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The nonlinear dynamics of a bistable energy harvesting system with colored noise disturbances

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Abstract

This paper deals with the nonlinear stochastic dynamics of a piezoelectric energy harvesting system subjected to a harmonic external excitation disturbed by a Gaussian colored noise. A parametric analysis is conducted, where the effects of the standard deviation and the correlation time of colored noise on the system response are investigated. The numerical results suggests a strong influence of noise on the system response for higher values of correlation time and standard deviation, and a low (noise level independent) influence for low values of correlation time.

Keywords: energy harvesting, nonlinear dynamics, stochastic dynamics, bistable systems, piezoelectricity

1. Introduction

Energy harvesting systems have been widely studied in literature on the past 15 years, because of their great appeal in many different applications: autonomous power supply to small electrical-electronic boarded equipment; secondary energy recovering in engineering; Internet-Of-Things (IoF) technologies in telecommunications; pacemarks in health equipment; among others.

Since the pioneering work of Cottone et al. [1], a special attention is given to the study of nonlinear energy harvesters, once the use of nonlinearities may enhance a lot power recovering capability in comparison to linear energy harvesting systems. For this reason, many recent works [2, 3, 4, 5, 6] has devoted attention to the subject.

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In the case of a vibrating harvester, its energetic efficiency is strongly influenced by the amplitude and frequency characteristics of the external excitation [7], and the introduction of a random component in the excitation source, for example, may raise the amount of harvested power [8, 9]. Investigate in detail how the characteristics of a forcing noise affect the response of a energy harvester is a relevant research problem in several areas of engineering and applied physics.

In this context, the present work deals with the nonlinear dynamics of a bistable piezo-magneto-elastic energy harvesting system subjected to an external forcing composed of a harmonic part disturbed by a random noise. The effects of the correlation time between harmonic forcing period and the random noise signal on harvester output voltage are investigated considering different levels of intensity for the noise disturbance.

2. Energy harvesting system

The energy harvesting system studied here, proposed by Erturk et al. [10], is the piezo-magneto-elastic beam, driven by a rigid base movement, that is illustrated in Figure 1.

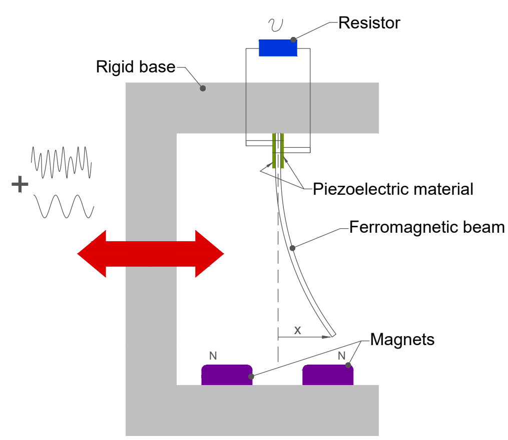


Figure 1: Piezo-magneto-elastic energy harvesting system.

Its nonlinear stochastic dynamics evolves according to the following system of stochastic differential equations

$$\ddot{X} + 2\xi\dot{X} - \frac{1}{2}X\left(1 - X^2\right) - \chi V = f\cos\Omega t + N_t, \qquad (1)$$

$$\dot{V} + \lambda V + \kappa \dot{X} = 0, \tag{2}$$

where the beam free edge displacement, its velocity and the system output voltage are represented by the random processes X, $\dot{X}$, and V, respectively; ξ means the mechanical damping ratio; χ and κ are the piezoelectric coupling terms; and λ is a reciprocal time constant; Such parameters are dimensionless, assumed as $\xi = 0.01$, $\chi = 0.05$, $\kappa = 0.5$, $\lambda = 0.05$, and the initial conditions are adopted as being $X_0 = 1$, $\dot{X}_0 = 0$ and $V_0 = 0$.

