Physical interpretation

March 12, 2018

We want to estimate the properties that particles need to have to emit the hard gamma radiation observed in the region $b \in (-6°, 6°)$, $\ell \in (-10°, 0°)$. Averaged over the ROI, the best-fit spectrum of the electrons shows no cutoff and we conclude, that electrons of at least $E_0 = 1\,\mathrm{TeV}$ need to be able to travel inside the whole volume of the ROI. We start with the diffusion equation

$$\frac{\partial \rho}{\partial t} - D\Delta\rho - \frac{\partial(b_{\mathrm{IC}}\rho)}{\partial E} = Q, \tag{1}$$

where we take into account diffusion, assuming a spacially constant diffusion coefficient $D(E) = D_0 \left(\frac{E}{1\,\mathrm{GeV}}\right)^{\delta}$, and energy loss via IC, where $b_{\mathrm{IC}}(E) = -\mathrm{d}E/\mathrm{d}t$. ρ is the electron density, Q is some distribution of sources. The diffusion equation has the solution

$$\rho(r, E, t) = \int dr_0^3 \int dE_0 \; [f(r, E, t, r_0, E_0, t_0)\; Q(r_0, E_0, t_0)] \tag{2}$$

with the Green's function

$$f(r, E, t, r_0, E_0, t_0) = \frac{1}{|b_{\mathrm{IC}}(E)|}\; e^{\frac{(r-r_0)^2}{4\lambda}}\; \frac{\delta(t - t_0 - \tau)}{(4\pi\lambda)^{3/2}}. \tag{3}$$

The Green's function was found with the help of the substitutions (Syrovat-skii)

$$t'(E, E_0) := t - \tau := t - \int_E^{E_0} \frac{\mathrm{d}E'}{b_{\mathrm{IC}}(E')}, \tag{4}$$

$$\lambda(E, E_0) = \int_E^{E_0} \frac{D(E')}{b_{\mathrm{IC}}(E')}\mathrm{d}E'. \tag{5}$$

We read off the typical diffusion distance

$$\langle x \rangle^2 = \sigma^2 = 2\lambda = 2\int_E^{E_0} \frac{D(E')}{b_{\mathrm{IC}}(E')}\mathrm{d}E' = 2D_0 \int_E^{E_0} \frac{1}{b_{\mathrm{IC}}(E')}\left(\frac{E'}{1\,\mathrm{GeV}}\right)^{\delta} \mathrm{d}E', \tag{6}$$

which is the distance that the electrons travel while cooling from energy E to E_0. For the spacially constant diffusion coefficient we insert the local values $D_0 = 3 \times 10^{28}\,\mathrm{cm}^2/\mathrm{s} = 100\,\mathrm{pc}^2/\mathrm{kyr}$ and $\delta = 0.4$ (Pulsars VS dark matter). $b_{\mathrm{IC}}(E)$ is calculated with gamma-spectra.py (function "ICS-Edot") via

$$b_{\mathrm{IC}}(E_{\mathrm{e}}) = c \int\int \sigma_{\mathrm{IC}} \left(E\frac{dN}{dE}\right)_{\mathrm{ISRF}} \mathrm{d}E_\gamma \mathrm{d}E_{\mathrm{ISRF}} \tag{7}$$

renaming the electron energy $E = E_e$; E_γ are the final gamma-ray photon energies. σ_{IC} is the IC-cross section taken from Blumenthal and Gould (eq. 2.48). The ISRF at the different latitudes, shown in Figure **??**, is taken from the files that Dima uploaded (GALPROP) and a 2.73 K thermal spectrum.

The resulting energy loss $b_{\mathrm{IC}}(E_{\mathrm{e}})$ is shown in Figure **??** as a function of electron energy. We observe that $b_{\mathrm{IC}}(E_{\mathrm{e}})$ is constant above a certain energy ($E_{\mathrm{e}} \approx 10^7$). The reason for this is probably that $b_{\mathrm{IC}}(E) \approx \Delta E/\Delta T \sim E_{\mathrm{e}}\,\sigma_{\mathrm{IC}}\,\rho_{\mathrm{ISRF}}$ and $\sigma_{\mathrm{IC}} \sim 1/E_{\mathrm{e}}$ for high energies, i.e

