

The original MLE problem for multivariate Student's t distributed data is

$$\underset{\boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu}{\text{minimize}} \quad \sum_{t=1}^T f_t(\boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu), \quad (1)$$

where $f_t(\boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu) = \frac{1}{2} \log \det(\boldsymbol{\Sigma}) + \frac{N+\nu}{2} \log \left(1 + \frac{1}{\nu} (\mathbf{x}_t - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x}_t - \boldsymbol{\mu})\right) - \log \Gamma\left(\frac{N+\nu}{2}\right) + \log \Gamma\left(\frac{\nu}{2}\right) + \frac{N}{2} \log \nu$. Since its direct minimization is complicated, the EM algorithm instead iteratively optimizes the Q function at iteration k :

$$\underset{\boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu}{\text{minimize}} \quad \sum_{t=1}^T Q_t(\boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu), \quad (2)$$

where $Q_t(\boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu) = \frac{1}{2} \log \det(\boldsymbol{\Sigma}) + \frac{\mathbb{E}_k[\tau_t]}{2} (\mathbf{x}_t - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x}_t - \boldsymbol{\mu}) + \frac{\nu}{2} \mathbb{E}_k[\tau_t] - \frac{\nu}{2} \mathbb{E}_k[\log \tau_t] - \frac{\nu}{2} \log \frac{\nu}{2} + \log \Gamma\left(\frac{\nu}{2}\right)$.

Now, if we want to put more weights on the recent observations related items, we may be interested in solving the following problems:

$$\underset{\boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu}{\text{minimize}} \quad \sum_{t=1}^T w_t f_t(\boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu), \quad (3)$$

where $\sum_{t=1}^T w_t = T$. Then the corresponding surrogate problem becomes

$$\underset{\boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu}{\text{minimize}} \quad \sum_{t=1}^T w_t Q_t(\boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu), \quad (4)$$

which is

$$\underset{\boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu}{\text{minimize}} \quad \frac{T}{2} \log \det(\boldsymbol{\Sigma}) + \sum_{t=1}^T w_t \left\{ \frac{\mathbb{E}_k[\tau_t]}{2} (\mathbf{x}_t - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x}_t - \boldsymbol{\mu}) + \frac{\nu}{2} \mathbb{E}_k[\tau_t] - \frac{\nu}{2} \mathbb{E}_k[\log \tau_t] \right\} - \frac{T\nu}{2} \log \frac{\nu}{2} + T \log \Gamma\left(\frac{\nu}{2}\right) \quad (5)$$