TxForest: Composable Memory Transactions over Filestores

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Abstract

Keywords

1. Introduction

Databases are a long-standing, effective technology for storing structured and semi-structured data. Using a database has many benefits, including transactions and access to rich set of data manipulation languages and toolkits.

downsides: heavy legacy, relational model is not always adequate

excerpt from [3]: Although database systems are equipped with more advanced and secure data management features such as transactional atomicity, consistency, durability, manageability, and availability, lack of high performance and throughput scalability for storage of unstructured objects, and absence of standard filesystem-based application program interfaces have been cited as primary reasons for content management providers to often prefer existing filesystems or devise filesystem-like solutions for unstructured objects.

cheaper and simpler alternative: store data directly as a collection of files, directories and symbolic links in a traditional filesystem.

examples of filesystems as databases

filesystems fall short for a number of reasons

Forest [1] made a solid step into solving this, by offering an integrated programming environment for specifying and managing filestores.

Although promising, the old Forest suffered two essential shortcomings:

- It did not offer the level of transparency of a typical DBMS. Users don't get to believe that they are working directly on the database (filesystem). they explicitly issue load/store calls, and instead manipulate in-memory representations and the filesystem independently. offline synchronization.
- It provided none of the transactional guarantees familiar from databases. transactions are nice: prevent concurrency and failure problems. successful transactions are guaranteed to run in serial order and failing transactions rollback as if they never occurred. rely on extra programmers' to avoid the hazards of concurrent updates. different hacks and tricks like creating lock files and storing data in temporary locations, that severely increase the complexity of the applications. writing concurrent programs is notoriously hard to get right. even more in the presence of laziness (original forest used the generally unsound Haskell lazy I/O)

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$[pads \mid \textbf{data}\ Balance = Balance\ Int \mid]$

$[forest \mid$
$\quad \textbf{type}\ Accounts = [\,a :: Account \mid a \leftarrow matches\ (GL\ \texttt{"*"})]$
$\quad \textbf{type}\ Account = \texttt{File}\ Balance$
$\mid]$

$\textbf{data}\ Balance = ...$
$\textbf{type}\ Balance\_md = ...$

$\textbf{data}\ Accounts$
$\textbf{instance}\ TxForest\ ()\ Accounts\ (FileInfo, [(FilePath, Account)])\ \textbf{wher}$

$\textbf{data}\ Account$
$\textbf{instance}\ TxForest\ ()\ Account\ ((FileInfo, Balance\_md), Balance)\ \textbf{wher}$

transactional filesystem use cases:

a directory has a group of files that must be processed and deleted and having the aggregate result written to another file.

software upgrade (rollback),

concurrent file access (beautiful account example?)

Specific use cases: LHC
Network logs
Dan's scientific data

2. Examples

3. The Forest Language

the forest description types

a forest description defines a structured representation of a semi-structured filestore.

each Forest declaraction is interpreted as: an expected on-disk shape of a filesystem fragment a transactional variable an ordinary Haskell type for the in-memory representation that represents the content of a variable

two expression quotations: non-monadic (e) vs monadic $< |e| >$

FileInfo for directories/files/symlinks.

4. Forest Transactions

The Forest description language introduced in the previous section describes how to specify the expected shape of a filestore as an allegorical Haskell type, independently from the concrete programming

```
[forest |
  type Universal_d = Directory
    { ascii_files  is [f :: TextFile      | f <- matches (GL "*"), (kind f_att ≡ AsciiK)]
    , binary_files is [b :: BinaryFile    | b <- matches (GL "*"), (kind b_att ≡ BinaryK)]
    , directories  is [d :: Universal_d   | d <- matches (GL "*"), (kind d_att ≡ DirectoryK)]
    , symlinks     is [s :: Link          | s <- matches (GL "*"), (isJust (symLink s_att))]
    }
|]
```

artifacts that are used to manipulate such filestores. We now focus on the key goal of this paper: the design of the Transactional Forest interface.

As we shall see, TxForest (for short) offers an elegant and powerful abstraction for concurrently manipulating structured filestores. We first describe general-purpose transactional facilities (Section 4.1). We then introduce transactional forest variables that allow programmers to interact with filestores (Section 4.2). We briefly touch on how programmers can verify, at any time, if a filestore conforms to its specification (Section 4.3), and finish by introducing analogues of standard file system operations over filestores (Section 4.4).

4.1 Composable transactions

As an embedded domain-specific language in Haskell, the inspiration for TxForest is the widely popular *software transactional memory* (STM) Haskell library, that provides a small set of composable operations to define the key components of a transaction. We now explain the intuition of each one of these mechanisms, cast in the context of TxForest.

Running transactions In TxForest, one runs a transaction by calling the $atomic$ function with type:[1]

$$atomic :: FTM\ a \rightarrow IO\ a$$

It receives a forest memory transaction, of type $FTM\ \ a$, and produces an $IO\ \ a$ action that executes the transaction atomically with respect to all other concurrent transactions, returning a result of type a. In the pure functional language Haskell, FTM and IO are called monads. Different monads are typically used to characterize different classes of computational effects. IO is the primitive Haskell monad for performing irrevocable I/O actions, including reading/writing to files or to mutable references, managing threads, etc. For example, consider the Haskell prelude functions:

$$getChar :: IO\ Char$$
$$putChar :: Char \rightarrow IO\ ()$$

These respectively read a character from the standard input and write a character to the standard output.

Conversely, our FTM monad denotes computations that are tentative, in the sense that they happen inside the scope of a transaction and can always be rolled back. As we discuss in the remainder of this section, these consist of STM-like transactional combinators, file system operations on Forest filestores, or arbitrary pure functions. Note that, since FTM and IO are different types, the Haskell type system effectively prevents non-transactional actions from being run inside of a transaction. This is a valuable guarantee and one that is not commonly found in transactional libraries for mainstream programming languages lacking a very expressive type system.

[1] For the original STM interface [2], substitute FTM by STM.

Blocking transactions To allow a transaction to *block* on a resource, TxForest provides a $retry$ operation with type:

$$retry :: FTM\ a$$

Conceptually, $retry$ cancels the current transaction, without emitting any errors, and schedules it to be retried at a later time. Since each transaction logs all the reads/writes that it performs on a filestore, an efficient implementation waits for another transaction to update the shared filestore fragments read by the blocked transaction before retrying.

Using $retry$ we can define a pattern for conditional transactions that waits on a condition to be verified before performing an action:

$$wait :: FTM\ Bool \rightarrow FTM\ a \rightarrow FTM\ a$$
$$wait\ p\ a = \mathbf{do}\ \{b \leftarrow p; \mathbf{if}\ b\ \mathbf{then}\ retry\ \mathbf{else}\ a\}$$

Note that $wait$ does not require a cycle; the transactional semantics handles consecutive retries.

Composing transactions Multiple transactions can be sequentially composed via the standard **do** notation. For example, we can write:

$$\mathbf{do}\ \{x \leftarrow ftm1; fmt2\ x\}$$

This runs a transaction $ftm1 : FTM\ \ a$ and passes its result to a transaction $ftm2 :: a \rightarrow FTM\ \ b$. Since the whole computation is itself a transaction, it will be performed indivisibly inside an $atomic$ block.

We can also compose transactions as *alternatives*, using the $orElse$ primitive:

$$orElse :: FTM\ a \rightarrow FTM\ a \rightarrow FTM\ a$$

This combinator performs a left-biased choice: It first runs transaction $ftm1$, tries $ftm2$ if $ftm1$ retries, and the whole transaction retries if $ftm2$ retries. For example, it might be used to read either one of two files depending on the current configuration of the file system.

Note that $orElse$ provides an elegant mechanism for nested transactions. At any point inside a larger transaction, we can tentatively perform a transaction $ftm1$ and rollback to the beginning (of the nested transaction) to try $ftm2$ in case $ftm1$ retries:

$$\mathbf{do}\ \{...; orElse\ ftm1\ ftm2; ...\}$$

Exceptions The last general-purpose feature of FTM transactions are *exceptions*. In Haskell, both built-in and user-defined exceptions are used to signal error conditions. We can $throw$ and $catch$ exceptions in the FTM monad in the same way as the IO monad:

$$throw :: Exception\ e \Rightarrow e \rightarrow FTM\ a$$
$$catch :: Exception\ e \Rightarrow FTM\ a \rightarrow (e \rightarrow FTM\ a) \rightarrow FTM\ a$$

For instance, a TxForest user may define a new $FileNotFound$ exception and write the following pseudo-code:

$$
\begin{array}{l}
tryRead = \mathbf{do} \\
\quad \{exists \leftarrow ...find\ file\ ... \\
\quad ; \mathbf{if}\ (not\ exists)\ \mathbf{then}\ throw\ FileNotFound\ \mathbf{else}\ return\ () \\
\quad ; ...read\ file...\}
\end{array}
$$

If the file in question is not found, then a $FileNotFound$ exception is thrown, aborting the current $atomic$ block (and hence the file is never read). Programmers can prevent the transaction from being aborted, and its effects discarded, by catching exceptions inside the transaction, e.g.:

$$
\begin{array}{l}
catch\ tryRead \\
\quad (\lambda FileNotFound \rightarrow return\ ...\ default...)\ tryRead
\end{array}
$$

4.2 Transactional variables

We have seen how to build transactions from smaller transactional blocks, but we still haven't seen concrete operations to manipulate *shared data*, a fundamental piece of any transactional mechanism. In vanilla Haskell STM, communication between threads is done via shared mutable memory cells called *transactional variables*. For a transaction to log all memory effects, transactional variables can only be explicitly created, read from or written to using specific transactional operations. Nevertheless, Haskell programmers can traverse, query and manipulate the content of transactional variables using the rich language of purely functional computations; since these don't have side-effects, they don't ever need to be logged or rolled back.