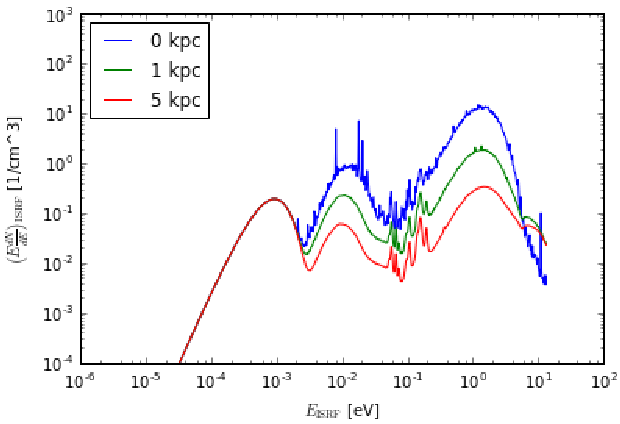

Figure 1: Density of the ISRF at different heights above the GP.

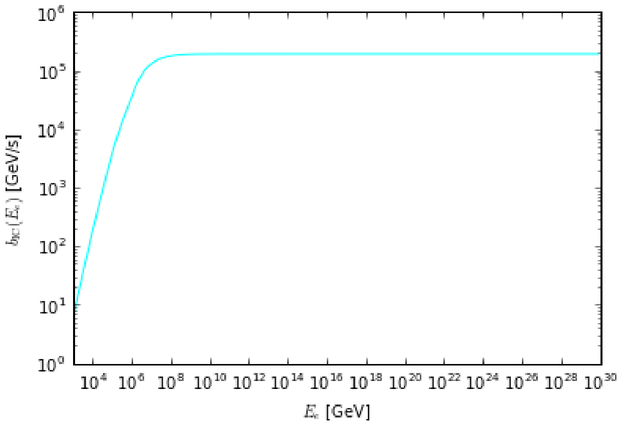

Figure 2: $b_{\text{IC}}(E_{\text{e}})$.

above some energy the IC cross section is constant.

For the initial electron energy, we choose $E_1 = 100\,\text{TeV}$, an energy which can be realistically achieved in SN-shock acceleration. Since we observe no cutoff to at least $E_0 = 1\,\text{TeV}$, we choose this as the final electron energy. Whith this input we find for the typical diffusion distance

$$\langle x\rangle = \sqrt{2D_0\int_{E_1}^{E_0}\frac{1}{b_{\text{IC}}(E')}\left(\frac{E'}{1\,\text{GeV}}\right)^{\delta}\mathrm{d}E'} = 1.5\,\text{kpc}. \tag{8}$$

Rearranging $b_{\text{IC}}(E_{\text{e}}) = -\frac{\mathrm{d}E}{\mathrm{d}t}$ we find a characteristic cooling time of

$$T_{\text{cool}} = \int_{E_0}^{E_1}\frac{1}{b_{\text{IC}}(E_{\text{e}})}dE_{\text{e}} = 320\,\text{kyr}. \tag{9}$$

Assuming a distance of 8 kpc to the ROI, we find a height of $\Delta H = 0.84\,\text{kpc}$ (6°) in latitude and a length of $\Delta L = 0.71\,\text{kpc}$ (5°) in longitude. Since the diffusion distance of the electrons exceeds the minimal distance from the center of the ROI to its border ($\Delta x = 0.7\,\text{kpc}$), the electrons cannot be confined. Neglecting cooling, the diffusion equation simplifies to

$$\frac{\partial\rho}{\partial t} - D\Delta\rho = Q, \tag{10}$$

where the Green's function can be found to be

$$f(r,E,t,r_0,t_0) = \frac{\Theta(t-t_0)}{(4\pi D(E)(t-t_0))^{3/2}}\;\mathrm{e}^{-\frac{(r-r_0)^2}{4D(E)(t-t_0)}}. \tag{11}$$

Reading off the diffusion distance

$$\langle x \rangle^2 = \sigma^2 = 2D(E)(t - t_0), \tag{12}$$

we find for the escape time of the electrons at 1 TeV

$$T_{\rm esc} := t - t_0 = \frac{\Delta x^2}{2D(E)} = 150\,\mathrm{kyr}. \tag{13}$$

The characteristic age of the particles must be shorter than the cooling and escape times, in this case shorter than 150 kyr. This is a relatively young population of electrons.

Since the energy losses of electrons exceed the energy losses of protons by far, the same conclusion applies for protons: They escape before they cool down to 1 TeV in less than 150 kyr.

