Basic Discrete structure

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Set:- A set is a collection or aggregate of definite and distinguishable objects selected by means of some rules or description.

A set is any well defined un-ordered collection of distinct object called as elements or members of the set.
For eg:-
a) All vowel alphabet
b) All zones of Nepal
c) All odd numbers

Notation:- In general capital letters A, B, C, D... are used to denote the set and small letters a, b, c, d ... are used to denote the members of the set.
For eg:-
$A = \{a, e, i, o, u\}$

Set representation
A set can be represented by four methods:

i) Description method:- In this method, a set is specified by a verbal description.
For eg:- The set S of numbers 1, 2, 3 is represented by S = the set of positive integers less than 4.

ii) Tabulation method:- In this method a set is specified by listing all elements in a set. Thus, we can write a set S of numbers 1, 2, 3 as:-

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$S = \{1, 2, 3\}$

Note that the set $\{1, 2, 3\}$ $\{2, 3, 1\}$ and $\{3, 1, 2\}$ are same.

iii) Rule method :- In this method a set is specified by stating a characteristics property common to all elements in the set. So, we can write a set S of elements $1, 2, 3$ as

$S = \{x \rightarrow x$ is an integer and $1 \leq x \leq 3\}$

It is read as: The set of all element x such that x is a positive integer less than 4.

The rule method is suitable when there is large no. of elements in the set.

For eg:-

$S = \{x \rightarrow x$ is name of people who use colgate, tooth paste in Bharatpur$\}$

Some set terminologies:-

1) Finite set:- A set consisting of a finite number of elements is called finite set.

For eg:- S = A set of days in a week.

2) Infinite set:- A set consisting of infinite number of elements is called infinite set.

For eg:- S = A set of all even ~~no~~ numbers.

$A \subseteq B$

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3) Empty set:- A set without any element is called empty or null or void set and usually denoted by the symbol ϕ or { }.

For eg:-

$S = \{x : x$ is the male students of Maiyadevi college$\}$

Note: {0}, {ϕ} is not empty set.

4) Unit set:- A set which contains only one element is called unit or singleton set.

For eg:- $S = \{0\}$, $N = \{2\}$

5) Universal set:- A universal set is the original set which contains all elements under consideration of a particular situation denoted by U.

6) Subset:- A set that contains of some or all elements of another set is called subset of that set A. set 'A' is subset of 'B' if each element of 'A' is also an element of 'B'. We represent the subset by $\subset$ symbol.

For eg:- $A \subset B$ (A is subset of B) if and only if $x \in A$ implies that $x \in B$

For eg:-

→ If $A = \{1, 2\}$ and $B = \{1, 2, 3\}$ then $A \subset B$.

→ Every set is subset of itself ie. $A \subset B$

→ NULL set is subset of any sets i.e $\phi \subset S$.

→ If $A = \{a, b, c\}$ and $B = \{b, a, c\}$ then $A \subset B$ and $B \subset A$ ie. $A = B$.

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7) Power set :- The set of possible subsets from any set is known as power set of that set.

For eg:- If $A = \{a\}$ the possible subsets are: ϕ and $\{a\}$

Hence, No. of subsets $= 2^1$

If $A = \{a, b\}$ that possible subsets are: $\phi, \{a\}, \{b\}, \{a, b\}$

Hence, No. of subsets $= 4 = 2^2$

If $A = \{a, b, c\}$ the possible subsets are: $\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}$

Hence, No. of subsets $= 8 = 2^3$

From above example, we conclude that "a set with n elements has 2^n subsets".

8) Ordered pair: A set of two elements 'a' and 'b' written as $\{a, b\}$ is called a pair.

9) Cartesian product :- The cartesian product of two non-empty sets A and B is defined as the set of all possible ordered pairs (a, b) such that $a \in A$ and $b \in B$.

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

In general,

$$A \times B \neq B \times A$$

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Q) Let $A = \{1, 2\}$ and $B = \{3, 4, 5\}$. Construct $A \times B$ and $B \times A$ then comment the result.

→ Soln

$A = \{1, 2\}$

$B = \{3, 4, 5\}$

$A \times B = \{(1,3) (1,4) (1,5) (2,3) (2,4) (2,5)\}$

$B \times A = \{(3,1) (3,2) (4,1) (4,2) (5,1) (5,2)\}$

From the above example,

$A \times B \neq B \times A$.

Practical lab-1: WAP to find the cartesian product of two sets.

```
#include <stdio.h>
# include <conio.h>
void main ()
{
int a[10], b[10], c[10], i, j, k;
for (i=1; i<=5; i++)
{
printf("Enter %d element of set A : ", i);
scanf("%d", &a[i]);
}
for(j=1; j<=5; j++)
{
printf("Enter %d element of set B: ", j);
scanf("%d", &b[j]);
}
```

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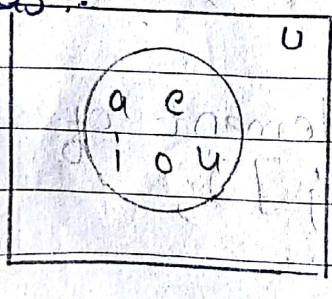
```
}
printf("\n cartesian product is:");
 printf("{ ");
 for (i=1; i<=5; i++)
 {
 for (j=1; j<=5; j++)
 {
 printf("(%d,%d)", a[i], b[j]);
 printf(", ");
 }
 }
 printf("} ");
 getch();
 }
```

venn-diagram

Set, subset and operation of set can be represented by diagram, such diagram is called venn-diagram. In sketching venn-diagram, we usually represent the universal set by a rectangle and its subset by a circle. Letus consider the set of vowels as V which is the subset of the universal set U, the English alphabets. Then V={a,e,i,o,u} is represented as:



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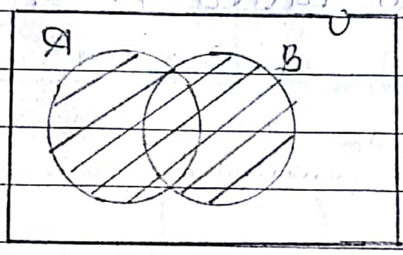
Set operations:

① Union:- The union of two sets A and B represented by $A \cup B$ is the set of only those elements which belongs to either A or B or both.

