Instructions: Complete the homework problems below on a *separate* sheet of paper (and not all jammed up between the questions). Each solution should be accompanied with supporting work or an explanation why the solution is correct. Your work will be graded on correctness as well as the clarity of your explanations.

(4pts) 1. Zombie Euler and Zombie Cauchy - two famous zombie mathematicians - have just signed up for Twitter accounts. After one day, Zombie Cauchy has more followers than Zombie Euler. Each day after that, the number of new followers of Zombie Cauchy is exactly the same as the number of new followers of Zombie Euler (and neither lose any followers). Prove, by mathematical induction that on every day after the first day, Zombie Cauchy will have more followers than Zombie Euler.

(4pts) 2. Find the largest number of points which a football team cannot get exactly using just 3-point field goals and 7-point touch downs (ignore the possibility of safeties, missed extra points, and two point conversions). Prove your answer is correct by mathematical induction.

(4pts) 3. Write down first 6 or so terms of the sequences generated by each of the following generating functions, using the fact that $\frac{1}{1-x}$ generates $1, 1, 1, 1, \ldots$. No explanation or work required.

   (a) $\dfrac{5}{1-x}$

   (b) $\dfrac{1}{1+2x}$

   (c) $\dfrac{1}{(1-x^2)^2}$

   (d) $\dfrac{1}{(1-x^2)^2} + \dfrac{5}{1-x}$

(4pts) 4. Find the generating function for the sequence $1, 4, 11, 34, 101, 304, \ldots$ using the fact that the sequence is recursively defined by $a_n = 2a_{n-1} + 3a_{n-2}$ with $a_0 = 1$ and $a_1 = 4$.

(4pts) 5. Find a generating function for the sequence $3, 4, 6, 10, 18, 34, 66, \ldots$. Hint: find the generating function for the difference between terms. Explain why your answer is correct.

(4pts-bns) 6. Bonus: Use the generating function for the sequence in question 5 to find a closed formula for that sequence.