Introduction to Exposure Rating

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05 May 2015

Exposure rating is a tool for insurance pricing that allocates premium to bands of damage ratios or severity of losses. First ideas were published in (Salzmann 1963). It is often used to price excess of loss reinsurance.

Exposure rating uses the loss experience of a similar portfolio of policies to estimate the expected losses of the portfolio to be covered. The method is frequently used as a benchmark when there is no sufficient credible claims history from the client.

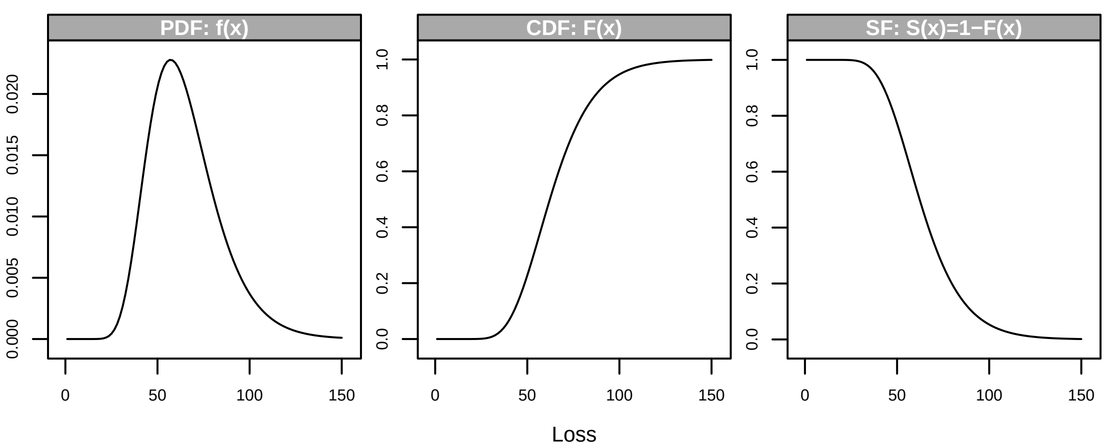
Loss Distribution

First lets assume we have perfect information, i.e. we know the loss distribution for a certain risk.

To keep it simple I assume the loss distribution is log-normal with a mean (M) of 65 and a coefficient of variation (CV) of 30%. The corresponding log-normal parameters μ and σ can be derived via:

$$\sigma^2 = \log(1 + CV^2) = 0.29$$
$$\mu = \log(M) - \sigma^2/2 = 4.13$$

The following chart shows the corresponding probability density curve $f(x)$, cumulative distribution function $F(x)$ and survival function $S(x) = 1 - F(x)$.



In the insurance context the survival function is often called *exceedance probability function*, as it describes the probability of exceeding a certain loss.

Expected Loss

Let's define X as the random variable that describes the ground up loss. The expected loss (in plain English the probability weighted sum of the losses) is then:

$$E[X] = \int_{-\infty}^{\infty} x\, f(x) dx$$

Assuming losses are not negative ($P(X < 0) = 0$) this simplifies to:

$$E[X] = \int_0^\infty x\, f(x)dx$$

Limited Expected Loss

Let's further assume that losses are limited by α in my contract. Then the limited expected value (LEV), written as $E[X \wedge \alpha]$, is given as

$$E[X \wedge \alpha] = \int_0^\alpha x\, f(x)dx + \int_\alpha^\infty \alpha\, f(x)dx$$

To evaluate this sum of integrals I recall that $F(x) = \int f(x)dx$ and $F(\infty) = 1$. Hence:

$$\int_\alpha^\infty \alpha\, f(x)dx = \alpha \int_\alpha^\infty f(x)dx = \alpha - \alpha F(\alpha)$$

