

Instructions: Complete the homework problems below on a *separate* sheet of paper (and not all jammed up between the questions). Each solution should be accompanied with supporting work or an explanation why the solution is correct. Your work will be graded on correctness as well as the clarity of your explanations.

- (4pts) 1. Let $A = \{1, 2, 3, \dots, 9\}$.
- (a) How many subsets of A contain only even numbers?
 - (b) How many subsets of A contain an even number of elements?
- (4pts) 2. For how many three digit numbers (100 to 999) is the *sum of the digits* even? (For example, 343 has an even sum of digits: $3 + 4 + 3 = 10$ which is even.) Explain.
- (8pts) 3. Gridtown USA, besides having excellent donut shoppes, is known for its precisely laid out grid of streets and avenues. Streets run east-west, and avenues north-south, for the entire stretch of the town, never curving and never interrupted by parks or schools or the like.
- (a) Suppose you live on the corner of 1st and 1st and work on the corner of 12th and 12th. How many blocks must you drive to get to work as quickly as possible?
 - (b) How many different routes can you take to work, assuming you want to get there as quickly as possible?
 - (c) Now suppose you want to stop and get a donut on the way to work, from your favorite donut shoppe on the corner of 8th st and 10th ave. How many routes to work, via the donut shoppe, can you take (again, ensuring the shortest possible route)?
 - (d) Disaster Strikes Gridtown: there is a pothole on 4th avenue between 5th and 6th street. How many routes to work can you take avoiding that unsightly (and dangerous) stretch of road?
 - (e) How many routes are there both avoiding the pothole and visiting the donut shoppe?
- (4pts) 4. How many 9-bit strings (that is, bit strings of length 9) are there which:
- (a) Start with the sub-string 101?
 - (b) Have weight 5 (i.e., contain exactly five 1's) and start with the sub-string 101?