Symbolically, $A \cup B = \{x : x \in A$ or $x \in B$ or $x \in A$ and $B\}$

Venn-diagram



$A \cup B$

It is also called 'A cup B'.

For eg:-

1) If $A = \{1, 3\}$ and $B = \{p, q, r\}$ then $A \cup B = \{1, 3, p, q, r\}$

ii) If $A = \{p, q, r\}$ and $B = \{p, c, s\}$ then $A \cup B = \{p, q, r, c, s\}$

2. Intersection: The intersection of two sets A and B denoted by $A \cap B$ is the set of only those element which belongs to both sets A and B.

Symbolically,

$A \cap B = \{x : x \in A$ and $x \in B\}$

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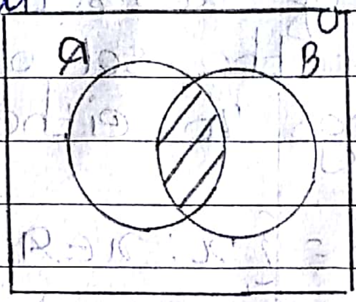
Venn-diagram



$A \cap B$

It is also called 'A cap B'.

for eg:-

1) If $A = \{2,3,4\}$ and $B = \{3,5\}$ then
$A \cap B = \{3\}$

2) If $M = \{$Ram, shyam, Hari, Krishna$\}$
$N = \{$shyam, Krishna, Mahesh$\}$
$M \cap N = \{$shyam, Krishna$\}$

3) If $P = \{1,3,5\}$ $Q = \{2,4\}$ then
$P \cap Q = \phi$

3) Complement: Let set A be the subset of universal set U then the complement of A with respect to U is the set of all those elements of U which donot belong to A denoted by $\bar{A}$ or A' or A^c.

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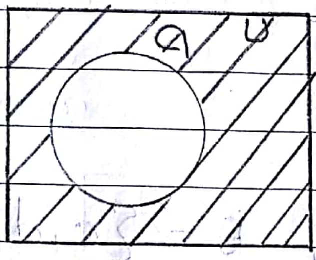
Symbolically,
$\bar{A} = \{x : x \in U \text{ and } x \notin A\}$

venn-diagram.



$\bar{A}$

For eg:-
1) If $U = \{1,2,3,4,5\}$ and $A = \{2,4\}$ then
$\bar{A} = \{1,3,5\}$

2) If U = set of English alphabets V = set of vowel letters then.
$\bar{V}$ = set of consonants.

Difference:- Let A and B be two sets and each set is the subset of a universal set U. Then A difference B denoted by A-B is the set of all those elements which belongs to A but not B.
Symbolically,
$A-B = \{x : x \in A \text{ and } x \in B\}$
$B-A = \{x : x \in B \text{ and } x \in A\}$

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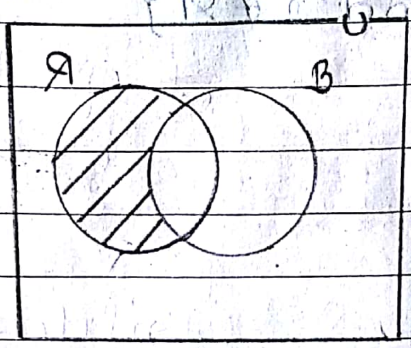
Venn-diagram.



A-B

For eg:- If A={a,b,x,y} B = {c,d,x,y} then
A-B = {a,b}
B-A = {c,d}
∴ A-B ≠ B-A.

Practical lab 2:
* WAP to print the union of any two set

```
#include <stdio.h>
#include <conio.h>
main( )
{
int a[10], b[10], c[10], m, n, i, j, k=0,
flag=0;
printf ("Enter no. of elements of set A: ");
scanf ("%d", &m);
for (i=0; i<m; i++)
{
printf ("Enter element of set A");
scanf ("%d", &a[i]);
}
printf ("Enter no. of elements of set B: ");
```

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```
scanf("%d", &n);
for(i=0; i<n; i++)
{
printf("Enter element of set B:");
scanf("%d", &b[i]);
printf("Union is:\n");
for(i=0; i<m; i++)
{
c[k] = a[i];
k++;
}
for(i=0; i<n; i++)
{
flag = 0;
for(j=0 ; j<m; j++)
{
if(b[i]==c[j])
{
flage=1;
break;
}
}
if(flag==0)
{
c[k] = b[i];
k++;
}}
printf("{");
for(i=0; i<k; i++)
{
```

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```
printf("%d", c[i]);
}
printf("}");
getch();
}
```

Set identities:

1) Identity law:

a) $A \cup \phi = A$

b) $A \cup U = U$

c) $A \cap \phi = \phi$

d) $A \cap U = A$

2) Idempotent law:

a) $A \cup A = A$

b) $A \cap A = A$

3) Commutative law:

a) $A \cup B = B \cup A$

b) $A \cap B = B \cap A$

4) Associative law:

a) $A \cup (B \cup C) = (A \cup B) \cup C$

b) $A \cap (B \cap C) = (A \cap B) \cap C$

5) Distributive law:

a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

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6. Complement law:
a) $\overline{\overline{A}} = A$
b) $A \cup \overline{A} = U$
c) $A \cap \overline{A} = \phi$
d) $\overline{\phi} = U$
e) $\overline{U} = \phi$

8) De-Morgan law
a) $(A \cup B)' = A' \cap B'$
b) $(A \cap B)' = A' \cup B'$

7. Set difference
a) $A - B = A \cap \overline{B}$
b) $U - A = \overline{A}$
c) $A - U = \phi$
d) $A - \phi = A$
e) $\phi - A = \phi$
f) $A - A = \phi$

Q) Given $A = \{1, 3, 5\}$, $B = \{0, 1, 2, 3\}$ and $C = \{0, 1, 5\}$ verify the distributive law.

