1. The Grinch sneaks into a room with 6 Christmas presents to 6 different people. He proceeds to switch the name-labels on the presents. How many ways could he do this if:

(2pts) (a) No present is allowed to end up with its original label?

Solution:

$$6! - \left[\binom{6}{1}5! - \binom{6}{2}4! + \binom{6}{3}3! - \binom{6}{4}2! + \binom{6}{5}1! - \binom{6}{6}0!\right]$$

(2pts) (b) Exactly 2 presents keep their original labels?

Solution:

$$\binom{6}{2}\left(4! - \left[\binom{4}{1}3! - \binom{4}{2}2! + \binom{4}{3}1! - \binom{4}{4}0!\right]\right)$$

(1pts) (c) Exactly 5 presents keep their original labels?

Solution: 0. Once 5 presents have their original label, there is only one present left and only one label left, so the 6th present must get its own label.

2. After a late night of math studying, you and your friends decide to go to your favorite tax-free fast food Mexican restaurant, *Burrito Chime*. You decide to order off of the dollar menu, which has 7 items. Your group has \$16 to spend (and will spend all of it).

(1pts) (a) How many different orders are possible? (The *order* in which the order is place does not matter - just which and how many of each item that is ordered.)

Solution: $\binom{22}{6}$ - there are 16 stars and 6 bars.

(1pts) (b) How many different orders are possible if you want to get at least one of each item?

Solution: $\binom{15}{6}$ - buy one of each item, using \$7. This leaves you \$11 to distribute to the 7 items, so 11 stars and 6 bars.

(3pts) (c) How many different orders are possible if you don't get more than 4 of any one item? Hint: this is tricky - get rid of the bad orders using PIE.

Solution:

$$\binom{22}{6} - \left[\binom{7}{1}\binom{17}{6} - \binom{7}{2}\binom{12}{6} + \binom{7}{3}\binom{7}{6}\right]$$