Instructions: The problems below are purely for you to practice. I will not collect these, but it is still a good idea to write out your solutions in full. Any of these problems or problems similar are fair game for quizzes and exams.

1. Find the generating function for each of the following sequences by relating them back to a sequence with known generating function.
   (a) $4, 4, 4, 4, 4, \ldots$
   (b) $2, 4, 6, 8, 10, \ldots$
   (c) $0, 0, 0, 2, 4, 6, 8, 10, \ldots$
   (d) $1, 5, 25, 125, \ldots$
   (e) $1, -3, 9, -27, 81, \ldots$
   (f) $1, 0, 5, 0, 25, 0, 125, 0, \ldots$
   (g) $0, 1, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0, 5, \ldots$

2. Find the sequence generated by the following generating functions:
   (a) $\dfrac{4x}{1-x}$
   (b) $\dfrac{1}{1-4x}$
   (c) $\dfrac{x}{1+x}$
   (d) $\dfrac{3x}{(1+x)^2}$
   (e) $\dfrac{1+x+x^2}{(1-x)^2}$ (Hint: multiplication)

3. Show how you can get the generating function for the triangular numbers in three different ways:
   (a) Take two derivatives of the generating function for $1, 1, 1, 1, 1, \ldots$
   (b) Use differencing.
   (c) Multiply two known generating functions.

4. Use differencing to find the generating function for $4, 5, 7, 10, 14, 19, 25, \ldots$

5. Find a generating function for the sequence with recurrence relation $a_n = 3a_{n-1} - a_{n-2}$ with initial terms $a_0 = 1$ and $a_1 = 5$.

6. Use the recurrence relation for the Fibonacci numbers to find the generating function for the Fibonacci sequence.

7. Use multiplication to find the generating function for the sequence of partial sums of Fibonacci numbers, $S_0, S_1, S_2, \ldots$ where $S_0 = F_0$, $S_1 = F_0 + F_1$, $S_2 = F_0 + F_1 + F_2$, $S_3 = F_0 + F_1 + F_2 + F_3$ and so on.

8. Find the generating function for the sequence with closed formula $a_n = 2(5^n) + 7(-3)^n$.

9. Find a closed formula for the nth term of the sequence with generating function $\dfrac{3x}{1-4x} + \dfrac{1}{1-x}$.

10. Find a_7 for the sequence with generating function $\dfrac{2}{(1-x)^2} \cdot \dfrac{x}{1-x-x^2}$