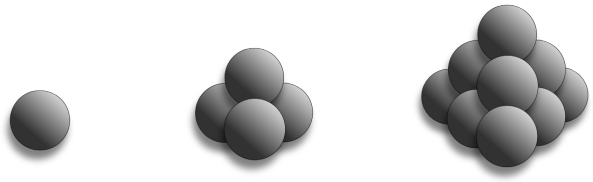
Math 228 **Homework 4** Due: Wed February 22, 2012

**Instructions**: Complete the homework problems below on a *separate* sheet of paper (and not all jammed up between the questions). Each solution should be accompanied with supporting work or an explanation why the solution is correct. Your work will be graded on correctness as well as the clarity of your explanations.

(4pts) 1. We have a formula $n$th term of an arithmetic sequence with first term $a$ and common difference $d$. It is $a_n = a + d(n-1)$. Find a closed formula for the sum of the first $n$ terms of the sequence: $S_n = a_1 + a_2 + \cdots + a_n$. Show your work.

(4pts) 2. The $n$th term of a geometric sequence with first term $a$ and common ratio $r$ is $a_n = a \cdot r^{n-1}$. Find a close formula for the sum of the first $n$ terms of the sequence: $S_n = a_1 + a_2 + \cdots + a_n$. Show your work.

(6pts) 3. In their down time, ghost pirates enjoy stacking cannonballs in triangular based pyramids (aka, tetrahedrons), like those pictured here:



Note, in the picture on the right, there are some cannonballs (actually just one) you cannot see.

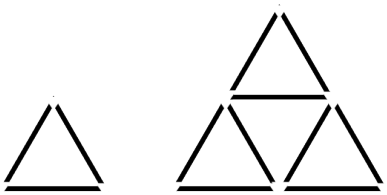
The pirates wonder how many cannonballs would be required to build a pyramid 15 layers high (thus breaking the world cannonball stacking record). Can you help?

(a) Let $P(n)$ denote the number of cannonballs needed to create a pyramid $n$ layers high. So $P(1) = 1$, $P(2) = 4$, and so on. Calculate $P(3)$, $P(4)$ and $P(5)$.

(b) Use polynomial fitting to find a closed formula for $P(n)$. Show your work.

(c) Answer the pirate's question: how many cannonballs do they need to make a pyramid 15 layers high?

(6pts) 4. If you have enough toothpicks, you can make a large triangular grid. Below, are the triangular grids of size 1 and of size 2. The size 1 grid requires 3 toothpicks, the size 2 grid requires 9 toothpicks.



(a) Let $t_n$ be the number of toothpicks required to make a size $n$ triangular grid. Write out the first 5 terms of the sequence $t_1, t_2, \ldots$.

(b) Find a recursive definition for the sequence. Explain why you are correct.

(c) Find a closed formula for the sequence. Explain why you are correct.

(4pts-bns) 5. Bonus: How many triangles (of all sizes and orientations) are contained in a size $n$ triangular grid? For example, there is one triangle in a size 1 grid, and 5 triangles in a size 2 grid.