Structural Estimation of Dynamic Stochastic
Optimizing Models of Intertemporal Choice
For Dummies!

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http://www.econ2.jhu.edu/people/ccarroll/SolvingMicroDSOPs-Slides.pdf

- Efficient Solution Methods for Canonical C problem

    - CRRA utility
    - Plausible (microeconomically calibrated) uncertainty
    - Life cycle or infinite horizon

- How To Add a Second Choice Variable

- Method of Simulated Moments Estimation of Parameters

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The Basic Problem at Date t

$$\max \ \mathbb{E}_t \left[\sum_{n=0}^{T-t} \beth^n \mathrm{u}(\mathbf{c}_{t+n}) \right]. \tag{1}$$

$$y_t = \boldsymbol{p}_t \boldsymbol{\theta}_t \tag{2}$$

$\mathbf{p}_{t+1} = \mathcal{G}_{t+1}\mathbf{p}_t$ - permanent labor income dynamics

$\log \boldsymbol{\theta}_{t+n} \sim \mathcal{N}(-\sigma_{\boldsymbol{\theta}}^2/2, \sigma_{\boldsymbol{\theta}}^2)$ - lognormal transitory shocks $\forall \ n > 0$

$$\tag{3}$$

Bellman Equation

$$\mathbf{v}_t(\mathbf{m}_t, \mathbf{p}_t) = \max_{\mathbf{c}_t} \ \mathrm{u}(\mathbf{c}_t) + \beth \mathbb{E}_t[\mathbf{v}_{t+1}(\mathbf{m}_{t+1}, \mathbf{p}_{t+1})] \tag{4}$$

$m-$ 'market resources' (net worth plus current income)

$p-$ permanent labor income

Trick: Normalize the Problem

$$
\begin{aligned}
\mathrm{v}_t(m_t) = \max_{c_t} \quad & \mathrm{u}(c_t) + \beta \mathbb{E}_t[\mathcal{G}_{t+1}^{1-\rho} \mathrm{v}_{t+1}(m_{t+1})] \\
& \text{s.t.} \\
a_t &= m_t - c_t \\
m_{t+1} &= \underbrace{(\mathrm{R}/\mathcal{G}_{t+1})}_{\equiv \mathcal{R}_{t+1}} a_t + \boldsymbol{\theta}_{t+1}.
\end{aligned}
\tag{5}
$$

where nonbold variables are bold ones normalized by $\boldsymbol{p}$:

$$
m_t = m_t / \boldsymbol{p}_t
\tag{6}
$$

Yields $\mathrm{c}_t(m)$ from which we can obtain

$$
c_t(m_t, \boldsymbol{p}_t) = \mathrm{c}_t(m_t / \boldsymbol{p}_t) \boldsymbol{p}_t
\tag{7}
$$

When Doesn't Normalization Work?

- Non-CRRA utility
- Non-Friedman (transitory/permanent) income process
    - e.g., AR(1)
    - But micro evidence is consistent with Friedman

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Trick: View Everything from End of Period

Define
$$v_{t^+}(a_t) = \mathbb{E}_t[\beta \mathcal{G}_{t+1}^{1-\rho} v_{t+1}(\mathcal{R}_{t+1} a_t + \boldsymbol{\theta}_{t+1})] \tag{8}$$

