

**Instructions:** Here are some extra practice problem on the logic and proofs stuff we have been doing. Again, these are just for you.

1. Write the negation, converse and contrapositive for each of the statements below.
  - (a) If the power goes off, then the food will spoil.
  - (b) If the door is closed, then the light is off.
  - (c)  $\forall x(x < 1 \rightarrow x^2 < 1)$
  - (d) For all natural numbers  $n$ , if  $n$  is prime, then  $n$  is solitary.
  - (e) For all functions  $f$ , if  $f$  is differentiable, then  $f$  is continuous.
  - (f) For all integers  $a$  and  $b$ , if  $a \cdot b$  is even, then  $a$  and  $b$  are even.
  - (g) For every integer  $x$  and every integer  $y$  there is an integer  $n$  such that if  $x > 0$  then  $nx > y$ .
  - (h) For all real numbers  $x$  and  $y$ , if  $xy = 0$  then  $x = 0$  or  $y = 0$ .
  - (i) For every student in Math 228, if they do not understand implications, then they will fail the exam.
2. Consider the statement: for all integers  $n$ , if  $n$  is even then  $8n$  is even.
  - (a) Prove the statement. What sort of proof are you using?
  - (b) Is the converse true? Prove or disprove.
3. Consider the statement: for all integers  $n$ , if  $n$  is odd, then  $7n$  is odd.
  - (a) Prove the statement. What sort of proof are you using?
  - (b) Prove the converse. What sort of proof are you using?
4. Consider the statement: for all integers  $a$  and  $b$ , if  $a$  is even and  $b$  is a multiple of 3, then  $ab$  is a multiple of 6.
  - (a) Prove the statement. What sort of proof are you using?
  - (b) State the converse. Is it true? Prove or disprove.
5. Prove that  $\log(7)$  is irrational.
6. Prove that there are no integer solutions to the equation  $x^2 = 4y + 3$ .