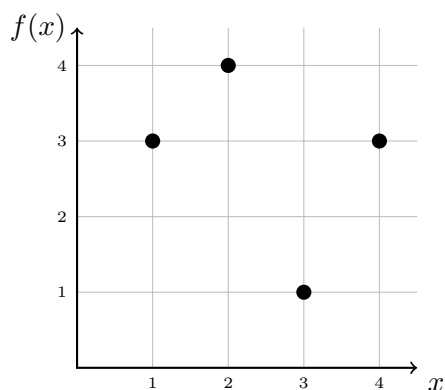


**Instructions:** The problems below are purely for you to practice. I will not collect these, but it is still a good idea to write out your solutions in full. Any of these problems or problems similar are fair game for quizzes and exams.

- Find all functions  $f : \{1, 2, 3\}$  to  $\{a, b\}$ . How many are there? How many are one-to-one? How many are onto? How many are both?
- Find all functions  $f : \{1, 2\}$  to  $\{a, b, c\}$ . How many are there? How many are one-to-one? How many are onto? How many are both?
- Consider the function  $f : \{1, 2, 3, 4, 5\} \rightarrow \{1, 2, 3, 4\}$  given by the table below:

$x$	1	2	3	4	5
$f(x)$	3	2	4	1	2

- Is  $f$  one-to-one? Explain.
  - Is  $f$  onto? Explain.
- Consider the function  $f : \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\}$  given by the graph below.



- Is  $f$  one-to-one? Explain.
- Is  $f$  onto? Explain.

- For each function given below, determine whether or not the function is one-to-one and whether or not the function is onto.
  - $f : \mathbb{N} \rightarrow \mathbb{N}$  given by  $f(n) = n + 4$ .
  - $f : \mathbb{Z} \rightarrow \mathbb{Z}$  given by  $f(n) = n + 4$ .
  - $f : \mathbb{Z} \rightarrow \mathbb{Z}$  given by  $f(n) = 5n - 8$ .
  - $f : \mathbb{Z} \rightarrow \mathbb{Z}$  given by  $f(n) = \begin{cases} n/2 & \text{if } n \text{ is even} \\ (n+1)/2 & \text{if } n \text{ is odd.} \end{cases}$
- Let  $A = \{1, 2, 3, \dots, 10\}$ . Consider the function  $f : \mathcal{P}(A) \rightarrow \mathbb{N}$  given by  $f(B) = |B|$ . So  $f$  takes a subset of  $A$  as an input and outputs the cardinality of that set.
  - Is  $f$  one-to-one? Prove your answer.
  - Is  $f$  onto? Prove your answer.
  - Find  $f^{-1}(1)$ .
  - Find  $f^{-1}(0)$ .

- (e) Find  $f^{-1}(12)$ .
7. Let  $A = \{n \in \mathbb{N} : 0 \leq n \leq 999\}$  be the set of all numbers with three or fewer digits. Define the function  $f : A \rightarrow \mathbb{N}$  by  $f(abc) = a + b + c$ , where  $a$ ,  $b$ , and  $c$  are the digits of the number in  $A$ . For example,  $f(253) = 2 + 5 + 3 = 10$ .
- Find  $f^{-1}(3)$ .
  - Find  $f^{-1}(28)$ .
  - Use one of the parts above to prove that  $f$  is not one-to-one.
  - Use one of the parts above to prove that  $f$  is not onto.
8. Find a set  $X$  and a function  $f : X \rightarrow \mathbb{N}$  so that  $f^{-1}(0) \cup f^{-1}(1) = X$ .
9. What can you deduce about the sets  $X$  and  $Y$  if you know,
- there is a one-to-one function  $f : X \rightarrow Y$ . Explain.
  - there is a onto function  $f : X \rightarrow Y$ . Explain.
  - there is a bijection  $f : X \rightarrow Y$ . Explain.
10. Suppose  $f : X \rightarrow Y$  is a function. Which of the following are possible? Explain.
- $f$  is one-to-one but not onto.
  - $f$  is onto but not one-to-one.
  - $|X| = |Y|$  and  $f$  is one-to-one but not onto.
  - $|X| = |Y|$  and  $f$  is onto but not one-to-one.
  - $|X| = |Y|$ ,  $X$  and  $Y$  are finite, and  $f$  is one-to-one but not onto.
  - $|X| = |Y|$ ,  $X$  and  $Y$  are finite, and  $f$  is onto but not one-to-one.
11. Consider the function  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  given by  $f(n) = \begin{cases} n + 1 & \text{if } n \text{ is even} \\ n - 3 & \text{if } n \text{ is odd.} \end{cases}$
- Is  $f$  one-to-one? Prove your answer.
  - Is  $f$  onto? Prove your answer.