Function field $\mathcal{F}_H = \mathcal{F} = {}^{K(x)[y]}/{}_{(y^2 - C(x))}$.

Construction of the field-homomorphism $\lambda_{\mathcal{Q}} : \mathcal{F} \to \overline{K}((t))$. Three cases as in EK1. (write this out, the only relevant difference is $\lambda_\infty(y) \mapsto t^{-5}\sqrt{\tau}$).

The Lemmas 1 through 7 from EK1 remain valid. (write these out and prove where necessary)

So given that $(f) = \frac{**}{2\infty}$ implies $f = \alpha + \beta x$ (Lemma 7) we consider the next two higher cases. (I will use the sum notation for the divisors next time).

Minilemma 1: If f is such that $\lambda_\infty(f) = \gamma t^e + \dots$ then $\lambda_\infty(\overline{f}) = -\gamma t^e + \dots$

Proof. Let $\lambda_\infty(\overline{f}) = \widetilde{\gamma} t^e + \dots$. Since $f + \overline{f} = 2p$ and $\mathrm{ord}_\infty(p)$ is always even for $p \in K(x)$, we must have $(\gamma + \widetilde{\gamma})t^{-3} + \cdots = \sigma t^{-2} + \dots$ so $\gamma = -\widetilde{\gamma}$. □

Lemma 2: Let $f \in \mathcal{F}^*$ with $(f) = \frac{***}{3\infty}$. Then $f = \alpha + \beta x$.

Proof. Let $f = p + qy$ with $p, q \in K(t)$. We have $\lambda_\infty(f + \overline{f}) = \sigma t^{-2} + \dots$ so $p = \frac{1}{2}(f + \overline{f}) = \alpha + \beta x$.

Now, consider $f - \overline{f} = 2qy$. We get $\lambda_\infty(f - \overline{f}) = 2\lambda_\infty(q) t^{-5}\sqrt{\tau} = 2\gamma t^{-3} + \dots$ and since $\sqrt{\tau}$ is a a series of the form $1 + \dots$ we must have $\lambda_\infty(q) = \gamma t^2 + \dots$

Suppose $q \neq 0$, this implies $\frac{1}{q} = \widetilde{\delta} + \widetilde{\epsilon} x$. Rewriting to accomodate for $q = 0$ gives $q = \frac{\epsilon}{x - \delta}$ with $\delta, \epsilon \in K$ as well so

$$f = \alpha + \beta x + \frac{\epsilon}{x - \delta} y$$

However, now $\lambda_{\mathcal{Q}}(f) = \alpha + \beta(t^2 + \delta) + \epsilon\sqrt{\mu}\frac{t\sqrt{\tau}}{t^2}$ for $\mathcal{Q} = (\delta, 0)$ which is equal to $\epsilon\sqrt{\mu} t^{-1} + \dots$. If $\epsilon \neq 0$ (μ is non zero anyway) this means that $\mathcal{Q}$ is a new (true) pole for f which contradicts $(f) = \frac{***}{3\infty}$, so q must be 0. □

Lemma 3: Let $f \in \mathcal{F}^*$ with $(f) = \frac{****}{4\infty}$. Then $f = \alpha + \beta x + \gamma x^2$.

Proof. (different function f): With $\lambda_\infty(f) = \gamma t^{-4} + \dots$ so $\lambda_\infty(f - \gamma x^2)$ being of the form $\sigma t^{-3} + \dots$ we fall into the case above and the claim follows. □

Now, suppose we have the equivalent of the Haupthilfssatz from EK1 (This has yet to be proven and of course and could turn out to be quite the behemoth):

Theorem For any $\mathbf{P}_1 = \{\mathcal{Q}_1, \mathcal{Q}_2\}, \mathbf{P}_2 = \{\mathcal{Q}_3, \mathcal{Q}_4\}$ such that $\mathbf{P}_1 + \mathbf{P}_2 = \mathbf{P}_3 = \{\mathcal{Q}_5, \mathcal{Q}_6\}$ we have a function f in $\mathcal{F}$ such that $(f) = \frac{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4}{\mathcal{Q}_5\mathcal{Q}_6\infty\infty}$.

Here's the main result in hopefully legible notation:

Theorem *(Associativity)*: Given

$$\begin{aligned} & \underbrace{(\mathbf{P}+\mathbf{Q})}_{=\mathbf{T}}+\mathbf{R}=\mathbf{S} \\ \text{and} \qquad & \mathbf{P}+\underbrace{(\mathbf{Q}+\mathbf{R})}_{=\mathbf{W}}=\mathbf{S}' \end{aligned}$$

with

$$\mathbf{P}=\{\mathcal{P}_1,\mathcal{P}_2\},\mathbf{Q}=\{\mathcal{Q}_1,\mathcal{Q}_2\},\mathbf{R}=\{\mathcal{R}_1,\mathcal{R}_2\},\mathbf{T}=\{\mathcal{T}_1,\mathcal{T}_2\},\mathbf{W}=\{\mathcal{W}_1,\mathcal{W}_2\},$$
$$\mathbf{S}=\{\mathcal{A},\mathcal{B}\},\mathbf{S}'=\{\mathcal{U},\mathcal{V}\},$$
$$\mathcal{A}=(a,*),\mathcal{B}=(b,*)$$

we have a function f with the following divisor:

$$(f)=\frac{\mathcal{P}_1\mathcal{P}_2\mathcal{Q}_1\mathcal{Q}_2}{\mathcal{T}_1\mathcal{T}_2\infty\infty}\frac{\mathcal{T}_1\mathcal{T}_2\mathcal{R}_1\mathcal{R}_2}{\mathcal{A}\mathcal{B}\infty\infty}\frac{\mathcal{U}\mathcal{V}\infty\infty}{\mathcal{P}_1\mathcal{P}_2\mathcal{W}_1\mathcal{W}_2}\frac{\mathcal{W}_1\mathcal{W}_2\infty\infty}{\mathcal{Q}_1\mathcal{Q}_2\mathcal{R}_1\mathcal{R}_2}=\frac{\mathcal{U}\mathcal{V}}{\mathcal{A}\mathcal{B}}.$$

This means that $\widetilde{f}=f(x-a)(x-b)$ has divisor $(\widetilde{f})=\frac{\overline{\mathcal{A}}\,\overline{\mathcal{B}}\,\mathcal{U}\,\mathcal{V}}{4\infty}$. By the last Lemma $\widetilde{f}$ must be of the form $\kappa(x-\alpha_1)(x-\alpha_2)\in\overline{K}[x]$ and so

$$f=\frac{\kappa(x-\alpha_1)(x-\alpha_2)}{(x-a)(x-b)}.$$

But this has divisor

$$(f)=\frac{\mathcal{M}\overline{\mathcal{M}}\mathcal{N}\overline{\mathcal{N}}}{\mathcal{A}\overline{\mathcal{A}}\mathcal{B}\overline{\mathcal{B}}}.$$

So either $\mathcal{A}=\mathcal{M}$ and $\mathcal{B}=\overline{\mathcal{M}}$ or $\mathcal{A}=\mathcal{M}$ and $\mathcal{B}=\mathcal{N}$. First case implies $\mathbf{S}$ and $\mathbf{S}'$ equivalent to 0, second case implies $(f)=\frac{\mathcal{A}\mathcal{B}}{\mathcal{A}\mathcal{B}}$ and in both cases we're done.