so
$$v_t(m_t) = \max_{c_t} \quad u(c_t) + \mathfrak{v}_t(m_t - c_t) \tag{9}$$

with FOC
$$u^c(c_t) = v_{t^+}^a(m_t - c_t). \tag{10}$$

and Envelope relation

$$u^c(c_t) = v_t^m(m_t) \tag{11}$$

Trick: Discretize the Risks

E.g. use an equiprobable 7-point distribution:
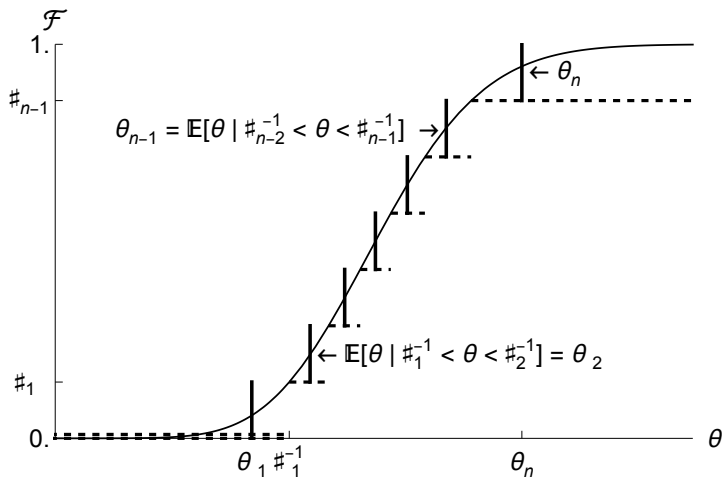
Trick: Discretize the Risks

$$\mathfrak{v}'_t(a_t) = \beta R \mathcal{G}_{t+1}^{-\rho} \left(\frac{1}{n}\right) \sum_{i=1}^{n} u'\left(c_{t+1}(\mathcal{R}_{t+1} a_t + \boldsymbol{\theta}_i)\right) \tag{12}$$

So for any particular $m_{\mathcal{T}-1}$ the corresponding $c_{\mathcal{T}-1}$ can be found using the FOC:

$$\mathrm{u}^c(c_t) = \mathrm{v}^a_{t+}(m_t - c_t). \tag{13}$$

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Trick: Interpolate a Consumption Rule

1. Define a grid of points $\vec{m}$ (indexed $m[i]$)
2. Use numerical rootfinder to solve $u'(c) = \mathfrak{v}'_t(m[i] - c)$
   - The c that solves this becomes $c[i]$
3. Construct interpolating function $\grave{c}$ by linear interpolation
   - 'Connect-the-dots'

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Trick: Interpolate a Consumption Rule

Example: $\vec{m}_{T-1} = \{0., 1., 2., 3., 4.\}$ (solid is 'correct' soln)
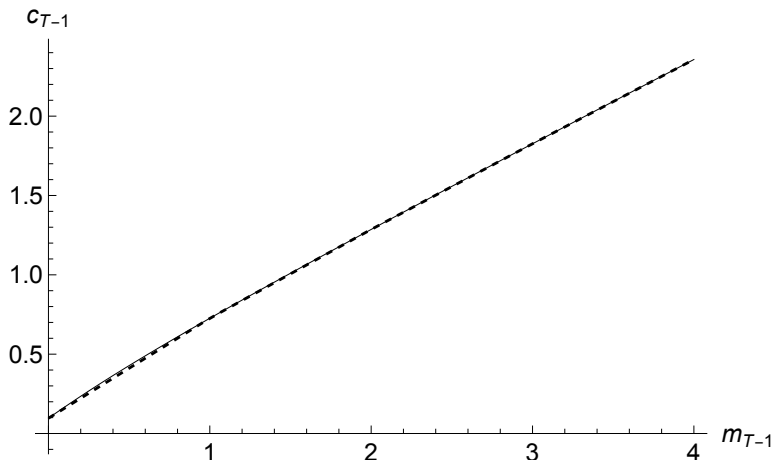
Problem: Numerical Rootfinding is *Slow*

Numerical search for values of c_{T-1} satisfying $u'(c) = \mathfrak{v}'_t(m[i] - c)$ at, say, 6 gridpoints of $\vec{m}_{T-1}$ may require hundreds or even thousands of evaluations of

$$\mathfrak{v}'_{T-1}(\overbrace{m_{T-1} - c_{T-1}}^{a_{T-1}}) = \beta_T \mathcal{G}_T^{1-\rho} \left(\frac{1}{n}\right) \sum_{i=1}^{n} (\mathcal{R}_T a_{T-1} + \boldsymbol{\theta}_i)^{-\rho}$$

Solution: The Method of Endogenous Gridpoints

- Define vector of *end-of-period* asset values $\vec{a}$
- For each $a[j]$ compute $\mathfrak{v}'_t(a[j])$

Each of these $\mathfrak{v}'_t[j]$ corresponds to a unique $c[j]$ via FOC:

$$
\begin{aligned}
c[j]^{-\rho} &= \mathfrak{v}'_t(a[j]) \\
c[j] &= \left(\mathfrak{v}'_t(a[j])\right)^{-1/\rho}
\end{aligned}
\tag{14}
$$

But the DBC says

$$
\begin{aligned}
a_t &= m_t - c_t \\
m[j] &= a[j] + c[j]
\end{aligned}
\tag{15}
$$

So computing $\mathfrak{v}'_t$ at a vector of $\vec{a}$ values has produced for us the corresponding $\vec{c}$ and $\vec{m}$ values at virtually no cost!

