Math 228 **Homework 3** Due: Wednesday, February 4

**Instructions**: Same rules as usual - turn in your work on separate sheets of paper. You must justify all your answers for full credit.

(8pts) 1. How many 9-bit strings (that is, bit strings of length 9) are there which:

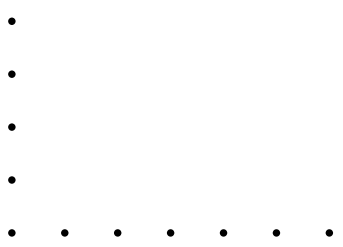
(a) Start with the sub-string 101? Explain.

(b) Have weight 5 (i.e., contain exactly five 1's) and start with the sub-string 101? Explain.

(c) Either start with 101 or end with 11 (or both)? Explain.

(d) Have weight 5 and either start with 101 or end with 11 (or both)? Explain.

(6pts) 2. How many triangles are there with vertices from the points shown below? Note, we are not allowing degenerate triangles - ones with all three vertices on the same line, but we do allow non-right triangles. Explain why your answer is correct. (HINT: you need at exactly two points on either the $x$- or $y$-axis, but don't over-count the right triangles.)



(8pts) 3. Gridtown USA, besides having excellent donut shoppes, is known for its precisely laid out grid of streets and avenues. Streets run east-west, and avenues north-south, for the entire stretch of the town, never curving and never interrupted by parks or schools or the like.

Suppose you live on the corner of 1st and 1st and work on the corner of 12th and 12th. Thus you must travel 22 blocks to get to work as quickly as possible.

(a) How many different routes can you take to work, assuming you want to get there as quickly as possible?

(b) Now suppose you want to stop and get a donut on the way to work, from your favorite donut shoppe on the corner of 8th st and 10th ave. How many routes to work, via the donut shoppe, can you take (again, ensuring the shortest possible route)?

(c) Disaster Strikes Gridtown: there is a pothole on 4th avenue between 5th and 6th street. How many routes to work can you take avoiding that unsightly (and dangerous) stretch of road?

(d) How many routes are there both avoiding the pothole and visiting the donut shoppe?

(8pts) 4. Recall that the formula for $P(n,k)$ is $\dfrac{n!}{(n-k)!}$. Your task here is to explain *why* this is the right formula.

(a) Suppose you have 12 chips, each a different color. How many different stacks of 5 chips can you make? Explain your answer and why it is the same as using the formula for $P(12,5)$.

(b) Using the scenario of the 12 chips again, what does 12! count? What does 7! count? Explain.

(c) Explain why it makes sense to divide 12! by 7! when computing $P(12,5)$ (in terms of the chips).

(d) Does your explanation work for numbers other than 12 and 5? Explain the formula $P(n,k) = \frac{n!}{(n-k)!}$ using the variables $n$ and $k$.