BACHELORTHESIS

Multi-modal Localization using Wi-Fi Signal Strength and 2D Range Finder

vorgelegt von

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Abstract

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too...

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Abbreviations

AMCL	**A**daptive **M**onte **C**arlo **L**ocalization
BFGS	**B**royden **F**letcher **G**oldfarb **S**hanno
GPS	**G**lobal **P**ositioning **S**ystem
L-BFGS	**L**imited-memory **B**royden **F**letcher **G**oldfarb **S**hanno
LIDAR	**LI**ght **D**etection **A**nd **R**anging
MAC-address	**M**edia **A**ccess **C**ontrol-address
RGB-D	**R**ed **G**reen **B**lue-**D**epth
ROS	**R**obot **O**perating **S**ystem
Rprop	**R**esilient back**prop**agation
SSID	**S**ervice **S**et **Id**entifier
Wi-Fi	**Wi**reless **Fi**delity

Glossary

BFGS An optimization algorithm, that can be used to find minima or maxima of functions. It uses both first order- and second order-derivative information, while not explicitly computing the second order-derivative.

control data Can be provided by the commands given to a robot, so for example its velocity. Alternatively often provided by sensors like odometers. Control data gives us information about the change of state of a robot.

environment measurement data Provided by sensors like laser range finders, cameras or sonars. The environment measurement data gives us information about the current state of the environment of a robot.

Gaussian process A collection of random variables, any finite number of which have a joint Gaussian distribution[Rasmussen and Williams, 2005, p. 13].

global localization Global localization describes localization without any prior information about the current position. That means a robot would have to figure out its current position at the start of the localization.

GPS A global positioning system that is using satellites. It can be used to determine the current location with an error of a few meters.

gradient descent An algorithm that uses first-order derivatives in order to find a local minimum of a function.

hyperparameter A hyperparameter is a parameter that can be learned. There is no need to set it manually.

kernel When used with Gaussian processes kernels are also referred to as covariance function. They are used to measure the difference between two inputs. They can lift the inputs into a higher space, while the computations still happen in the input space. Thus one can save computation time by applying using kernels. They can be used whenever an operation can be represented by a dot product.

kidnapped robot problem Describes the problem when a robot is taken and brought to a different location while a localization is running. The problem is that the robot is now localized in the wrong position and thus the localization fails. Kidnapped robot problem is often used interchangeably with localization failure.

L-BFGS The limited-memory variant of the BFGS algorithm.

laser range finder Sensor that uses laser beams to detect the distance of obstacles.

LIDAR Short for "Light Detection and Ranging". It is a type of sensor, that is using light in order to measure the distance to obstacles in the environment. It can be used to create 3D representations of areas.

log-likelihood It is the natural logarithm of a likelihood function. In some cases they are easier to work with than the original likelihood function. The logarithm is monotonically increasing and has the maximum at the same point as the likelihood function. Often the partial derivatives of the log-likelihood function is easier to compute than the partial derivatives of the likelihood function. Thus it is attractive to use in order to optimize hyperparameters by searching the maximum of a function.

MAC-address Short for media access control-address. This address is a commonly used identifier in networks. So for example a Wi-Fi receiver has its own MAC-address as does the access point. The MAC-address is unique for each interface.

Monte Carlo localization Robot self-localization using probability. Its basis is the recursive state estimation implemented with a particle filter.

odometer An odometer is a common sensor used in order to measure the distance traveled.

particle Particles are generally a small part of larger system. Here particles are used to refer to samples that represent parts of a state belief.

posterior A posterior is also called posterior probability distribution. It is a distribution created from the prior distribution after taking further evidence into account that wasn't considered when creating the prior.

prior A prior is also called prior probability distribution. It is a distribution that was created by making certain assumptions before taking further evidence into account. When the distribution is transformed by taking that further evidence into account, it is called posterior, or posterior distribution.

recursive state estimation Using incoming data a current state is computed recursively, by using the previous state. The state is represented by a probability density function.

regression Inferring the relationship between different variables from available scalars from said variables. In case of the example of the linear regression the result is a linear function. In the example of Gaussian process regression the result is a Gaussian process.

RGB-D sensor A sensor that is able to sense colors and depth.

Rprop Short for resilient backpropagation. Using first-order derivatives the algorithm can be used to find a local minimum. It only uses the sign of resulting partial derivatives when searching for the minimum.

SSID Short for service set identifier. It is used by Wi-Fi access points. The identifier is not unique. It is used to signal different access point that belong to the same Wi-Fi network.

training set A set of data that was recorded in order to discover the relationship between different variables. To discover those relationships regression can be used.

Chapter 1

Introduction

1.1   Motivation

Localization is an important problem in robotics. It is needed in order to solve a wide range of problems. Even simple tasks in households require a robot to be able to determine its own position in the environment. It needs to be able to get from point A to point B. Being able to do this, a robot could for example bring a fresh cup of coffee and return the used cup back to the kitchen. It can help with cleaning tasks[Pinheiro et al., 2015]. It can also be highly important in industrial environments. Hamburg has one of the world's largest and busiest ports. In order to bring the shipping containers from one place to the other autonomous transport vehicles can be used. These are just a few of many real world examples, where localization is used.

Another modern example of using localization is the autonomous vehicle. The Global Positioning System(GPS)[Misra and Enge, 2011] can give us a good idea about the position of the car. But GPS is too inaccurate for the high demands of autonomous vehicles. Levinson et al. [2007] for example used Light Detection and Ranging(LIDAR)[Weitkamp, 2005] additionally to the GPS to vastly improve the localizations accuracy. The LIDAR is a vision based sensor that is similar to a radar. It can be used to map out nearby obstacles. The LIDAR is not well suited to estimate the position on a large map with no prior information, so the GPS can be used to provide a rough position estimate. On the other hand the GPS is inaccurate, so once a rough position is estimated, the LIDAR can help to narrow the position down.

In robotics another similar sensor to the LIDAR, a 2D laser range finder is often used for localization. With the laser range finder the robot is also able to detect the distance of obstacles near itself. There are many situations where a robot can not rely on GPS. In many buildings its signal is obstructed. Localizing without any prior information about the robot's position is called global localization. There are existing approaches to do this with just a vision-based sensor like the LIDAR or a laser range finder. Usually when localizing only a small part of the map

is taken into account, because we have a rough idea about the position. When using global localization with only a laser range finder the whole map is taken into account at the beginning. But these methods have the downside that they are often unreliable and lead to localization failures that are hard to recover from. This is especially true for environments that don't have many unique characteristics. When localizing with a laser range finder one usually creates a map with the existing obstacles and the outline of the room. Often many parts of these maps look very similar. The less distinctive features the map has, the more likely it is that the global localization with a laser range finder fails. Many places indoors can cause those problems. Many rooms are symmetric, so it can be hard to differentiate between the walls. Many buildings have rooms with the same outline, like for example many office-environments. Office environments can also be quite large, and the larger the space that has to be taken into account the more likely it is, that a global localization only using a laser range finder will fail. So this can cause problems especially in buildings.

In most buildings there are a lot of different Wi-Fi[IEEE, 2012] networks. The networks MAC-address provides a unique identifier for each signal. The signal strengths can be a good indicator about how far the wireless access point is away. And while one signal alone wouldn't be enough to infer the position multiple signals from different access points can be used to do just that.

This could be used to enhance the localization with laser scanners. When at the start no position estimation is given it could be computed with the Wi-Fi data. Also in case of a localization failure the Wi-Fi signals could be used to infer the correct position.

This combines the strengths of both the position estimation with Wi-Fi signals and the localization with a laser scanner. The Wi-Fi signals have a unique identifier and will only be observed in a certain range on a map. The outline of a wall could be nearly identical in different places, but overall the localization with a laser scanner is very accurate once the true position was determined.

In Chapter 2 we will explain the basics of probability based localization, with the so called Monte Carlo localization[Dellaert et al., 1999] as an example. Furthermore we will explore how a Wi-Fi receiver can be used with the Monte Carlo localization and explain how Gaussian process[Rasmussen and Williams, 2005] regression can be used for this purpose. Chapter 3 details the implementation of a Wi-Fi position estimation using the knowledge from Chapter 2 about Monte Carlo localization and Gaussian process regression. In Chapter 4 experiments and results regarding the usage of the Wi-Fi position estimation will be discussed. In chapter 5 we will reflect the results and come to a conclusion. Lastly chapter 6 provides an outlook for future applications and possibilities.

1.2 Related Work

One of the most popular methods for localization is the Monte Carlo localization. We are going to use this method, both for localization via a 2D range finder, and parts of it also to estimate the position via Wi-Fi. Thrun et al. [2005] give an extensive overview over the Monte Carlo localization.

In order for the Monte Carlo localization to work, one needs a measurement model for the sensor that is used. Thrun et al. [2005] provide models that can be used for a 2D range finder. In order to estimate the position with a Wi-Fi receiver, one needs to implement a different measurement model.

There have been different approaches to solving the localization problem with Wi-Fi signals. Serrano et al. [2004] used a propagation model to estimate the signal strength at a given point on the map. The advantage of this technique is that only the positions of the access points have to be known. But it is a lot more inaccurate than other approaches. The problem is that inside a building the Wi-Fi signal is often obstructed by obstacles, like walls, furniture or humans. This makes the propagation model very inaccurate and thus also leads to inaccurate position estimations.

A different approach to the problem uses a map that was created beforehand. Biswas and Veloso [2010] recorded Wi-Fi signals a in grid like fashion, recording in a certain interval. In order to interpolate the Wi-Fi signal from the recordings, linear interpolation was used. The drawback here is that the Wi-Fi signals have to be recorded at certain positions on the map. This makes the process of creating the map more complicated and time consuming.

In order to create something akin to a map from recorded Wi-Fi data, using regression proved to be useful. As it turns out Gaussian processses are well suited for this kind of problem. Rasmussen and Williams [2005] give an extensive overview of how they can be used in machine learning.

Ferris et al. [2006] use Gaussian processes for building the map and for localization. This means the signal strength can be recorded at random places. Thus the needed data for creating the map can be recorded while the robot drives around or even does other tasks. In our approach we want to determine whether such a method can be used for global localization in an office environment with a 2D laser range finder and a wheeled robot. There are already existing investigations into a global localization with Wi-Fi data, but in different contexts. Duvallet and Tews [2008] used it in an industrial environment to enhance the previously developed laser beacon localization[Tews et al., 2007]. The most striking difference here is the environment, which is large and spacious compared to the often cramped spaces used as offices. Also the number of Wi-Fi signals that can be observed in office spaces is usually a lot higher than in industrial environments. Ito et al. [2014] used Wi-Fi for global localization for an iPhone 4 with an RGB-D sensor, so a sensor like the Microsoft Kinect for example that can sense colors and depth. For the RGB-D sensor they used a map created from the floor-plan, that is less accurate

then the maps usually used. Here an obvious disparity to our test environment is the platform used. While the phone is used in conjunction with an RGB-D sensor, which is comparable to a 2D laser range finder, the phone does not have any wheels that can provide odometry. Here they resort to an alternative for the odometry method using the RGB-D sensor.

Chapter 2

Basics

This chapter summarizes the basics for robot self-localization. At first in section 2.1 we will explain how uncertainty can complicate the task of localization and how probability can be helpful to approach this problem. At first we will introduce the recursive state estimation and building on this move on to an implementation of it using a particle filter. Afterwards we will show how said particle filter can be used for robot self-localization with the Monte Carlo localization. In the next section we will explain the problem of global localization and then introduce the kidnapped robot problem and a possible solution.

In section 2.2 we will look into the aspects of Wi-Fi that are important for the Wi-Fi position estimation.

Section 2.3 will revolve around the Wi-Fi sensor model, that can be used with the Monte Carlo localization. We will define what a Gaussian process is and derive it from a Bayesian treatment of the linear regression.

2.1 Localization

2.1.1 Uncertainty

Uncertainty plays a big role in robotics. By dealing with the real world this inherently introduces unpredictability. The robot's environment is dynamic and can change constantly. Even the robot itself changes its own environment by acting in it. Robots usually have a variety of sensors. They are used to get information about the state of their environment. The sensors used in robotics have certain limitations. They can only provide an inexact interpretation of the world around them and this is something we need to keep in mind. When an odometer tracks a robots path it will never be exact. When a laser scanner measures the distance to an obstacle we have to expect that there will be an error margin. The sensors have to deal with physical limitations when trying to capture the state of their

environment. Often they are subject noise. In localization 2D maps get used very often. They are a model of the real world. But once again it is only an approximate model that strips away a lot of information from the real world and thus introduces more uncertainty[Thrun et al., 2005, p. 3-4].

Probability can be a helpful tool to circumvent this problem. When localizing instead of trying to find the pose of the robot directly we test how likely it is that the robot is in different hypothesized poses. So, using all the information we get from the sensors we compute which poses are the most likely ones[Thrun et al., 2005, p. 5].

2.1.2 Recursive State Estimation

Thrun et al. [2005] divide the data usually collected by robots into two different categories:

1. Environment measurement data, denoted $z_{t_1:t_2}$ for data from time t_1 to time t_2

2. Control data, denoted $u_{t_1:t_2}$ for data from time t_1 to time t_2

The first one gives us information about the current state of the environment, provided by sensors like cameras, laser range scanners or Wi-Fi receivers. The second kind of data gives us information about the change of state. A common example for this is the velocity of a robot, usually measured by an odometer[Thrun et al., 2005, p. 22-23].