The external excitation is compound a harmonic force with amplitude and frequency respectively given by $f = 0.115$ and $\Omega = 0.8$, and disturbance component N_t, assumed as a zero-mean Gaussian colored noise with covariance function

$$\texttt{cov}_{N_t}(t_1, t_2) = \sigma \exp\left(-\frac{|t_2 - t_1|}{\tau_{corr}}\right), \tag{3}$$

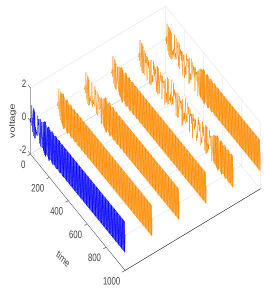
where τ_{corr} means the correlation time and σ the colored noise standard deviation. The parameters values are assumed as percentages of harmonic forcing amplitude and frequency, respectively.

3. Numerical experimentation

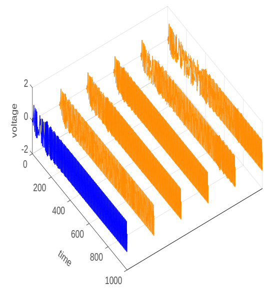
For the present analysis, the noise correlation times are set as $\tau_{corr} = \eta\,\tau_\Omega$, where $\tau_\Omega = 2\pi/\Omega$ and $\eta = \{1\%,\ 50\%,\ 100\%\}$. The noise standard deviation values are assumed as 1%, 25% and 50% of harmonic amplitude f. In each case, a Monte Carlo simulation with 256 samples is performed. The realizations of noise are generated by deterministic approximations, obtained by the Karhunen-Loève expansion. This method allows the dynamics to be integrated with the standard 4th-order Runge-Kutta method with automatic time step.

Figure 2 presents different voltage time series sampled from Monte Carlo simulations, for $\sigma/f = 1\%$, $\sigma/f = 25\%$ and $\sigma/f = 50\%$, and several values of correlation times. Depicted on blue, the reader see the voltage response for a pure harmonic forcing. The corresponding mean and standard deviation values, for the are presented on Figures 3 and 4, respectively.

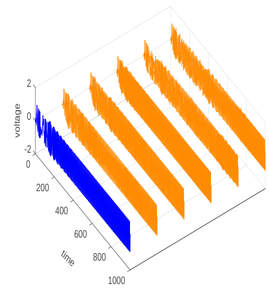
The analysis of the Monte Carlo mean for the harvester output voltages on Figure 3 suggests a longer chaotic transient for the higher correlation times, specially for the maximum noise incidence, on $\tau_{corr}/\tau_\Omega = 100\%$. In this case, the distortion observed in the voltage mean curve points the presence of more non-periodic time series during analyzed interval, despite of low level of noise. As in the previous cases, the results analysis for higher



