Math 228 **Practice Problems 2: Sets**
Hints and Answers Spring 2012

Instructions: The problems below are purely for you to practice. I will not collect these, but it is still a good idea to write out your solutions in full. Any of these problems or problems similar are fair game for quizzes and exams.

1. (a) $A \cap B = \{3, 4, 5\}$.
   (b) $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$.
   (c) $A \setminus B = \{1, 2\}$.
   (d) Yes.
   (e) No.

2. (a) $A \cap B = \{4, 6, 8, 10, 12\}$
   (b) $A \cup B = \{x \in \mathbb{N} \ : \ (3 \leq x \leq 13) \vee x \text{ is even}\}$. (the set of all natural numbers which are either even or between 3 and 13 inclusive).
   (c) $B \cap C = \emptyset$.
   (d) $B \cup C = \mathbb{N}$.

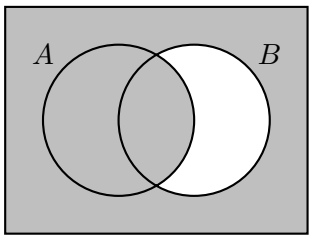
3. For example, $A = \{2, 3, 5, 7, 8\}$ and $B = \{3, 5\}$.

4. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5, \{1, 2, 3\}\}$

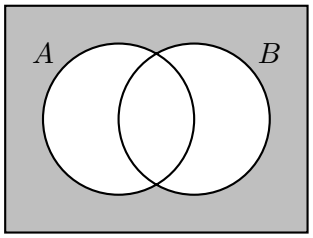
5. (a) No.
   (b) No.
   (c) $2\mathbb{Z} \cap 3\mathbb{Z}$ is the set of all integers which are multiples of both 2 and 3 (so multiples of 6). Therefore $2\mathbb{Z} \cap 3\mathbb{Z} = \{x \in \mathbb{Z} \ : \ \exists y \in \mathbb{Z}(x = 6y)\}$.
   (d) $2\mathbb{Z} \cup 3\mathbb{Z}$.

6. The set of primes.

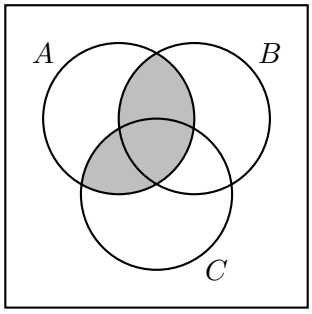
7. (a) $A \cup \overline{B}$:



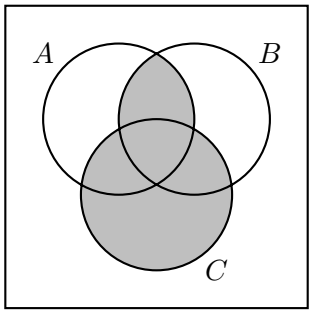
   (b) $\overline{(A \cup B)}$:



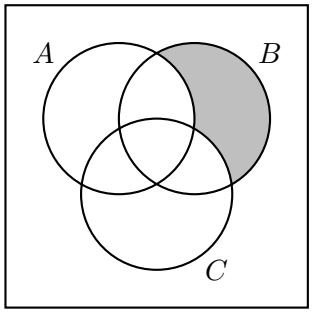
   (c) $A \cap (B \cup C)$:



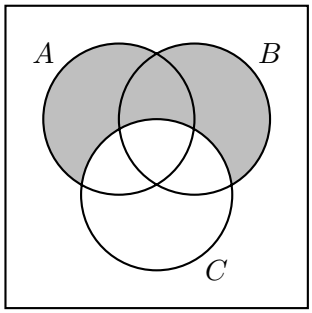
   (d) $(A \cap B) \cup C$:



   (e) $\overline{A} \cap B \cap \overline{C}$:



   (f) $(A \cup B) \setminus C$:



8. For example, $A \cup B \cap \overline{(A \cap B)}$. Note that $\overline{A \cap B}$ would almost work, but also contain the area outside of both circles.

9. (a) 34.
   (b) 103.
   (c) 8.

10. $\mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$.

11. There are 10 singletons. There are 45 doubletons (because $45 = 9 + 8 + 7 + \cdots + 2 + 1$).

12. $\{2, 3, 5\}, \{1, 2, 3, 5\}, \{2, 3, 4, 5\}, \{2, 3, 5, 6\}, \{1, 2, 3, 4, 5\}, \{1, 2, 3, 5, 6\}, \{2, 3, 4, 5, 6\}$, and $\{1, 2, 3, 4, 5, 6\}$.

13. For example $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7, 8, 9\}$.

14. For example, $A = \{1, 2, 3\}$ and $B = \{2, 3, 4, 5\}$.

15. $0 \leq |A \cap B| \leq 10$ and $15 \leq |A \cup B| \leq 25$.

16. $|A \cup B| + |A \cap B| = 13$

17. If R is the set of red cards and F is the set of face cards, we have $|R \cup F| = |R| + |F| - |R \cap F|$. There are 6 cards which are both red and a face card, so $|R \cup F| = 32$.

18. 39.

19. $|(A \cup C) \cap \overline{B}| = 44$

20. One possibility: $(A \cup B) \cap C$.