

# Finite Element Methods HW1

**1a** We observe quite easily that  $f(0) = 0$  while  $f(1) = 1$ . So our node points are  $\{0, 1\}$  and the node values are also  $\{0, 1\}$ . Thus  $\lambda_0(x) = \frac{1-x}{1-0} = 1 - x$  and  $\lambda_1(x) = \frac{x-0}{1-0} = x$ .

$$\pi f(x) = x$$

Our plot is:

In [ ]:

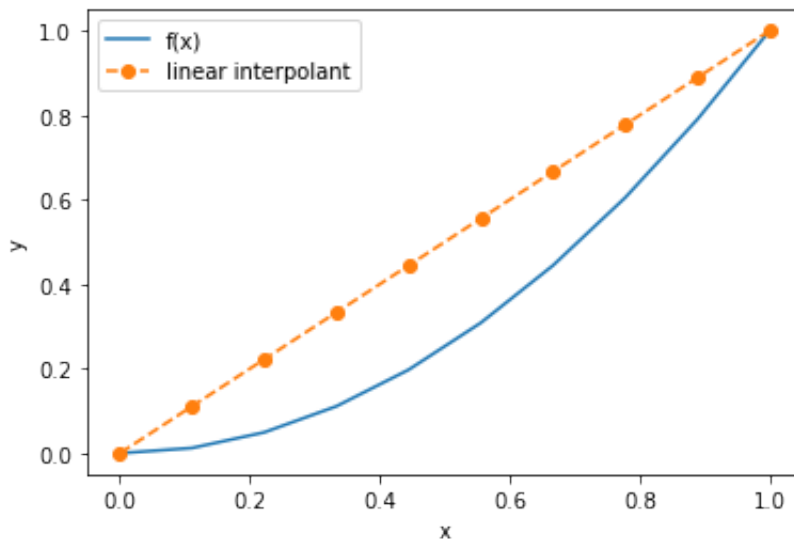
```
import numpy as np
import matplotlib.pyplot as plt

x = np.linspace(0,1,10)
y1 = x*x
y2 = x

plt.plot(x,y1)
plt.plot(x,y2,'--', marker = 'o')
plt.legend(["f(x)", "linear interpolant"])
plt.xlabel("x")
plt.ylabel("y")
plt.show
```

*#import numpy*  
*#import pyplot from matplotlib*  
*#setup x as a 1D vector*  
*#we plot y=x^2*  
*#we setup the linear interpolant*  
*#we plot the first y*  
*#we plot the second y with markers*  
*#give a plot legend*  
*#label our axes*

Out [ ]: <function matplotlib.pyplot.show(close=None, block=None)>



**1b** We easily see that  $f(0) = 0$  while also  $f(1) = 0$ . Clearly our linear interpolant is  $y = 0$ .

Our graph is:

In [ ]:

```

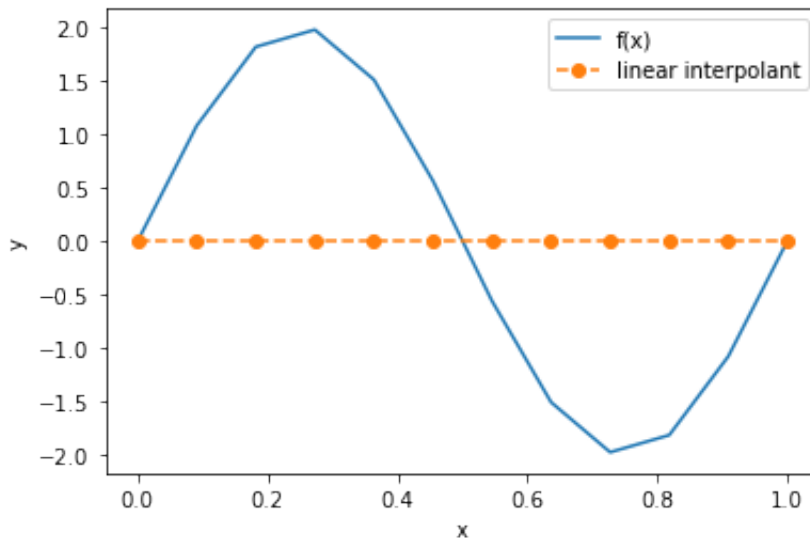
x = np.linspace(0,1,12)
y1 = 2*np.sin(2*np.pi*x)
y2 = x*0

plt.plot(x,y1)
plt.plot(x,y2,'--', marker = 'o')
plt.legend(["f(x)", "linear interpolant"])
plt.xlabel("x")
plt.ylabel("y")
plt.show

```

*#setup x as a 1D vector*  
*#we plot  $y=x^2$*   
*#we plot our linear inter*  
  
*#we plot our function*  
*#we plot our interpolant*  
*#we give a legend*  
*#we label our axes*

Out[ ]: &lt;function matplotlib.pyplot.show(close=None, block=None)&gt;

**2a** Consider the following code:

In [ ]:

```

x1 = np.linspace(0,1,10)
y1 = x1*x1+1

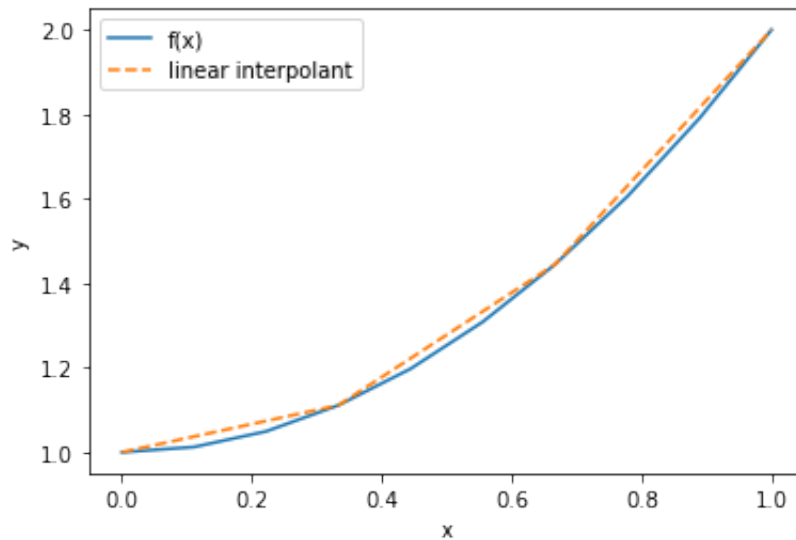
x2 = np.linspace(0,1,4)
y2 = x2*x2+1

plt.plot(x1,y1,x2,y2,'--')
plt.legend(["f(x)", "piecewise linear interpolant"])
plt.xlabel("x")
plt.ylabel("y")
plt.show

```

*#we create a 1d v*  
*#we plot our func*  
  
*#we create a 1d v*  
*#this is fewer no*  
  
*#we plot our grap*  
*#we create a lege*  
*#label our axes*

Out[ ]: <function matplotlib.pyplot.show(close=None, block=None)>



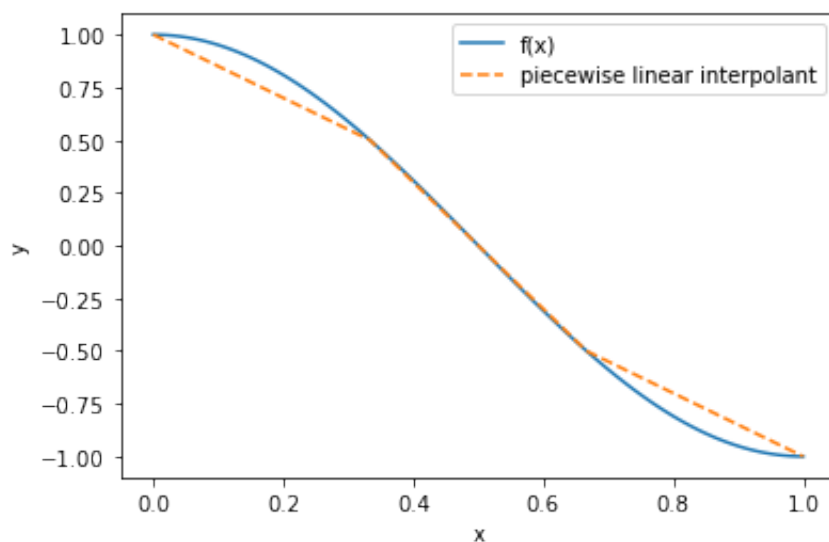
**2b** Consider the following diagram:

```
In [ ]: x1 = np.linspace(0,1,100)           #we create a 1
        y1 = np.cos(np.pi*x1)             #we plot our f

        x2 = np.linspace(0,1,4)           #we create a 1
        y2 = np.cos(np.pi*x2)             #this is fewer

        plt.plot(x1,y1,x2,y2,'--')         #we plot our g
        plt.legend(["f(x)", "piecewise linear interpolant"]) #we create our
        plt.xlabel("x")                   #we label axes
        plt.ylabel("y")
        plt.show
```

Out[ ]: <function matplotlib.pyplot.show(close=None, block=None)>



**3a** Analytically  $\int_0^1 x^2 dx = [\frac{x^3}{3} + c]_0^1 = \frac{1}{3}$

**3b** For the midpoint rule, midpoint is:  $m = \frac{1+0}{2} = \frac{1}{2}$ , and so we have  
 $J \approx f(0.5) \times 1 = 0.25$