$\Rightarrow$ From,

distributive law, $A \cup (B \cap C) = (A \cup B) \cap C$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Taking L.H.S.

$A \cup (B \cap C) = A \cup \{0, 1, 2, 3\} \cap \{0, 1, 5\}$

$= A \cup \{0, 1\}$

$= \{1, 3, 5\} \cup \{0, 1\}$

$= \{0, 1, 3, 5\}$

R.H.S.

$(A \cup B) = \{1, 3, 5\} \cup \{0, 1, 2, 3\}$

$= \{0, 1, 2, 3, 5\}$

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$(A \cup C) = \{1,3,5\} \cup \{0,1,5\}$
$= \{0,1,3,5\}$

Now,
$(A \cup B) \cap (A \cup C)$
$\{0,1,2,3,5\}$ $\{0,1,3,5\}$
$\{0,1,3,5\}$
proved//

ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Taking L.H.S.
$A \cap (B \cup C) = \{1,3,5\} \cap \{0,1,2,3,5\}$

$= \{1,3,5\}$

Taking R.H.S.
$(A \cap B) \cup (A \cap C) = \{1,3\} \cup \{1,5\}$
$= \{1,3,5\}$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ proved//

2. Given the sets.
$U = \{x : x$ is the positive Integer less than 16$\}$
$A = \{5,10,15\}$
$B = \{2,4,6,8,10\}$
$C = \{1,5,9,11,15\}$

Find.
a) $A \cap B$ b) $A \cup B \cup C$
$\rightarrow \{10\}$ $\Rightarrow \{1,2,4,5,6,8,9,10,11,15\}$

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c) $A \cap B \cap C$

$= \phi$

d) $(\overline{A \cap B \cap C})$

$\Rightarrow \{1, 2, 4, 5, 6, 8, 9, 10, 11, 15\}$

e) $A' \cap B'$

$\Rightarrow \{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14\} \cap \{1, 3, 5, 7, 9, 11, 12, 13, 14, 15\}$

$= \{1, 3, 7, 9, 11, 12, 13, 14\}$

f) $A' \cup C$

$= \{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14\} \cup \{1, 5, 9, 11, 15\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14\}$

g) $A' \cup B$

$\Rightarrow \{1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

h) $A' \cap B$

$\Rightarrow \{2, 4, 6, 8\}$

Q Prove that $\overline{A \cap B} = \bar{A} \cup \bar{B}$ using set builder notation

$\Rightarrow$ Soln

$\overline{A \cap B} = \{x : x \notin (A \cap B)\}$

$= \{x : \neg (x \in A \wedge x \in B)\}$

$= \{x : \neg (x \in A) \vee \neg (x \in B)\}$

$= \{x : x \notin A \vee x \notin B\}$

$= \{x : x \in \bar{A} \vee x \in \bar{B}\}$

$= (\bar{A} \cup \bar{B})$ proved//

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1) prove that $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Sol^n

$\overline{A \cup B} = \{x : x \notin (A \cup B)\}$

$= \{x : 7(x \in A) \cup \{x \in B)\}$

$= x : 7(x \in A) \cap 7(x \in B)\}$

$= \{x : x \notin A \wedge x \notin B\}$

$= \{x : x \in \overline{A} \wedge x \in \overline{B}\}$

$= \overline{A} \cap \overline{B}$

2) Let A, B, C are sets show that

$\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$

⇒ Sol^n

$\overline{A \cup (B \cap C)} = \overline{A} \cap (\overline{B \cap C})$ [De-Morgan's law]

$= \overline{A} \cap (\overline{B} \cup \overline{C})$ [second De-morgan law for intersec]

$= (\overline{B} \cup \overline{C}) \cap \overline{A}$ [commutative for intersec]

$= (\overline{C} \cup \overline{B}) \cap \overline{A}$ [" for union]

proved

Inclusion and Exclusion:

Let A and B be any two disjoint sets then we extensively use inclusion exclusion principle. Given sets A and B the union of A and B is given by

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Similarly,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

* There are 345 students at a college who have taken the course in calculus, 212 who have taken the course in discrete maths and 188 who have taken course in both calculus and discrete maths. How many students have taken the course in either calculus or discrete maths?

Sol^n

$n(C) = 345$

$n(D) = 212$

$n(C \cap D) = 188$

$n(C \cup D) = ?$

$$n(C \cup D) = n(C) + n(D) - n(C \cap D)$$

$$= 345 + 212 - 188$$

$$= 369$$

Computer representation of set:

We specify the arbitrary ordering of elements of U, for instance, $a_1, a_2 \ldots a_n$ represent a subset A of U with bit string of length n where i^{th} bit in this string is 1 if

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$a_i \in A$ and 0 if $a_i \notin A$
For eg:
Let U $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
find.

a) Bit string which represent set of all odd integers.

b) Bit string which represent set of all even integers.

c) Bit string which represent the set not exceeding 5 in U.

Sol^n

a) $S = \{1010101010\}$

b) $S = \{0101010101\}$

c) $S = \{1111100000\}$

2. The bit string for the set $A = \{1, 3, 5, 7, 9\}$ is $\{1010101010\}$ what is the bit string and set of $\bar{A}$?

→ We have,

$A = \{1, 3, 5, 7, 9\}$

i.e. $A = \{1010101010\}$

$\bar{A} = \{0101010101\}$

$\bar{A} = \{2, 4, 6, 8, 10\}$

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The bit strings of the set A = {1, 2, 3, 4, 5} and B = {1, 3, 5, 7, 9} are 1111100000 and 1010101010. Find union and intersection of set using given bit strings.

