Structural Estimation of Dynamic Stochastic Optimizing Models of Intertemporal Choice For Dummies!

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http://www.econ2.jhu.edu/people/ccarroll/SolvingMicroDSOPs-Slides.pdf

- Efficient Solution Methods for Canonical C problem
  - CRRA utility
  - Plausible (microeconomically calibrated) uncertainty
  - Life cycle or infinite horizon
- How To Add a Second Choice Variable
- Method of Simulated Moments Estimation of Parameters

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The Basic Problem at Date t

$$\max \ \mathbb{E}_t \left[\sum_{n_\theta=0}^{T-t} \beth^{n_\theta} u(c_{t+n}) \right], \tag{1}$$

$$y_t = \boldsymbol{p}_t \theta_t \tag{2}$$

$$\begin{aligned} \mathrm{R}_t &= \mathrm{R} \ \forall \ t && \text{- constant interest factor} = 1 + \mathrm{r} \\ \boldsymbol{p}_{t+1} &= \Gamma_{t+1} \boldsymbol{p}_t && \text{- permanent labor income dynamics} \\ \log \ \theta_{t+n} &\sim \mathcal{N}(-\sigma_\theta^2/2, \sigma_\theta^2) && \text{- lognormal transitory shocks} \ \forall \ n > 0. \end{aligned} \tag{3}$$

Bellman Equation

$$\mathrm{v}_t(m_t, \boldsymbol{p}_t) \quad = \quad \max_{c_t} \; u(c_t) + \mathbb{E}_t[\beth \mathrm{v}_{t+1}(m_{t+1}, \boldsymbol{p}_{t+1})] \tag{4}$$

m	–	'market resources' (net worth plus current income)
$\boldsymbol{p}$	–	permanent labor income

Trick: Normalize the Problem

$$
\begin{aligned}
\mathrm{v}_t(m_t) &= \max_{c_t} \quad u(c_t) + \mathbb{E}_t[\beta \Gamma_{t+1}^{1-\rho} \mathrm{v}_{t+1}(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t \\
m_{t+1} &= \underbrace{(\mathsf{R}/\Gamma_{t+1})}_{\equiv \mathcal{R}_{t+1}} a_t + \theta_{t+1}
\end{aligned} \tag{5}
$$

where nonbold variables are bold ones normalized by $\boldsymbol{p}$:

$$
m_t = \boldsymbol{m}_t / \boldsymbol{p}_t \tag{6}
$$

Yields $\mathrm{c}_t(m)$ from which we can obtain

$$
\boldsymbol{c}_t(\boldsymbol{m}_t, \boldsymbol{p}_t) = \mathrm{c}_t(\boldsymbol{m}_t/\boldsymbol{p}_t)\boldsymbol{p}_t \tag{7}
$$

When Doesn't Normalization Work?

- Non-CRRA utility
- Non-Friedman (transitory/permanent) income process
  - e.g., AR(1)
  - But micro evidence is consistent with Friedman

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Trick: View Everything from End of Period

Define

$$\mathfrak{v}_t(a_t) = \mathbb{E}_t[\beta\Gamma_{t+1}^{1-\rho}\mathrm{v}_{t+1}(\mathcal{R}_{t+1}a_t + \theta_{t+1})] \tag{8}$$

so

$$\mathrm{v}_t(m_t) = \max_{c_t} \; u(c_t) + \mathfrak{v}_t(m_t - c_t) \tag{9}$$

with FOC

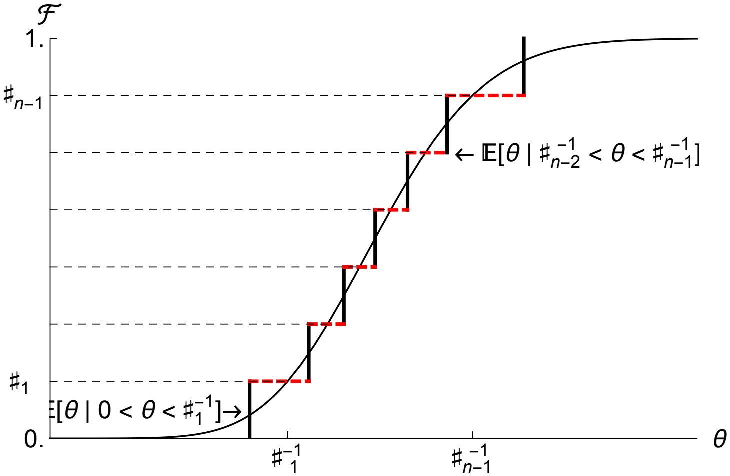
$$u'(c_t) = \mathfrak{v}'_t(m_t - c_t). \tag{10}$$