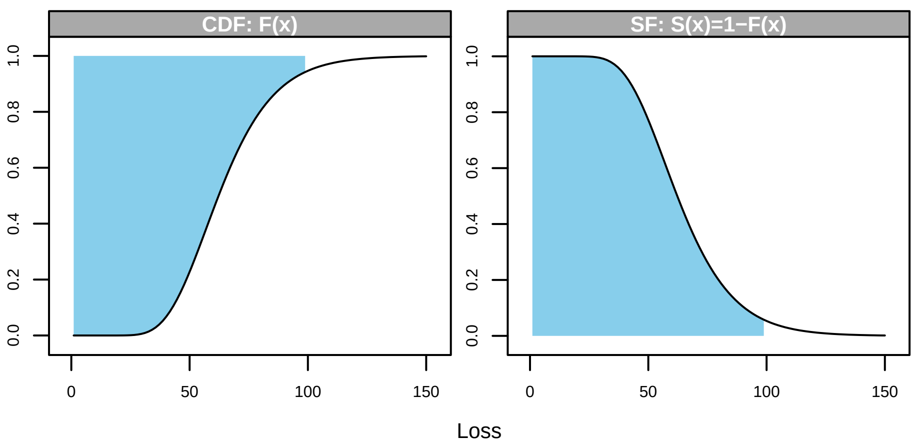
The integration by parts theorem $\int u(x)v'(x)\, dx = u(x)v(x) - \int v(x)\, u'(x)dx$ helps me with the first part of the integral:

$$\int_0^\alpha x\, f(x)dx = \alpha F(\alpha) - \int_0^\alpha F(x)dx$$

Therefore:

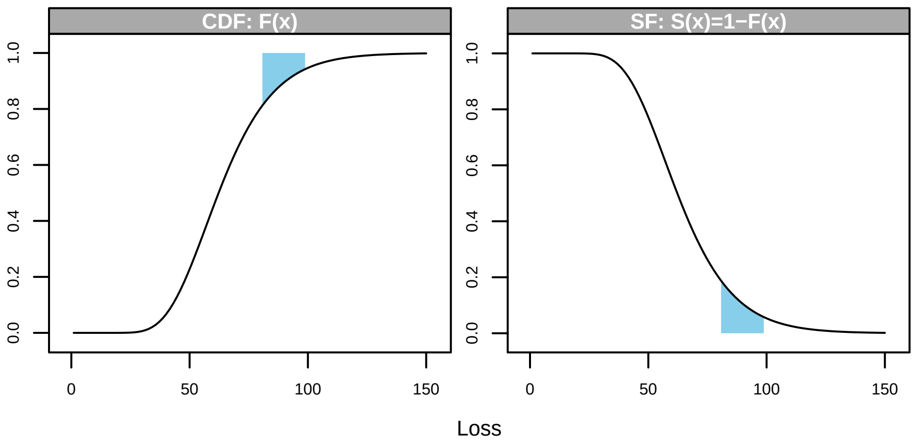
$$E[X \wedge \alpha] = \alpha - \int_0^\alpha F(x)dx = \int_0^\alpha 1 - F(x)dx = \int_0^\alpha S(x)dx$$

The integral describes the area above the CDF up to α, or the area under the survival function up to α. In the example below the limit α was set to 100.



Loss Cost of a Layer

Suppose I want to insure the layer of claims between 80 and 100.



The expected loss cost would be the difference between the limited expected values of $E[X \wedge 100] - E[X \wedge 80]$.

```
S <- function(x){ 1 - plnorm(x, mu, sigma) }
(lyr <- integrate(S, 0, 100)$value - integrate(S, 0, 80)$value)
```

```
## [1] 2.22814
```

Increased Limit Factor

On the other hand the ratio of the original loss cost with a limit of 100 to a limit of 80 is called an increased limit factor (ILF):

```
(ILF <- integrate(S, 0, 100)$value / integrate(S, 0, 80)$value)
```

```
## [1] 1.03592
```

Therefore we would expect the LEV to increase by 3.6% as the limit increases from 80 to 100.

Note ILFs are often used for pricing casualty business.

Exposure curves

From the loss distribution we could read of the expected loss for any layer. However, often we will not have that level of information about the risk.