From these we can interpolate as before to construct $\grave{c}_t(m)$.

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Why Directly Approximating v_t is a Bad Idea

Principles of Approximation

- Hard to approximate things that approach ∞ for relevant m
  - Not a prob for Rep Agent models: 'relevant' m's are $\approx$ SS

- Hard to approximate things that are highly nonlinear

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Approximate Something That Would Be Linear in PF Case

Perfect Foresight Theory:

$$c_t(m) = (m + \mathfrak{h}_t)\underline{\kappa}_t \tag{16}$$

for market resources m and end-of-period human wealth $\mathfrak{h}$.

This is why it's a good idea to approximate c_t

Bonus: Easy to debug programs by setting $\sigma^2 = 0$ and testing whether numerical solution matches analytical!

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But What if You *Need* the Value Function?

Perfect foresight value function:

$$
\begin{aligned}
\bar{v}_t(m_t) &= u(\bar{c}_t)\mathbb{C}_t^T \\
&= u(\bar{c}_t)\underline{\kappa}_t^{-1} \\
&= u((\blacktriangle m_t + \blacktriangle\mathfrak{h}_t)\underline{\kappa}_t)\underline{\kappa}_t^{-1} \\
&= u(\blacktriangle m_t + \blacktriangle\mathfrak{h}_t)\underline{\kappa}_t^{1-\rho}\underline{\kappa}_t^{-1} \\
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$$

where the second line uses the fact demonstrated in Carroll (2023)
that $\mathbb{C}_t = \kappa_t^{-1}$.
This can be transformed as

$$
\begin{aligned}
\bar{\Lambda}_t &\equiv ((1-\rho)\bar{v}_t)^{1/(1-\rho)} \\
&= c_t(\mathbb{C}_t^T)^{1/(1-\rho)} \\
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Approximate Slope Too

Carroll (2023) shows that $\mathfrak{c}_t^m$ exists everywhere.

Define *consumed* function and its derivative as

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\begin{aligned}
\mathfrak{c}_t(a) &= (\mathfrak{v}_t'(a))^{-1/\rho} \\
\mathfrak{c}_t^a(a) &= -(1/\rho)\left(\mathfrak{v}_t'(a)\right)^{-1-1/\rho}\mathfrak{v}_t''(a)
\end{aligned}
\tag{19}
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and using chain rule it is easy to show that

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\mathfrak{c}_t^m = \mathfrak{c}_t^a/(1 + \mathfrak{c}_t^a)
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To Implement: Modify Prior Procedures in Two Ways

1. Construct $\vec{c}_t^m$ along with $\vec{c}_t$ in EGM algorithm

2. Approximate $c_t(m)$ using piecewise Hermite polynomial

   • Exact match to both level and derivative at set of points

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Problem: c̆ Below Bottom m Gridpoint and Extrapolation

Consider what happens as a_{T-1} approaches $\underline{a}_{T-1} \equiv -\boldsymbol{\theta}\mathcal{R}_T^{-1}$,

$$\lim_{a\downarrow\underline{a}_{T-1}} \mathfrak{v}'_{T-1}(a) = \lim_{a\downarrow\underline{a}_{T-1}} \beta R \mathcal{G}_T^{-\rho}\left(\frac{1}{n}\right)\sum_{i=1}^{n}(a\mathcal{R}_T + \boldsymbol{\theta}_i)^{-\rho}$$

$$= \infty$$

This means our lowest value in $\vec{a}_{T-1}$ should be $> \underline{a}_{T-1}$.