and Envelope relation

$$u'(c_t) = \mathrm{v}'_t(m_t) \tag{11}$$

Trick: Discretize the Risks

E.g. use an equiprobable 7-point distribution:



Trick: Discretize the Risks

$$\mathfrak{v}'_t(a_t) = \beta \mathsf{R} \Gamma_{t+1}^{-\rho} \left(\frac{1}{n}\right) \sum_{i=1}^{n} u'\left(\mathrm{c}_{t+1}(\mathcal{R}_{t+1} a_t + \theta_i)\right) \quad (12)$$

So for any particular m_{T-1} the corresponding c_{T-1} can be found using the FOC:

$$u'(c_t) = \mathfrak{v}'_t(m_t - c_t). \quad (13)$$

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Trick: Interpolate a Consumption Rule

1. Define a grid of points $\vec{m}$ (indexed $m[i]$)
2. Use numerical rootfinder to solve $u'(c) = \mathfrak{v}'_t(m[i] - c)$
   - The c that solves this becomes $c[i]$
3. Construct interpolating function $\grave{c}$ by linear interpolation
   - 'Connect-the-dots'

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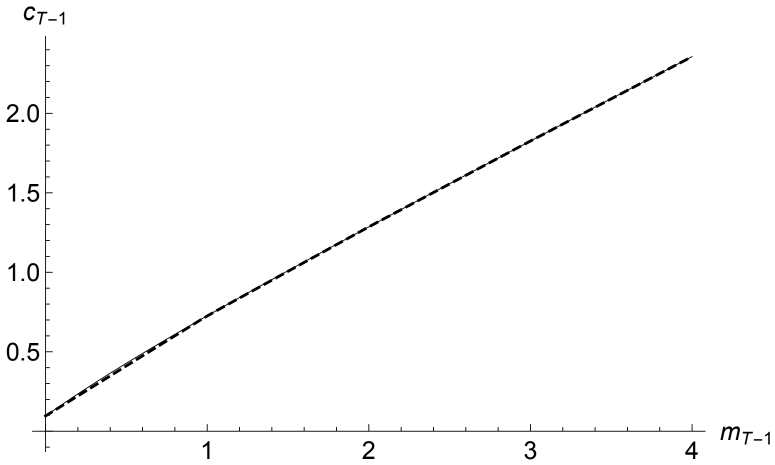
Trick: Interpolate a Consumption Rule

Example: $\vec{m}_{T-1} = \{0., 1., 2., 3., 4.\}$ (solid is 'correct' soln)



Problem: Numerical Rootfinding is *Slow*

Numerical search for values of c_{T-1} satisfying $u'(c) = \mathfrak{v}'_t(m[i] - c)$ at, say, 6 gridpoints of $\vec{m}_{T-1}$ may require hundreds or even thousands of evaluations of

$$\mathfrak{v}'_{T-1}(\overbrace{m_{T-1} - c_{T-1}}^{a_{T-1}}) \quad = \quad \beta_T \Gamma_T^{1-\rho} \left(\frac{1}{n}\right) \sum_{i=1}^{n} \frac{(\mathcal{R}_T a_{T-1} + \theta_i)^{1-\rho}}{1-\rho}$$

Solution: The Method of Endogenous Gridpoints

- Define vector of *end-of-period* asset values $\vec{a}$
- For each $a[j]$ compute $\mathfrak{v}'_t(a[j])$

Each of these $\mathfrak{v}'_t[j]$ corresponds to a unique $c[j]$ via FOC:

$$c[j]^{-\rho} = \mathfrak{v}'_t(a[j]) \tag{14}$$

$$c[j] = \left(\mathfrak{v}'_t(a[j])\right)^{-1/\rho} \tag{15}$$

But the DBC says

$$a_t = m_t - c_t \tag{16}$$

$$m[j] = a[j] + c[j] \tag{17}$$

So computing $\mathfrak{v}'_t$ at a vector of $\vec{a}$ values has produced for us the corresponding $\vec{c}$ and $\vec{m}$ values at virtually no cost!