In the context of TxForest, shared data is not stored in-memory, but instead on the filestore. It is illuminating to quote the STM paper [2]:

> "We study internal concurrency between threads interacting through memory [...]; we do not consider here the questions of external interaction through storage systems or databases."

We consider precisely the question of external interaction with a file system. Two transactions may communicate, e.g., by reading from or writing to the same file or possibly a list of files within a directory. To facilitate this interaction, the TxForest compiler generates an instance of the $TxForest$ type class (and corresponding types) for each Forest declaration:

$$\begin{array}{l}\textbf{class}\ \mathit{TxForest}\ \mathit{args}\ \mathit{ty}\ \mathit{rep}\mid \mathit{ty}\to \mathit{rep}, \mathit{ty}\to \mathit{args}\ \textbf{where}\\ \quad \mathit{new} \quad\quad\quad :: \mathit{args}\to \mathit{FilePath}\to \mathit{FTM}\ \mathit{fs}\ \mathit{ty}\\ \quad \mathit{read} \quad\quad\quad :: \mathit{ty}\to \mathit{FTM}\ \mathit{rep}\\ \quad \mathit{writeOrElse} :: \mathit{ty}\to \mathit{rep}\to b\\ \quad\quad\quad\quad\quad\quad \to (\mathit{Manifest}\to \mathit{FTM}\ \mathit{fs}\ b)\to \mathit{FTM}\ \mathit{fs}\ b\end{array}$$

In this signature, ty is an opaque transactional variable type that uniquely identifies a user-declared Forest type. Each transactional variables provides a window into the filesystem, shaped as a plain Haskell representation type rep. The representation type closely follows the declared Forest type, with additional file-content metadata for directories, files and symbolic links; directories have representations of type $(\mathit{FileInfo}, \mathit{dir\_rep})$ and basic types have representations of type $((\mathit{FileInfo}, \mathit{base\_md}), \mathit{base\_rep})$, for base representation $\mathit{base\_rep}$ and metadata $\mathit{base\_md}$.

Creation The transactional forest programming style makes no distinction between data on the file system and in-memory. Anywhere inside a transaction, users can declare a new transactional variable, with argument data pertaining to the forest declaration and rooted at the argument path in the file system. This operation does not have any effect on the file system and just establishes the schema to which a filestore should conform.

Reading Users can $read$ data from a filestore by reading the contents of a transactional variable. Imagine that we want to retrieve the balance of a particular account from a directory of accounts as specified in Figure **??**:

$$\begin{array}{l}\textbf{do}\\ \quad \mathit{accs} :: \mathit{Accounts}\leftarrow \mathit{new}\ ()\ \texttt{"/var/db/accounts"}\\ \quad (\mathit{accs\_info}, \mathit{accs\_rep})\leftarrow \mathit{read}\ \mathit{accs}\\ \quad \textbf{let}\ \mathit{acc1} :: \mathit{Account} = \mathit{fromJust}\ (\mathit{lookup}\ \texttt{"account1"}\ \mathit{accs\_rep})\\ \quad ((\mathit{acc1\_info}, \mathit{acc1\_md}), \mathit{Balance}\ \mathit{balance})\leftarrow \mathit{read}\ \mathit{acc1}\\ \quad \mathit{return}\ \mathit{balance}\end{array}$$

The corresponding generated Haskell functions and types appear in Figure **??**. In the background, this is done by lazily traversing the directories, files and symbolic links mentioned in the top-level forest description. The second line reads the account directory and generates a list of accounts, which can be manipulated with standard list operations to find the desired account. An account is itself a transactional variable, which can be read in the same way. Note that the file holding the balance of `"account1"` is only read in the fourth line. The type signatures elucidate the type of each transactional variable.

Programmers can control the degree of laziness in a forest description by adjusting the granularity of Forest declarations. For instance, if we have chosen to inline the type of $Account$ in the description as follows:

$$\begin{array}{l}[\mathit{forest}\mid\\ \quad \textbf{type}\ \mathit{Accounts} = [a :: \texttt{File}\ \mathit{Balance}\mid a\leftarrow \mathit{matches}\ (\mathit{GL}\ \texttt{"*"})]\\ \mid]\end{array}$$

Then reading the accounts directory would also read the file content of all accounts, since the balance of each account would not be encapsulated behind a transactional variable (as in Figure **??**.

Writing Users can modify a filestore by writing new content to a transactional variable. The $writeOrElse$ function accepts additional arguments to handle possible conflicts, which may arise due to data dependencies in the Forest description that cannot be statically checked by the type system. If these dependencies are not met, the data is not a valid representation of a filestore. If the write succeeds, the file system is updated with the new data and a default value of type b is returned. If the write fails, a user-supplied alternate function is executed instead. The function takes a $Manifest$ describing the tentative modifications to the file system and a report of the inconsistencies. We can easily define more convenient derived forms of $writeOrElse$:

$$\begin{array}{l}\quad \text{-- optional write}\\ \mathit{tryWrite} :: \mathit{TxForest}\ \mathit{args}\ \mathit{ty}\ \mathit{rep}\Rightarrow \mathit{ty}\to \mathit{rep}\to \mathit{FTM}\ ()\\ \mathit{tryWrite}\ t\ v = \mathit{writeOrElse}\ t\ v\ ()\ (\mathit{const}\ (\mathit{return}\ ()))\\ \quad \text{-- write or restart the transaction}\\ \mathit{writeOrRetry} :: \mathit{TxForest}\ \mathit{args}\ \mathit{ty}\ \mathit{rep}\Rightarrow \mathit{ty}\to \mathit{rep}\to ()\to \mathit{FTM}\ ()\\ \mathit{writeOrRetry}\ t\ v = \mathit{writeOrElse}\ t\ v\ ()\ (\mathit{const}\ \mathit{retry})\\ \quad \text{-- write or yield an error}\\ \mathit{writeOrThrow} :: (\mathit{TxForest}\ \mathit{args}\ \mathit{ty}\ \mathit{rep}, \mathit{Exception}\ e)\Rightarrow \mathit{ty}\to \mathit{rep}\to ()\\ \mathit{writeOrThrow}\ t\ v\ e = \mathit{writeOrElse}\ t\ v\ ()\ (\mathit{const}\ (\mathit{throw}\ e))\end{array}$$

A typical example of an inconsistent representation is when a Forest description refers to the same file more than once, to describe it in multiple ways, and the user attempts to write conflicting data in each occurrence. For instance, in Forest we may describe a symbolic link to an ASCII file both as a $SymLink$ and a $Text$ file:

$$\begin{array}{l}\textbf{type}\ \mathit{Folder} = \mathit{Directory}\ \{\\ \quad \{\mathit{link}\quad \mathit{is}\ \texttt{"README"} :: \mathit{SymLink}\\ \quad , \mathit{notes}\ \mathit{is}\ \texttt{"README"} :: \mathit{File}\ \mathit{Text}\\ \quad , \ldots\}\end{array}$$

Here, the file information for the $link$ and $notes$ fields must match.

Akin to reactive environments like spreadsheets [4], each write takes *immediate* effect on the (transactional snapshot of the) file system: an update on a variable is automatically propagated to the file system, eventually triggering the update of other variables dependent on common parts of the file system. We can observe this data flow by defining two accounts pointing to the same file and writing to one of them:

$$\begin{array}{l}\mathit{acc1} :: \mathit{Account}\leftarrow \mathit{new}\ ()\ \texttt{"/var/db/accounts/account"}\\ \mathit{acc2} :: \mathit{Account}\leftarrow \mathit{new}\ ()\ \texttt{"/var/db/accounts/account"}\\ (\mathit{acc\_md}, \mathit{Balance}\ \mathit{balance})\leftarrow \mathit{read}\ \mathit{acc2}\\ \mathit{tryWrite}\ \mathit{acc1}\ (\mathit{acc\_md}, \mathit{Balance}\ (\mathit{balance}+1))\\ (\mathit{acc\_md}', \mathit{Balance}\ \mathit{balance}')\leftarrow \mathit{read}\ \mathit{acc2}\end{array}$$

By incrementing the balance of $acc1$, we are implicitly incrementing the balance of $acc2$ (if the write succeeds, then $balance' = balance + 1$). Note that even if we attempt to write different balances to each variable, in sequence:

$$
\begin{array}{l}
tryWrite\ acc2\ (acc\_md, Balance\ 10) \\
tryWrite\ acc1\ (acc\_md, Balance\ 20) \\
(\_, Balance\ balance1) \leftarrow read\ acc1 \\
(\_, Balance\ balance2) \leftarrow read\ acc2
\end{array}
$$

it is always the case that $balance1 \equiv balance2 \equiv 20$, and there is no inconsistency since the first write propagates to both variables before the second write occurs.