**3c** For trapezoidal rule, we have  $J \approx \frac{f(0)+f(1)}{2} \times h = \frac{0+1}{2} \times 1 = \frac{1}{2}$

**3d** Finally for Simpsons we have  $J \approx \frac{f(0)+4f(0.5)+f(1)}{6} = 0 + 1 + 16 = \frac{1}{3}$

**3e** We divide into two intervals  $[0, 0.5]$  and  $[0.5, 1]$ .

With midpoint rule we have:  $J \approx \frac{0+0.5}{2} + \frac{0.5+1}{2} = 0.25 + 0.75 = 2$

With trapezoidal we have:

$$J \approx \frac{f(0)+f(0.5)}{2} \times \frac{1}{2} + \frac{f(0.5)+f(1)}{2} \times \frac{1}{2} = \frac{0.25}{2} \times \frac{1}{2} + \frac{0.25+1}{2} \times \frac{1}{2} = 0.375$$

With Simpsons we have:

$$\frac{f(0)+4f(0.25)+f(0.5)}{6} \times \frac{1}{2} + \frac{f(0.5)+4f(0.75)+f(1)}{6} \times \frac{1}{2} \approx 0.0416 + 0.2916 = 0.3332$$

**3f** Observe that  $f''(x) = 2$  which means that  $|f''(x)| \leq 2$ . Thus we can say:

*For one interval:*

$$|E_M| \leq 2 \frac{(1-0)^3}{24 \times 1^2} = \frac{1}{12} \text{ where } E_M \text{ is the error in the midpoint rule.}$$

$$|E_T| \leq 2 \frac{(1-0)^3}{12 \times 1^2} = \frac{1}{6} \text{ where } E_T \text{ is the error in the trapezoidal rule.}$$

$$|E_M| \leq 2 \frac{(1-0)^5}{180 \times 1^4} = \frac{1}{90} \text{ where } E_S \text{ is the error in the Simpson's method.}$$

*For two intervals:*

$$|E_M| \leq 2 \frac{(1-0)^3}{24 \times 2^2} = \frac{1}{48}$$

$$|E_T| \leq 2 \frac{(1-0)^3}{12 \times 2^2} = \frac{1}{24}$$

$$|E_M| \leq 2 \frac{(1-0)^5}{180 \times 2^4} = \frac{1}{1440}$$

**4** We represent as a matrix and rref:

```
In [ ]: from sympy import *

A = Matrix([[1,0,0],[0,1,0],[-.5,0,3.5]]) #we take coefficients and create
A
```

```
Out[ ]: 
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -0.5 & 0 & 3.5 \end{bmatrix}$$

```

```
In [ ]: A.rref() #we rref this matrix hoping to get I.
```

```
Out[ ]: (Matrix([
  [1, 0, 0],
  [0, 1, 0],
  [0, 0, 1]]),
  (0, 1, 2))
```

Thus they form a basis.

We define our functions as:

```
In [ ]: def f_4a(x):
        return 2*x+1

        def f_4b(x):
            return x*x*x
```

Our load assembler is:

```
In [ ]: #COPIED FROM DR KAMAN'S SAMPLE PROGRAMS

def LoadAssembler(x,f):
    n = len(x)
    b = np.zeros((n,1))
    for i in range(0,n-1):
        h = x[i+1]-x[i]
        b[i] = b[i] + f(x[i]) * (h/2)
        b[i+1] = b[i+1] + f(x[i+1]) * (h/2)
    return b

#we create a variable to
#initialize vector b to b
#loop over i that goes to
#h is the distance from t
#each entry in b is f of
#we calculate the next b
#return b
```

Our mass assembler is:

In [ ]:

#COPIED FROM DR KAMAN'S SAMPLE PROGRAMS

```

def MassAssembler(x):
    M = np.zeros((len(x),len(x)))
    n = len(x)
    for i in range(0,n-1):
        h = x[i+1]-x[i]
        M[i,i] = M[i,i] + (h/3)
        M[i,i+1] = M[i,i+1] +(h/6)
        M[i+1,i] = M[i+1,i] +(h/6)
        M[i+1,i+1] = M[i+1,i+1] + (h/3)
    return M

```

*#initialize m as an nxn m*  
*#n is as long as x*  
*#we iterate to fill out t*  
*#h is the distance from t*  
*#the diagonals are all h/*  
*#the superdiagonals are h/*  
*#the subdiagonals are h/6*  
*#we calculate the next di*  
*#return M*

Our main function for number 4a and 4b is:

In [ ]:

```

x = np.linspace(-1,1,10)
M= MassAssembler(x)
b = LoadAssembler(x,f_4a)

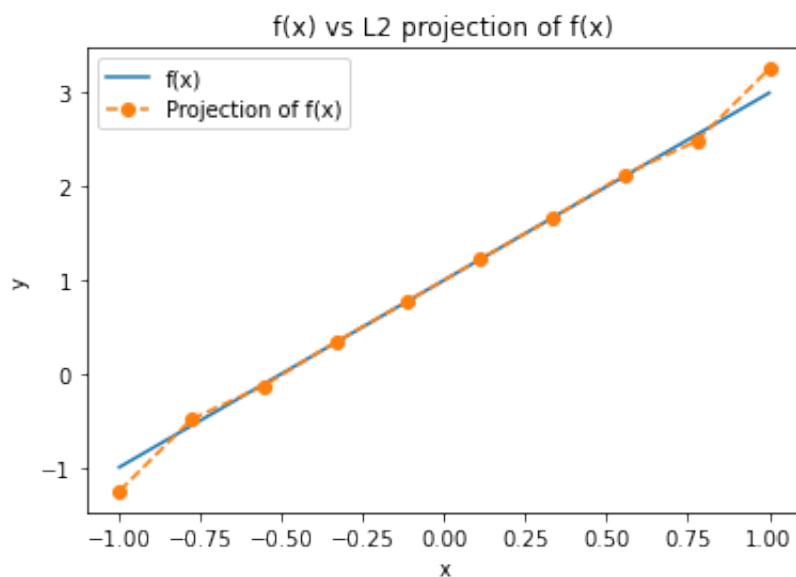
Phf = np.linalg.inv(M).dot(b)

plt.plot(x,f_4a(x))
plt.plot(x,Phf,'--',marker = 'o')
plt.legend(["f(x)", "Projection of f(x)"])
plt.xlabel("x")
plt.ylabel("y")
plt.title("f(x) vs L2 projection of f(x)")
plt.show

```

*#create 1D ve*  
*#We calculate*  
*#we calculate*  
*#solve the li*  
*#we plot func*  
*#use -- and m*  
*#create legen*  
*#label axes*  
*#create title*

Out[ ]: &lt;function matplotlib.pyplot.show(close=None, block=None)&gt;



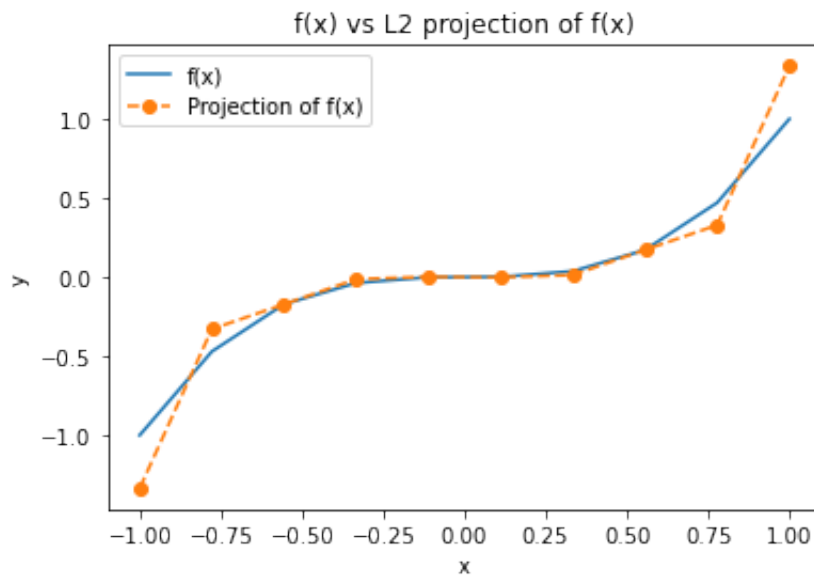
For 4b we have:

```
In [ ]: M = MassAssembler(x)                                #We calculate the Mass matrix
        b = LoadAssembler(x,f_4b)                          #we calculate the load vector

        Phf = np.linalg.inv(M).dot(b)                       #solve the linear system

        plt.plot(x,f_4b(x))                                  #plot our function
        plt.plot(x,Phf,'--',marker = 'o')                   #plot projection with -- and o
        plt.legend(["f(x)", "Projection of f(x)"])           #plot legend
        plt.xlabel("x")                                       #plot axes
        plt.ylabel("y")
        plt.title("f(x) vs L2 projection of f(x)")           #create title
        plt.show
```

```
Out[ ]: <function matplotlib.pyplot.show(close=None, block=None)>
```