Sol^n

Given

A = {1, 2, 3, 4, 5} i.e. 1111100000
B = {1, 3, 5, 7, 9} i.e. 1010101010

A ∪ B =
1111100000
1010101010
1111101010

A ∪ B = {1, 2, 3, 4, 5, 7, 9}

A ∩ B =
1111100000
1010101010
1010100000

∴ A ∩ B = {1, 3, 5}

Lab

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3. WAP to print the intersection of two set

```
#include <stdio.h>
#include <conio.h>
main()
{
int a[10], b[10], c[10], i, j, k, m, n, flag=0;
printf("Enter no of elements of set A:");
scanf("%d", &m);
for(i=0; i<m; i++)
{
printf("Enter element of set ");
scanf("%d", &a[i]);
}
printf("Enter element of set B:");
scanf("%d", &n);
for(i=0; i<n; i++)
{
printf("Enter element of set ");
scanf("%d", &b[i]);
}
for(i=0; i<m; i++)
{
flag=0;
for(j=0; j<n; j++)
{
if (a[i]==b[j])
{
flag=1;
break;
```

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```
    }
  }
if (flag==1)
 {
  c[k]=a[i];
   k++;
   }
  }
if (k==0)
 {
printf(" set is null");
 printf("{");
 }
 else
  {
printf("Intersection is:");
printf("{ ");
for(i=0; i<k; i++)
 {
  printf("%d", c[i]);
   }
 }
 printf("} ");
  getch();
   }
```

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1) Let $A = \{1,2,3,4,5\}$ $B = \{2,3,6\}$ Find.
a) $A \cup B$ b) $A \cap B$ c) $A - B$ d) $B - A$

⇒ a) $A \cup B$
Here, $A = \{1,2,3,4,5\}$
$B = \{2,3,6\}$
Now,
$A \cup B = \{1,2,3,4,5\} \cup \{2,3,6\}$
$= \{1,2,3,4,5,6\}$

b) $A \cap B$
⇒ $A \cap B = \{1,2,3,4,5\} \cap \{2,3,6\}$
$= \{2,3\}$

c) $A - B$
⇒ $A - B = \{1,2,3,4,5\} - \{2,3,6\}$
$= \{1,4,5\}$

d) $B - A$
⇒ $B - A = \{2,3,6\} - \{1,2,3,4,5\}$
$= \{\cancel{2}, 6\}$

2. Let A and B be sets. show that
a) $A \cup B = B \cup A$
⇒ Sol^n
$A \cup B = \{x : x \in (A \cup B)\}$
$= \{x : x \in A \vee x \in B\}$
$= \{x : x \in B \vee x \in A\}$
$= \{x : x \in (B \cup A)\} = B \cup A$

proved

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b) $A \cap B = B \cap A$

⇒ Sol^n $A \cap B = \{x : x \in (A \cap B)\}$

$= \{x : x \in A \wedge x \in B\}$

$= \{x : x \in B \wedge x \in A\}$

$= \{x : x \in (B \cap A)\}$

$= B \cap A$

proved//

3. Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$ and $A \cap B = \{3, 6, 9\}$

⇒ Sol^n Here,

$A - B = \{1, 5, 7, 8\} \Rightarrow A_0 = \{1, 5, 7, 8\}$

$B - A = \{2, 10\}$

$B_0 = \{2, 10\}$

$A \cap B = \{3, 6, 9\}$

Now,

$A = A_0 \cup (A \cap B)$

$= \{1, 5, 7, 8\} \cup \{3, 6, 9\}$

$= \{1, 3, 5, 6, 7, 8, 9\}$

$B = B_0 \cup (A \cap B)$

$= \{2, 10\} \cup \{3, 6, 9\}$

$= \{2, 3, 6, 9, 10\}$

4. Show that if A, B, C are three sets then:

a) $\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$

⇒ $\overline{A \cup B \cup C} = \{x : x \notin (A \cup B \cup C)\}$

$= \{x : \neg(x \in A \vee x \in B \vee x \in C)\}$

$= \{x : \neg(x \in A) \wedge \neg(x \in B) \wedge \neg(x \in C)\}$

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$\Rightarrow \{x : x \notin A \wedge x \notin B \wedge x \notin C\}$

