Instructions: The problems below are purely for you to practice. I will not collect these, but it is still a good idea to write out your solutions in full. Any of these problems or problems similar are fair game for quizzes and exams.

1. Make a truth table for the statement $(P \vee Q) \to (P \wedge Q)$.
2. Make a truth table for the statement $\neg P \wedge (Q \to P)$. What can you conclude about P and Q if you know the statement is true?
3. Make a truth table for the statement $\neg P \to (Q \wedge R)$.
4. Determine whether the following two statements are logically equivalent: $\neg(P \to Q)$ and $P \wedge \neg Q$. Explain how you know you are correct.
5. Are the statements $P \to (Q \vee R)$ and $(P \to Q) \vee (P \to R)$ logically equivalent?
6. Consider the statement about a party, "If it's your birthday or there will be cake, then there will be cake."
   (a) Translate the above statement into symbols. Clearly state which statement is P and which is Q.
   (b) Make a truth table for the statement.
   (c) Assuming the statement is true, what (if anything) can you conclude if there will be cake?
   (d) Assuming the statement is true, what (if anything) can you conclude if there will not be cake?
   (e) Suppose you found out that the statement was a lie. What can you conclude?
7. Suppose P and Q are the statements: P: Jack passed math. Q: Jill passed math.
   (a) Translate "Jack and Jill both passed math" into symbols.
   (b) Translate "If Jack passed math, then Jill did not" into symbols.
   (c) Translate "$P \vee Q$" into English.
   (d) Translate "$\neg(P \wedge Q) \to Q$" into English.
   (e) Suppose you know that if Jack passed math, then so did Jill. What can you conclude if you know that:
      i. Jill passed math?
      ii. Jill did not pass math?
8. Geoff Poshingten is out at a fancy pizza joint, and decides to order a calzone. When the waiter asks what he would like in it, he replies, "I want either pepperoni or sausage, and if I have sausage, I must also include quail. Oh, and if I have pepperoni or quail then I must also have ricotta cheese."
   (a) Translate Geoff's order into logical symbols.
   (b) The waiter knows that Geoff is either a liar or a truth-teller (so either everything he says is false, or everything is true). Which is it?
   (c) What, if anything, can the waiter conclude about the ingredients in Geoff's desired calzone?

9. Consider the statement "If Oscar eats Chinese food, then he drinks milk."
   (a) Write the converse of the statement.
   (b) Write the contrapositive of the statement.
   (c) Is it possible for the contrapositive to be false? If it was, what would that tell you?
   (d) Suppose the original statement is true, and that Oscar drinks milk. Can you conclude anything (about his eating Chinese food)? Explain.
   (e) Suppose the original statement is true, and that Oscar does not drink milk. Can you conclude anything (about his eating Chinese food)? Explain.

10. Simplify the following statements (so that negation only appears right before variables).
    (a) $\neg(P \to \neg Q)$
    (b) $(\neg P \vee \neg Q) \to \neg(\neg Q \wedge R)$
    (c) $\neg((P \to \neg Q) \vee \neg(R \wedge \neg R))$
    (d) It is false that if Sam is not a man then Chris is a woman, and that Chris is not a woman.

11. Translate into symbols. Use $E(x)$ for "x is even" and $O(x)$ for "x is odd."
    (a) No number is both even and odd.
    (b) One more than any even number is an odd number.
    (c) There is prime number that is even.
    (d) Between any two numbers there is a third number.
    (e) There is no number between a number and one more than that number.

12. Translate into English:
    (a) $\forall x(E(x) \to E(x+2))$
    (b) $\forall x \exists y(\sin(x) = y)$
    (c) $\forall y \exists x(\sin(x) = y)$
    (d) $\forall x \forall y(x^3 = y^3 \to x = y)$

13. Simplify the statements (so negation appears only directly next to predicates).
    (a) $\neg \exists x \forall y(\neg O(x) \vee E(y))$
    (b) $\neg \forall x \neg \forall y \neg(x < y \wedge \exists z(x < z \vee y < z))$
    (c) There is a number n for which no other number is either less n than or equal to n.
    (d) It is false that for every number n there are two other numbers which n is between.

14. Consider the statement "for all integers a and b, if $a + b$ is even, then a and b are even"
    (a) Write the contrapositive of the statement
    (b) Write the converse of the statement
    (c) Write the negation of the statement.
    (d) Is the original statement true or false? Prove your answer.
    (e) Is the contrapositive of the original statement true or false? Prove your answer.
    (f) Is the converse of the original statement true or false? Prove your answer.
    (g) Is the negation of the original statement true or false? Prove your answer.

15. Prove that $\sqrt{3}$ is irrational.