Math 228 **Homework 8**

Instructions: Same rules as usual - turn in your work on separate sheets of paper. You must justify all your answers for full credit.

(8pts) 1. For each sequence given below, find a closed formula for a_n, the nth term of the sequence (assume the first terms are a_0) by relating it to another sequence for which you already know the formula. In each case, briefly say how you got your answers.

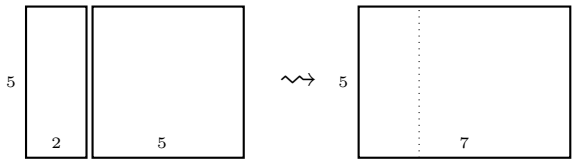
(a) 4, 5, 7, 11, 19, 35, ...

(b) 0, 3, 8, 15, 24, 35, ...

(c) 6, 12, 20, 30, 42, ...

(d) 0, 2, 7, 15, 26, 40, 57, ... (Cryptic Hint: these might be called "house numbers")

(8pts) 2. Starting with any rectangle, we can create a new, larger rectangle by attaching a square to the longer side. For example, if we start with a 2×5 rectangle, we would glue on a 5×5 square, forming a 5×7 rectangle:



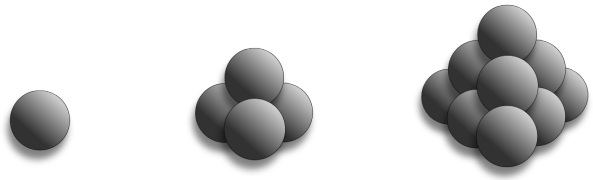
(a) Create a sequence of rectangles using this rule starting with a 1×2 rectangle. Then write out the sequence of *perimeters* for the rectangles (the first term of the sequence would be 6, since the perimeter of a 1×2 rectangle is 6 - the next term would be 10).

(b) Repeat the above part this time starting with a 1×3 rectangle.

(c) Find recursive formulas for each of the sequences of perimeters you found in parts (a) and (b). Don't forget to give the initial conditions as well.

(d) Are the sequences arithmetic? Geometric? If not, are they *close* to being either of these (for example, are the differences *almost* constant)? Explain.

3. In their down time, ghost pirates enjoy stacking cannonballs in triangular based pyramids (aka, tetrahedrons), like those pictured here:



Note, in the picture on the right, there are some cannonballs (actually just one) you cannot see. The next picture would have 4 cannonballs you cannot see.

The pirates wonder how many cannonballs would be required to build a pyramid 15 layers high (thus breaking the world cannonball stacking record). Can you help?

(2pts) (a) Let $P(n)$ denote the number of cannonballs needed to create a pyramid n layers high. So $P(1) = 1$, $P(2) = 4$, and so on. Calculate $P(3)$, $P(4)$ and $P(5)$.

(4pts) (b) Use polynomial fitting to find a closed formula for $P(n)$. Show your work.

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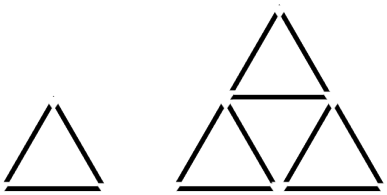
(2pts) (c) Answer the pirate's question: how many cannonballs do they need to make a pyramid 15 layers high?

(6pts) 4. If you have enough toothpicks, you can make a large triangular grid. Below, are the triangular grids of size 1 and of size 2. The size 1 grid requires 3 toothpicks, the size 2 grid requires 9 toothpicks.



(a) Let t_n be the number of toothpicks required to make a size n triangular grid. Write out the first 5 terms of the sequence $t_1, t_2, \ldots$.

(b) Find a recursive definition for the sequence. Explain why you are correct.

(c) Find a closed formula for the sequence. Explain why you are correct.

(4pts-bns) 5. Bonus: How many triangles (of all sizes and orientations) are contained in a size n triangular grid? For example, there is one triangle in a size 1 grid, and 5 triangles in a size 2 grid.