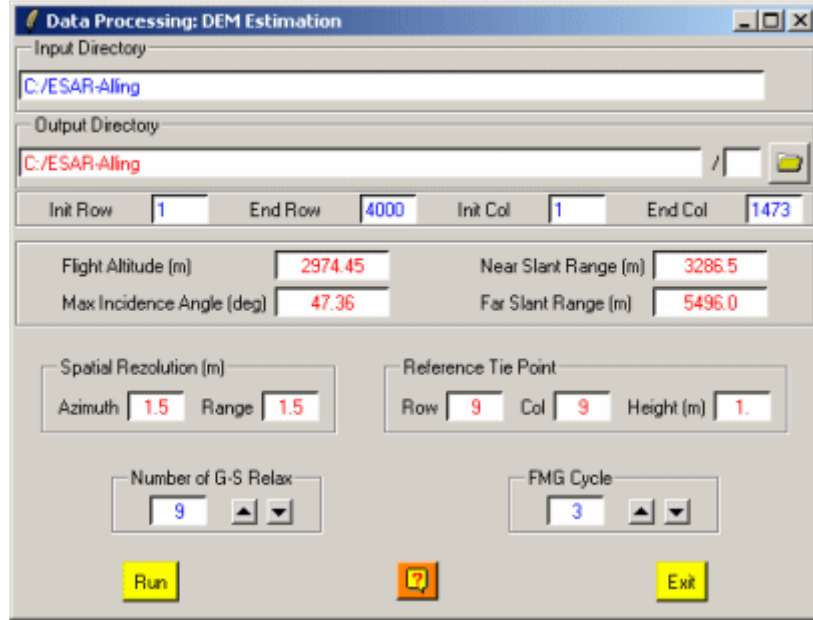


DEM Estimation



Description:

The basic flowchart of this approach is shown in Figure 1; there are three main steps which are polarization orientation angle (POA) shift estimation, slope estimation and height estimation. Following the chart this paragraph will introduce them briefly.

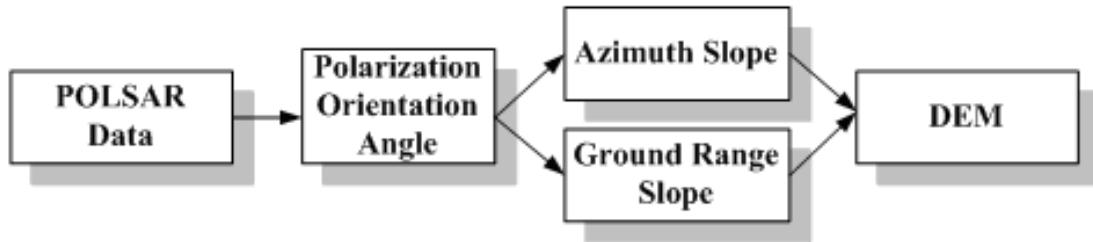


Figure 1 The basic flowchart of DEM estimation using single-pass POLSAR data

POA Estimation Using Topography-POA Shift Model & CPM [1]

The circular polarization method (CPM) has a mathematical expression to calculate the POA shift θ induced by terrain slope.

$$\theta' = \frac{1}{4} \left[\tan^{-1} \left(\frac{-4 \operatorname{Re} \langle (S_{HH} - S_{VV}) S_{HV}^* \rangle}{-\langle |S_{HH} - S_{VV}|^2 \rangle + 4 \langle |S_{HV}|^2 \rangle} \right) + \pi \right] \quad (1a)$$

$$\theta = \begin{cases} \theta' & \theta' \leq \pi/4 \\ \theta' - \pi/2 & \theta' > \pi/4 \end{cases} \quad (1b)$$

The soundness of equation (1) depends on the topography—POA shift model developed by Lee et al [2], where ω is azimuth slope, β is ground range slope, η is radar incidence angle.

$$\tan \theta = \frac{\tan \omega}{\sin \eta - \cos \eta \tan \beta} \quad (2)$$

This model is limited by terrain surface cover and radar wavelength. If the imaging area is heavily forested and L band or higher frequency band POLSAR data are used here, the POA shift result will be affected by small size things more, such as leaves and branches, not the terrain variation. P band POLSAR data is necessary to forested area, and L band is enough to barely or slightly covered area, such as vegetation or corps. It is should be noticed that the reliable POA shift estimation is the foundation of this DEM estimation method. If the POLSAR data frequency and the ground cover doesn't agree with the topography—POA shift model, this DEM estimation method will fail.