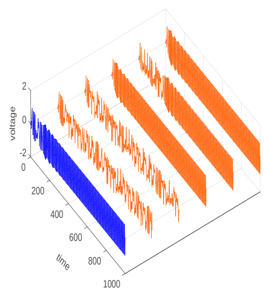
(a) $\tau_{corr}/\tau_\Omega = 1\%$



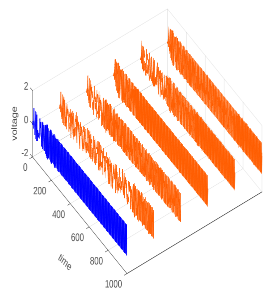
(b) $\tau_{corr}/\tau_\Omega = 50\%$



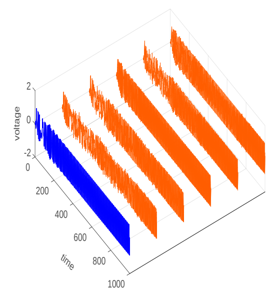
(c) $\tau_{corr}/\tau_\Omega = 100\%$



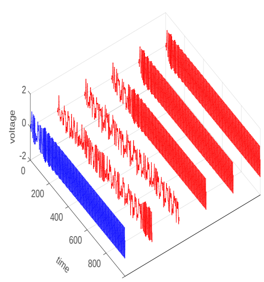
(d) $\tau_{corr}/\tau_\Omega = 1\%$



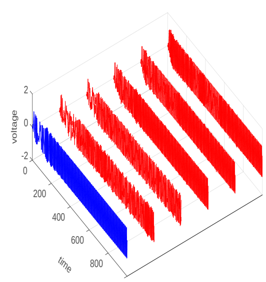
(e) $\tau_{corr}/\tau_\Omega = 50\%$



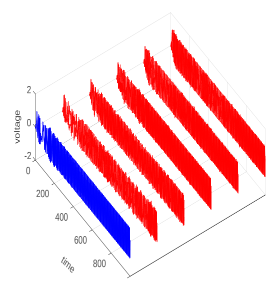
(f) $\tau_{corr}/\tau_\Omega = 100\%$



(g) $\tau_{corr}/\tau_\Omega = 1\%$



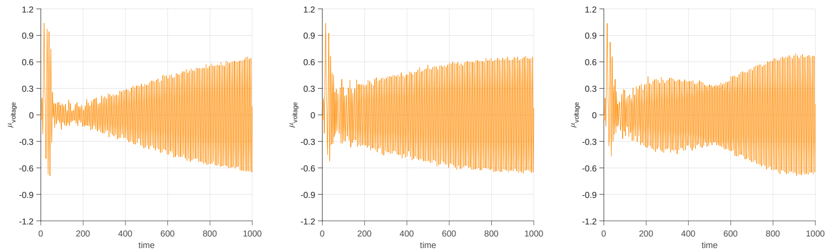
(h) $\tau_{corr}/\tau_\Omega = 50\%$



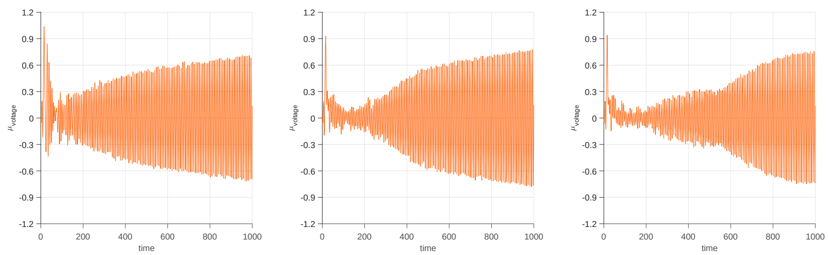
(i) $\tau_{corr}/\tau_\Omega = 100\%$

Figure 2: Typical samples of voltage time series for $\sigma/f = 1\%$ (top), $\sigma/f = 25\%$ (middle) and $\sigma/f = 50\%$ (bottom), and different values of correlation time. On blue, the deterministic case, considering harmonic forcing.

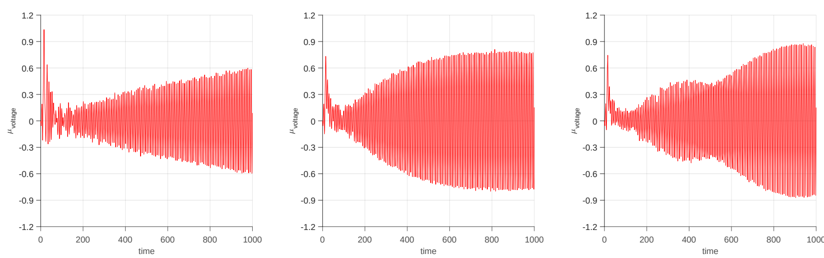
correlation times, with $\sigma/f = 25\%$, suggests longer transient states on voltage dynamics, what could be inferred trough the distortions on mean voltage signal. Although, the standard deviation curves points to less variations on output voltages regards the mean behavior, pointing to a typical pattern. A



(a) $\tau_{corr}/\tau_\Omega = 1\%$ (b) $\tau_{corr}/\tau_\Omega = 50\%$ (c) $\tau_{corr}/\tau_\Omega = 100\%$



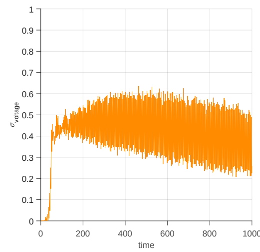
(d) $\tau_{corr}/\tau_\Omega = 1\%$ (e) $\tau_{corr}/\tau_\Omega = 50\%$ (f) $\tau_{corr}/\tau_\Omega = 100\%$



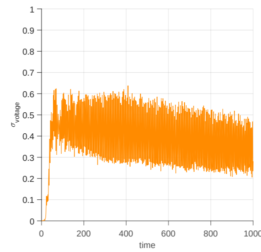
(g) $\tau_{corr}/\tau_\Omega = 1\%$ (h) $\tau_{corr}/\tau_\Omega = 50\%$ (i) $\tau_{corr}/\tau_\Omega = 100\%$

Figure 3: Voltage time series mean for $\sigma/f = 1\%$ (top), $\sigma/f = 25\%$ (middle) and $\sigma/f = 50\%$ (bottom), and different values of correlation time.

