

Instructions: Same rules as usual - turn in your work on separate sheets of paper. You must justify all your answers for full credit.

- (4pts) 1. In a recent survey, 30 students reported whether they liked their potatoes Mashed, French-fried, or Twice-baked. 15 liked them mashed, 20 liked French fries, and 9 liked twice baked potatoes. Additionally, 12 students liked both mashed and fried potatoes, 5 liked French fries and twice baked potatoes, 6 liked mashed and baked, and 3 liked all three styles. How many students *hate* potatoes? Explain why your answer is correct.
- (4pts) 2. For how many three digit numbers (100 to 999) is the *sum of the digits* even? (For example, 343 has an even sum of digits: $3 + 4 + 3 = 10$ which is even.) Explain.
- (8pts) 3. Let $A = \{1, 2, 3, \dots, 9\}$.
- How many subsets of A are there? That is, find $|\mathcal{P}(A)|$. Explain.
 - How many subsets of A contain exactly 5 elements? Explain.
 - How many subsets of A contain only even numbers? Explain.
 - How many subsets of A contain an even number of elements? Explain.
- (8pts) 4. How many 9-bit strings (that is, bit strings of length 9) are there which:
- Start with the sub-string 101? Explain.
 - Have weight 5 (i.e., contain exactly five 1's) and start with the sub-string 101? Explain.
 - Either start with 101 or end with 11 (or both)? Explain.
 - Have weight 5 and either start with 101 or end with 11 (or both)? Explain.
- (6pts) 5. How many triangles are there with vertices from the points shown below? Note, we are not allowing degenerate triangles - ones with all three vertices on the same line, but we do allow non-right triangles. Explain why your answer is correct. (HINT: you need at exactly two points on either the x - or y -axis, but don't over-count the right triangles.)