Slope Estimation Using Compensation-Lambertian Method[3]

This method is developed for azimuth and ground range slope estimation using single-pass POLSAR data. The basic idea of this approach is to combine the polarization compensation technique and Lambertian backscattering model. The intensity of SAR image pixel will be modified after compensation. The variation can be used to calculate the azimuth slope using Lambertian backscattering model. First of all, it's necessary to select the intensity term from stokes matrix which should agree with Lambertian model after compensation, growing bigger. We choose A_1 profile in polarization synthesize response 3D plot as the intensity in Lambertian model:

$$I = A_1 = \frac{m_{22} + m_{33}}{2} + \frac{\sqrt{(m_{22} - m_{33})^2 + 4m_{23}^2}}{2} \sin(4\varphi - \tan^{-1}(\frac{m_{33} - m_{22}}{2m_{23}})) \quad (3)$$

The differential equation of (3) with POA φ is used to develop the mathematical expression of circular polarization method, where m_{xy} is stokes matrix term. The Lambertian model is modified by Paquerault et al ^[4] to relate the slant range radar image intensity and the slope (azimuth slope and ground range slope).

$$I(\omega, \beta) = K\sigma_0 R_g R_a \frac{\cos(\eta) \sin^2(\eta + \beta)}{\cos(\eta + \beta)} \cos(\omega) \quad (4)$$

K is SAR system constant, σ_0 is unknown backscattering coefficient, R_g and R_a are ground range and azimuth resolution. All above parameters are independent with radar incidence angle η , azimuth slope ω and ground range slope β .

After using polarization compensation equation (5) ^[5] where θ is POA shift, we will get a new polarization coherency matrix T^{new}

$$T^{new} = UTU^T, U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta \\ 0 & -\sin 2\theta & \cos 2\theta \end{bmatrix} \quad (5)$$

From equation(2), the azimuth slope ω can be set to zero when POA shift θ is compensated to zero using (5). Equation (4) becomes

$$I(0, \beta) = K\sigma_0 R_g R_a \frac{\cos(\eta) \sin^2(\eta + \beta)}{\cos(\eta + \beta)} \quad (6)$$

Combining equation (3), (4) and (5), the azimuth slope ω can be computed by:

$$\omega = \arccos\left(\frac{I(\omega, \beta)}{I(0, \beta)}\right) = \arccos\left(\frac{2m_{22}}{m_{22} + m_{33} + \sqrt{(m_{22} - m_{33})^2 + 4m_{23}^2}}\right) \quad (7)$$

It is should be noted that the sign of ω can't be decided by (7), but the sign of POA shift θ is

the same with ω under a limitation (8), which is from (2) and (9).

$$\eta \geq \beta \quad (8)$$

$$\frac{\tan \omega}{\tan \theta} = \frac{\sin(\eta - \beta)}{\cos \beta} \quad (9)$$

This limitation can be satisfied to all of the negative ground range slope area and most of the positive one.

Bring ω , θ and η into (2), ground range slope β can be obtained by

$$\beta = \tan^{-1} \left(\frac{\sin \eta - \tan \omega / \tan \theta}{\cos \eta} \right) \quad (10)$$

As a result of $\tan \omega / \tan \theta$ in (10) enlarges the error of ω and θ , ground range slope β is not accurate for relative flat area. For solving this problem, it is necessary to add a revising step. The difference between neighbor pixels along ground range direction is valid to estimate height using full multigrid method (FMG), the correction procedure makes use of incorrect slope value to estimate height and gets the revised azimuth slope and ground range slope value from the differences of height along azimuth and ground range directions respectively.

$$\omega_r(x, y) \approx \arctan \left(\frac{H(x, y) - H(x-1, y)}{R_a} \right) \quad (11a)$$

$$\beta_r(x, y) \approx \arctan \left(\frac{H(x, y) - H(x, y-1)}{R_g} \right) \quad (11b)$$

In equations (11a) and (11b), H is height, ω_r and β_r is the azimuth and ground range slope after revising, x and y is the row and column number of matrix, R_a and R_g is the azimuth and ground range resolution.

If the value range of slope is known before processing, it will help us to make the estimated slope more accurate.

Height Estimation Using FMG Method ^[6]

In this part, we will introduce least squares slope to height estimating approach using full multigrid numerical method. This approach has been utilized to phase unwrapping in [6].

