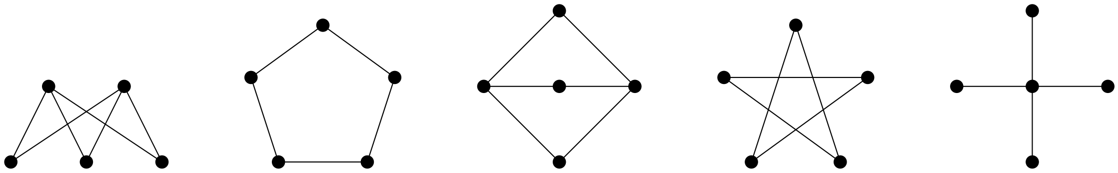
Math 228 **Graph Sameness** Monday, April 6

Main Question: What does it mean for two graphs to be the same?

1. Which (if any) of the graphs below are the same?



2. The graphs above are unlabeled. Usually we think of a graph as having a specific set of vertices. Which (if any) of the graphs below are the same?

3. Actually, all the graphs we have seen above are just *drawings* of graphs. A graph is really an abstract mathematical object consisting of two sets V and E where E is a set of 2-element subsets of V.

   Are the graphs below the same or different?

   Graph 1: $V = \{a, b, c, d, e\}$, $E = \{\{a, b\}, \{a, c\}, \{a, d\}, \{a, e\}, \{b, c\}, \{d, e\}\}$.

   Graph 2: $V = \{v_1, v_2, v_3, v_4, v_5\}$, $E = \{\{v_1, v_3\}, \{v_1, v_5\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_3, v_5\}, \{v_4, v_5\}\}$.

For each of the following, try to give two <u>different</u> unlabeled graphs with the given properties, or explain why doing so is impossible.

4. Two different trees with the same number of vertices and the same number of edges.

5. Two different graphs with 8 vertices all of degree 2.

6. Two different graphs with 5 vertices all of degree 4.

7. Two different graphs with 5 vertices all of degree 3.

When a connected graph can be drawn without any edges crossing, it is called *planar*. When a planar graph is drawn in this way, it divides the plane into regions called *faces*.

8. Two different planar graphs with the same number of vertices, edges, and faces.

9. Two different planar graphs with the same number of vertices and edges, but a different number of faces.

Graph Definitions

Graph: A collection of *vertices*, some of which are connected by *edges*. More precisely, a pair of sets V and E where V is a set of vertices and E is a set of 2-element subsets of V.

Adjacent: Two vertices are *adjacent* if they are connected by an edge. Two edges are *adjacent* if they share a vertex.

Bipartite graph: A graph for which it is possible to divide the vertices into two disjoint sets such that there are no edges between any two vertices in the same set.

Complete bipartite graph: A bipartite graph for which every vertex in the first set is adjacent to every vertex in the second set.

Complete graph: A graph with edges connecting every pair of vertices.

Connected: A graph is *connected* if there is a path from any vertex to any other vertex.

Chromatic number: The minimum number of colors required in a proper vertex coloring of the graph.

Cycle: A path (see below) that starts and stops at the same vertex, but contains no other repeated vertices.

Degree of a vertex: The number of edges connected to a vertex is called the *degree* of the vertex.

Euler path: A path which uses each edge exactly once.

Euler circuit: An Euler path which starts and stops at the same vertex.

Face: A region in the plane created by a planar graph drawn without edges crossing.

Multigraph: A *multigraph* is just like a graph but can contain multiple edges between two vertices as well as single edge loops (that is an edge from a vertex to itself).

Neighbor: The neighbors of a vertex are all the vertices adjacent to it.

Path: A sequence of vertices such that consecutive vertices (in the sequence) are adjacent (in the graph). A path in which no vertex is repeated is called *simple*.

Planar: A graph is planar if it is possible to draw it (in the plane) without any edges crossing.

Subgraph: We say that H is a subgraph of G if every vertex and edge of H is also a vertex or edge of G. We say H is an *induced* subgraph of G if every vertex of H is a vertex of G and for pair of vertices in H are adjacent in H if and only if they are adjacent in G.

Tree: A (connected) graph with no cycles. (A non-connected graph with no cycles is called a *forest*.) The vertices in a tree with degree 1 are called *leaves*.

Vertex coloring: An assignment of colors to each of the vertices of a graph. A vertex coloring is *proper* if adjacent vertices are always colored differently.