---

```

%L2-Projection of f(x) = 1 using n= 5
function L2Projector1D()
n = 5; % number of subintervals
h = 1/n; % mesh size
x = 0:h:1; % mesh
M = MassAssembler1D(x); %assemble mass
b = LoadAssembler1D(x, @Fool); %assemble load
% Fool is a small routine, it can be inlined
% Fool = inline('x.*sin(x)','x')
% b = LoasAssembler1D(x,Fool)
Pf = M\b; %solve linear system
plot(x,Pf,'r--o',x, Fool(x),'g:','LineWidth',2)% plot L^2 projection
legend('L2 projection','f(x)','Location','northwest')
hold on

```

$n =$

5

$M =$

0	0	0	0	0	0
0	0	0	0	0	0
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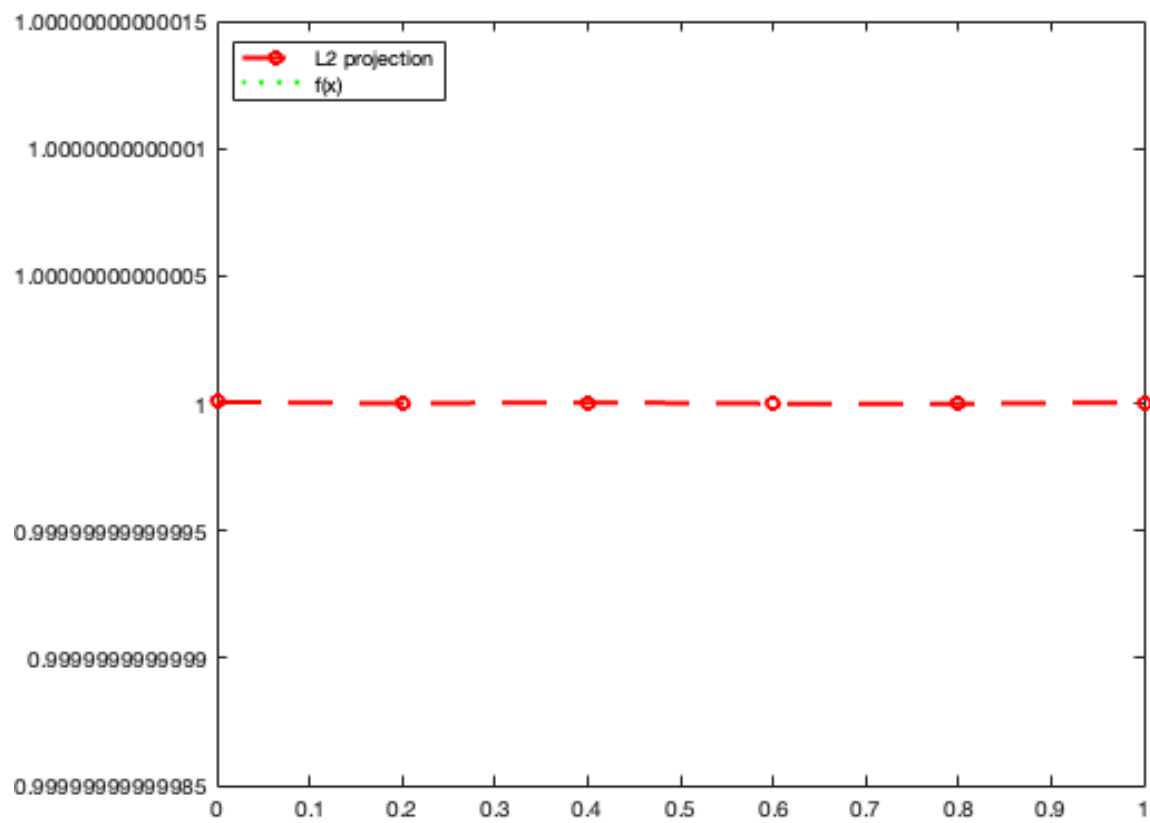
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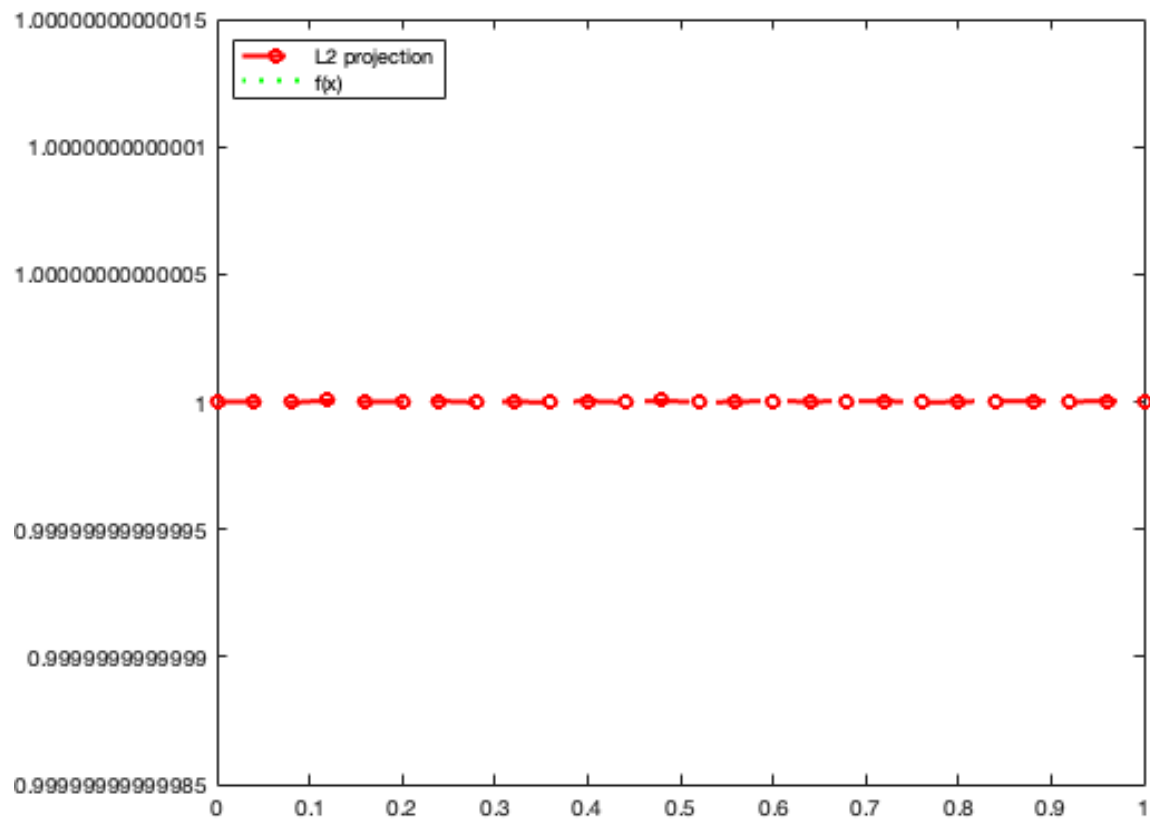
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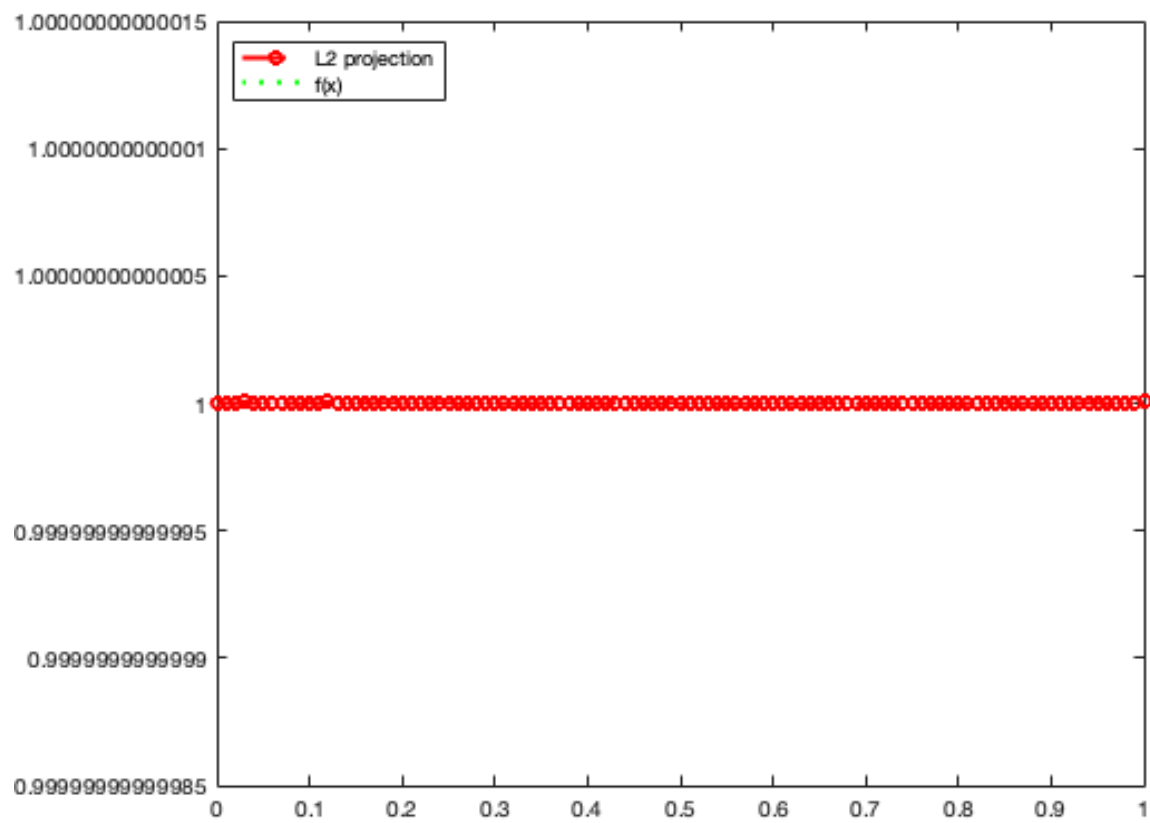
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```

