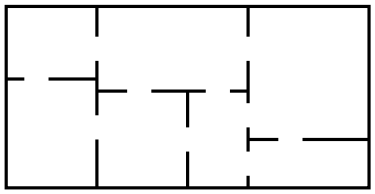
Math 228 **Homework 11** Due: Wednesday, April 22

Instructions: Same rules as usual - turn in your work on separate sheets of paper. You must justify all your answers for full credit.

1. Edward A. Mouse has just finished his brand new house. The floor plan is shown below:



(3pts) (a) Edward wants to give a tour of his new pad to a lady-mouse-friend. Is it possible for them to walk through every doorway exactly once? If so, in which rooms must they begin and end the tour? Explain.

(2pts) (b) Is it possible to tour the house visiting each room exactly once (not necessarily using every doorway)? Explain.

(3pts) (c) After a few mouse-years, Edward decides to remodel. He would like to add some new doors between the rooms he has. Of course, he cannot add any doors to the exterior of the house. Is it possible for each room to have an odd number of doors? Explain.

2. A group of 10 friends decides to head up to a cabin in the woods (where nothing could possibly go wrong). Unfortunately, a number of these friends have dated each other in the past, and things are still a little awkward. To get the cabin, they need to divide up into some number of cars, and no two people who dated should be in the same car.

(3pts) (a) What is the smallest number of cars you need if all the relationships were strictly heterosexual? Represent an example of such a situation with a graph. What kind of graph do you get?

(3pts) (b) Because a number of these friends dated there are also conflicts between friends of the same gender, listed below. Now what is the smallest number of conflict-free cars they could take to the cabin?

Friend	A	B	C	D	E	F	G	H	I	J
Has a conflict with	B E J	A D G	H J	B F	A I	D J	B	C I	E H J	A C F I

(2pts) (c) What do these questions have to do with coloring?

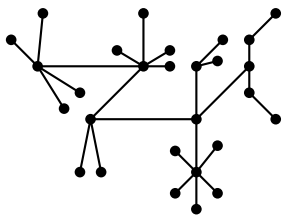
(6pts) 3. We say that a graph has a *Hamilton path* if there is a path which visits each vertex exactly once (you do not need to use every edge in the path).

(a) Suppose a graph has a Hamilton path. What is the maximum number of vertices of degree one the graph can have? Explain why your answer is correct.

(b) Find a graph which does not have a Hamilton path even though no vertex has degree one. Explain why your example works.

4. Prove the chromatic number of any tree is two.

(2pts) (a) Describe a procedure to color the tree below.



(3pts) (b) Prove that your procedure from part (a) always works for any tree.

(3pts) (c) Now, prove using induction that every tree has chromatic number 2.

(4pts-bns) 5. BONUS: King Arthur has hired a consultant, Sir Cumference, to plan a seating arrangement for Arthur and nine of his knights to sit around the round table. This would not be a difficult task, were it not for the jealousies and petty rivalries that exist among the knights. Arthur insists that Lancelot should sit on his right and that Kay should sit on his left; Bedivere refuses to sit next to anyone but Lionel and Tristan; Gawain won't sit next to Tristan, Lancelot, or Lionel; Gareth won't sit next to Galahad, Lancelot, or Kay; Perceval objects to sitting next to Galahad, Lancelot, or Lionel; Tristan refuses to sit next to Lancelot, Perceval or Kay; Galahad will sit next to anyone except Gawain and Kay; and Lionel will not sit next to Galahad. The other two knights are not particular about whom they sit next to.
Help Sir Cumference find a suitable seating arrangement and explain what this problem has to do with graphs.