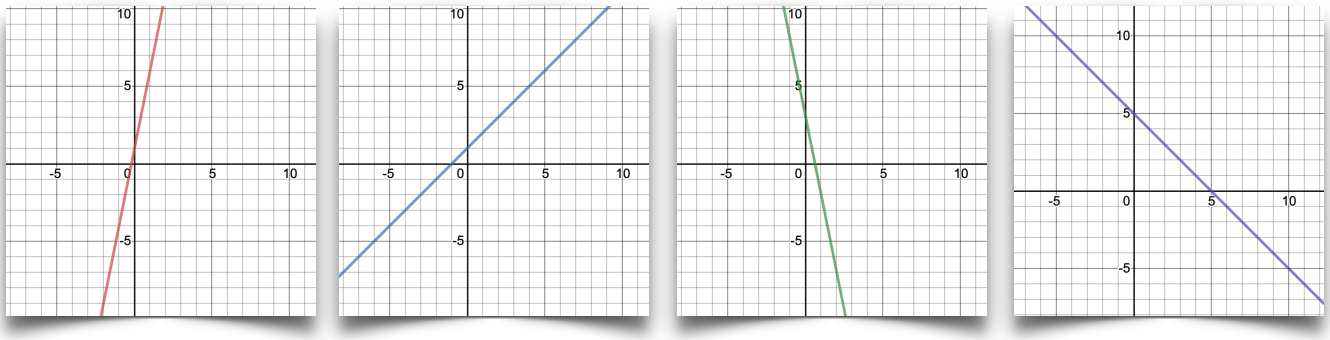
Linear Equations

Equations from two points
Equations from a point and a slope

Linear equations are degree one polynomials. There are of the form $y = m\,x + b$. There are generally two pieces of information that define a linear equation, that is *slope*, usually denoted *m,* (how steep the line is), and the *y-intercept*, denoted *b* (where the graph crosses the y-axis).

The effect of the slope is how steep the graph is. The sign of the slope determines whether the graph is leaning right or left:



$y = 5x + 1$ $y = x + 1$ $'y = -5x + 3$ $y = -x + 5$

In face, you are highly encouraged to play around with this **graph**.

One thing we need to understand is how to find a linear equation satisfying certain constraints. There two types of constraints under which you will have to find linear equations: *a linear equation given two points* and *a linear equation given a slope and a point*.

Finding Linear Equations from Two Points

Given two points (x_1, y_1) and (x_2, y_2) we are asked to find a linear equation between these two points. Notice that a linear equation, has two parts a *slope* and a *y-intercept*. We start at the beginning by first finding the slope, which is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Once we are done we pick an arbitrary point along the line (x, y), and use that to find the lope again:

$$\frac{y - y_1}{x - x_1} = m$$

This, with minor simplification, is our equation: We look at a couple of examples:

Example: Find the equation of the line between (5,6) and (7,11).

The slope of this line would be: $\frac{11-6}{7-5} = \frac{5}{2}$. Now, let (x, y) be some arbitrary point along the line. The slope between (x, y) and any other point should be $\frac{5}{2}$. In that case

$$\begin{aligned} \frac{y-11}{x-7} &= \frac{5}{2} \\ 2y - 22 &= 5x - 35 \qquad \text{cross-multiply} \\ 2y &= 5x - 35 + 22 \\ 2y &= 5x - 13 \\ y &= \frac{5}{2}x - \frac{13}{2} \end{aligned}$$

Example: Find the equation of the line between (9,7) and $(0, -6)$:

Slope is: $\frac{7-(-6)}{9-0} = \frac{13}{9}$. Similar arguments yield:

$$\begin{aligned} \frac{y-(-6)}{x-0} &= \frac{13}{9} \\ 9y + 54 &= 13x \\ 9y &= 13x - 54 \\ y &= \frac{13}{9}x - 6 \end{aligned}$$

Finding Linear Equations from a Slope and a Point

The other constraint is to find a linear equation given a slope and a point. This is easier than the previous examples. Pick an arbitrary point on the line, say (x, y). Notice that the slope between the given point and (x, y) should match the given slope. This gives us a way to find the equation satisfying the constraint. We look at examples:

Example: Find the equation of the line going through (6,1) and having slope $\frac{5}{6}$.

Pick an arbitrary point on the line, (x, y), and we try and find the slope between that and (6,1), which gives us:

$$\begin{aligned}\frac{y-1}{x-6} &= \frac{5}{6}\\ 6y-6 &= 5x-30\\ 6y &= 5x-24\\ y &= \frac{5}{6}x-4\end{aligned}$$

Example: Find the equation of the line going through $(-1,9)$ with slope $-\frac{6}{5}$.

We use the same argument as above:

$$\begin{aligned}\frac{y-9}{x+1} &= -\frac{6}{5}\\ 5y-45 &= -6x-6\\ 5y &= -6x+39\\ y &= -\frac{6}{5}x+\frac{39}{5}\end{aligned}$$