Instructions: Complete the homework problems below on a *separate* sheet of paper (and not all jammed up between the questions). This is to be turned in and graded, so make sure your work is neat and easy to ready - there is nothing wrong with using a separate sheet of paper for each problem. Each solution should be accompanied with supporting work or an explanation why the solution is correct. All proofs should be written out in paragraph form. Your work will be graded on correctness as well as the clarity of your explanations.

(6pts) 1. For each of the following statements, write the converse, contrapositive, and the negation.

(a) $\exists x \forall y (x \geq 2y \rightarrow x > y + 1)$

(b) For all integers a and b, if a is even and b is odd, then $a + b$ is odd.

(5pts) 2. Prove the statement: For all integers n, if $5n$ is odd, then n is odd. Clearly state the style of proof you are using.

(5pts) 3. Prove the statement: For all integers a, b, and c, if $a^2 + b^2 = c^2$, then a or b is even. Hint: try a proof by contradiction.

(4pts) 4. Order matters with quantifiers! Sometimes.

(a) Find a formula φ such that $\forall x \exists y\, \varphi$ is true but $\exists y \forall x\, \varphi$ is false.

(b) Explain why you cannot find a formula ψ such that $\forall x \exists y\, \psi$ is false but $\exists y \forall x\, \psi$ is true.

The formulas φ and ψ can be predicates or multiple predicates connected by logical connectives, containing the (free) variables x and y. For example, φ might be $E(x) \wedge O(y)$ (although this does not satisfy part (a)).