%L2-Projection of f(x) = x^2(x-1)(1-2x) using n= 5
function L2Projector1D()
n = 5; % number of subintervals
h = 1/n; % mesh size
x = 0:h:1; % mesh
M = MassAssembler1D(x); %assemble mass
b = LoadAssembler1D(x, @Fool); %assemble load
% Fool is a small routine, it can be inlined
% Fool = inline('x.*sin(x)','x')
% b = LoasAssembler1D(x,Fool)
Pf = M\b; %solve linear system
plot(x,Pf,'r--o',x, Fool(x),'g:','LineWidth',2)% plot L^2 projection
legend('L2 projection','f(x)','Location','northwest')
hold on

```

$n =$

5

$M =$

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

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-0.0192

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$-0.0192$

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$0.0288$

$y =$

$0.0288$

$y =$

$0.0768$

$y =$

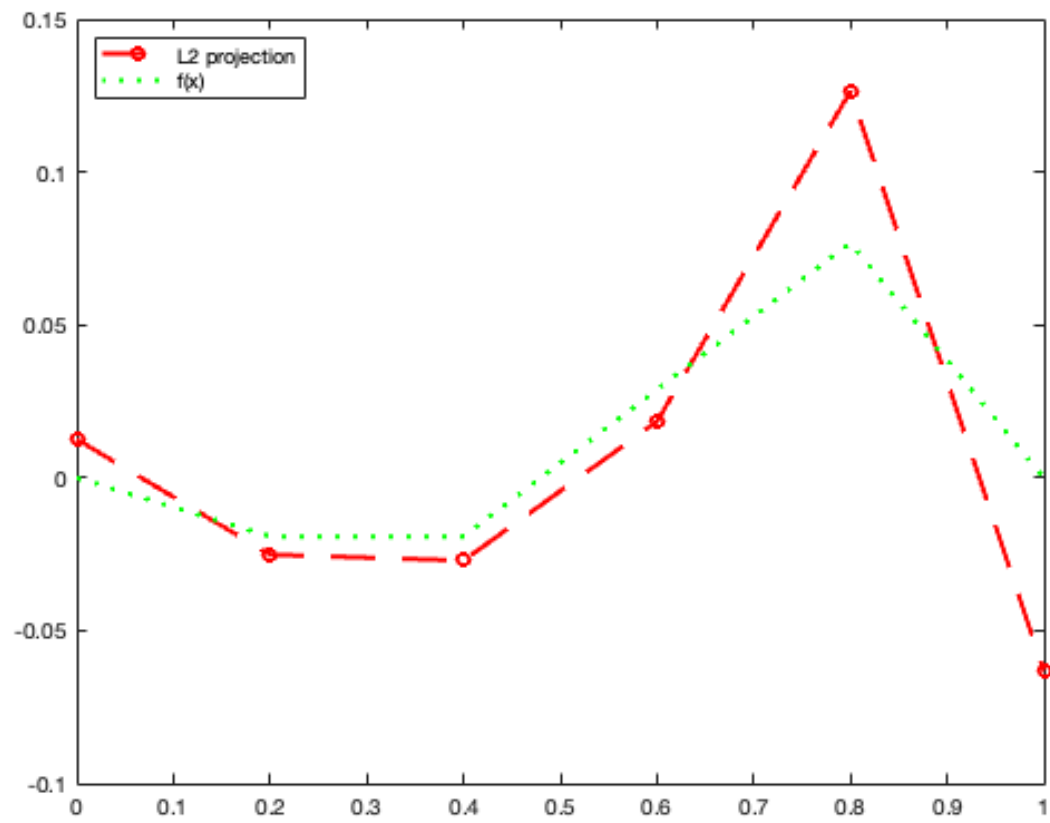
$0.0768$

$y =$

$0$

$y =$

$0 \quad -0.0192 \quad -0.0192 \quad 0.0288 \quad 0.0768 \quad 0$



*Published with MATLAB® R2021b*







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$$-0.0248$$

$$y =$$

$$-0.0248$$

$$y =$$

$$-0.0251$$

---

$$y =$$

$$-0.0251$$

$$y =$$

$$-0.0232$$

$$y =$$

$$-0.0232$$

$$y =$$

$$-0.0192$$

$$y =$$

$$-0.0192$$

$$y =$$

$$-0.0130$$

$$y =$$

$$-0.0130$$

$$y =$$

$$-0.0048$$

$$y =$$

$$-0.0048$$

$$y =$$

$$0.0052$$

$$y =$$

$$0.0052$$

---

 $y =$

$0.0166$

$y =$

$0.0166$

$y =$

$0.0288$

$y =$

$0.0288$

$y =$

$0.0413$

$y =$

$0.0413$

$y =$

$0.0533$

$y =$

$0.0533$

$y =$

$0.0639$

$y =$

$0.0639$

$y =$

---

0.0721

$y =$

0.0721

$y =$

0.0768

$y =$

0.0768

$y =$

0.0768

$y =$

0.0768

$y =$

0.0706

$y =$

0.0706

$y =$

0.0569

$y =$

0.0569

$y =$

0.0339

$y =$

---

0.0339

$y =$

0

$y =$

Columns 1 through 7

0	-0.0014	-0.0049	-0.0096	-0.0146	-0.0192	-0.0228
---	---------	---------	---------	---------	---------	---------

Columns 8 through 14

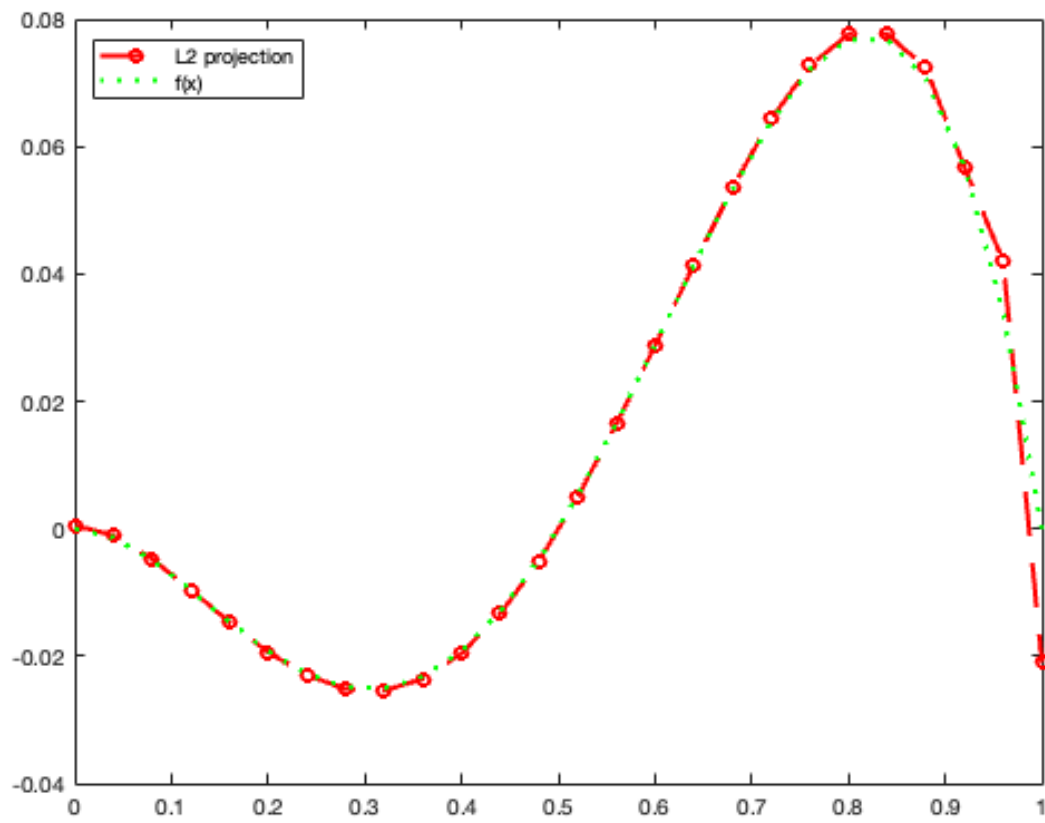
-0.0248	-0.0251	-0.0232	-0.0192	-0.0130	-0.0048	0.0052
---------	---------	---------	---------	---------	---------	--------

Columns 15 through 21

0.0166	0.0288	0.0413	0.0533	0.0639	0.0721	0.0768
--------	--------	--------	--------	--------	--------	--------

Columns 22 through 26

0.0768	0.0706	0.0569	0.0339	0
--------	--------	--------	--------	---



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Columns 14 through 26

---

3



Columns 27 through 39

---

5



Columns 40 through 52

---

7





Columns 53 through 65

---

9



Columns 66 through 78

---

11



[illegible]

Columns 92 through 101

---

14







---

$$-9.7020e-05$$

$$y =$$

$$-9.7020e-05$$

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$$-3.7632e-04$$

$$y =$$

$$-3.7632e-04$$

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$$-8.2062e-04$$

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$$-8.2062e-04$$

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$$-0.0014$$

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$$-0.0014$$

$$y =$$

$$-0.0021$$

$$y =$$

$$-0.0021$$

$$y =$$

$$-0.0030$$

$$y =$$

---

-0.0030

$y =$

-0.0039

$y =$

-0.0039

$y =$

-0.0049

$y =$

-0.0049

$y =$

-0.0060

$y =$

-0.0060

$y =$

-0.0072

$y =$

-0.0072

$y =$

-0.0084

$y =$

-0.0084

---

 $y =$

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$y =$

$-0.0096$

$y =$

$-0.0109$

$y =$

$-0.0109$

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$-0.0121$

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$-0.0121$

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$-0.0134$

$y =$

$-0.0146$

$y =$

$-0.0146$

$y =$

$-0.0158$

---

$$y =$$
$$-0.0158$$

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$$-0.0170$$

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$$-0.0170$$

$$y =$$
$$-0.0181$$

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$$-0.0181$$

$$y =$$
$$-0.0192$$

$$y =$$
$$-0.0192$$

$$y =$$
$$-0.0202$$

$$y =$$
$$-0.0202$$

$$y =$$
$$-0.0211$$

$$y =$$
$$-0.0211$$

---

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$-0.0220$

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$-0.0234$

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$-0.0240$

$y =$

$-0.0245$

$y =$

$-0.0245$

$y =$

---

-0.0248

$y =$

-0.0248

$y =$

-0.0251

$y =$

-0.0251

$y =$

-0.0252

$y =$

-0.0252

$y =$

-0.0252

$y =$

-0.0252

$y =$

-0.0251

$y =$

-0.0251

$y =$

-0.0248

$y =$

---

-0.0248

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-0.0244

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-0.0239

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-0.0232

