.82**
	SK	1335.5	**18.26**		SK	2991.8	**55.20**
LPA	OR	155.90	**16.83**	MSF	OR	55.96	**6.96**
	TW	1433.9	**100.31**		TW	867.54	**72.51**
	US	49.11	**2.77**		US	**25.42**	29.96
	EU	276.20	**25.57**		EU	**64.86**	68.66
	UK	299.62	**11.06**		UK	55.25	**29.74**
	SK	OT	**78.25**		SK	477.72	**86.84**
RC	OR	–	**12.49**	CL	OR	–	**20.33**
	TW	–	**140.16**		TW	–	OT
	US	–	**1.31**		US	–	**1.22**
	EU	–	**2.75**		EU	–	**2.39**
	UK	–	**14.65**		UK	–	**420.12**
	SK	–	**176.78**		SK	–	OT

The baseline results for SCC, BCC and MSF are tested on Pregel+, and the baseline results for LPA are tested on PowerGraph. Baselines for RC and CL are missed since none of the other frameworks provided an implementation.

that we only consider LLoCs in the core functions, while ignoring the comments, input/output expressions, and data structure (e.g., the graph) definitions. Gemini fails to express most algorithms since its programming model is most limited. PowerGraph needs lots of code for TC since it does not provide the serialization/de-serialization semantics for users to exchange neighbor-lists. Although Pregel+ is able to express some complex non-ISVP algorithms (e.g., SCC and BCC), it is usually intractable. In fact, its algorithms for these applications are decomposed into several parts, with each part constitutes of an individual algorithm (a sub-algorithm) and needs all necessary functions to be programmed. This is obviously not friendly to the users. Moreover, since every sub-algorithm needs its own implementation for parsing the input from the previous sub-algorithm and outputting data for the next sub-algorithm (which are not counted in the LLoCs), the actual lines of code are further more than the results in Table I (e.g., 811 lines of code in total for SCC and 3017 lines for BCC). The algorithm decomposition also results in poor performance.

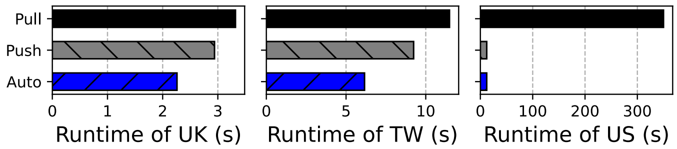


Fig. 3: Execution time (in seconds) of BFS on three datasets.

Although all compared frameworks evaluated are claimed to provide succinct interface, and have been widely used, Flash requires less effort in implementation than other works in all tested cases, showing that Flash achieves the best productivity when expressing both the ISVP and non-ISVP algorithms. We provide more examples in the full version [1] for the readers to further judge the succinctness and readability.

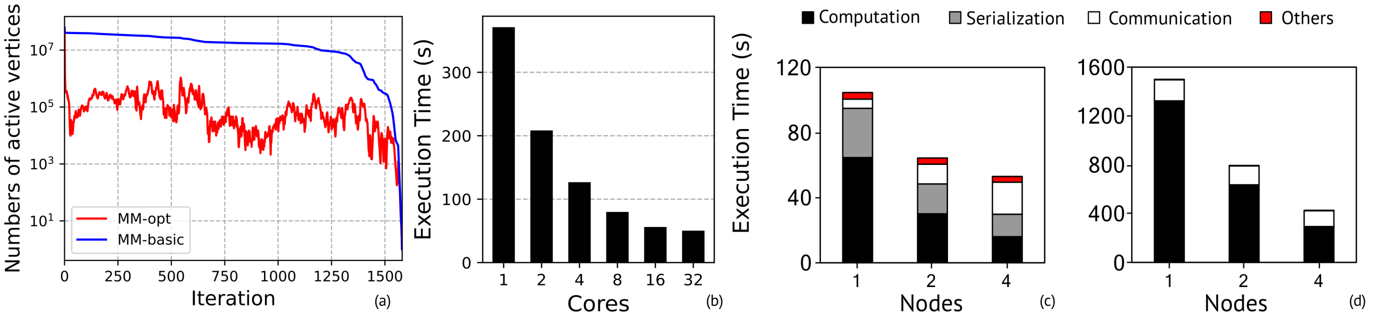


Fig. 4: (a) Number of active vertices in all iterations for MM-basic and MM-opt on TW. (b) Execution time of TC on TW with varying cores. Execution time of TC on TW (c) and CL on UK (d) with varying nodes.

D. Micro Benchmarks

Dual update propagation model. Adaptive switching between the *pull* (dense) mode and the *push* (sparse) mode according to the density improves the performance of FLASH significantly. We demonstrate the effectiveness of adaptive switching by compare its performance with either *push* or *pull*.

Figure 3 shows the execution time of BFS on three different graphs under different modes, as expected, the performance gap is quite significant. For TW and UK, the sparse mode outperforms the dense mode, while our dual update propagation scheme achieves the best performance. For the US graph, since it is sparse with a very low average degree, our adaptive switching falls into the sparse mode all the time, while the dense mode consumes much longer execution time.

Advanced implementation. The high expressiveness of FLASH does not only allow users to implement algorithms with less effort, but also allows users to implement the advanced version of some algorithms for higher performance. Consider the MM application as an example, we implement a basic algorithm in FLASH (MM-basic), as well as an optimized one (MM-opt), as described in Section III-B. Other frameworks cannot implement MM-opt since they do not support to define arbitrary edges sets. The advanced algorithm has higher efficiency than the basic one, which means it will touch less vertices and edges during the execution. Figure 4 (a) compares the number of active vertices (size of the frontier) in all iterations for both algorithms on the TW dataset. The significant reduction in active vertices leads to a considerable speedup of 70.1×. This is also the main reason that FLASH significantly outperforms other works, showing that the ability to express more advanced algorithms is absolutely necessary.

