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Answer 1

a.

$a(a \cup c)^*b(a \cup c)^*b(a \cup c)^*cc$

b.

$(bb \cup bc \cup cb \cup cc)^*(ab \cup ac \cup ba \cup ca)(bb \cup bc \cup cb \cup cc)^*$

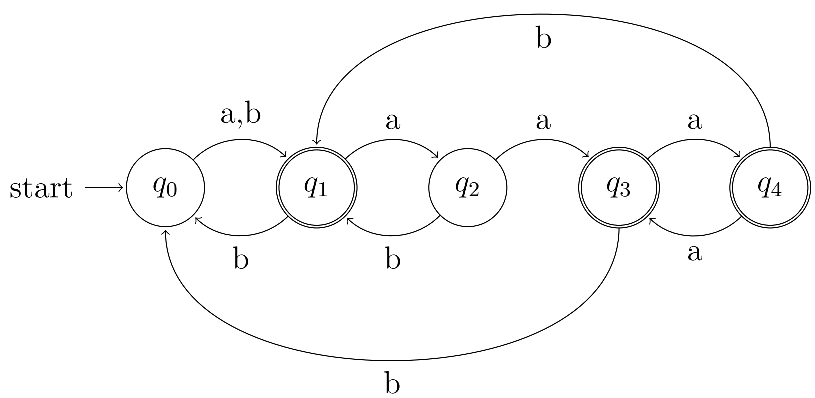
c.

$(a^*b^*(ba)a^*b^*(ba))^*(a^*b^*)$

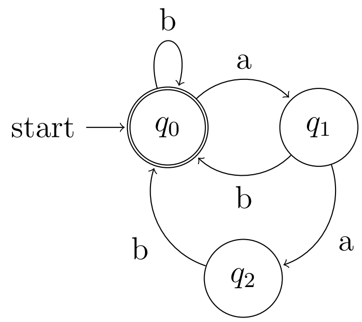
Answer 2

a.

First construct a DFA that accepts the strings of odd length. Then construct another DFA that accepts the strings ending with aaa. The union of these two DFAs is an NFA. Converting that NFA into its DFA equivalent and minimizing its states, we have the following:



b.



Answer 3

The answer is no. We may end up with a non-regular language. To see this, consider the languages of the form $L_i = \{a^i b^i\}, i \in N$. Such languages are; for instance, the following:

$$L_0 = \{e\}, L_1 = \{ab\}, L_2 = \{aabb\}, ...$$

Since the one and only element of such sets is a regular expression, the languages L_i are **regular**. Now let's consider the infinite union of L_i, and call it L_u.

$$L_u = \bigcup_{i=0}^{\infty} L_i = \{e\} \cup \{ab\} \cup \{aabb\} \cup ... = \{a^i b^i : i \geq 0\}$$

The last set is a **non-regular** language by the application of pumping lemma! The proof is given on p. 89 of the textbook(Example 2.4.2).

Answer 4

We'll prove this by contradiction. Assume the language represented by the given set is regular. Then, according to the pumping lemma, there exists an integer $m \geq 1$ such that for every $w \in L$ and $|w| \geq m$ there exist strings x, y, z with $y \neq e$, $|xy| \leq m$ such that for every $i \geq 0$ $xy^i z \in L$.

Keeping in mind that $\boldsymbol{m}$ is the integer in the pumping lemma, now we have to pick a word with $w \in L$. **Pick $\boldsymbol{w = b^m c^{2m}}$**. It is clear that $|w| \geq m$, so we can proceed to the next step and say that the three strings x, y, z such that $y \neq e$ and $|xy| \leq m$ in the pumping lemma are $\boldsymbol{y = b^k, x = b^{m-k} z = c^{2m}}$ with $\boldsymbol{k > 0}$ in our case. With these arrangements for x, y, z, we must have $xy^i z \in L$ for all $i \geq 0$.

Now consider $xy^0 z$, that is, consider $xz = b^{m-k} c^{2m}$. According to the final step of the lemma, this string is in L, a contradiction! To see the contradiction, remember $k > 0$ so there is no way $xz \in L$. Hence our assumption in the beginning that L was regular is wrong.

