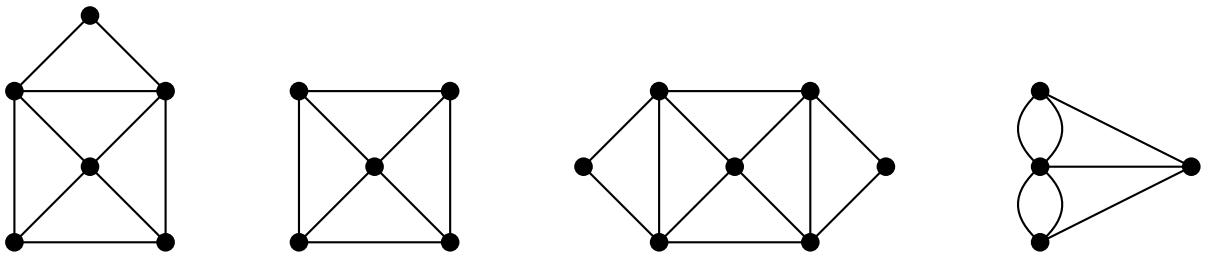
An Euler path is a path which uses every edge exactly once. An Euler circuit is an Euler path which starts and stops at the same vertex. Your goal is to find a quick way to check whether a graph has an Euler path and/or circuit.

1. Which of the graphs below have Euler paths? Which have Euler circuits?

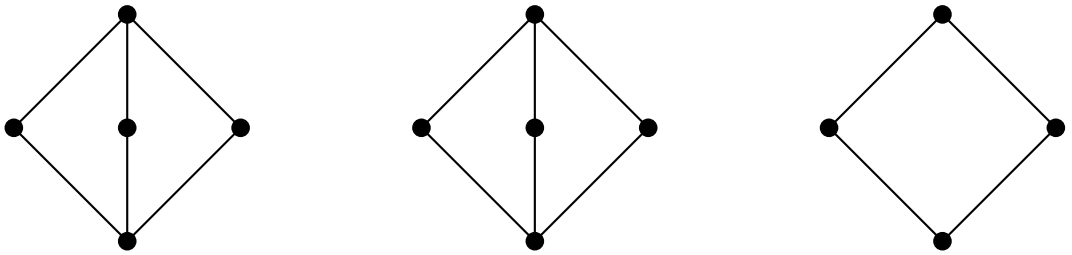


2. For each of the graphs below, add at least 3 edges to get a graph which has:

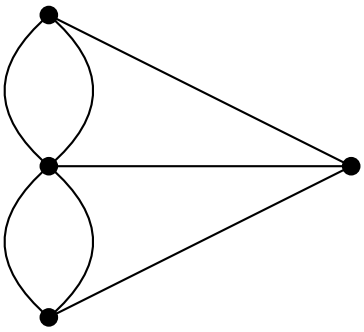
   (a) an Euler circuit

   (b) an Euler path, but not a circuit

   (c) no Euler path

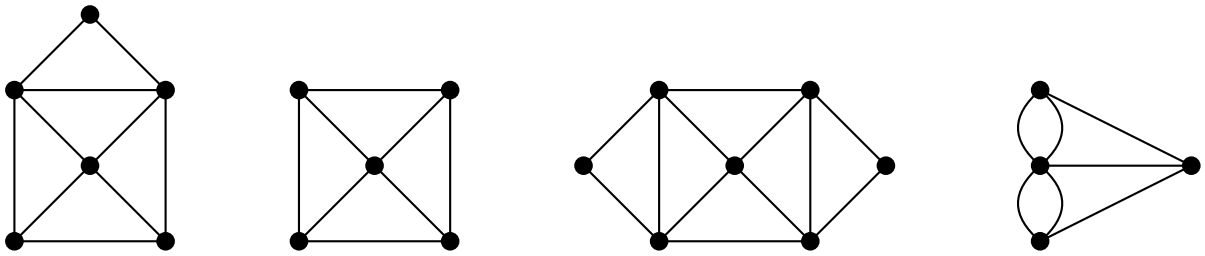


3. What is the smallest number of bridges you must add to Königsberg to get an Euler path? What about an Euler circuit?



The *degree* of a vertex is the number edges connected to it. Is there a connection between the degrees of vertices and the existence of Euler paths and circuits? Let's find out.

4. List the degrees of each vertex of the graphs below. Which graphs had Euler paths? Which had Euler circuits? What does this have to do with the vertex degrees?



5. Is it possible for a graph with a degree 1 vertex to have an Euler circuit? If so, draw one. If not explain why not. What about an Euler path?

6. What if every vertex of the graph has degree 2. Is there an Euler path? An Euler circuit? Draw some graphs.

7. Below is *part* of a graph. Even though you can only see some of the vertices, can you deduce whether the graph will have an Euler path or circuit?