E. Scalability and Time Breakdown

Now, we examine the scalability of FLASH in terms of both intra-node and inter-node. We first compare the performance of FLASH while varying the cores of each node as 1, 2, 4, 8, 16, 32. Figure 4 (b) presents the execution time of running TC on the TW dataset, which shows a reasonable trend of scalability, achieving speedup of 1.8×, 2.9×, 4.7×, 6.7× and 7.5× at 2, 4, 8, 16, 32 cores, respectively. The other cases show similar trends, except the cases that the communication time dominates the execution such as running BFS on the US graph, on which the scalability is poor for all frameworks. The reduction on execution time after 4 cores slows down since when more cores are used, the scheduling cost and memory contention inside a node increase.

We also conduct experiments to evaluate the inter-node scalability. Figure 4 (c) and (d) shows the execution time of TC on TW and CL on UK using varying number of nodes, with each node running 32 cores. The speedup is 2.0× and 3.5× respectively when increasing the cluster size form 1 node to 4 nodes. FLASH scales better on CL since it contains a relatively large amount of computation. Actually, increasing the number of nodes reduces computation time but leads to more communication. This is also the common pattern that limits the scalability of all kinds of distributed graph processing frameworks. To validate this, we conduct a piecewise breakdown analysis that mainly divides the overall execution time into: (1) computation time, with the overlap part of computation and communication counted; (2) communication time, including network transferring and waiting time; (3) serialization time; and (4) time to do others, such as constructing the data structure and initialization. We found that with the increase of the cluster size, the overall execution time is decreased since the computation time decreases nearly linearly, while the communication and serialization time account to more and more percentages of the total time.

VI. CONCLUSIONS

In this paper, we present FLASH, a framework for programming distributed graph processing algorithms. We track three essential features for graph frameworks from exploring more advanced and complex graph algorithms: expressiveness, productivity and efficiency. FLASH proposes a new high-level programing model, which provides stronger expressiveness and exposes a succinct and user-friendly interface at the same time, leading to easy programming for a wide variety of graph algorithms. We provide an efficient implementation of FLASH based on a novel system architecture and a middleware named FLASHWARE. Experimental results demonstrate the outstanding performance of FLASH compared to existing frameworks. In addition to the algorithms discussed in this paper, we believe that other algorithms can also benefit from our framework, since the huge potentials it revealed.

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APPENDIX A
EXPRESSING OTHER PROGRAMMING MODELS

FLASH has the ability to simulate the traditional vertex-centric programming models. Since other vertex-centric programming models such as GAS [9] and Gemini [11] are more limited in expressiveness compared to the original model proposed by Pregel [6], we try to simulate the Pregel-like model using FLASH. As a consequence, it is possible to port existing vertex-centric programs in our model.

The typical execution workflow of the vertex-centric model proceeds in synchronized iterations (or *supersteps*). As shown in Algorithm 7, in each superstep, all active vertices (called *frontiers*) execute the same user-defined vertex function in parallel, which receives a set of messages as input ($inbox$) and can produce one or more messages as output ($outbox$). At the end of a superstep, the runtime receives the messages from the $outbox$ of each vertex and computes the set of active vertices for the next superstep. The execution will terminate when there are no active vertices or when some convergence condition is met.

As shown in Algorithm 8, the local computation in each superstep of the vertex-centric model can be implemented in FLASH through a VERTEXMAP function, which processes the $inbox$ and produces the updated value as well as the $outbox$ for each vertex, and the EDGEMAP function adds the message to the $inbox$ of the target from the $outbox$ of the source.

Algorithm 7 VERTEX-CENTRIC MODEL SEMANTICS

```
$A = V$
while SIZE($A$) $> 0$ do
    parfor $v \in A$ do
        $v.inbox$ = RECEIVEMESSAGES($v$)
        $(value^{new}, v.outbox)$ = COMPUTE($v.value, v.inbox$)
        $v.value = value^{new}$
```

Algorithm 8 SIMULATING VERTEX-CENTRIC USING FLASH

```
function LOCAL($v$)
    $(v.value, v.outbox)$ = COMPUTE($v.value, v.inbox$)
    return $v$

function UPDATE($s, d$)
    add the messages (from $s$ to $d$) to $d.inbox$ from $s.outbox$
    return $d$

function MERGE($t, d$)
    $d.inbox = d.inbox \cup t.inbox$
    return $d$

$A = V$
while SIZE($A$) $> 0$ do
    $A$ = EDGEMAP($A, E$, CTRUE, UPDATE, CTRUE, MERGE)
    $A$ = VERTEXMAP($A$, CTRUE, LOCAL)
```

APPENDIX B
APPLICATIONS

Besides BFS and BC, whose algorithms are demonstrated in Section III, we now give more example applications and describe their implementations in FLASH, to show the strong expressiveness of our programming model.

A. Connected Components (CC)

A weakly connected component is a maximal subgraph of a graph such that for every pair of vertices in it, there is an undirected path connecting them. In existing vertex-centric models, the standard method for calculating CC is label propagation. In this algorithm, each vertex is attached with a property which represents its component label, being its own vertex id initially. In the subsequent supersteps, a vertex will update its label if it receives a smaller id and then it propagates this id to all its neighbors. This ISVP-based algorithm is both simple and scalable, but not necessarily efficient. As the label is propagated only one hop at a time, it may require many iterations to converge, especially for graphs that have large diameters. This algorithm is implemented in FLASH as Algorithm 9 shows.

Algorithm 9 CONNECTED COMPONENTS

```
function INIT($v$):
    $v.cc = v.id$
    return v
function CHECK($s, d$):
    return $s.cc < d.cc$
function UPDATE($s, d$):
    $d.cc = min(d.cc, s.cc)$
    return $d$

$U$ = VERTEXMAP($V$, CTRUE, INIT)
while SIZE($U$) $\neq 0$ do
    $U$ = EDGEMAP($U, E$, CHECK, UPDATE, CTURE, UPDATE)
```

An optimized CC algorithm [20] is proposed to overcome the problem of Algorithm 9. It utilizes a parent pointer $p(v)$ for each vertex to maintain a tree (forest) structure, with each rooted tree represents a connected component of the graph. In each iteration, the algorithm uses $StarDetection$ to identify stars (tree of depth one), in which every vertex points to one common rooted vertex (the rooted vertex is self-pointing); then it merges stars that are connected via some edges using two $StarHooking$ operations; finally, it applies $PointerJumping$ to assign $p(v) = p(p(v))$. When the algorithm terminates, there must be isolated stars, each standing for one connected component with the rooted vertex as its id.