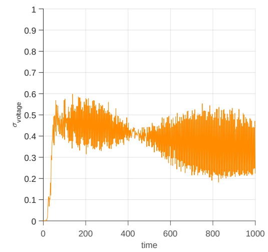
comparison between the results with $\tau_{corr}/\tau_\Omega = 1\%$ suggests that low correlation times reduce the effects of a higher noise incidence over the harmonic forcing. For $\sigma/f = 50\%$, while the mean output voltage behavior follows those on the previous cases, the standard deviation curves for higher correlation times suggests less variations between the mean curve shape and a typical voltage time series. The comparison between the standard deviation results for different levels of noise reinforces the correlation time influence over the voltage. But note that, even when noise standard deviation is high,



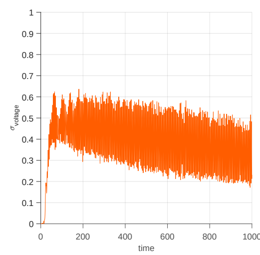
(a) $\tau_{corr}/\tau_{\Omega} = 1\%$



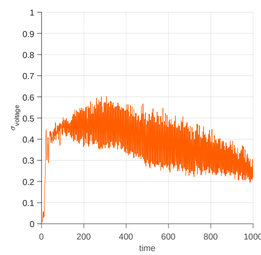
(b) $\tau_{corr}/\tau_{\Omega} = 50\%$



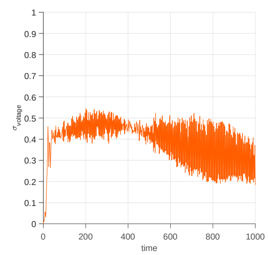
(c) $\tau_{corr}/\tau_{\Omega} = 100\%$



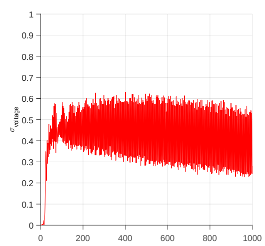
(d) $\tau_{corr}/\tau_{\Omega} = 1\%$



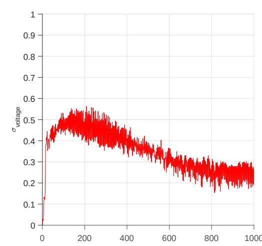
(e) $\tau_{corr}/\tau_{\Omega} = 50\%$



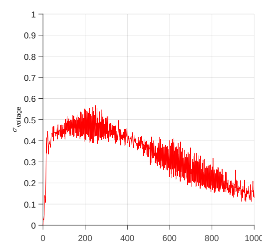
(f) $\tau_{corr}/\tau_{\Omega} = 100\%$



(g) $\tau_{corr}/\tau_{\Omega} = 1\%$



(h) $\tau_{corr}/\tau_{\Omega} = 50\%$



(i) $\tau_{corr}/\tau_{\Omega} = 100\%$

Figure 4: Voltage time series standard deviation for $\sigma/f = 1\%$ (top), $\sigma/f = 25\%$ (middle) and $\sigma/f = 50\%$ (bottom), and different values of correlation time.

a low correlation time suppress its effects over system voltage response.

4. Final Remarks

The paper analyzed the influence of a random noise component on external harmonic forcing acting over a piezoelectric energy harvesting system. Numerical simulations considered different values noise correlation times and standard deviations, from which noise incidence and magnitude were tested. The results suggests a strong influence of noise signal over the voltage results for higher correlation times and standard deviations, what implies in longer chaotic transient states. Lower correlation times reduces the impacts of the higher noise magnitudes. For future works, authors intent to carry out more detailed analysis by expanding the noise correlation time and coefficient of variation and observation windows.

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