

1. Solve the recurrence relations below. Use telescoping if possible, if not, try iteration. If that also won't work, use the characteristic root technique.

(a) $a_n = 2a_{n-1} + 1$ with $a_0 = 0$ (this was the recursion for the Tower of Hanoi).

(b) $F_n = F_{n-1} + F_{n-2}$ with $F_0 = 0$ and $F_1 = 1$.

2. **Colorful Tiles:** You have access to 1×1 tiles which come in 2 different colors and 1×2 tiles which come in 3 different colors. We want to figure out how many different 1×10 path designs we can make out of these tiles.

(a) Find the first few terms of the sequence a_1, a_2, a_3, \dots where a_n is the number of different $1 \times n$ paths designs. Think about how you can find a_3 from a_2 and a_1 .

(b) Find a recursive definition for the sequence. That is, given a recurrence relation and initial conditions.

(c) Solve the recurrence relation subject to the initial conditions.

(d) How many different 1×10 paths can you design?