$= \{x : x \in \bar{A} \cap \bar{B} \cap \bar{C}\}$

$= \bar{A} \cap \bar{B} \cap \bar{C}$

ii) $\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$

$\Rightarrow$ Soln

$\overline{A \cap B \cap C} = \{x : x \notin (A \cap B \cap C)\}$

$= \{x : \neg(x \in A \wedge x \in B \wedge x \in C)\}$

$= \{x : x \notin A \vee x \notin B \vee x \in C\}$

$= \{x : x \in \bar{A} \cup \bar{B} \cup \bar{C}\}$

$= \bar{A} \cup \bar{B} \cup \bar{C}$

proved//

5. Let $A = \{0, 2, 4, 6, 8, 10\}$ $B = \{0, 1, 2, 3, 4, 5, 6\}$ and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find:

a) $A \cap B \cap C$

$\Rightarrow$ Here,

$A = \{0, 2, 4, 6, 8, 10\}$

$B = \{0, 1, 2, 3, 4, 5, 6\}$

$C = \{4, 5, 6, 7, 8, 9, 10\}$

Now,

$A \cap B \cap C = \{4, 6\}$

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b) $A \cup B \cup C$

⇒ $A \cup B \cup C = \{0,1,2,3,4,5,6,7,8,9,10\}$

c) $(A \cup B) \cap C$

⇒ Here,

$A \cup B = \{0,2,4,6,8,10\} \cup \{0,1,2,3,4,5,6\}$
$= \{0,1,2,3,4,5,6,8,10\}$

Also,

$(A \cup B) \cap C = \{0,1,2,3,4,5,6,8,10\} \cap \{4,5,6,7,8,9,10\}$
$= \{4,5,6,8,10\}$

d) $(A \cap B) \cup C$

⇒ Here,

$(A \cap B) = \{0,2,4,6\}$

Also,

$(A \cap B) \cup C = \{0,2,4,6\} \cup \{4,5,6,7,8,9,10\}$
$= \{0,2,4,5,6,7,8,9,10\}$

6. Suppose that the universal set is $U = \{1,2,3,4,5,6,7,8,9,10\}$. Find the set specified by each of these bit strings.

a) 1111001111

⇒ $\{1,2,3,4,7,8,9,10\}$

b) 0101111000

⇒ $\{2,4,5,6,7\}$

c) 1000010011

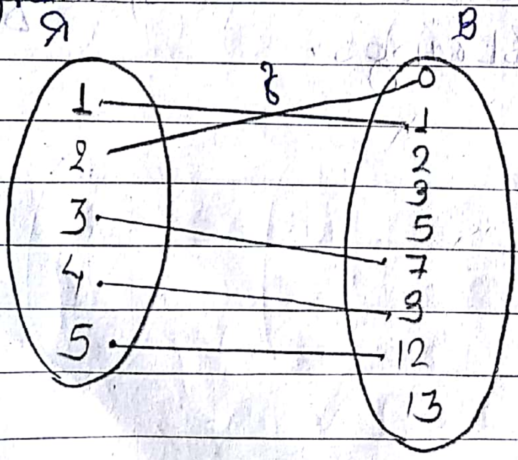
⇒ $\{1,6,9,10\}$

Function:

Let A and B be two non-empty sets. A function f from A to B is a set of ordered pairs with the property that for each element x in A there is a unique element y in B. The set A is called the domain of the function and set B is called co-domain. If $(x, y) \in f$, it is customary to write $y = f(x)$, y is called the image of x and x is the pre-image of y. The set consisting all the images of the elements of A under the function f is called range of f which is denoted by $f(A)$.

For example:

Let $A = \{1, 2, 3, 4, 5\}$ $B = \{0, 2, 3, 5, 7, 9, 12, 13\}$ and $f = \{(1,1)(2,0)(3,7)(4,9)(5,12)\}$ then f is a function from A to B because each element of A has an unique image in B which can be expressed by diagram.



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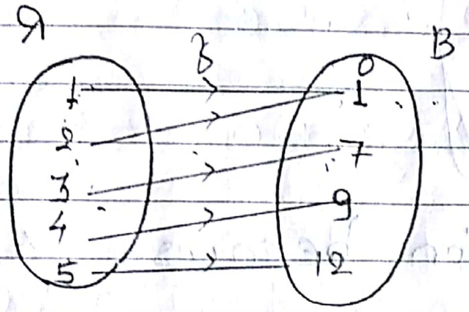
Q) Identify whether the following are functions are If not give reason.

→ Given,

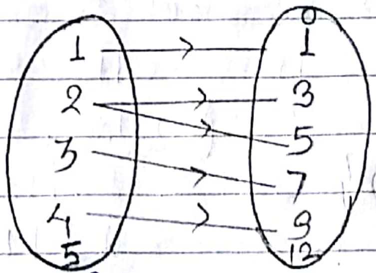
$A = \{1, 2, 3, 4, 5\}$ $B = \{0, 1, 2, 3, 5, 7, 9, 12, 13\}$

i) $f = \{(1,1)(2,1)(3,7)(4,9)(5,12)\}$



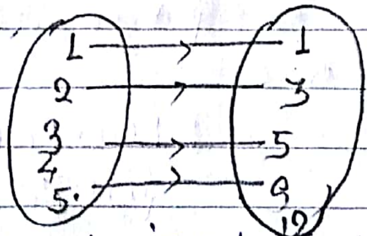
∵ It is function because each element x in A there is unique element y in b.

ii) $f = \{(1,1)(2,3)(2,5)(3,7),(4,9)\}$



∵ It is not function because domain have 2 image in range.

