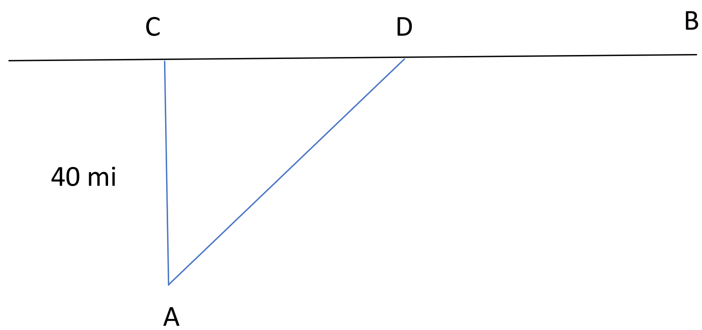
**PROBLEM SET**

**DIFFERENTIAL CALCULUS**

**TIME RATES**

1. A man in a motor boat at A receives a message at noon calling him to B(see figure). A bus making 40 mph leaves C, bound for B at 1:00 PM > If AC = 40 miles, what must be the speed of the boat to enable the man to catch the bus.




2. Water is flowing into a vertical cylindrical tank at the rate of 24 cu.ft. per minute. If the radius of the tank is 4 ft., how fast is the surface rising?
3. Water flows into a vertical cylindrical tank at the rate of 12 cu.ft., the surface rises at 6 inch per minute. Find the radius of the tank.
4. A triangular through is 10 ft long, 6 ft across the top and 3 ft deep. If the water flows in at the rate of 12 cu.ft per min., find how fast is the surface is rising when the water is 6 in deep.
5. A ladder 20 ft. long leans against a vertical wall. If the top slides down at the rate of 2 ft. per second, find the rate of change of the slope of the ladder.
6. A man 6 ft. tall walks away from a lamp post 16 ft. high at the rate of 5 mph., find how fast does the shadow lengthen.
7. A boy on a bike rides north 5 miles, then turns east. If he rides 10 mph, at what rate was his distance to the starting point changing 2 hours after he left that point.
8. A train starting at noon, travels north at 40 mph. Another train, starting from the same point at 2 PM travels east at 50 mph, how fast is the distance between them increasing.
9. A trapezoidal through is 10 ft long, 4 ft wide at the top, 2 ft wide at the bottom and 2 ft deep. If water flows in at 10 cu. ft per minute, find how fast the water surface is rising , when the water is 6 inches deep.

10. For the through in problem 9, find how fast the water surface is rising when the water is 1 ft. deep.
11. A light at eye level stands 20 ft. from a house and 15 ft from the path leading from the house to the street. A man walks along the path at 6 ft per sec. How fast does his shadow move along the wall when he is 5 ft. from the house.
12. In problem 11, when the man is 5 ft from the house, find the rate of change of that portion of his shadow that lies on the ground.
13. A light is placed on the ground 30 ft. from the building. A man 6 ft tall walks from the light toward the building at the rate of 5 ft per sec. Find the rate at which the length of his shadow on the wall is changing when he is 15 ft. from the building.
14. Solve problem 13 if the light is 10 ft. above the ground.
15. One city, A, is 30 miles north and 55 miles east of another city B. At noon, a car starts west from A at 40 mph, at 12:10 PM, another car starts east from B at 60 mph. Find in two ways when the cars will be nearest together.
16. One city, E, is 20 miles north and 20 miles east of another city F. At noon, a car starts south from E at 40 mph; at 12:10 PM, another car starts east from F at 60 mph. Find the rate at which the cars approach each other between 12:10 PM and 12:30 PM. What happens at 12:30 PM?
17. From a car travelling east at 40 mph, an airplane travelling horizontally north at 100 mph is visible 1 mile east, 2 miles south, and 2 miles up. Find when the two will be nearest together.
18. For problem 17, find how fast the two will be separating after a long time.
19. An arc light hangs at a height of 30 ft. above the center of a street60 ft wide. A man 6 ft tall walks along the sidewalk at the rate of 4 ft. per sec. How fast is his shadow lengthening when he is 40 ft. up the street.
20. For problem 19, how fast is the tip of the shadow moving.

**MAXIMA AND MINIMA**

1. The sum of two numbers is k. Find the minimum value of the sum of their squares.
2. The sum of two positive numbers is 2. Find the smallest value possible for the sum of the cube of one number and the square of the other.
3. Find two numbers whose sum is $a$, if the product of one to the square of the other is to be a minimum.
4. A rectangular lot is bounded at the back by a river. No fence is needed along the river and there is to be 24-ft opening in front. If the fence along the front costs $1.50 per foot, along the sides $1 per foot, find the dimensions of the largest lot which can be thus fenced in for $300.
5. A rectangular field of fixed area is to be enclosed and divided into three lots by parallels to one of the sides. What should be the relative dimensions of the field to make the amount of fencing minimum?
6. Find the volume of the largest box that can be made by cutting equal squares out of the corners of a piece of cardboard of dimensions 15 inches by 24 inches, and then turning up the sides.
7. The strength of a rectangular beam is proportional to the breadth and the square of the depth. Find the shape of the largest beam that can be cut from a log of given size.
8. The stiffness of a rectangular beam is proportional to the breadth and the cube of the depth. Find the shape of the stiffest beam that can be cut from a log of given size.
9. Find the rectangle of maximum perimeter inscribed in a given circle.
10. If the hypotenuse of the right triangle is given, show that the area is maximum when the triangle is isosceles.
11. Find the most economical proportions for a covered box of fixed volume whose base is a rectangle with one side three times as long as the other.
12. Find the most economical proportions for a cylindrical cup
13. Find the most economical proportions for a box with an open top and a square base
14. From a strip of tin 14 inches a trapezoidal gutter is to be made by bending up the sides at an angle of 45°. Find the width of the base for greatest carrying capacity.
15. The base of a covered box is a square. The bottom and back are made of pine, the remainder of oak. If oak is m times as expensive as pine, find the most economical proportion.
16. Find the dimension of the largest rectangular building that can be placed on a right-triangular lot, facing one of the perpendicular sides.

17. A lot has the form of a right triangle, with perpendicular sides 60 and 80 feet long. Find the length and width of the largest rectangular building that can be erected, facing the hypotenuse of the triangle.
18. Find the circular cone of maximum volume inscribed in a sphere of radius a.
19. A man in a motorboat at A (Figure 42) receives a message at noon calling him to B. A bus making 40 miles per hour leaves C, bound for B, at 1:00 PM. If AC = 40 miles, what must be the speed of the boat to enable the man to catch the bus.

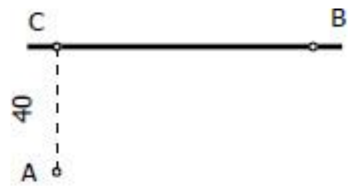



**Figure 42**

20. Find the shortest distance from the point (5, 0) to the curve $2y^2 = x^3$.