Instead, we will have to infer information from others risks that share similar loss characteristics as a function of the overall exposure, assuming that the relative loss size distribution is independent of the individual risk characteristic.

Hence, we will require a view on the expected full value loss cost for the overall exposure.

Normalising loss experience using TIV and MPL

To make risks more comparable we look at ratio of losses to the underlying exposure, where the exposure is given as the sum insured (SI), or better the total insured value (TIV), or perhaps as the maximum probable loss (MPL).

Other definitions and measures are also popular, such as possible maximum loss (PML), estimate maximum loss (EML), or maximum foreseeable loss (MFL).

While the TIV and SI are straightforward to understand, the other metrics can be little more challenging to assess.

Suppose we insure a big production plant with of several buildings against fire. It is unlikely that a fire will destroy the whole facility, instead it is believed that the distance between the building will ensure that fires can be contained in a local area. Hence the MPL might be only the highest value of any of those buildings.

Shifting from the metric from loss amounts to damage ratio (loss as a % of the exposure metric) allows us to compare and benchmarks risks.

Analysing deductibles

Most insurance policies are written with a deductible, so that the insured will cover the first $X of the losses.

Thus, the reinsurer needs to understand, by how much the claims burden is reduced for a given deductible.

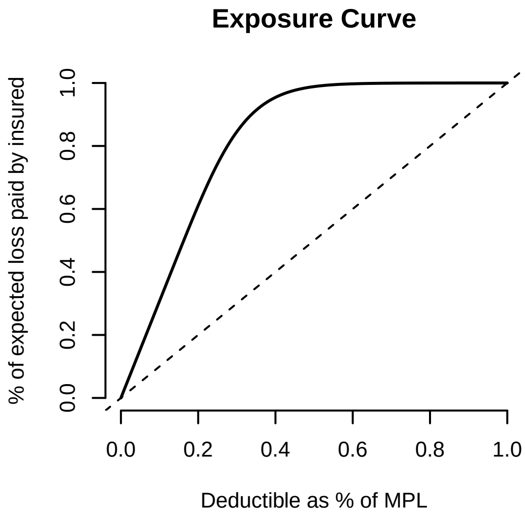
From the previous section we have learned how to calculate the limited expected value, which is the loss cost to the insured.

The exposure curve (or also called *first loss curve* or *original loss cure*) is defined as the proportion of the LEV for a given deductible d compared to the overall expected claims cost $E[X]$:

$$G(d) = \frac{E[X \wedge d]}{E[X]}$$

For our example, using a log-normal distribution and a MPL of 200 we get:

```
MPL <- 200
ExpectedLoss <- 65
Deductible <- seq(0, MPL, 1)
G <- sapply(Deductible, function(x){
  LEVx <- integrate(S, 0, x)$value
  LEVx/ExpectedLoss
})
plot(Deductible/MPL, G,
     t="l", bty="n", lwd=1.5,
     main="Exposure Curve",
     xlab="Deductible as % of MPL",
     ylab="% of expected loss paid by insured",
     xlim=c(0,1), ylim=c(0,1))
abline(a=0, b=1, lty=2)
```



The steepness of the curve is related to the severity of the loss distribution. The closer the curve is to the diagonal the greater the proportion of large loss.

If all losses were total losses then the exposure curve would be identical with the diagonal.

Different perils and exposure will have different exposure curves. Well known exposure curves are those used by Swiss Re and Lloyd's.

