Math 228 **Homework 6**

Instructions: Same rules as usual - turn in your work on separate sheets of paper. You must justify all your answers for full credit.

(8pts) 1. Gridtown USA, besides having excellent donut shoppes, is known for its precisely laid out grid of streets and avenues. Streets run east-west, and avenues north-south, for the entire stretch of the town, never curving and never interrupted by parks or schools or the like.

Suppose you live on the corner of 1st and 1st and work on the corner of 12th and 12th. Thus you must travel 22 blocks to get to work as quickly as possible.

(a) How many different routes can you take to work, assuming you want to get there as quickly as possible?

(b) Now suppose you want to stop and get a donut on the way to work, from your favorite donut shoppe on the corner of 8th st and 10th ave. How many routes to work, via the donut shoppe, can you take (again, ensuring the shortest possible route)?

(c) Disaster Strikes Gridtown: there is a pothole on 4th avenue between 5th and 6th street. How many routes to work can you take avoiding that unsightly (and dangerous) stretch of road?

(d) How many routes are there both avoiding the pothole and visiting the donut shoppe?

(6pts) 2. Suppose you own a fezzes and b bow ties. Of course, a and b are both greater than 1.

(a) How many combinations of fez and bow tie can you make? You can wear only one fez and one bow tie at a time. Explain.

(b) Explain why the answer is *also* $\binom{a+b}{2} - \binom{a}{2} - \binom{b}{2}$. (If this is what you claimed the answer was in part (a), try it again.)

(c) Use your answers to parts (a) and (b) to give a combinatorial proof of the identity

$$\binom{a+b}{2} - \binom{a}{2} - \binom{b}{2} = ab$$

3. Consider the identity:

$$k\binom{n}{k} = n\binom{n-1}{k-1}$$

(2pts) (a) Is this true? Try it for a few values of n and k.

(2pts) (b) Use the formula for $\binom{n}{k}$ to give an algebraic proof of the identity.

(4pts) (c) Give a combinatorial proof of the identity. Hint: How many ways can you select a chaired committee of k people from a group of n people?

Writing Assignment: Please turn in the writing assignment below separately from the rest of your homework, NOT stapled to the other problems.

(8pts) 4. In this writing assignment, I would like you to reflect on how you solve counting problems, and how you decide if your answer is correct. To do this, consider the following counting problem:

How many 10-digit numbers contain exactly four 1's, three 2's, two 3's and one 4?

Write about how you personally go about answering this question. I am more interested in your process than in the correct answer (although a correct answer will be worth 2 bonus points). Tell me about what you think about when you see the question, what your first guess is, and how you decide whether your answer is correct. This should take 1-2 pages.