Discrete Mathematics Study Center

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- Let a be the proposition "The water is contaminated" and b be the proposition "The dolphins are sick". Translate the following expressions into English.

1. $a \rightarrow b$
2. $\neg b \rightarrow \neg a$
3. $b \vee \neg a$

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- Translate the following logical propositions into English expressions. Let a be the proposition "Jim told the truth", b be "I will jump up and down", c be "the gardener is innocent", and d be "the lawyer charges too much money".

1. $(a \wedge \neg c) \rightarrow b$
2. $\neg b \rightarrow (\neg a \wedge d)$
3. $(d \wedge c) \rightarrow \neg b$

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- In common English, often we intend to mean something that is different from what we say. For example, determine whether the following sentences intend to use an exclusive or logical "or." Explain your answer.

1. To be or not to be?
2. The menu comes with soup or salad.

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- Determine whether or not the associative law holds for $\oplus$ (exclusive or).

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- Find a logical proposition that is equivalent to $p \oplus q$ but that only uses the connectives $\vee$, $\wedge$, $\neg$. Prove your answer is correct.

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- Is the statement "I always lie" a proposition? Justify your answer.

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- One grocery store advertises that, "good food is not cheap" and another grocery store advertises that, "cheap food is not good." Are they both saying the same thing or different things? Justify your answer.

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- Determine if the following are tautologies, contradictions or contingencies. You may use truth tables.

1. $((a \vee b) \wedge (\neg a \vee c)) \rightarrow (b \vee c)$
2. $\neg(a \oplus b) \leftrightarrow ((a \wedge b) \vee \neg(a \vee b))$
3. $(\neg a \rightarrow \neg b) \rightarrow (\neg b \rightarrow \neg a)$

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- Determine if the following are tautologies, contradictions or contingencies. You cannot use truth tables to justify your answers. Use either logical equivalences or some other means that does not use truth tables.

1. $(\neg a \rightarrow (b \rightarrow c)) \leftrightarrow (b \rightarrow (a \vee c))$
2. $((a \vee c) \wedge (b \vee c)) \vee ((\neg c \vee \neg b) \wedge (\neg c \vee a))$
3. $((a \vee b) \wedge (\neg a \vee c)) \rightarrow (b \vee c)$
4. $\neg(a \oplus b) \leftrightarrow (a \leftrightarrow b)$

Show/Hide Solution

- Let a be the proposition "Google is a good search engine", b be the proposition "The web page is found". Translate the following expressions into English.

  1. $\neg a \wedge \neg b$
  2. $\neg b \rightarrow \neg a$
  3. $b \vee \neg a$
  4. $a \rightarrow b$

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- Is the statement "Help me understand this logic stuff" a proposition? Justify your answer.

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- Let p be the proposition "Linux is an operating system", q be the proposition "Firefox is a web browser". Translate the following expressions into English.

  1. $\neg q \rightarrow \neg p$
  2. $q \vee \neg p$
  3. $p \wedge \neg q$
  4. $q \rightarrow p$

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- Translate the following English expressions into logical statements. You must explicitly state what are the atomic propositions and then show their logical relation.

  1. If the hard drive crashes then the data is lost.
  2. The infrared scanner detects motion only if the intruder is in the room or the scanner is defective.
  3. The circuit board is overheating and the mouse has two buttons.
  4. If the server is down and the network connection is lost then email is not available but I play games.

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- Translate the following logical propositions into English expressions. Let m be the proposition "Tom eats pizza", n be "Tom has indigestion", s be "Tom plays football", and w be "Tom cannot run".

  1. $(s \wedge m) \rightarrow (n \vee w)$
  2. $m \rightarrow (n \vee \neg s)$
  3. $\neg w \rightarrow (\neg m \wedge s)$

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- Determine whether or not the following statements are true and explain why.

  1. If $((2+2=4)$ and $(7^2=48))$ then $6+3=8$.
  2. $6-1=5$ and $((5-3=1)$ or $(2+5=7))$.
  3. $((6-1=5)$ and $(5-3=1))$ or $2+5=7$.
  4. If $(2+4=7)$ then $(3+5=9)$.

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- Prove or disprove the following: (you may use truth tables, or logical equivalences or any other valid form of argument).

  1. $p \wedge q$ is equivalent to $\neg p \vee \neg q$
  2. $\neg(p \vee \neg q)$ is equivalent to $\neg p \vee q$
  3. $\neg(p \rightarrow \neg q)$ is equivalent to $p \wedge \neg q$
  4. $p \wedge (q \vee r)$ is equivalent to $(p \wedge q) \vee (p \wedge r)$
  5. $p \oplus q$ is equivalent to $\neg p \oplus \neg q$

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- Determine if the following are tautologies, contradictions or contingencies. Show your work. You may use truth tables.

  1. $(a \wedge b) \rightarrow (a \vee b)$
  2. $(\neg a \wedge b) \wedge (a \vee \neg b)$
  3. $(\neg c \rightarrow \neg b) \wedge \neg(c \vee \neg b)$
  4. $(\neg a \vee \neg b) \leftrightarrow ((a \vee b) \wedge \neg(a \vee \neg b))$

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- Determine if the following are tautologies, contradictions or contingencies. You cannot use truth tables to justify your answers. Use either logical equivalences or some other means that does not use truth tables. Show your work.

1. $(p \wedge q) \rightarrow (p \rightarrow q)$
2. $((m \vee n) \wedge (\neg m \vee w)) \rightarrow (n \vee w)$
3. $\neg((a \rightarrow \neg b) \rightarrow (b \rightarrow \neg a))$
4. $(\neg a \rightarrow (b \rightarrow c)) \rightarrow (b \rightarrow (a \vee c))$

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- State the converse, inverse and contrapositive of the following both logically and in English:

1. I will use an iPod only if it has a 100 GB hard drive.
2. If I use a cell phone then I will annoy people.

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- Consider the following propositions:

1. [A:] The computer in the lab uses Linux.
2. [B:] A hacker breaks into the computer.
3. [C:] The data on the computer is lost.

Translate the following propositions into English:

1. $(A \rightarrow \neg B) \wedge (\neg B \rightarrow \neg C)$.
2. $\neg(A \vee \neg B)$.
3. $C \iff (\neg A \wedge B)$.

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- Assume the following propositions are defined:

1. [T:] Tom is a good tennis player.
2. [P:] Tom plays every day.
3. [S:] Tom practices his serve.

Translate the following propositions into logical notation:

1. Although Tom practices his serve, since he does not play everyday, he is not a good tennis player.
2. Tom is not a good tennis player unless he plays every day and practices his serve.
3. Tom is a good tennis player who practices his serve but does not play every day.
4. A necessary condition for Tom to be a good tennis player is that he play every day and practices his serve.
5. Tom is a good tennis player who plays every day and practices his serve.

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- Prove that the proposition $((A \rightarrow B) \wedge (\neg C \rightarrow \neg B)) \rightarrow (\neg A \vee C)$ is a tautology.

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- Prove the proposition $(\neg(C \rightarrow A) \vee ((\neg B \rightarrow \neg A) \wedge (C \rightarrow B))) \rightarrow (\neg A \vee B)$ is a tautology using the basic equivalence relations. You should not use truth tables, but you must derive the result. Essentially, use the rules outlined in your textbook to show that this statement is always true.

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