(4pts) 1. In a recent survey, 30 students reported whether they liked their potatoes Mashed, French-fried, or Twice-baked. 15 liked them mashed, 20 liked French fries, and 9 liked twice baked potatoes. Additionally, 12 students liked both mashed and fried potatoes, 5 liked French fries and twice baked potatoes, 6 liked mashed and baked, and 3 liked all three styles. How many students *hate* potatoes? Explain why your answer is correct.

Solution: Using the principle of inclusion/exclusion, the number of students who like their potatoes in at least one of the ways described is

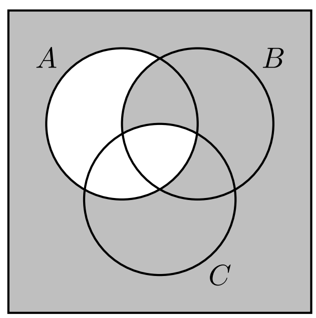
$$15 + 20 + 9 - 12 - 5 - 6 + 3 = 24.$$

Therefore there are $30 - 24 = 6$ students who do not like potatoes. You can also do this problem with a Venn diagram.

(6pts) 2. Consider the set $S = \overline{A} \cup (B \cap \overline{C})$.

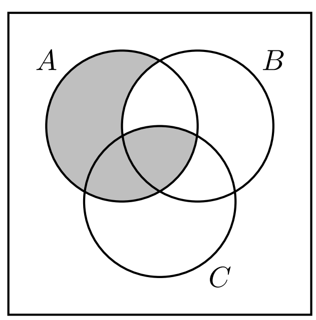
(a) Draw a Venn diagram for the set S.

Solution:



(b) Use the Venn diagram from part (a) to draw a Venn diagram for $\overline{S}$.

Solution:



(c) Use the Venn diagram from part (b) to express $\overline{S}$ in terms for A, B and C. Your answer should have bars only over single letters.

Solution: $A \cap (\overline{B} \cup C)$

(4pts) 3. Let A, B, and C be sets. Suppose $A \subseteq B$ and $B \subseteq C$. Does this mean $A \subseteq C$? Explain why or why not.

> **Solution:** Yes. Let $x \in A$. Since $A \subseteq B$, we know then that $x \in B$. since $B \subseteq C$ then $x \in C$. So every element of A is also an element of C, which is just to say $A \subseteq C$.

(6pts) 4. Consider the function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 2n + 3$.

(a) Is f one-to-one? Explain.

> **Solution:** f is one-to-one. Let x and y be integers, and suppose $f(x) = f(y)$. So $2x + 3 = 2y + 3$, which implies that $2x = 2y$ and as such $x = y$. Thus the only way to get the same output is to start with the same input, which is to say f is one-to-one.

(b) Is f onto? Explain.

> **Solution:** f is not onto. For example, the integer 2 in the codomain is not in the range, as the output of f is always odd.

(c) Let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = 2x + 3$. Is g onto? Explain.

> **Solution:** g is onto (even though the rule defining g is the same as the one for f and f is not onto). Let $y \in \mathbb{R}$ be an element of the codomain. We must find a value x in the domain such that $2x + 3 = y$. Take $x = \frac{y-3}{2}$. This is a real number, so in the domain. Of course, $2(\frac{y-3}{2}) + 3 = y$ so this x works. Since every real number is in the range, g is onto.