Suppose we construct c̆ by linear interpolation:

$$\mathring{c}_{T-1}(m) = \mathring{c}_{T-1}(\vec{m}_{T-1}[1]) + \mathring{c}'_{T-1}(\vec{m}_{T-1}[1])(m - \vec{m}_{T-1}[1])$$

True c is strictly concave $\Rightarrow \exists m^- > \underline{m}_{T-1}$ for which
$m^- - \mathring{c}_{T-1}(m^-) < \underline{a}_{T-1}$

Solution: Hard-Code the Bottom Point

Theory says that

$$\lim_{m \downarrow \underline{m}_{T-1}} c_{T-1}(m) = 0$$

$$\lim_{m \downarrow \underline{m}_{T-1}} c^m_{T-1}(m) = \bar{\kappa}_{T-1}$$

(21)

1. Redefine $\vec{a}$ *relative* to $\underline{a}_{T-1}$

2. Construct corresponding $\vec{m}_{T-1}$ and $\vec{c}_{T-1}$

3. Prepend $\underline{m}_{T-1}$ to $\vec{m}_{T-1}$

4. Prepend 0. to $\vec{c}_{T-1}$

5. Prepend $\bar{\kappa}_{T-1}$ to $\vec{\kappa}_{T-1}$

then proceed as before.

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5. Prepend $\bar{\kappa}_{T-1}$ to $\vec{\kappa}_{T-1}$

then proceed as before.

Solution: Hard-Code the Bottom Point

Theory says that

$$
\begin{aligned}
\lim_{m \downarrow \underline{m}_{T-1}} c_{T-1}(m) &= 0 \\
\lim_{m \downarrow \underline{m}_{T-1}} c^m_{T-1}(m) &= \bar{\kappa}_{T-1}
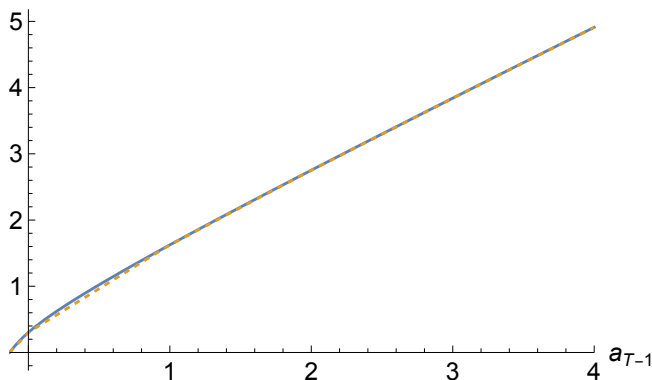\end{aligned}
\tag{21}
$$

1. Redefine $\vec{a}$ *relative* to $\underline{a}_{T-1}$
2. Construct corresponding $\vec{m}_{T-1}$ and $\vec{c}_{T-1}$
3. Prepend $\underline{m}_{T-1}$ to $\vec{m}_{T-1}$
4. Prepend 0. to $\vec{c}_{T-1}$
5. Prepend $\bar{\kappa}_{T-1}$ to $\vec{\kappa}_{T-1}$

then proceed as before.

Trick: Improving the a Grid

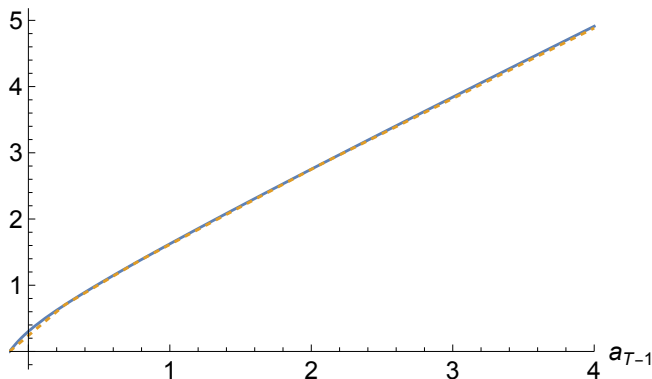Grid Spacing: Uniform

$(\mathrm{u}'_{T-1}(a_{T-1}))^{-1/\rho}, \grave{c}_{T-1}(a_{T-1})$

Trick: Improving the a Grid

Grid Spacing: Same $\{\underline{a}, \bar{a}\}$ But Triple Exponential $e^{e^{e^{\cdots}}}$ Growth

$(\mathfrak{u}'_{T-1}(a_{T-1}))^{-1/\rho}, \grave{c}_{T-1}(a_{T-1})$

The Method of Moderation

- Further improves speed and accuracy of solution
- See my talk at the conference!