4.3 Validation

As Forest lays a structured view on top of a semi-structured file system, a filestore does not need to conform perfectly to an associated Forest description. Behind the scenes, TxForest lazily computes a summary of such discrepancies. These may flag, for example, that a mandatory file does not exist or an arbitrarily complex user-defined Forest constraint is not satisfied. Validation is not performed unless explicitly demanded by the user. At any point, a user can $validate$ a transactional variable and its underlying filestore:

$$validate :: TxForest\ args\ ty\ rep \Rightarrow ty \rightarrow FTM\ ForestErr$$

The returned $ForestErr$ reports a top-level error count and the topmost error message:

$$
\begin{array}{l}
\mathbf{data}\ ForestErr = ForestErr \\
\quad \{ numErrors :: Int \\
\quad , errorMsg \quad :: Maybe\ ErrMsg \}
\end{array}
$$

We can always make validation mandatory and validation errors fatal by encapsulating any error inside a $ForestError$ exception:

$$
\begin{array}{l}
validRead :: TxForest\ args\ ty\ rep \Rightarrow ty \rightarrow FTM\ rep \\
validRead\ ty = \mathbf{do} \\
\quad rep \leftarrow read\ ty \\
\quad err \leftarrow validate\ ty \\
\quad \mathbf{if}\ numErrors\ err \equiv 0 \\
\quad\quad \mathbf{then}\ return\ rep \\
\quad\quad \mathbf{else}\ throw\ (ForestError\ err)
\end{array}
$$

4.4 Standard file system operations

To better understand the TxForest interface, we now discuss how to perform common operations on a Forest filestore.

Creation/Deletion Given that validation errors are not fatal, a $read$ always returns a representation. For example, if a user tries to read the balance of a non-existent account:

$$
\begin{array}{l}
\mathbf{do} \\
\quad badAcc :: Account \leftarrow new\ ()\ \texttt{"/var/db/accounts/account"} \\
\quad (acc\_info, Balance\ balance) \leftarrow read\ badAcc
\end{array}
$$

then acc_info will hold invalid file information and $balance$ a default value (implemented as 0 for Int values). Perhaps less intuitive is how to create a new account; we create a new variable (that if read would hold default data) and write new valid file information and some balance:

$$
\begin{array}{l}
newAccount\ path\ balance = \mathbf{do} \\
\quad newAcc :: Account \leftarrow new\ ()\ path \\
\quad tryWrite\ newAcc\ (validFileInfo\ path, Balance\ balance)
\end{array}
$$

Deleting an account is dual to creating one; we write invalid file information and the default balance to the corresponding variable:

$$
\begin{array}{l}
delAccount\ acc = \mathbf{do} \\
\quad tryWrite\ acc\ (invalidFile, Balance\ 0)
\end{array}
$$

The takeaway lesson is that the $FileInfo$ metadata actually determines whether a directory, file or symbolic link exists or not in the file system, since we cannot infer that from the data alone (e.g., an empty account has the same balance as a non-existent account). This also reveals less obvious data dependencies: For valid paths the $fullpath$ in the metadata must match the path to which the representation corresponds in the description, and for invalid paths the representation data must match the Forest-generated default data. Since this can become cumbersome to ensure manually, we provide a general function that conveniently removes a filestore, named after the POSIX `rm` operation:

$$rm :: TxForest\ args\ ty\ rep \Rightarrow ty \rightarrow FTM\ ()$$

Copying A user can copy an account from a source path to a target path as follows:

$$
\begin{array}{l}
copyAccount\ srcpath\ tgtpath = \mathbf{do} \\
\quad src :: Account \leftarrow new\ ()\ srcpath \\
\quad tgt :: Account \leftarrow new\ ()\ tgtpath \\
\quad (info, balance) \leftarrow read\ src \\
\quad tryWrite\ tgt\ (info\ \{fullpath = tgtpath\}, balance)
\end{array}
$$

The pattern is to create a variable for each path, and copy the content with an updated $fullpath$. Copying a directory of accounts follows the same pattern but is more complicated, in that we also have to recursively copy underlying accounts and update all the metadata accordingly. Therefore, we provide an analogue to the POSIX `cp` operation that attempts to copy the content of a representation into another:

$$
\begin{array}{l}
cpOrElse\ ::\ TxForest\ args\ ty\ rep \Rightarrow ty \rightarrow ty \rightarrow b \\
\qquad\qquad \rightarrow (Manifest \rightarrow rep \rightarrow FTM\ fs\ b) \rightarrow FTM\ fs\ b
\end{array}
$$

Unlike rm, $copyOrElse$ is only a best-effort operation that may fail due to arbitrarily complex data dependencies in the Forest description. Such dependencies necessarily hold in the source representation for the source arguments but may not for the target arguments. Similarly to $writeOrElse$, we provide $tryCopy$, $copyOrRetry$ and $copyOrThrow$ operations with the expected type signatures.

For an example of what might go wrong while copying, consider the following description for accounts parameterized by a template name:

$$
\begin{array}{l}
[forest\ | \\
\quad \mathbf{type}\ NamedAccounts\ (acc :: String) = \\
\quad\quad [a :: Account\ |\ a \leftarrow matches\ (GL\ (acc +\!\!+ \texttt{"*"}))] \\
\ |]
\end{array}
$$

This specification has an implicit data dependency that all the account files listed in the in-memory representation have names matching the Glob pattern. Thus, trying to copy between filestores with different templates would effectively fail, as in:

$$
\begin{array}{l}
\mathbf{do} \\
\quad src :: Accounts \leftarrow new\ \texttt{"account"}\ \texttt{"/var/db/accounts"} \\
\quad tgt :: Accounts \leftarrow new\ \texttt{"acc"}\ \texttt{"/var/db/accs"} \\
\quad tryCopy\ src\ tgt
\end{array}
$$

5. Implementation

We now delve into how Transactional Forest can be efficiently implemented. The current implementation is available from the project website (`forestproj.org`) and is done completely in Haskell. We split our presentation into three possible designs, with increasing levels of incremental support and complexity.

5.1 Transactional Forest

Original STM interface We have implemented TxForest as a domain-specific variant of `STM` Haskell [2], and inherit the same transactional mechanism based on *optimistic concurrency control*: each transaction runs in a (possibly) different thread and keeps a thread-local log of reads and writes (including the tentatively-written data) to *shared resources*, and reads within a transaction first consult its log so that they see preceding writes. Once finished, each transaction validates its log against previous transactions that committed before its starting time and, only if no write-read conflicts are detected, commits its writes permanently; otherwise, it is re-executed. These validate-and-commit operations are guaranteed to run $atomic$ally in respect to all other threads by acquiring per-shared-resource locks according to a global total order (no locks are used during the transaction's execution): the transaction waits on the sorted sequence of read resources to be free (to ensure that it sees the commits of concurrently writing transactions) and acquires the sorted sequence of written resources. They are *disjoint-access parallel* (meaning that transactions with non-overlapping writes run in parallel) and *read parallel* (meaning that transactions that only read from the same resources run in parallel).

Blocking transactions ($retry$) validate their log and register themselves in wait-queues attached to each read address; updating transactions unblock any pending waiters. Nested transactions ($orElse$) work similarly to normal transactions: writes are recorded only to a nested log and reads consult all the logs of the nested and enclosing transactions. Validation a nested transaction also implies validating all enclosing transactions. Exceptional transactions ($throw$) must also validate the log before raising an exception to the outside world; on success, they rollback all modifications except for new thread-local memory allocations (to consistently handle exceptions that carry information created inside the transaction). If the first alternative retries, then the second alternative is attempted; if both retry, then both logs are validated and the thread will wait on the union of the read resources. A more detailed account, including a complete formal semantics, is given in [2].

Transaction logs The main difference from `STM` Haskell to TxForest is that the shared resources are not mutable memory cells in the traditional sense, but paths in the file system. [2] This is to say that, although users manipulate structured representations of filestores, all the in-memory data structures are local to each transaction, and only file system operations need to be logged. The concurrent handling of file paths, however, is subtle in the presence of symbolic links –the identity of a path is not unique (as different paths may refer to the same real path) nor stable (since the real path depends on the current symbolic link configuration)– making it harder to identify conflicts between transactions and to properly lock resources. For example, one transaction may read a file whose path is concurrently modified by other transaction. Therefore, our transaction logs keep special track of symbolic link modifications and we perform all file operations over "canonical" file paths, calculated against the transaction log while marking each resolved link as read.

Round-tripping functions In TxForest, each transactional variable is an in-memory data structure that reflects the content of a particular filestore, declared from the respective Forest description type with given arguments and root path. Behind the scenes, the transactional engine is responsible for preserving the abstraction, and keeping each variable "in sync" with the latest transactional snapshot of the file system, such that changes on the file system are propagated to the affected in-memory filestore variables, and writes to variables move the file system snapshot forward. This task is performed by a coupled pair of $load$ and $store$ functions. Their precise definitions and formal semantics is given in Appendix A. Informally, each Forest variable is implemented as a "thunk" that lazily computes visible data (denoting the content and metadata of the filestore) and hidden data (remembering errors during validation). The $load$ function for a top-level variable generates a top-level thunk that, once evaluated, recursively reads data from the (transactional snapshot of the) filesystem, building a thunk for each level of structure. Error information is computed behind an additional thunk to guarantee that validation is only performed when explicitly demanded by the user. The $store$ function strictly traverses a nested structure of in-memory thunks and updates the (transactional snapshot of the) file system to reflect the same content, returning an additional $validator$ function that tests the updated file system for inconsistencies during storing. These two functions are carefully designed so that they preserve data on round trips: loading a filestore and immediately storing it back always succeeds and keeps the file system unchanged; and storing succeeds as long as loading the updated file system yields the same in-memory representation.

