

Instructions: Here are 3 possible extra credit problems on induction, each worth 15 points. You can complete any two you like, for up to 30 points added to your homework total (thus improving your homework/quiz grade). Clearly indicate which 2 of the 3 problems you have attempted. These are due the last day of classes: Friday, May 3.

- (15pts) 1. **30 Blood-thirsty Pirates:** A ship of 30 pirates has recently acquired a large chest of pirate treasure that they must split among themselves. As is tradition, the captain must propose a split of the treasure, and then the entire crew (including the captain) votes on whether to accept the proposal. If at least half of the crew accepts the proposal, it passes, the gold is divided as the captain said, and the pirates go on their merry way. If less than half of the crew accepts the proposal, the captain must walk the plank, at which time the next in command takes over the captain's hat and makes a new proposal, starting the process over again.¹

Assuming all the pirates are perfectly logical, greedy (so will always vote to get the most gold possible) and blood-thirsty (so will vote to dispose of the captain if it makes no difference for their gold-take), how should the captain divide the loot so as to keep as much gold for himself as possible? Prove your answer using induction.

- (15pts) 2. **30 Red-eyed Monks:** There is a remote island where 30 monks make their home. Each monk has either green or red eyes, and while every monk can see the eye color of every other monk, there are no reflective surfaces in which monks could see their own eye color. Additionally, the monks have taken a vow of silence so there is no way for the monks to tell each other about their eye color.

The monastery is under a strange spell: if ever a monk discovers that he has red eyes, on midnight of that day, he will disappear in a puff of smoke.

One day, you visit the island and discover that in fact all 30 monks have red eyes. Upon leaving, you mention to the entire group, "at least one of you has red eyes." What happens to the monks, and when? Prove your answer using induction.

- (15pts) 3. **30 Black-eyed Peas:** You and a friend play the following game. You start with 30 raw black-eyed peas in a pile. A move consists of taking 1, 2 or 3 peas from the pile. Play alternates until all the peas are gone. The last player to move (take some number of peas) is the winner.

Do you want to play first or second? Describe the winning strategy, and prove your correct, using induction.

¹It is safe to assume all the pirates have a particular rank, so whenever a current captain is thrown overboard, there is no dispute as to who takes over.