Activity 1: Odds are... awesome

Our goal: find a nice formula for the sum of the first n odd numbers.

1. First, find the sum of the first n odd numbers for small values of n. Fill in the missing bits below:

   (a) $n = 1$: $1 = \underline{1}$

   (b) $n = 2$: $1 + 3 =$

   (c) $n = 3$: $1 + 3 + 5 =$

   (d) $n = 4$:

   (e) $n = 5$:

   (f) $n = 9$:

2. Perhaps you now have a guess for a formula for the sum of the first n odd numbers. Before you write that down, how could we write the sum of the first n odd numbers "the long way"? Last time, I asked you to compute the sum of the first 100 odd number: $1 + 3 + 5 + \cdots + 199$. What should you replace 199 with to express the sum of the first n odd numbers.

3. Now write down your *conjecture*. How confident are you that you are correct? What could you do to become more confident?

4. Become more confident! In your group, decide on a way to convince a skeptical math student that your formula is correct. This math student is so skeptical that just showing her that your formula works for a few more values of n will have no effect.

Activity 2: Seeing dots

The goal: investigate which numbers of dots can be arranged in which ways.

1. Find a common way (perhaps in a particular shape) to arrange the following numbers of dots: 1, 4, 9, 16, and 25. (Your way should not work for 3 or 6 or 17 dots, for example.) What is the next number of dots which can be arranged in this way?

2. Earlier, you found that the number 16 can be expressed as $1+3+5+7$. In the dot representation for 16 you gave above, locate collections of 1, 3, 5, and 7 dots. Will something similar work for 25 dots?

3. Find a common way to arrange the following numbers of dots: 1, 3, 6, 10, 15, and 21. What is the next number of dots which can be arranged in this way? What should we call these numbers?

4. Notice that 16 can also be expressed as $6 + 10$. Can you represent this fact using the dot arrangements from questions 1 and 3? Do something similar for 25 dots.

Activity 3: ⟨insert witty title involving sums here⟩

Our goal: Investigate the sum $1 + 2 + 3 + \cdots + n$.

1. Find sums for small values of n.

   $n = 1$:

   $n = 2$:

   $n = 3$:

   $n = 4$:

   $n = 5$:

   $n = 6$:

2. Let's call the sum of the first n positive integers $T(n)$. What does T stand for?

3. Find $T(100)$.

4. Find a closed formula for $T(n)$. How confident are you that you are correct?

5. Find a formula for $T(n) + T(n + 1)$. (You might want to look back at activity 2, part 4.)

Activity 4: Triangles, Squares, and Cubes - oh my!

Our goal: We have seen what happens when we add up odd numbers and positive integers. What happens when we add up perfect cubes?

1. Begin by gathering some data:

   $1^3 =$

   $1^3 + 2^3 =$

   $1^3 + 2^3 + 3^3 =$

   $1^3 + 2^3 + 3^3 + 4^3 =$

   $1^3 + 2^3 + 3^3 + 4^3 + 5^3 =$

2. Make a conjecture.

3. What does your conjecture say the sum of the first 7 cubes should be?

4. What is the sum of the first 7 cubes? Did your conjecture work?

5. How convinced are you that your conjecture is correct? Can you think of an argument that would convince your skeptical friend?