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MIE1603H: Project Report
Vehicle Routing Model for the Military Aircraft Mission Planning Problem (MAMPP)

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1 INTRODUCTION AND LITERATURE REVIEW

We studied the Military Aircraft Mission Planning Problem (MAMPP), that was firstly introduced by Quttineh *et al.* [2], where a fleet of aircrafts need to attack a set of ground targets in a way that the total expected effect is maximized. The expected effect on a target can be calculated according to the weapon being used (decided in advanced) and on the direction of the impact of the weapon. The mission planning, meaning this problem involves three types of decisions : choice of attack directions, assignment of aircraft against targets and scheduling of each aircraft's assigned tasks. Generally, mission planning involves several tasks such as surveillance, backup support, rescue assistance or attack (in this case). Many military routing examples such as this one can be found in [3] and [4].

The MAMPP is in fact a generalized vehicle routing problem with precedence relationships and synchronization in time and position between multiple vehicles (aircrafts). Synchronization in a vehicle routing problem is a very challenging and appealing topic. Most recent work on the synchronization constraints has been done by Drexl [5], who presented two mixed-integer programming (MIP) formulations and five branch-and-cut algorithms. Moreover, a recent survey in synchronization in vehicle routing has also been given by Drexl [6]. In the case of the MAMPP, the synchronization that is being taken into account is operation synchronization that decides on the time and location of some interaction between the aircrafts. There are four more different types of synchronizations that are not considered in this problem which are : task synchronization, movement synchronization, load synchronization and resource synchronization.

Since the problem is clearly part of the vehicle routing problem (VRP) class, it can be easily described through a network. The attacks and illuminations can be described as nodes, each of which is associated with an expected effect on the target. Furthermore, we can introduce dummy nodes associated with crossings of the entry and exit lines of the target scene, where the possible aircraft movements can be represented by arcs. Since there are precedence constraints, some of the arcs can be eliminated from the network. Also, having the constraint of attacking/illuminating each target only once results the problem into a network that only contains arcs between different targets. Each of the arcs has two attributes, expected effect and a travel time. The diameter of a target scene is usually of the order of 100km, the distances between targets are of the order of few kilometers and typically a mission involves 6-8 targets and 4-6 aircrafts.

2 Problem Definition

The mixed-integer programming model for the MAMPP is already familiar from the work of Quttineh *et al.* [2] and [1]. Before exploring the time-indexed model, we first present the variables and notation used in the following three tables respectively.

2.1 Notation Used

$$x_{ij}^r := \begin{cases} 1 & \text{if aircraft } r \text{ traverses arc } (i,j)\ , \\ 0 & \text{if otherwise .} \end{cases}$$

$$y_{is}^r := \begin{cases} 1 & \text{if node } i \text{ is visited by aircraft } r \text{ in time periods } s\ , \\ 0 & \text{if otherwise .} \end{cases}$$

$t_{end} :=$ the time the last aircraft passes the exit line

Table1 : Indices and sets	
Notation	**Description**
R	fleet of aircraft,r
M	set of targets m to be attacked
N	set of nodes in the network, excluding the origin and destination
G, G_m	set of all sectors for all targets,and for target m respectively
N_m^A, N_m^I	set of feasible attack A, and illumination I nodes,respectively for target m
A	set of arcs in the network(including origin and destination) and set of arcs s.t. node
A_g, I_g	set of arcs (i,j)s.t. node j is an attack node or illumination node in sector g,respectively
P	set of ordered pairs of targets s.t. the attack on target m cannot precede the attack on target n
S	set of time periods within a discretized planning horizon

Table2 : Parameters	
Notation	**Description**
c_{ij}^r	expected effect of the attack, for arcs (i,j) with $i \in N_m^A$-arcs leaving attack nodes
S_{ij}^r	time needed for aircraft r to traverse arc (i,j) expressed in number of time periods
T_s	ending time of period s
Γ^r	armanent(military equipment) capacity of aircraft r
q_m	weapon capacity needed towards target m
μ	positive parameter that puts weights on mission time span against expected efforts on targets

These decision variables, sets and parameters define the following problem formulation.