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Imposing 'Artificial' Borrowing Constraints

$$
\begin{aligned}
v_{T-1}(m_{T-1}) &= \max_{c_{T-1}} u(c_{T-1}) + \mathbb{E}_{T-1}[\beta \mathcal{G}_T^{1-\rho} v_T(m_T)] \\
&\text{s.t.} \\
a_{T-1} &= m_{T-1} - c_{T-1} \\
m_T &= \mathcal{R}_T a_{T-1} + \boldsymbol{\theta}_T \\
a_{T-1} &\geq 0.
\end{aligned}
$$

Define $\grave{c}_t^*$ as soln to unconstrained problem. Then

$$
\grave{c}_{T-1}(m_{T-1}) = \min[m_{T-1}, \grave{c}_{T-1}^*(m_{T-1})]. \tag{22}
$$

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Imposing 'Artificial' Borrowing Constraints

Point where constraint makes transition from binding to not is

$$u'(m_{T-1}^{\#}) = \mathfrak{v}'_{T-1}(0.)$$
$$m_{T-1}^{\#} = \left(\mathfrak{v}'_{T-1}(0.)\right)^{-1/\rho}$$

Procedure is very easy:

- Add 0. as first point in $\vec{a}$
- $\Rightarrow \vec{m}[1] = m_{T-1}^{\#}$
- Above $m_{T-1}^{\#}$, $\grave{c}_{T-1}(m)$ obtained as before
- Below $m_{T-1}^{\#}$, $\grave{c}_{T-1}(m) = m$

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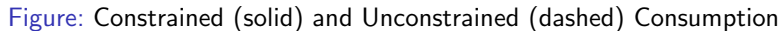
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Imposing 'Artificial' Borrowing Constraints

Figure: Constrained (solid) and Unconstrained (dashed) Consumption

Recursion: Period t Solution Given Period $t + 1$

1 Construct

$$
\begin{aligned}
c_{\bar{t},i} &= \left(v'_{t^+}(a_{t,i})\right)^{-1/\rho}, \\
&= \left(\beta\mathbb{E}_t\left[R\mathcal{G}_{t+1}^{-\rho}(\grave{c}_{t+1}(\mathcal{R}_{t+1}a_{t,i} + \boldsymbol{\theta}_{t+1}))^{-\rho}\right]\right)^{-1/\rho},
\end{aligned}
\tag{23}
$$

2 Call the result $\vec{c}_t$ and generate the corresponding $\vec{m}_t = \vec{c}_t + \vec{a}_t$

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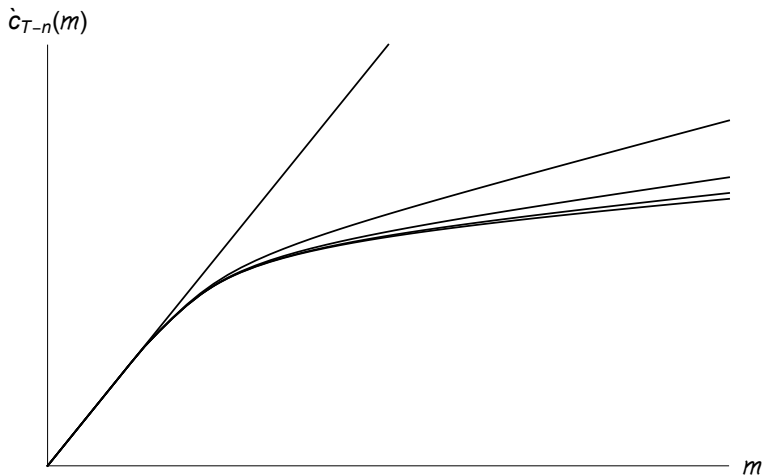# Consumption Rules $\grave{c}_{T-n}$ Converge

