

Introduction

Welcome to Discrete Mathematics. If this is your first time encountering the subject, you will probably find discrete mathematics quite different from other math subjects. You might not even know what discrete math is! Hopefully this short introduction will shed some light on what the subject is about and what you can expect as you move forward in your study of it.

1 What is Discrete Mathematics?

dis·crete / **dis'krēt**.

Adjective: Individually separate and distinct.

Synonyms: separate - detached - distinct - abstract.

This is not an easy question to answer. Part of the challenge is that discrete math is a very broad description which encapsulates a large number of subjects in mathematics. In this course we will study four main topics: *combinatorics* (the theory of ways things *combine*, in particular, how to count these ways), *sequences*, *logic*, and *graph theory*. However, there are other topics that belong under the discrete umbrella, including computer science, abstract algebra, number theory, game theory, probability, and geometry (some of these, particularly the last two, have both discrete and non-discrete variants).

If you are familiar with some of these subjects you might start to notice some common themes and that might help understand what discrete math is about. Another way we might start to get a feel for the subject is to consider the types of problems you solve using discrete math. Here are a few examples.

1. The most popular mathematician in the world is throwing a party for all of his friends. As a way to kick things off, they decide that everyone should shake hands. Assuming all 10 people at the party each shake hand with every other person (but not themselves, obviously) exactly once, how many handshakes take place?
2. At the warm-up event for Oscar's All Star Hot Dog Eating Contest, Al ate one hot dog. Bob then showed him up by eating three hot dogs. Not to be outdone, Carl ate five. This continued with each contestant eating two more hot dogs than the previous contestant. How many hot dogs did Zeno (the 26th and final contestant) eat? How many hot dogs were eaten all together?
3. While walking through a fictional forest, you encounter three trolls. Each is either a *knight*, who always tells the truth, or a *knave*, who always lies. The trolls will not let you pass until you correctly identify each as either a knight or a knave. Each troll makes a single statement:

Troll 1: If I am a knave then there are exactly two knights here.

Troll 2: Troll 1 is lying.

Troll 3: Either we are all knaves or at least one of us is a knight.

Which troll is which?

4. Back in the days of yore, five small towns decided they wanted to build roads directly connecting each pair of towns. While the towns had plenty of money to build roads as long and as winding as they wished, it was very important that the roads not intersect with each other (as stop signs had not yet been invented). Also, tunnels and bridges were not allowed. Is it possible for each of these town to build a road to each of the four other towns without creating any intersections?