Exponential Functions

Exponential functions describe a wider array of phenomena in real life. These are function that grow at a certain factor, as opposed to a certain constant term, as with linear functions.

Assume a certain virus is so contagious that every person infects two other people every week. This means that every week, the number of people who have the virus doubles. Thus after one week we have twice the number of people infected, after two weeks we have four times the number of infected initially, and after eight weeks we have 256 times the number of people infected initially. This is *much much* faster than how a linear function would increase.

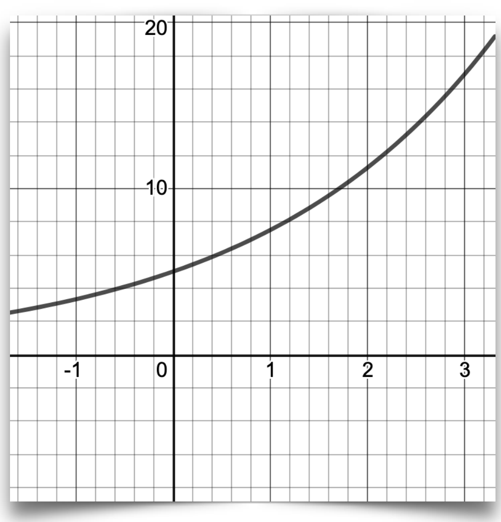
Exponential functions look like the following:

Growth/Decay rate

$$y = ax^r$$

Initial value

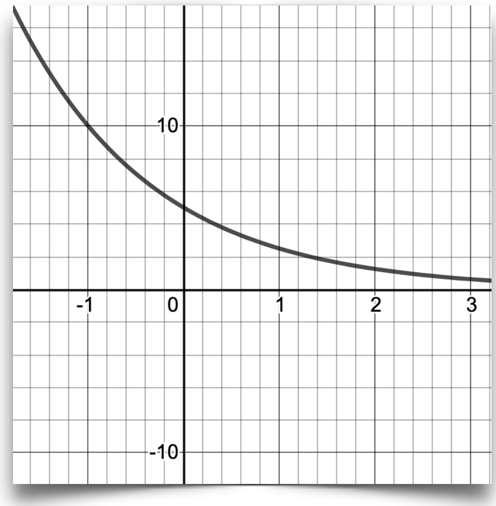
So for example the following is an exponential equation: $y = 5(1.5)^x$, with an initial value of 5 and a growth rate of 1.5. This is what the graph looks like:



$y = 5(1.5)^x$

Notice that the y-intercept is actually $y = 5$. You can think of this in terms of time. At time $t = 0$, there was 5 of whatever is growing exponentially. This means, for example at the start (time $t = 0$) there was five people infected.

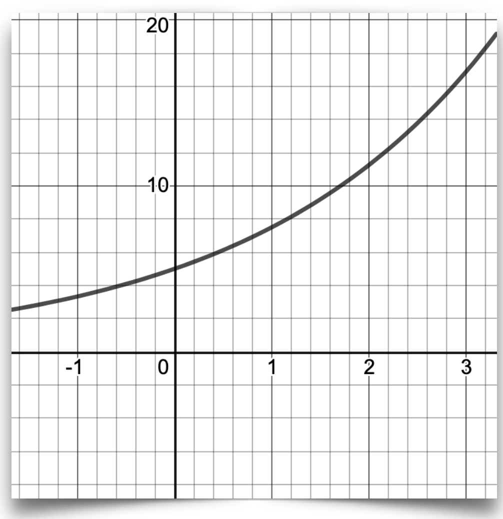
In face, there's more. If the growth factor is < 0, this means that the graph is decreasing. Take a look at the graph of $y = 5(0.5)^x$. Again, these are also exponential functions, but these look radically different from exponential function. This is an example of *exponential decay*. The growth factor, when then the growth factor < 1 is called the *decay factor*.



$y = 5(.5)^x$

There is a fundamental relationship between an exponential function and its growth/decay factor. The growth/decay factor refers to the factor that every point on the graph is multiplied by to get the next value. Take our previous example:

Notice that the y value at $x = 1$ is 7.5. The value at $x = 2$ is 11.25. Observe that $\frac{11.25}{7.5} = 1.5$. Notice again, that the value at $x = 0.5$ is ≈ 6.12, and at $x = 1.5$ is ≈ 9.19. Observe that $\frac{9.19}{6.12} \approx 1.5$. This is true of any two points along the graph separated by a value of 1.



$y = 5(1.5)^x$

What happens if we take two points with a distance of 2? Notice that at $x = 1$, the y-value is 7.5, and at $x = 3$, the value of y is 16.875. If we divide we get $\frac{16.876}{7.5} = 2.25 = 1.5^2$.

This is true for any exponential function.