The current state is denoted x_t. At this point we will rely on probability for the reasons already explained. So, the probability of state x_t is the following: $p(x_t|x_{0:t-1}, z_{1:t-1}, u_{1:t})$. This takes all previous states, environment measurement data and control data into account. But if we assume that state x_t is a sufficient summary of everything that happened before time step t, then we can omit most data:

$$p(x_t|x_{0:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t|x_{t-1}, u_t) \qquad (2.1)$$

The probability that a certain measurement was observed can be described by $p(z_t|x_{0:t}, z_{1:t-1}, u_{1:t})$. Once again we can omit most data:

$$p(z_t|x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t|x_t) \qquad (2.2)$$

Equation 2.1 is known as the state transition probability and equation 2.2 is known as the measurement probability. The state transition probability takes the control data into account. As we will see for localization this means it is used to reflect the movement of the robot. The measurement probability on the other hand takes the measurement data into account. For localization this means it is used to reflect the information the robot has about its environment at the moment[Thrun et al., 2005, p. 24-25].

The state of a robot is represented by so called belief distributions.

$$\overline{bel}(x_t) = p(x_t|z_{1:t-1}, u_{1:t}) \tag{2.3}$$

Equation 2.3 is often referred to as prediction. It doesn't take the last environment measurement into account.

$$bel(x_t) = p(x_t|z_{1:t}, u_{1:t}) \tag{2.4}$$

The posterior from equation 2.4 on the other hand does take the last measurement into account[Thrun et al., 2005, p. 25-26].

Once again, we don't want to take all past data from all time steps into account. So we are going to simplify both equations for our purposes. $bel(x_t)$ can be simplified by using Bayes rule and equation 2.2:

$$
\begin{aligned}
p(x_t|z_{1:t}, u_{1:t}) &= \frac{p(z_t|x_t, z_{1:t-1}, u_{1:t})p(x_t|z_{1:t-1}, u_{1:t})}{p(z_t|z_{1:t-1}, u_{1:t})} \\
&= \eta p(z_t|x_t, z_{1:t-1}, u_{1:t})p(x_t|z_{1:t-1}, u_{1:t}) \\
bel(x_t) &= \eta p(z_t|x_t)\overline{bel}(x_t)
\end{aligned}
\tag{2.5}
$$

Here η is simply a constant, stemming from the fact that $p(z_t|z_{1:t-1}, u_{1:t})$ is independent of the state x_t and so no matter what value x_t takes on, it will always stay the same.

$\overline{bel}(x_t)$ can be simplified for our purposes, using the theorem of total probability and equation 2.1[Thrun et al., 2005, p. 31-33].

$$
\begin{aligned}
p(x_t|z_{1:t-1}, u_{1:t}) &= \int p(x_t|x_{t-1}, z_{1:t-1}, u_{1:t})p(x_{t-1}|z_{1:t-1}, u_{1:t})dx_{t-1} \\
&= \int p(x_t|u_t, x_{t-1})p(x_{t-1}|z_{1:t-1}, u_{1:t-1})dx_{t-1} \\
\overline{bel}(x_t) &= \int p(x_t|u_t, x_{t-1})bel(x_{t-1})dx_{t-1}
\end{aligned}
\tag{2.6}
$$

In the second step in equation 2.6 we make use of the fact the u_t is not needed to infer the probability of state x_{t-1} and therefore simply omit it. 2.6 and 2.5 are used in the Bayes filter algorithm.

Algorithm 1 Bayes_filter [Thrun et al., 2005, p. 27]

1: **procedure** BAYES_FILTER($bel(x_{t-1}), u_t, z_t$)
2:     **for** all x_t **do**
3:         $\overline{bel}(x_t) = \int p(x_t|u_t, x_{t-1})bel(x_{t-1})dx_{t-1}$
4:         $bel(x_t) = \eta p(z_t|x_t)\overline{bel}(x_t)$
5:     **end for**
6:     **return** $bel(x_t)$
7: **end procedure**

The algorithm gets the distribution of belief $bel(x_{t-1})$ from the last time step, the new environment data z_t and control data u_t from this time step t. The belief gets updated. At first with the control data, to make sure that the change of state gets reflected in the distribution before we apply our knowledge from the environment data to our distribution. In line 4 the belief gets updated with the measurement data too. Now all available data was used to create the new belief distribution. The belief distribution can be used to infer the state x_t.

Figure 2.1 shows a simple example how the algorithm could be used for localization. It shows a robot that is able to if it stands in front of a door. It reflects the nature of the control data and measurement data. Whenever the robot moved and the control data is used to update the belief, the belief still spikes in the correct position, but the spike becomes weaker. We have to take into account that the odometer could give us imprecise measurements. This is the reason why we use the measurement data too. Even if we had the correct position in the beginning, when we only use control data, after driving around for some time the localization will be off by a huge margin. The measurement data is there to correct this. If we look at both control data and measurement data in a certain time interval the uncertainty induced by moving around is small enough that we can compensate for it by using the measurement data.

The figure also provides a good example as to why we use probabilities. In figure 2.1 (b) the robot senses a door. There are three different doors, but the robot can't be sure which door it is, so it simply applies the same probabilities to all three doors instead of randomly choosing one. Then it also can't be sure about the exact position in front of these doors because the data could be imprecise. So in front of each door the distribution has bell-shaped spikes to reflect this.

Another good example why probabilities work so well can be observed in (c). The robot moved and of course the control data is also imprecise. So the spikes move according to the control data, but they get weaker and more spread out.

Now (d) shows a nice example how the data from the time steps before get taken into account. The spikes move according to the control data. Now the robot observes a door once again, just like in (b). But because the belief has only a spike in front of the correct door, the new belief has a high and distinct spike in front of the correct door after taking the measurement data into account.

2.1.3 Particle Filter

The particle filter is a non-parametric solution to implement the Bayes filter. Here the posterior distribution $bel(x_t)$ is represented by finitely many samples[Thrun et al., 2005, p. 85].

The posterior distribution, so the particles, are denoted as:

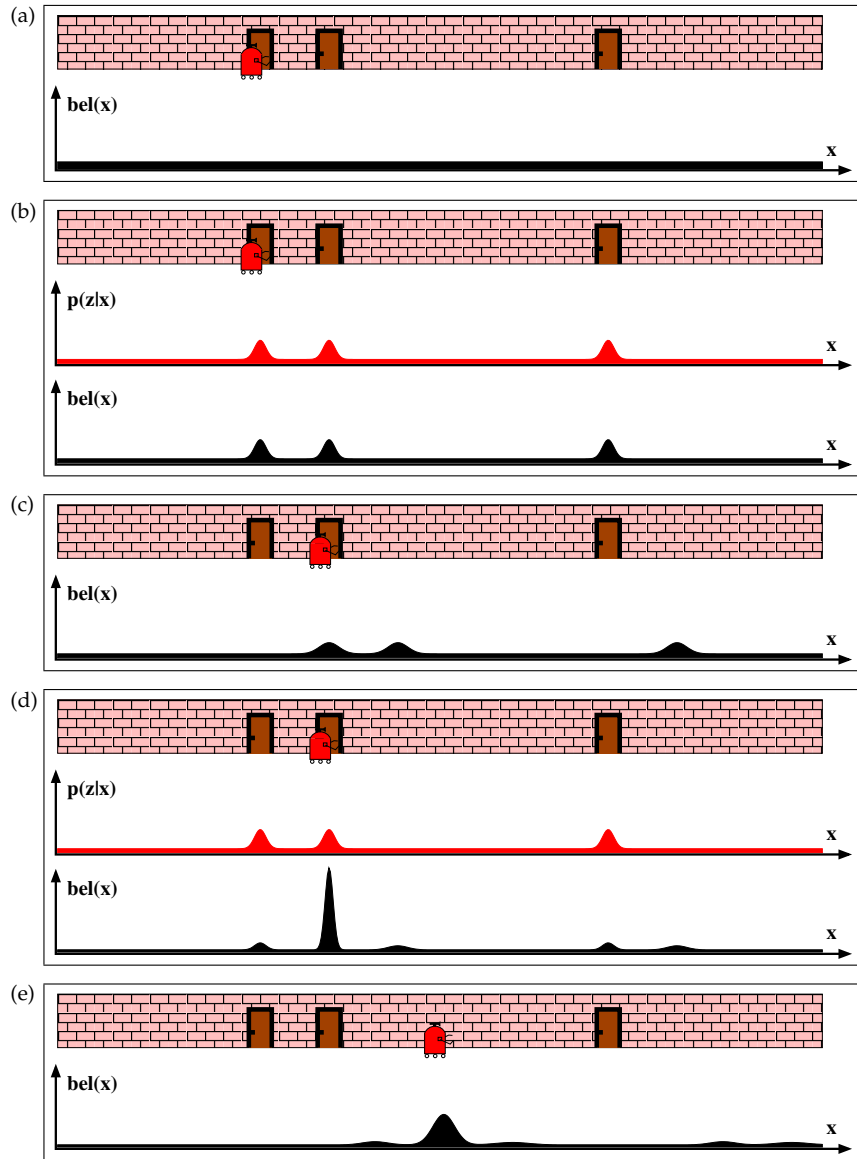$$X_t = x_t^{[1]}, x_t^{[2]}, ..., x_t^{[M]} \tag{2.7}$$

FIGURE 2.1: A simple example to explain how Bayes filter works. In (a) the belief is evenly distributed. In (b) the robot observes the door and the measurement data reflects this and updates $bel(x)$ accordingly. In (c) the robot moves to the right and the beliefs structure shifts to the right as well, but the peaks become lower, because the control data could be imprecise. In (d) the robot takes the new measurement data into account. The belief clearly spikes at the correct position. In (e) the robot moved again. The spike is still in the correct position, but it is lower[Thrun et al., 2005, p. 6].

Each particle represents a possible state hypothesis at time t. M is the number of particles[Thrun et al., 2005, p. 96-97].

Algorithm 2 Particle_filter [Thrun et al., 2005, p. 98]

1: **procedure** PARTICLE_FILTER(X_{t-1}, u_t, z_t)
2:     $\bar{X}_t = X_t = \emptyset$
3:     **for** $m = 1$ to M **do**
4:         sample $x_t^{[m]} \sim p(x_t|u_t, x_{t-1}^{[m]})$
5:         $w_t^{[m]} = p(z_t|x_t^{[m]})$
6:         $\bar{X}_t = \bar{X}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$
7:     **end for**
8:     **for** $m = 1$ to M **do**
9:         draw i with probability $\propto w_t^{[i]}$
10:       add $x_t^{[i]}$ to X_t
11:     **end for**
12:     **return** X_t
13: **end procedure**

So the belief $bel(x_t)$ is approximated by particle set X_t. The probability for a state hypothesis to be included is ideally proportional to its Bayes filter posterior $bel(x_t)$[Thrun et al., 2005, p. 98].

$$x_t^{[m]} \sim p(x_t|z_{1:t}, u_{1:t}) \tag{2.8}$$

Just like in the Bayes filter the posterior X_t is computed recursively from the set X_{t-1}.

On line 4 of algorithm 2 the equivalent to $\overline{bel}(x_t)$ is computed. The state $x_t^{[m]}$ is generated based on particle $x_{t-1}^{[m]}$ and control data u_t. Based on $p(x_t|u_t, x_{t-1}^{[m]})$ new particles are sampled. This is done for every particle in the set X_{t-1}. In this step the most recent information from the control data gets added.

Line 5 is then the equivalent to computing $bel(x_t)$. $w_t^{[m]}$ is the weight of particle m at time t. The higher the weight the more likely it is that the state of the particle is representative of the real state.

On line 9 new particles get drawn. This happens according to the weights w_t. The higher the weight, the higher the chance of the particle to be drawn. The particles that were unlikely to represent the real state won't get drawn and only the ones that have a high weight survive.

2.1.4 Monte Carlo Localization

The particle filter can be used for localization[Thrun et al., 2005, p. 252]. Figure 2.2 shows a simple example. This figure is similar to figure 2.1. But here the
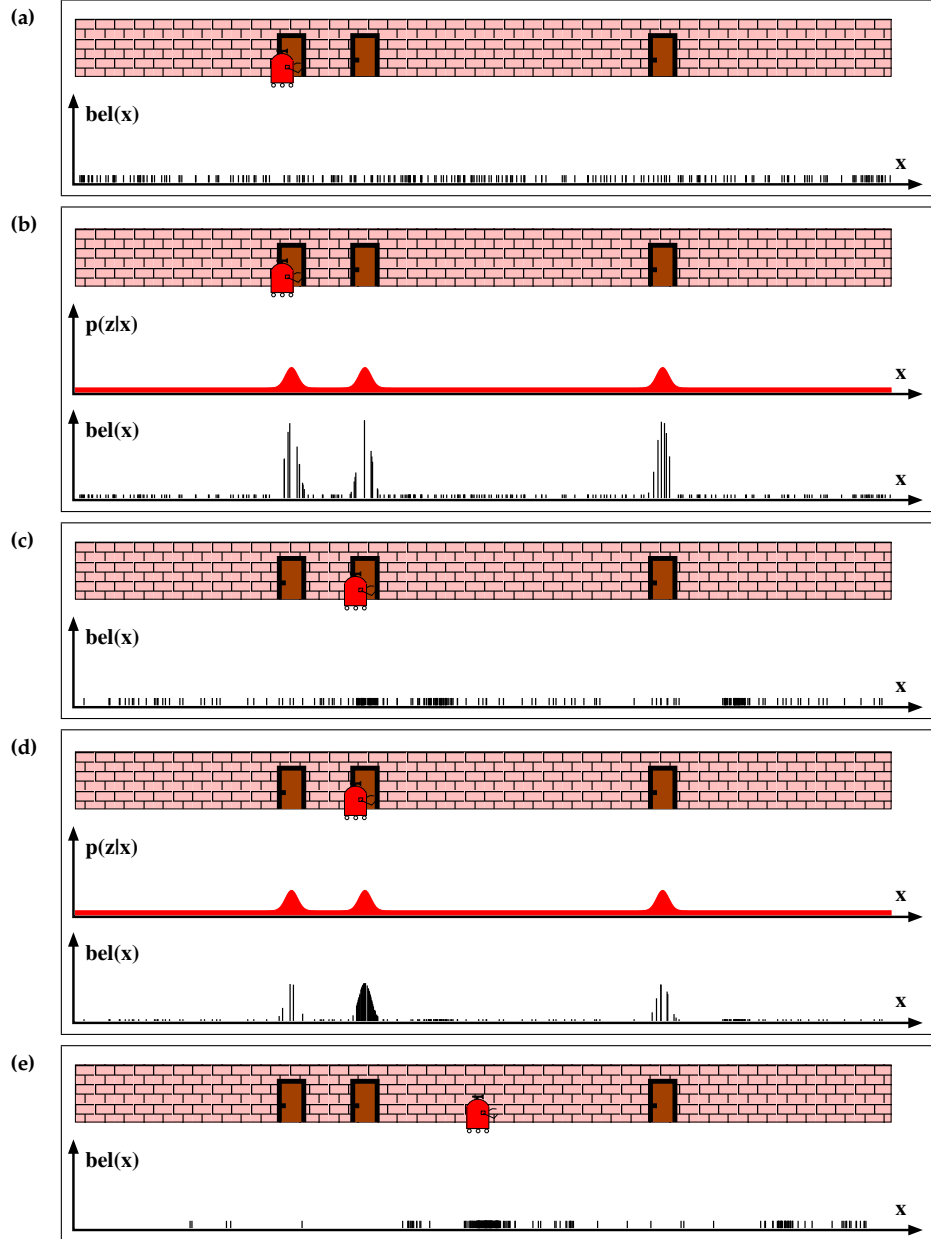
FIGURE 2.2: This is a simple example of the localization realized with a particle filter. It is similar to figure 2.1, but this time the belief is not a curve but instead represented by the distribution of particles[Thrun et al., 2005, p. 251].

