

1. You have 11 identical mini key-lime pies to give to 4 children. However, you don't want any kid to get more than 3 pies. How many ways can you distribute the pies?
 - (a) How many ways are there to distribute the pies without any restriction?
 - (b) Let's get rid of the ways that one or more kid gets too many pies. How many ways are there to distribute the pies if Al gets too many pies? What if Bruce gets too many? Or Cat? Or Dent?
 - (c) What if two kids get too many pies? How many ways can this happen? Does it matter which two kids you pick to overfeed?
 - (d) Is it possible that three kids get too many pies? If so, how many ways can this happen?
 - (e) How should you combine all the numbers you found above to answer the original question?
2. Suppose now you have 13 pies and 7 children. No child can have more than 2 pies. How many ways can you distribute the pies?

3. Consider all functions $f : \{1, 2, 3, 4, 5\} \rightarrow \{1, 2, 3, 4, 5\}$. How many of the *bijections* have the property that $f(x) \neq x$ for any $x \in \{1, 2, 3, 4, 5\}$?

Using an approach similar to the previous questions, you claim that the answer is:

$$5! - \left[\binom{5}{1} 4! - \binom{5}{2} 3! + \binom{5}{3} 2! - \binom{5}{4} 1! + \binom{5}{5} 0! \right]$$

Explain why this is correct.

4. Recall that a *surjection* is a function for which every element of the codomain is in the range. How many of the functions $f : \{1, 2, 3, 4, 5\} \rightarrow \{1, 2, 3, 4, 5\}$ are surjective?