Instructions: The problems below are purely for you to practice. I will not collect these, but it is still a good idea to write out your solutions in full. Any of these problems or problems similar are fair game for quizzes and exams.

1. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5, 6, 7\}$ and $C = \{2, 3, 5\}$.
   (a) Find $A \cap B$.
   (b) Find $A \cup B$.
   (c) Find $A \setminus B$.
   (d) Is $C \subseteq A$?
   (e) Is $C \subseteq B$?

2. Let $A = \{x \in \mathbb{N} : 3 \leq x \leq 13\}$, $B = \{x \in \mathbb{N} : x \text{ is even}\}$, and $C = \{x \in \mathbb{N} : x \text{ is odd}\}$.
   (a) Find $A \cap B$.
   (b) Find $A \cup B$.
   (c) Find $B \cap C$.
   (d) Find $B \cup C$.

3. Find an example of sets A and B such that $A \cap B = \{3, 5\}$ and $A \cup B = \{2, 3, 5, 7, 8\}$.

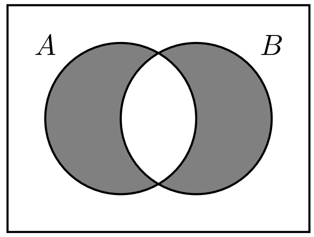
4. Find an example of sets A and B such that $A \subseteq B$ and $A \in B$.

5. Recall $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ (the integers). Let $\mathbb{Z}^+$ be the positive integers. Let $2\mathbb{Z}$ be the even integers, $3\mathbb{Z}$ be the multiples of 3, and so on.
   (a) Is $\mathbb{Z}^+ \subseteq 2\mathbb{Z}$?
   (b) Is $2\mathbb{Z} \subseteq \mathbb{Z}^+$?
   (c) Find $2\mathbb{Z} \cap 3\mathbb{Z}$. Describe the set in words, and also in symbols (using a $:$ symbol).
   (d) Express $\{x \in \mathbb{Z} : \exists y \in \mathbb{Z}(x = 2y \vee x = 3y)\}$ as a union or intersection of two sets above.

6. Let A_2 be the set of all multiples of 2 except for 2. Let A_3 be the set of all multiples of 3 except for 3. And so on, so that A_n is the set of all multiple of n except for n, for any $n \geq 2$. Describe (in words) the set $\overline{A_2 \cup A_3 \cup A_4 \cup \cdots}$

7. Draw a Venn diagram to represent each of the following:
   (a) $A \cup \overline{B}$
   (b) $\overline{(A \cup B)}$
   (c) $A \cap (B \cup C)$
   (d) $(A \cap B) \cup C$
   (e) $\overline{A} \cap B \cap \overline{C}$
   (f) $(A \cup B) \setminus C$

8. Describe a set in terms of A and B which has the following Venn diagram:



9. Find the cardinalities:
   (a) Find $|A|$ when $A = \{4, 5, 6, \ldots, 37\}$
   (b) Find $|A|$ when $A = \{x \in \mathbb{Z} \; : \; -2 \leq x \leq 100\}$
   (c) Find $|A \cap B|$ when $A = \{x \in \mathbb{N} \; : \; x \leq 20\}$ and $B = \{x \in \mathbb{N} \; : \; x \text{ is prime}\}$

10. Let $A = \{a, b, c\}$. Find $\mathcal{P}(A)$.

11. Let $A = \{1, 2, \ldots, 10\}$. How many subsets of A contain exactly one element (i.e., how many *singleton* subsets are there). How many *doubleton* (containing exactly two elements) are there?

12. Let $A = \{1, 2, 3, 4, 5, 6\}$. Find all sets $B \in \mathcal{P}(A)$ which have the property $\{2, 3, 5\} \subseteq B$.

13. Find an example of sets A and B such that $|A| = 4$, $|B| = 5$ and $|A \cup B| = 9$.

14. Find an example of sets A and B such that $|A| = 3$, $|B| = 4$ and $|A \cup B| = 5$.

15. If $|A| = 10$ and $|B| = 15$, what is the largest possible value for $|A \cap B|$? What is the smallest? What are the possible values for $|A \cup B|$?

16. If $|A| = 8$ and $|B| = 5$, what is $|A \cup B| + |A \cap B|$?

17. In a regular deck of playing cards there are 26 red cards and 12 face cards. Explain in terms of sets why there are only 32 cards which are either red or a face card.

18. A group of college students were asked about their TV watching habits. Of those surveyed, 28 students watch *House*, 19 watch *Castle* and 24 watch re-runs of *24*. Additionally, 16 watch *House* and *Castle*, 14 watch *House* and *24* and 10 watch *Castle* and *24*. There are 8 students who watch all three shows. How many students surveyed watched at least one of the shows?

19. Find $|(A \cup C) \cap \overline{B}|$ provided $|A| = 50$, $|B| = 45$, $|C| = 40$, $|A \cap B| = 20$, $|A \cap C| = 15$, $|B \cap C| = 23$ and $|A \cap B \cap C| = 12$.

20. Using the same data as the previous question, describe a set with cardinality 26.