In this algorithm, virtual edges are generated to maintain the tree structure. Since the messages are not always along the original edges, it could not be implemented in the models that do not support communication beyond neighborhood. While in FLASH, this algorithm is expressed without much effort, as shown in Algorithm 10. Algorithm 10 converges much faster than Algorithm 9, for example, it takes only 7 iterations on the US dataset, while Algorithm 9 takes 6262 iterations. As a consequence, it brings a significant speedup up to an order of magnitude. FLASH provides some pre-defined operators for conveniently defining edge sets, for example, in this algorithm, edges with targets in U are expressed as `join(E, U)`, edges between $v \in U$ and $v.p$ are expressed as `join(U, p)` or `join(p, U)`.

Algorithm 10 OPTIMIZED CONNECTED COMPONENTS

```
function INIT($v$):
    $v.p = v.id, v.f = v.id, v.s = false$
    return v

function UPDATE1($s, d$):
    $d.p = min(d.p, s.id)$,
    return $d$

function UPDATE2($s, d$):
    $d.s = true$, return $d$

function FILTER1($v$): return $(v.p == v.id)$ and $(v.s == false)$
function LOCAL1($v$):
    $v.p = INF$, return $v$

function FILTER2($v$): return $v.p == INF$
function LOCAL2($v$):
    $v.p = v.id$, return $v$

function LOCALS($v$):
    $v.s = true$, return $v$
function FS1($s, d$): return $s.p \neq d.p$
function M($s, d$):
    $d.s = false$, return $d$
function FS2($s, d$): return $(s.s == false)$ and $(d.s == true)$
function STARDETECTION($U$):
    $U$ = VERTEXMAP($U$, CTRUE, LOCALS)
    EDGEMAPDENSE($V, join(p, U)$, FS1, M, CTRUE)
    EDGEMAPSPARSE($U, join(join(U, p), p)$, CTRUE, M, CTRUE, M)
    EDGEMAPDENSE($V, join(p, U)$, FS2, M, CTRUE)

function FILTERH1($v$): return $v.s == true$
function LOCALH1($v, cond$):
    $v.f = (cond?v.p : INF)$
    return $v$
function H1($s, d$):
    $d.f = min(d.f, s.p)$
    return $d$
function FH2($s, d$):
    return $(s.p \neq s.id)$ and $(s.f \neq INF)$ and $(s.f \neq s.p)$
function H2($s, d$):
    $d.f = min(d.f, s.f)$
    return $d$
function FILTERH2($v$):
    return $(v.p == v.id)$ and $(v.f \neq INF)$ and $(v.f \neq v.p)$
function LOCALH2($v$):
    $v.p = v.f$, return $v$
function STARHOOKING($U, cond$):
    $U$ = VERTEXMAP($U$, FILTERH1, LOCALH1.bind($cond$))
    EDGEMAPDENSE($V, join(E, U)$, FS1, H1, CTRUE)
    EDGEMAPSPARSE($U, join(U, p)$, FH2, H2, CTRUE, H2)
    VERTEXMAP($U$, FILTERH2, LOCALH2)

function UPDATEJ($s, d$):
    $d.p = s.p$, return $d$
function POINTERJUMPING($U$):
    return EDGEMAPDENSE($V, join(p, U)$, CTRUE, UPDATEJ, CTRUE)

$U$ = VERTEXMAP($V$, CTRUE, INIT)
EDGEMAPDENSE($U, E$, CTRUE, UPDATE1, CTRUE)
EDGEMAPSPARSE($U, join(U, p)$, CTRUE, UPDATE2, CTRUE, UPDATE2)
$U$ = VERTEXMAP($V$, FILTER1, LOCAL1)
$U$ = EDGEMAPDENSE($V, join(E, U)$, CTRUE, UPDATE1, CTRUE)
$U$ = MINUS($V$, VERTEXMAP($V$, FILTER2, LOCAL2)
while SIZE($U$) $\neq 0$ do
    STARDETECTION($U$), STARHOOKING($A, true$)
    STARDETECTION($U$), STARHOOKING($A, false$)
    $U$ = POINTERJUMPING($U$)
```

Algorithm 11 MAXIMAL MATCHING

```
function INIT($v$):
    $v.s = -1, v.p = -1$
    return v

function COND($v$): return $v.s == -1$
function UPDATE($s, d$):
    $d.p = max(d.p, s.id))$
    return $d$
function R1($t, d$):
    $d.p = max(d.p, t.p)$
    return $d$

function CHECK($s, d$): return $(s.p == d.id)$ and $(d.p == s.id)$
function UPDATE2($s, d$):
    $d.s = s.id$
    return $d$
function R2(t, d): return $t$

$U$ = VERTEXMAP($V$, CTRUE, INIT)
while SIZE($U$) $\neq 0$ do
    $U$ = VERTEXMAP($U$, COND, INIT)
    $U$ = EDGEMAP($U, E$, CTRUE, UPDATE, COND, R1)
    EDGEMAP($U, E$, CHECK, UPDATE2, COND, R2)
```

