Math 228 **Homework 6** Due: Wednesday, March 4

Instructions: Same rules as usual - turn in your work on separate sheets of paper. You must justify all your answers for full credit.

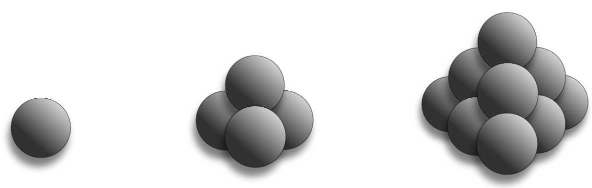
(6pts) 1. Consider the sequence $5, 11, 19, 29, 41, 55, \ldots$. Assume $a_1 = 5$.

(a) Find a closed formula for a_n, the nth term of the sequence, by writing each term as a sum of a sequence. Hint: first find a_0, but ignore it when collapsing the sum.

(b) Find a closed formula again, this time using either polynomial fitting or the characteristic root technique (whichever is appropriate). Show your work.

(c) Find a closed formula once again, this time by recognizing the sequence as a modification to some well known sequence(s). Explain.

2. In their down time, ghost pirates enjoy stacking cannonballs in triangular based pyramids (aka, tetrahedrons), like those pictured here:



Note, in the picture on the right, there are some cannonballs (actually just one) you cannot see. The next picture would have 4 cannonballs you cannot see.

The pirates wonder how many cannonballs would be required to build a pyramid 15 layers high (thus breaking the world cannonball stacking record). Can you help?

(2pts) (a) Let $P(n)$ denote the number of cannonballs needed to create a pyramid n layers high. So $P(1) = 1$, $P(2) = 4$, and so on. Calculate $P(3)$, $P(4)$ and $P(5)$.

(4pts) (b) Use polynomial fitting to find a closed formula for $P(n)$. Show your work.

(2pts) (c) Answer the pirate's question: how many cannonballs do they need to make a pyramid 15 layers high?

(8pts) 3. Consider the sequences $2, 5, 12, 29, 70, 169, 408, \ldots$ (with $a_0 = 2$).

(a) Describe the rate of growth of this sequence.

(b) Find a recursive definition for the sequence.

(c) Find a closed formula for the sequence.

(d) If you look at the sequence of differences between terms, and then the sequence of second differences, the sequence of third differences, and so on, will you ever get a constant sequence? Explain how you know.

4. Let a_n be the number of $1 \times n$ tile designs can you make using 1×1 squares available in 4 colors and 1×2 dominoes available in 5 colors.

(3pts) (a) First, find a recurrence relation to describe the problem. Explain why the recurrence relation is correct (in the context of the problem).

(2pts) (b) Write out the first 6 terms of the sequence $a_1, a_2, \ldots$.

(3pts) (c) Solve the recurrence relation. That is, find a closed formula for a_n.