

Instructions: The problems below are purely for you to practice. I will not collect these, but it is still a good idea to write out your solutions in full. Any of these problems or problems similar are fair game for quizzes and exams.

1. Consider functions $f : \{1, 2, 3, 4\} \rightarrow \{a, b, c, d, e, f\}$.
 - (a) How many functions are there total?
 - (b) How many functions are one-to-one?
 - (c) How many functions are onto?
 - (d) How many functions have the property that $f(1) \neq a$ or $f(2) \neq b$, or both?
2. Consider sets A and B with $|A| = 10$ and $|B| = 17$.
 - (a) How many functions $f : A \rightarrow B$ are there?
 - (b) How many functions $f : A \rightarrow B$ are one-to-one?
3. Consider sets A and B with $|A| = 10$ and $|B| = 5$. How many functions $f : A \rightarrow B$ are onto?
4. Let $A = \{1, 2, 3, 4, 5\}$. How many one-to-one functions $f : A \rightarrow A$ have the property that for each $x \in A$, $f(x) \neq x$?
5. Ten ladies of a certain age drop off their red hats at the hat check of a museum. As they are leaving, the hat check attendant gives the hats back randomly. What is the probability that exactly six of the ladies receive their own hat (and the other four do not)?
6. After gym class you are tasked with putting the 14 identical dodge-balls away into 5 bins.
 - (a) How many ways can you do this if there are no restrictions?
 - (b) How many ways can you do this if each bin must contain at least one dodge-ball?
 - (c) How many ways can you do this if no bin can hold more than 6 balls?
7. How many integer solutions are there to the equation $x + y + z = 8$ for which
 - (a) x , y , and z are all positive?
 - (b) x , y , and z are all non-negative?
 - (c) x , y , and z are all greater than -3 .
8. When playing Yahtzee, you roll five regular 6-sided dice. How many different outcomes are possible from a single roll? The order of the dice does not matter.
9. How many integer solutions to $x_1 + x_2 + x_3 + x_4 = 25$ are there for which $x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$ and $x_4 \geq 4$?
10. You have 9 presents to give to your 4 kids. How many ways can this be done if
 - (a) The presents are identical, and each kid gets at least one present?
 - (b) The presents are identical, and some kids might get no presents?
 - (c) The presents are unique, and some kids might get no presents?
 - (d) the presents are unique and each kid gets at least one present?

11. For each of the following counting problems, say whether the answer is

A. $\binom{10}{4}$ B. $P(10, 4)$ C. Neither

If you answer is “Neither,” say what the answer should be instead.

- (a) How many shortest lattice paths are there from $(0, 0)$ to $(10, 4)$?
- (b) If you have 10 bow ties, and you want to select 4 of them for next week, how many choices do you have?
- (c) Suppose you have 10 bow ties and you will wear one on each of the next 4 days. How many choices do you have?
- (d) If you want to wear 4 of your 10 bow ties next week (Monday through Sunday), how many ways can this be accomplished?
- (e) Out of a group of 10 classmates, how many ways can you rank your top 4 friends?
- (f) If 10 students come to their professor’s office but only 4 can fit at a time, how different combinations of 4 students can see the prof first?
- (g) How many 4 letter words can be made from the first 10 letters of the alphabet?
- (h) How many ways can you make the word “cake” from the first 10 letters of the alphabet?
- (i) How many ways are there to distribute 10 apples among 4 children?
- (j) If you have 10 kids (and live in a shoe) and 4 types of cereal, how many ways can your kids eat breakfast?
- (k) How many ways can you arrange exactly 4 ones in a string of 10 binary digits?
- (l) You want to select 4 single digit numbers as your lotto picks. How many choices do you have?
- (m) 10 kids want ice-cream. You have 4 varieties. How many ways are there to give the kids as much ice-cream as they want?
- (n) How many 1-1 functions are there from $\{1, 2, \dots, 10\}$ to $\{a, b, c, d\}$?
- (o) How many onto functions are there from $\{1, 2, \dots, 10\}$ to $\{a, b, c, d\}$?
- (p) Each of your 10 bow ties match 4 pairs of suspenders. How many outfits can you make?
- (q) After the party, the 10 kids each choose one of 4 party-favors. How many outcomes?
- (r) How many 6-elements subsets are there of the set $\{1, 2, \dots, 10\}$
- (s) How many ways can you split up 11 kids into 5 teams?
- (t) How many solutions are there to $x_1 + x_2 + \dots + x_5 = 6$ where each x_i is non-negative?
- (u) Your band goes on tour. There are 10 cities within driving distance, but only enough time to play 4 of them. How many choices do you have for the cities on your tour?
- (v) In how many different ways can you play the 4 cities you choose?
- (w) Out of the 10 breakfast cereals available, you want to have 4 bowls. How many ways can you do this?
- (x) There are 10 types of cookies available. You want to make a 4 cookie stack. How many different stacks can you make?
- (y) From you home at $(0,0)$ you want to go to either the donut shop at $(5,4)$ or the one at $(3,6)$. How many paths could you take?
- (z) How many 10-digit numbers do not contain a sub-string of 4 repeated digits?