Transactional variables In TxForest, each transaction keeps a local file system snapshot with a unique thread-local version number and a log of tentative updates over the real file system. When users create a new transactional variable, the $load$ function is called to create the corresponding thunk. Note that, due to laziness, no data is actually loaded. These thunks, acting as transactional variables, can be concurrently accessed by multiple transactions. When a transaction $read$s a transactional variable, the associated level of data is loaded from the current file system snapshot, and the resulting value is added to a (per-transaction) per-variable memoization table mapping file system versions to values. We implement them as weak hash tables, allowing the Haskell garbage collector to purge entries for versions other than the current one.[3] A call to $writeOrElse$ starts by making a copy of the current file system snapshot and adding an entry the the memoization table of the respective variable mapping a newly generated unique file system version to the newly written value. It then invokes the $store$ function (remembering the creation-time arguments and root path) to update the file system log (under the new version) and runs the resulting validator; on inconsistencies, the transaction rolls back to the backed up file system snapshot and executes the user-supplied alternative action instead.

5.2 Incremental Transactional Forest

We have described all the components for a complete implementation of TxForest, but not a very efficient one. To understand its limitations, imagine that a transaction maintains two completely unrelated variables and reads the first, writes new content to the second, and reads the first again. Since the two reads occur between a file system change, our simple memoization mechanism will fail, and the same content will be redundantly loaded from the file system twice. Even worse, writes in TxForest force a deep evaluation of the in-memory filestore, compromising the convenient laziness properties of the runtime system. For example, if a transaction reads a directory variable and immediately writes the read value to the same variable, the underlying $store$ function will strictly traverse the filestore to redundantly overwrite sub- files and directories with their old content, even though nothing has actually changed.

Round-tripping functions The problem in both examples is that the $load$ and $store$ functions used by the runtime system are agnostic to modifications on the file system or on the in-memory

[2] STM maintains a log with the old value held in a memory cell and the new value written to it by the transaction, and validation test if they are pointer-equal. We do not remember old content of file paths, nor test for equality.

[3] Operations that support rollback, like $writeOrElse$ and $orElse$, also require preserving entries for the backed up file system version.

data, and execute from scratch every time with a running "footprint" proportional to the size of the Forest description. Being Forest an embedded DSL in Haskell, we can exploit domain-specific knowledge to design incremental round-tripping functions, intuitively named $load_\triangle$ and $store_\triangle$, with "footprint" proportional to the size of the actual updates. Especially since transaction are already equipped with the machinery to keep logs of modifications, we can extend our runtime system to make use of the incremental functions in place of their non-incremental counterparts. The formal ingredients for an incremental Forest load/store semantics are developed in Appendix B.

File system updates fs log keeps all the changes over the real fs. we need to know the changes between particular tx-local fs versions.

the key to any IC algorithm is to exploit locality in the updates to update only structures affected by the update

the fact that the filesystem is a graph, due to symlinks, brings additional chanllenges for incremental algorithms. because although the FS is a graph, we can't have it efficiently materialized, without traversing the whole FS. with symlinks, a change in a remote part of the FS tree may eventually affect, in non-obvious ways, another branch in the tree. therefore, FS deltas are algo graphs. but like the FS graph, we can't materialize them without traversing the full FS.

the runtime keeps a global record of all the symbolic links found under a root path that declares the subtree of the file system over which transactions conventionally operate. this table can be computed once, when booting the runtime system, and maintained by running transactions. although finding symlinks is an expensive task that involves traversing the whole file system tree, this initial cost is less relevant if we assume the database-centric model, in which the runtime system operates as an always-on server that accepts txs issued by client processes.

Filestore updates

Transactional variables read computes a diff and calls loadDelta, write computes a diff and calls storeDelta

forest specs "share" the whole FS, so its normal for them to interfere with one another. problem with 1st approach: ic loading: some change occurs between two reads, for instance, two completely unrelated variables; read spec1, write spec2, read spec1 (our simple cache mechanism fails to prevent recomputation)

exploit DSL information to have incrementality; intra-transaction

5.3 Log-structured Transactional Forest

problem with 2nd approach: tx1 reads a variable; tx2 reads the same variable

exploit (DSL info +) FS support to have incrementality

read-only transactions require no synchronization

6. Evaluation

although Haskell is a great language laboratory, we are already paying a severe performance overhead if efficiency is the only concern.

even the Haskell STM is implemented in C

7. Related Work

transactional filesystems (user-space vs kernel-space) `http://www.fuzzy.cz/en/articles/transactional-file-systems`
`http://www.fsl.cs.sunysb.edu/docs/valor/valor_fast2009.pdf`
`http://www.fsl.cs.sunysb.edu/docs/amino-tos06/amino.pdf`

libraries for transactional file operations: `http://commons.apache.org/proper/commons-transaction/file/index.html`
`https://xadisk.java.net/`
`https://transactionalfilemgr.codeplex.com/`

tx file-level operations (copy,create,delete,move,write) schema somehow equivalent to using the unstructured universal Forest representation

but what about data manipulation: transactional maps,etc?

8. Conclusions

transactional variables do not descend to the content of files. pads specs are read/written in bulk. e.g., append line to log file. extend pads.

References

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A. Forest Semantics

$$F^*(r \,/\, u) = \begin{cases} F^*(r') & \text{if } F(F^*(r) \,/\, u) = (i, \texttt{Link}\ r') \\ F^*(r) \,/\, u & \text{otherwise} \end{cases}$$
$$F^*(\cdot) = \cdot$$

$$\frac{}{r \in \cdot} \qquad \frac{}{r \in r} \qquad \frac{r \in r'}{r \,/\, u \in r'}$$

$$F{\searrow}r \triangleq F|_{\{\forall\, r'.\ F^*(r') \in r\}}$$

$$r_1 \in^*_F r_2 = \forall\, r \in r_1.\ F^*(r) \in F^*(r_2)$$

$F \underset{rs}{=} F' = \forall\, r \in rs.\ F{\searrow}r = F'{\searrow}r$

$\mathit{Err}\ a = (\mathsf{M}\ \mathit{Bool}, a)$

s	$\mathcal{R}[\![s]\!]$	$\mathcal{C}[\![s]\!]$
$\mathsf{M}\,s$	$\mathsf{M}\,(\mathit{Err}\ (\mathcal{R}[\![s]\!]))$	$\mathsf{M}\,(\mathit{Err}\ (\mathcal{R}[\![s]\!]))$
$k^{\tau_2}_{\tau_1}$	$\mathit{Err}\ (\tau_2, \tau_1)$	(τ_2, τ_1)
$e :: s$	$\mathcal{R}[\![s]\!]$	$\mathcal{C}[\![s]\!]$
$\langle x : s_1, s_2\rangle$	$\mathit{Err}\ (\mathcal{R}[\![s_1]\!], \mathcal{R}[\![s_2]\!])$	$(\mathcal{C}[\![s_1]\!], \mathcal{C}[\![s_2]\!])$
$\{s \mid x \in e\}$	$\mathit{Err}\ [\mathcal{R}[\![s]\!]]$	$[\mathcal{C}[\![s]\!]]$
$\mathsf{P}\,(e)$	$\mathit{Err}\ ()$	$()$
$s?$	$\mathit{Err}\ (\mathit{Maybe}\ (\mathcal{R}[\![s]\!]))$	$\mathit{Maybe}\ (\mathcal{C}[\![s]\!])$

$\mathcal{R}[\![\cdot]\!]$ is the internal in-memory representation type of a forest declaration; $\mathcal{C}[\![\cdot]\!]$ is the external type of content of a variables that users can inspect/modify

$\mathit{err}(a) = \texttt{do}\ \{ e \leftarrow \texttt{get}\ a; (a_{err}, v) \leftarrow e; \texttt{return}\ a_{err} \}$
$\mathit{err}(a_{err}, v) = \texttt{return}\ a_{err}$
$\mathit{valid}(v) = \texttt{do}\ \{ a_{err} \leftarrow \mathit{err}\ v; e_{err} \leftarrow \texttt{get}\ a_{err}; e_{err} \}$

$v_1\ {}_{\Theta_1}{\sim}_{\Theta_2}\ v_2$ denotes value equivalence modulo memory addresses, under the given environments. $e_1\ {}_{\Theta_1}{\sim}_{\Theta_2}\ e_2$ denotes expression equivalence by evaluation modulo memory addresses, under the given environments.

$v_1\ {}_{\Theta_1}\overset{\text{err}}{\sim}_{\Theta_2}\ v_2$ denotes value equivalence (ignoring error information) modulo memory addresses, under the given environments.

$\boxed{\Theta; \varepsilon; r; s \vdash \texttt{load}\ F \Rightarrow \Theta'; v}$ "Under heap Θ and environment ε, load the specification s for filesystem F at path r and yield a representation v."

