Homework 2

Math 228 **Solutions** Spring 2013

(6pts) 1. (a) Make a truth table for the statement $P \to (\neg Q \vee R)$.

Solution:

P	Q	R	$\neg Q \vee R$	$P \to (\neg Q \vee R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

(b) If Tommy **lies** when he says, "if I ate pizza, then either I didn't eat cucumber sandwiches or I did eat raisins," what can you conclude about what Tommy ate? Explain.

Solution: The statement made is the same as the one we made a truth table for above. If the statement is a lie, then we are in the case(s) in which the statement is false. This turns out to be only the second case, so we see that P and Q are true and R is false. Therefore Tommy ate pizza a cucumber sandwiches, but not raisins.

(6pts) 2. Can you distribute conjunctions over disjunctions? Disjunctions over conjunctions? Let's find out. Remember, two statements are logically equivalent if they are true in exactly the same cases.

(a) Are the statements $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge (P \vee R)$ logically equivalent?

Solution: Yes they are. We prove this by showing that their truth tables are identical:

P	Q	R	$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

(b) Are the statements $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee (P \wedge R)$ logically equivalent?

Solution: It works again. Here are the two truth tables which prove it:

P	Q	R	$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

(6pts) 3. Use De Morgan's Laws, and any other logical equivalence facts you know to simplify the following statements. Show all your steps, justifying each. Your final statements should have negations only appear directly next to the propositional variables (P, Q, etc.), and no double negations.

(a) $\neg((\neg P \wedge Q) \vee \neg(R \vee \neg S))$.

Solution: $\neg((\neg P \wedge Q) \vee \neg(R \vee \neg S))$
$\neg(\neg P \wedge Q) \wedge \neg\neg(R \vee \neg S)$ by De Morgan's law.
$\neg(\neg P \wedge Q) \wedge (R \vee \neg S)$ by double negation.
$(\neg\neg P \vee \neg Q) \wedge (R \vee \neg S)$ by De Morgan's law.
$(P \vee \neg Q) \wedge (R \vee \neg S)$ by double negation.

(b) $\neg((\neg P \to \neg Q) \wedge (\neg Q \to R))$ (careful with the implications).

Solution: We will need to convert the implications to disjunctions so we can apply De Morgan's law:
$\neg((\neg P \to \neg Q) \wedge (\neg Q \to R))$
$\neg((\neg\neg P \vee \neg Q) \wedge (\neg\neg Q \vee R))$ by implication/disjunction equivalence.
$\neg((P \vee \neg Q) \wedge (Q \vee R))$ by double negation.
$\neg(P \vee \neg Q) \vee \neg(Q \vee R)$ by De Morgan's law.
$(\neg P \wedge \neg\neg Q) \vee (\neg Q \wedge \neg R)$ by De Morgan's law.
$(\neg P \wedge Q) \vee (\neg Q \wedge \neg R)$ by double negation.

(6pts) 4. Can you chain implications together? That is, if $P \to Q$ and $Q \to R$, does that means the $P \to R$? Can you chain more implications together? Let's find out:

(a) Prove that the following is a valid argument form:
$$\begin{array}{rc} & P \to Q \\ & Q \to R \\ \hline \therefore & P \to R \end{array}$$

Solution: Consider the truth table:

P	Q	R	$P \to Q$	$Q \to R$	$P \to R$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

Notice that both $P \to Q$ and $Q \to R$ are true in rows 1, 5, 7 and 8. In each of these rows, $P \to R$ is also true. So whenever the premises are true, so in the conclusion. Thus the argument form is valid.

(b) Prove that the following is a valid argument form:
$$\begin{array}{rc} & P_1 \to P_2 \\ & P_2 \to P_3 \\ & \vdots \\ & P_8 \to P_9 \\ \hline \therefore & P_1 \to P_9. \end{array}$$
I suggest you don't go through the trouble of writing out a 512 row truth table. You should still be able to explain why this is argument form is valid (using part (a)).

Solution: From the first two premises, we can conclude that $P_1 \to P_3$, using the same reasoning as in part (a). The next line of the argument form is $P_3 \to P_4$. Since we know that $P_1 \to P_3$ is true as well, we can conclude (using part (a) again) that $P_1 \to P_4$. And so on. Eventually we will find that $P_1 \to P_8$. This combined with the last premise ($P_8 \to P_9$) allows us to conclude that $P_1 \to P_9$.

(6pts) 5. Consider the statement, "if you study logic, then you will be happy."

(a) Rephrase the implication in at least 3 different ways. At least one of the ways should use necessary/sufficient language.

Solution: The implication is equivalent to each of the following:

- You will be happy if you study logic.
- You will study logic only if you are happy.
- To be happy it is sufficient to study logic.
- Being happy necessarily follows from studying logic.
- If you are unhappy, then you have not studied logic.
- Either you don't study logic or you are happy.

(b) State the converse of the implication, and rephrase the converse in at least 3 different ways.

Solution: The converse is, "if you are happy, then you study logic." This is equivalent to each of the following:

- You study logic if you are happy.
- You are happy only if you study logic.
- To be happy it is necessary to study logic.
- To study logic it is sufficient to be happy.
- If you don't study logic, then you are not happy.
- Either are you not happy or you do not study logic.