Figure: Converging $\grave{c}_{T-n}(m)$ Functions for $n = \{1, 5, 10, 15, 20\}$

Portfolio Choice

Now the consumer has a choice between a risky and a safe asset.
The portfolio return is

$$\begin{aligned}
\mathfrak{R}_{t+1} &= R(1 - \varsigma_t) + \mathbf{R}_{t+1}\varsigma_t \\
&= R + (\mathbf{R}_{t+1} - R)\varsigma_t
\end{aligned} \tag{24}$$

so (setting $\mathcal{G} = 1$) the maximization problem is

$$\mathbf{v}_t(m_t) = \max_{\{c_t, \varsigma_t\}} \quad \mathbf{u}(c_t) + \beta \mathbb{E}_t[\mathbf{v}_{t+1}(m_{t+1})]$$

$$\text{s.t.}$$

$$\mathfrak{R}_{t+1} = R + (\mathbf{R}_{t+1} - R)\varsigma_t$$

$$m_{t+1} = (m_t - c_t)\mathfrak{R}_{t+1} + \theta_{t+1}$$

$$0 \leq \varsigma_t \leq 1,$$

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so (setting $\mathcal{G} = 1$) the maximization problem is

$$v_t(m_t) = \max_{\{c_t, \varsigma_t\}} \quad u(c_t) + \beta \mathbb{E}_t[v_{t+1}(m_{t+1})]$$

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$$\Re_{t+1} = R + (\mathbf{R}_{t+1} - R)\varsigma_t$$

$$m_{t+1} = (m_t - c_t)\Re_{t+1} + \boldsymbol{\theta}_{t+1}$$

$$0 \leq \varsigma_t \leq 1,$$

Portfolio Choice

The FOC with respect to c_t now yields an Euler equation

$$u^c(c_t) = \mathbb{E}_t[\beta \mathfrak{R}_{t+1} u^c(c_{t+1})]. \tag{25}$$

while the FOC with respect to the portfolio share yields

$$0 = \mathbb{E}_t[v_{t+1}^m(m_{t+1})(\mathbf{R}_{t+1} - R)a_t]$$
$$= a_t \mathbb{E}_t\left[u^c\left(c_{t+1}(m_{t+1})\right)(\mathbf{R}_{t+1} - R)\right].$$

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&= a_t \mathbb{E}_t\left[\mathrm{u}^c\left(\mathrm{c}_{t+1}(m_{t+1})\right)(\mathbf{R}_{t+1} - \mathrm{R})\right].
\end{aligned}$$

Convergence

When the problem satisfies certain conditions (Carroll (2023)), it defines a 'converged' consumption rule with a 'target' ratio $\check{m}$ that satisfies:

$$\mathbb{E}_t[m_{t+1}/m_t] = 1 \text{ if } m_t = \check{m} \tag{26}$$

Define the target m implied by the consumption rule c_t as $\check{m}_t$.

Then a plausible metric for convergence is to define some value ϵ and to declare the solution to have converged when

$$|\check{m}_{t+1} - \check{m}_t| < \epsilon \tag{27}$$

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Trick: Coarse then Fine θ

1. Start with coarse grid for θ (say, 3 points)

2. Solve to convergence; call period of convergence n

3. Construct finer grid for θ (say, 7 points)

4. Solve for period $T - n - 1$ assuming $\grave{c}_{T-n}$

5. Continue to convergence

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5. Continue to convergence

Trick: Coarse then Fine $\vec{a}_{T-1}$

1. Start with coarse grid for $\vec{a}$ (say, 5 gridpoints)

2. Solve to convergence; call period of convergence n

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Life Cycle Maximization Problem

$$v_t(m_t) = \max_{c_t} \quad u(c_t) + \beth \mathcal{L}_{t+1}\hat{\beta}_{t+1}\mathbb{E}_t[(\boldsymbol{\Psi}_{t+1}\mathcal{G}_{t+1})^{1-\rho}v_{t+1}(m_{t+1})]$$

