Poroelasticitiy Tests

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Contents

1 Introduction

The PorousFlow module includes the ability to couple fluid flow to solid mechanics, and thus includes poroelasticity, which is the theory of a fully-saturated single-phase fluid with constant bulk density and constant viscosity coupled to small-strain isotropic elasticity.

There is one important difference between the theories, however. The time-derivative terms of poroelasticity are

$$\frac{1}{M}\dot{P} + \alpha\dot{\epsilon}_{ii} \ , \tag{1.1}$$

where M is the Biot modulus:

$$\frac{1}{M} = \frac{(1-\alpha)(\alpha-\phi)}{K} + \frac{\phi}{K_{\mathrm{f}}} \ , \tag{1.2}$$

P is the fluid porepressure, α is the Biot coefficient, ϵ_{ii} is the volumetric strain, ϕ is the porosity, K is the solid (drained) bulk modulus, and K_{f} is the fluid bulk modulus. Evidently from Eqn (1.2), the Biot modulus, M, should evolve with time as the porosity evolves. Indeed, the terms in Eqn (1.1) are derived from the continuity equation $\partial(\phi\rho)/\partial t + \phi\rho\dot{\epsilon}_{ii}$ using the evolution of ϕ (and that $\rho = \rho_0 \exp(P/K_{\mathrm{f}})$). However, in the standard analytical solutions of poroelasticity theory, the Biot modulus, M is considered fixed.

The PorousFlow module allows porosity to vary with fluid porepressure and volumetric strain, so usually the Biot modulus would vary too, causing differences with the analytical solutions of poroelasticity. Therefore, PorousFlow offers a porosity relationship that evolves porosity in such a way as to keep M fixed. This is called `PorousFlowPorosityHMBiotModulus`.

PorousFlow is also built with finite strains in mind, whereas poroelasticity is not. Therefore, in comparisons with solutions from poroelasticity theory, either the strain should be kept small, or the various finite-strain switches in PorousFlow should be turned off (they are all on by default).

2 Simple tests

2.1 Volumetric expansion due to increasing porepressure

The porepressure within a fully-saturated sample is increased:

$$P_{\mathrm{f}} = t \ . \tag{2.1}$$

Zero mechanical pressure is applied to the sample's exterior, so that no Neumann BCs are needed on the sample. No fluid flow occurs since the porepressure is increased uniformly throughout the sample

The effective stresses should then evolve as $\sigma_{ij}^{\mathrm{eff}} = \alpha t\delta_{ij}$, and the volumetric strain $\epsilon_{00} + \epsilon_{11} + \epsilon_{22} = \alpha t/K$. MOOSE produces this result correctly.

2.2 Undrained oedometer test

A cubic single-element fully-saturated sample has roller BCs applied to its sides and bottom. All the sample's boundaries are impermeable. A downwards (normal) displacement, u_z, is applied to its top, and the rise in porepressure and effective stress is observed. (Here z denotes the direction normal to the top face.) There is no fluid flow in the single element.

Under these conditions, assuming constant porosity, and denoting the height (z length) of the sample by L:

$$\begin{aligned} P_{\mathrm{f}} &= -K_f \log(1 - u_z) \ , \\ \sigma_{xx}^{\mathrm{eff}} &= (K - \tfrac{2}{3}G)u_z/L \ , \\ \sigma_{zz}^{\mathrm{eff}} &= (K + \tfrac{4}{3}G)u_z/L \ . \end{aligned} \tag{2.2}$$

2.3 Porepressure generation of a confined sample

A single-element fully-saturated sample is constrained on all sides and its boundaries are impermeable. Fluid is pumped into the sample via a source s (kg.s^{-1}.m^{-3}) and the rise in porepressure is observed.

Denoting the strength of the source by s (units are s^{-1}), the expected result is

$$\begin{aligned}
\text{fluid mass} &= \text{fluid mass}_0 + st \ , \\
\sigma_{ij}^{\mathrm{eff}} &= 0 \ , \\
P_{\mathrm{f}} &= K_{\mathrm{f}} \log(\rho\phi/\rho_0) \ , \\
\rho &= \rho_0 \exp(P_{\mathrm{f}}/K_f) \ , \\
\phi &= \alpha + (\phi_0 - \alpha) \exp\left(P_{\mathrm{f}}(\alpha - 1)/K\right) \ . \qquad (2.3) \\
& \qquad (2.4)
\end{aligned}$$

Here K is the solid bulk modulus.

2.4 Porepressure generation of an unconfined sample

A single-element fully-saturated sample is constrained on all sides, except its top. All its boundaries are impermeable. Fluid is pumped into the sample via a source s ($\mathrm{kg.s^{-1}.m^{-3}}$)) and the rise in the top surface, the porepressure, and the stress are observed.

Regardless of the evolution of porosity, the following ratios result

$$\begin{aligned}
\sigma_{xx}/\epsilon_{zz} &= K - 2G/3 \ , \\
\sigma_{zz}/\epsilon_{zz} &= K + 4G/3 \ , \\
P/\epsilon_{zz} &= (K + 3G/3 + \alpha^2 M)/\alpha - \alpha M \ . \qquad (2.5)
\end{aligned}$$

where K is the undrained bulk modulus, G the shear modulus, α the Biot coefficient, and M is the initial Biot modulus. MOOSE produces these results when using the `PorousFlowPorosityHM` material.

However, if the Biot modulus, M, is held fixed as the porosity evolves, and the source is

$$s = S\rho_0 \exp(P/K_{\mathrm{f}}) \ , \qquad (2.6)$$

with S being a *constant* volumetric source ($\mathrm{m^3.s^{-1}.m^{-3}}$) then

$$\begin{aligned}
\epsilon_{zz} &= \frac{\alpha M st}{K + 4G/3 + \alpha^2 M} \ , \\
P &= M(st - \alpha\epsilon_{zz}) \ , \\
\sigma_{xx} &= (K - 2G/3)\epsilon_{zz} \ , \\
\sigma_{zz} &= (K + 4G/3)\epsilon_{zz} \ . \qquad (2.7)
\end{aligned}$$

MOOSE produces these results when using the `PorousFlowPorosityHMBiotModulus` material.

3 Terzaghi consolidation

This is documented at http://mooseframework.org/wiki/PhysicsModules/TensorMechanics/TensorMe
The PorousFlow Material PorousFlowPorosityHMBiotModulus must be used.

4 Mandel's consolidation of a drained medium

This is documented at `http://mooseframework.org/wiki/PhysicsModules/TensorMechanics/TensorMe`
The PorousFlow Material `PorousFlowPorosityHMBiotModulus` must be used.