Assuming that the errors in the input slopes are Gaussian random variable, the integrated height values also have Gaussian errors ^[7]. Given the azimuth and ground range slope difference format by (11) on a rectangular grid defined by $0 \leq x \leq M-1$ and $0 \leq y \leq N-1$, we seek a height solution $H(x, y)$ on the same grid. The sum of height errors is a χ^2 statistic.

$$\chi^2 = \sum_{x,y} (H(x, y) - H(x-1, y) - \Delta_a(x, y))^2 + \sum_{x,y} (H(x, y) - H(x, y-1) - \Delta_g(x, y))^2 \quad (12)$$

$$\Delta_a(x, y) = R_a \cdot \tan(\omega(x, y)) \quad , \quad \Delta_g(x, y) = R_g \cdot \tan(\beta(x, y)) \quad (13)$$

Differentiating error function χ^2 with respect to $H(x, y)$ and setting the results equal to zero, the discrete Poisson equation of this problem can be written as (14).

$$(H(x+1, y) - 2H(x, y) + H(x-1, y)) + (H(x, y+1) - 2H(x, y) + H(x, y-1)) = \rho(x, y) \quad (14)$$

The source function on the right hand of (14) is the function of input slope.

$$\rho(x, y) = \rho_a(x, y) + \rho_g(x, y)$$

$$\rho_a(x,y) = \Delta_a(x+1,y) - \Delta_a(x,y) \quad , \quad \rho_g(x,y) = \Delta_g(x,y+1) - \Delta_g(x,y) \quad (15)$$

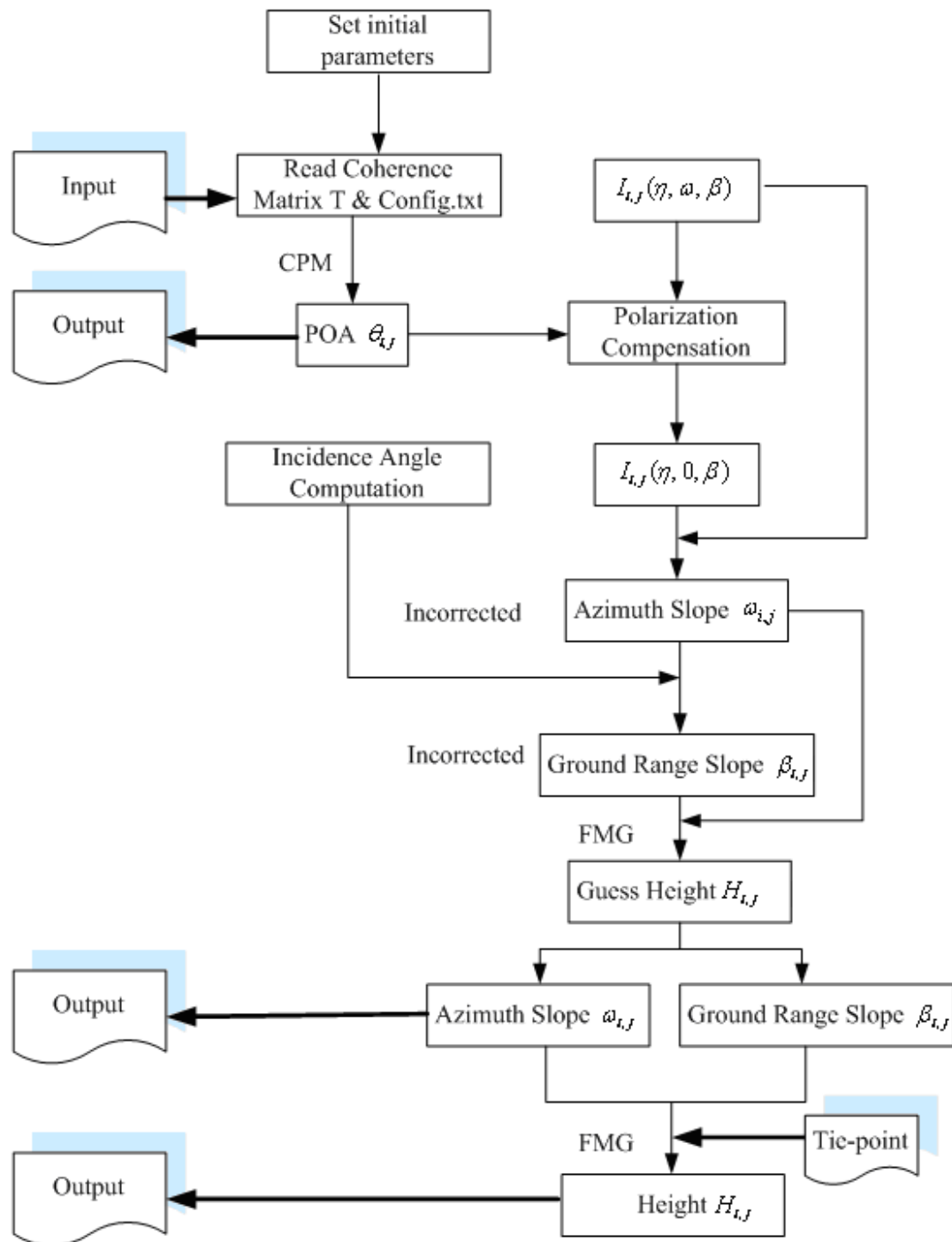
The classical numerical methods for getting Poisson equation solution such as Gauss-Seidel relaxation and successive over relaxation (SOR) can remove the spatial high-frequency components of the error quickly but the low-frequency components slowly. The multigrid approach provides a scheme for transforming the low-frequency components of the error on the fine grid into high-frequency components on the coarser grid, which will be removed quickly by Gauss-Seidel relaxation.