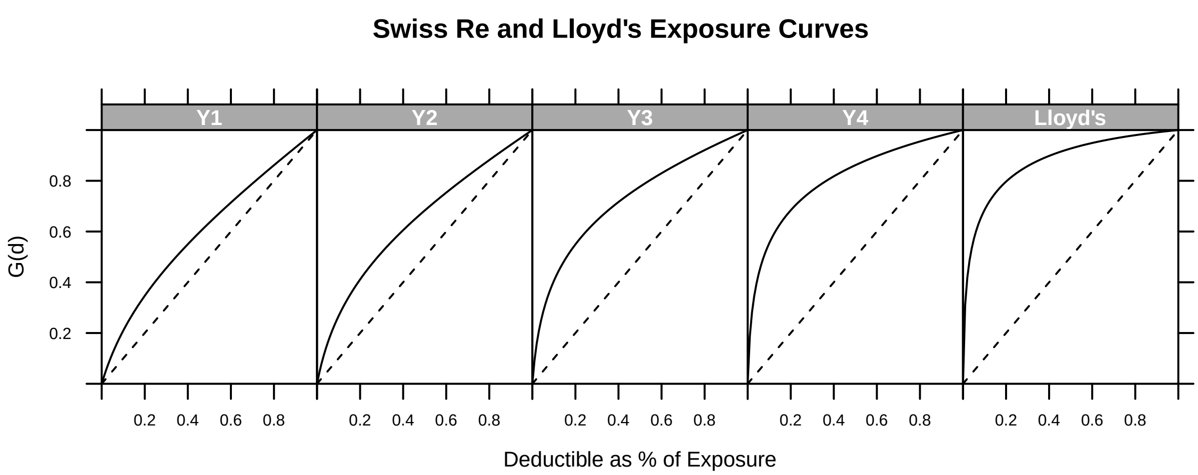
The Swiss Re curves were developed based on data from the 1950's - 1960's. They are also known as the *Gasser curves Y1- Y4*, named after Peter Gasser. The origin of the Lloyd's exposure curve is unknown. Some say it is based on fire losses during World War II.

Stefan Bernegger showed that those exposure curves can be approximated with the Mawell-Boltzmann, Bose-Einstein and Fermi-Diarc (MBBEFD) distributions (Bernegger 1997).

These distributions are of the form

$$G(x) = \frac{\ln(a + b^x) - \ln(a+1)}{\ln(a+b) - \ln(a+1)}, \text{ with } x \in [0,1] \text{ and } b > 0$$

The Swiss Re and Lloyd's curves can be approximate as a function of only one parameter c, as shown by Bernegger. See also the functions `swissRe` and `mbbefdExposure` as part of this package.



Typical uses cases of those curves with parameter c are summarised in the table below, taken from the Swiss Re paper *Exposure rating* (Guggisberg 2004).:

Gasser	**Parameter c**	**Scope of application**	**Basis**
Y1	1.5	Personal lines	Sum Insured
Y2	2.0	Commercial lines (small-scale)	Sum Insured
Y3	3.0	Commercial lines (medium-scale)	Sum Insured
	3.1	Captive business interruption	MPL
	3.4	Captive property damage / BI	MPL
	3.4	Captive property damage	MPL
Y4	4.0	Industrial and large commercial	MPL
Lloyd's	5.0	Lloyd's: Industry	Top location

It is believed that smaller risks have a higher proportion of server losses than larger risks, relative to the sum insured.

Example

The following example data is taken from the earlier Swiss Re technical paper above.

It is the aim to find the risk premium for a per risk WXL cover (fire only) with the limits

$$\text{CHF } 3,500,000 \text{ xs CHF } 1,500,000$$

The total gross net premium income (GNPI) is CHF $95,975,000$ in 2004, with an expected loss ratio of 55%. Therefore we have a view on the mean expected loss cost.

The risk profile given by the cedent is from the year 2002. However, instead of re-indexing the historical data to 2004, we back-index the data to 2002 with a factor of $457/550 = 0.83$:

$$\text{CHF } 2,908,182 \text{ xs CHF } 1,246,364$$

The data presented below is from the client, with pre-selected exposure curves, in this case the parameter c of the MBBEFD curve.

The policies have been grouped into different exposure bands. The mean MPL is simply the average of the lower and upper band.