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-0.0224

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-0.0215

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$$-0.0204$$

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$$-0.0179$$

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$$-0.0164$$

$$y =$$

$$-0.0148$$

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$$-0.0148$$

$$y =$$

$$-0.0130$$



---

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$$y = -0.0048$$

$$y = -0.0024$$

$$y = -0.0024$$

---

$$y =$$

$$0$$

$$y =$$

$$0$$

$$y =$$

$$0.0025$$

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$$0.0025$$

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$$0.0052$$

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$$0.0079$$

$$y =$$

$$0.0079$$

$$y =$$

$$0.0107$$

$$y =$$

$$0.0107$$

$$y =$$

---

*0.0136*

*y* =

*0.0136*

*y* =

*0.0166*

*y* =

*0.0166*

*y* =

*0.0196*

*y* =

*0.0196*

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*0.0226*

*y* =

*0.0226*

*y* =

*0.0257*

*y* =

*0.0257*

*y* =

*0.0288*

*y* =

---

*0.0288*

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*0.0319*

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*0.0319*

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*0.0351*

*y* =

*0.0351*

*y* =

*0.0382*

*y* =

*0.0382*

*y* =

*0.0413*

*y* =

*0.0413*

*y* =

*0.0444*

*y* =

*0.0444*

---

 $y =$

$0.0474$

$y =$

$0.0474$

$y =$

$0.0504$

$y =$

$0.0504$

$y =$

$0.0533$

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$0.0588$

$y =$

$0.0588$

$y =$

$0.0614$

---

 $y =$

$0.0614$

$y =$

$0.0639$

$y =$

$0.0639$

$y =$

$0.0662$

$y =$

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$0.0683$

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$0.0683$

$y =$

$0.0703$

$y =$

$0.0703$

$y =$

$0.0721$

$y =$

$0.0721$

---

$$y =$$

$$0.0736$$

$$y =$$

$$0.0736$$

$$y =$$

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$$0.0760$$

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$$0.0760$$

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$$0.0768$$

$$y =$$

$$0.0768$$

$$y =$$

$$0.0773$$

$$y =$$

$$0.0773$$

$$y =$$

---

*0.0775*

*y* =

*0.0775*

*y* =

*0.0773*

*y* =

*0.0773*

*y* =

*0.0768*

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*0.0768*

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*0.0759*

*y* =

*0.0759*

*y* =

*0.0746*

*y* =

*0.0746*

*y* =

*0.0728*

*y* =



---

0.0728

$y =$

0.0706

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0.0706

$y =$

0.0680

$y =$

0.0680

$y =$

0.0648

$y =$

0.0648

$y =$

0.0611

$y =$

0.0611

$y =$

0.0569

$y =$

0.0569

---

 $y =$

$0.0521$

$y =$

$0.0521$

$y =$

$0.0467$

$y =$

$0.0467$

$y =$

$0.0406$

$y =$

$0.0406$

$y =$

$0.0339$

$y =$

$0.0339$

$y =$

$0.0265$

$y =$

$0.0265$

$y =$

$0.0184$

---

$y =$

0.0184

$y =$

0.0096

$y =$

0.0096

$y =$

0

$y =$

Columns 1 through 7

0 -0.0001 -0.0004 -0.0008 -0.0014 -0.0021 -0.0030

Columns 8 through 14

-0.0039 -0.0049 -0.0060 -0.0072 -0.0084 -0.0096 -0.0109

Columns 15 through 21

-0.0121 -0.0134 -0.0146 -0.0158 -0.0170 -0.0181 -0.0192

Columns 22 through 28

-0.0202 -0.0211 -0.0220 -0.0228 -0.0234 -0.0240 -0.0245

Columns 29 through 35

-0.0248 -0.0251 -0.0252 -0.0252 -0.0251 -0.0248 -0.0244

Columns 36 through 42

-0.0239 -0.0232 -0.0224 -0.0215 -0.0204 -0.0192 -0.0179

Columns 43 through 49

-0.0164 -0.0148 -0.0130 -0.0111 -0.0091 -0.0070 -0.0048

Columns 50 through 56

-0.0024 0 0.0025 0.0052 0.0079 0.0107 0.0136

---

*Columns 57 through 63*

0.0166	0.0196	0.0226	0.0257	0.0288	0.0319	0.0351
--------	--------	--------	--------	--------	--------	--------

*Columns 64 through 70*

0.0382	0.0413	0.0444	0.0474	0.0504	0.0533	0.0561
--------	--------	--------	--------	--------	--------	--------

*Columns 71 through 77*

0.0588	0.0614	0.0639	0.0662	0.0683	0.0703	0.0721
--------	--------	--------	--------	--------	--------	--------

*Columns 78 through 84*

0.0736	0.0750	0.0760	0.0768	0.0773	0.0775	0.0773
--------	--------	--------	--------	--------	--------	--------

*Columns 85 through 91*

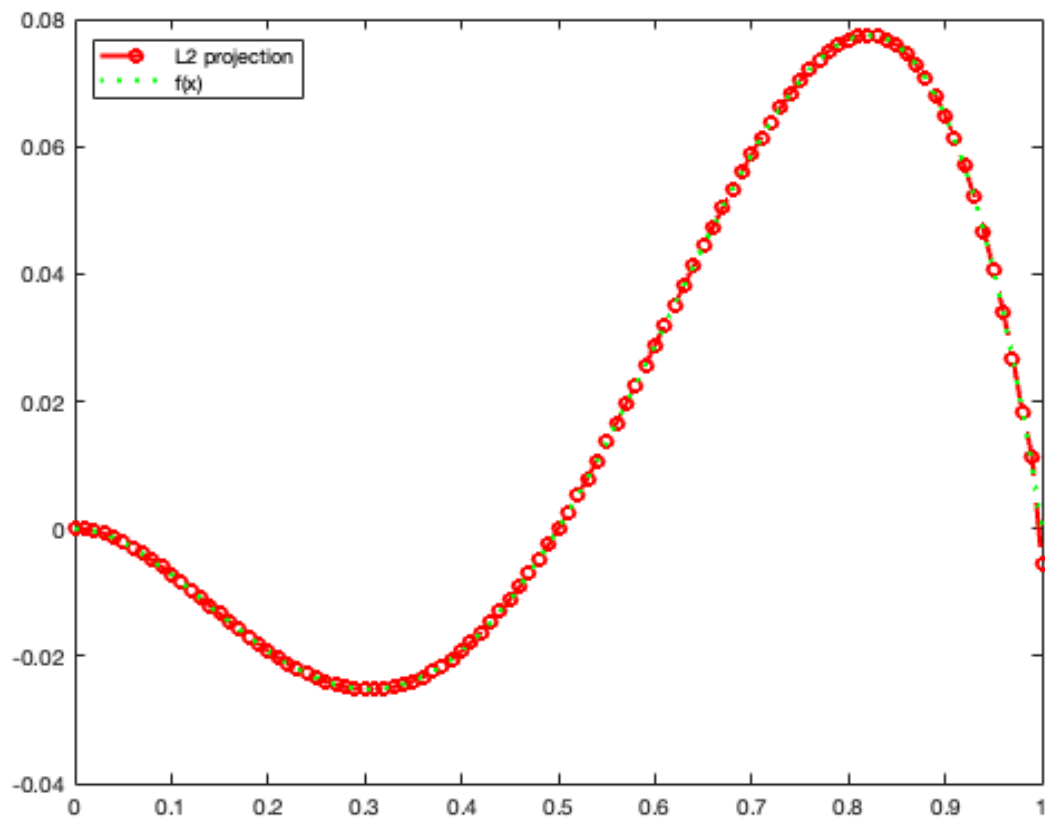
0.0768	0.0759	0.0746	0.0728	0.0706	0.0680	0.0648
--------	--------	--------	--------	--------	--------	--------

*Columns 92 through 98*

0.0611	0.0569	0.0521	0.0467	0.0406	0.0339	0.0265
--------	--------	--------	--------	--------	--------	--------

*Columns 99 through 101*

0.0184	0.0096	0
--------	--------	---



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---