2.2 Mathematical Formulation

The problem is mathematically formulated as follows:

$$\max \quad \sum_{r\in R}\sum_{(i,j)\in A} c_{ij}^r x_{ij}^r - \mu t_{end}$$

$$\text{s.t.} \quad \sum_{(o,j)\in A} x_{oj}^r = 1 \qquad \forall r \in R \tag{2.1}$$

$$\sum_{(i,d)\in A} x_{id}^r = 1 \qquad \forall r \in R \tag{2.2}$$

$$\sum_{(i,k)\in A} x_{ik}^r = \sum_{(k,j)\in A} x_{kj}^r \qquad \forall k \in N, \forall r \in R \tag{2.3}$$

$$\sum_{r\in R}\sum_{g\in G_m}\sum_{(i,j)\in A_g} x_{ij}^r = 1 \qquad \forall m \in M \tag{2.4}$$

$$\sum_{r\in R}\sum_{g\in G_m}\sum_{(i,j)\in I_g} x_{ij}^r = 1 \qquad \forall m \in M \tag{2.5}$$

$$\sum_{r\in R}\sum_{(i,j)\in A_g} x_{ij}^r = \sum_{r\in R}\sum_{(i,j)\in I_q} x_{ij}^r \qquad \forall g \in G \tag{2.6}$$

$$\sum_{g\in G_m}\sum_{(i,j)\in A_g\cup I_g} x_{ij}^r \le 1 \qquad \forall m \in M, \forall r \in R \tag{2.7}$$

$$\sum_{m\in M}\sum_{g\in G_m}\sum_{(i,j)\in A_g} q_m x_{ij}^r \le \Gamma^r \qquad \forall r \in R \tag{2.8}$$

$$y_{o0}^r = 1 \qquad \forall r \in R \tag{2.9}$$

$$\sum_{t=s+S_{ij}^r}^{|S|} y_{jt}^r \ge x_{ij}^r + y_{is}^r - 1 \qquad \forall (i,j) \in A, s \in \{0\}\cup S, \forall r \in R \tag{2.10}$$

$$\sum_{s\in S} y_{ks}^k = \sum_{(k,j)\in A} x_{kj}^r \qquad \forall k \in N, \forall r \in R \tag{2.11}$$

$$\sum_{r\in R}\sum_{i\in N_m^A} y_{is}^r = \sum_{r\in R}\sum_{i\in N_m^I} y_{is}^r \qquad \forall m \in M, \forall s \in S \tag{2.12}$$

$$\sum_{r\in R}\sum_{t=s}^{|S|}\sum_{i\in N_m^A} y_{it}^r \ge \sum_{r\in R}\sum_{i\in N_n^A} y_{is}^r \qquad \forall (m,n) \in P, s \in S \tag{2.13}$$

$$\sum_{i\in N_m^A}\sum_{t=1}^{s-1} y_{it}^t + \sum_{i\in N_m^A} y_{is}^r \le 1 \qquad \forall (m,n) \in P, s \in S, r \in R \tag{2.14}$$

$$\sum_{s\in S} y_{is}^r \le 1 \qquad \forall i \in N\cup\{o,d\}, \forall r \in R \tag{2.15}$$

$$\sum_{s\in\{0\}\cup S} T_s y_{ds}^r \le t_{end} \qquad \forall r \in R \tag{2.16}$$

The problem is multi-objective optimization problem. The primary objective is to maximize the total expected effect against all targets. In order to achieve this , some long flight paths within the target scene might be used which usually expose the air crafts to a higher risk of being detected. That is why a secondary objective is introduced in order to limit the mission time span. The value of μ in the objective is usually chosen by a mission planner.

2.3 DESCRIPTION OF CONSTRAINTS

Constraint (2.1) ensures that each aircraft enters the target scene via the origin, while constraint (2.2) ensures that each aircraft leaves the target scene via the destination node. Constraint (2.3) is the node balance equation for each of the air crafts. Constraint (2.4) ensures that each target will be attached exactly once, while constraint (2.5) ensures that each target will be illuminated exactly once. Constraint (2.6) synchronizes these two tasks to the same sector (**synchronization constraint**). Constraint (2.7) ensures that each aircraft can visit each target at most once. Constraint (2.8) gives the maximum capacity of the armanent (military equipment). Furthermore, constraint (2.9) ensures that each aircraft leaves the origin (enters the target zone) at time zero. Constraint (2.10) ensures that , if aircraft r is visiting node j right after node i then the time of visiting node j cannot be earlier than the time of visiting node i plus the time for the aircraft to traverse the arc (i, j) . Constraint (2.11) enforces that if node i is not visited by an aircraft , no outgoing arc from that node can be traversed by an aircraft. Constraint (2.12) is yet again a **synchronization constraint**, that ensures that an attach and illumination of a target need to be synchronized in time. Constraint (2.13) defines the precedence restrictions on the attacking times of pairs of targets. In a similar fashion,constraint (2.14) defines the precedence restrictions of an individual aircraft. Finally, constraint (2.16) defines the total mission time.