From these we can interpolate as before to construct $\grave{c}_t(m)$.

Solution: The Method of Endogenous Gridpoints

- Define vector of *end-of-period* asset values $\vec{a}$
- For each $a[j]$ compute $\mathfrak{v}_t'(a[j])$

Each of these $\mathfrak{v}_t'[j]$ corresponds to a unique $c[j]$ via FOC:

$$
\begin{aligned}
c[j]^{-\rho} &= \mathfrak{v}_t'(a[j]) && (14)\\
c[j] &= \left(\mathfrak{v}_t'(a[j])\right)^{-1/\rho} && (15)
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Why Directly Approximating v_t is a Bad Idea

Principles of Approximation

- Hard to approximate things that approach ∞ for relevant m
    - Not a prob for Rep Agent models: 'relevant' m's are $\approx$ SS
- Hard to approximate things that are highly nonlinear

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Approximate Something That Would Be Linear in PF Case

Perfect Foresight Theory:

$$\mathrm{c}_t(m) \quad = \quad (m + \mathfrak{h}_t)\underline{\kappa}_t \tag{18}$$

for market resources m and end-of-period human wealth $\mathfrak{h}$.

This is why it's a good idea to approximate c_t

Bonus: Easy to debug programs by setting $\sigma^2 = 0$ and testing whether numerical solution matches analytical!

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But What if You *Need* the Value Function?

Perfect foresight value function:

$$
\begin{aligned}
\bar{\mathrm{v}}_t(m_t) &= u(\bar{c}_t)\mathbb{C}_t^T && (19)\\
&= u(\bar{c}_t)\underline{\kappa}_t^{-1} && (20)\\
&= u((\blacktriangle m_t + \blacktriangle \mathfrak{h}_t)\underline{\kappa}_t)\underline{\kappa}_t^{-1} && (21)\\
&= u(\blacktriangle m_t + \blacktriangle \mathfrak{h}_t)\underline{\kappa}_t^{1-\rho}\underline{\kappa}_t^{-1} && (22)\\
&= u(\blacktriangle m_t + \blacktriangle \mathfrak{h}_t)\underline{\kappa}_t^{-\rho} && (23)
\end{aligned}
$$

where the second line uses the fact demonstrated in Carroll (Forthcoming) that $\mathbb{C}_t = \kappa_t^{-1}$.
This can be transformed as

$$
\begin{aligned}
\bar{\Lambda}_t &\equiv ((1-\rho)\bar{\mathrm{v}}_t)^{1/(1-\rho)}\\
&= c_t(\mathbb{C}_t^T)^{1/(1-\rho)}\\
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Approximate Slope Too

Carroll (Forthcoming) shows that c_t^m exists everywhere.

Define *consumed* function and its derivative as

$$\mathfrak{c}_t(a) = (\mathfrak{v}_t'(a))^{-1/\rho} \tag{25}$$

$$\mathfrak{c}_t^a(a) = -(1/\rho)\left(\mathfrak{v}_t'(a)\right)^{-1-1/\rho}\mathfrak{v}_t''(a) \tag{26}$$

and using chain rule it is easy to show that

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To Implement: Modify Prior Procedures in Two Ways

1. Construct $\vec{\mathrm{c}}_t^m$ along with $\vec{\mathrm{c}}_t$ in EGM algorithm
2. Approximate $\mathrm{c}_t(m)$ using piecewise Hermite polynomial
   - Exact match to both level and derivative at set of points

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Problem: $\grave{c}$ Below Bottom m Gridpoint and Extrapolation