Algorithm 12 OPTIMIZED MAXIMAL MATCHING

```
function INIT($v$):
    $v.s = -1, v.p = -1$
    return v
function LOCAL($v$):
    $v.s = v.p$
    return v

function F1($s, d$): return $s.s == -1$
function M1($s, d$):
    $d.p = max(d.p, s.id))$
    return $d$
function COND($v$): return $v.s == -1$
function R1($t, d$):
    $d.p = max(d.p, t.p)$
    return $d$

function F2($s, d$): return $d.p == s.id$
function M2($s, d$):
    $d.s = s.id$
    return $d$
function R2($t, d$): return $t$

function M3($s, d$): return $d$

$U$ = VERTEXMAP($V$, CTRUE, INIT)
while SIZE($U$) $\neq 0$ do
    $U$ = VERTEXMAP($U$, COND, INIT)
    EDGEMAPDENSE($V, join(E, U)$, F1, M1, COND, R1)
    $A$ =EDGEMAPSPARSE($U, join(U, p)$, F2, M2, COND, R2)
    $B$ =EDGEMAPSPARSE($A, join(A, p)$, F2, M2, COND, R2)
    $U$ =EDGEMAPSPARSE(UNION($A, B$), $E$, F2, M3, COND, M3)
```

B. Maximal Matching (MM)

In graph theory, a matching in an undirected graph is a set of edges without common vertices. And a maximal matching is a matching of a graph G that is not a subset of any other matching. The task of this application is to find an arbitrary maximal matching.

This problem could be solved in a greedy algorithm which always tries to build a match for each unmatched vertex (called temporary match) in the first phase. The "tie breaking" is done by following the largest vertex id. And then, if two vertices are matched each other, they will be added to the result in the second phase, means the temporary match is successful. This algorithm executes iteratively until no more vertices could be added to the result. Algorithm 11 shows how to implement this algorithm in FLASH.

Also, we can implement an optimized version for this algorithm (named MM-opt), as shown in Algorithm 12. In every step, we need to conduct computation for an unmatched vertex only if its temporary matched vertex is matched successful in the last iteration (line 28). Otherwise, its temporary matched vertex should not be changed. This is done by executing the EDGEMAPSPARSE operation from successfully matched vertices targeting specific neighbors (line 31). This algorithm is not supported by other frameworks since they do not support the users to define arbitrary edge sets.

C. Maximal Independent Set (MIS)

In graph theory, a set of vertices constitutes an independent set if and only if any two of the vertices that contained in it do not have an edge connecting them. A maximal independent set S is then a set of vertices that: (1) constitutes an independent set; and (2) there does not exist another independent set S' that is a proper superset of S (i.e., $S \subset S'$).

Maximal Independent Set (MIS) is an important and widely-used graph application, whose output is an arbitrary maximal independent set of the input graph. It is difficult to be implemented in a message-passing model and hence is not provided by most existing vertex-centric graph processing systems. To the best of our knowledge, GPS [12] is the only graph processing system that contains a distributed MIS implementation, which is based on Luby's classic parallel algorithm [39]. GPS is an open-source Pregel implementation from Stanford Infolab, which was reported to be 12× faster than Giraph [7].

On the contrary, the algorithm for MIS provided by FLASH is implemented intuitively, as Algorithm 13 shows. Initially, all vertices are set to be available (to set $v.d = false$, line 22). In each step, we find all available vertices (with $v.b == true$, line 24) and add them to the result (line 25). $v.r$ is used to decide the priorities between neighbors in the same iteration. Once a vertex is added to the result, the neighbors of it should be labeled as not available immediately (to set $ngh.d = true$, line 26-27). This algorithm terminates when there are no more vertices could be added to the result.

Algorithm 13 MAXIMAL INDEPENDENT SET

```
 1: function INIT($v$):
 2:     $v.d = false, v.b = true, v.r = v.deg * |V| + v.id$
 3:     return v
 4:
 5: function COND1($v$): return $v.b == true$
 6: function F1($s, d$): return $(s.d == false)$ and $(s.r < d.r)$
 7: function UPDATE($s, d$):
 8:     $d.b = false$
 9:     return $d$
10:
11: function COND2($v$): return $v.d == false$
12: function UPDATE2($s, d$): return $d$
13: function R2($t, d$):
14:     $d.d = true$
15:     return $d$
16:
17: function FILTER($v$): return $v.b == false$
18: function LOCAL($v$):
19:     $v.b = true$
20:     return v
21:
22: $A$ = VERTEXMAP($V$, CTRUE, INIT)
23: while SIZE($U$) $\neq 0$ do
24:     EDGEMAPDENSE($V, join(E, A)$, F1, UPDATE1, COND1, R1)
25:     $B$ = VERTEXMAP($A$, COND1)
26:     $C$ = EDGEMAPSPARSE($B, E$, CTRUE, UPDATE2, COND2, R2)
27:     $A$ = VERTEXMAP(MINUS($A, C$), FILTER, LOCAL)
```

D. Triangle Counting (TC)

Triangle Counting (TC) is a basic problem that is used as a subroutine of many important social network analysis algorithms. This application counts the number of triangles in an undirected graph, where a triangle is formed by three vertices and edges between each pair of them. The implementations of this application are provided by some vertex-centric graph frameworks. For example, there are two versions of implementations of TC in PowerGraph. The optimized one implements the "hash-table" version of "edge-iterator" algorithm described in [40].

To fit this algorithm in FLASH, we give an implementation as described by Algorithm 14, which is very simple and readable. The function CHECK is for performance consideration, and it ensures that every triangle is counted only once, instead of 3 times if it is not provided. PowerGraph needs lots of code for TC since it does not provide the serialization/deserialization semantics for users to exchange neighbor-lists. Gemini [11] does not support to implement this algorithm, since it limits the vertex properties to be fixed-length but the neighbor-lists should be maintained in this application.

