Practice Problems 1: Logic

Math 228 **Hints and Answers** Spring 2012

1.

P	Q	$(P \vee Q) \to (P \wedge Q)$
T	T	T
T	F	F
F	T	F
F	F	T

2.

P	Q	$\neg P \wedge (Q \to P)$
T	T	F
T	F	F
F	T	F
F	F	T

If the statement is true, then both P and Q are false.

3. Hint: Like above, only now you will need 8 rows instead of just 4.

4. Make a truth table for each and compare. The statements are logically equivalent.

5. Again, make two truth tables. The statements are logically equivalent.

6. (a) P: it's your birthday; Q: there will be cake. $(P \vee Q) \to Q$
   (b) Hint: you should get three T's and one F.
   (c) Only that there will be cake.
   (d) It's your birthday!
   (e) The cake is a lie.

7. (a) $P \wedge Q$
   (b) $P \to \neg Q$
   (c) Jack passed math or Jill passed math (or both).
   (d) If Jack and Jill did not both pass math, then Jill did.
   (e) i. Nothing else.
   ii. Jack did not pass math either.

8. (a) Three statements: $P \vee S$, $S \to Q$, $(P \vee Q) \to R$. You could also connect the first two with a $\wedge$.
   (b) He cannot be lying about all three sentences, so he is telling the truth.
   (c) No matter what, Geoff wants ricotta. If he doesn't have quail, then he must have pepperoni but not sausage.

9. Consider the statement "If Oscar eats Chinese food, then he drinks milk."
   (a) If Oscar drinks milk, then he eats Chinese food.
   (b) If Oscar does not drink milk, then he does not eat Chinese food.
   (c) Yes. The original statement would be false too.
   (d) Nothing. The converse need not be true.
   (e) He does not eat Chinese food. The contrapositive would be true.

10. (a) $P \wedge Q$

(b) $(P \vee Q) \vee (Q \wedge \neg R)$
(c) F. Or $(P \wedge Q) \wedge (R \wedge \neg R)$
(d) Either Sam is a woman and Chris is a man, or Chris is a woman.

11. (a) $\neg \exists x(E(x) \wedge O(x))$
(b) $\forall x(E(x) \rightarrow O(x+1))$
(c) $\exists x(P(x) \wedge E(x))$ (where $P(x)$ means "x is prime")
(d) $\forall x \forall y \exists z(x < z < y \vee y < z < x)$
(e) $\forall x \neg \exists y(x < y < x+1)$

12. (a) Any even number plus 2 is an even number.
(b) For any x there is a y such that $\sin(x) = y$. In other words, every number x is in the domain of sine.
(c) For every y there is an x such that $\sin(x) = y$. In other words, every number y is in the range of sine (which is false).
(d) For any numbers, if the cubes of two numbers are equal, then the numbers are equal.

13. (a) $\forall x \exists y(O(x) \wedge \neg E(y))$
(b) $\exists x \forall y(x \geq y \vee \forall z(x \geq z \wedge y \geq z))$
(c) There is a number n for which every other number is strictly greater than n.
(d) There is a number n which is not between any other two numbers.

14. (a) For all integers a and b, if a or b are not even, then $a+b$ is not even.
(b) For all integers a and b, if a and b are even, then $a+b$ is even.
(c) There are numbers a and b such that $a+b$ is even but a and b are not both even.
(d) False. For example, $a = 3$ and $b = 5$. $a+b = 8$, but neither a nor b are even.
(e) False, since it is equivalent to the original statement.
(f) True. Let a and b be integers. Assume both are even. Then $a = 2k$ and $b = 2j$ for some integers k and j. But then $a + b = 2k + 2j = 2(k+j)$ which is even.
(g) True, since the statement is false.

15. Suppose $\sqrt{3}$ were rational. Then $\sqrt{3} = \frac{a}{b}$ for some integers a and $b \neq 0$. Without loss of generality, assume $\frac{a}{b}$ is reduced. Now

$$3 = \frac{a^2}{b^2}$$
$$b^2 3 = a^2$$

So a^2 is a multiple of 3. This can only happen if a is a multiple of 3, so $a = 3k$ for some integer k. Then we have

$$b^2 3 = 9k^2$$
$$b^2 = 3k^2$$

So b^2 is a multiple of 3, making b a multiple of 3 as well. But this contradicts our assumption that $\frac{a}{b}$ is in lowest terms.