Progress Report

Shakil Rafi

University of Arkansas

March 13, 2022

Table of Contents

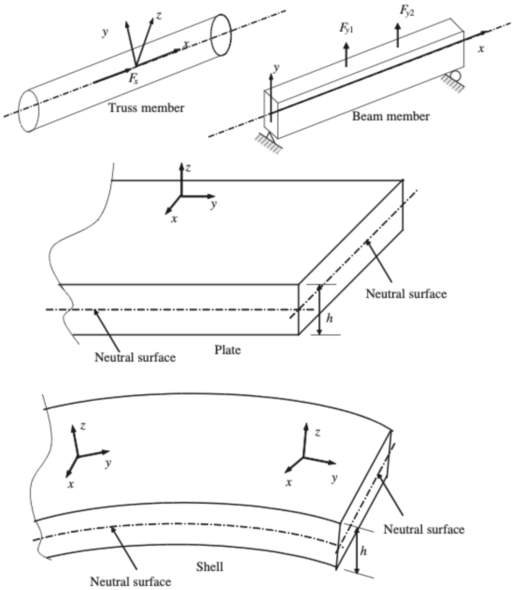
What to Solve

We seek to solve dynamic equilibrium equations. The formulation from Liu and Quek assumes among other things:

1. *Linear Elastic* Deformation grows proportionally with external force.
2. *Isotropic* Material property is direction independent.

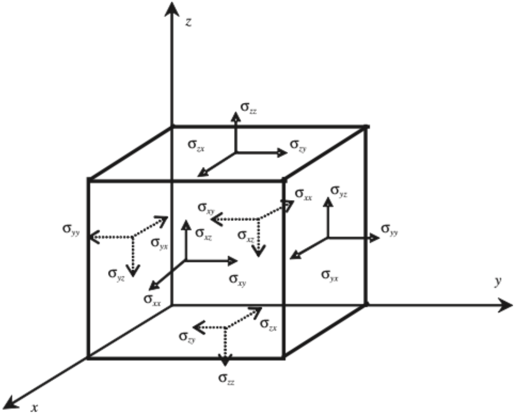
We distinguish between four kinds of objects: beams, trusses, plates and shells.

What to Solve



What to Solve

Taking a queue from Liu and Quek, we start off by defining DEE for an idealized infinitesimal, linearly elastic, isotropic material.



Observe that because of equilibrium $\sigma_{xy} = \sigma_{yx}; \sigma_{xz} = \sigma_{zx}; \sigma_{yz} = \sigma_{yz}$
Which give sus the stress tensors: $\sigma^T = \{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{yz}, \sigma xz, \sigma xy\}$

What to Solve

We also get six strain components $\epsilon^T = \{\epsilon_{xx}, \epsilon_{yy}, \epsilon zz, \gamma_{yz}, \gamma_{xz}, \gamma_{xy}$ Where:

$$\epsilon_{xx} = \frac{\partial u}{\partial x}; \quad \epsilon_{yy} = \frac{\partial v}{\partial y}; \quad \epsilon_{zz} = \frac{\partial w}{\partial z};$$

$$\gamma_{xy} = 2\epsilon_{xy} = \frac{\partial u}{\partial_y} + \frac{\partial v}{\partial x};$$

$$\gamma_{xy} = 2\epsilon_{xz} = \frac{\partial u}{\partial_z} + \frac{\partial w}{\partial x};$$

$$\gamma_{yz} = 2\epsilon_{yz} = \frac{\partial v}{\partial_z} + \frac{\partial w}{\partial y};$$

What to Solve

Provided $\mathbf{U} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$ the displacement matrix we get the system $\epsilon = \mathbf{LU}$

Where the differential operator is: $\mathbf{L} = \begin{pmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{pmatrix}$