To give you a feel for the type and variety of questions discrete mathematics can help us solve, here are a few sample problems.

1. While walking through a fictional forest, you encounter three trolls. Each is either a *knight*, who always tells the truth, or a *knave*, who always lies. The trolls will not let you pass until you correctly identify each as either a knight or a knave. Each troll makes a single statement:

   Troll 1: Only one of us is a knave.

   Troll 2: No, only one of us is a knight.

   Troll 3: We are all knaves.

   Which troll is which?

2. Back in the days of yore, five small towns decided they wanted to build roads connecting each pair of towns. While the towns had plenty of money to build roads as long as they wished, it was very important that the roads not intersect with each other (as stop signs had not yet been invented). Also, tunnels and bridges were not allowed. Is it possible for each of these town to build a road to each of the four other towns without creating any intersections?

3. At the warm-up event for Oscar's All Star Hot Dog Eating Contest, Al ate one hot dog. Bob then showed him up by eating three hot dogs. Not to be outdone, Carl ate five. This continued with each contestant eating two more hot dogs than the previous contestant.

   (a) How many hot dogs did Zeno (the 26th and final contestant) eat?

   (b) How many hot dogs were eaten that day all together?

4. Bruce and Sam find themselves by a fountain with only two empty water bottles - one can hold exactly 3 gallons, the other exactly 5 gallons. They know that they must somehow measure 4 gallons of water, else catastrophe strikes. How can they accomplish their task?

   Homework Assignment: For Wednesday, carefully write up a solution to the knight and knave troll puzzle above. The solution should consist of both the answer (which troll is which) and a careful argument for why that answer must be correct. Treat this like a mini essay. Your write-up should be somewhere between two and five paragraphs. You might find it easier to type this assignment, but neatly hand-written is also acceptable.