iii) $f = \{(1,1)(2,3)(3,5)(5,9)\}$



∵ It is not function.

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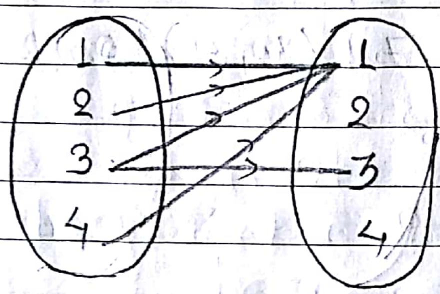
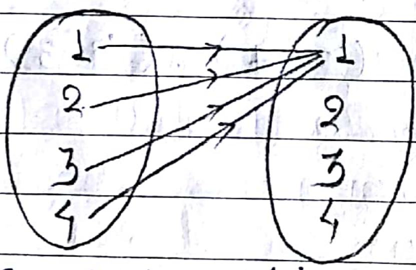
Q) Let $X = \{1,2,3,4\}$. Determine whether or not each relation below is a function from X into X or not.

i) $f = \{(1,1)(2,1)(3,1)(4,1)(3,3)\}$



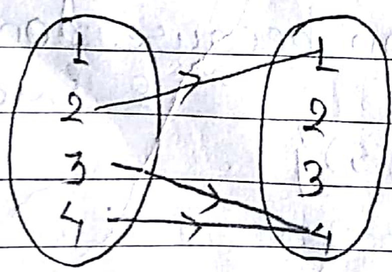
It is not function because 3 have two image.

ii) $f = \{(1,1)(2,1)(3,1)(4,1)\}$



It is a function.

iii) $f = \{(2,1)(3,4)(4,4)\}$



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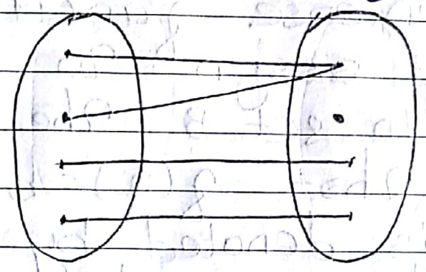
Injective (one to one) function:

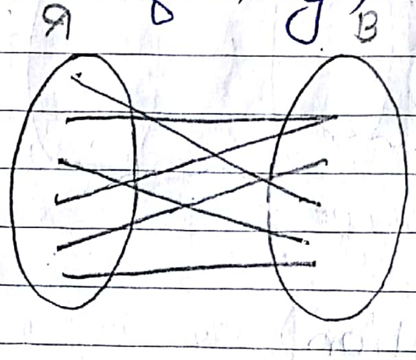
A function from A to B is one to one if for all $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$ implies $x_1 = x_2$. We can express that x is one to one using quantifiers as $\forall x_1 \forall x_2$ $(f(x_1) = f(x_2)) \Rightarrow x_1 = x_2$ where UOD is the domain of the function.



Surjective function (onto)

A function f from A to B is onto function if every element of B is the image of some element in A. We can express that f is surjective using quantifiers as:

$\forall y \exists x \, (f(x) = y)$



Bijective function

A function f from A to B is bijective function if it is both injective and surjective.

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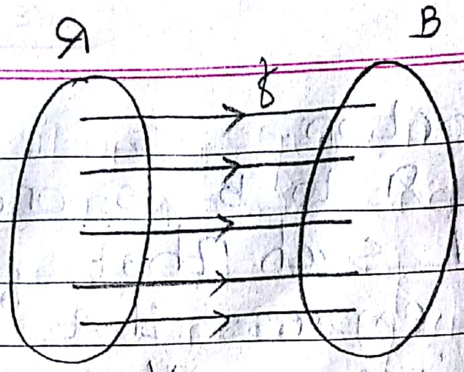
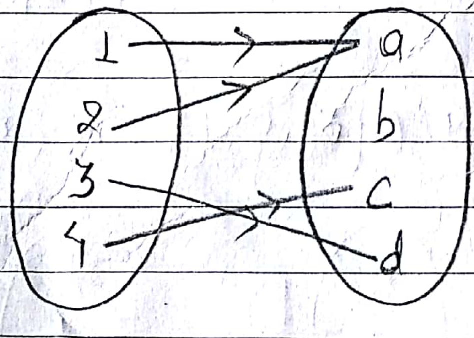


Inverse of function:

Let $f: A \to B$ be a function which is bijective, then the inverse function of f is the function that assign to an element b belonging to set B the unique element a in A such that $f(a) = b$. The inverse function of f is denoted by f^{-1}.
Hence, $f^{-1}(b) = a$ when $f(a) = b$.

Q) Let $A = \{1, 2, 3, 4\}$ $B = \{a, b, c, d\}$ and let $f = \{(1, a)\ (2, a)\ (3, d)\ (4, c)\}$. show that f is a function and state with a reason, whether f is invertible or not.

⇒ Sol^n



It is a function

Now, to find the inverse of f i.e. f^{-1}

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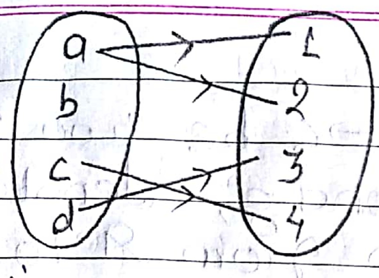


$\therefore f^{-1}$ is not a function because the object has a two images $\{1,2\}$.

Q) Find the inverse of the functions:

i) $f(x) = 3x - 7$

ii) $f(x) = x^2$

i) $f(x) = 3x-7$

$y = 3x - 7$

$\frac{y+7}{3} = x$

$\therefore f^{-1}(y) = \frac{y+7}{3}$ Interchange the value.

$\therefore f^{-1}(x) = \frac{x+7}{3}$

ii) $f(x) = x^2$

Let $y = x^2$

$x = \sqrt{y}$

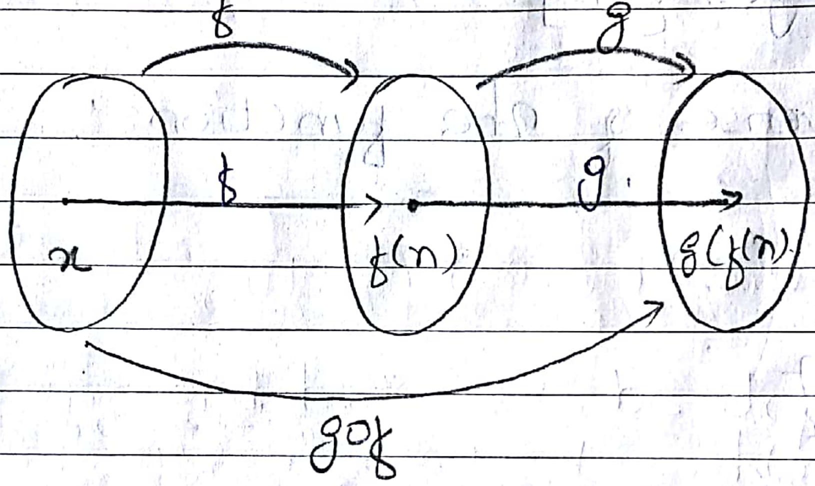
Interchanging y and x

$y = \sqrt{x}$

$f^{-1}(x) = \sqrt{x}$

Composition of function:

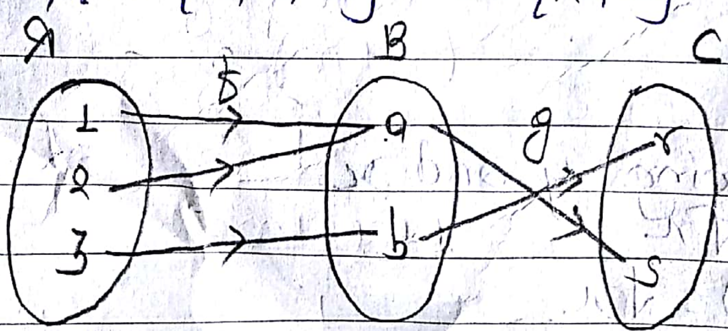
Let $f: A \to B$ and $g: B \to C$ be two functions. The composition of f and g, denoted by $g \circ f$ is a new function from A to C defined by $g \circ f(x) = g(f(x))$ for all $x \in A$