The total energy density in electrons with energies between $E_0 = 1\,\mathrm{GeV}$ and $E_1 = 1\,\mathrm{TeV}$ necessary to produce the observed emission, can be found via integrating the electron spectrum that was found in the fit of the IC-spectrum to the low-energy model residual:

$$\frac{\mathrm{d}E_{\rm tot}}{\mathrm{d}V} = \int_{E_0}^{E_1} \left(E\frac{\mathrm{d}N}{\mathrm{d}E}\right)_{\rm e} \mathrm{d}E \tag{14}$$

and equivalently for protons. Since it is more convenient with my scripts to calculate this energy density in the already defined latitude stripes $b \in (-6^\circ, -2^\circ)$, $(-2^\circ, 2^\circ)$ and $(2^\circ, 6^\circ)$, I calculate $\frac{\mathrm{d}E_{\rm tot}}{\mathrm{d}V}$ in the three regions for negative longitudes separately and calculate the total energy content by multiplying each energy density with the corresponding size of the region. For negative longitdudes I find:

$(-6^\circ, -2^\circ)$

$$\begin{aligned}
\left(\frac{\mathrm{d}E_{\rm tot}}{\mathrm{d}V}\right)_{\rm e} &= 51.424\,543\,159\,3\,\mathrm{meV/cm^3} = 8.239\,118\,924\,63 \times 10^{-14}\,\mathrm{erg/cm^3}, \\
\rightarrow E_{\rm tot,e} &= \left(\frac{\mathrm{d}E_{\rm tot}}{\mathrm{d}V}\right)_{\rm e} \cdot 8\,\mathrm{kpc}(\tan(6^\circ) - \tan(2^\circ)) \cdot (8\,\mathrm{kpc}\tan(10^\circ))^2 \\
&= 8.239 \times 10^{-14}\,\mathrm{erg/cm^3} \cdot 3.2766 \times 10^{64}\,\mathrm{cm^3} = 2.700 \times 10^{51}\,\mathrm{erg}. \\
\left(\frac{\mathrm{d}E_{\rm tot}}{\mathrm{d}V}\right)_{\rm p} &= 569.938\,986\,511\,\mathrm{meV/cm^3} = 9.131\,427\,915\,86 \times 10^{-13}\,\mathrm{erg/cm^3}, \\
\rightarrow E_{\rm tot,p} &= 9.1314 \times 10^{-13}\,\mathrm{erg/cm^3} \cdot 3.2766 \times 10^{64}\,\mathrm{cm^3} = 2.992 \times 10^{52}\,\mathrm{erg}.
\end{aligned} \tag{15}$$

$(-2^\circ, 2^\circ)$

$$\begin{aligned}
\left(\frac{\mathrm{d}E_{\rm tot}}{\mathrm{d}V}\right)_{\rm e} &= 4.535\,661\,462\,62\,\mathrm{meV/cm^3} = 7.266\,929\,737\,54 \times 10^{-15}\,\mathrm{erg/cm^3}, \\
\rightarrow E_{\rm tot,e} &= \left(\frac{\mathrm{d}E_{\rm tot}}{\mathrm{d}V}\right)_{\rm e} \cdot 8\,\mathrm{kpc}\ 2\tan(2^\circ) \cdot (8\,\mathrm{kpc}\tan(10^\circ))^2 \\
&= 7.300 \times 10^{-15}\,\mathrm{erg/cm^3} \cdot 3.265 \times 10^{64}\,\mathrm{cm^3} = 2.3726 \times 10^{50}\,\mathrm{erg}. \\
\left(\frac{\mathrm{d}E_{\rm tot}}{\mathrm{d}V}\right)_{\rm p} &= 376.300\,738\,236\,\mathrm{meV/cm^3} = 6.029\,001\,607\,56 \times 10^{-13}\,\mathrm{erg/cm^3}, \\
\rightarrow E_{\rm tot,p} &= 6.029 \times 10^{-13}\,\mathrm{erg/cm^3} \cdot 3.265 \times 10^{64}\,\mathrm{cm^3} = 1.9684 \times 10^{52}\,\mathrm{erg}.
\end{aligned} \tag{16}$$