```

%L2-Projection of f(x) = arctan((x-0.5)/0.1) using n= 5
function L2Projector1D()
n = 5; % number of subintervals
h = 1/n; % mesh size
x = 0:h:1; % mesh
M = MassAssembler1D(x); %assemble mass
b = LoadAssembler1D(x, @Fool); %assemble load
% Fool is a small routine, it can be inlined
% Fool = inline('x.*sin(x)','x')
% b = LoasAssembler1D(x,Fool)
Pf = M\b; %solve linear system
plot(x,Pf,'r--o',x, Fool(x),'g:','LineWidth',2)% plot L^2 projection
legend('L2 projection','f(x)','Location','northwest')
hold on

```

$n =$

5

$M =$

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

$y =$

-1.3734

$y =$

-1.2490

$y =$

-1.2490

$y =$

-0.7854

$y =$

---

$-0.7854$

$y =$

$0.7854$

$y =$

$0.7854$

$y =$

$1.2490$

$y =$

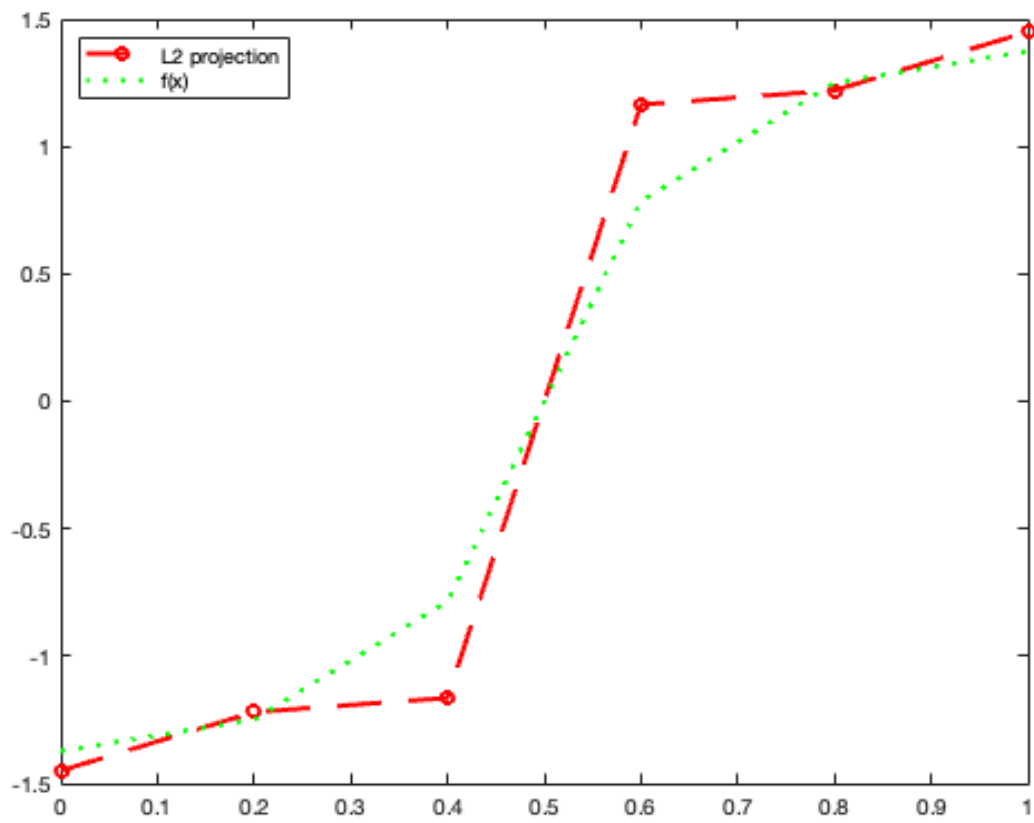
$1.2490$

$y =$

$1.3734$

$y =$

$-1.3734 \quad -1.2490 \quad -0.7854 \quad 0.7854 \quad 1.2490 \quad 1.3734$



*Published with MATLAB® R2021b*



---