particles and their weights are used to represent the belief distribution. In (a), (c) and (e) only the distribution of the particles is shown, but not their weights. It shows how after some iterations the particles are clustered around the positions that are most likely the real position. But this also involves the disadvantage that after some iterations the particles will only be clustered around one spot, so we only observe this particular part of the whole belief. This is both an advantage and a disadvantage. It means we save computing power, because we only compute a small part of the whole belief distribution. But it also means that after some iterations we will only observe one particular point of the belief and disregard the rest. In certain situations this can lead to localization failures, which can't be resolved unless we tweak the algorithm. How this happens and how those situations can be salvaged will be explained in the following section 2.1.6.

There is control data that is usually provided by an odometer. The measurement data is usually provided by sensors like laser range finders. In our specific case the Wi-Fi receiver will be another source of measurement data.

A sample motion model and a measurement model are needed for the Monte Carlo localization to work. The sample motion model takes u_t and $x_{t-1}^{[m]}$ as input. It takes the odometer's data into account and samples new particles based on that. In simpler terms this just means that the particles are moved according to what we can infer from the control data available. If a robot drove a few meters to the right the particles should move to the right as well. This step does not take the measurement data into account yet.

That happens in the next step. The measurement model takes the newly sampled particle $x_t^{[m]}$ and measurement data z_t and computes a weight. The higher the weight, the more likely is it according to the model that the particle is representative of the real pose.

These models are different for different kind of sensors. Later on we will show the used measurement model for the Wi-Fi receiver in-depth.

Algorithm 3 Monte_Carlo_Localization [Thrun et al., 2005, p. 252]

1: **procedure** Monte_Carlo_Localization(X_{t-1}, u_t, z_t, m)
2:     $\bar{X}_t = X_t = \emptyset$
3:     **for** $m = 1$ to M **do**
4:         $x_t^{[m]} = \textbf{sample\_motion\_model}(u_t, x_{t-1}^{[m]})$
5:         $w_t^{[m]} = \textbf{measurement\_model}(z_t, x_{t-1}^{[m]}, m)$
6:         $\bar{X}_t = \bar{X}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$
7:     **end for**
8:     **for** $m = 1$ to M **do**
9:         draw i with probability $\propto w_t^{[i]}$
10:        add $x_t^{[i]}$ to X_t
11:     **end for**
12:     **return** X_t
13: **end procedure**

Algorithm 3 is the Monte Carlo localization. Note how similar it is to the particle filter from algorithm 2. The only difference is how the new set of particles is sampled in line 4 and how the weights are computed in line 5. This happens according to the sample_motion_model and the measurement_model.
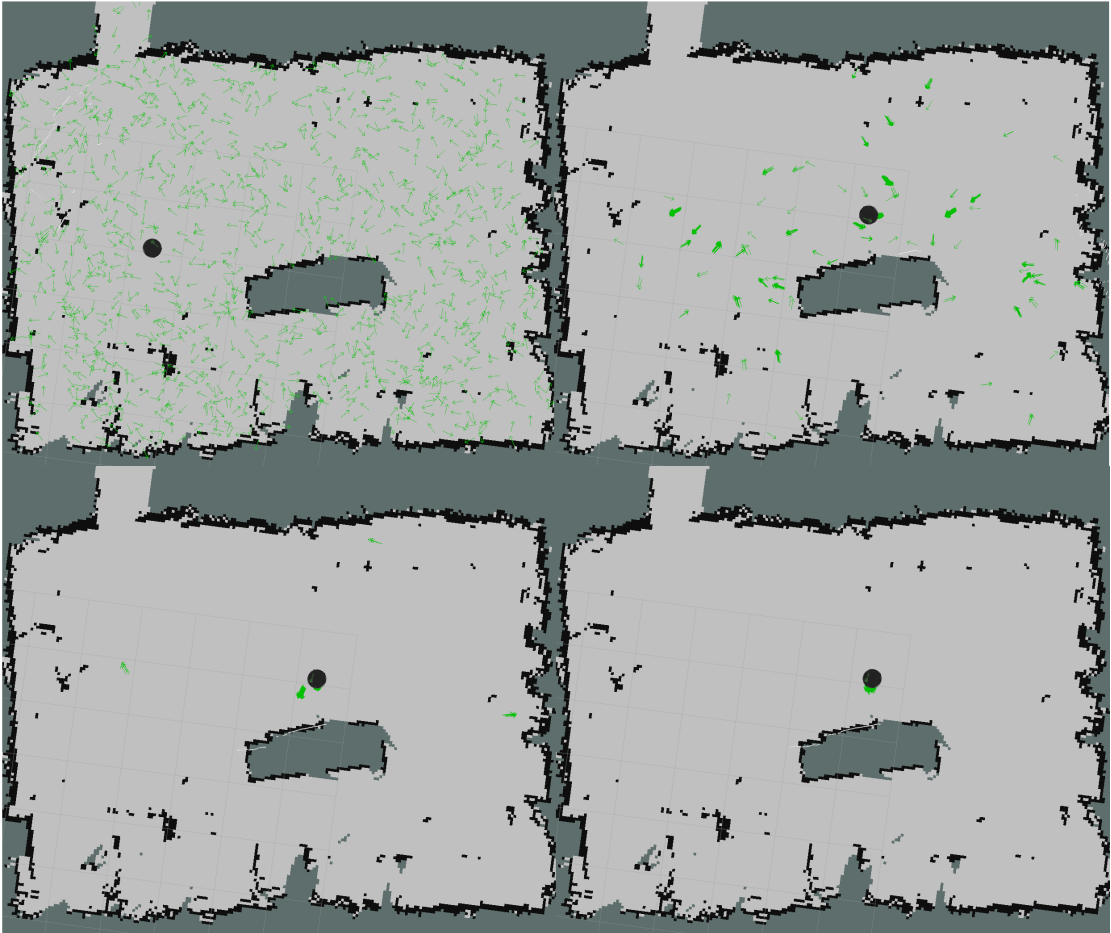
2.1.5 Global Localization

FIGURE 2.3: Example of global localization in our laboratory. The black circle represents the robot. The green arrows represent the position and orientation of the particles. In the upper left pictures the particles are spread over the entire state space. In the subsequent pictures the robot rotates to observe its environment. Slowly the particles converge to the correct location on the map and thus it is possible to infer the correct position of the robot.

There is local localization and there is global localization. In the local localization problem the position at the beginning is already known. The particles are already clustered around the correct position at the beginning. The task then is just to track the position from there on out. In the global localization problem we have no information about the robot's location at the start.

A common approach to solve this problem is to spread the particles over the entire map at the start. After some iterations of weighing and sampling the particles they are supposed to be clustered around the real state. And in many situations this . There are some situations where this can pose a problem though.

Many buildings have multiple rooms with similar structures. This can lead to the creation of multiple clusters and the pose is ambiguous. Many rooms are symmetric. This too can lead to clusters on multiple locations.

2.1.6 Kidnapped Robot Problem

The kidnapped robot problem occurs when, while performing localization, the robot is taken and placed at a different location. It won't be able to register where it went in most cases. Wheeled robots usually use odometers for the control data and once the robot is taken up and not on the ground they won't register anything anymore.

This leads to problems when the particles are already clustered around one pose. When sampling new particles this cluster will persist and there will not be any particles on the rest of the map. To recover from such failures one can introduce a certain number of random particles into the set[Thrun et al., 2005, p. 256].

We could introduce a certain number of random particles in every iteration. But we can also use the measurement model and the generated weights as an indicator how likely the current pose is. For this purpose we calculate the average weight of the current iteration[Thrun et al., 2005, p. 257].

$$\frac{1}{M} \sum_{m=1}^{M} w_t^{[m]} \approx p(z_t|z_{1:t-1}, u_{1:t}, m) \tag{2.9}$$

This already gives us a good idea, but we don't want to rely on only a single iteration to decide the induction of random particles. There could be an unusually high amount of noise or other reasons for inaccurate measurements. So we take the average over multiple iterations into account. We keep track of two different values:

$$w_{slow} = w_{slow} + \alpha_{slow}(w_{avg} - w_{slow})$$
$$w_{fast} = w_{fast} + \alpha_{fast}(w_{avg} - w_{fast}) \tag{2.10}$$

α_{slow} and α_{fast} are decay rates and constants that have to be set beforehand. w_{slow} is the long term measurement probability and w_{fast} is the short term probability. This means that past weighting averages have a higher influence on w_{slow} then they do on w_{fast}.

During the resampling process random particles are drawn with probability:

$$\max\{0.0, 1.0 - w_{fast}/w_{slow}\} \tag{2.11}$$

The higher the long term probability w_{slow} is compared to w_{fast} the more likely it is that random particles get introduced. So that means the lower the recent weights are compared to the older ones, the higher the probability that random particles are drawn[Thrun et al., 2005, p. 258-259].

2.2 Wi-Fi

A Wi-Fi signal is spread from a base station known as access point. Each signal carries a unique identifier called MAC-address, which can be used to set the different signals apart. The Service Set Identifier (SSID) on the other hand isn't unique and there are often multiple access points distributing Wi-Fi signals under the same SSID. This makes the MAC-address relevant and the SSID irrelevant for the Wi-Fi position estimation[IEEE, 2012].

The Wi-Fi signal strength is measured in dBm. According to our own experience the range of the signal is between about -90 dbm up to-40 dbm. The higher the number the better the signal. For our purposes the stability of the data transfer via the Wi-Fi connections is unimportant. We are only interested the received signal strength.

The access points send their signal on so called channels. Wi-Fi signals are spread on different frequency ranges. The ranges usually used are 2.4 GHz and 5 GHz. These frequency bands are then further divided into channels. So each Wi-Fi signal is spread on a specific channel. The more Wi-Fi signals in a close range send on the same channel, the more they interfere with each other.

Our goal is to fetch the MAC-address and the corresponding signal strength from every available Wi-Fi network, not only the one the robot is connected to. This can be achieved by actively scanning the different Wi-Fi channels. In order to receive the data a probe request is sent to the available access points. Each access point answers. This has to be done on each supported channel. Sending out the request, waiting for the answer and repeating that procedure for each channel means that it can take a few seconds to complete a whole scan.

2.3 Wi-Fi Sensor Model

2.3.1 Overview

As discussed earlier in order for the Monte Carlo localization to work with a certain type of sensor, we need to provide a measurement model for it. Our goal is to create one for a Wi-Fi receiver. For this purpose we chose to create a map of the Wi-Fi signal strengths. Doing this for Wi-Fi signals is different from creating a map of obstacles for example. For an obstacle map a point on the map can only

have 3 different states: free, occupied and unknown. But Wi-Fi signals change constantly from position to position. This makes it more complicated to create an accurate map. It isn't feasible to record the Wi-Fi signal strengths on every single point of the map. Like we discussed the scanning process alone will take a few seconds. Thus we choose to record a fair amount of data points and use regression to interpolate a likely value for the Wi-Fi signal strengths at every point on the map. Wi-Fi signals can be unpredictable in the way they spread. With no obstacles between robot and access point it is reasonable to expect the signal to constantly get weaker the further we go away. But in buildings there will be walls and other obstacles between the access points and the receiver.

Gaussian processes are a very flexible solution for this kind of problem. They are non-parametric and using hyperparameter optimization it is possible to fit them to the Wi-Fi signals. As we will show with a Gaussian process it is possible to compute a Gaussian distribution for every coordinate on the map. So given a coordinate x and y we can compute a Gaussian distribution with a mean and variance. The resulting Gaussian distribution is a model of the probability of the occurrence of a signal strength at that coordinate. This property makes it very well suited to use it as measurement model. We are not interested in the most likely signal strength observed at a certain point on the map, but in the probability that a given signal strength is observed. This is needed in order to compute the weights for particles of the Monte Carlo localization, so a Gaussian process is very well suited for this task.

In the further subsections we will explain how Gaussian processes work and how we can use them to create the Wi-Fi map and how we can use them for the measurement model.