Max MPL '000	Mean MPL Gross Loss '000	Gross Premium '000	Exposure Curve Parameter c
150	75	33,434	1.5
250	200	14,568	1.5
400	325	6,324	1.5
600	500	4,584	2
800	700	3,341	2
1,000	900	1,405	2
1,250	1,125	1,169	3
1,500	1,375	683	3
1,750	1,625	613	3
2,000	1,875	554	3
2,500	2,250	700	4
3,000	2,750	552	4
4,000	3,500	1,194	4

Max MPL '000	Mean MPL Gross Loss '000	Gross Premium '000	Exposure Curve Parameter c
5,500	4,750	1,490	4
9,000	7,250	4,177	4
12,500	10,750	3,527	4
18,000	15,250	3,249	4
24,000	21,000	2,712	4
36,000	30,000	2,588	4
48,000	42,000	1,988	4
72,000	60,000	657	4
90,000	81,000	1,918	4

In order to use the exposure curves we need the expected loss and the deductible as % of the MPL.

We have to consider three scenarios:

Losses below deductible

Losses with a maximum MPL below the deductible are not relevant and hence no reinsurance premium needs to be calculated.

Losses above deductible, but below limit

As an example I use the band that has a maximum MPL of CHF 4m, with a mean MPL gross loss of CH 3.5m and gross premium of CH 1.194m.

The client would retain 1,246/3,500=35.6% of the maximum MPL. Using our exposure curve we can read of that this would reduce our expected claims burden by 79.5%.

Therefore the reinsurance premium for this band is 1,194k · 20.5% =244.8k.

Losses above limit

Here I select the band with a maximum MPL of CHF 90m and a mean gross MPL of CHF 81m. Because of the limit of CHF 4154.5 only that amount matters. From the exposure curve we can read of again the average proportion of loss paid by the cedant 0.8%.

To calculate the premium we first have to derive the proportion of premium that is attached to the risk below the limit: 4,155k/8,100k · 1,918k=98.4k.

Multiplying the premium with the reinsurer's share of the loss gives us a premium of CHF 23.4k.

Rate on Line

For the overall portfolio we can calculate the rate on line, which is the loss cost of the layer compared to the total loss cost, using the expected loss ratio of 55%:

```
XL_ROL <- function(Deductible, Limit, MaxMPL,
                   MeanMPL, GrossPremium, C, ULR){
  DedPerMPL <- Deductible / ifelse(MaxMPL < Deductible, Deductible,
                                   ifelse(MaxMPL<Limit, MaxMPL, Limit))
```

```r
  LossShare <- 1 - apply(cbind(DedPerMPL, C), 1,
                         function(x){
                           mbbefdExposure(x[1],
                                          b=swissRe(x[2])['b'],
                                          g=swissRe(x[2])['g'])
                           })
  NetPremium <- GrossPremium * ifelse(MaxMPL < Limit, 1,
                                      Limit/MaxMPL)
  XL_Premium <- NetPremium * LossShare
  # Rate on Line
  sum(XL_Premium)/sum(NetPremium)*ULR
}

rol <- XL_ROL(Deductible = D,
              Limit = L,
              MaxMPL = ClientData$MaxMPL,
              MeanMPL = ClientData$MeanMPLGrossLoss,
              GrossPremium = ClientData$GrossPremium,
              C=ClientData$ExposureCurve, ULR=0.55)
```

Applying the function to our data we get a rate on line of 1.55%.

Note

Differences to the Swiss Re paper are a result of rounding.

References

Bernegger, S. 1997. "The Swiss Re Exposure Curves and the MBBEFD Distribution Class." *ASTIN Bulletin* 27 (1): 99–111. http://www.casact.net/library/astin/vol27no1/99.pdf.

Guggisberg, Daniel. 2004. *Exposure Rating*. Swiss Re. http://media.cgd.swissre.com/documents/pub_exposure_rating_en.pdf.

Salzmann, Ruth E. 1963. "Rating by Layer of Insurance." *PCAS* L: 15–26. https://www.casact.org/pubs/proceed/proceed63/63015.pdf.