Algorithm 14 TRIANGLE COUNTING

```
function INIT($v$):
    $v.count = 0, v.out = \{\}$
    return v

function CHECK($s, d$):
    return $(s.deg > d.deg)$ or $((s.deg == d.deg)$ and $(s.id > d.id))$
function UPDATE1($s, d$):
    add $s.id$ to $d.out$
    return $d$
function R1($t, d$):
    $d.out = d.out \cup t.out$
    return $d$

function UPDATE2($s, d$):
    $d.count = d.count + intersact(s.out, d.out)$
    return $d$
function R2($t, d$):
    $d.count = d.count + t.count$
    return $d$

$U$ = VERTEXMAP($V$, CTRUE, INIT)
$U$ = EDGEMAP($U, E$, CHECK, UPDATE1, CTURE, R1)
$U$ = EDGEMAP($U, E$, CTRUE, UPDATE2, CTURE, R2)
```

E. Graph Coloring (GC)

Graph Coloring is a classic algorithmic problem in graph theory, which has been studied since the early 1970s. A graph coloring is an assignment of labels (also called "colors") to elements of a graph G subject to certain constraints. In the simplest form of this problem, it is a way of coloring the vertices of a graph such that no two adjacent vertices are of the same color; this is called a vertex coloring, which we consider in this paper.

This application is computationally hard. It is a NP-complete problem to decide if a given graph admits a k-coloring for a given k except for the cases $k \in \{0, 1, 2\}$. While in our case, we only try to find a practicable solution which uses as few colors as possible. This algorithm is based on the greedy strategy: in each step, every vertex will choose a color which is smallest and has not been used by its neighbors. The algorithm executes iteratively, until there is not any vertex changes its color in an iteration. Algorithm 14 demonstrates its implementation in FLASH.

An optimization of this algorithm is to use the asynchronous execution like that in PowerGraph [9], in which the algorithm can converge much faster. This is also the main reason that PowerGraph outperforms FLASH in some cases for this application. However, the asynchronous execution may result in more colors used, i.e., a practicable but sub-optimal solution compared to the result of a BSP-base algorithm. For the graph processing frameworks that do not support to define properties with unfixed-length on the vertices, this algorithm is not possible to be expressed directely, such as in Gemini [11] and Ligra [21].

Algorithm 15 GRAPH COLORING

```
function INIT($v$):
    $v.c = 0, v.colors = \{\}$
    return v

function F1($s, d$):
    return $(s.deg > d.deg)$ or $((s.deg == d.deg)$ and $(s.id > d.id))$
function UPDATE1($s, d$):
    add $s.c$ to $d.colors$
    return $d$
function R1($t, d$):
    $d.colors = d.colors \cup t.colors$
    return $d$

function LOCAL1($v$):
    for $i \in \{0, 1, 2, ...\}$ do
        if $(i \notin v.colors)$ then
            $v.cc = i$
            return v

function FILTER($v$): return $v.c \neq v.cc$
function LOCAL2($v$):
    $v.c = v.cc$
    return v

$U$ = VERTEXMAP($V$, CTRUE, INIT)
while SIZE($U$) $\neq 0$ do
    $U$ = EDGEMAP($V, E$, F1, UPDATE1, CTURE, R1)
    $U$ = VERTEXMAP($V$, CTRUE, LOCAL1)
    $U$ = VERTEXMAP($V$, FILTER, LOCAL2)
```

F. K-Core Decomposition (KC)

K-core decomposition is a well-established metric which partitions a graph into layers from external to more central vertices. It is widely used in real-world applications, such as to

Algorithm 16 K-CORE DECOMPOSITION

```
function INIT($v$):
    $v.d = v.deg$
    return v

function FILTER($v, k$): return $v.d < k$
function LOCAL($v, k$):
    $v.core = k - 1$
    return v

function UPDATE($s, d$):
    $d.d = d.d - 1$
    return $d$

$U$ = VERTEXMAP($V$, CTRUE, INIT)
for $k = \{1, 2, ..., n\}$ do
    while true do
        $A$ = VERTEXMAP($U$, FILTER.bind($k$), LOCAL.bind($k$))
        if (SIZE($A$) $== 0$) then
            break
        $U$ = MINUS($U, A$)
        EDGEMAPDENSE($A, E$, CTRUE, UPDATE, CTRUE)
    if (SIZE($U$) $== 0$) then
        break
```

Algorithm 17 OPTIMIZED K-CORE DECOMPOSITION

```
function INIT($v$):
    $v.core = v.deg$
    return v

function LOCAL1($v$):
    $v.cnt = 0, v.c = \{0\}$
    return v

function F1($s, d$):
    return $s.core \geq d.core$
function UPDATE1($s, d$):
    $d.cnt = d.cnt + 1$
    return $d$
function R1($t, d$):
    $d.cnt = d.cnt + t.cnt$
    return $d$

function FILTER($v$):
    return $v.cnt < v.core$

function UPDATE2($s, d$):
    $d.c[min(d.core, s.core)] = d.c[min(d.core, s.core)] + 1$
    return $d$

function LOCAL2($v$):
    $sum = 0$
    while $sum + v.c[v.core] < v.core$ do
        $sum = sum + v.c[v.core]$
        $v.core = v.core - 1$
    return v

$U$ = VERTEXMAP($V$, CTRUE, INIT)
while SIZE($U$) $\neq 0$ do
    $U$ = VERTEXMAP($V$, CTRUE, LOCAL1)
    $U$ = EDGEMAP($U, E$, F1, UPDATE1, CTRUE, R1)
    $U$ = VERTEXMAP($U$, FILTER)
    $U$ = EDGEMAPDENSE($V, join(E, U)$, CTRUE, UPDATE2, CTRUE)
    $U$ = VERTEXMAP($U$, CTRUE, LOCAL2)
```

study the clustering structure in social network analysis [41], in network visualization [43] and in bioinformatics [42]. In graph theory, a k-core of a graph G is a maximal connected sub-graph in which all vertices have the degree of at least k. Equivalently, it is one of the connected components of the subgraph of G formed by repeatedly deleting all vertices of degree less than k.

Ligra provides an algorithm for this application. In each step, it iterates over all remaining active vertices; and for each active vertex, removes it if induced degree $< k$, any vertex removed has core-number $(k-1)$ (i.e., is part of the $(k-1)$-core, but not k-core); this algorithm stops once no vertices are removed. When it finishes, vertices remaining are in the k-core. We follow this algorithm and implement it in FLASH's programming model, as Algorithm 16 shows. Besides, we further implemented an optimized algorithm in FLASH, please refer to [44] for more details of the algorithm. Although it is a shared-memory algorithm, we redesign and implement it in our model, as Algorithm 17 shows. This algorithm significantly outperforms the basic one, achieving speedups of up to two orders of magnitude.

