1a) This ODE is not directly integrable, is seprable, but is not linear.

b) $f(x,y) = |y|$, so a solution exists, because $f(x,y)$ is continuous.

c) $f_y(x,y) = \pm 1$, which is continuous, so the solution is unique

2) $P = (2x + y\cos xy)$ $\quad Q = (x\cos xy + 2y)$

$$\frac{\delta P}{\delta y} = (-xy\sin xy) \qquad \frac{\delta Q}{\delta x} = (-xy\sin xy)$$

$$\frac{\delta P}{\delta y} = \frac{\delta Q}{\delta x},$$ so the ODE is exact

$$\frac{\delta f}{\delta x} = P(x,y)$$
$$= 2x + y\cos xy$$

$f(x,y) = x^2 + \sin(xy) + g(y)$ where g is a function of y

$$f_y(x,y) = Q(x,y)$$
$$= 2y + x\cos xy$$
$$= x\cos xy + g'(y)$$

so $g'(y) = 2y$

$\Rightarrow g(y) = y^2 + c$

so $f(x,y) = x^2 + y^2 + \sin xy + c$

The implicit solution is

$x^2 + y^2 + \sin(xy) = C$ where $C = -c$

Using the IVP $y(0) = 1$

$\Rightarrow 0 + 1 + 0 = C$

$1 = C$

$C = 1$

The solution to the IVP is

$$x^2 + y^2 + \sin(xy) = 1$$

3) [Homogenous solution]: (Characteristic equation)

$$\lambda^2 - 3\lambda + 2 = 0$$

$$(\lambda - 2)(\lambda - 1) = 0$$

$$\lambda = 2 \text{ or } 1$$

so the homogenous solution is

$$y_H = Ae^x + Bxe^{2x}$$

[Particular solution]: Guess $y_{P_2} = ax^2 + bx + c$, sub into the ODE

$$(2a) - 3(2ax + b) + 2(ax^2 + bx + c) = -2x^2$$

$$2a - 6ax - 3b + 2ax^2 + 2bx + 2c = -2x^2$$

$$\Rightarrow \quad 2a = -2, \quad -6a + 2b = 0 \quad 2a - 3b + 2c = 0$$

$$\Rightarrow \quad a = -1, \; b = -3, \; c = -3.5$$

so $y_{P_2} = -x^2 - 3x + 3.5$

$y_{P_1} = xke^x$ for a constant k

$$k(2e^x + xe^x) + 3k(e^x + xe^x) + 2kxe^x = e^x$$

$$2ke^x + kxe^x - 3ke^x + 3kxe^x + 2kxe^x = e^x$$

$$K = 1$$

$$y_{P_1} = xe^x$$

the general solution is

$$y = y_H + y_{P_1} + y_{P_2}$$

$$= Ae^x + Bxe^{2x} + xe^x - x^2 - 3x - 3.5$$

d) The frequency of the beats is

$$\frac{\frac{-W+W_0}{2}}{2\pi} = \frac{\frac{-2+\sqrt{14/3}}{2}}{2\pi}$$

$$= \frac{-1+\sqrt{14/3}/2}{2\pi}$$

$$= -\frac{1}{2\pi} + \frac{\sqrt{14/3}}{4\pi}$$

$$= -\frac{1}{2\pi} + \frac{\sqrt{14}}{4\pi\sqrt{3}}$$

7a) Substitute $y = x^r$ into the equation

$$(r-1)(r)x^2 x^{r-2} - 2r(x)(x^{r-1}) + 2x^r = 0$$

$$\implies ((r-1)(r) - 2r + 2)(x^r) = 0$$

$$\implies r^2 - 3r + 2 = 0$$

$$(r-2)(r-1) = 0$$

$$r = 1 \text{ or } 2$$

so $y_1 = x^2$, $y_2 = x$

b) $W(x) = \begin{vmatrix} x & 1 \\ x^2 & 2x \end{vmatrix}$

$$= x^2$$

$$x^2 y'' - 2xy' + 2y = 0$$

$$\implies y'' - 2x^{-1}y' + 2x^{-2}y = 0$$

$$\implies p(x) = 2x^{-1}$$

so

$$\frac{dW}{dx} - 2x^{-1}W = 0 \quad (\text{sub } W = x^2 \text{ into this})$$

$$LHS = 2x - 2x^{-1}(x^2)$$

$$= 2x - 2x$$

$$= 0$$

$$= RHS (\text{verified})$$