Regaining Control

with Indexed Monads

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flatMap(Oslo) 2018

Who am I?

- Scala 3 Compiler Engineer

Who am I?

- Scala 3 Compiler Engineer
- Klarna

Functional State

The Canonical Example

```
type Seed = Long

def rng(seed: Seed): (Seed, Long)

def rbg(seed: Seed): (Seed, Boolean) = {
  val (newSeed, rand) = rng(seed)
  (newSeed, rand > 0L)
}
```

Adding Three Random Numbers

```scala
val s0 = 0L

val (s1, r0) = rng(s0)
val (s2, r1) = rng(s1)
val (_,  r2) = rng(s2)

r0 + r1 + r2
// res0: Long = 3318706044697439873
```

The Canonical Example

Avoid passing the state?

The Canonical Example

Avoid passing the state?

Get rid of boilerplate?

The Canonical Example

```
S ⇒ (S, A)
```

The State Monad

```scala
case class State[S, A](run: S ⇒ (S, A)) extends AnyVal

val nextLong: State[Seed, Long] = State(seed ⇒ rng(seed))

def nextBool: State[Seed, Boolean] = ???
```

Map

We'd like to implement map in such a way that we do not affect S

```
State[S, A] ⇒ State[S, B]
```

The State Monad

```scala
case class State[S, A](run: S ⇒ (S, A)) extends AnyVal {

  def map[B](f: A ⇒ B): State[S, B] = State {
    s0 ⇒ {
      val (s1, a) = run(s0)
      (s1, f(a))
    }
  }

}
```

The State Monad

```scala
val nextBool: State[Seed, Boolean] = nextLong.map(_ > 0L)
```

The State Monad

How do we get rid of the explicit state passing?

The State Monad

We want to reason about the A value in `State[S, A]`

(without having to worry about S!)

The State Monad

We sort of want to pull the value out, to *bind* it...

The State Monad

```scala
case class State[S, A](run: S ⇒ (S, A)) extends AnyVal {

  // ...

  def flatMap[B](f: A ⇒ State[S, B]): State[S, B] = State {
    s0 ⇒ {
      val (s1, a) = run(s0)
      f(a).run(s1)
    }
  }

}
```

Adding Three Random Numbers

```scala
val addition: State[Seed, Long] = for {
  r0 ← nextLong
  r1 ← nextLong
  r2 ← nextLong
} yield r0 + r1 + r2

addition.run(0L)
// res1: (Seed, Long) = (-7280499659394350823,3318706044697439873)
```

Cooler stuff

```scala
case class Customer(id: Long, debt: Long, name: String)

val randomCustomer: State[Seed, Customer] =
  for {
    id      ← nextLong
    debt    ← nextLong
    isHuman ← nextBool
    name    = if (isHuman) "Kim" else "Mark Zuckerberg"
  } yield Customer(id, debt, name)

randomCustomer.run(1L)._2
// res2: Customer = Customer(1,7806831264735756412,Mark Zuckerberg)
```

Are we there yet?

Stack safety?

What about effects?

What about effects?

```scala
import cats.effect.IO

def getNonce(seed: Seed): IO[(Seed, Long)] =
  IO(rng(seed))

val nextNonce: State[Seed, Long] =
  State(seed ⇒ getNonce(seed))
// <console>:18: error: type mismatch;
//  found   : cats.effect.IO[(Seed, Long)]
//     (which expands to)  cats.effect.IO[(Long, Long)]
//  required: (Seed, Long)
//     (which expands to)  (Long, Long)
//          State(seed ⇒ getNonce(seed))
//                                      ^
```

StateT

```scala
case class StateT[F[_], S, A](val run: S ⇒ F[(S, A)])

val nextNonce: StateT[IO, Seed, Long] =
  StateT(seed ⇒ getNonce(seed))
```

Stack Safety

Now depends on F[_]

Requirements on F[_]

`Functor[F]` and `FlatMap[F]`

for `map` and `flatMap`

State in Cats

```scala
import cats.Eval

type State[S, A] = StateT[Eval, S, A]
```

Where is my indexed Monad?

Also, what are indexed Monads?

Thus Far

```
S ⟹ (S, A)
```

What if?

```
I ⇒ (O, A)
```

Indexed State Monad

```scala
case class IxState[I, O, A](run: I ⇒ (O, A))
```

Yet Another Naive Implementation

```scala
case class IxState[I, O, A](run: I ⇒ (O, A)) {

  def map[B](f: A ⇒ B): IxState[I, O, B] = IxState {
    i ⇒ {
      val (o, a) = run(i)
      (o, f(a))
    }
  }

}
```

Yet Another Naive Implementation

```scala
case class IxState[I, O, A](run: I ⇒ (O, A)) {

  // ...

  def flatMap[OO, B](f: A ⇒ IxState[O, OO, B]): IxState[I, OO, B] =
    IxState {
      i ⇒ {
        val (o, a) = run(i)
        f(a).run(o)
      }
    }
}
```

Chained State Transitions

```
IxState[S1, S2, A] ⇒
IxState[S2, S3, B] ⇒
IxState[S3, S4, C] ...
```

Now we can model state transitions!

```scala
sealed trait OrderStatus
case class Initiated() extends OrderStatus
case class Received()  extends OrderStatus
case class Packed()    extends OrderStatus
case class Shipped()   extends OrderStatus
case class Delivered() extends OrderStatus
```

Helper Functions

```scala
object IxState {
  def set[I, O](o: O): IxState[I, O, Unit] =
    IxState(_ ⇒ (o, ()))
}
```

Helper Functions

```scala
def received: IxState[Initiated, Received, Unit] =
  IxState.set(Received())

def packed: IxState[Received, Packed, Unit] =
  IxState.set(Packed())

def shipped: IxState[Packed, Shipped, Unit] =
  IxState.set(Shipped())

def delivered: IxState[Shipped, Delivered, Unit] =
  IxState.set(Delivered())
```

Usage

```scala
val order = for {
  _ ← received
  _ ← packed
  _ ← shipped
  _ ← delivered
} yield ()

order.run(Initiated())
// res3: (Delivered, Unit) = (Delivered(),())
```

Static errors!

```
for {
  _ ← delivered
  _ ← packed
} yield ()
// <console>:19: error: type mismatch;
//  found   : IxState[Received,Packed,Unit]
//  required: IxState[Delivered,?,?]
//          _ ← packed
//              ^
```

Cats

```scala
class IndexedStateT[F[_], SA, SB, A](val runF: F[SA ⇒ F[(SB, A)]])
```

Wait a minute, this looks familiar...

StateT in Cats

```scala
import cats.data.IndexedStateT

type StateT[F[_], S, A] = IndexedStateT[F, S, S, A]
```

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Passing state explicitly

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$$S \Rightarrow (S, A)$$

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State[S, A]

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```
StateT[F[_], S, A]
```

The Epifani

$$State[S, A] =$$
$$StateT[Eval, S, A]$$

The Epifani

`IndexedStateT[F[_], SA, SB, A]`

The Epifani

```
StateT[F[_], S, A] =
IndexedStateT[F, S, S, A]
```