G. Strongly Connected Components (SCC)

A directed graph is said to be strongly connected if there is a path in each direction between each pair of vertices of the graph. The strongly connected components (SCC) of an arbitrary directed graph form a partition into subgraphs that are themselves strongly connected. As an important metric to measure the connectivity of a graph, it is often used as a building block in social network analysis.

It is challenging to design vertex-centric algorithms for SCC. Although there are simple sequential algorithms for computing it based on depth-first search (DFS), it cannot be applied to design parallel algorithms for computing SCC. As far as we know, Pregel+ is the only existing graph processing framework that has provided an implementation for the SCC application, thus we evaluate this implementation as a baseline in our paper. The algorithm it implemented is proposed in [45], which consists of a series of PPAs (practical Pregel algorithms). Since each individual PPA need all the necessary functions to be programmed, it is obviously not friendly to the users. Moreover, every PPA need its own implementation for parsing the input from the previous part and outputting data for the next part. Therefore, the actual lines of code for SCC are extremely high. In fact, there are 811 lines of code in total for expressing this SCC algorithm in Pregel+.

In FLASH, we implement the parallel coloring algorithm which is proposed in [46] for finding strongly connected components. It executes three phases for each superstep: in the first phase, to identify trivial SCCs; and then, in the second phase, coloring the vertices by the maximum id of the vertex that can reach it; and in the third phase, the algorithm detects one SCC for each color, by doing a traversal in the transpose of the graph and limiting the traversal to only the vertices with this color. Algorithm 18 shows the implementation of this algorithm in our framework, which is much more succinct than the algorithm in Pregel+.

Algorithm 18 STRONGLY CONNECTED COMPONENTS

```
function INIT($v$):
    $v.scc = -1$
    return $v$

function LOCAL1($v$):
    $v.fid = v.id$
    return $v$

function F1($s, d$):
    return $s.fid < v.fid$
function M1($s, d$):
    $d.fid = min(d.fid, s.fid)$
    return $d$
function COND1($v$):
    return $v.scc == -1$
function R1($t, d$):
    $d.fid = min(d.fid, t.fid)$
    return $d$

function FILTER2($v$):
    return $v.fid == v.id$
function LOCAL2($v$):
    $v.scc = v.id$
    return $v$

function F2($s, d$):
    return $s.scc == d.fid$
function M2($s, d$):
    $d.scc = d.fid$
    return $d$
function COND2($v$):
    return $v.scc == -1$
function R2($t, d$):
    return $t$

function FILTER3($v$):
    return $v.scc == -1$

$A =$ VERTEXMAP($V$, CTRUE, INIT)
while SIZE($A$) $\neq 0$ do
                                                        ▷ phase 1
    $B =$ VERTEXMAP($A$, CTRUE, LOCAL1)
    while SIZE($B$) $\neq 0$ do
        $B =$ EDGEMAP($B, join(E, A)$, F1, M1, COND1, R1)
                                                        ▷ phase 2
    $B =$ VERTEXMAP($A$, FILTER2, LOCAL2)
    while SIZE($B$) $\neq 0$ do
        $B =$ EDGEMAP($B, join(reverse(E), A)$, F2, M2, COND2, R2)
    $A =$ VERTEXMAP($V$, FILTER3)
```

H. Biconnected Components (BCC)

In graph theory, a biconnected graph is a connected and non-separable graph, meaning that if any one vertex of it is removed, the graph will remain connected. Therefore, a biconnected graph has no articulation vertices (also called cut vertices). The biconnected component (BCC) of a graph G is a maximal subgraph of it which is biconnected. And any connected graph can be decomposed into a tree of biconnected components. The use of biconnected components the is very important in the field of networking, because of its property of redundancy.

As same with SCC, the implementation of BCC is only available in Pregel+, while all other existing frameworks did not implement it. However, the implementation in Pregel+ is

Algorithm 19 BICONNECTED COMPONENTS

```
function INIT($v$):
    $v.cid = v.id, v.d = v.deg$
    $v.dis = -1, v.p = -1, v.bcc = -1$
    return $v$

function F1($s, d$):
    return $(s.d > d.d)$ $or$ $((s.d == d.d))$ $and$ $(s.cid > d.cid))$
function UPDATE1($s, d$):
    $d.cid = s.cid, d.d = s.d$
    return $d$
function R1($t, d$):
    $flag = (t.d > d.d)$ $or$ $((t.d == d.d))$ $and$ $(t.cid > d.cid))$
    if $(flag)$ then
        $d.cid = t.cid, d.d = t.d$
    return $d$

function FILTER1($v$): return $v.cid == v.id$
function LOCAL1($v$):
    $v.dis = 0$
    return $v$

function UPDATE2($s, d$):
    $d.dis = s.dis + 1$
    return $d$
function COND2($v$): return $v.dis == -1$
function R2($t, d$): return $t$

function F3($s, d$): return $s.dis == d.dis - 1$
function UPDATE3($s, d$):
    $d.p = s.id$
    return $d$
function COND3($v$): return $v.p == -1$
function R3($t, d$): return $t$

function F4($s, d$):
    return $(s.id > d.id)$ $and$ $(d.p \neq s.id)$ $and$ $(s.p \neq s.id)$
function JOINEDGES($V, E, f$):
    for $e \in E$ do
        $a = get(e.s),\ b = get(e.d)$
        if F4($a, b$) then
            $dsu\_union(f, a, b)$
            while $a \neq b$ do
                $da = a.dis,\ db = b.dis$
                $dp = get(a.p),\ db = get(b.p)$
                if $(da \geq db)$ then
                    if $(pa \neq pb)$ then
                        $dsu\_union(f, pa, a)$
                    $a = pa$
                if $(db \geq da)$ then
                    if $(pa \neq pb)$ then
                        $dsu\_union(f, pa, a)$
                    $b = pb$

function LOCAL3($v, f'$):
    $v.bcc = dsu\_find(f', v)$
    return $v$

$A$ = VERTEXMAP($V$, CTRUE, INIT)
while SIZE($A$) $\neq 0$ do                                   ▷ CC round
    $A$ = EDGEMAP($A, E$, F1, UPDATE1, CTRUE, R1)
$A$ = VERTEXMAP($V$, FILTER1, LOCAL1)
while SIZE($A$) $\neq 0$ do                                   ▷ BFS round
    $A$ = EDGEMAP($A, E$, CTRUE, UPDATE2, COND2, R2)
EDGEMAP($V, E$, F3, UPDATE3, COND3, R3)
$f = dsu(V)$
JOINEDGES ($V, E, f$)                                          ▷ join edges
$f'$ = REDUCE ($f$)                                             ▷ reduce
$A$ = VERTEXMAP($V$, CTRUE, LOCAL3.bind($f'$))
```