$$\text{s.t.}$$

$$a_t = m_t - c_t$$

$$m_{t+1} = a_t \underbrace{\left(\frac{R}{\boldsymbol{\Psi}_{t+1}\mathcal{G}_{t+1}}\right)}_{\equiv \mathcal{R}_{t+1}} + \boldsymbol{\theta}_{t+1}$$

$$\mathcal{L}_t^{t+n} : \quad \text{probability to } \mathcal{L}\text{ive until age } t+n \text{ given alive at age } t$$

$$\hat{\beta}_t^{t+n} : \quad \text{age-varying discount factor between ages } t \text{ and } t+n$$

$$\Psi_t : \quad \text{mean-one shock to permanent income}$$

$$\beth : \quad \text{time-invariant 'pure' discount factor}$$

Details follow Cagetti (2003)

- Parameterization of Uncertainty

- Probability of Death

- Demographic Adjustments to β

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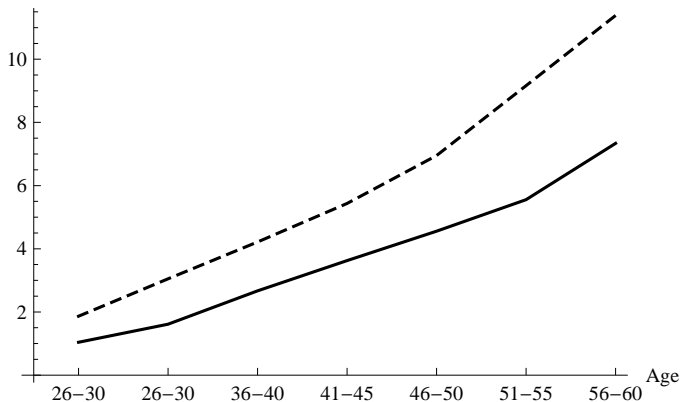- Demographic Adjustments to β

Empirical Wealth Profiles

Figure: *m* from SCF (means (dashed) and medians (solid))

Simulated Moments

Given a set of parameter values $\{\rho, \beth\}$:

- Start at age 25 with empirical m data
- Draw shocks using calibrated $\sigma_{\psi}^2, \sigma_{\theta}^2$
- Consume according to solved c_t

$\Rightarrow m$ distribution by age

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$\Rightarrow m$ distribution by age

Choose What to Simulate

```
GapEmpiricalSimulatedMedians[ρ,⌐]:=
[                       ConstructcFuncLife[ρ,⌐];
Simulate;
```

$$\sum_i^N \omega_i \, |\varsigma_i^\tau - \mathbf{s}^\tau(\xi)|$$

```
];
```

Calculate Match Between Theory and Data

$$\xi = \{\rho, \beth\} \tag{28}$$

solve

$$\min_{\xi} \sum_i^N \omega_i \left| \varsigma_i^\tau - \mathbf{s}^\tau(\xi) \right| \tag{29}$$

Bootstrap Standard Errors (Horowitz (2001))

Yields estimates of

Table: Estimation Results

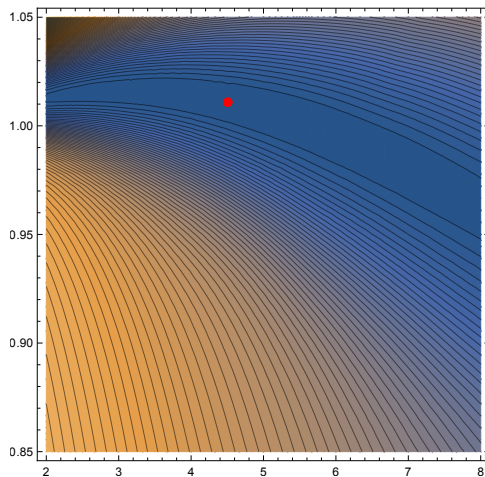| ρ	⊐
4.68	1.00
(0.13)	(0.00)

Contour Plot

Figure: Point Estimate and Height of Minimized Function

References I

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HOROWITZ, JOEL L. (2001): "The Bootstrap," in Handbook of Econometrics, ed. by James J. Heckman, and Edward Leamer, vol. 5. Elsevier/North Holland.