Validity Range of the Topography-POA Shift Model

Land Cover Type	Band
Bare or vegetation	P, L
Forest	P

References:

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- 2 J.S. Lee, D.L. Schuler, T.L. Ainsworth, and W.-M. Boerner. Polarization Orientation Estimation and Applications: a review[C]. IGRASS'2003: 428-430.
- 3 Yang LI, Wen HONG, Fang CAO, Yan-ping WANG, Yi-rong WU, "Estimation of Terrain Slope Using A Compensation-Lambertian Method from Single-Pass POLSAR Data", IEEE 27th International Geoscience and Remote Sensing Symposium, Boston, Massachusetts, U.S.A, July. 2008.
- 4 S. Paquereault, H. Maitre. A New Method for Backscatter Model Estimation and Elevation Map Computation Using Radarclinometry[C]. In: Proc. EUROPTO-SPIE Conf. SAR image Analysis, Modeling and Technique III, Barcelona, Spain, 1998, vol.3479, pp.230-241.
- 5 J.S. Lee, D.L. Schuler, T.L. Ainsworth. Polarimetric SAR Data Compensation for Terrain Azimuth Slope Variation. IEEE Transactions on Geoscience and Remote Sensing [J]. 2000, 38(5):2153-2163.
- 6 Mark D. Pritt. Phase Unwrapping by Means of Multigrid Techniques for Interferometric SAR [J]. IEEE Transactions on Geosciences and Remote Sensing, 1996, 34(3):728-738.
- 7 JIN Ya-Qiu and LUO Lin. Terrain Topographic Inversion Using Single-Pass Polarimetric SAR Image Data [J]. SPIE volume 4894, 2003, 425:433.



Comments:

Parameters written in Red can be modified directly by the user from the keyboard.

Input/Output Arguments:

Input Directory	Indicates the location of the considered Main Directory (MD) containing the polarimetric data sets to be processed.
Output Directory	Indicates the location of the processed data output directory. The default value is set automatically to : Main Directory (MD) .

Output Image Number of Rows/Columns:

The output image numbers of rows and columns are initialised to the input data set dimensions.

Input Parameters:

Flight Altitude	Flight altitude information obtained from the acquisition track information file
Max Incidence Angle	Maximum Incidence Angle information obtained from the acquisition track information file.
Near Slant Range	Near Slant Range value information obtained from the acquisition track information file.
Far Slant Range	Far Slant Range value information obtained from the acquisition track information file.
Spatial Resolution	Spatial Resolution (Range, Azimuth) value information obtained from the acquisition track information file.
Reference Tie Point (Row)	Image reference point row number. The default value is set automatically to: 9 .
Reference Tie Point (Col)	Image reference point col number. The default value is set automatically to: 9 .
Reference Tie Point (Height)	Image reference height value. The default value is set automatically to: 1 .

Number of G-S Relax:

Number of Range and Azimuth coarsest grid value. The default value is set automatically to: **9**.

FMG Cycle:

The FMG algorithm is used twice in this method: slope correction the first time, and height estimation the second time. The default value is set automatically to: **3**.

Output Files:

The output files are:

- [orientation_cir.bin](#): The polarization orientation angle (POA) shift estimation using circular polarization method (CPM). (Unit: degree)
 - [slope_a.bin](#): The azimuth slope. (Unit: degree)
 - [slope_r.bin](#): The ground range slope. (Unit: degree)
 - [height.bin](#): The absolute height referring to the tie point. (Unit: meter)
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