Consider what happens as a_{T-1} approaches $\underline{a}_{T-1} \equiv -\underline{\theta}\mathcal{R}_T^{-1}$,

$$\begin{aligned} \lim_{a \downarrow \underline{a}_{T-1}} \mathfrak{v}'_{T-1}(a) &= \lim_{a \downarrow \underline{a}_{T-1}} \beta R \Gamma_T^{-\rho} \left(\frac{1}{n}\right) \sum_{i=1}^{n} (a\mathcal{R}_T + \theta_i)^{-\rho} \\ &= \infty \end{aligned}$$

This means our lowest value in $\vec{a}_{T-1}$ should be $> \underline{a}_{T-1}$.

Suppose we construct $\grave{c}$ by linear interpolation:

$$\grave{c}_{T-1}(m) = \grave{c}_{T-1}(\vec{m}_{T-1}[1]) + \grave{c}'_{T-1}(\vec{m}_{T-1}[1])(m - \vec{m}_{T-1}[1])$$

True c is strictly concave $\Rightarrow \exists m^- > \underline{m}_{T-1}$ for which $m^- - \grave{c}_{T-1}(m^-) < \underline{a}_{T-1}$

Solution: Hard-Code the Bottom Point

Theory says that

$$\lim_{m \downarrow \underline{m}_{T-1}} \mathrm{c}_{T-1}(m) = 0 \tag{28}$$

$$\lim_{m \downarrow \underline{m}_{T-1}} \mathrm{c}^{m}_{T-1}(m) = \bar{\kappa}_{T-1} \tag{29}$$

1. Redefine $\vec{a}$ *relative* to $\underline{a}_{T-1}$
2. Construct corresponding $\vec{m}_{T-1}$ and $\vec{c}_{T-1}$
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$$\lim_{m \downarrow \underline{m}_{T-1}} \mathrm{c}_{T-1}(m) = 0 \tag{28}$$

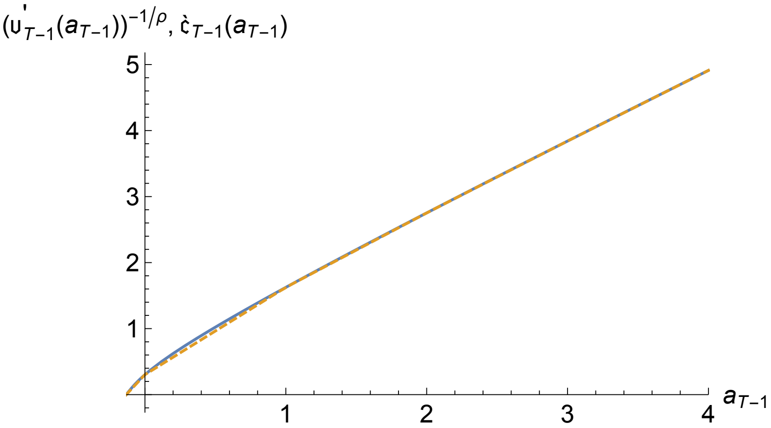
$$\lim_{m \downarrow \underline{m}_{T-1}} \mathrm{c}^{m}_{T-1}(m) = \bar{\kappa}_{T-1} \tag{29}$$

1. Redefine $\vec{a}$ *relative* to $\underline{a}_{T-1}$
2. Construct corresponding $\vec{m}_{T-1}$ and $\vec{c}_{T-1}$
3. Prepend $\underline{m}_{T-1}$ to $\vec{m}_{T-1}$
4. Prepend 0. to $\vec{c}_{T-1}$
5. Prepend $\bar{\kappa}_{T-1}$ to $\vec{\kappa}_{T-1}$

then proceed as before.