Algorithm 20 LABEL PROPAGATION

```
function INIT($v$):
    $v.set = \{\}$
    return v

function UPDATE1($s, d$):
    add $s.c$ to $d.set$
    return $d$
function R1($t, d$):
    $d.set = d.set \cup t.set$
    return $d$

function LOCAL1($v$):
    $max = 0$, $count = \{0\}$
    for $i \in v.set$ do
        $count[i] = count[i] + 1$
        if $(count[i] > max)$ then
            $max = count[i]$
            $v.cc = i$
    return v

function FILTER($v$):
    return $v.c \neq v.cc$
function LOCAL2($v$):
    $v.c = v.cc$, $v.set = \{\}$
    return v

$U$ = VERTEXMAP($V$, CTRUE, INIT)
for $iters \in \{1, 2, .., MaxIters\}$ do
    $U$ = EDGEMAP($V, E$, CTRUE, UPDATE1, CTRUE, R1)
    $U$ = VERTEXMAP($U$, CTRUE, LOCAL1)
    $U$ = VERTEXMAP($U$, FILTER, LOCAL2)
```

based on the algorithm proposed in [45], which is extremely complex for programming, taking more than 3000 lines of code in total.

While in FLASH, due to its high expressiveness, we could implement the algorithm for BCC as [47] proposed. This algorithm relies on an initial BFS traversal which creates a BFS tree. And an articulation vertex can be identified in the BFS tree by the fact that it has at least a single child vertex which does not have a path to any other vertex on the same BFS level as the articulation vertex that does not pass through the articulation vertex. Algorithm 19 shows the implementation of this algorithm in our framework, in which dsu_find and dsu_union are pre-defined functions provided by FLASH, to implement the disjoint set (union find algorithm) which is often used in graph applications (e.g., it is also utilized in MSF, as we will present). As we can see, this algorithm is more succinct and readable compared with the implementation in Pregel+.

I. Label Propagation (LPA)

The label propagation algorithm (LPA) is a well-known semi-supervised machine learning algorithm that assigns labels (also called classifications) to previously unlabeled vertices in the graph. At the start of the algorithm, typically, a small subset of the vertices have labels. And then, these labels are propagated iteratively to the unlabeled vertices during this algorithm [48]. The solution that it produces is not unique. It requires little prior information, and has advantages in time performance when compared with other clustering algorithms. As a consequence, the LPA algorithm is often used for finding communities in social network analysis [49].

This algorithm can be expressed in FLASH easily, with the implementation shown in Algorithm 20. We suppose that every vertex owns a label in the beginning ($v.c$), which is provided by the input data or it will be initialized as 0. In each of the following supersteps, every vertex checks all of its neighbors, and changes its label to the most frequent one among its neighbors. This algorithm terminates after executing for a number of iterations.

Similar with GC, Gemini does not support this algorithm because it limits the vertex properties to be fixed-length. PowerGraph has provided this LPA algorithm, while it is $9.3\times$ $27.1\times$ slower than FLASH. As we analyze, this performance gap comes from our efficient system implementation, although both frameworks use the same algorithm.

J. Minimum Spanning Forest (MSF)

Minimum Cost Spanning Tree/Forest (MSF) is an application that calculates a spanning tree of a connected, undirected, weighted graph. This tree should connect all the vertices of the graph with the minimum total weight of its edges. For unconnected graphs, we calculate a tree for every connected component, that is to say, to calculate a minimum spanning forest. MSF is an important graph application that is used directly in the design of networks and invoked as a subroutine in many other algorithms, such as [50], [51], [52]. As a result, sequential MSF algorithms have been well studied, for example, the Kruskal's algorithm [54]. However, distributed MSF is usually very complex. Actually, according to our survey, a very few existing graph processing frameworks have provided a distributed MSF implementation. For our evaluation, we consider Pregel+, who implemented MSF based on the parallel algorithm proposed in [53].

Algorithm 21 MINIMUM SPANNING FOREST

```
function F1($s, d$):
    return $s.id > d.id$
function UPDATE1($s, d, w, H$):
    add $(s, d, w)$ to $H$
    return $d$

function F2($s, d, f$):
    return $dsu\_find(f, s) \neq dsu\_find(f, d)$
function UPDATE2($s, d, w, f$):
    $dsu\_union(f, s, d)$
    return $d$

function KRUSKAL($V, E$):
    $f = dsu(V), msf = \{\}$
    sort $E$ by $w$
    for $(s, d, w) \in E$ do
        if F2($s, d, f$) then
            UPDATE2($s.d, w, f$)
            add $(s, d, w)$ to $msf$
    return $msf$

$H = \{\}$
EDGEMAPDENSE($V, E$, F1, UPDATE1.bind($H$), CTURE)
$msf$ = KRUSKAL($V, H$)
$H'$ = REDUCE($msf$)
$res$ = KRUSKAL($V, H'$)
```

Thanks to the strong expressiveness of our programming model, and a series of auxiliary operators provided by FLASH, we could implement the sequential Kruskal's algorithm with few changes. The Kruskal's algorithm is typically more efficient than the algorithm of [53]. Algorithm 21 gives a brief presentation of our implementation, in which dsu_find and dsu_union are pre-defined functions provided by FLASH, to implement the disjoint set (union find algorithm) which is often used in graph applications. Obviously, it is much easier to program compared to [53], as we demonstrate by counting the LLoCs in our paper.

