Math 228 **Homework 10**

Instructions: Same rules as usual - turn in your work on separate sheets of paper. You must justify all your answers for full credit.

(4pts) 1. Find a generating function for the sequence $3, 4, 6, 10, 18, 34, 66, \ldots$. Hint: find the generating function for the difference between terms. Explain why your answer is correct.

(4pts) 2. Find the generating function for the sequence $1, 4, 11, 34, 101, 304, \ldots$ using the fact that the sequence is recursively defined by $a_n = 2a_{n-1} + 3a_{n-2}$ with $a_0 = 1$ and $a_1 = 4$.

(4pts) 3. Zombie Euler and Zombie Cauchy - two famous zombie mathematicians - have just signed up for Twitter accounts. After one day, Zombie Cauchy has more followers than Zombie Euler. Each day after that, the number of new followers of Zombie Cauchy is exactly the same as the number of new followers of Zombie Euler (and neither lose any followers). Explain how a proof by mathematical induction can show that on every day after the first day, Zombie Cauchy will have more followers than Zombie Euler. That is, explain what the base case and inductive case are, and why they together prove that Zombie Cauchy will have more followers on the 4th day.

Special Induction Instructions: For the rest of the homework problems, you should first give a rough sketch of the argument (i.e., say *why* induction will work in this case) and then also give a formal proof by induction (starting with, "Let $P(n)$ be the statement...").

(6pts) 4. Find the largest number of points which a football team cannot get exactly using just 3-point field goals and 7-point touchdowns (ignore the possibilities of safeties, missed extra points, and two point conversions). Prove your answer is correct by mathematical induction.

(6pts) 5. Prove, by mathematical induction, that $F_0 + F_1 + F_2 + \cdots + F_n = F_{n+2} - 1$, where F_n is the nth Fibonacci number ($F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$).

(6pts) 6. Prove that every natural number is either a Fibonacci number or can be written as the sum of distinct Fibonacci numbers. Use strong induction. Hint: To write 32 as the sum of distinct Fibonacci numbers, you can first look for the largest Fibonacci number less than 32. In this case, that's 21, so if you can write 11 as the sum of distinct Fibonacci numbers, you will be done.