Cartesian Tree Sorting

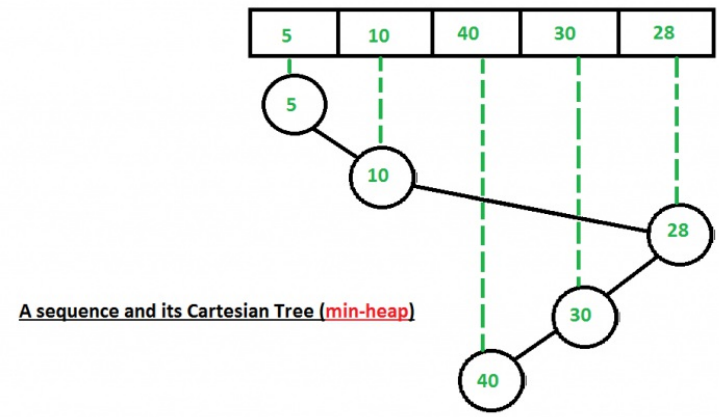
Prerequisites : Cartesian Tree

Cartesian Sort is an Adaptive Sorting as it sorts the data faster if data is partially sorted. In fact, there are very few sorting algorithms that make use of this fact.

For example consider the array {5, 10, 40, 30, 28}. The input data is partially sorted too as only one swap between "40" and "28" results in a completely sorted order. See how Cartesian Tree Sort will take advantage of this fact below.

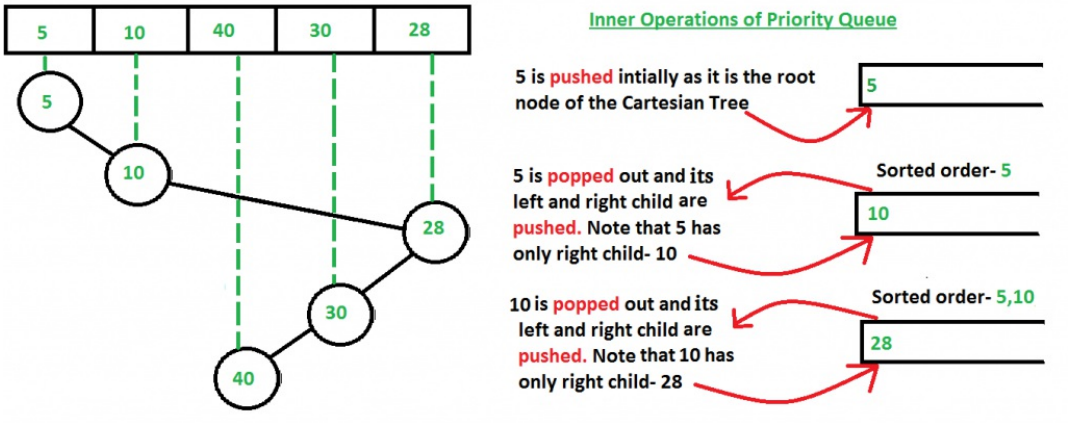
Below are steps used for sorting.

Step 1 : Build a (min-heap) Cartesian Tree from the given input sequence.



Step 2 : Starting from the root of the built Cartesian Tree, we push the nodes in a priority queue.
Then we pop the node at the top of the priority queue and push the children of the popped node in the priority queue in a pre-order manner.

1. Pop the node at the top of the priority queue and add it to a list.
2. Push left child of the popped node first (if present).
3. Push right child of the popped node next (if present).



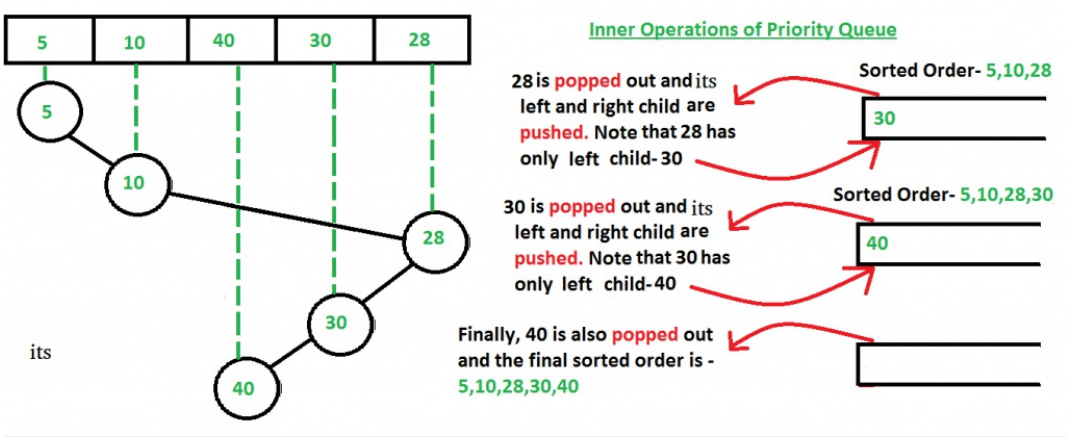
How to build (min-heap) Cartesian Tree?
Building min-heap is similar to building a (max-heap) Cartesian Tree (discussed in previous post), except the fact that now we scan upward from the node's parent up to the root of the tree until a node is found whose value is smaller (and not larger as in the case of a max-heap Cartesian Tree) than the current one and then accordingly reconfigure links to build the min-heap Cartesian tree.

