Math 228 **Homework 7**

Instructions: Same rules as usual - turn in your work on separate sheets of paper. You must justify all your answers for full credit.

1. After a late night of math studying, you and your friends decide to go to your favorite tax-free fast food Mexican restaurant, *Burrito Chime*. You decide to order off of the dollar menu, which has 7 items. Your group has \$16 to spend (and will spend all of it).

(2pts) (a) How many different orders are possible? Explain. (The *order* in which the order is place does not matter - just which and how many of each item that is ordered.)

(2pts) (b) How many different orders are possible if you want to get at least one of each item? Explain.

(4pts) (c) How many different orders are possible if you don't get more than 4 of any one item? Explain. Hint: get rid of the bad orders using PIE.

(6pts) 2. Consider functions $f : \{1, 2, 3, 4, 5\} \to \{0, 1, 2, \ldots, 9\}$.

(a) How many of these functions are strictly increasing? Explain. (A function is strictly increasing provided if $a < b$, then $f(a) < f(b)$.)

(b) How many of the functions are non-decreasing? Explain. (A function is non-decreasing provided if $a < b$, then $f(a) \leq f(b)$.)

(6pts) 3. Rephrase each of the following counting problems in terms of counting functions. That is, describe a collection of functions so that each function corresponds to an outcome of the counting problem. Specifically, say what the domain and codomain of the functions are in terms of the counting problem, and say whether you are counting all functions, injective functions, or surjective functions.

(a) The 10 members of Math Club all decide to each pick one of the 15 math club meetings to give a presentation (each of which will take the entire meeting time). How many ways can this happen?

(b) Over the next seven days, you plan to finish a box of 24 different types of chocolates, having at least one chocolate each day. How many ways can this happen?

(c) At the end of the semester, you assign grades to each student in your class. There are 12 possible grades, and you have 30 students in your class. How many ways are there to do this?

4. The Grinch sneaks into a room with 6 Christmas presents to 6 different people. He proceeds to switch the name-labels on the presents. How many ways could he do this if:

(4pts) (a) No present is allowed to end up with its original label? Explain what each term in your answer represents.

(4pts) (b) Exactly 2 presents keep their original labels? Explain.

(2pts) (c) Exactly 5 presents keep their original labels? Explain.