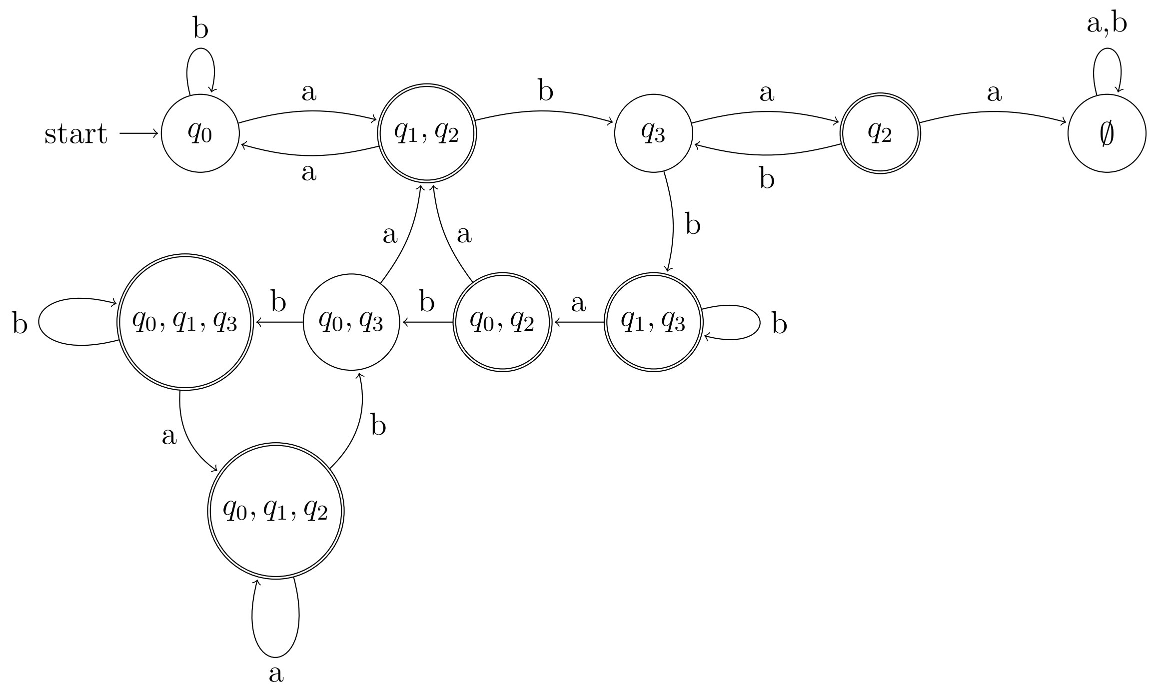
Answer 5

Assume L_1, L_2 are two regular languages. We want to prove that $L_1 \setminus L_2$ is also regular. We can play with this expression in the following way:

$$\begin{aligned} L_1 \setminus L_2 &= \{x \in L_1 \wedge x \notin L_2\} \text{ by definition of set minus} \\ &= \{x \mid x \in L_1 \wedge x \in \overline{L_2}\} \text{ by definition of complement} \\ &= L_1 \cap \overline{L_2} \end{aligned}$$

Now recall that there exists an NFA for any regular language. So we can construct finite automata for both L_1 and L_2. Furthermore, it is given in the textbook p. 78 that finite automaton languages are closed under intersection and complement. Thus, $L_1 \setminus L_2 = L_1 \cap \overline{L_2}$ is an accepted language by some finite automaton. Now that we have the finite automaton for $L_1 \cap \overline{L_2}$, we can always come up with a regular expression by eliminating the states of the finite automaton one by one, with the help of union, kleene star, and concatenation operators. What we end up with is a regular language. Hence, $L_1 \setminus L_2$ is regular.

Answer 6



This DFA with ten states is unfortunately minimal. If we apply the algorithm given in the textbook p. 99, initially, for example, the classes of $\equiv_0$ are $\{\{q_1, q_2\}, \{q_1, q_3\}, q_2, \{q_0, q_1, q_3\}, \{q_0, q_2\},$ $\{q_0, q_1, q_2\}\}$ and $\{q_0, q_3, \emptyset, \{q_0, q_3\}\}$.
Then, for $\equiv_1$, we have, $\{q_0\}$, $\{q_3, \{q_0, q_3\}\}$, $\{\emptyset\}$,$\{q_2, \{q_1, q_2\}\}$, $\{\{q_1, q_3\}, \{q_0, q_1, q_3\}\}$, $\{\{q_0, q_2\},$ $\{q_0, q_1, q_2\}\}$. After the termination of the algorithm, in the end($\equiv_4$), it turns out that no two states are in the same equivalence class. This means what we have started with is minimal.