Why not to use only priority queue?
One might wonder that using priority queue would anyway result in a sorted data if we simply insert the numbers of the input array one by one in the priority queue (i.e- without constructing the Cartesian tree).

But the time taken differs a lot.

Suppose we take the input array – {5, 10, 40, 30, 28}

If we simply insert the input array numbers one by one (without using a Cartesian tree), then we may have to waste a lot of operations in adjusting the queue order everytime we insert the numbers (just like a typical heap performs those operations when a new number is inserted, as



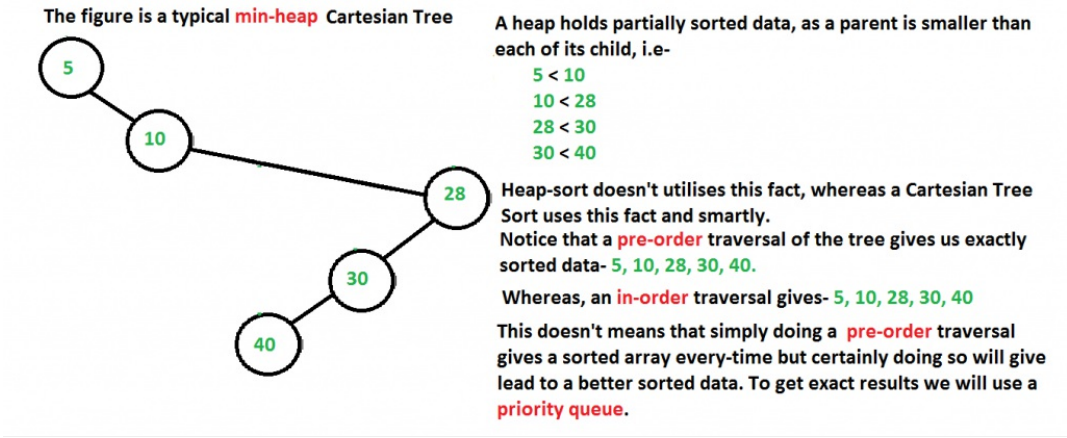
priority queue is nothing but a heap).

Whereas, here we can see that using a Cartesian tree took only 5 operations (see the above two figures in which we are continuously pushing and popping the nodes of Cartesian tree), which is linear as there are 5 numbers in the input array also. So we see that the best case of Cartesian Tree sort is O(n), a thing where heap-sort will take much more number of operations, because it doesn't make advantage of the fact that the input data is partially sorted.

Why pre-order traversal?

The answer to this is that since Cartesian Tree is basically a heap- data structure and hence follows all the properties of a heap. Thus the root node is always smaller than both of its children. Hence, we use a pre-order fashion popping-and-pushing as in this, the root node is always pushed earlier than its children inside the priority queue and since the root node is always less than both its child, so we don't have to do extra operations inside the priority queue.

