Math 228 **Homework 9**

Instructions: Same rules as usual - turn in your work on separate sheets of paper. You must justify all your answers for full credit.

(6pts) 1. Solve the recurrence relation $a_n = a_{n-1} + 3$ using:

(a) Telescoping. Show your work.

(b) Iteration. Show you work.

2. Let a_n be the number of $1 \times n$ tile designs can you make using 1×1 tiles available in 4 colors and 1×2 tiles available in 5 colors.

(3pts) (a) First, find a recurrence relation to describe the problem. Explain why the recurrence relation is correct (in the context of the problem).

(2pts) (b) Write out the first 6 terms of the sequence $a_1, a_2, \ldots$.

(3pts) (c) Solve the recurrence relation. That is, find a closed formula for a_n.

(6pts) 3. Consider the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}$.

(a) Find the general solution to the recurrence relation (beware the repeated root).

(b) Find the solution when $a_0 = 1$ and $a_1 = 2$.

(c) Find the solution when $a_0 = 1$ and $a_1 = 8$.

(10pts) 4. Write down first 6 or so terms of the sequences generated by each of the following generating functions, using the fact that $\frac{1}{1-x}$ generates $1, 1, 1, 1, \ldots$. In each case, briefly explain how you arrived at your answer.

(a) $\dfrac{5}{1-x}$

(b) $\dfrac{1}{1+2x}$

(c) $\dfrac{1}{(1-x^2)^2}$

(d) $\dfrac{1}{1+2x} + \dfrac{5}{1-x}$

(e) $\dfrac{1}{1+2x} \cdot \dfrac{5}{1-x}$