Instructions: The problems below are purely for you to practice. I will not collect these, but it is still a good idea to write out your solutions in full. Any of these problems or problems similar are fair game for quizzes and exams.

1. Find the closed formula for each of the following sequences by relating them to a well know sequence. Assume the first term given is a_1.
   (a) $2, 5, 10, 17, 26, \ldots$
   (b) $0, 2, 5, 9, 14, 20, \ldots$
   (c) $8, 12, 17, 23, 30, \ldots$
   (d) $1, 5, 23, 119, 719, \ldots$

2. The Fibonacci sequence is $0, 1, 1, 2, 3, 5, 8, 13, \ldots$ (where $F_0 = 0$).
   (a) Give the recursive definition for the sequence.
   (b) Write out the first few terms of the sequence of partial sums.
   (c) Give a closed formula for the sequence of partial sums in terms of F_n (for example, you might say $F_0 + F_1 + \cdots + F_n = 3F_{n-1}^2 + n$, although that is definitely not correct).

3. Write out the first few terms of the sequence given by $a_1 = 3$; $a_n = 2a_{n-1} + 4$. Then find a recursive definition for the sequence $10, 24, 52, 108, \ldots$.

4. Write out the first few terms of the sequence given by $a_n = n^2 - 3n + 1$. Then find a closed formula for the sequence (starting with a_1) $0, 2, 6, 12, 20, \ldots$.

5. Consider the sequence $8, 14, 20, 26, \ldots,$.
   (a) What is the next term in the sequence?
   (b) Find a formula for the nth term of this sequence, assuming $a_1 = 8$.
   (c) Find the sum of the first 100 terms of the sequence: $\sum_{k=1}^{100} a_k$.

6. Consider the sequence $1, 7, 13, 19, \ldots, 6n + 7$.
   (a) How many terms are there in the sequence?
   (b) What is the second-to-last term?
   (c) Find the sum of all the terms in the sequence.

7. Find $5 + 7 + 9 + 11 + \cdots + 521$.

8. Find $5 + 15 + 45 + \cdots + 5 \cdot 3^{20}$

9. Find $1 - \frac{2}{3} + \frac{4}{9} - \cdots + \frac{2^{30}}{3^{30}}$

10. Find x and y such that $27, x, y, 1$ is part of an arithmetic sequence. Then find x and y so that the sequence is part of a geometric sequence. (x and y might not be integers.)

11. Use summation ($\sum$) or product ($\prod$) notation to rewrite the following.
    (a) $2 + 4 + 6 + 8 + \cdots + 2n$
    (b) $1 + 5 + 9 + 13 + \cdots + 425$
    (c) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{50}$

(d) $2 \cdot 4 \cdot 6 \cdot \cdots \cdot 2n$

(e) $(\frac{1}{2})(\frac{2}{3})(\frac{3}{4}) \cdots (\frac{100}{101})$

12. Expand the following sums and products. That is, write them out the long way.

    (a) $\sum_{k=1}^{100}(3+4k)$

    (b) $\sum_{k=0}^{n} 2^k$

    (c) $\sum_{k=2}^{50} \frac{1}{(k^2-1)}$

    (d) $\prod_{k=2}^{100} \frac{k^2}{(k^2-1}$

    (e) $\prod_{k=0}^{n}(2+3k)$

13. Use polynomial fitting to find the formula for the nth term of the following sequences:

    (a) 2, 5, 11, 21, 36,. . .

    (b) 0, 2, 6, 12, 20, . . .

14. Can you use polynomial fitting to find the formula for the nth term of the sequence 4, 7, 11, 18, 29, 47, . . . ? Explain why or why not.

15. Find the next 2 terms in the sequence $3, 5, 11, 21, 43, 85 \ldots$.. Then give a recursive definition for the sequence. Finally, use the characteristic root technique to find a closed formula for the sequence.

16. Solve the recurrence relation $a_n = a_{n-1} + 2^n$ with $a_0 = 5$.

17. Show that 4^n is a solution to the recurrence relation $a_n = 3a_{n-1} + 4a_{n-2}$.

18. Find the solution to the recurrence relation $a_n = 3a_{n-1} + 4a_{n-2}$ with initial terms $a_0 = 2$ and $a_1 = 3$.

19. Find the solution to the recurrence relation $a_n = 3a_{n-1} + 4a_{n-2}$ with initial terms $a_0 = 5$ and $a_1 = 8$.

20. Solve the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$.

    (a) What is the solution if the initial terms are $a_0 = 1$ and $a_1 = 2$?

    (b) What do the initial terms need to be in order for $a_9 = 30$?

    (c) For which x are there initial terms which make $a_9 = x$?

21. Solve the recurrence relation $a_n = 3a_{n-1} + 10a_{n-2}$ with initial terms $a_0 = 4$ and $a_1 = 1$.