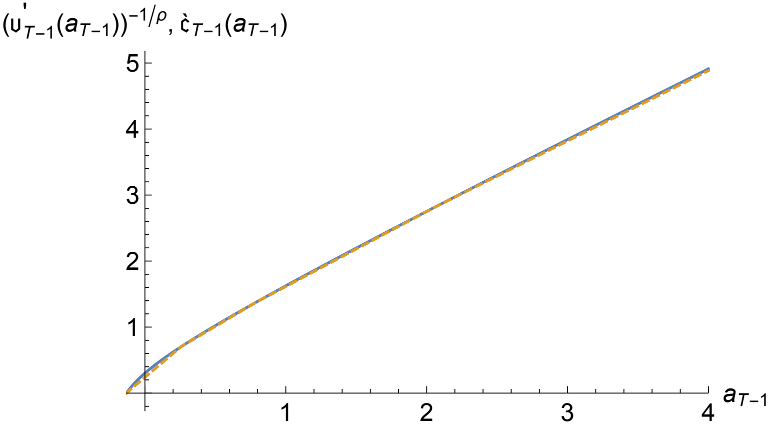
Trick: Improving the a Grid

Grid Spacing: Uniform



Trick: Improving the a Grid

Grid Spacing: Same $\{\underline{a}, \bar{a}\}$ But Triple Exponential $e^{e^{e^{\cdots}}}$ Growth



The Method of Moderation

- Further improves speed and accuracy of solution
- See my talk at the conference!

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Imposing 'Artificial' Borrowing Constraints

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\begin{aligned}
\mathrm{v}_{T-1}(m_{T-1}) &= \max_{c_{T-1}} \; u(c_{T-1}) + \mathbb{E}_{T-1}[\beta \Gamma_T^{1-\rho} \mathrm{v}_T(m_T)] \\
&\text{s.t.} \\
a_{T-1} &= m_{T-1} - c_{T-1} \\
m_T &= \mathcal{R}_T a_{T-1} + \theta_T \\
a_{T-1} &\geq 0.
\end{aligned}
$$

Define $\grave{\mathrm{c}}_t^*$ as soln to unconstrained problem. Then

$$
\grave{\mathrm{c}}_{T-1}(m_{T-1}) = \min[m_{T-1}, \grave{\mathrm{c}}_{T-1}^*(m_{T-1})]. \tag{30}
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Imposing 'Artificial' Borrowing Constraints

Point where constraint makes transition from binding to not is

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\begin{aligned}
u'(m^{\#}_{T-1}) &= \mathfrak{v}'_{T-1}(0.) \\
m^{\#}_{T-1} &= \left(\mathfrak{v}'_{T-1}(0.)\right)^{-1/\rho}
\end{aligned}
$$

Procedure is very easy:

- Add 0. as first point in $\vec{a}$
- $\Rightarrow \vec{m}[1] = m^{\#}_{T-1}$
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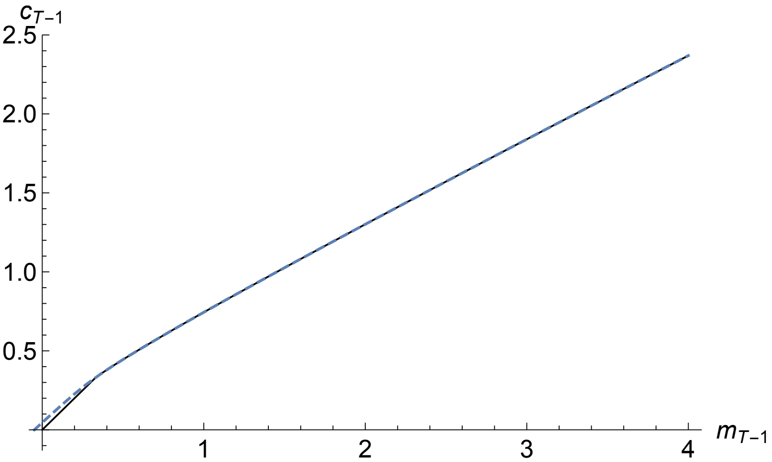

Figure: Constrained (solid) and Unconstrained (dashed) Consumption

Recursion: Period t Solution Given Period $t+1$

1. Construct

$$\begin{aligned} \mathfrak{c}_{t,i} &= \left(\mathfrak{v}'_t(a_{t,i})\right)^{-1/\rho}, & (31) \\ &= \left(\beta \mathbb{E}_t \left[\mathsf{R}\Gamma_{t+1}^{-\rho}(\grave{\mathrm{c}}_{t+1}(\mathcal{R}_{t+1} a_{t,i} + \theta_{t+1}))^{-\rho} \right]\right)^{-1/\rho} & (32) \end{aligned}$$