3 RESULTS

3.1 INSTANCES

We used a rather simplified version of the real data provided in [2], due to the fact that the following data were not available.

- **A** : Set of arcs in the network. The most important data for this problem. The papers we referred had got their data from an industrial partner who used real time data for solving this problem. Unfortunately, this data was not publicly available so we had to generate our own set of arcs. We used a naive approach for generating A. We set a random limit to the number of arcs and used a function to add arcs to A until that particular number is reached. For example if the random number was 220, we generated 220 random arcs.
- A_g and I_g: Similarly, A_g and I_g was not available and we have to randomly choose a subset of A.

For our instances we decided to use 2 sectors per target instead of 6 (in the paper) with attack and illumination nodes set at 2 per sector . Also, all of our scenarios include only 2 air crafts and 2 targets because of simplicity. The twelve instances that we used are shown in the following table:

- **Sequence column -** in order to describe given precedence relations between targets, we introduce the notation {1234} for no precedences at all among the four targets, while for example {12|34} means that targets 1 and 2 must be visited before targets 3 and 4.

Table1: Instances Parameters and Results															
#	R	M	Sequence	N	A	A_g	I_g	Γ	G	G_m	N_m^A	N_m^I	q_m	μ	$Objective$
1	2	2	{1|2}	16	240	2	2	2	4	2	2	2	1	0.005	10.045
2	2	2	{2|1}	16	240	2	2	3	4	2	2	2	1	0.005	9.045
3	2	2	{1|2}	16	240	2	2	2	4	2	2	2	1	0.08	9.64
4	2	2	{2|1}	16	240	3	2	2	4	2	3	2	1	0.08	9.64
5	2	2	{1|2}	16	230	3	2	3	4	2	3	2	1	0.005	9.035
6	2	2	{2|1}	16	230	2	3	2	4	2	2	3	1	0.005	9.040
7	2	2	{1|2}	16	230	2	2	2	4	1	2	2	1	0.08	10.040
8	2	2	{2|1}	16	230	2	2	2	4	2	2	2	1	0.08	11.480
9	2	2	{1, 2}	16	230	2	2	2	4	2	2	2	1	0.005	7.025
10	2	2	{1, 2}	16	230	2	2	2	4	2	2	2	1	0.08	10.560
11	2	2	{1, 2}	16	240	2	2	2	4	2	2	2	1	0.005	7.030
12	2	2	{1, 2}	16	240	2	2	2	4	2	2	2	1	0.08	8.560

Sample Solution :
xvalues:
x[1,13,17] = 1
x[1,6,13] = 1
x[1,0,6] = 1
x[2,0,1] = 1
x[2,1,9] = 1
x[2,9,17] = 1
y values:
y[1,0,0] = 1
y[1,6,3] = 1
y[1,13,6] = 1
y1,17,9] = 1
y[2,0,0] = 1
y[2,1,3] = 1
y[2,9,6] = 1
y[2,17,9] = 1
Objective = 9.0
So, the above sample solution can be interpreted in the following way: The first xvalue indicates that flight 1 travelled from node 13 to node 17. The second yvalue indicates that flight 1 travelled from node 13 to node 17Their paths can be written as follows:
1 : 0 → 6 → 13 → 17
2 : 0 → 1 → 9 → 17

3.2 Results for the Objective Value for the 12 Instances

On the graph are shown the results from the fifteen instances concerning the multi objective function. This is a huge problem with a large number of variables and constraints so it would be interesting in the future to investigate the influence of modifying the parameters extensively. The results might not be consistent with the ones shown in the paper that we were investigating, since we used different instances.

4 Proposed Methods for Future Work

In this section we explore the different techniques we propose to approach and improve the aforementioned problem.

1. Extend the target modelling by allowing attack options on different discrete altitude layers. At the moment the problem is solved at one altitude layer but for example we can consider each 1000ft as a different possible altitude layer for an attack.

2. Besides the five different mentioned cuts in [5], it would be useful to develop problem-specific infeasible path(route) cuts to obtain a better lower bound. Another possibility is to generate Benders cuts as one of the proposed cuts in [5].

3. Generate feasible solutions with meta-heuristics ([7] currently working on) that can be used to provide high quality initial columns for the restricted master problem in [1]. Or just efficient meta-heuristics for the proposed problem.

With this, we see a great opportunity to explore this problem beyond the course project requirements.

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