```

%L2-Projection of f(x) = arctan((x-0.5)/0.01) using n= 5
function L2Projector1D()
n = 5; % number of subintervals
h = 1/n; % mesh size
x = 0:h:1; % mesh
M = MassAssembler1D(x); %assemble mass
b = LoadAssembler1D(x, @Fool); %assemble load
% Fool is a small routine, it can be inlined
% Fool = inline('x.*sin(x)','x')
% b = LoasAssembler1D(x,Fool)
Pf = M\b; %solve linear system
plot(x,Pf,'r--o',x, Fool(x),'g:','LineWidth',2)% plot L^2 projection
legend('L2 projection','f(x)','Location','northwest')
hold on

```

$n =$

5

$M =$

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

$y =$

-1.5508

$y =$

-1.5375

$y =$

-1.5375

$y =$

-1.4711

$y =$

---

*-1.4711*

*y =*

*1.4711*

*y =*

*1.4711*

*y =*

*1.5375*

*y =*

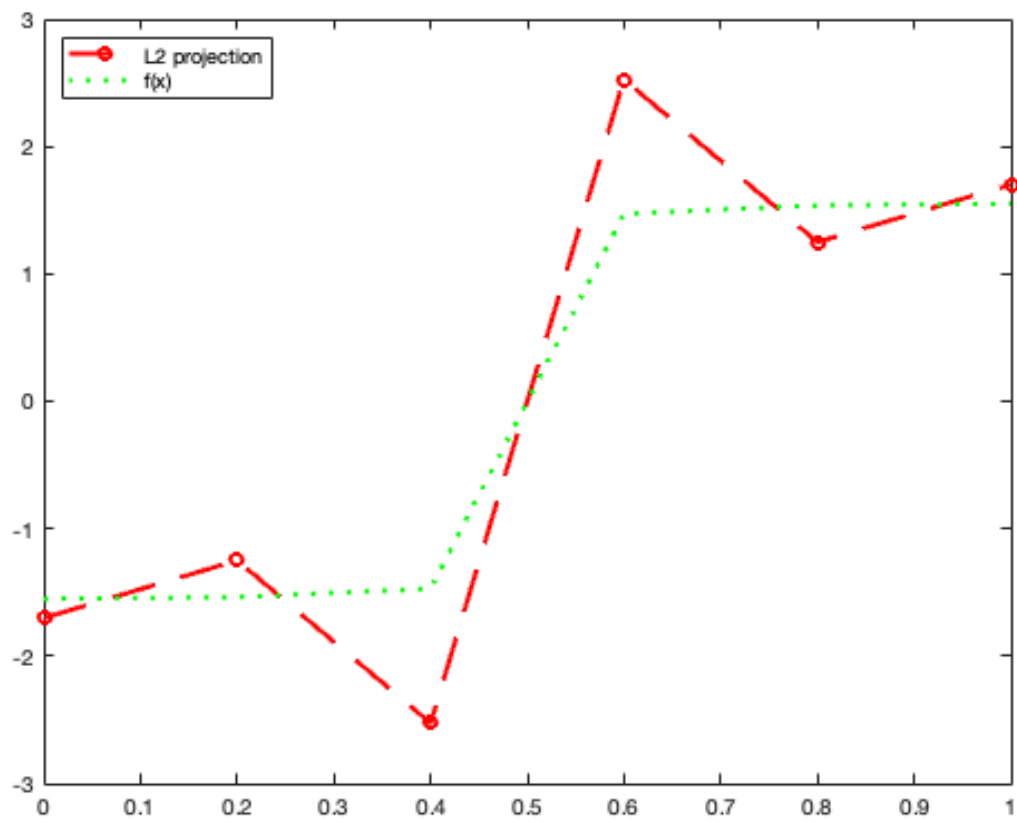
*1.5375*

*y =*

*1.5508*

*y =*

*-1.5508   -1.5375   -1.4711   1.4711   1.5375   1.5508*



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---

$$y =$$

$$-1.3135$$

$$y =$$

$$-1.3135$$

$$y =$$

$$-1.2847$$

$$y =$$

$$-1.2847$$

$$y =$$

$$-1.2490$$

$$y =$$

$$-1.2490$$

$$y =$$

$$-1.2036$$

$$y =$$

$$-1.2036$$

$$y =$$

$$-1.1442$$

$$y =$$

$$-1.1442$$

$$y =$$

$$-1.0637$$

---

 $y =$

$-1.0637$

$y =$

$-0.9505$

$y =$

$-0.9505$

$y =$

$-0.7854$

$y =$

$-0.7854$

$y =$

$-0.5404$

$y =$

$-0.5404$

$y =$

$-0.1974$

$y =$

$-0.1974$

$y =$

$0.1974$

$y =$

$0.1974$

---

 $y =$

$0.5404$

$y =$

$0.5404$

$y =$

$0.7854$

$y =$

$0.7854$

$y =$

$0.9505$

$y =$

$0.9505$

$y =$

$1.0637$

$y =$

$1.0637$

$y =$

$1.1442$

$y =$

$1.1442$

$y =$



---

1.2036

$y =$

1.2036

$y =$

1.2490

$y =$

1.2490

$y =$

1.2847

$y =$

1.2847

$y =$

1.3135

$y =$

1.3135

$y =$

1.3371

$y =$

1.3371

$y =$

1.3567

$y =$

---

1.3567

$y =$

1.3734

$y =$

Columns 1 through 7

-1.3734   -1.3567   -1.3371   -1.3135   -1.2847   -1.2490   -1.2036

Columns 8 through 14

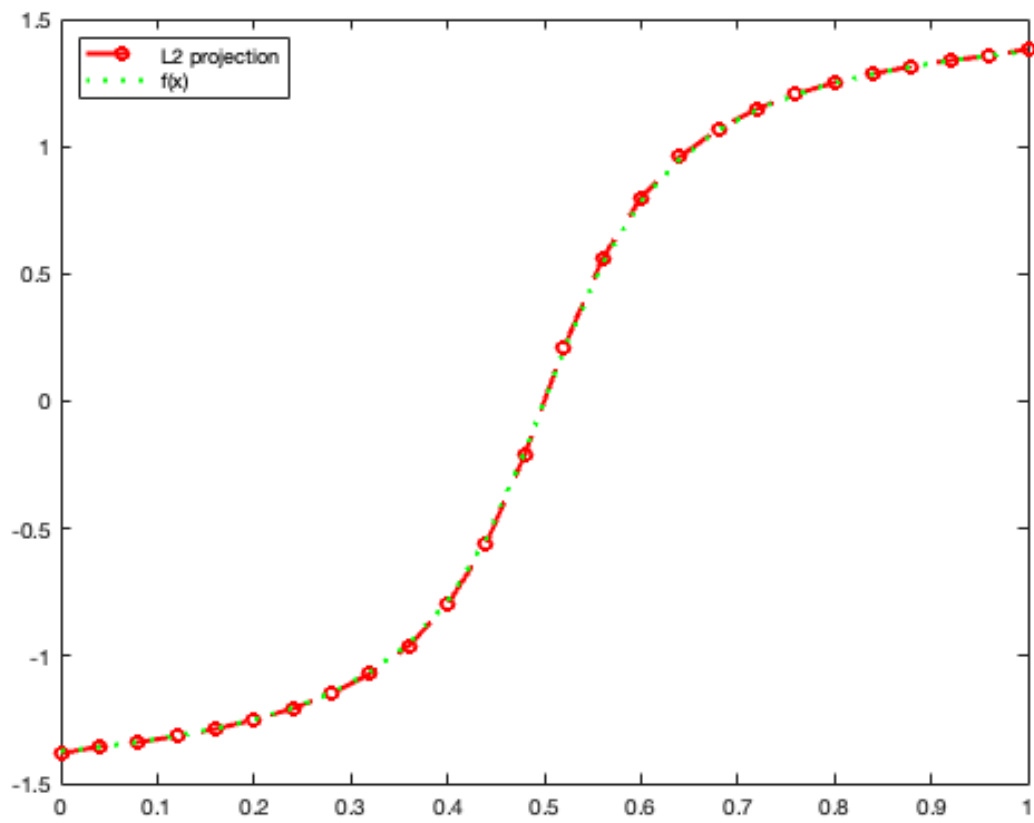
-1.1442   -1.0637   -0.9505   -0.7854   -0.5404   -0.1974   0.1974

Columns 15 through 21

0.5404   0.7854   0.9505   1.0637   1.1442   1.2036   1.2490

Columns 22 through 26

1.2847   1.3135   1.3371   1.3567   1.3734



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---

$$y =$$

$$-1.5445$$

$$y =$$

$$-1.5445$$

$$y =$$

$$-1.5414$$

$$y =$$

$$-1.5414$$

$$y =$$

$$-1.5375$$

$$y =$$

$$-1.5375$$

$$y =$$

$$-1.5324$$

$$y =$$

$$-1.5324$$

$$y =$$

$$-1.5254$$

$$y =$$

$$-1.5254$$

$$y =$$

$$-1.5153$$

---

$$y =$$

$$-1.5153$$

$$y =$$

$$-1.4995$$

$$y =$$

$$-1.4995$$

$$y =$$

$$-1.4711$$

$$y =$$

$$-1.4711$$

$$y =$$

$$-1.4056$$

$$y =$$

$$-1.4056$$

$$y =$$

$$-1.1071$$

$$y =$$

$$-1.1071$$

$$y =$$

$$1.1071$$

$$y =$$

$$1.1071$$

---

 $y =$

$1.4056$

$y =$

$1.4056$

$y =$

$1.4711$

$y =$

$1.4711$

$y =$

$1.4995$

$y =$

$1.4995$

$y =$

$1.5153$

$y =$

$1.5153$

$y =$

$1.5254$

$y =$

$1.5254$

$y =$



---

1.5324

$y =$

1.5324

$y =$

1.5375

$y =$

1.5375

$y =$

1.5414

$y =$

1.5414

$y =$

1.5445

$y =$

1.5445

$y =$

1.5470

$y =$

1.5470

$y =$

1.5491

$y =$

---

1.5491

$y =$

1.5508

$y =$

Columns 1 through 7

-1.5508   -1.5491   -1.5470   -1.5445   -1.5414   -1.5375   -1.5324

Columns 8 through 14

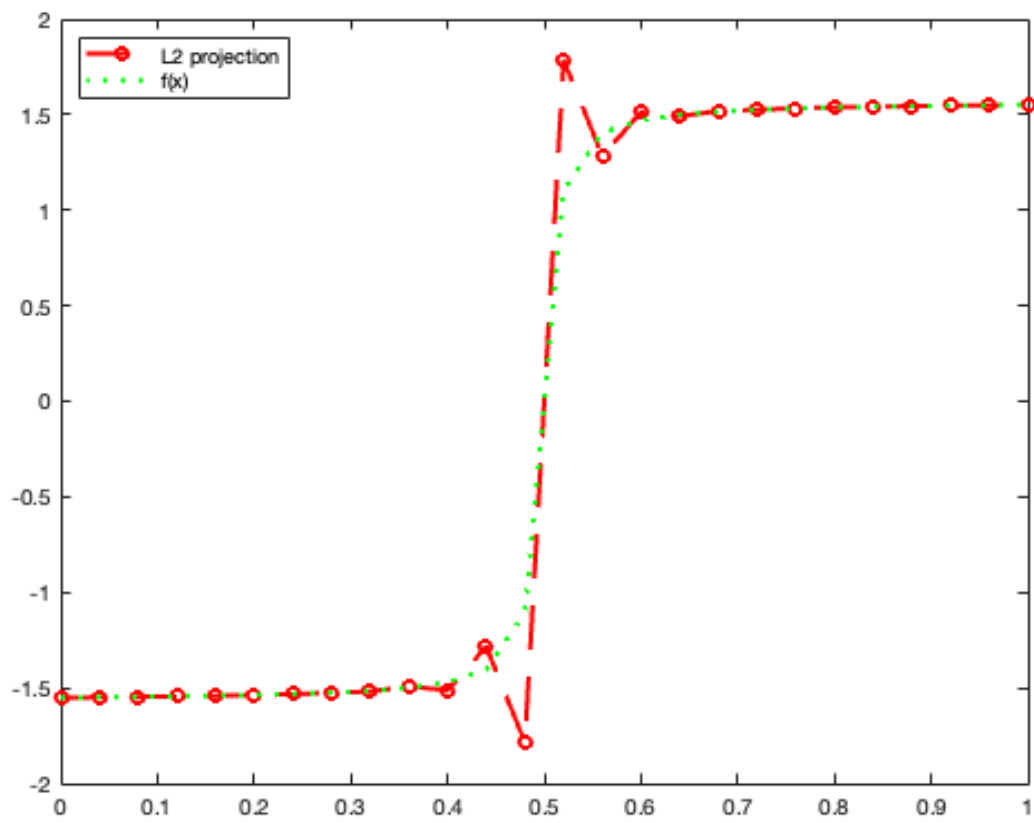
-1.5254   -1.5153   -1.4995   -1.4711   -1.4056   -1.1071   1.1071

Columns 15 through 21

1.4056   1.4711   1.4995   1.5153   1.5254   1.5324   1.5375

Columns 22 through 26

1.5414   1.5445   1.5470   1.5491   1.5508



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Columns 14 through 26

---

3



Columns 27 through 39

---

5





Columns 40 through 52

---

7



Columns 53 through 65

---

9



Columns 66 through 78

---

11



[illegible]



Columns 92 through 101

---

14





---

-1.3695

$y =$

-1.3695

$y =$

-1.3654

$y =$

-1.3654

$y =$

-1.3612

$y =$

-1.3612

$y =$

-1.3567

$y =$

-1.3567

$y =$

-1.3521

$y =$

-1.3521

$y =$

-1.3473

$y =$

---

-1.3473