2.3.2 Gaussian Processes

We have a dataset D of n observations of the form $D = \{(\mathbf{x_i}, y_i) | i = 1, ..., n\}$. In our example the $\mathbf{x_i}$ is a coordinate on the map and y_i is the associated Wi-Fi signal strength. Now in regression we want to infer the function values for new inputs. So we want to get a function f from the dataset D. But how do we infer function f? We need to make previous assumptions about the function because otherwise all functions that cross all training inputs and the corresponding values would be equally valid. The two common methods to achieve this are: [Rasmussen and Williams, 2005, p. 2]

1. Restrict function f to certain classes of functions.
2. Put prior probabilities on all possible functions.

When using the first solution depending on what classes are chosen, they can be a bad fit. For example one can imagine that linear functions would be a bad fit for Wi-Fi data. They are not flexible enough. On the other hand it can happen

that the classes chosen lead to overfitting when they are too flexible[Rasmussen and Williams, 2005, p. 2].

In the second solution we would need to put a prior on all possible functions. The possible functions are infinitely many, so this seems like a difficult task. This is where the Gaussian process can help us.

A stochastic process is a generalization of the probability distribution. While the probability distribution concerns scalars and vectors, the process concerns functions[Rasmussen and Williams, 2005, p. 2]. Here we will focus on processes that are Gaussian. The reason for that is that is that it makes the computations required a lot easier. One can think of a function $f(x)$ as a vector, where each entry specifies a value for a specific input x. These vectors would be infinitely large. When we want to infer a function value from a Gaussian process at finitely many points we can just ignore the infinite other points. This makes it computationally feasible to use Gaussian processes for regression[Rasmussen and Williams, 2005, p. 2].
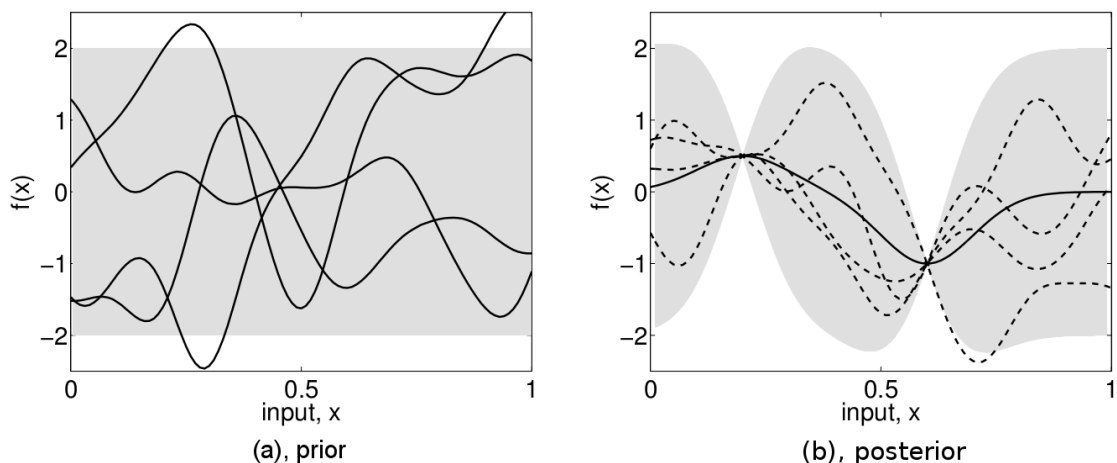

FIGURE 2.4: An example of how the prior distribution of functions works. In (a) functions were drawn from the prior. As one can see they have similar characteristics. In (b) the posterior is shown. Two points were observed. The solid line is the mean prediction. The shaded region is twice the standard deviation at each input value x[Rasmussen and Williams, 2005, p. 3]
.

To give a better intuition on how inferring the function f from the dataset D works we will give an example in form of figure 2.4. As one can see in (a) the functions are drawn without any observations. In (b) two observations were made. The functions go through the observed values. The thick line signifies the mean prediction. But this method also gives as a variance at each input value. The closer the input value is to an observation, the lower the standard deviation indicated by the gray shadows. The obvious reason is that we are more certain about the true function value the closer we are to the observations we made.

When using Gaussian processes the form of the drawn functions is determined by a covariance function[Rasmussen and Williams, 2005, p. 4]. This function determines the functions that are considered for inference. So a different covariance function would have created a different mean and variance and the functions drawn from the prior in (a) would have looked different. This makes choosing a fitting covariance function important. A better fit produces a more accurate result. But covariance functions usually aren't static. There are parameters. But these can be learned, that is why they are called hyperparameters. So while we still have to choose a fitting covariance function for the Gaussian process they can be fit to the data by determining the right hyperparameters. How to do this will be discussed later in this section. In the following section we will define what a Gaussian process is and how it can be used for regression.

Regression   Regression is a method to determine a relationship between a number of variables, using data was recorded beforehand. So in our example we want to know the relationship between the Wi-Fi signal strength and the coordinates on the map. We are going to use Gaussian process regression to predict the likelihood of Wi-Fi signals at certain coordinates based on the Wi-Fi signals that we observed beforehand. To explain how we do this with Gaussian processes we are going to start with a simpler regression and go on from there.

We are going to look at the linear model from a Bayesian perspective.

Once again we have a training set D of n observations in the form of $D = \{(\mathbf{x_i}, y_i) | i = 1, ..., n\}$. To make things easier we are going to introduce vector $\mathcal{X}$ and y to form $D = (\mathcal{X}, y)$.

Now the standard linear regression model with Gaussian noise has the following form:

$$f(\mathbf{x}) = \mathbf{x^T w}$$
$$y = f(\mathbf{x}) + \varepsilon \tag{2.12}$$

Here $f(\mathbf{x})$ is the function value. $\mathbf{w}$ is a vector of weights, which act as the parameters of the linear function. y are the observed function values. Because the real values won't be exactly on the graph of $f(\mathbf{x})$ the noise term ε is added[Rasmussen and Williams, 2005, p. 8].

The noise term is of the form of a Gaussian distribution with zero mean and variance σ_n^2.

$$\varepsilon \sim \mathcal{N}(0, \sigma_n^2) \tag{2.13}$$

From this we infer the likelihood $p(\mathbf{y}|\mathbf{X}, \mathbf{w})$, so the probability density of the observations $\mathbf{y}$ given the weights $\mathbf{w}$ and the training inputs $\mathbf{X}$[Rasmussen and Williams, 2005, p. 9].

$$p(\mathbf{y}|\mathbf{X}, \mathbf{w}) = \prod_{i=1}^{n} p(y_i|\mathbf{x_i}, \mathbf{w}) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{(y_i - \mathbf{x_i}^T\mathbf{w})^2}{2\sigma_n^2}\right)$$
$$= \frac{1}{(2\pi\sigma_n)^{n/2}} \exp\left(-\frac{1}{2\sigma_n^2}|\mathbf{y} - \mathbf{X}^T\mathbf{w}|^2\right) = \mathcal{N}(\mathbf{X}^T\mathbf{w}, \sigma_n^2 I) \tag{2.14}$$

We can form the likelihood to a Gaussian distribution with mean $\mathbf{X}^T\mathbf{w}$, which is $f(x)$, and a variance of $\sigma_n^2 I$, which resembles the noise term ε.

Because we use the Bayesian formalism we need to specify a prior over the weights. This prior expresses what we believe their nature is like before we look at the observations. For this we use another Gaussian distribution. It will have a mean of zero and the variance is the covariance matrix Σ_p[Rasmussen and Williams, 2005, p. 9].

$$\mathbf{w} \sim \mathcal{N}(0, \Sigma_p) \tag{2.15}$$

To be able to predict new values, we need to know the weights. This means we want to infer the value of the weights from the training set. This is called the posterior and has the form $p(\mathbf{w}|\mathbf{X}, \mathbf{y})$. We can use Bayes' theorem in order to achieve this.

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{marginal likelihood}}$$
$$p(\mathbf{w}|\mathbf{y}, \mathbf{X}) = \frac{p(\mathbf{y}|\mathbf{X}, \mathbf{w})p(\mathbf{w})}{p(\mathbf{y}|\mathbf{X})} \tag{2.16}$$

$p(\mathbf{y}|\mathbf{X})$ is independent of the weights and therefore it is just a normalizing constant. Using the theorem of total probability it takes on the following form:

$$p(\mathbf{y}|\mathbf{X}) = \int p(\mathbf{y}|\mathbf{x}, \mathbf{w})p(\mathbf{w})d\mathbf{w} \tag{2.17}$$

Now leaving out the normalizing constant and only concentrating on the likelihood and prior we are able to form the posterior into a normal distribution, by using equations 2.14 and 2.15.

$$p(\mathbf{w}|\mathbf{X}, \mathbf{y}) \propto \exp\big(-\frac{1}{2\sigma_n^2}(\mathbf{y} - \mathbf{X}^T\mathbf{w})^T(\mathbf{y} - \mathbf{X}^T\mathbf{w})\big) \exp\big(-\frac{1}{2}\mathbf{w}^T\Sigma_p^{-1}\mathbf{w}\big)$$
$$\propto \exp\big(-\frac{1}{2}(\mathbf{w} - \bar{\mathbf{w}})^T(\frac{1}{\sigma_n^2}\mathbf{X}\mathbf{X}^T + \Sigma_p^{-1})(\mathbf{w} - \bar{\mathbf{w}})\big) \tag{2.18}$$

Here $\bar{\mathbf{w}} = \sigma_n^{-2}(\sigma_n^{-2}\mathbf{X}\mathbf{X}^T + \Sigma_p^{-1})^{-1}\mathbf{X}\mathbf{y}$. Now the resulting Gaussian distribution is the following:

$$p(\mathbf{w}|\mathbf{X}, \mathbf{y}) \sim \mathcal{N}(\bar{\mathbf{w}} = \frac{1}{\sigma_n^2}A^{-1}\mathbf{X}\mathbf{y}, A^{-1}) \tag{2.19}$$

with $A = \sigma_n^{-2}\mathbf{X}\mathbf{X}^T + \Sigma_p^{-1}$[Rasmussen and Williams, 2005, p. 9].

Now we want to predict new values. We have a new input $\mathbf{x}_*$ with the function value $f_* \triangleq f(\mathbf{x}_*)$. In order to achieve this we use the posterior distribution and average over all possible parameter values[Rasmussen and Williams, 2005, p. 11].

$$p(f_*|\mathbf{x}_*, \mathbf{X}, \mathbf{y}) = \int p(f_*|\mathbf{x}_*, \mathbf{w})p(\mathbf{w}|\mathbf{X}, \mathbf{y})d\mathbf{w}$$
$$= \mathcal{N}(\frac{1}{\sigma_n^2}\mathbf{x}_*^T A^{-1}\mathbf{X}\mathbf{y}, \mathbf{x}_*^T A^{-1}\mathbf{x}_*) \tag{2.20}$$

This is called the predictive distribution. With this distribution one can predict the function values for new inputs $\mathbf{x}_*$. But of course this solution is still very limited. It still only considers linear functions and won't be flexible enough for the Wi-Fi data.

But there is a trick we can apply in order to fix this flaw. We can simply project the inputs into high dimensional space by using a set of basis functions and apply the linear regression model in that space. A simple example would be $\phi(x) = (1, x, x^2, x^3, ...)$. As long as the function is independent of $\mathbf{w}$ one can still apply the linear regression to it[Rasmussen and Williams, 2005, p. 11].

We will look closer at the basis function at a later point, for now it is simply given by $\phi(x)$. This changes the model for linear regression as follows[Rasmussen and Williams, 2005, p. 12].

$$f(\mathbf{x}) = \phi(\mathbf{x})^T \mathbf{w} \tag{2.21}$$

Now applying the basis function to predictive distribution from equation 2.20 can easily be done, by just applying the basis function to the inputs.

$$f_* | \mathbf{x}_*, \mathbf{X}, \mathbf{y} \sim \mathcal{N}(\frac{1}{\sigma_n^2}\phi(\mathbf{x}_*)^T A^{-1}\Phi\mathbf{y}, \phi(\mathbf{x}_*)^T A^{-1}\phi(\mathbf{x}_*)) \tag{2.22}$$

Here $\Phi = \Phi(\mathbf{X})$ and $A = \sigma_n^{-2}\Phi\Phi^T + \Sigma_p^{-1}$.

One drawback here is A^{-1}. We would have to invert A which is a $N \times N$ matrix, where N is the size of the feature space. But we can rewrite the formula[Rasmussen and Williams, 2005, p. 12].

$$f_* | \mathbf{x}_*, \mathbf{X}, \mathbf{y} \sim \mathcal{N}(\phi_*^T \Sigma_p \Phi(K + \sigma_n^2 I)^{-1}\mathbf{y}, \phi_* \Sigma_p \phi_* - \phi_*^T \Sigma_p \Phi(K + \sigma_n^2 I)^{-1}\Phi^T \Sigma_p \phi_*) \tag{2.23}$$

Here $\phi(\mathbf{x}_*) = \phi_*$ and $K = \Phi^T \Sigma_p \Phi$.

Now we will introduce function $k(\mathbf{x_p}, \mathbf{x_q})$ which is called the covariance or kernel function.

$$\begin{aligned} k(\mathbf{x_p}, \mathbf{x_q}) &= \phi(\mathbf{x_p})^T \Sigma_p \phi(\mathbf{x_q}) \\ &= \psi(\mathbf{x_p}) \cdot \psi(\mathbf{x_q}) \end{aligned} \tag{2.24}$$

The fact that we can rewrite this function as a dot product makes it possible to apply the so called kernel trick[Rasmussen and Williams, 2005, p. 12]. We can define a function $k(\mathbf{x_p}, \mathbf{x_q})$ and then replace all occurrences by it. The kernel trick is used to lift the inputs into a higher dimensional space, while making all computations still in the input space. This saves a lot of computation time and memory. So we are going to place the kernel function where we can in equation 2.23.

$$f_* | \mathbf{x}_*, \mathbf{X}, \mathbf{y} \sim \mathcal{N}(\mathbf{k}_*^T (K + \sigma_n^2 I)^{-1}\mathbf{y}, k(\mathbf{x}_*, \mathbf{x}_*) - \mathbf{k}_*^T (K + \sigma_n^2 I)^{-1}\mathbf{k}_*) \tag{2.25}$$

with $\mathbf{k}_*$ denoting a vector of covariances between the test point and the training points, and K being the covariance matrix of all training points, so $K = K(\mathbf{X}, \mathbf{X})$.

