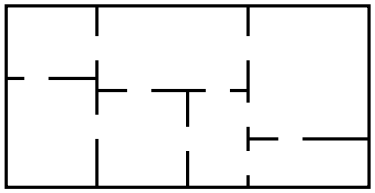
Math 228 **Homework 4**

Instructions: Same rules as usual - turn in your work on separate sheets of paper. You must justify all your answers for full credit.

1. Edward A. Mouse has just finished his brand new house. The floor plan is shown below:



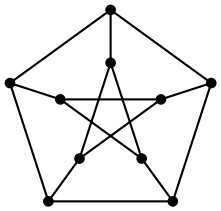
(3pts) (a) Edward wants to give a tour of his new pad to a lady-mouse-friend. Is it possible for them to walk through every doorway exactly once? If so, in which rooms must they begin and end the tour? Explain.

(2pts) (b) Is it possible to tour the house visiting each room exactly once? Explain.

(3pts) (c) After a few mouse-years, Edward decides to remodel. He would like to add some new doors between the rooms he has. Of course, he cannot add any doors to the exterior of the house. Is it possible for each room to have an odd number of doors? Explain.

(4pts) 2. Suppose you are at a party with 19 of your closest friends (so including you, there are 20 people there). Explain why there must be least two people at the party who are friends with the same number of people at the party. Assume friendship is always reciprocated.

(4pts) 3. Prove that the *Petersen graph* (below) is not planar. Hint: what is the length of the shortest circuit?



4. A group of 10 friends decides to head up to a cabin in the woods (where nothing could possibly go wrong). Unfortunately, a number of these friends have dated each other in the past, and things are still a little awkward. To get the cabin, they need to divide up into some number of cars, and no two people who dated should be in the same car.

(3pts) (a) What is the smallest number of cars you need if all the relationships were strictly heterosexual? Represent an example of such a situation with a graph. What kind of graph do you get?

(3pts) (b) What is the smallest number of cars you need if the relationships could be represented by the Petersen graph (above)? Assume each person is represented by a vertex, and two people have dated if there is an edge between their vertices. Explain.

(2pts) (c) What do these questions have to do with coloring?

(6pts) 5. We say that a graph has a *Hamilton path* if there is a path which visits each vertex exactly once (you do not need to use every edge in the path).

(a) Suppose a graph has a Hamilton path. What is the maximum number of vertices of degree one the graph can have? Explain why your answer is correct.

(b) Find a graph which does not have a Hamilton path even though no vertex has degree one. Explain why your example works.