Activity 1: Odds are...awesome

Our goal: find a nice formula for the sum of the first n odd numbers.

1. First, find the sum of the first n odd numbers for small values of n. Fill in the missing bits below:

   (a) $n = 1$: $1 = 1$

   (b) $n = 2$: $1 + 3 =$ ____

   (c) $n = 3$: $1 + 3 + 5 =$ ____

   (d) $n = 4$: ________________ $=$ ____

   (e) $n = 5$: ___________________ $=$ ____

   (f) $n = 6$: _____________________ $=$ ____

2. Perhaps you now have a guess for a formula for the sum of the first n odd numbers. What is the sum of the first 10 odd numbers? Check your answer.

3. Now write down your *conjecture* (i.e., guess). Write it both as an English sentence, and then also as an equation. The equation will contain "$+\cdots+$" in it.

4. Is your conjecture correct? How do you know? You don't need a complete answer to this, but spend 3 minutes thinking about your reasons for believing your conjecture.

Activity 2: Seeing dots

The goal: investigate which numbers of dots can be arranged in which ways.

1. For each of the numbers below, arrange that many dots into a geometric shape. The basic shape should be the same for all cases, and not work if I gave you 17 or 3 or 42 dots, for example.

   1 dot   4 dots   9 dots   16 dots   25 dots

2. Earlier, you found that the number 16 can be expressed as $1+3+5+7$. In the dot representation for 16 you gave above, locate collections of 1, 3, 5, and 7 dots. Can you do so in such a way that can be extended to 25 dots?

3. Find a common way to arrange the numbers of dots below. What is the next number of dots which can be arranged in this way? What should we call these numbers?

   1 dot   3 dots   6 dots   10 dots   15 dots

4. Notice that 16 can also be expressed as $6 + 10$. Can you represent this fact using the dot arrangements from questions 1 and 3? Do something similar for 25 dots.

Activity 3: $\langle$insert witty title involving sums here$\rangle$

Our goal: Investigate the sum $1 + 2 + 3 + \cdots + n$.

1. Find sums for small values of n.

   $n = 1$:

   $n = 2$:

   $n = 3$:

   $n = 4$:

   $n = 5$:

   $n = 6$:

2. Let's call the sum of the first n positive integers $T(n)$ (what does T stand for?). Find $T(100)$.

3. Find a closed formula for $T(n)$. Check that it works with a few examples.

4. Find a formula for $T(n) + T(n + 1)$. (You might want to look back at activity 2, part 4.)

Activity 4: Triangles, Squares, and Cubes - oh my!

Our goal: We have seen what happens when we add up odd numbers and positive integers. What happens when we add up perfect cubes?

1. Begin by gathering some data:

   $1^3 =$

   $1^3 + 2^3 =$

   $1^3 + 2^3 + 3^3 =$

   $1^3 + 2^3 + 3^3 + 4^3 =$

   $1^3 + 2^3 + 3^3 + 4^3 + 5^3 =$

2. Make a conjecture. Again, write your conjecture both as an English sentence and an equation.

3. What does your conjecture say the sum of the first 7 cubes should be? Check your answer.

4. How convinced are you that your conjecture is correct? How would you convince a skeptical friend that you are correct? Yes, that skeptical friend is you!