2. Call the result $\vec{c}_t$ and generate the corresponding $\vec{m}_t = \vec{c}_t + \vec{a}_t$
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Consumption Rules $\grave{c}_{T-n}$ Converge

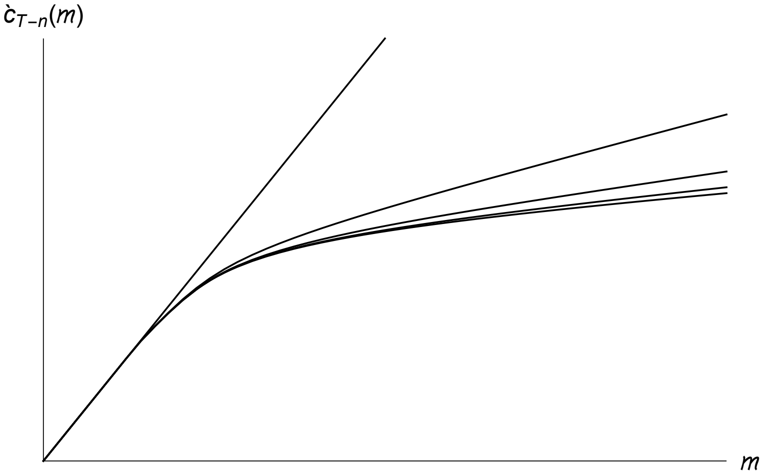

Figure: Converging $\grave{c}_{T-n}(m)$ Functions for $n = \{1, 5, 10, 15, 20\}$

Portfolio Choice

Now the consumer has a choice between a risky and a safe asset. The portfolio return is

$$\mathbb{R}_{t+1} = \mathsf{R}(1-\varsigma_t) + \mathbf{R}_{t+1}\varsigma_t \tag{33}$$

$$= \mathsf{R} + (\mathbf{R}_{t+1} - \mathsf{R})\varsigma_t \tag{34}$$

so (setting $\Gamma = 1$) the maximization problem is

$$\begin{aligned} \mathrm{v}_t(m_t) &= \max_{\{c_t,\varsigma_t\}} \quad u(c_t) + \beta\mathbb{E}_t[\mathrm{v}_{t+1}(m_{t+1})] \\ &\text{s.t.} \\ \mathbb{R}_{t+1} &= \mathsf{R} + (\mathbf{R}_{t+1} - \mathsf{R})\varsigma_t \\ m_{t+1} &= (m_t - c_t)\mathbb{R}_{t+1} + \theta_{t+1} \\ 0 \leq \varsigma_t &\leq 1, \end{aligned}$$

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Portfolio Choice

The FOC with respect to c_t now yields an Euler equation

$$u'(c_t) \quad = \quad \mathbb{E}_t[\beta \mathbb{R}_{t+1} u'(c_{t+1})]. \tag{35}$$

while the FOC with respect to the portfolio share yields

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Convergence

When the problem satisfies certain conditions (Carroll (Forthcoming)), it defines a 'converged' consumption rule with a 'target' ratio $\check{m}$ that satisfies:

$$\mathbb{E}_t[m_{t+1}/m_t] = 1 \text{ if } m_t = \check{m} \quad (37)$$

Define the target m implied by the consumption rule c_t as $\check{m}_t$.

