

Instructions: Complete the homework problems below on *separate* sheets of paper (and not all jammed up between the questions). This is to be turned in and graded, so make sure your work is neat and easy to read - there is nothing wrong with using a separate sheet of paper for each problem. Each solution should be accompanied with supporting work or an explanation why the solution is correct. Your work will be graded on correctness as well as the clarity of your explanations.

- (9pts) 1. Let $A = \{2, 4, 6, 8\}$. Suppose B is a set with $|B| = 5$.
- (a) What are the smallest and largest possible values of $|A \cup B|$? Explain.
 - (b) What are the smallest and largest possible values of $|A \cap B|$? Explain.
 - (c) What are the smallest and largest possible values of $|A \times B|$? Explain.
- (6pts) 2. Let A , B and C be sets. Suppose that $A \subseteq B$ and $B \subseteq C$. Does this mean that $A \subseteq C$? Explain how you know (i.e., prove your answer).
- (10pts) 3. For each scenario below, explain how it could be interpreted as a function. Specifically, say what the domain and codomain should be and why you made the choice you did. Then decide whether the function could be injective, surjective or even bijective. Explain what assumptions you would have to make in each case or why it would be impossible.
- (a) The 10 members of Math Club all decide to each pick one of the 15 math club meetings to give a presentation (each of which will take the entire meeting time).
 - (b) Over the next seven days, you plan to finish a box of 24 different types of chocolates.
- (5pts) 4. Make up a scenario like the two above for which the function *must* be a bijection. Explain what it is about your scenario which forces the function to be bijective.
- (3 (bonus)) 5. BONUS! Find an example of a set A with $|A| = 3$ which contains only other sets and has the following property: for all sets $B \in A$, we also have $B \subseteq A$. Explain why your example works. (FYI: sets that have this property are called *transitive*.)