$\boxed{s = \mathsf{M}\,s_1}$

$$\frac{a \notin \texttt{dom}(\Theta) \quad a_{err} \notin \texttt{dom}(\Theta) \quad e = \varepsilon; r; \mathsf{M}\,s \vdash \texttt{load}\ F \qquad e_{err} = \texttt{do}\ \{ e_1 \leftarrow \texttt{get}\ a; v_1 \leftarrow e_1; \mathit{valid}\ v_1 \}}{\Theta; \varepsilon; r; \mathsf{M}\,s \vdash \texttt{load}\ F \Rightarrow \Theta[a_{err} : e_{err}, a : e]; (a_{err}, a)}$$

$\boxed{s = k}$

$$\frac{a_{err} \notin \texttt{dom}(\Theta) \quad \Theta; \texttt{load}_k(\varepsilon, F, r) \Rightarrow \Theta'; (b, v)}{\Theta; \varepsilon; r; k \vdash \texttt{load}\ F \Rightarrow \Theta'[a_{err} : \texttt{return}\ b]; (a_{err}, v)}$$

$$\texttt{load}_{\texttt{File}}(\varepsilon, F, r) \begin{cases} \texttt{return}\ (\mathit{True}, (i, u)) & \text{if } F(r) = (i, \texttt{File}\ u) \\ \texttt{return}\ (\mathit{False}, (i_{\texttt{invalid}}, \texttt{""})) & \text{otherwise} \end{cases}$$

$$\texttt{load}_{\texttt{Dir}}(\varepsilon, F, r) \begin{cases} \texttt{return}\ (\mathit{True}, (i, us)) & \text{if } F(r) = (i, \texttt{Dir}\ us) \\ \texttt{return}\ (\mathit{False}, (i_{\texttt{invalid}}, \{\,\})) & \text{otherwise} \end{cases}$$

$$\texttt{load}_{\texttt{Link}}(\varepsilon, F, r) \begin{cases} \texttt{return}\ (\mathit{True}, (i, r')) & \text{if } F(r) = (i, \texttt{Link}\ r') \\ \texttt{return}\ (\mathit{False}, (i_{\texttt{invalid}}, \cdot)) & \text{otherwise} \end{cases}$$

$\boxed{s = e :: s_1}$

$$\frac{\Theta; [\![r \,/\, e]\!]^{\varepsilon}_{\mathit{Path}} \Rightarrow \Theta'; r' \quad \Theta; \varepsilon; r'; s \vdash \texttt{load}\ F \Rightarrow \Theta''; v}{\Theta; \varepsilon; r; e :: s \vdash \texttt{load}\ F \Rightarrow \Theta''; v}$$

$\boxed{s = \langle x : s_1, s_2\rangle}$

$$\frac{\begin{array}{c}\Theta;\varepsilon;r;s_1 \vdash \mathtt{load}\ F \Rightarrow \Theta_1; v_1\\ \Theta_1;\varepsilon[x\mapsto v_1];r;s_2 \vdash \mathtt{load}\ F \Rightarrow \Theta_2; v_2\\ e_{err} = \mathtt{do}\ \{\, b_1 \leftarrow \mathit{valid}(v_1); b_2 \leftarrow \mathit{valid}(v_2); \mathtt{return}\ (b_1 \wedge b_2)\,\}\end{array}}{\Theta;\varepsilon;r;\langle x : s_1, s_2\rangle \vdash \mathtt{load}\ F \Rightarrow \Theta_2[a_{err} : e_{err}]; (a_{err}, (v_1, v_2))}$$

$\boxed{s = \mathsf{P}\ e}$

$$\frac{a_{err} \notin \mathtt{dom}(\Theta)}{\Theta;\varepsilon;r;\mathsf{P}\ e \vdash \mathtt{load}\ F \Rightarrow \Theta[a_{err} : [\![e]\!]^{\varepsilon}_{Bool}]; (a_{err}, ())}$$

$\boxed{s = s_1?}$

$$\frac{r \notin \mathtt{dom}(F) \quad a_{err} \notin \mathtt{dom}(\Theta)}{\Theta;\varepsilon;r;s? \vdash \mathtt{load}\ F \Rightarrow \Theta[a_{err} : \mathtt{return}\ \mathit{True}]; (a_{err}, \mathit{Nothing})}$$

$$\frac{r \in \mathtt{dom}(F) \quad a_{err} \notin \mathtt{dom}(\Theta') \quad \Theta;\varepsilon;r;s \vdash \mathtt{load}\ F \Rightarrow \Theta'; v}{\Theta;\varepsilon;r;s? \vdash \mathtt{load}\ F \Rightarrow \Theta[a_{err} : \mathit{valid}(v)]; (a_{err}, \mathit{Just}\ v)}$$

$\boxed{s = \{s_1 \mid x \in e\}}$

$$\frac{\begin{array}{c}a_{err} \notin \mathtt{dom}(\Theta) \quad \Theta; [\![e]\!]^{\varepsilon}_{\{\tau\}} \Rightarrow \Theta'; \{\, t_1, .., t_k\,\}\\ \Theta'; \forall\, i \in \{1,...,k\}.\ \mathtt{do}\ \{v_i \leftarrow \varepsilon[x \mapsto t_i]; r; s \vdash \mathtt{load}\ F; \mathtt{return}\ \{t_i \mapsto v_i\}\} \Rightarrow \Theta''; \mathit{vs}\\ e_{err} = \forall\, i \in \{1,...,k\}.\ \mathtt{do}\ \{\, b_i \leftarrow \mathit{valid}(\mathit{vs}(t_i)); \mathtt{return}\ (\bigwedge\ b_i)\,\}\end{array}}{\Theta;\varepsilon;r;\{s \mid x \in e\} \vdash \mathtt{load}\ F \Rightarrow \Theta''[a_{err} : e_{err}]; (a_{err}, \mathit{vs})}$$

$\boxed{\Theta;\varepsilon;r;s \vdash \mathtt{store}\ F\ v \Rightarrow \Theta'; (F', \phi')}$ "Under heap Θ and environment ε, store the representation v for the specification s on filesystem F at path r and yield an updated filesystem F' and a validation function ϕ'."

$\boxed{s = \mathsf{M}\, s_1}$

$$\frac{\begin{array}{c}\Theta(a) = e \quad \Theta; e \Rightarrow \Theta'; (a_{err}, v)\\ \Theta';\varepsilon;r;s \vdash \mathtt{store}\ F\ v \Rightarrow \Theta''; (F', \phi')\end{array}}{\Theta;\varepsilon;r;\mathsf{M}\, s \vdash \mathtt{store}\ F\ a \Rightarrow \Theta''; (F', \phi')}$$

$\boxed{s = k}$

$$\frac{\Theta; \mathtt{store}_k(\varepsilon, F, r, (d, v)) \Rightarrow \Theta'; (F', \phi)}{\Theta;\varepsilon;r;k \vdash \mathtt{store}\ F\ (a_{err}, (d, v)) \Rightarrow \Theta'; (F', \phi)}$$

$$\mathtt{store}_{\mathtt{File}}(\varepsilon, F, r, (i, u)) \begin{cases} \mathtt{return}\ (F[r := (i, \mathtt{File}\ u)], \lambda F'.\ F'(r) = (i, \mathtt{File}\ u)) & \text{if } i \neq i_{\mathtt{invalid}}\\ \mathtt{return}\ (F[r := \bot], \lambda F'.\ F'(r) \neq (\_, \mathtt{File}\ \_)) & \text{if } i = i_{\mathtt{invalid}} \wedge F(r) = (\_, \mathtt{File}\ \_)\\ \mathtt{return}\ (F, \lambda F'.\ F'(r) \neq (\_, \mathtt{File}\ \_)) & \text{if } i = i_{\mathtt{invalid}} \wedge F(r) \neq (\_, \mathtt{File}\ \_)\end{cases}$$

$$\mathtt{store}_{\mathtt{Dir}}(\varepsilon, F, r, (i, \{\, u_1, ..., u_n\,\})) \begin{cases} \mathtt{return}\ (F[r := (i, \mathtt{Dir}\ \{\, u_1, ..., u_n\,\})], \lambda F'.\ F'(r) = (i, \mathtt{Dir}\ \{\, u_1, ..., u_n\,\})) & \text{if } i \neq i_{\mathtt{invalid}}\\ \mathtt{return}\ (F[r := \bot], \lambda F'.\ F'(r) \neq (\_, \mathtt{Dir}\ \_)) & \text{if } i = i_{\mathtt{invalid}} \wedge F(r) = (\_, \mathtt{Di}\\ \mathtt{return}\ (F, \lambda F'.\ F'(r) \neq (\_, \mathtt{Dir}\ \_)) & \text{if } i = i_{\mathtt{invalid}} \wedge F(r) \neq (\_, \mathtt{Di}\end{cases}$$

$$\mathtt{store}_{\mathtt{Link}}(\varepsilon, F, r, (i, r')) \begin{cases} \mathtt{return}\ (F[r := (i, \mathtt{Link}\ r')], \lambda F'.\ F'(r) = (i, \mathtt{Link}\ r')) & \text{if } i \neq i_{\mathtt{invalid}}\\ \mathtt{return}\ (F[r := \bot], \lambda F'.\ F'(r) \neq (\_, \mathtt{Link}\ \_)) & \text{if } i = i_{\mathtt{invalid}} \wedge F(r) = (\_, \mathtt{Link}\ \_)\\ \mathtt{return}\ (F, \lambda F'.\ F'(r) \neq (\_, \mathtt{Link}\ \_)) & \text{if } i = i_{\mathtt{invalid}} \wedge F(r) \neq (\_, \mathtt{Link}\ \_)\end{cases}$$

$\boxed{s = e :: s_1}$

$$\frac{\begin{array}{c}\Theta; [\![e]\!]^{\varepsilon}_{Path} \Rightarrow \Theta'; r'\\ \Theta';\varepsilon;r';s \vdash \mathtt{store}\ F\ v \Rightarrow \Theta''; (F', \phi')\end{array}}{\Theta;\varepsilon;r;e :: s \vdash \mathtt{store}\ F\ v \Rightarrow \Theta''; (F', \phi')}$$

$\boxed{s = \langle x : s_1, s_2\rangle}$

$$\frac{\begin{array}{c}\Theta;\varepsilon;r;s_1 \vdash \mathtt{store}\ F\ v_1 \Rightarrow \Theta_1; (F_1, \phi_1)\\ \Theta_1;\varepsilon[x \mapsto v_1];r;s_2 \vdash \mathtt{store}\ F\ v_2 \Rightarrow \Theta_2; (F_2, \phi_2)\\ \phi = \lambda F'.\ \phi_1(F') \wedge \phi_2(F')\end{array}}{\Theta;\varepsilon;r;\langle x : s_1, s_2\rangle \vdash \mathtt{store}\ F\ (a_{err}, (v_1, v_2)) \Rightarrow \Theta_2; (F_1 +\!\!+ F_2, \phi)}$$

