The Kakeya Needle Problem

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Intro
Given a needle, what is the smallest area needed to spin the needle 360°?

History
This problem was first posed by Soichi Kakeya in 1917.

Start with the obvious

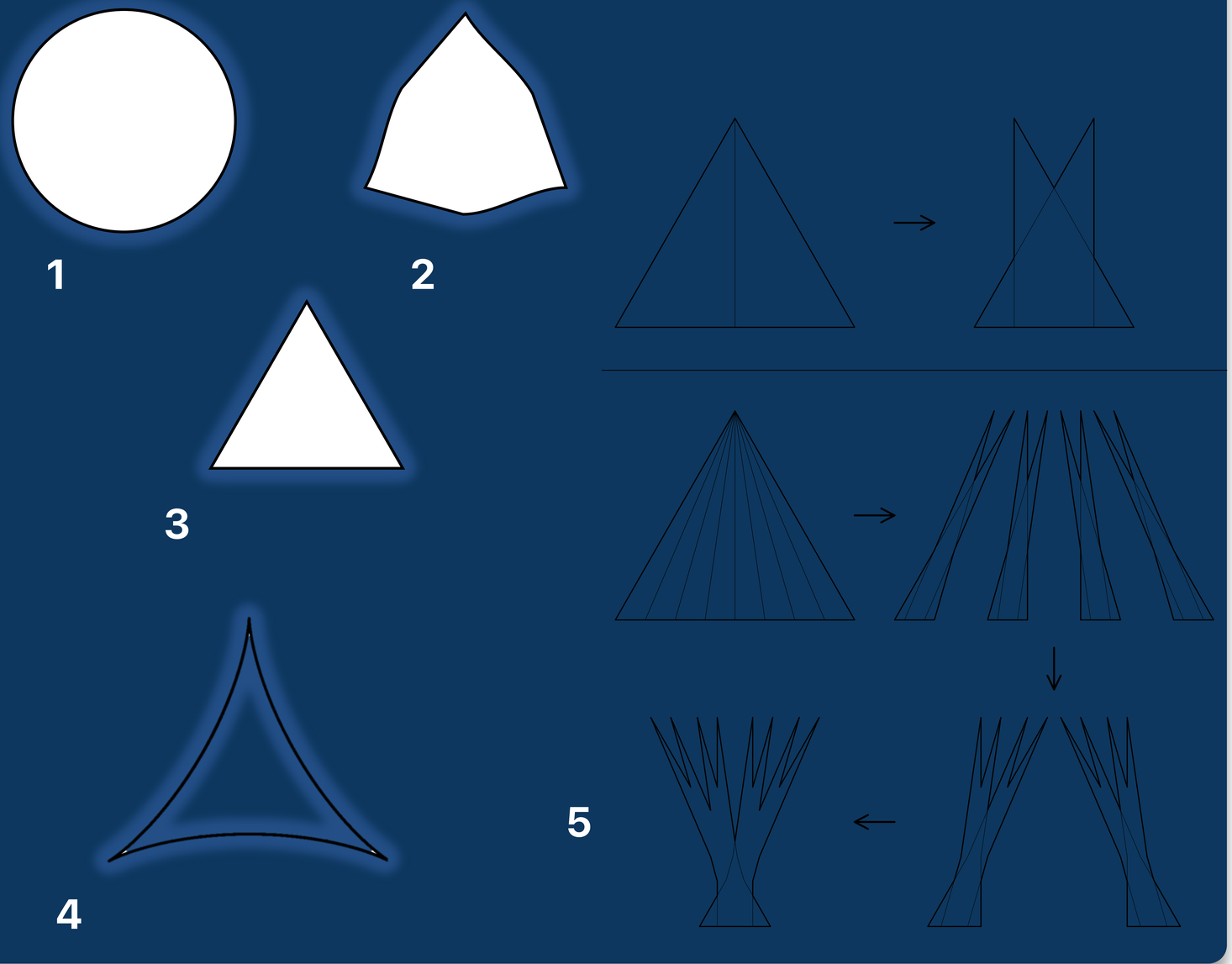
We can easily see that we can spin a needle inside a circle. Shape 1.

If we “shave off the sides” to make an inflated equilateral triangle. Shape 2.

What if we shaved the sides all the way to an equilateral triangle? Shape 3.

In 1920 Pal showed that an equilateral triangle is the smallest convex shape that we can spin a needle in.

What is the smallest area needed to spin a needle?



Can we make the are area smaller?

We may form a deltoid. Shape 4.

Is there a general way to construct smaller and smaller shapes?

Yes, with Perron trees. Shape 5. We can make the area arbitrarily small.

How can we extend this?

We can extend it in 3D. What is the smallest by volume shape where we can spin a needle by 360°, in any direction.

What are the properties of this fractal shape that we get?

References and more info