1. Let $A = \{x^2 : x \in \mathbb{N}\}$.
   (a) Describe the set A in a couple other ways - in words and by listing (some of) the elements.

   (b) Classify each of the following as true, false, or meaningless.

   i. $4 \in A$ ii. $4 \subseteq A$ iii. $\{4\} \in A$ iv. $\{4\} \subseteq A$

2. Let A be any set. Classify each of the following as always true, sometimes false, or meaningless.

   (a) $\emptyset \in A$ (c) $A \in \mathcal{P}(A)$ (e) $\emptyset \in \mathcal{P}(A)$

   (b) $\emptyset \subseteq A$ (d) $A \subseteq \mathcal{P}(A)$ (f) $\emptyset \subseteq \mathcal{P}(A)$

3. Are the following statements true for all sets A and B? If so, explain why. If not, give a counter-example.
   (a) $A \cup B \subseteq B$

   (b) $A \cap B \subseteq B$

   (c) $A \subseteq A \cup B$

   (d) $A \subseteq A \cap B$

Set Theory Notation

Symbol:	Read:	Example:
$\{, \}$	braces	$\{1, 2, 3\}$. The braces enclose the elements of a set. This is the set which contains the numbers 1, 2 and 3.
$:$	such that	$\{x : x > 2\}$ is the set of all x such that x is greater than 2.
$\in$	is an element of	$2 \in \{1, 2, 3\}$ asserts that 2 is one of the elements in the set $\{1, 2, 3\}$. However, $4 \notin \{1, 2, 3\}$.
$\subseteq$	is a subset of	$A \subseteq B$ asserts that every element of A is also an element of B.
$\subset$	is a proper subset of	$A \subset B$ asserts that every element of A is also an element of B, but $A \neq B$.
$\cap$	intersection	$A \cap B$ is the *set* of all elements which are elements of both A and B.
$\cup$	union	$A \cup B$ is the *set* of all elements which are elements of A or B or both.
$\setminus$	set difference	$A \setminus B$ is the *set* of all elements of A which are not elements of B.
$\overline{A}$	compliment (of A)	$\overline{A}$ is the set of everything which is not an element of A. The A can be any set here.
$\lvert A \rvert$	cardinality (of A)	$\lvert\{4, 5, 6\}\rvert = 3$ because there are 3 elements in the set. Sometimes we say $\lvert A \rvert$ is the *size* of A.

Special sets

$\emptyset$	The *empty set* is the set which contains no elements.
$\mathcal{U}$	The *universe set* is the set of all elements.
$\mathbb{N}$	The set of natural numbers. That is, $\mathbb{N} = \{0, 1, 2, 3 \ldots\}$
$\mathbb{Z}$	The set of integers. $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$
$\mathbb{Q}$	The set of rational numbers.
$\mathbb{R}$	The set of real numbers.
$\mathcal{P}(A)$	The *power set* of any set A is the set of all subsets of A.