Math 228 **Homework 8** Due: Wednesday, April 1

**Instructions**: Same rules as usual - turn in your work on separate sheets of paper. You must justify all your answers for full credit.

(3pts) 1. Your "friend" has shown you a "proof" he wrote to show that $1 = 3$. Here is the proof:

*Proof.* We want to show that $1 = 3$. Of course we can do anything to one side of an equation as long as we also do it to the other side. So subtract 2 from both sides. This gives $-1 = 1$. Now square both sides, to get $1 = 1$. And we all agree this is true. Thus $1 = 3$. QED

Carefully explain what is wrong with this proof using what we know about logic. Hint: First identify the implication which follows from the proof.

(5pts) 2. Tommy Flanagan was telling you what he ate yesterday afternoon. He tells you, "I had either popcorn or raisins. Also, if I had cucumber sandwiches, then I had soda. But I didn't drink soda or tea." Of course you know that Tommy is the worlds worst liar, and everything he says is false. What did Tommy eat?

Justify your answer by writing all Tommy's statements using sentence variables ($P, Q, R, S, T$), taking their negations, and using these to deduce what Tommy actually ate.

(6pts) 3. Use De Morgan's Laws, and any other logical equivalence facts you know to simplify the following statements. Show all your steps. Your final statements should have negations only appear directly next to the sentence variables or predicates ($P$, $Q$, $E(x)$, etc.), and no double negations. It would be a good idea to use only conjunctions, disjunctions, and negations.

(a) $\neg((\neg P \wedge Q) \vee \neg(R \vee \neg S))$.

(b) $\neg((\neg P \to \neg Q) \wedge (\neg Q \to R))$ (careful with the implications).

(8pts) 4. Can you chain implications together? That is, if $P \to Q$ and $Q \to R$, does that means the $P \to R$? Can you chain more implications together? Let's find out:

(a) Prove that the following is a valid argument form:

$$\begin{array}{rr} & P \to Q \\ & Q \to R \\ \hline \therefore & P \to R \end{array}$$

(b) Prove that the following is a valid argument form for any $n \geq 2$:

$$\begin{array}{rc} & P_1 \to P_2 \\ & P_2 \to P_3 \\ & \vdots \\ & P_{n-1} \to P_n \\ \hline \therefore & P_1 \to P_n. \end{array}$$

I suggest you don't go through the trouble of writing out a $2^n$ row truth table. Instead, you should use part (a) and mathematical induction.

(8pts) 5. Consider the statement: $\forall x \forall y (x - y \geq 2 \to \exists z (y < z \wedge z < x))$.

(a) Explain what this statement says in words. Is the statement true?

(b) State the contrapositive of the original statement. Do so both in words and in symbols.

(c) State the converse of the original statement. Is the converse true?

(d) State the negation of the original statement. Do so both in words and in symbols (simplifying as much as possible).