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Answer 1

a)

p	q	$\neg p$	$q \implies \neg p$	$p \iff q$	$(q \implies \neg p) \iff (p \iff q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

b)

p	q	r	$p \vee q$	$p \implies r$	$(p \vee q) \wedge$ $(p \implies r)$	$(q \implies r)$	$(p \vee q)\wedge$ $(p \implies r) \wedge$ $(q \implies r)$	$[(p \vee q)\wedge$ $(p \implies r) \wedge$ $(q \implies r)] \implies r$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	F	F	T
F	F	T	F	T	F	T	F	T
F	F	F	F	T	F	T	F	T

Answer 2

$\neg p \implies (q \implies r) \equiv \neg\neg p \vee (q \implies r)$ rule 1 from table 7
$\neg\neg p \vee (q \implies r) \equiv p \vee (q \implies r)$ double negation law from table 6
$p \vee (q \implies r) \equiv p \vee (\neg q \vee r)$ rule 1 from table 7
$p \vee (\neg q \vee r) \equiv (p \vee \neg q) \vee r$ associative law from table 6
$(p \vee \neg q) \vee r \equiv (\neg q \vee p) \vee r$ commutative law from table 6
$(\neg q \vee p) \vee r \equiv \neg q \vee (p \vee r)$ associative law from table 6
$\neg q \vee (p \vee r) \equiv \neg\neg q \implies (p \vee r)$ rule 3 from table 7
$\neg\neg q \implies (p \vee r) \equiv q \implies (p \vee r)$ double negation law from table 6

Answer 3

a) $\forall x L(x, Burak)$

b) $\forall x L(Hazal, x)$

c) $\forall x \exists y L(x, y)$

d) $\neg \exists x \forall y L(x, y)$

e) $\forall y \exists x L(x, y)$

f) $\neg \exists x L(x, Burak) \land \neg \exists y L(y, Mustafa)$

g) $\exists x \exists y (y \neq x \land \forall z (L(Ceren, z) \implies (z = x \lor z = y)))$

h) $\exists y \forall x (L(x, y) \land \exists z \forall t (L(t, z) \implies z = y))$

i) $\neg \exists x L(x, x)$

j) $\exists x \exists y (y \neq x \land L(x, x) \land L(x, y) \land \forall z (L(x, z) \implies (z = y \lor z = x)))$

Answer 4

1	$\mid\ p \lor q \lor r$	
2	$\mid\ \underline{\neg p}$	
3	$\mid\ \mid\ \underline{p}$	
4	$\mid\ \mid\ \mid\ \underline{\neg(q \lor r)}$	
5	$\mid\ \mid\ \mid\ \neg p$	R, 2
6	$\mid\ \mid\ \mid\ p$	R, 3
7	$\mid\ \mid\ q \lor r$	$\neg$E, 4
8	$\mid\ \mid\ \underline{q \lor r}$	
9	$\mid\ \mid\ q \lor r$	R, 8
10	$\mid\ q \lor r$	$\lor$E, 1, 3, 8

Answer 5

1	$\forall x(P(x) \implies Q(x))$	
2	$\underline{\exists x Q(x) \implies R(y)}$	
3	$\vert\ \underline{P(a)}$	
4	$\vert\ P(a) \implies Q(a)$	$\forall$E, 1
5	$\vert\ Q(a)$	$\Rightarrow$E, 3, 4
6	$\vert\ \exists x Q(x)$	$\exists$I, 5
7	$\vert\ R(y)$	$\Rightarrow$E, 2, 6
8	$P(a) \implies R(y)$	$\Rightarrow$I, 3, 7
9	$\forall x(P(x) \implies R(y))$	$\forall$I, 8

Answer 6

Let's prove that $(p \vee r) \vdash (\neg p \implies r)$ and call it *lemma 1.*

1	$\underline{p \vee r}$	
2	$\vert\ \underline{\neg p}$	
3	$\vert\ \vert\ \underline{p}$	
4	$\vert\ \vert\ \vert\ \underline{\neg r}$	
5	$\vert\ \vert\ \vert\ \neg p$	R, 2
6	$\vert\ \vert\ \vert\ p$	R, 3
7	$\vert\ \vert\ r$	$\neg$E, 4
8	$\vert\ \vert\ \underline{r}$	
9	$\vert\ \vert\ r$	R, 8
10	$\vert\ r$	$\vee$E, 1, 3, 8
11	$\neg p \implies r$	$\Rightarrow$I, 2, 10

Let's prove that $(\neg q \vee r) \vdash (q \implies r)$ and call it *lemma 2.*

1	$\underline{\neg q \vee r}$	
2	$\vert\ \underline{q}$	
3	$\vert\ \vert\ \underline{\neg q}$	
4	$\vert\ \vert\ \vert\ \underline{\neg r}$	
5	$\vert\ \vert\ \vert\ q$	R, 2
6	$\vert\ \vert\ \vert\ \neg q$	R, 3
7	$\vert\ \vert\ r$	$\neg$E, 4
8	$\vert\ \vert\ \underline{r}$	
9	$\vert\ \vert\ r$	R, 8
10	$\vert\ r$	$\vee$E, 1, 3, 8
11	$q \implies r$	$\Rightarrow$I, 2, 10

Now let's get back to the original problem.

1	$p \vee r$	
2	$\neg q \vee r$	
3	$(p \implies q)$	
4	$\neg(\neg p \vee q)$	
5	p	
6	q	$\Rightarrow$E, 3, 5
7	$\neg p \vee q$	$\vee$I, 6
8	$\neg(\neg p \vee q)$	R, 4
9	$\neg p$	$\neg$I, 5
10	$\neg p \vee q$	$\vee$I, 9
11	$\neg(\neg p \vee q)$	R, 4
12	$\neg p \vee q$	$\neg$E, 4
13	$\neg p \implies r$	lemma 1
14	$q \implies r$	lemma 2
15	$\neg p$	
16	r	$\Rightarrow$E, 13, 15
17	q	
18	r	$\Rightarrow$E, 14, 17
19	r	$\vee$E, 12, 15, 17
20	$(p \implies q) \implies r$	$\Rightarrow$I, 3, 19