Equation 2.25 now enables us, given the training data $\mathbf{X}$ and $\mathbf{y}$, to compute a Gaussian distribution for a new data point $\mathbf{x}_*$. For the Wi-Fi data that would mean that given previously recorded coordinates $\mathbf{X}$ and corresponding Wi-Fi signal strengths $\mathbf{y}$ from one access point and a new coordinate $\mathbf{x}_*$ we would could compute Gaussian distribution. Now given a newly observed Wi-Fi signal strength from the same access point, we would be able to determine a likelihood that it was observed at coordinate $\mathbf{x}_*$, simply by using the computed Gaussian distribution. This is exactly what we need to compute the weights for the Monte Carlo localization from section 2.1.4.

Now we have everything we need to fully define the Gaussian process and then predict values from it.

According to Rasmussen and Williams [2005, p. 13] a Gaussian process is fully defined as follows:

$$\mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}')) \tag{2.26}$$

The mean function $m(\mathbf{x})$ is usually defined as 0.

The kernel function $k(\mathbf{x}, \mathbf{x}')$, also called covariance function, is an important aspect for the Gaussian process. There are many possibilities to choose from for kernel functions. The functions considered as kernel functions have to be symmetric and positive semi-definite.

But while there are many functions that could be used as kernel, we are going to focus on one of the most popular ones. The radial basis function kernel. (RBF kernel). Ferris et al. [2006] showed that this kernel is very well suited for the usage with Wi-Fi signal strengths.

$$k(x_p, x_q) = \sigma_f^2 \exp(-\frac{1}{2l^2}(x_p - x_q)^2) + \sigma_n^2 \delta_{pq} \tag{2.27}$$

Here δ_{pq} is the Kronecker delta, that is 1 whenever $p = q$ and 0 else. This particular kernel function results in a very smooth graph. There are three so called hyperparameters. These are variables, but they don't have to be set manually, but can actually be learned. How will be discussed in the next section. The hyperparameters here are the lengthscale l, the signal variance σ_f^2 and the noise variance σ_n^2.

Hyperparameter Optimization   The Gaussian process regression is a parameterless regression method. But we have to deal with so called hyperparameters. The advantage here is that we can use optimization algorithms in order to find values that are working well. In order to find those values we take the log-likelihood and maximize it. The log-likelihood function gives us an indication how well the Gaussian process fits to the data. The higher the value of the log-likelihood function the better the closer the Gaussian process resembles the given data.

The log-likelihood is defined by the following function[Rasmussen and Williams, 2005, p. 113].

$$log\, p(\mathbf{y}|\mathbf{X}, \theta) = -\frac{1}{2}\mathbf{y}^T(K + \sigma_n^2 I)^{-1}\mathbf{y} - \frac{1}{2}log\,|K + \sigma_n^2 I| - \frac{n}{2}log\, 2\pi \qquad (2.28)$$

Here θ are the hyperparameters l, σ_n^2 and σ_f^2. In order to use optimization algorithms we also need the partial derivatives of the function[Rasmussen and Williams, 2005, p. 114].

$$\frac{\partial}{\partial\theta_j}log\, p(\mathbf{y}|\mathbf{X}, \theta) = \frac{1}{2}tr\left((K^{-1}\mathbf{y})(K^{-1}\mathbf{y})^T\frac{\partial K}{\partial\theta_j}\right) \qquad (2.29)$$

So, we need a method that uses the information from the log-likelihood function and its gradient, and maximizes the value by adjusting the hyperparameter values. Some examples of algorithms that are suitable to optimize the hyperparameters are the gradient descent[Shewchuk, 1994], BFGS or L-BFGS[Liu and Nocedal, 1989]. Blum and Riedmiller [2013] propose to use resilient backpropagation (Rprop) to solve this problem. They show that the algorithm has a similar performance as L-BFGS. But it has the advantage over L-BFGS and similar methods that it is easier to implement.

Like most methods the algorithm requires not only the function itself, but also the gradient. But it doesn't use the second order derivatives or an approximation thereof, which leads to shorter computation times per iteration[Blum and Riedmiller, 2013].

In every iteration the hyperparameters θ are updated depending on the sign of the derivative:

$$\theta_i^{(t+1)} = \theta_i^{(t)} - sign\left(\frac{\partial J^{(t)}}{\partial\theta_i}\right)\Delta_i^{(t)} \qquad (2.30)$$

Δ_i is the update-value. Depending on the change of the sign the hyperparameter, Δ_i is either increased by a factor of $\eta^+ > 0$ or decreased by a factor $0 < \eta^- < 1$.

$$\Delta_i^{(t)} = \begin{cases} \eta^+ \cdot \Delta_i^{(t-1)}, \text{ if } \dfrac{\partial J}{\partial\theta_i}^{(t-1)} \cdot \dfrac{\partial J}{\partial\theta_i}^{(t)} > 0 \\[2ex] \eta^- \cdot \Delta_i^{(t-1)}, \text{ if } \dfrac{\partial J}{\partial\theta_i}^{(t-1)} \cdot \dfrac{\partial J}{\partial\theta_i}^{(t)} < 0 \\[2ex] \Delta_i^{(t-1)}, \text{ else} \end{cases} \qquad (2.31)$$

The initial update value is set to Δ_0 and is bounded by Δ_{min} and Δ_{max}. The parameters have to be specified, but there are values that work for most cases[Blum and Riedmiller, 2013].

So in order to get good values for the hyperparameters we need to apply the algorithm to the partial derivatives as specified in equation 2.29. Like most optimization algorithms it tries to minimize the function value, but we need to maximize it.

The higher the log-likelihood is the better the fit. So we simply use the negative log-likelihood and its negative partial derivatives.

The algorithms will run for a set number of iterations or until every partial derivative is 0. At every iteration we check the negative log-likelihood and compare it with the best result we found yet. If the new result is smaller we save the current hyperparameters. At the end of this process the Gaussian process will fit well to the training data[Blum and Riedmiller, 2013].

2.4   Chapter Summary

At the beginning of this chapter we explained the basics of localization. We moved from the uncertainty involved in robotics to the recursive state estimation to its implementation with a particle filter. At the end of section 2.1 we showed the Monte Carlo localization that uses particle filters. For the Monte Carlo localization to actually work we need sensor models. For a Wi-Fi receiver Gaussian processes are well suited. So, in the second half of this chapter we determined what a Gaussian process is and showed how it can be used for regression by deriving it from the linear regression. Even though the Gaussian process regression is a parameterless method, there are hyperparameters that have to be optimized. We explained how this can be done using Rprop.

In the next chapter we will use this knowledge to explain our implementation of the Wi-Fi position estimation, by using Gaussian processes and the Monte Carlo localization.

Chapter 3

Implementation

This chapter discusses the implementation of the parts needed for a functioning Wi-Fi position estimation. In section 3.1 we will introduce the Robot Operating System(ROS) and explain its basic components. In section 3.2 we will explain how the required Wi-Fi data gets fetched and published for other parts of the program. Section 3.3 gives a short overview about the used implementation of the Monte Carlo localization for the 2D laser range finder. In section 3.4 we will show how the published Wi-Fi and position data from the robot is collected and stored for further use with the Wi-Fi position estimation. The section 3.5 details the implementation of the Gaussian process for the Wi-Fi position estimation. And at last section 3.6 will tie the previous chapters together and explain the Wi-Fi position estimation as a whole.

3.1   ROS

The Wi-Fi position estimation was implemented with the ROS-Framework and run on a TurtleBot 2. ROS[Fernandez et al., 2015] is short for Robot Operating System. It supports a wide variety of robots and was designed so that different robots and environments have a common basis to make it easier to share.

The software in ROS is divided into packages. So each package usually has a different functionality or purpose. An example used in this project is the AMCL package and the created package for the Wi-Fi position estimation. These packages contain nodes. The nodes are processes. Before the nodes can be executed a master has to be started. On this master the nodes are registered, so that different nodes can see each other and can communicate with each other.

For the purpose of communication between the nodes there are topics and services. Topics can be either published or subscribed to. For example, there is the AMCL node, that publishes a pose estimation and a Wi-Fi publisher node, that publishes the Wi-Fi signal strengths, and the Wi-Fi data recorder is subscribed to these nodes so it can save the signal strengths and the corresponding position on the
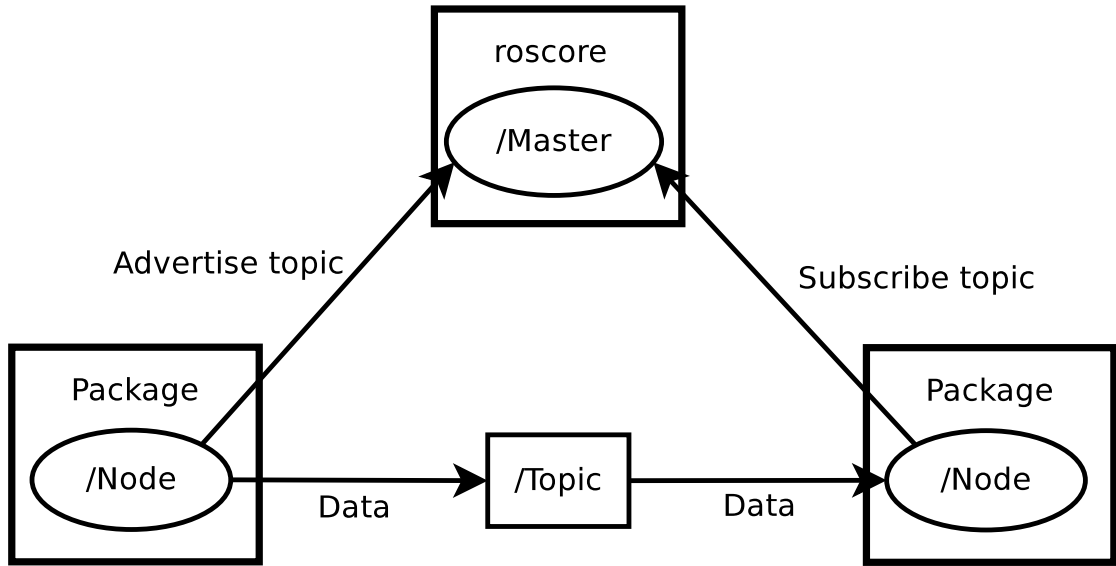
FIGURE 3.1: Relationship of the basic parts of the ROS environment.

map. A service can be offered or called by a node. For example amcl offers global localization via service. This means services are only performed, whenever they are called, while the data published on topics gets usually published constantly, no matter what.

3.2   Wi-Fi Data Publisher

The first step to realize the position estimation is to actually get the Wi-Fi data. For this we use an active Wi-Fi scan. We wait for the result and handle the data. Then the SSIDs, MAC-addresses and signal strengths are published. This is procedure gets repeated until the node is stopped. As already discussed the scan can take up to a few seconds. Of course this means that the rate at which the node publishes new Wi-Fi data is limited by the time the scan takes as well.

3.3   AMCL

We use the AMCL package(http://wiki.ros.org/amcl, accessed on 25.10.2016) for the localization using the 2D range finder. The package contains an implementation of the Monte Carlo localization and furthermore provides the needed measurement model for the laser range finder. The package implements the following specific algorithms from Thrun et al. [2005]:

- sample_motion_model_odometry

- beam_range_finder_model

- likelihood_field_range_finder_model

- Augmented_MCL

- KLD_Sampling_MCL

When started AMCL publishes the pose of the robot determined by the localization. This pose contains the x and y coordinate and the orientation as a quaternion on the given map. The poses also contain a covariance matrix, that represents the spread of the particles around the determined position.

The package has multiple use cases in this project. For one it is used for the data collection, so that we know where on the map the data was recorded. It is also used in conjunction with the Wi-Fi position estimation, to test how well it works as an aid for the global localization.

The last use case is to use the Wi-Fi position estimation in case of a localization failure. We can use the method discussed in section 2.1.6 as an indicator if there is a localization failure. Using the value from equation 2.11 we can infer the quality of the localization. Usually AMCL does only compute this value, but does not publish it so that other nodes can use it. AMCL was slightly modified, so that it publishes that value under the identifier "/max_weight".

3.4   Data Collection
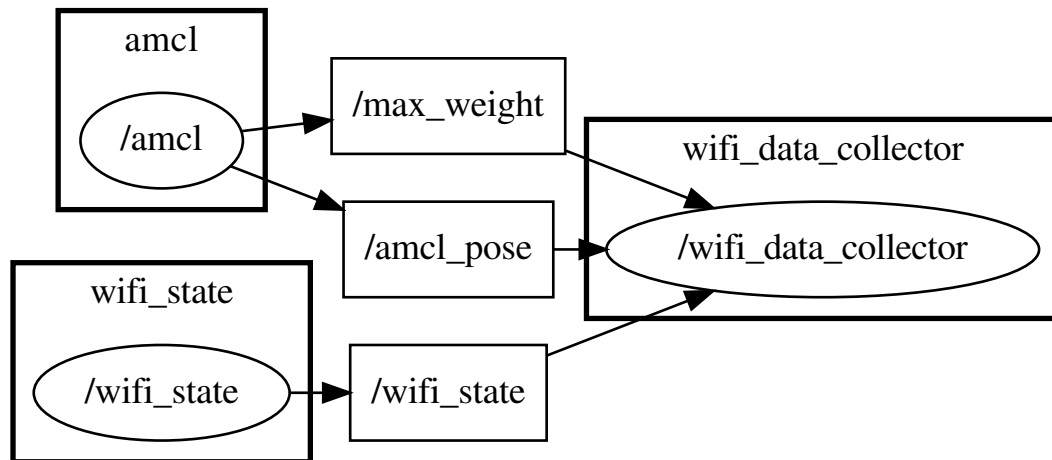

FIGURE 3.2: This shows the nodes and their topics the Wi-Fi_data_collector subscribed to and how it stores the data.