$\boxed{s = \mathsf{P}\ e}$

$$\frac{\phi = \lambda F'.\ \mathit{True}}{\Theta; \varepsilon; r; \mathtt{P}\ e \vdash \mathtt{store}\ F\ (a_{err}, ()) \Rightarrow \Theta; (F, \phi)}$$

$\boxed{s = s_1?}$

$$\frac{\phi = \lambda F'.\ r \notin \mathtt{dom}(F')}{\Theta; \varepsilon; r; s? \vdash \mathtt{store}\ F\ (a_{err}, \mathit{Nothing}) \Rightarrow \Theta; (F[r := \bot], \phi)}$$

$$\frac{\Theta; \varepsilon; r; s \vdash \mathtt{store}\ F\ v \Rightarrow \Theta'; (F_1, \phi_1) \quad \phi = \lambda F'.\ \phi_1(F') \wedge r \in \mathtt{dom}(F')}{\Theta; \varepsilon; r; s? \vdash \mathtt{store}\ F\ (a_{err}, \mathit{Just}\ v) \Rightarrow \Theta; (F_1, \phi)}$$

$\boxed{s = \{ s_1 \mid x \in e \}}$

$$\frac{\Theta; [\![e]\!]^{\varepsilon}_{\{\tau\}} \Rightarrow \Theta'; ts \quad vs = \{ t_1 \mapsto v_1, ..., t_k \mapsto v_k \} \quad \phi = \lambda F'.\ ts = \{ t_1, ..., t_k \} \wedge \bigwedge\ \phi_i(F') \quad \Theta'; \forall\, i \in \{1, ..., k\}.\ \mathtt{do}\ \{ (F_i, \phi_i) \leftarrow \varepsilon[x \mapsto v_i]; r; s \vdash \mathtt{store}\ F\ v_i; \mathtt{return}\ (F_1 +\!\!+ \ldots +\!\!+ F_k, \phi) \} \Rightarrow \Theta''; F'\ \phi'}{\Theta; \varepsilon; r; \{ s \mid x \in e \} \vdash \mathtt{store}\ F\ (a_{err}, vs) \Rightarrow \Theta; (F', \phi')}$$

Proposition 1 (Load Type Safety). *If* $\Theta; \varepsilon; r; s \vdash \mathtt{load}\ F \Rightarrow \Theta'; v'$ *and* $\mathcal{R}[\![s]\!] = \tau$ *then* $\vdash v : \tau$.

Theorem A.1 (LoadStore). *If*

$$\Theta; \varepsilon; r; s \vdash \mathtt{load}\ F \Rightarrow \Theta'; v$$
$$\Theta''; \varepsilon; r; s \vdash \mathtt{store}\ F\ v' \Rightarrow \Theta'''; (F', \phi')$$
$$v \ {}_{\Theta'}\!\overset{err}{\sim}_{\Theta''}\ v'$$

then $F = F'$ *and* $\phi'(F')$.

Theorem A.2 (StoreLoad). *If*

$$\Theta; \varepsilon; r; s \vdash \mathtt{store}\ F\ v \Rightarrow \Theta'; (F', \phi')$$
$$\Theta'; \varepsilon; r; s \vdash \mathtt{load}\ F \Rightarrow \Theta''; v'$$

then $\phi'(F')$ *iff* $v \ {}_{\Theta'}\!\overset{err}{\sim}_{\Theta''}\ v'$

stronger than the original forest theorem: store validation only fails for impossible cases (when representation cannot be stored to the FS without loss)

weaker in that we don't track consistency of inner validation variables; equality of the values is modulo error information. in a real implementation we want to repair error information on storing, so that it is consistent with a subsequent load.

the error information is not stored back to the FS, so the validity predicate ignores it.

B. Forest Incremental Semantics

Note that:

- We have access to the old filelesystem, since filesystem deltas record the changes to be performed.
- We do not have access to the old environment, since variable deltas record the changes that already occurred.

$\delta_F ::= \mathtt{addFile}(r, u) \mid \mathtt{addDir}(r) \mid \mathtt{addLink}(r, r') \mid \mathtt{rem}(r) \mid \mathtt{chgAttrs}(r, i) \mid \delta_{F_1}; \delta_{F_2} \mid \emptyset$

$\delta_v ::= \mathsf{M}_{\delta_a}\ \delta_{v_1} \mid \delta_{v_1} \otimes \delta_{v_2} \mid \{ t_i \mapsto \delta_{\bot v_i} \} \mid \delta_{v_1}? \mid \emptyset \mid \Delta$
$\delta_{\bot v} ::= \bot \mid \delta_v$

$\Delta_v ::= \emptyset \mid \Delta$

$\mathtt{addFile}(r', u) \searrow_F r \triangleq$ **if** $r' \in^*_F r$ **then** $\mathtt{addFile}(r', u)$ **else** $\emptyset$
$\mathtt{addDir}(r') \searrow_F r \triangleq$ **if** $r' \in^*_F r$ **then** $\mathtt{addDir}(r')$ **else** $\emptyset$
$\mathtt{addLink}(r', r'') \searrow_F r \triangleq$ **if** $r' \in^*_F r$ **then** $\mathtt{addLink}(r', r'')$ **else** $\emptyset$
$\mathtt{rem}(r') \searrow_F r \triangleq$ **if** $r' \in^*_F r$ **then** $\mathtt{rem}(r')$ **else** $\emptyset$
$\mathtt{chgAttrs}(r', i) \searrow_F r \triangleq$ **if** $r' \in^*_F r$ **then** $\mathtt{chgAttrs}(r', i)$ **else** $\emptyset$
$(\delta_{F_1}; \delta_{F_2}) \searrow_F r \triangleq \delta_{F_1} \searrow_F r; \delta_{F_2} \searrow_{F_1} r$ **where** $F_1 = (\delta_{F_1} \searrow_F r)\ F$
$\emptyset \searrow_F r \triangleq \emptyset$

$\Theta; v \xrightarrow{\delta_v} \Theta'; v'$

the value delta maps v to v'

monadic expressions only read from the store and perform new allocations; they can't modify existing addresses.

For any expression application $e\ \Theta = (\Theta', v)$, we have $\Theta = \Theta \cap \Theta'$.

errors are computed in the background

$$\frac{a' \notin \mathtt{dom}(\Theta)}{\Theta; \delta_a; \Delta_e \vdash a : e \Rightarrow \Theta[a' : e]; (a', \Delta)} \quad \frac{}{\Theta; \emptyset; \Delta_e \vdash a : e \Rightarrow \Theta[a : e]; (a, \Delta)} \quad \frac{}{\Theta; \emptyset; \emptyset \vdash a : e \Rightarrow \Theta; (a, \emptyset)}$$

$\boxed{\Theta; \varepsilon; \Delta_\varepsilon; r; s \vdash \mathtt{load}_\Delta\ F\ v\ \delta_F\ \delta_v \Rightarrow \Theta'; (v', \Delta'_v)}$ "Under heap Θ, environment ε and delta environment Δ_ε, incrementally load the specification s for the original filesystem F and original representation v, given filesystem changes δ_F and representation changes δ_v, to yield an updated representation v' with changes Δ'_v.

$$\frac{\Delta_\varepsilon|_{fv(s)} = \emptyset \quad \delta_F \searrow_F r = \emptyset}{\Theta; \varepsilon; \Delta_\varepsilon; r; s \vdash \mathtt{load}_\Delta\ F\ v\ \delta_F\ \emptyset \Rightarrow \Theta; (v, \emptyset)}$$

$$\frac{\Theta; \varepsilon; r; s \vdash \mathtt{load}\ (\delta_F\ F) \Rightarrow \Theta'; v'}{\Theta; \varepsilon; \Delta_\varepsilon; r; s \vdash \mathtt{load}_\Delta\ F\ v\ \delta_F\ \delta_v \Rightarrow \Theta'; (v', \Delta)}$$

$\boxed{s = \mathsf{M}\, s_1}$

$$\frac{\begin{array}{c}\Theta(a) = e \quad \Theta; e \Rightarrow \Theta'; (a_{err}, v) \\ \Theta'; \varepsilon; \Delta_\varepsilon; r; s \vdash \mathtt{load}_\Delta\ F\ v\ \delta_F\ \delta_v \Rightarrow \Theta''; (v', \Delta_v) \quad v = v'\end{array}}{\Theta; \varepsilon; \Delta_\varepsilon; r; \mathsf{M}\, s \vdash \mathtt{load}_\Delta\ F\ a\ \delta_F\ (\mathsf{M}_\emptyset\ (\emptyset \otimes \delta_v)) \Rightarrow \Theta''; (a, \emptyset)}$$

$$\frac{\begin{array}{c}\Theta(a) = e \quad \Theta; e \Rightarrow \Theta'; (a_{err}, v) \\ \Theta'; \varepsilon; \Delta_\varepsilon; r; s \vdash \mathtt{load}_\Delta\ F\ v\ \delta_F\ \delta_v \Rightarrow \Theta_1; (v', \Delta_v) \\ \Theta_1; \delta_{a_{err}}; \Delta_v \vdash a_{err} : \mathit{valid}\ v' \Rightarrow \Theta_2; (a'_{err}, \Delta_{a_{err}}) \\ \Theta_2; \delta_a; \Delta_{a_{err}} \vdash a : \mathtt{return}\ (a'_{err}, v') \Rightarrow \Theta_3; (a', \Delta_a)\end{array}}{\Theta; \varepsilon; \Delta_\varepsilon; r; \mathsf{M}\, s \vdash \mathtt{load}_\Delta\ F\ a\ \delta_F\ (\mathsf{M}_{\delta_a}\ (\delta_{a_{err}} \otimes \delta_v)) \Rightarrow \Theta_3; (a', \Delta_a)}$$

