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Answer 2.17

This is a conditional probability problem. Let's have the following events.

$$\begin{array}{ll} SX = \{\text{Part supplied by X}\} & DX = \{\text{Part supplied by X and defective}\} \\ SY = \{\text{Part supplied by Y}\} & DY = \{\text{Part supplied by Y and defective}\} \\ SZ = \{\text{Part supplied by Z}\} & DZ = \{\text{Part supplied by Z and defective}\} \end{array}$$

Then we compute the probabilities of these events given the information in the question.

$$\begin{array}{ll} P\{SX\} = 0.24 & P\{DX\} = (0.24)(0.05) = 0.012 \\ P\{SY\} = 0.36 & P\{DY\} = (0.36)(0.10) = 0.036 \\ P\{SZ\} = 0.40 & P\{DZ\} = (0.40)(0.06) = 0.024 \end{array}$$

Let us finally denote the event $CHDP$–the computer has a defective part. But this is equivalent to saying a part supplied by X is defective, or a part supplied by Y is defective, or a part supplied by Z is defective. Hence $CHDP = \{DX \text{ or } DY \text{ or } DZ\}$, and $P\{CHDP\} = P\{DX\} + P\{DY\} + P\{DZ\} = 0.072$ since these are mutually exclusive events.

What we are looking for is $P\{DZ \mid CHDP\} = \dfrac{P\{DZ \cap CHDP\}}{P\{CHDP\}} = \dfrac{P\{DZ\}}{P\{CHDP\}}$

Note that since DZ is a subset of $CHDP$, their intersection is DZ. When numbers are substituted into the equation, the answer is

$$\frac{0.024}{0.072} = \frac{1}{3}$$

Answer 2.29

Let us denote the event $F_k = \{\text{The keyword has been found in k of the first 4 searched databases.}\}$. We are interested in computing $P\{F_2 \cup F_3 \cup F_4\} = P\{F_2\} + P\{F_3\} + P\{F_4\}$, since these are mutually exclusive events. The only thing to do now is to find a way to compute $P\{F_k\}$. We can do so by calculating the number of favorable outcomes over the number of total outcomes, $\dfrac{N_F}{N_T}$.

The number of total outcomes for F_k is $\dbinom{9}{4}$, since there are 9 databases from which we can choose, and we want to choose only 4 of them.

The number of favorable outcomes for F_k is $\binom{5}{k}\binom{4}{4-k}$, since we should choose k out of 5 databases with the keyword, and $4-k$ out of 4 databases without the keyword. Hence,

$$P\{F_k\} = \frac{\binom{5}{k}\binom{4}{4-k}}{\binom{9}{4}}$$

Now $P\{F_2\} + P\{F_3\} + P\{F_4\}$ yields

$$\frac{\binom{5}{2}\binom{4}{2} + \binom{5}{3}\binom{4}{1} + \binom{5}{4}\binom{4}{0}}{\binom{9}{4}} = \frac{60+40+5}{126} = 0.8\overline{3}$$

Answer 3.6

$E(X)$ is $\sum xP(x)$, where x denotes a value the random variable X can take, and P_x denoting its probability. Back to our question, we see that the random variable X can take either the value 0 or 1 since the tested two blocks may not have any error($X = 0$), or one of the tested blocks may have an error($X = 1$). Thus, $E(X) = \sum xP(x) = 0P(0) + 1P(1) = P(1)$. The probability that the random selected two blocks has 1 error is the answer.
Again, $\frac{N_F}{N_T}$. There are $\binom{6}{2} = 15$ total outcomes, this is N_T. For N_F, we should choose the 1 block with the error, plus another block without the error, hence $N_F = \binom{1}{1}\binom{5}{1} = 5$. Thus, the answer is $\frac{5}{15} = \frac{1}{3}$

Answer 3.7

Let X_k be a random variable that shows the number of home runs scored in the kth game. We are interested in another random variable $Y = X_1 + X_2$, given the X_k's distribution.

Now let's compute $E(Y) = E(X_1 + X_2)$. Using the linearity of expectation, we have $E(Y) = E(X_1) + E(X_2) = 2E(X)$. $E(X)$ is $0(0.4) + 1(0.4) + 2(0.2) = 0.8$ from the table given in the question. Hence $E(Y) = 2(0.8) = 1.6$.

For the $Var(Y)$, $Var(Y) = Var(X_1 + X_2) = Var(X_1) + Var(X_2) = 2Var(X)$, since X_1 and X_2 are independent events. $Var(X) = (0-0.8)^2(0.4) + (1-0.8)^2(0.4) + (2-0.8)^2(0.2) = 0.56$. Hence $Var(Y) = 2(0.56) = 1.12$.

If we are not allowed to use the above theorems, here is an alternative solution.

The total number of home runs in two games can be 0, 1, 2, 3, or 4; since there might be 0,1, or 2 home runs in a single game. Let $P_k(x)$ denote the probability of x home runs in the kth game. Now we are ready to find a distribution table for two games, just like the table given in the question.

$$P(0) = P_1(0)P_2(0) = (0.4)(0.4) = 0.16$$

$$P(1) = P_1(0)P_2(1) + P_1(1)P_2(0) = (0.4)(0.4) + (0.4)(0.4) = 0.32$$

$$P(2) = P_1(2)P_2(0) + P_1(0)P_2(2) + P_1(1)P_2(1) = (0.2)(0.4) + (0.4)(0.2) + (0.4)(0.4) = 0.32$$

$$P(3) = P_1(2)P_2(1) + P_1(1)P_2(2) = (0.2)(0.4) + (0.4)(0.2) = 0.16$$

$$P(4) = P_1(2)P_2(2) = (0.2)(0.2) = 0.04$$

As a check, note that $0.16 + 0.32 + 0.32 + 0.16 + 0.04 = 1$, so we are likely to have exhausted all the cases. This transforms into the following table.

y	0	1	2	3	4
$P(y)$	0.16	0.32	0.32	0.16	0.04

$E(Y) = 0(0.16) + 1(0.32) + 2(0.32) + 3(0.16) + 4(0.04) = 1.60$
$Var(Y) = (1.6)^2(0.16) + (0.6)^2(0.32) + (-0.4)^2(0.32) + (-1.4)^2(0.16) + (-2.4)^2(0.04) = 1.12$