$y =$

-1.3423

$y =$

-1.3423

$y =$

-1.3371

$y =$

-1.3371

$y =$

-1.3316

$y =$

-1.3316

$y =$

-1.3258

$y =$

-1.3258

$y =$

-1.3198

$y =$

-1.3198

---

 $y =$

$-1.3135$

$y =$

$-1.3135$

$y =$

$-1.3068$

$y =$

$-1.3068$

$y =$

$-1.2998$

$y =$

$-1.2998$

$y =$

$-1.2925$

$y =$

$-1.2925$

$y =$

$-1.2847$

$y =$

$-1.2847$

$y =$

$-1.2766$

---

 $y =$

$-1.2766$

$y =$

$-1.2679$

$y =$

$-1.2679$

$y =$

$-1.2588$

$y =$

$-1.2588$

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$-1.2490$

$y =$

$-1.2490$

$y =$

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Columns 1 through 7

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Columns 8 through 14

-1.3423   -1.3371   -1.3316   -1.3258   -1.3198   -1.3135   -1.3068

Columns 15 through 21

-1.2998   -1.2925   -1.2847   -1.2766   -1.2679   -1.2588   -1.2490

Columns 22 through 28

-1.2387   -1.2278   -1.2161   -1.2036   -1.1903   -1.1760   -1.1607

Columns 29 through 35

-1.1442   -1.1264   -1.1071   -1.0863   -1.0637   -1.0391   -1.0122

Columns 36 through 42

-0.9828   -0.9505   -0.9151   -0.8761   -0.8330   -0.7854   -0.7328

Columns 43 through 49

-0.6747   -0.6107   -0.5404   -0.4636   -0.3805   -0.2915   -0.1974

Columns 50 through 56

-0.0997        0    0.0997    0.1974    0.2915    0.3805    0.4636

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*Columns 57 through 63*

0.5404    0.6107    0.6747    0.7328    0.7854    0.8330    0.8761

*Columns 64 through 70*

0.9151    0.9505    0.9828    1.0122    1.0391    1.0637    1.0863

*Columns 71 through 77*

1.1071    1.1264    1.1442    1.1607    1.1760    1.1903    1.2036

*Columns 78 through 84*

1.2161    1.2278    1.2387    1.2490    1.2588    1.2679    1.2766

*Columns 85 through 91*

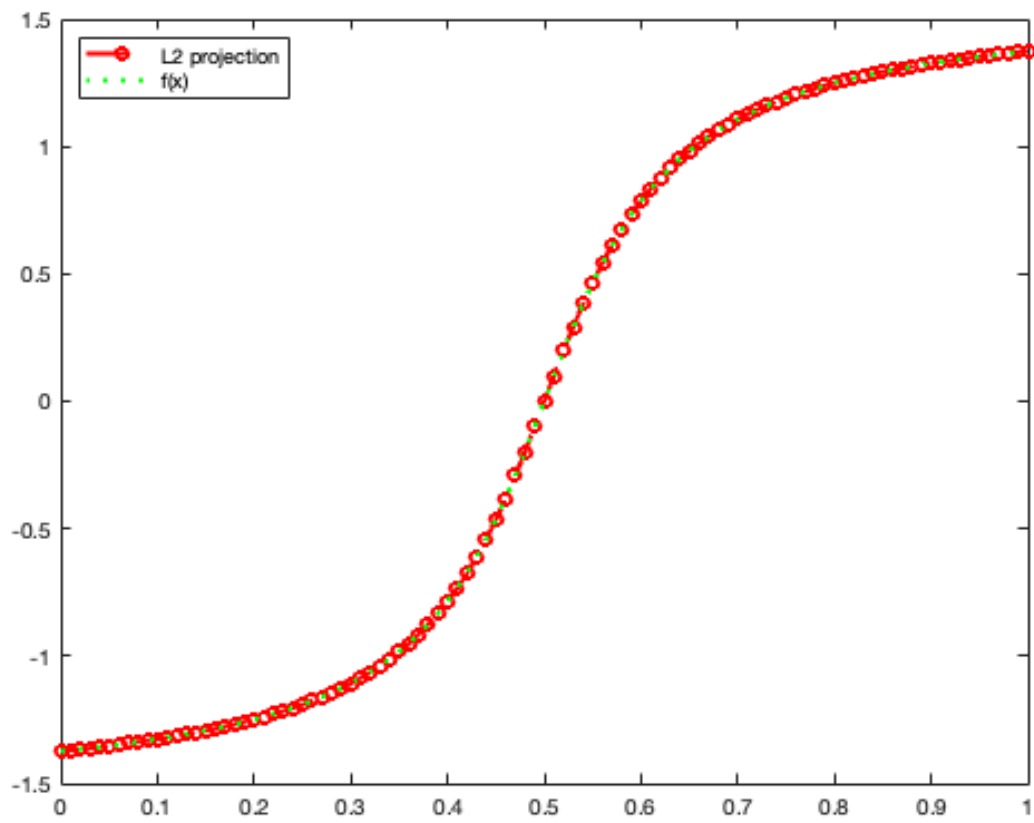
1.2847    1.2925    1.2998    1.3068    1.3135    1.3198    1.3258

*Columns 92 through 98*

1.3316    1.3371    1.3423    1.3473    1.3521    1.3567    1.3612

*Columns 99 through 101*

1.3654    1.3695    1.3734



*Published with MATLAB® R2021b*



[illegible]



Columns 14 through 26

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3



Columns 27 through 39

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5



Columns 40 through 52

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Columns 53 through 65

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9



Columns 66 through 78

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11



[illegible]

Columns 92 through 101

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14



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Columns 8 through 14

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Columns 15 through 21

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Columns 22 through 28

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Columns 29 through 35

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Columns 36 through 42

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Columns 43 through 49

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Columns 50 through 56

-0.7854        0    0.7854    1.1071    1.2490    1.3258    1.3734

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*Columns 57 through 63*

1.4056    1.4289    1.4464    1.4601    1.4711    1.4801    1.4877

*Columns 64 through 70*

1.4940    1.4995    1.5042    1.5084    1.5120    1.5153    1.5182

*Columns 71 through 77*

1.5208    1.5232    1.5254    1.5273    1.5292    1.5308    1.5324

*Columns 78 through 84*

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*Columns 85 through 91*

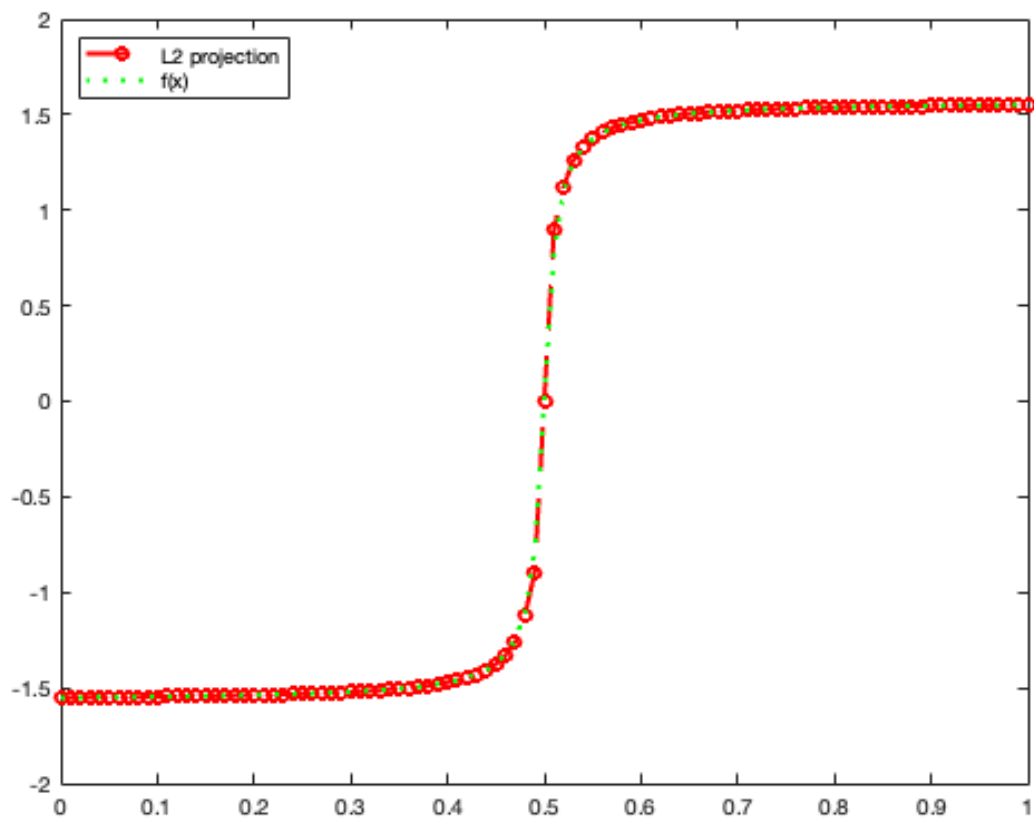
1.5414    1.5422    1.5430    1.5438    1.5445    1.5452    1.5458

*Columns 92 through 98*

1.5464    1.5470    1.5475    1.5481    1.5486    1.5491    1.5495

*Columns 99 through 101*

1.5500    1.5504    1.5508



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