$\boxed{s = e :: s_1}$

$$\frac{\begin{array}{c}\Delta_\varepsilon|_{fv(s)} = \emptyset \quad \Theta; [\![r\ /\ e]\!]^{\varepsilon}_{Path} \Rightarrow \Theta'; r' \\ \Theta'; \varepsilon; \Delta_\varepsilon; r'; e :: s \vdash \mathtt{load}_\Delta\ F\ v\ \delta_F\ \delta_v \Rightarrow \Theta''; (v', \Delta_v)\end{array}}{\Theta; \varepsilon; \Delta_\varepsilon; r; e :: s \vdash \mathtt{load}_\Delta\ F\ v\ \delta_F\ \delta_v \Rightarrow \Theta''; (v', \Delta_v)}$$

$\boxed{s = \langle x : s_1, s_2 \rangle}$

$$\frac{\begin{array}{c}\Theta; \varepsilon; \Delta_\varepsilon; r; s_1 \vdash \mathtt{load}_\Delta\ F\ v_1\ \delta_F\ \delta_{v_1} \Rightarrow \Theta_1; (v'_1, \Delta_{v_1}) \\ \Theta_1; \varepsilon[x \mapsto v'_1]; \Delta_\varepsilon[x \mapsto \Delta_{v_1}]; r; s_2 \vdash \mathtt{load}_\Delta\ F\ v_2\ \delta_F\ \delta_{v_2} \Rightarrow \Theta_2; (v'_2, \Delta_{v_2}) \\ \Theta_2; \delta_{a_{err}}; (\Delta_{v_1} \wedge \Delta_{v_2}) \vdash a_{err} : \mathtt{do}\ \{ b_1 \leftarrow \mathit{valid}\ v'_1; b_2 \leftarrow \mathit{valid}\ v'_2; \mathtt{return}\ (b_1 \wedge b_2) \} \Rightarrow \Theta'; (a'_{err}, \Delta_{a_{err}})\end{array}}{\Theta; \varepsilon; \Delta_\varepsilon; r; \langle x : s_1, s_2 \rangle \vdash \mathtt{load}_\Delta\ F\ (a_{err}, (v_1, v_2))\ \delta_F\ (\delta_{a_{err}} \otimes (\delta_{v_1} \otimes \delta_{v_2})) \Rightarrow \Theta'; ((a'_{err}, (v'_1, v'_2)), \Delta_{a_{err}})}$$

$\boxed{s = \mathsf{P}\, e}$

$$\frac{\Delta_\varepsilon|_{fv(e)} = \emptyset}{\Theta; \varepsilon; \Delta_\varepsilon; r; \mathsf{P}\, e \vdash \mathtt{load}_\Delta\ F\ v\ \delta_F\ \emptyset \Rightarrow \Theta; (v, \emptyset)}$$

$\boxed{s = s_1?}$

$$\frac{r \notin \mathtt{dom}(\delta_F\ F) \quad \Theta; \delta_{a_{err}}; \delta_v \vdash a_{err} : \mathtt{return}\ \mathit{True} \Rightarrow \Theta'; (a'_{err}, \Delta_{a_{err}})}{\Theta; \varepsilon; \Delta_\varepsilon; r; s? \vdash \mathtt{load}_\Delta\ F\ (a_{err}, \mathit{Nothing})\ \delta_F\ (\delta_{a_{err}} \otimes \delta_v) \Rightarrow \Theta'; ((a_{err}, \mathit{Nothing}), \Delta_{a_{err}})}$$

$$\frac{\begin{array}{c}r \in \mathtt{dom}(\delta_F\ F) \quad \Theta; \varepsilon; \Delta_\varepsilon; r; s \vdash \mathtt{load}_\Delta\ F\ v\ \delta_F\ \delta_v \Rightarrow \Theta'; (v', \Delta_v) \\ \Theta; \delta_{a_{err}}; \Delta_v \vdash a_{err} : \mathit{valid}(v') \Rightarrow \Theta'; (a'_{err}, \Delta_{a_{err}})\end{array}}{\Theta; \varepsilon; \Delta_\varepsilon; r; s? \vdash \mathtt{load}_\Delta\ F\ (a_{err}, \mathit{Just}\ v)\ \delta_F\ (\delta_{a_{err}} \otimes \delta_v?) \Rightarrow \Theta'; ((a_{err}, \mathit{Just}\ v'), \Delta_{a_{err}})}$$

$\boxed{s = \{ s \mid x \in e \}}$

$$\frac{\begin{array}{c}\Theta; [\![e]\!]^{\varepsilon}_{\{\tau\}} \Rightarrow \Theta_1; \{ t_1, ..., t_k \} \\ \Theta_1; \forall\, i \in \{1, ..., k\}.\ \mathtt{do}\ \{ (v_i, \Delta_{v_i}) \leftarrow \varepsilon; \Delta_\varepsilon; r; s \vdash_x \mathtt{load}_\Delta\ F\ vs\ \delta_F\ \delta_{vs}; \mathtt{return}\ (\{ t_i \mapsto v_i \}, \bigwedge\ \Delta_{v_i}) \} \Rightarrow \Theta_2; (vs', \Delta_{vs}) \\ \Theta_2; \delta_{a_{err}}; \Delta_{vs} \vdash a_{err} : \forall\, i \in \{1, ..., k\}.\ \mathtt{do}\ \{ b_i \leftarrow \mathit{valid}(vs'(t_i)); \mathtt{return}\ (\bigwedge\ b_i) \} \Rightarrow \Theta'; (a'_{err}, \Delta_{a_{err}})\end{array}}{\Theta; \varepsilon; \Delta_\varepsilon; r; \{ s \mid x \in e \} \vdash \mathtt{load}_\Delta\ F\ (a_{err}, vs)\ \delta_F\ (\delta_{a_{err}} \otimes \delta_{vs}) \Rightarrow \Theta'; ((a'_{err}, vs'), \Delta_{a_{err}})}$$

$$\frac{t \in \mathtt{dom}(vs) \quad \Theta; \varepsilon[x \mapsto t]; \Delta_\varepsilon[x \mapsto \emptyset]; r; s \vdash \mathtt{load}_\Delta\ F\ vs(t)\ \delta_F\ \delta_{vs}(t) \Rightarrow \Theta'; (v', \Delta_v)}{\Theta; \varepsilon; \Delta_\varepsilon; r; s \vdash_x \mathtt{load}_\Delta\ F\ (t, vs)\ \delta_F\ \delta_{vs} \Rightarrow \Theta'; (v', \Delta_v)}$$

$$\frac{t \notin \mathtt{dom}(vs) \quad \Theta;\varepsilon;r;s \vdash \mathtt{load}\ (\delta_F\ F) \Rightarrow \Theta';v'}{\Theta;\varepsilon;\Delta_\varepsilon;r;s \vdash_x \mathtt{load}_\Delta\ F\ (t,vs)\ \delta_F\ \delta_{vs} \Rightarrow \Theta';(v',\Delta)}$$

$\boxed{\Theta;\varepsilon;\Delta_\varepsilon;r;s \vdash \mathtt{store}_\Delta\ F\ v\ \delta_F\ \delta_v \Rightarrow \Theta';(F',\phi')}$ "Under heap Θ, environment ε and delta environment Δ_ε, store the representation v for the specification s on filesystem F at path r, given filesystem changes δ_F and representation changes δ_v, and yield an updated filesystem F' and a filesystem validation function ϕ'."

$$\frac{\begin{array}{c}\Delta_\varepsilon|_{fv(s)} = \emptyset \quad \delta_F \searrow_F r = \emptyset \\ \Theta;\varepsilon;r;s \vdash \mathtt{sense}\ v \Rightarrow rs \quad \phi = \lambda F'.\ F \underset{rs}{=} F'\end{array}}{\Theta;\varepsilon;\Delta_\varepsilon;r;s \vdash \mathtt{store}_\Delta\ F\ v\ \delta_F\ \emptyset \Rightarrow \Theta;(F,\phi)}$$

$$\frac{\Theta;\varepsilon;r;s \vdash \mathtt{store}\ (\delta_F\ F)\ v \Rightarrow \Theta';(F',\phi')}{\Theta;\varepsilon;\Delta_\varepsilon;r;s \vdash \mathtt{store}_\Delta\ F\ v\ \delta_F\ \delta_v \Rightarrow \Theta';(F',\phi')}$$

$\boxed{s = \mathsf{M}\, s_1}$

$$\frac{\begin{array}{c}\Theta(a) = e \quad \Theta;e \Rightarrow \Theta';(a_{err},v) \\ \Theta';\varepsilon;\Delta_\varepsilon;r;s \vdash \mathtt{store}_\Delta\ F\ v\ \delta_F\ \delta_v \Rightarrow \Theta'';(F',\phi')\end{array}}{\Theta;\varepsilon;\Delta_\varepsilon;r;\mathsf{M}\,s \vdash \mathtt{store}_\Delta\ F\ a\ \delta_F\ (\mathsf{M}_{\delta_a}\ (\delta_{a_{err}} \otimes \delta_v)) \Rightarrow \Theta'';(F',\phi')}$$