In the beginning of the algorithm, a minimum spanning forest is calculated inside each worker using the Kruskal's algorithm. And then the auxiliary operator REDUCE is used to reduce these local results in a new edge set. And at last, the Kruskal's algorithm is called again to get the final forest. This algorithm is correct because it can be easily proved that, if an edge is not used in the MSF of a subgraph, it is also not needed in the generation of the whole graph's MSF. In the Kruskal's algorithm, the edges have to be processed in the order of the weight. As a consequence, parallel processing is not available, so we process these edges through directly calling the condition checking function F1 and the mapping function UPDATE2, instead of using the EDGEMAP function. Although this implementation has a sacrifice in the parallelism, since this algorithm is more efficient, FLASH is still faster than Pregel+ in most cases.

K. Rectangle Counting (RC)

A rectangle in a graph is a cycle of length 4 (which consists of four vertices and four edges). Rectangles are most elementary sub-structures in the graphs. Therefore, as same with triangle counting, rectangle counting (RC) has many important applications in the real-world, including biological

Algorithm 22 RECTANGLE COUNTING

```
function INIT($v$):
    $v.count = 0$
    $v.out = \{\}$, $v.out_l = \{\}$
    return v

function UPDATE1($s, d$):
    if $(s.id > d.id)$ then
        add $s.id$ to $d.out_l$
    add $s.id$ to $d.out$
    return $d$
function R1($t, d$):
    $d.out = d.out \cup t.out$
    $d.out_l = d.out_l \cup t.out_l$
    return $d$

function F2($s.d$): return $s.id < d.id$
function UPDATE2($s, d$):
    $t = intersact(s.out_l, d.out)$
    $d.count = d.count + t * (t - 1)/2$
function R2($t, d$):
    $d.count = d.count + t.count$
    return $d$

$U$ = VERTEXMAP($V$, CTRUE, INIT)
$U$ = EDGEMAP($U, E$, CTRUE, UPDATE1, CTURE, R1)
$U$ = EDGEMAP($U, join(E, E)$, F2, UPDATE2, CTURE, R2)
```

Algorithm 23 K-CLIQUE COUNTING

```
function INIT($v$):
    $v.count = 0, v.out = \{\}$
    return $v$

function F1($s, d$):
    return $(s.deg > d.deg)$ or $((s.deg == d.deg)$ and $(s.id > d.id))$
function UPDATE1($s, d$):
    add $s.id$ to $d.out$
    return $d$
function R1($t, d$):
    $d.out = d.out \cup t.out$
    return $d$

function FILTER($v$):
    return SIZE$(v.out) \geq (k - 1)$

function COUNTING($cand, lev, k$):
    if $(lev == k)$ then
        return SIZE$(cand)$
    $t = 0$
    for $u \in cand$ do
        $cand' = intersect(cand, get(u).out)$
        if (SIZE$(cand') \geq k - lev - 1$) then
            $t = t+$COUNTING$(cand', lev + 1, k)$
    return t

function CL($v, k$):
    $v.count$ = COUNTING$(v.out, 1, k)$
    return $v$

$U$ = VERTEXMAP($V$, CTURE, INIT)
$U$ = EDGEMAP($U, E$, F1, UPDATE1, CTURE, R1)
$U$ = VERTEXMAP($U$, FILTER.bind($k$))
$U$ = VERTEXMAP($U$, CTURE, CL.bind($k$))
```

network analysis, social network analysis and so on. However, in existing graph processing frameworks, none has provided an implementation for this problem. A close analysis shows that they fail to give an efficient implementation for this application since accessing the two-hop neighbors is not supported in vertex-centric frameworks.

With the programming model of FLASH, we can design an algorithm to count rectangles in a graph. Similar to the algorithm for triangle counting, it is also based on counting the intersection of two vertices' neighbor sets every time. But in this algorithm, the two sets are from two vertices that are two-hop neighbors instead of immediate neighbors. Algorithm 22 gives the implementation for this algorithm in FLASH, in which $join(E, E)$ (line 26) is the pre-defined edge set which represents two hop neighbors. By defining this custom edge set for the EDGEMAP function, this RC algorithm is expressed easily. We distinguish $v.out$ (the neighbor set of v) and $v.out_l$ (the set containing neighbors that the ids are larger than $v.id$) to ensure that each rectangle is counted only once. The sum of $v.count$ for all vertices is the total number of rectangles in the input graph G.

L. K-Clique Counting (CL)

In graph theory, a clique of an undirected graph G is a subset of vertices with each pair of vertices are adjacent. A k-clique is then a clique containing k vertices. In the k-clique counting (CL) problem, the input is an undirected graph G and a number k, and the task is to count the number of k-cliques. For our evaluation, the performance results are tested under the setting of k to be 4. Since this problem is a basic metric for analyzing the topology of a graph, it has many important applications in social networks, bioinformatics, computational chemistry and other areas. However, the k-clique counting application is not provided yet in existing distributed graph processing frameworks as we surveyed.

[26] proposed a novel parallel algorithm for k-clique counting, and we port this algorithm in our model, as Algorithm 23 shows. In the beginning of the algorithm, each vertex maintains a set to record its neighbors (out), and the set $cand$ records the potential neighbors to complete the clique, which is $v.out$ initially. The counting step is completed through a recursive procedure. With every recursive call, a new candidate vertex u from $cand$ is added to the clique and $cand$ is pruned to contain only neighbors of u. To access the neighbors of an arbitrary vertex u, the get function which the FLASHWARE exposes is called immediately. The counts obtained from recursive calls are aggregated to get the total count for the central vertex.