$(2°, 6°)$

$$
\begin{aligned}
\left(\frac{\mathrm{d}E_\mathrm{tot}}{\mathrm{d}V}\right)_\mathrm{e} &= 40.288\,834\,549\,1\,\mathrm{meV/cm^3} = 6.454\,981\,975\,37 \times 10^{-14}\,\mathrm{erg/cm^3}, \\
\rightarrow E_\mathrm{tot,e} = \left(\frac{\mathrm{d}E_\mathrm{tot}}{\mathrm{d}V}\right)_\mathrm{e} &\cdot 8\,\mathrm{kpc}(\tan(6°) - \tan(2°)) \cdot (8\,\mathrm{kpc}\tan(10°))^2 \\
&= 6.455 \times 10^{-14}\,\mathrm{erg/cm^3} \cdot 3.2766 \times 10^{64}\,\mathrm{cm^3} = 2.115 \times 10^{51}\,\mathrm{erg}. \\
\left(\frac{\mathrm{d}E_\mathrm{tot}}{\mathrm{d}V}\right)_\mathrm{p} &= 483.107\,848\,803\,\mathrm{meV/cm^3} = 7.740\,239\,922\,76 \times 10^{-13}\,\mathrm{erg/cm^3}, \\
\rightarrow E_\mathrm{tot,p} &= 7.740 \times 10^{-13}\,\mathrm{erg/cm^3} \cdot 3.2766 \times 10^{64}\,\mathrm{cm^3} = 2.5362 \times 10^{52}\,\mathrm{erg}.
\end{aligned}
\tag{17}
$$

For comparison, here the values for positive longitudes:

$(-6°, -2°)$
$\left(\frac{\mathrm{d}E_\mathrm{tot}}{\mathrm{d}V}\right)_\mathrm{e} = 75.287\,171\,870\,2\,\mathrm{meV/cm^3} = 1.206\,233\,297\,23 \times 10^{-13}\,\mathrm{erg/cm^3}$,
$\left(\frac{\mathrm{d}E_\mathrm{tot}}{\mathrm{d}V}\right)_\mathrm{p} = 409.818\,683\,448\,\mathrm{meV/cm^3} = 6.566\,018\,214\,31 \times 10^{-13}\,\mathrm{erg/cm^3}$.

$(-2°, 2°)$
$\left(\frac{\mathrm{d}E_\mathrm{tot}}{\mathrm{d}V}\right)_\mathrm{e} = 126.295\,061\,78\,\mathrm{meV/cm^3} = 2.023\,469\,669\,68 \times 10^{-13}\,\mathrm{erg/cm^3}$,
$\left(\frac{\mathrm{d}E_\mathrm{tot}}{\mathrm{d}V}\right)_\mathrm{p} = 182.614\,888\,747\,\mathrm{meV/cm^3} = 2.925\,812\,643\,85 \times 10^{-13}\,\mathrm{erg/cm^3}$.

$(2°, 6°)$
$\left(\frac{\mathrm{d}E_\mathrm{tot}}{\mathrm{d}V}\right)_\mathrm{e} = 18.173\,497\,221\,7\,\mathrm{meV/cm^3} = 2.911\,714\,828\,9 \times 10^{-14}\,\mathrm{erg/cm^3}$,
$\left(\frac{\mathrm{d}E_\mathrm{tot}}{\mathrm{d}V}\right)_\mathrm{p} = 368.008\,317\,724\,\mathrm{meV/cm^3} = 5.896\,142\,403\,43 \times 10^{-13}\,\mathrm{erg/cm^3}$.

So the total energy in the region $b \in (-6°, 6°)$, $\ell \in (-10°, 0°)$, right of the GC, in electrons and protons is

$$E_\mathrm{tot,e} = 2.700 \times 10^{51}\,\mathrm{erg} + 2.3726 \times 10^{50}\,\mathrm{erg} + 2.115 \times 10^{51}\,\mathrm{erg} = 5.052 \times 10^{51}\,\mathrm{erg}, \tag{18}$$

$$E_\mathrm{tot,p} = 2.992 \times 10^{52}\,\mathrm{erg} + 1.9684 \times 10^{52}\,\mathrm{erg} + 2.5362 \times 10^{52}\,\mathrm{erg} = 7.497 \times 10^{52}\,\mathrm{erg}. \tag{19}$$

Note that the gas density that was assumed for the hadronic emission ($n_\mathrm{H} = 1\,/\mathrm{cm^3}$) is fairly low. The gas density close to the GC is probably by one order of magnitude higher. Therefore, the total energy contained in protons would be similar to the energy contained in electrons. Assuming an energy output of 1×10^{49} erg in electrons/protons per SN, the total energy in electrons would correspond to around 50 SNe.