Then a plausible metric for convergence is to define some value ϵ and to declare the solution to have converged when

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Trick: Coarse then Fine θ

1. Start with coarse grid for θ (say, 3 points)
2. Solve to convergence; call period of convergence n
3. Construct finer grid for θ (say, 7 points)
4. Solve for period $T - n - 1$ assuming $\grave{c}_{T-n}$
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Life Cycle Maximization Problem

$$
\begin{aligned}
\mathrm{v}_t(m_t) &= \max_{c_t} \left\{ u(c_t) + \beth \aleph_{t+1} \hat{\beta}_{t+1} \mathbb{E}_t [(\psi_{t+1} \Gamma_{t+1})^{1-\rho} \mathrm{v}_{t+1}(m_{t+1})] \right\} \\
&\text{s.t.} \\
a_t &= m_t - c_t \\
m_{t+1} &= a_t \underbrace{\left(\frac{\mathsf{R}}{\psi_{t+1} \Gamma_{t+1}} \right)}_{\equiv \mathcal{R}_{t+1}} + \theta_{t+1}
\end{aligned}
$$

$\aleph_s$: probability alive (not dead) until age s given alive at age $s-1$

$\hat{\beta}_s$: time-varying discount factor between age $s-1$ and s

Ψ_s : mean-one shock to permanent income

$\beth$: time-invariant discount factor

Details follow Cagetti (2003)

- Parameterization of Uncertainty
- Probability of Death
- Demographic Adjustments to β

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Empirical Wealth Profiles

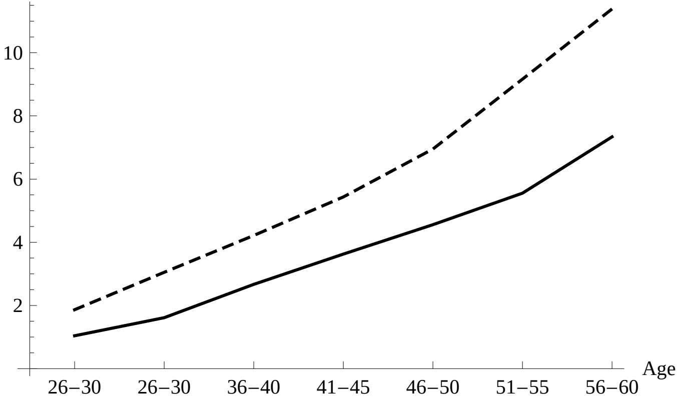

Figure: m from SCF (means (dashed) and medians (solid))

Simulated Moments

Given a set of parameter values $\{\rho, \beth\}$:

- Start at age 25 with empirical *m* data
- Draw shocks using calibrated σ_{ψ}^2,σ_{θ}^2
- Consume according to solved c_t

$\Rightarrow$ *m* distribution by age

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Choose What to Simulate

$$
\begin{aligned}
&\texttt{GapEmpiricalSimulatedMedians}[\rho, \gimel] := \\
&\quad [\quad \texttt{ConstructcFuncLife}[\rho, \gimel]; \\
&\quad\quad \texttt{Simulate}; \\
&\quad\quad \sum_{i}^{N} \omega_i \left| \varsigma_i^{\tau} - \mathrm{s}^{\tau}(\xi) \right| \\
&\quad];
\end{aligned}
$$

Calculate Match Between Theory and Data

$$\xi = \{\rho, \beth\} \tag{39}$$

solve

$$\min_{\xi} \sum_{i}^{N} \omega_i \left| \varsigma_i^{\mathcal{T}} - \mathrm{s}^{\mathcal{T}}(\xi) \right| \tag{40}$$

Bootstrap Standard Errors (Horowitz (2001))

Yields estimates of

Table: Estimation Results

ρ	$\beth$
4.68	1.00
(0.13)	(0.00)

Contour Plot

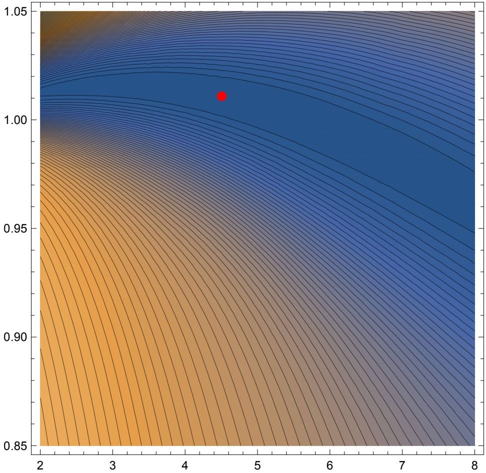

Figure: Point Estimate and Height of Minimized Function

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