Logarithm

A logarithm turns multiplication into addition. We can see this by observing what log does to the *product* of two numbers. It turns that into a *sum* of the logs of the individual numbers:

$$\log(ab) = \log(a) + \log(b)$$

Log also does the inverse for the inverse of multiplication, in other words, log turns division into subtraction:

$$\log(\frac{a}{b}) = \log(a) - \log(b)$$

Log also converts the multiplicative identity to the additive identity.

$$\log(1) = 0$$

In fact, the rules of log also look like the opposite of the rules of exponents. We will see here:

Rule	Exponents	Result	Example
Identity	$\log_a(1), \forall a \in \mathbb{R}$	0	$\log_5(1) = 0$
Inverse	$\log_a(\frac{1}{a})$	-1	$\log_5(\frac{1}{5}) = -1$
Power	$(a^b)^c, \forall a, b, c \in \mathbb{R}$	a^{bc}	$(5^2)^3 = 5^6$
Product	$a^b \times a^c, \forall a, b, c \in \mathbb{R}$	a^{b+c}	$5^2 \times 5^3 = 5^5$
Product (with same base)	$a^b \times c^b, \forall a, b, c \in \mathbb{R}$	$(a \times c)^b$	$5^2 \times 6^2 = (5 \times 6)^2 = 30^2$
Negative Power	$a^{-1}, \forall a \in \mathbb{R}$	$\frac{1}{a}$	$5^{-1} = \frac{1}{5}$
Fractional power	$a^{\frac{1}{b}}, \forall a, b \in \mathbb{R} \backslash \{0\}$	$\sqrt[b]{a}$	$5^{\frac{1}{3}} = \sqrt[3]{5}$
Quotient	$a^b \div a^c, \forall a, b, c \in \mathbb{R}$	$\frac{a^b}{a^c}$	$\frac{5^7}{5^3} = 5^{7-3} = 5^4$
Fractional power	$a^{\frac{b}{c}}, \forall a, b, c \in \mathbb{R}$	$\sqrt[c]{a^b}$	$5^{\frac{2}{5}} = \sqrt[5]{5^2}$