Q) Let $A = \{1, 2, 3\}$ $B = \{a, b\}$ and $C = \{r, s\}$ and $f: A \to B$ defined by $f(1) = a$, $f(2) = a$, $f(3) = b$ and $g: B \to C$ defined by $g(a) = s$ $g(b) = r$. Find the composition function $g \circ f: A \to C$

⇒ Soln

Given, $A = \{1, 2, 3\}$ $B = \{a, b\}$ $C = \{r, s\}$



$g \circ f(1) = g(a) = s$
$g \circ f(2) = g(a) = s$
$g \circ f(3) = g(b) = r$

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Q) Let the function f and g be defined by $f(x) = 2x+1$ and $g(x) = x^2 - 2$. Find $g \circ f$.

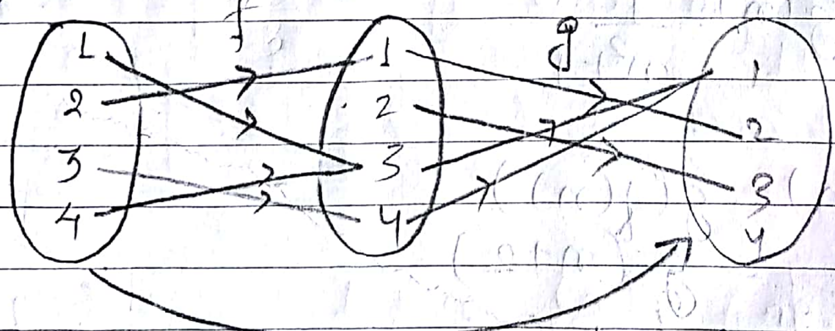
⇒ Here,

$$f(x) = 2x+1$$
$$g(x) = x^2 - 2$$
$$g \circ f(x) = ?$$
$$g \circ f(x) = g(f(x))$$
$$= g(2x+1)$$
$$= (2x+1)^2 - 2$$
$$= 4x^2 + 4x + 1 - 2$$
$$= 4x^2 + 4x - 1$$

$\therefore g \circ f(x) = 4x^2 + 4x - 1$

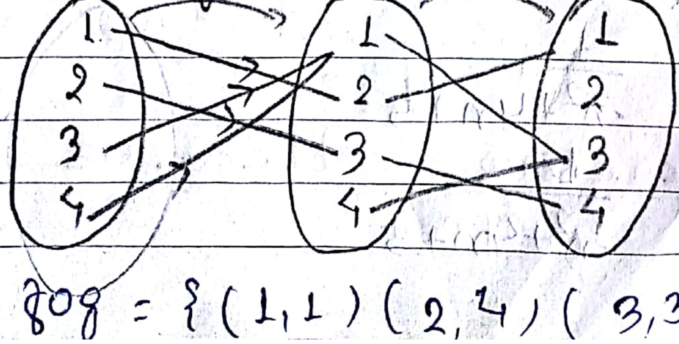
Q) Let $V = \{1, 2, 3, 4\}$ and let $f = \{(1,3)(2,1)(3,4)(4,3)\}$ and $g = \{(1,2)(2,3)(3,1)(4,1)\}$. Find

a) $g \circ f$



$g \circ f = \{(1,1)(2,2)(3,1)(4,1)$

b) $f \circ g$



$f \circ g = \{(1,1)(2,4)(3,3)(4,3)\}$

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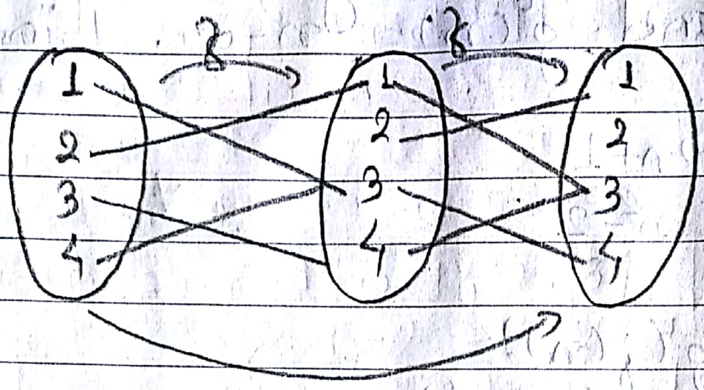
c) $g \circ f$



$g \circ f$.

$\therefore \quad g \circ f = \{(1,4)\ (2,3)\ (3,3)\ (4,4)\}$

Q) Let f, g and h : R ⟶ R be defined as : $f(x) = x+2$, $g(x) = \frac{1}{x^2+1}$, $h(x) = 3$. Find

a) $g \circ f(x)$.

= Here, $f(x) = x+2$

$g(x) = \frac{1}{x^2+1}$

Now,

$$g \circ f(x) = g(f(x))$$

$$= g(x+2)$$

$$= \frac{1}{(x+2)^2+1}$$

$$= \frac{1}{x^2+4x+5}$$

$$\therefore \quad g \circ f(x) = \frac{1}{x^2+4x+5}$$

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b. $gof^{-1}og(x)$

$\Rightarrow$ Here, $f(x) = x+2$

$y = x+2$

$y-2 = x$

Interchanging the value

$\therefore f^{-1}(x) = x-2$

Now, $gof^{-1}og(x) = gof^{-1}(x+2)$

$= g(x+\cancel{2}-\cancel{2})$

$= \frac{1}{x^2+1}$

c) $hogog(x)$

$\Rightarrow$ $hogof(x) = hog(f(x))$

$= hog(x+3)$

$= h\left(\frac{1}{(x+3)^2+1}\right)$

$= 3.$

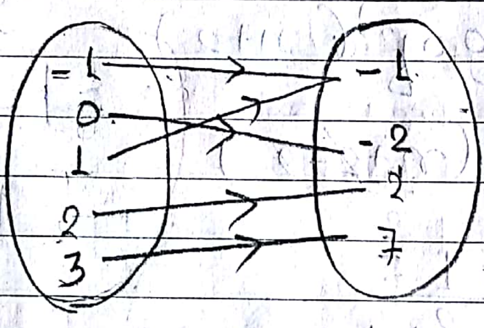
e) If $x = x^2 - 2$ and the domain of function is $\{-1, 0, 1, 2, 3\}$. Find the range of function. Is it one to one.

$\Rightarrow$ Here,

$$x = x^2 - 2$$

Domain : $\{-1, 0, 1, 2, 3\}$

Range = $\{-1, -2, -1, 2, 7\}$



$\therefore$ It is not one to one function

f) If the function $f: R \to R$ is defined by $f(x) = x^2 + 1$ for $x \in R$ and $x \geq 0$. Find $f^{-1}(8)$ and $f^{-1}(17)$

$\Rightarrow$ Soln

Given, $f(x) = x^2 + 1$ Let $f(x) = y$

$\therefore y = x^2 + 1$ $\therefore y = f^{-1}(x)$

$$y - 1 = x^2$$

$$x = \sqrt{y-1}$$

Interchanging the value

$$y = \sqrt{x-1}$$

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$\therefore f^{-1}(x) = \sqrt{x-1}$

Now,

$f^{-1}(8) = \sqrt{8-1}$
$= \sqrt{7}$

$\therefore f^{-1}(17) = \sqrt{17-1}$
$= \pm 4$

Functions in computer science:

1) Floor function:- Let x is a real number then $\lfloor x \rfloor$ is called a floor function of x, assigns to a real number x the largest integer that is less than or equal to x. This function rounds x down to the closest integer less than or equal to x. The floor function is often also called the greater integer function.

For eg:-

i) $\lfloor 8 \rfloor = 8$,
ii) $\lfloor 6.02 \rfloor = 6$
iii) $\lfloor -8.5 \rfloor = -9$
iv) $\lfloor -4 \rfloor = -4$
v) $\left\lfloor \frac{1}{2} \right\rfloor = 0$
vi) $\left\lfloor -\frac{1}{2} \right\rfloor = -1$

II) ceiling function:- If x is a real number $\lceil x \rceil$ is called the ceiling function of x,

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assign to the real number x the smallest integer that is greater than or equal to x. This function rounds x up to the closest integer greater than or equal to x.

for eg:-

i) $\lceil 8 \rceil = 8$

v) $\lceil \frac{1}{2} \rceil = 1$

ii) $\lceil 6.02 \rceil = 7$

vi) $\lceil -\frac{1}{2} \rceil = 0$

iii) $\lceil -8.5 \rceil = -8$

iv) $\lceil -4 \rceil = -4$

Q) Calculate

i) $\lfloor 8.3 \rfloor$
$\Rightarrow$ 8

ii) $\lfloor -8.7 \rfloor$
$\Rightarrow$ -9

iii) $\lceil -5.9 \rceil$
$\Rightarrow$ -5

iv) $\lceil \frac{1}{3} \rceil$
$\Rightarrow$ 1

v) $\lceil \log_2 51 \rceil$
$= \lceil \frac{\log 51}{\log 2} \rceil \Rightarrow 6$

vi) $\lceil \log_3 29 \rceil$
$\Rightarrow$ 4

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Q) Prove or disprove the statement $\lceil x+y \rceil = \lceil x \rceil + \lceil y \rceil$

⇒ Let, $x = 1.2$

$y = 3.2$

$\lceil x+y \rceil = \lceil x \rceil + \lceil y \rceil$

$\lceil 1.2 + 3.2 \rceil = \lceil 1.2 \rceil + \lceil 3.2 \rceil$

$\lceil 4.4 \rceil = 2 + 4$

$5 = 6$

Hence, $\lceil x+y \rceil \neq \lceil x \rceil + \lceil y \rceil$ disproved//

Fuzzy set:

Let X be a space of points with a generic element of X denoted by x. Thus $X = \{x\}$. A fuzzy set A in X is characterized by a membership function $f_A(x)$ which associates with each point in X a real number in the interval [0, 1] with the value of $f_A(x)$ at x representing the grade of membership of x in A. Thus, the nearer the value of $f_A(x)$ to unity, the higher the grade of membership of x in A.

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2. Union:- The membership functions of the union of two fuzzy sets A and B with membership functions μ_A and μ_B respectively is defined as the maximum of two individual membership functions which is called maximum criteria.

$$\mu_{A \cup B} = \max(\mu_A, \mu_B)$$

X	Y	Max (x,y)
0	0	0
0	1	1
1	0	1
1	1	1
0.2	0.5	0.5
0.7	0.2	0.7
0.6	0.6	0.6

Complement

The membership function of the complement of a Fuzzy set A with membership function μ_A is defined as the negation of the specified membership function. This is called the negation criteria.

$$\mu_A' = 1 - \mu_A$$

X	1-x
0	1
1	0
0.2	0.8
0.7	0.3
0.6	0.4