In order to collect the data, the aforementioned Wi-Fi data publisher is used in combination with AMCL. We need to know the position of the robot on the map, if we want to use the data later on to build a map of the signal strengths. AMCL

publishes the needed pose of the robot and the Wi-Fi data publisher the needed MAC-addresses and signal strengths. So the data collection node updates the pose whenever there is a new one and whenever new Wi-Fi data is published it stores the data together with the pose in csv files. For each MAC-address there is a separate file.
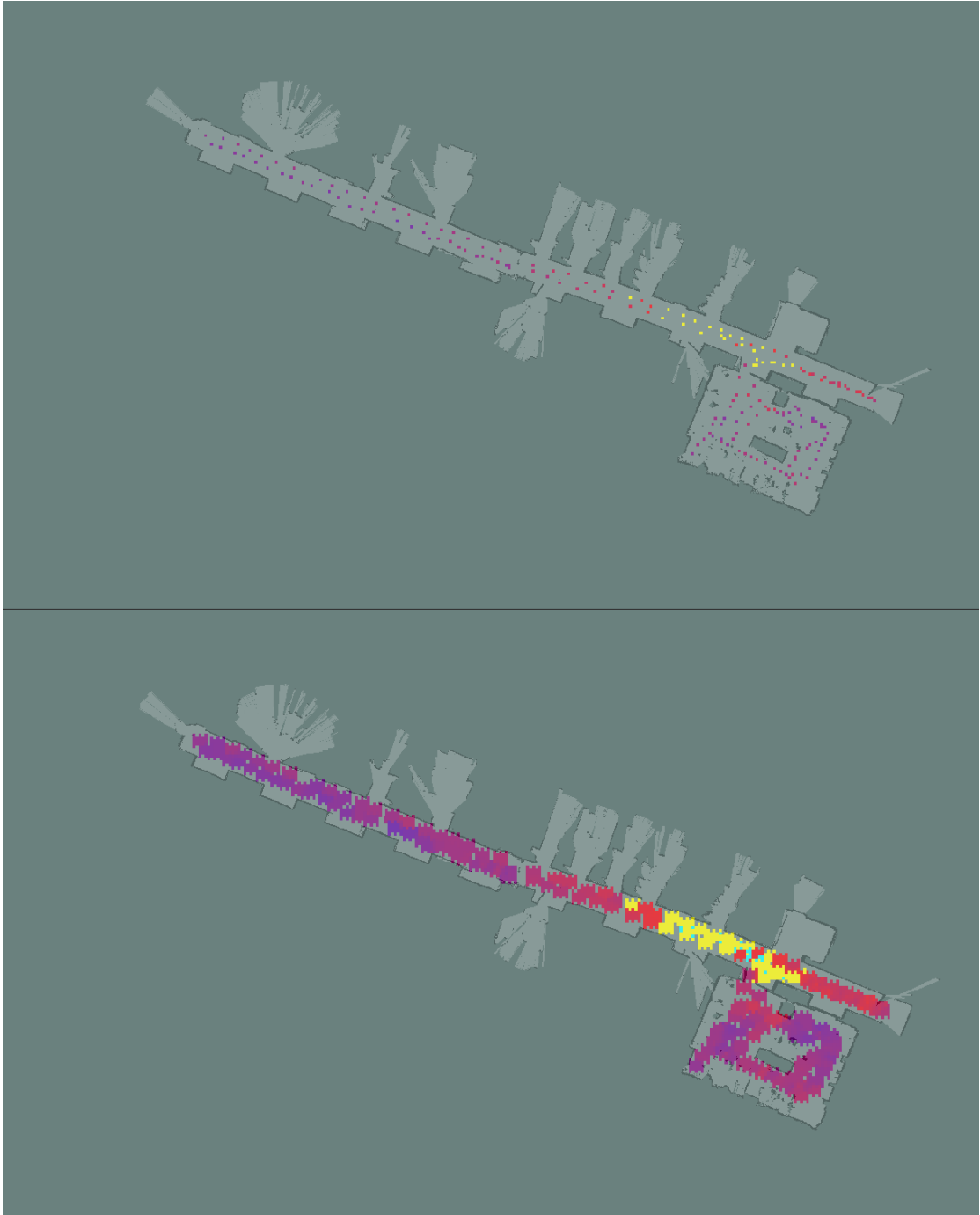

FIGURE 3.3: Both pictures represent Wi-Fi data from one access point, recorded on the hallway. Above each dot is a certain recording at the position on the map. Beneath we interpolated the data, to highlight the behavior of the signal strength. The colors from strong to weak signal strength: yellow, light blue, red, purple, blue.

3.5 Gaussian Process

The model from section 2.3 was used to implement the Gaussian process. A change was made to the formula for the kernel. The reason for this is that we want to prevent the hyperparameters from taking on negative values. This means we either have to apply an optimization algorithm that works for constrained problems, or make sure the parameters can't take on negative values. The first solution would mean we would have to use a more complicated algorithm, therefore we decided to take the second approach.

The covariance function from equation 2.27 is changed to the following form:

$$cov(y_p, y_q) = \sigma_f \exp\left(-\frac{|x_p - x_q|^2}{l}\right) + \sigma_n \delta_{pq} \tag{3.1}$$

In order to make sure that the hyperparameters can only take on positive values they are substituted by the following terms:

$$\sigma_f = exp(2\theta_1)$$
$$l = exp(\theta_2) \tag{3.2}$$
$$\sigma_n = exp(\theta_3)$$

This leads to changes for the gradient too:

$$\frac{\partial cov}{\partial \theta_1} = 2\sigma_f exp\left(-\frac{|x_p - x_q|^2}{l}\right)$$
$$\frac{\partial cov}{\partial \theta_2} = -\sigma_f exp\left(-\frac{|x_p - x_q|^2}{l}\right)\left(-\frac{|x_p - x_q|^2}{l}\right) \tag{3.3}$$
$$\frac{\partial cov}{\partial \theta_3} = \sigma_n \delta_{pq}$$

Now the optimization algorithm is applied to θ_1, θ_2 and θ_3. When these become negative, σ_f, l and σ_n will stay positive.

The Gaussian process is not a node, but a simple class. In order to create a Gaussian process usually a path to a folder with csv-files is provided. These files then contain training data, so positions and corresponding signal strengths. Each MAC-address usually has its own csv-file named after itself, so that the Gaussian process can distinguish them.

The implementation of the Gaussian process was done with the Eigen-library(http://eigen.tuxfam accessed on 28.10.2016). There are many matrix- and vector-operations needed and the Eigen-library is well suited for these kind of problems, as it provides matrix- and vector-classes with a number of useful algorithms for them.

For the optimization of the hyperparameters the Rprop-algorithm detailed in section 2.3.2 was implemented in a separate class. So once a Gaussian process was
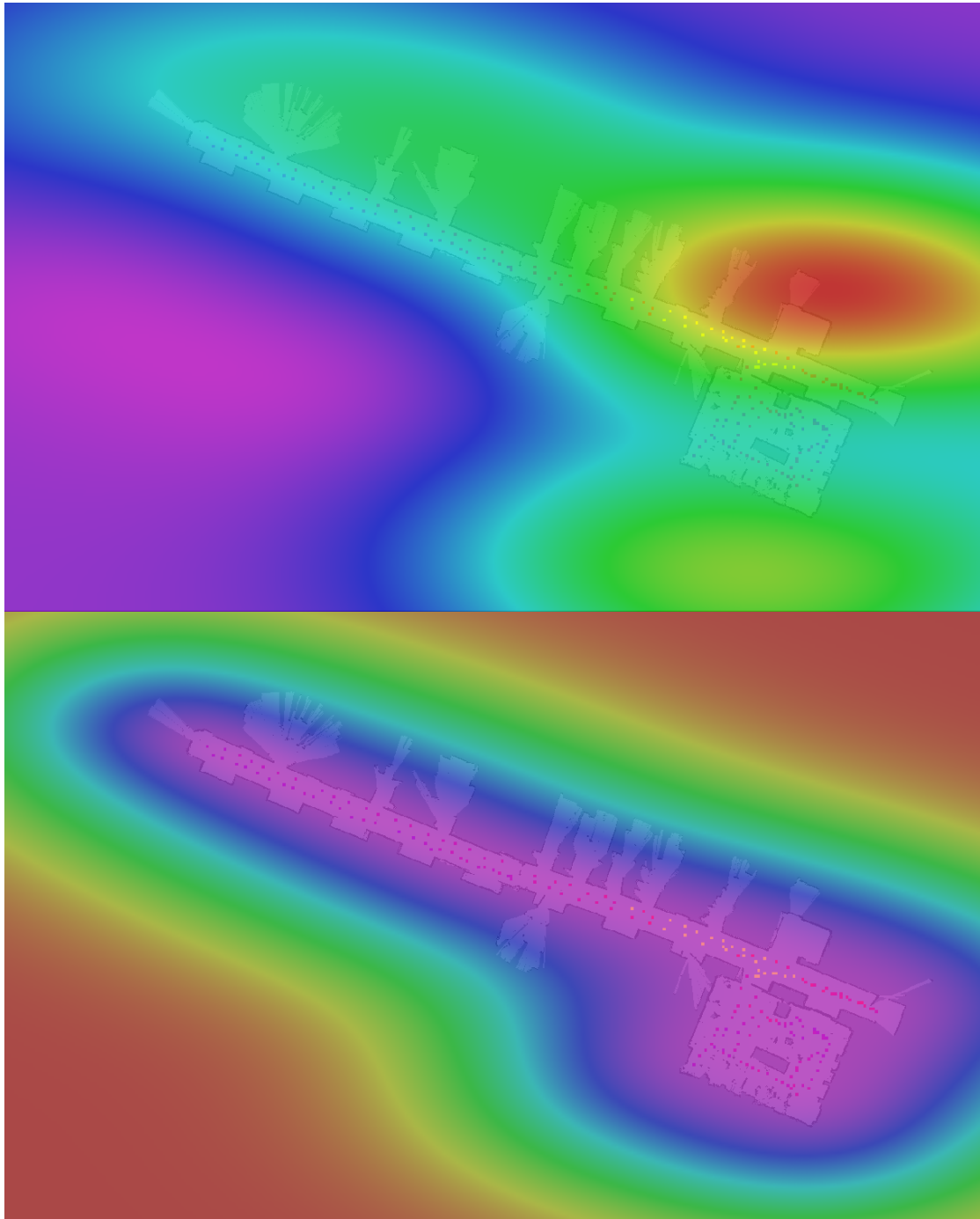
FIGURE 3.4: The picture above represents the mean of a Gaussian process that was trained for the data from figure 3.3. Below is the corresponding variance of the same process. As one can see the variance is very low close to the region where the data was recorded. This happens because we are more confident about the signal strengths in these regions. The further away we go, the higher the variance.

created, the Rprop-class can be used in order to train the Gaussian process, so that it fits the training data.
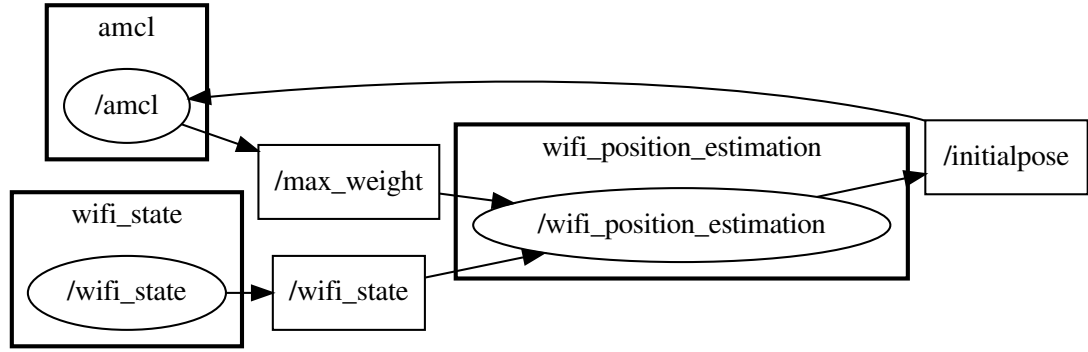
3.6   Wi-Fi Position Estimation

FIGURE 3.5: A basic diagram of the Wi-Fi localization and the subscribed and published topics.

The Wi-Fi position estimation uses the data that was collected to create a number of Gaussian processes. For every network there exists data for, a new Gaussian process is created.

Since we only try to estimate the position we basically only replicate the first step of the global Monte Carlo localization. So particles are spread across the map.

The Wi-Fi state publisher node will provide us with the current Wi-Fi signal strengths of the different networks. We will use this data with the coordinates from the particles to compute the weights for each particle.

The processes get the coordinates, and the signal strength that corresponds to the MAC-address of the data that the particular Gaussian process was created with. For each particle the weights are computed by multiplying the computed values from each MAC-address. So for n MAC-addresses we get the following formula to compute the weight:

$$w(x_*) = \prod_{i=1}^{n}(p(z_t^{[i]}|x_*)) \tag{3.4}$$

At the end the highest weight for a particular position determines which position is the most likely one. This position is then sent to AMCL. AMCL will use it as a new start position and will proceed the localization from those coordinates.

