The goal of this activity is to practice reading set theory notation and to start counting with sets. $\mathbb{N}$ denotes the set of natural numbers, namely $\{0, 1, 2, \ldots\}$.

1. Find the cardinality of each set below.
   (a) $A = \{3, 4, \ldots, 15\}$.

   (b) $B = \{n \in \mathbb{N} \ : \ 2 < n \leq 200\}$.

   (c) $C = \{n \leq 100 \ : \ n \in \mathbb{N} \wedge \exists m \in \mathbb{N}(n = 2m + 1)\}$.

2. Find two sets A and B for which $|A| = 5$, $|B| = 6$ and $|A \cup B| = 9$. What is $|A \cap B|$?

3. Find sets A and B with $|A| = |B|$ such that $|A \cup B| = 7$ and $|A \cap B| = 3$. What is $|A|$?

4. Let $A = \{1, 2, \ldots, 10\}$. Define $\mathcal{B}_2 = \{B \subseteq A \ : \ |B| = 2\}$. Find $|\mathcal{B}_2|$.

5. For any sets A and B, define $AB = \{ab \ : \ a \in A \wedge b \in B\}$. If $A = \{1, 2\}$ and $B = \{2, 3, 4\}$, what is $|AB|$? What is $|A \times B|$ (where $A \times B$ is the usual *Cartesian product* of A and B).

Set Theory Notation

Symbol:	Read:	Example:
$\{,\}$	braces	$\{1,2,3\}$. The braces enclose the elements of a set. This is the set which contains the numbers 1, 2 and 3.
$:$	such that	$\{x : x > 2\}$ is the set of all x such that x is greater than 2.
$\in$	is an element of	$2 \in \{1,2,3\}$ asserts that 2 is one of the elements in the set $\{1,2,3\}$. However, $4 \notin \{1,2,3\}$.
$\subseteq$	is a subset of	$A \subseteq B$ asserts that every element of A is also an element of B.
$\subset$	is a proper subset of	$A \subset B$ asserts that every element of A is also an element of B, but $A \neq B$.
$\cap$	intersection	$A \cap B$ is the *set* of all elements which are elements of both A and B.
$\cup$	union	$A \cup B$ is the *set* of all elements which are elements of A or B or both.
$\times$	cross, or Cartesian product	$A \times B$ is the set of all ordered pairs (a,b) with $a \in A$ and $b \in B$.
$\setminus$	set difference	$A \setminus B$ is the *set* of all elements of A which are not elements of B.
$\overline{A}$	compliment (of A)	$\overline{A}$ is the set of everything which is not an element of A. The A can be any set here.
$\|A\|$	cardinality (of A)	$\|\{4,5,6\}\| = 3$ because there are 3 elements in the set. Sometimes we say $\|A\|$ is the *size* of A.
Logic symbols:		
$\wedge$	and	$x \in A \wedge x \notin B$ means x is both in the set A **and** also not in B.
$\vee$	or	$x \in A \vee x \notin B$ asserts that x is an element of A **or** not an element of B, or both.
$\neg$	not	Another way to write $x \notin A$ is $\neg x \in A$.
$\forall$	for all	$\forall x(x \geq 0)$ claims that for every number is greater than 0.
$\exists$	there exists	$\exists x(x < 0)$ claims that there are negative numbers (there exists a number such that it is less than 0).