Practice Problems 4: Sequences

Math 228 **Hints and Answers** Spring 2012

1. (a) $a_n = n^2 + 1$
   (b) $a_n = \frac{n(n+1)}{2} - 1$
   (c) $a_n = \frac{(n+2)(n+3)}{2} + 2$
   (d) $a_n = (n+1)! - 1$ (where $n! = 1 \cdot 2 \cdot 3 \cdots n$)
2. (a) $F_n = F_{n-1} + F_{n-2}$ with $F_0 = 0$ and $F_1 = 1$.
   (b) $0, 1, 2, 4, 7, 12, 20, \ldots$
   (c) $F_0 + F_1 + \cdots + F_n = F_{n+2} - 1$
3. $3, 10, 24, 52, 108, \ldots$. The recursive definition for $10, 24, 52, \ldots$ is $a_n = 2a_{n-1} + 4$ with $a_1 = 10$.
4. $-1, -1, 1, 5, 11, 19, \ldots$ Thus the sequence $0, 2, 6, 12, 20, \ldots$ has closed formula $a_n = (n+1)^2 - 3(n+1) + 2$.
5. (a) 32.
   (b) $a_n = 8 + 6(n-1)$
   (c) 30500.
6. (a) $n + 2$ terms.
   (b) $6n + 1$.
   (c) $\frac{(6n+8)(n+2)}{2}$
7. 68117
8. $\frac{5 - 5 \cdot 3^{21}}{-2}$
9. $\frac{1 + \frac{2^{31}}{3^{31}}}{5/3}$
10. For arithmetic: $x = 55/3$, $y = 29/3$. For geometric: $x = 9$ and $y = 3$.
11. (a) $\displaystyle\sum_{k=1}^{n} 2k$
    (b) $\displaystyle\sum_{k=1}^{107} (1 + 4(k-1))$
    (c) $\displaystyle\sum_{k=1}^{50} \frac{1}{k}$
    (d) $\displaystyle\prod_{k=1}^{n} 2k$
    (e) $\displaystyle\prod_{k=1}^{100} \frac{k}{k+1}$
12. (a) $\displaystyle\sum_{k=1}^{100} (3 + 4k) = 7 + 11 + 15 + \cdots + 403$

(b) $\sum_{k=0}^{n} 2^k = 1 + 2 + 4 + 8 + \cdots + 2^n$

(c) $\sum_{k=2}^{50} \frac{1}{(k^2-1)} = 1 + \frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \cdots + \frac{1}{2499}$

(d) $\prod_{k=2}^{100} \frac{k^2}{(k^2-1} = \frac{4}{3} \cdot \frac{9}{8} \cdot \frac{16}{15} \cdots \frac{10000}{9999}$

(e) $\prod_{k=0}^{n} (2+3k) = (2)(5)(8)(11)(14)\cdots(2+3n)$

13. (a) Hint: third differences are constant, so $a_n = an^3 + bn^2 + cn + d$. Use the terms of the sequence to solve for a, b, c, and d.

    (b) $a_n = n^2 - n$

14. No. The sequence of differences is the same as the original sequence so no differences will be constant.

15. 171 and 341. $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 3$ and $a_1 = 5$. Closed formula: $a_n = \frac{8}{3}2^n + \frac{1}{3}(-1)^n$

16. By telescoping or iteration. $a_n = 3 + 2^{n+1}$

17. We claim $a_n = 4^n$ works. Plug it in: $4^n = 3(4^{n-1}) + 4(4^{n-2})$. This works - just simplify the right hand side.

18. By the Characteristic Root Technique. $a_n = 4^n + (-1)^n$.

19. $a_n = \frac{13}{5}4^n + \frac{12}{5}(-1)^n$

20. The general solution is $a_n = a + bn$ where a and b depend on the initial conditions.

    (a) $a_n = 1 + n$

    (b) For example, we could have $a_0 = 21$ and $a_1 = 22$.

    (c) For every x - take $a_0 = x - 9$ and $a_1 = x - 8$.

21. $a_n = \frac{19}{7}(-2)^n + \frac{9}{7}5^n$