AMCL will reset its own particles and spread them around that location. How far the particles are spread is determined by a covariance matrix. These values are not computed, but set manually.
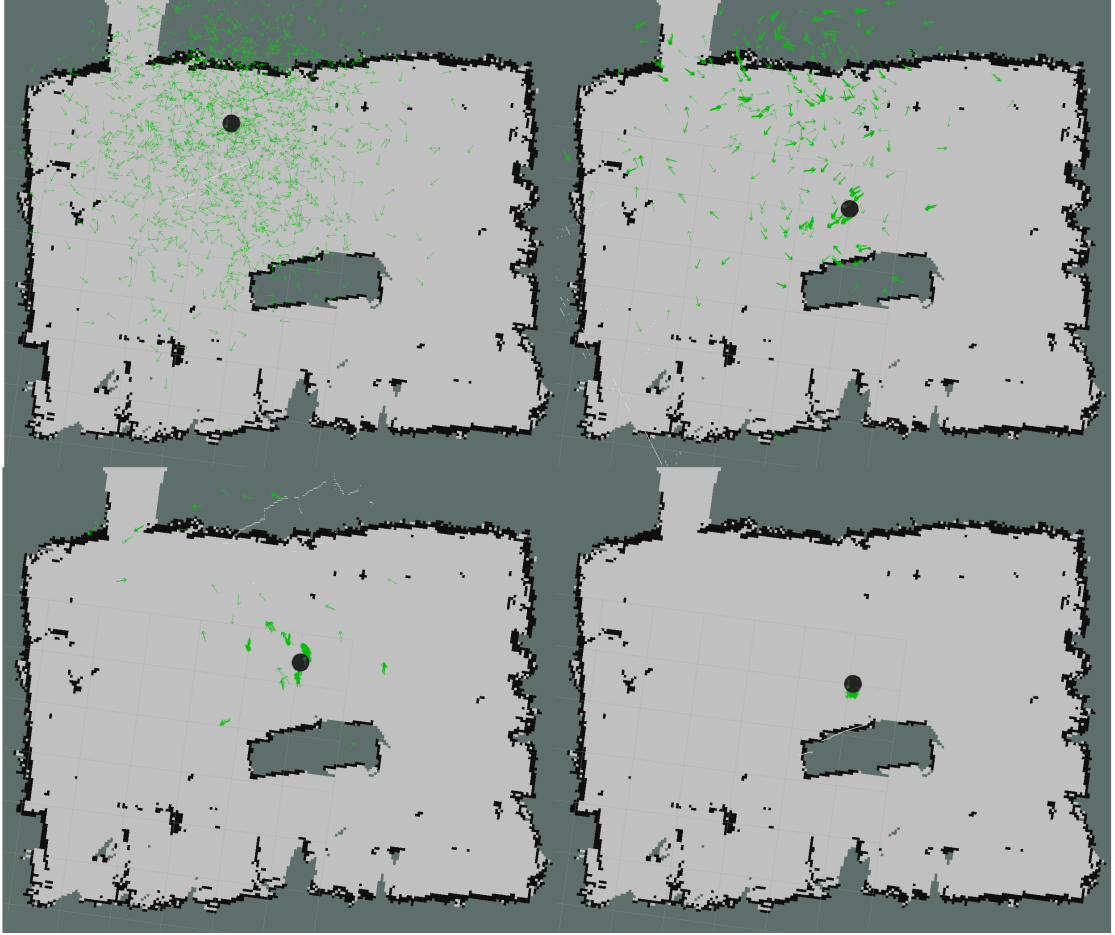
FIGURE 3.6: Example of using the Wi-Fi position estimation for global local-
ization in our laboratory. The black circle represents the robot and the green
arrows the position and orientation of the particles of AMCL using the 2D laser
range finder. In the upper left picture the Wi-Fi position estimation was per-
formed already. Compared to figure 2.3 one can see that the particles are a lot
more concentrated and not spread over the entire state space. The position is off
by a certain margin. In the following pictures the robot is turning around itself
to observe its environment with the 2D laser range finder. The particles are
quickly clustered around the correct location and the thus the correct position
of the robot can be inferred.

So the first use case would be the manual initiation of the position estimation in
order to give AMCL a seed for the localization. This would be similar to a global
localization. The robot doesn't know where it is on the map and has to deduce
its own position with the available data.

Another use case would be in case of a localization failure. To determine a local-
ization failure we use the "/max_weight" published by AMCL using the 2D laser
range finder.

We specify a threshold. If the computed value from equation 2.11 is bigger than the
threshold, the Wi-Fi position estimation is triggered. A position will be computed

and sent to AMCL as a new start position.

3.7 Using the Wi-Fi Position Estimation

FIGURE 3.7: The Turtlebot used for testing the software and carrying out the experiments in chapter 4.

FIGURE 3.8: The hallway where we tested the software and carried out the experiments in chapter 4.

In the preceding sections we explained the implementation of all the parts of the system. So this chapter will discuss the appliance of said software in the real world. The hardware used for testing the software can be seen in figure 3.7. It is a Turtlebot 2 with a Kinect. To complement the Kinect there is also the laser range finder "Hokuyo UTM-30LX" installed. The environment we tested the software in was mainly a long hallway that can be seen in figure 3.8. According to Microsoft the Kinect has a scanning range of 4 meters, but the data is still usable some meters beyond[Andersen et al., 2015]. The laser range finder has a scanning range of 30 meters[las, 2012], so it is vastly superior for our purposes, because it can see a lot further in the long hallway.

The goal now is to put the Wi-Fi position estimation to use. The first step here is to first collect data in the environment the robot is supposed to localize itself. This can be done with AMCL from section 3.3 with the laser range finder, the Wi-Fi publisher from section 3.2 and the Wi-Fi data collector from section 3.4. So one needs to start the nodes one after another. It is also important to make sure that AMCLs localization is close to the real position, so it is advisable to set the position manually at the start. The data collector offers different configurations. An important one is whether the Wi-Fi data should always be collected or only when standing still. In our tests we decided for the latter. To configure the node the launch file for the node can be customized. It contains the used parameters.

So when the nodes are started the Wi-Fi data can be collected by calling the rosservice "/start_recording" with the value "true". Now the data is recorded according to the chosen configuration. Another node not mentioned yet is the "map_traverser_node". This node gets a list of x- and y-coordinates as parameter and then drives to these positions one after the other. This node can also be used to record the data. Another method is to operate the robot via keyboard with the "turtlebot_teleop" package(http://wiki.ros.org/turtlebot_teleop, last visited 27.10.2016). The Wi-Fi and position data is then saved in comma separated values(csv)-files. The Wi-Fi data can also be visualized by using occupancy grids. Calling the service "/publish_wifi_maps" publishes the occupancy grids that can be used to visualize the currently collected data. The topics the occupancy grids are published to are named after the data's MAC-address and an identifier of the nature of the visualization. There are four identifiers:

- ln: normalized with the minimum and maximum of only this access point.

- li: normalized with the minimum and maximum of only this access point and interpolated.

- gn: normalized with the minimum and maximum of all access points.

- gi: normalized with the minimum and maximum of all access points and interpolated.

The collected Wi-Fi data can be used as training data for the Gaussian process. The Wi-Fi position estimation node is given a parameter that contains the path

to a folder with the csv-files containing the training data. There is a launch file that contains these parameters and should be customized to point to the correct folder containing the recorded Wi-Fi data.

Once started the node either trains the Gaussian processes created from the training data, or if this was already done when previously running the node it loads the already optimized hyperparameters. The optimized hyperparameters are saved as csv-files as well, in a subfolder of the folder containing the training data. Once the node has been initialized fully it can be used to approximate the position of the robot. In order to send the position estimation to AMCL the service called "compute_amcl_start_point" can be used. Another parameter given to the Wi-Fi position estimation node is a threshold for the "/max_weight" value published by AMCL. Is the value over this threshold the Wi-Fi position estimation is triggered to recover from the localization failure.

3.8 Chapter Summary

In this chapter we discussed the implementation of all parts needed for the Wi-Fi position estimation. At first we explained the ROS-framework used for the implementation of the system. Afterwards the parts needed to collect the Wi-Fi data were shown, with the Wi-Fi data publisher, AMCL and the Wi-Fi data collector. The collected data could then be used as training data. In order to implement the position estimation first the implementation of the Gaussian process was discussed and afterwards its application for the Wi-Fi position estimation node. At last we were showing the particular hardware used and expounded on how to use the implemented software.

We now have a system that is able to estimate the position via Wi-Fi data. In the next chapter we will use it in different experiments to determine how effective it is for the proposed tasks.

Chapter 4

Experiments and Results

In order to determine how well the Wi-Fi position estimation works, we carried out a number of experiments. But at first we will analyze the collected Wi-Fi data in section 4.1.1. In section 4.1.2 we will assess the accuracy of the Wi-Fi position estimation by itself. The following section 4.1.3 contains two experiments that determine how well the Wi-Fi position estimation can aid AMCL with a laser range finder. In the first experiment we evaluate how well it can help with the global localization. In the third and last experiment we inspected how well the localization failure detection works.

4.1 Data Analysis and Experiments

4.1.1 Wi-Fi data

First let's examine the Wi-Fi data and look at how it behaves. Figure 4.1 shows an example of the Wi-Fi strength of one access point. The signal reaches its peak close to the access point, marked by the yellow color. The further away the data was collected the weaker the signal was. In this example the access point was in the same room and there are basically no obstacles between the access point and the robot at any position. Turning to a different example from figure 4.2. As one can see the signal never reaches the strength of figure 4.1. But still the peak is at the center of the recorded data. To both sides then it gets weaker until it the Wi-Fi receiver can't pick it up anymore. It is reasonable to assume that the signal originates from an access point on another floor, so either from a hallway above or below. Still even though the signal is obstructed it is evenly distributed. This is probably due to the fact that no matter where the robot recorded the signal on the hallway, the signal was always obstructed by walls and thus was weakened equally.

Now figure 4.3 paints a different picture. We can see a clear peak in the upper right corner of the room. And indeed the access point is installed there behind
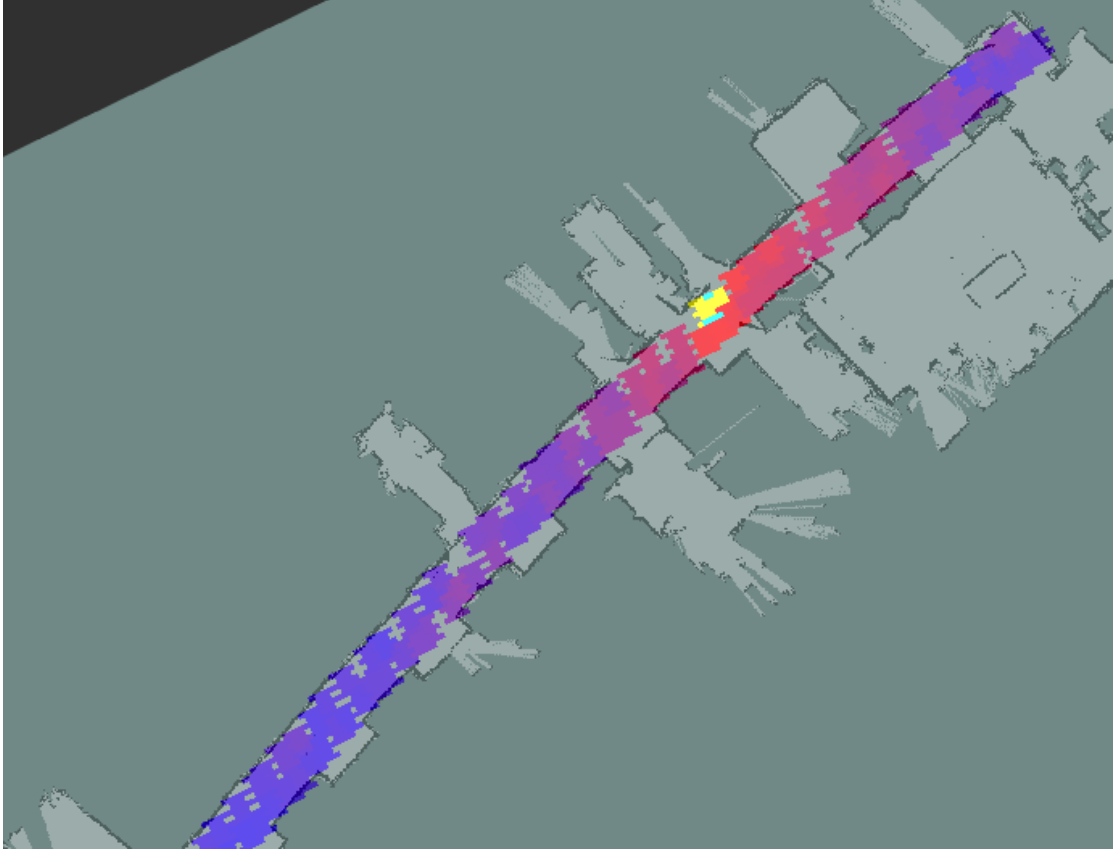
FIGURE 4.1: Visualization of Wi-Fi data collected on a hallway. The access point the signal originated from is on the same floor.

the wall. But the signal doesn't weaken gradually like it was on the floor. Even though in its entirety it does indeed get weaker the further away it was recorded, there are local peaks in different places.

This is probably due to the nature of this room. There are tables, chairs and a couch, which can all lead to the obstruction of the signal. Especially because the recording robot was relatively small.

4.1.2 Wi-Fi Position Estimation accuracy

For the Wi-Fi position estimation to be actually useful it is important that it is accurate to a certain extent. It is not necessary to achieve accuracy on the level of a few centimeters, but it can only be useful if it can reduce the possible positions to a radius of a few meters. The first experiment was designed to compare the actual position of the robot with the estimated position from the Wi-Fi data.

