

Instructions: Same rules as usual - turn in your work on separate sheets of paper. You must justify all your answers for full credit.

- (3pts) 1. Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.
- (3pts) 2. Prove that if $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $ac \equiv bd \pmod{n}$. Hint: rewrite each congruence as an equation, then multiply the equations.
- (6pts) 3. Find the remainder when $42^{2013} + 2013^{42}$ is divided by 5. Use modular arithmetic, and explain how you got your answer.
- (6pts) 4. Solve the following congruences for x . Give the general solution, as well as all solutions with $0 \leq x \leq 20$.
- (a) $75x \equiv 41 \pmod{4}$
- (b) $10x + 7 \equiv 3 \pmod{12}$
- (6pts) 5. Solve the following Diophantine equations. Describe all solutions.
- (a) $7x + 46y = 100$.
- (b) $55x + 42y = 47$
- (6pts) 6. The hugely popular math-rock band *Fibonacci's Rabbits* recently performed on campus. Tickets were \$21, unless you knew the password (the closed formula for the n th Fibonacci number), in which case tickets were only \$13. All together, the revenues from ticket sales came to \$12855. How many total tickets were sold, assuming the number of discounted tickets was as large as possible.