Practice Problems 7: Counting

Math 228 **Solutions** Spring 2012

1. 255

2. 8

3. 15

4. (a) $2^8 = 256$. You have two choices for each tie - wear it or don't.
   (b) You have 7 choices for regular ties (the 8 choices less the "no regular tie" option) and 31 choices for bow ties (32 total minus the "no bow tie" option). Thus total you have $7 \cdot 31 = 217$.
   (c) $\binom{2}{3}\binom{5}{3} = 30$
   (d) $5! = 120$

5. (a) 16 is the number of choices you have if you want to watch one movie, either a comedy or horror flick.
   (b) 63 is the number of choices you have if you will watch two movies, first a comedy and then a horror.

6. (a) 8^5, since you select from 8 letters 5 times.
   (b) $P(8,5) = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$. After selecting a letter, you have fewer letters to select for the next one.
   (c) 64 - you need to select the 4th and 5th letters.
   (d) $64 + 64 - 0 = 128$. There are 64 words which start with "aha" and another 64 words that end with "bah." Perhaps we over counted the words that both start with "aha" and end with "bah" but since the words are only 5 letters long, there are no such words.
   (e) $P(8,5) - 3 \cdot P(5,2) = 6660$ - all the words minus the bad ones. The taboo word can be in any of three positions (starting with letter 1, 2, or 3) and for each position we must choose the other two letters (from the remaining 5 letters)

7. $\binom{10}{6} + \binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10} = 386$

8. Use the binomial theorem. $\binom{14}{9} + \binom{15}{6}2^9$.

9. (a) $2^6 = 64$
   (b) $2^3 = 8$. We need to select yes/no for each of the remaining three elements.
   (c) $2^3 = 8$. We need to decide yes/no for the three non-prime elements.
   (d) $2^6 - 2^3 = 56$. There are 8 subsets which do not contain any odd numbers.
   (e) 9. We need to select one odd (3 choices) and one even (3 choices).

10. (a) $\binom{14}{7}$
    (b) $\binom{6}{2}\binom{8}{5}$
    (c) $\binom{14}{7} - \binom{6}{2}\binom{8}{5}$

11. (a) $\binom{10}{3}$
    (b) 2^{10}
    (c) $P(10,5)$

12. $\binom{7}{2}\binom{7}{2}$

13. (a) 5 (you need to skip one dot the top and the bottom).
    (b) $\binom{7}{2}$ - once you select the two dots on the top, the bottom two are determined.
    (c) This is tricky - you need to worry about running out of space. One way to count: break into cases by the location of the top left corner. You get $\binom{7}{2} + (\binom{7}{2} - 1) + (\binom{7}{2} - 3) + (\binom{7}{2} - 6) + (\binom{7}{2} - 10) + (\binom{7}{2} - 15)$
    (d) All of them

14. (a) $\binom{20}{4}\binom{16}{4}\binom{12}{4}\binom{8}{4}\binom{4}{4}$

    (b) $5!\binom{15}{3}\binom{12}{3}\binom{9}{3}\binom{6}{3}\binom{3}{3}$

15. $9!$ (there are 10 people seated around the table, but it does not matter where King Arthur sits, only who sits to his left, two seats to his left, and so on).