Proposal for a booth of the Kakeya Needle Conjecture

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1 The Problem

Let $I \subseteq \mathbb{R}$ be a unit interval, which practically speaking is a needle in our booth. Given a connected set $K \subseteq \mathbb{R}^d$, which in our demonstrated could be printed shape on a piece of letter size paper. Is there a minimum area that shape K can take, and what is it? Propoese by Soichi Kakeya.

For detailed descriptions of the problem, these resources will come in handy:

1. `https://math.uchicago.edu/~may/REU2021/REUPapers/Fox.pdf`
2. `https://en.wikipedia.org/wiki/Kakeya_set#Results`
3. `https://www.youtube.com/watch?v=IM-n9c-ARHU`
4. `https://www.youtube.com/watch?v=j-dce6QmVAQ`

2 Why this problem?

There are several reasons we may want to use this problem:

1. This problem is very easy to explain
2. This requires very little equipment see Section 3.
3. This is often not a common problem students would be familiar with. Typical problems like four color or bridges of Königsberg are already something advanced high school students may be familiar with.
4. It is somewhat actively being researched unlike the four color or bridges of Königsberg problem.

3 Equipment Needed

We need two equipments:

1. Several pieces of letter size paper with increasingly smaller area Kakeya sets, multiple copies of each.
2. A sewing needle. Size 10 "Sharps" recommended, although given the age of students something less sharp and larger may be needed. Visibility and size may pose problem for students with accessibility needs.
3. (Optional) A poster

4 Engagement pathway for students

Here is a step by step proposed pathway for engaging with students

1. Explain the problem to students
2. Give them the obvious solution a circle
3. Ask if we canmake it smaller
4. Give them the deltoid example
5. Engage by asking if we can make it smaller?
6. Show them some of the "spiky" looking Kakeya sets
7. (for very enthusiastic students) We may show general construction methods and event talk about Hausdorff or Minkowski dimensions. Or not.
8. Give the sheets of paper with Kakeya sets away to students, they make cool posters! Or fantastic tattoo ideas!