Refer to the below figure for better understanding-




```
// A C++ program to implement Cartesian Tree sort
// Note that in this program we will build a min-heap
// Cartesian Tree and not max-heap.
#include<bits/stdc++.h>
using namespace std;

/* A binary tree node has data, pointer to left child
  and a pointer to right child */
struct Node
{
   int data;
   Node *left, *right;
};

// Creating a shortcut for int, Node* pair type
typedef pair<int, Node*> iNPair;

// This function sorts by pushing and popping the
// Cartesian Tree nodes in a pre-order like fashion
void pQBasedTraversal(Node* root)
{
   // We will use a priority queue to sort the
   // partially-sorted data efficiently.
   // Unlike Heap, Cartesian tree makes use of
   // the fact that the data is partially sorted
   priority_queue <iNPair, vector<iNPair>, greater<iNPair>> pQueue;
  pQueue.push (make_pair (root->data,root));
```

```
    // Resembles a pre-order trave
rse as first data
    // is printed then the left and t
hen right child.
    while (! pQueue.empty())
    {
     iNPair popped_pair = pQue
ue.top();
   printf("%d ",popped_pair.firs
t);

     pQueue.pop();

  if (popped_pair.second->left !
= NULL)
       pQueue.push (make_pair(
popped_pair.second->left->data
,
                          popped_pai
r.second->left));

       if (popped_pair.second->ri
ght != NULL)
        pQueue.push (make_pair(
popped_pair.second->right->da
ta,
                             popped_
pair.second->right));
    }

    return;
}


Node *buildCartesianTreeUtil(i
nt root, int arr[],
         int parent[], int leftchild[
], int rightchild[])
{
    if (root == -1)
       return NULL;

    Node *temp = new Node;

    temp->data = arr[root];
    temp->left = buildCartesianT
reeUtil(leftchild[root],
               arr, parent, leftchild,
rightchild);

    temp->right = buildCartesian
TreeUtil(rightchild[root],
               arr, parent, leftchild,
rightchild);

    return temp ;
}

// A function to create the Cartes
ian Tree in O(N) time
Node *buildCartesianTree(int ar
r[], int n)
{
    // Arrays to hold the index of
parent, left-child,
    // right-child of each number i
n the input array
    int parent[n],leftchild[n],right
child[n];

    // Initialize all array values as
-1
    memset(parent, -1, sizeof(par
```

```
ent));
   memset(leftchild, -1, sizeof(le
ftchild));
   memset(rightchild, -1, sizeof(
rightchild));

   // 'root' and 'last' stores the in
dex of the root and the
   // last processed of the Cartes
ian Tree.
   // Initially we take root of the
Cartesian Tree as the
   // first element of the input ar
ray. This can change
   // according to the algorithm
   int root = 0, last;

   // Starting from the second ele
ment of the input array
   // to the last on scan across th
e elements, adding them
   // one at a time.
   for (int i=1; i<=n-1; i++)
   {
      last = i-1;
      rightchild[i] = -1;

      // Scan upward from the no
de's parent up to
      // the root of the tree until a
 node is found
      // whose value is smaller th
an the current one
      // This is the same as Step
2 mentioned in the
      // algorithm
      while (arr[last] >= arr[i] &
& last != root)
         last = parent[last];

      // arr[i] is the smallest elem
ent yet; make it
      // new root
      if (arr[last] >= arr[i])
      {
         parent[root] = i;
         leftchild[i] = root;
         root = i;
      }

      // Just insert it
      else if (rightchild[last] == -
1)
      {
         rightchild[last] = i;
         parent[i] = last;
         leftchild[i] = -1;
      }

      // Reconfigure links
      else
      {
         parent[rightchild[last]] =
 i;
         leftchild[i] = rightchild[la
st];
         rightchild[last]= i;
         parent[i] = last;
      }

   }
```

```
    // Since the root of the Cartesi
an Tree has no
    // parent, so we assign it -1
    parent[root] = -1;

    return (buildCartesianTreeUti
l(root, arr, parent,
                        leftchild,
rightchild));
}

// Sorts an input array
int printSortedArr(int arr[], int
n)
{
    // Build a cartesian tree
    Node *root = buildCartesian
Tree(arr, n);

    printf("The sorted array is-\n
");

    // Do pr-order traversal and in
sert
    // in priority queue
    pQBasedTraversal(root);
}

/* Driver program to test above
functions */
int main()
{
    /*  Given input array- {5,10,4
0,30,28},
        it's corresponding unique C
artesian Tree
        is-

        5
         \
          10
           \
            28
           /
          30
         /
        40
    */

    int arr[] = {5, 10, 40, 30, 28};
    int n = sizeof(arr)/sizeof(arr[0
]);

    printSortedArr(arr, n);

    return(0);
}
```

Output :

```
The sorted array is-
5 10 28 30 40
```

Time Complexity : O(n) best-case behaviour (when the input data is partially sorted), **O(n log n)** worst-case behavior (when the input data is not partially sorted)

Auxiliary Space : We use a priority queue and a Cartesian tree data structure. Now, at any moment of time the size of the priority queue doesn't exceeds the size of the input array, as we are constantly pushing and popping the nodes. Hence we are using O(n) auxiliary space.

References :

https://en.wikipedia.org/wiki/Adaptive_sort

http://11011110.livejournal.com/283412.htmlhttp://gradbot.blogspot.in/2010/06/cartesian-tree-sort.htmlhttp://www.keithschwarz.com/interesting/code/?dir=cartesian-tree-sorthttps://en.wikipedia.org/wiki/Cartesian_tree#Application_in_sorting

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Quy • a month ago

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I got a result:

The sorted array is-
1 2 3 8 4 5 34 44 88 87 45 51

Rachit Belwariar ➚ Quy • a month ago

The algorithm is completely fine. The bug is in the way we are using priority queue, as we have made a priority queue of pointers to the node of Cartesian Tree, instead of data . So it is sorting it according to the address of the nodes of Cartesian Tree.
This will be fixed soon. Thanks for pointing out.

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