To determine the actual position amcl is used. The position was determined at the start of the localization, so that it is as accurate as possible. The test environment was a long hallway. The hallway can be seen in figure 4.1 This would be the kind
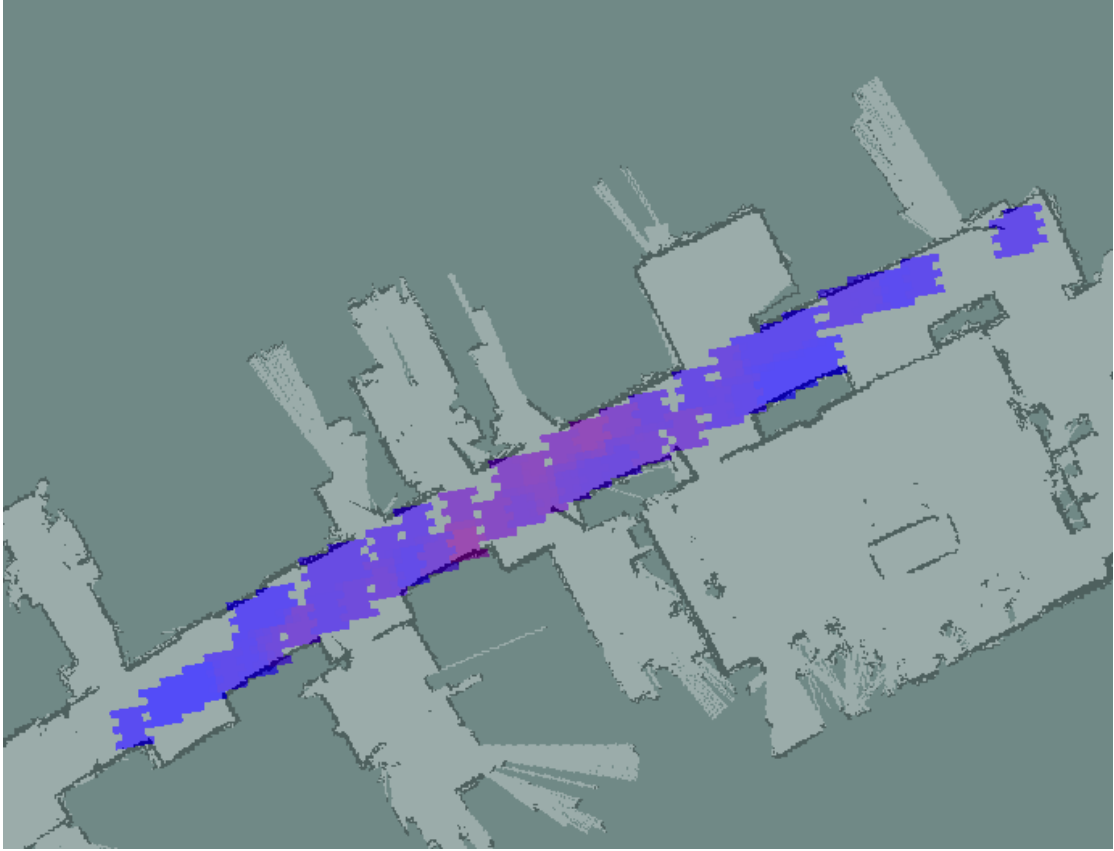
FIGURE 4.2: Visualization of Wi-Fi data collected on a hallway. In this example the access point is on a different floor.

of environment where the Wi-Fi position estimation could give an advantage over the usual approach to global localization because there aren't any characteristics that stand out and there is a lot of symmetry.

The robot drives to different points on the map that were chosen beforehand. When it arrived the robot will execute the Wi-Fi position estimation. The result will be compared with the coordinates given by amcl. The difference between the two results will be computed.

The result of the experiment was, that on average the difference to the ground truth was 1.81 meters.

4.1.3   Amcl with Wi-Fi Position Estimation

Global Localization   The proposed potential use for the position estimation was that it could give a starting position for amcl. This experiment will answer in how far the position estimation can aid amcl. Once again a hallway is used. A set of goals for the robot to drive to is used. When a goal was reached the two different methods for global localization are compared.
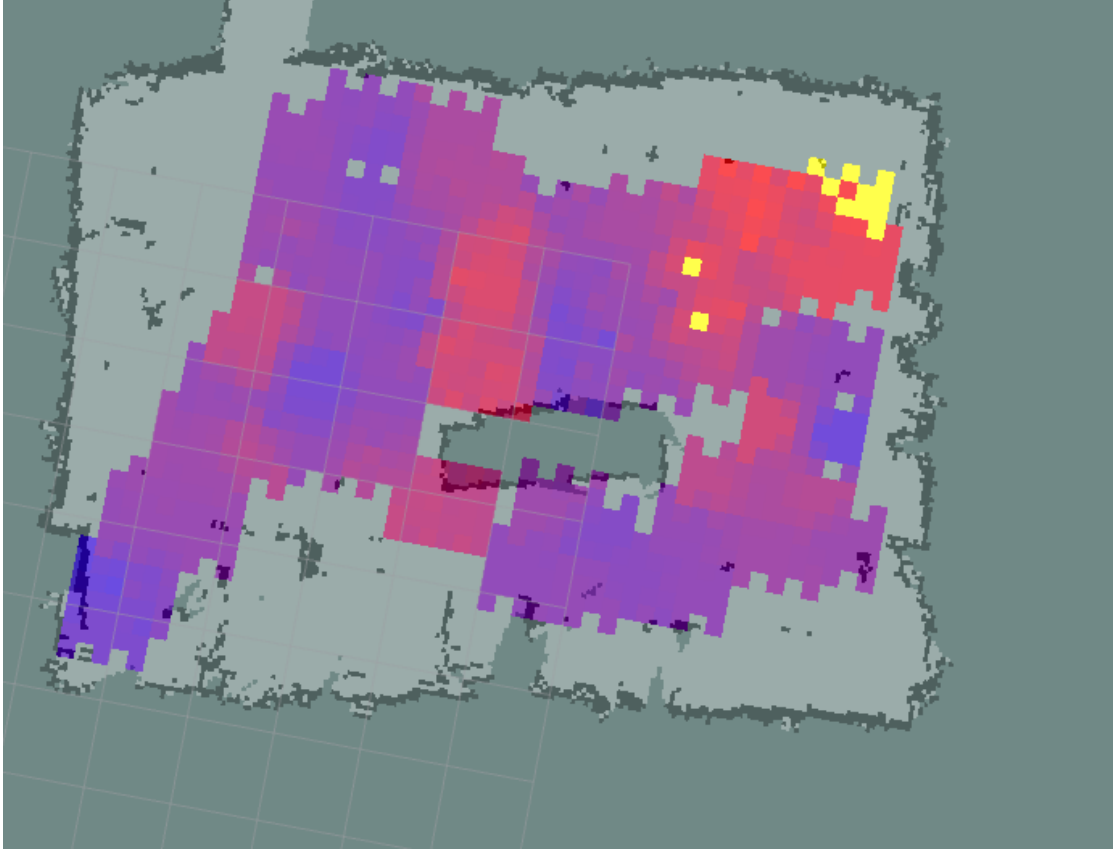
FIGURE 4.3: Visualization of Wi-Fi data collected in a room.

One method uses the typical global localization where the particles are spread over the entire map and then converge to one pose. The other method will use the Wi-Fi position estimation as a seed and will lead to a much smaller area for the particles to be spread out in.

When the estimation was performed the robot will turn 360 degrees and then drive to the next planned goal. From there the procedure is repeated.

At the arrival at the next goal the difference to the actual position was measured. Using the Wi-Fi position estimation the mean error was 3.39 meters. With the global localization the mean error was 17.34 meters.

While observing the global localization using the Wi-Fi position estimation when executing the experiment some notable behaviors occurred. All in all in 7 of 19 tries the robot was able to approximately find the correct location. In some cases the failure could be explained by the fact that the Wi-Fi position estimation was too far away from the true position. In other cases the robot localized itself in the wrong direction. Of course the reason for this is that with the Wi-Fi data we are not able to infer any information about the orientation of the robot. The hallway looks very similar on both sides, so it is easy to reconstruct as to why the localization was failing. In figure 4.2 one can see that on one side of the hallway the wall is built straight, while on the other side in a certain interval the wall gives way

to a bit more space. So sometimes it happened that the robot was localizing itself next to one of these spaces, even though it was actually next to the space before or behind it. So even when we are able to narrow down the possible position of the robot on the map, the laser range finder based localization can still get confused by the structure of the room.

Kidnapped Robot Problem   Another case where the position estimation can be used in AMCL, is when the localization fails. This kind of problem is called the kidnapped robot problem.

In this experiment the robot will drive up and down the hallway. The values from equation 2.10 are used to compare the weights in the long term, with the weights in the short term. This means the robot needs to be localized correctly for some time, so that it can be detected that the robot was kidnapped. So at first the robot will drive up and down the hallway with the help of AMCL using the laser range finder. We will record this with a rosbag. Once finished the rosbag can be replayed and a new instance of AMCL can be applied to the recorded data. At first we will initialize the new instance of AMCL to the correct location. After driving up and down for some time AMCL will be given an initial position at a random point on the map. So this will result in a localization failure. Meanwhile AMCL still gets the data from the robot driving up and down the hallway from the rosbag. Since AMCL now has the wrong location, the "/max_weight" value should rise and signal this localization failure or kidnapping of the robot.

We now want to know if the Wi-Fi position estimation would be triggered. For that to happen we set a threshold of 0.1 for the "/max_weight" topic. This value is computed by AMCL as explained in section 2.1.6.

In 10 tries the Wi-Fi position estimation was triggered every time by the time the robot was at the other end of the hallway. The mechanic that is used is the same as in the previous experiment, so a new the new computed position is given as a seed for AMCL. The accuracy of this mechanic was already discussed in the previous chapter.

4.2   Chapter Summary

We looked at the Wi-Fi data and its behavior. When there is are no obstacles in between the access point and the robot, the signal gradually gets weaker the further the robot is away. In rooms with many obstructions we saw that the signal is less predictable. The first experiment was designed to measure the Wi-Fi position estimation's accuracy, that resulted in an average error of about 1.81 meters. In the next experiment we used this mechanic to enhance the global localization of AMCL. The computed position is given as a seed for AMCL. The result was an improvement over AMCL's usual global localization, with an error of 3.39 meters with using the Wi-Fi position estimation compared to an error of

17.34 meters without it. The last experiment was designed to test the localization failure detection. It worked in 10 out of 10 tries. In the following chapter we will evaluate the results of the experiments and draw our conclusions from them.

Chapter 5

Conclusion

The Wi-Fi position estimation can be realized with Gaussian processes. The results of the first experiment had a mean error of around 1.81 meters to the ground truth. As explained the position is computed by spreading particles over the map and then choosing the one with the highest weight. The Wi-Fi position estimation can be helpful for the Monte Carlo localization with a laser range finder. The experiment showed that in the case of global localization the Wi-Fi position estimation can lead to a more accurate position estimation. With the Wi-Fi position estimation the error to the ground truth after driving for a few seconds was 3.39 meters, while it was 17.34 meters for the global localization of amcl.

But of course there are drawbacks to the Wi-Fi position estimation. For one scanning for new data takes a few seconds. That means when the robot drives around the newest data for an access point can already be seconds old and thus be already meters away from the current position. Another problem is the computation time for the Gaussian processes. When using the Wi-Fi position estimation for global localization it can take a few seconds as well to get a result. The factors here are the number of access points available and the numbers of particles used. To use it for the global localization a large number of particles are needed. Of course this also depends on the size of the map. But once again when the robot drives around it can happen that the result of the Wi-Fi position estimation is already based on data that is tens of seconds old and thus based on a different position on the map.

These are limitations that will pose problems when the Wi-Fi position estimation is actually to be used in real world applications. As for the global localization it can still be usable. The robot needs to know its own position before driving around and the Wi-Fi position estimation can help with that. So it will not be a problem when the robot has to stand while computing it. In the case of a localization failure, so the kidnapped robot problem, the robot is usually already doing a task, so it is probable that the robot will drive around and as discussed this can pose a problem for the position estimation. So when the a localization failure is detected the robot would need to stop its task and stand still until a position was computed and continue afterwards.

Another problem can arise when the Wi-Fi position estimation computes a position that is too far off from the true position. Since the detection for the localization failure is based on the difference between the short- and long-term average weights it likely that this failure will not be recognized.

Chapter 6

Outlook

Thus far only a position estimation was realized. While this can be helpful for the Monte Carlo localization with a laser scanner, it was shown that there are still a lot of problematic limitations. A sensor measurement model for the Wi-Fi receiver was implemented. This could be used directly in amcl. For the Monte Carlo localization it is no problem to incorporate different sensors. The weights can simply be computed by computing the product of all sensor measurement models. But once again the biggest obstacle here is the time requirement for both the Wi-Fi scan and the computation time of the Wi-Fi measurement model. As for the Wi-Fi measurement model this problem could be eased by the fact that amcl uses a lot less particles when it converged to a position. Another trick that could be used is to omit some access points per iteration. But even then the time a Wi-Fi scan takes would pose a problem. Also the question if incorporating the Wi-Fi data into the localization would lead to a better localization still stands. With the measurement model used here for the Wi-Fi receiver it is not possible to determine the direction of the robot, but only its position. The localization with a laser range finder is already reliable once the true pose was found, so the contribution of Wi-Fi data to this problem remains questionable.

Similar concerns stand for a localization realized purely with the Wi-Fi measurement model, without a laser scanner. Once again the time a Wi-Fi scan takes and the computation time of the Wi-Fi sensor model are too high.

As for the use of the Wi-Fi position estimation for the global localization, we still see possibilities for optimizations. Right now particles are spread over the entire map. So with a bigger map one either heightens the particle count, or one accepts the worse position estimation. But with the recorded Wi-Fi signals beforehand and the current recorded Wi-Fi signals one can approximate the area, where these recordings take place. So by looking at the MAC-addresses observed and then looking at the area where these particular MAC-addresses were observed one can at least do an approximation of the area where the robot is and then only spread the particles there. Of course this only works if one assumes that the training data covers the area where the robot is positioned, but in most cases that should be given.

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