Math 228 **Homework 4** Due: Wednesday, Feb 11

Instructions: Same rules as usual - turn in your work on separate sheets of paper. You must justify all your answers for full credit.

(6pts) 1. Suppose you own x fezzes and y bow ties. Of course, x and y are both greater than 1.

(a) How many combinations of fez and bow tie can you make? You can wear only one fez and one bow tie at a time. Explain.

(b) Explain why the answer is *also* $\binom{x+y}{2} - \binom{x}{2} - \binom{y}{2}$. (If this is what you claimed the answer was in part (a), try it again.)

(c) Use your answers to parts (a) and (b) to give a combinatorial proof of the identity

$$\binom{x+y}{2} - \binom{x}{2} - \binom{y}{2} = xy$$

(6pts) 2. Consider the identity:

$$k\binom{n}{k} = n\binom{n-1}{k-1}$$

(a) Is this true? Try it for a few values of n and k.

(b) Use the formula for $\binom{n}{k}$ to give an algebraic proof of the identity.

(c) Give a combinatorial proof of the identity. Hint: How many ways can you select a team of k people from a group of n people *and* select one of them to be the team captain?

(6pts) 3. After a late night of math studying, you and your friends decide to go to your favorite tax-free fast food Mexican restaurant, *Burrito Chime*. You decide to order off of the dollar menu, which has 7 items. Your group has \$16 to spend (and will spend all of it).

(a) How many different orders are possible? Explain. (The *order* in which the order is placed does not matter - just which and how many of each item that is ordered.)

(b) How many different orders are possible if you want to get at least one of each item? Explain.

(c) How many different orders are possible if you don't get more than 4 of any one item? Explain. Hint: get rid of the bad orders using PIE.

(6pts) 4. Consider functions $f : \{1, 2, 3, 4, 5\} \to \{0, 1, 2, \ldots, 9\}$.

(a) How many of these functions are strictly increasing? Explain. (A function is strictly increasing provided if $a < b$, then $f(a) < f(b)$.)

(b) How many of the functions are non-decreasing? Explain. (A function is non-decreasing provided if $a < b$, then $f(a) \leq f(b)$.)

(6pts) 5. The Grinch sneaks into a room with 6 Christmas presents to 6 different people. He proceeds to switch the name-labels on the presents. How many ways could he do this if:

(a) No present is allowed to end up with its original label? Explain what each term in your answer represents.

(b) Exactly 2 presents keep their original labels? Explain.

(c) Exactly 5 presents keep their original labels? Explain.