Homework4 for Mathmatical Image Process

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1 Wavelet Based Image Processing

Image Restoration can be considered as solving the inverse problem of

$$Au = f + \eta$$

There are two approaches to process images by wavelets:

- **Analysis Approach**

$$\min_u ||\lambda \cdot Wu||_1 + \frac{1}{2} ||Au - f||_2^2$$

- **Balanced Approach**

$$\min_\alpha ||\lambda \cdot \alpha||_1 + \frac{1}{2} \left|\left|AW^T\alpha - f\right|\right|_2^2 + \frac{\kappa}{2} \left|\left|(I - WW^T)\alpha\right|\right|_2^2$$

To solve the analysis approach method, we used Split Bregman method to do the optimization problem:

Algorithm 1 Split Bregman For Analysis Approach

set d_0 and b_0

repeat

update:

- $u_k = (A^TA + \mu I)^{-1}(A^Tf + \mu W^T(d_k - b_k))$
- $d_{k+1} = \mathscr{T}_{\lambda/\mu}(Wu_{k+1} + b_k)$
- $b_{k+1} = b_k + \delta(Wu_{k+1} - d_{k+1})$

until $\frac{||Wu_{k+1} - d_{k+1}||}{||f||_2} < tol$

To solve the balanced approach problem we use the ISTA algorithm:

Algorithm 2 ISTA For Balanced Approach

set α_0

repeat

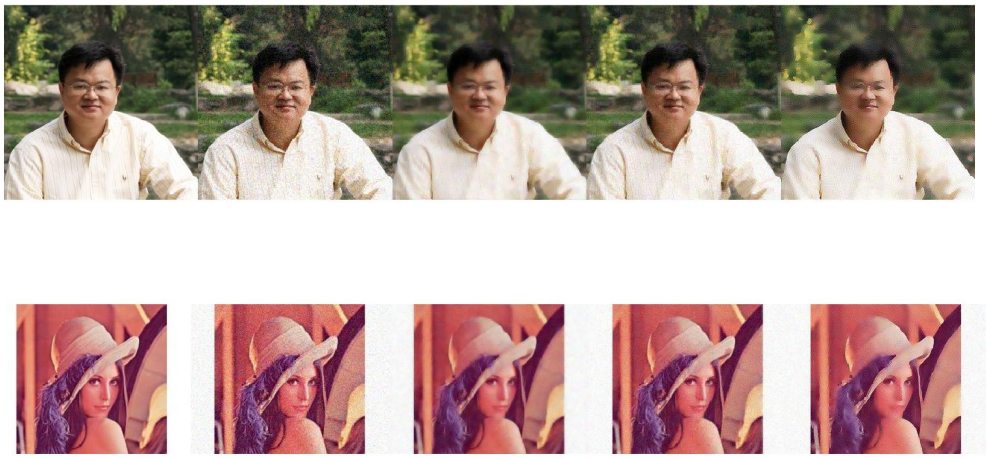
update:

- $g_k = \alpha_k - \nabla F_2(\alpha_k)/L$
- $\alpha_{k+1} = \mathscr{T}_{\lambda/L}(g_k)$

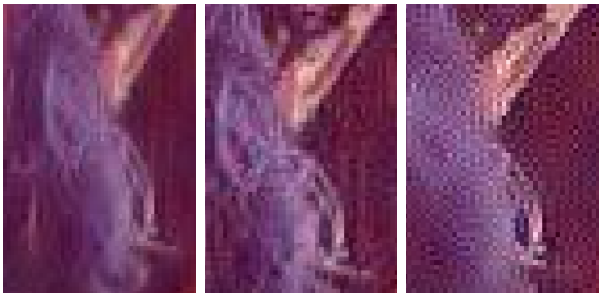
until end

2 Wavlet Based Image Denoising

In the denoising problem, the linear operator A here is id, results is shown bellow:



Test1. Image Denoising Test:(a) Original Image (b) Noised Image (c) Analysis Approach (d) Balanced Approach (e) TV model



Test2. Detail Of The Denoised Image:(a) Analysis Approach (b) Balanced Approach (c) TV model

Obviously, the wavelet based image denoising holds more detials.

3 Wavelet Based Deblur

Test3. Image Debluring Test:(a) Original Image (b) Blured Image With Noise (c) Analysis Approach (d) Balanced Approach (e) TV model

4 ReadMe

The functions in the folder **profunc** are functions to process the images, for denoising there are three functions:

- denoise_TV.m
- denoise_anl.m
- denoise_wav.m(balanced approach)

For debluring, the code of TV model is in blur.m and others are in deblur_wav.m

The **test_another.m** and test_wavelet.m are the test enterance.