$\boxed{s = e :: s_1}$

$$\frac{\begin{array}{c}\Delta_\varepsilon|_{fv(s)} = \emptyset \quad \Theta;[\![r\ /\ e]\!]^\varepsilon_{Path} \Rightarrow \Theta';r' \\ \Theta';\varepsilon;\Delta_\varepsilon;r';e :: s \vdash \mathtt{store}_\Delta\ F\ v\ \delta_F\ \delta_v \Rightarrow \Theta'';(F',\phi')\end{array}}{\Theta;\varepsilon;\Delta_\varepsilon;r;e :: s \vdash \mathtt{store}_\Delta\ F\ v\ \delta_F\ \delta_v \Rightarrow \Theta'';(F',\phi')}$$

$\boxed{s = \langle x : s_1, s_2\rangle}$

$$\frac{\begin{array}{c}\Theta;\varepsilon;\Delta_\varepsilon;r;s_1 \vdash \mathtt{store}_\Delta\ F\ v_1\ \delta_F\ \delta_{v_1} \Rightarrow \Theta_1;(F_1',\phi_1') \\ \Theta_1;\varepsilon[x \mapsto v_1];\Delta_\varepsilon[x \mapsto \delta_{v_1}];r;s_2 \vdash \mathtt{store}_\Delta\ F\ v_2\ \delta_F\ \delta_{v_2} \Rightarrow \Theta_2;(F_2',\phi_2') \\ \phi = \lambda F'.\ \phi_1'(F_1') \wedge \phi_2'(F_2')\end{array}}{\Theta;\varepsilon;\Delta_\varepsilon;r;\langle x : s_1, s_2\rangle \vdash \mathtt{store}_\Delta\ F\ (a_{err},(v_1,v_2))\ \delta_F\ (\delta_{a_{err}} \otimes (\delta_{v_1} \otimes \delta_{v_2})) \Rightarrow \Theta_2;((F_1 +\!\!+ F_2),\phi)}$$

$\boxed{s = \mathsf{P}\ e}$

$$\frac{\phi = \lambda F'.\ \mathtt{return}\ \mathit{True}}{\Theta;\varepsilon;\Delta_\varepsilon;r;\mathsf{P}\ e \vdash \mathtt{store}_\Delta\ F\ v\ \delta_F\ \delta_v \Rightarrow \Theta;(F,\phi)}$$

$\boxed{s = s_1?}$

$$\frac{r \notin \mathtt{dom}(\delta_F\ F) \quad \phi = \lambda F'.\ r \notin \mathtt{dom}(F')}{\Theta;\varepsilon;\Delta_\varepsilon;r;s? \vdash \mathtt{store}_\Delta\ F\ (a_{err},\mathit{Nothing})\ \delta_F\ (\delta_{a_{err}} \otimes \emptyset) \Rightarrow \Theta;(F,\phi)}$$

$$\frac{\begin{array}{c}r \in \mathtt{dom}(\delta_F\ F) \quad \Theta;\varepsilon;\Delta_\varepsilon;r;s \vdash \mathtt{store}_\Delta\ F\ v\ \delta_F\ \delta_v \Rightarrow \Theta';(F_1,\phi_1) \\ \phi = \lambda F'.\ \phi_1(F') \wedge e \in \mathtt{dom}(F')\end{array}}{\Theta;\varepsilon;\Delta_\varepsilon;r;s? \vdash \mathtt{store}_\Delta\ F\ (a_{err},\mathit{Just}\ v)\ \delta_F\ (\delta_{a_{err}} \otimes \delta_v?) \Rightarrow \Theta';(F_1,\phi)}$$

$\boxed{s = \{s \mid x \in e\}}$

$$\frac{\begin{array}{c}\Theta;[\![e]\!]^\varepsilon_{\{\tau\}} \Rightarrow \Theta';ts \quad vs = \{t_1 \mapsto v_1,\dots,t_k \mapsto v_k\} \\ \phi = \lambda F'.\ ts = \{t_1,\dots,t_k\} \wedge \bigwedge\ \phi_i(F') \\ \Theta_1;\forall t_i \in \mathtt{dom}(vs).\ \mathtt{do}\ \{(F_i,\phi_i) \leftarrow \varepsilon[x \mapsto t_i];\Delta_\varepsilon[x \mapsto \emptyset];r;s \vdash \mathtt{store}_\Delta\ F\ vs(t_i)\ \delta_F\ \delta_{vs}(t_i);\mathtt{return}\ (F_1 +\!\!+ \dots +\!\!+ F_k,\phi)\} \Rightarrow \Theta_2;(F',\phi')\end{array}}{\Theta;\varepsilon;\Delta_\varepsilon;r;\{s \mid x \in e\} \vdash \mathtt{store}_\Delta\ F\ (a_{err},vs)\ \delta_F\ (\delta_{a_{err}} \otimes \delta_{vs}) \Rightarrow \Theta_2;(F',\phi')}$$

$\boxed{\Theta;\varepsilon;r;s \vdash \mathtt{sense}\ v \Rightarrow rs}$ "Sensitivity of a forest specification in respect to a representation"

$$\frac{\Theta(a) = e \quad \Theta;e \Rightarrow \Theta';v \quad \Theta';\varepsilon;r;s \vdash \mathtt{sense}\ v \Rightarrow rs}{\Theta;\varepsilon;r;\mathsf{M}\ s \vdash \mathtt{sense}\ a \Rightarrow rs}$$

$$\frac{\Theta;\varepsilon;r;s \vdash \mathtt{sense}\ v \Rightarrow rs}{\Theta;\varepsilon;r;e :: s \vdash \mathtt{sense}\ v \Rightarrow \{r\} \cup rs}$$

$$\frac{\Theta;\varepsilon;r;s_1 \vdash \mathtt{sense}\ v_1 \Rightarrow rs_1 \quad \Theta;\varepsilon[x \mapsto v_1];r;s_2 \vdash \mathtt{sense}\ v_2 \Rightarrow rs_2}{\Theta;\varepsilon;r;\langle x : s_1, s_2\rangle \vdash \mathtt{sense}\ (a_{err},(v_1,v_2)) \Rightarrow rs_1 \cup rs_2}$$

$$\frac{}{\Theta;\varepsilon;r;\mathsf{P}\ e \vdash \mathtt{sense}\ v \Rightarrow \{\,\}}$$

$$\frac{}{\Theta; \varepsilon; r; s? \vdash \texttt{sense}\ (a_{err}, \mathit{Nothing}) \Rightarrow \{\, r \,\}}$$

$$\frac{\Theta; \varepsilon; r; s \vdash \texttt{sense}\ v \Rightarrow rs}{\Theta; \varepsilon; r; s? \vdash \texttt{sense}\ (a_{err}, \mathit{Just}\ v) \Rightarrow \{\, r \,\} \cup rs}$$

$$\frac{vs = \{\, t_1 \mapsto v_1, ..., t_k \mapsto v_k \,\} \quad \forall\, 1 \in \{i, ..., k\}.\ \Theta; \varepsilon[x \mapsto t_i]; r; s \vdash \texttt{sense}\ v_i \Rightarrow r_i}{\Theta; \varepsilon; r; \{s \mid x \in e\} \vdash \texttt{sense}\ (a_{err}, vs) \Rightarrow \bigcup\ r_i}$$

Theorem B.1 (Incremental Load Soundness)**.** *If*

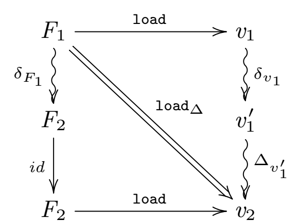
$$\Theta; \varepsilon; r; s \vdash \texttt{load}\ F_1 \Rightarrow \Theta_1; v_1$$

$$\Theta_1; v_1 \xrightarrow{\delta_{v_1}} \Theta_2; v_1'$$

$$\Theta_2; \varepsilon'; \Delta_\varepsilon; r; s \vdash \texttt{load}_\Delta\ F_1\ v_1'\ \delta_{F_1}\ \delta_{v_1} \Rightarrow \Theta_3; (v_2, \Delta_{v_1'})$$

$$\Theta_1; \varepsilon'; r; s \vdash \texttt{load}\ (\delta_{F_1}\ F_1) \Rightarrow \Theta_4; v_3$$

then $v_2\ {}_{\Theta_3}\!\overset{err}{\sim}_{\Theta_4}\ v_3$ *and* $valid(v_2)\ {}_{\Theta_3}\!\overset{err}{\sim}_{\Theta_4}\ valid(v_3)$*.*



Lemma 1 (Incremental Load Stability)**.** $\Theta; \varepsilon; \Delta_\varepsilon; r; \mathsf{M}\ s \vdash \texttt{load}_\Delta\ F\ a\ \delta_F\ (\mathsf{M}_\emptyset\ \delta_v) \Rightarrow \Theta'; (a, \Delta_a)$

Theorem B.2 (Incremental Store Soundness)**.** *If*

$$\Theta; \varepsilon; r; s \vdash \texttt{store}\ F\ v_1 \Rightarrow \Theta_1; (F_1, \phi_1)$$

$$\Theta_1; v_1 \xrightarrow{\delta_{v_1}} \Theta_2; v_2$$

$$\Theta_2; \varepsilon'; \Delta_\varepsilon; r; s \vdash \texttt{store}_\Delta\ F_1\ v_2\ \delta_{F_1}\ \delta_{v_1} \Rightarrow \Theta_3; (F_2, \phi_2)$$

$$\Theta_2; \varepsilon'; r; s \vdash \texttt{store}\ (\delta_{F_1}\ F_1)\ v_2 \Rightarrow \Theta_4; (F_3, \phi_3)$$

then $F_2 = F_3$ *and* $\phi_2(F_2) = \phi_3(F_3)$*.*