Essentials (Based on past papers):

(1) Basic definitions of ∇ operators (grad, div, and curl) and knowing how to sketch vector fields.

(2) Index Notation stuff:

- $\mathbf{a}\cdot\mathbf{b} = a_i b_i$
- $\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$
- δ_{ij} is symmetric (i.e $\delta_{ij} = \delta_{ji}$) and has an index substitution property: $\delta_{ij}a_j = a_i$, which means with 2 dummy indicies $\delta_{ij}a_i b_j = \mathbf{a}\cdot\mathbf{b}$.
- $\epsilon_{ijk} = \begin{cases} 0 & \text{if any of } i, j \text{ or } k \text{ are equal} \\ 1 & \text{if } (i,j,k) = (1,2,3), (2,3,1), (3,1,2) \\ -1 & \text{if } (i,j,k) = (1,3,2), (3,2,1), (2,1,3) \end{cases}$
- ϵ_{ijk} represents the i element of the vector product, i.e. $\epsilon_{ijk}a_j b_k = (\mathbf{a}\times\mathbf{b})_i$
- $\epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$.
- $\nabla_i = \frac{\partial}{\partial x_i} = \partial_i$, index notation forms of grad,div, and curl can easily be gotten from a mixture of this and the above.
- A note on ∂_i, remember this still *differentiates,* so it follows rules such as the product rule: $\partial_i F_i g = g\partial_i F_i + F_i \partial_i g$.

(3) Basic multiple integration rules, including how to change orders, other things include:

- Change of basis, if you change to a basis where $x = x(u,v), y = y(u,v)$ then you must multiply by the *Absolute Value* of Jacobian of the transormation given by:

$$J = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix}$$
$$dxdy = \text{abs}(J)dudv$$

- The above expands to 3 dimensions in a trivially similar way, where if $x = x(u,v,w), y = y(u,v,w), z = z(u,v,w)$ then you simply have:

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$
$$dxdydz = \text{abs}(J)dudvdw$$

- COMMON TRANSFORMS

$$\text{Plane Polars: } (R,\theta), x = R\cos\theta, y = R\sin\theta, J = R$$
$$\text{Cylindrical Polars: } (R,\theta,z), x = R\cos\theta, y = R\sin\theta, z = z, J = R$$
$$\text{Spherical Polars: } (r,\theta,\phi), x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta, J = r^2\sin\theta$$

Curve, Line, and Surface integrals:

- There are 2 types of line integrals. The first is if you are given scalar function, i.e. $f(x,y,z) = x^2 + y^2 = 1$ which is simply paramererised by $x = \cos t, y = \sin t, z = 0)$, this is solved then by $\int_{t_1}^{t_2} f(\mathbf{x}(t))ds$ where ds is:
$$\left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right)^{\frac{1}{2}} dt$$

- The next type is scalar line integrals of a vector field around a curve. these are written as:
$$\int\limits_C \mathbf{F}\cdot d\mathbf{x} = \int F_1dx + F_2dy + F_3dz$$
The method when you have a curve that is not simply parameterisable but has 2 bounds. i.e. (from a past paper) $x^3 \le y \le x$, $0 \le x \le 1$ is to essentially split it into 2 integrals going all the way around the curve, for the example given we are going counter-clockwise around the curve so we have:
$$\oint\limits_C = \oint\limits_{C_1} + \oint\limits_{C_2}$$
$$C_1 = y = x^3, \text{limits: } [0,1]$$
$$C_2 = y = x,\ \text{limits: } [1,0]$$
this can now be easily solved using elementary change of variables.

- For these sketching the segments is VERY helpful.

- The work down by a force moving a particle around a curve is given by $\int_C \boldsymbol{F}\cdot d\mathbf{x}$

- DEFINITIONS AND PROPERTIES OF CONSERVATIVE FORCES:
$$\oint\limits_C \boldsymbol{F}\cdot d\mathbf{x} = 0 \text{ around any curve}$$
$$\int\limits_P^Q \boldsymbol{F}\cdot d\mathbf{x}\text{is independent of path between } P\&Q$$
$$\boldsymbol{\nabla}\times\boldsymbol{F} = 0 \text{ at each point}$$
$$\boldsymbol{F} = \nabla\phi \text{ where } \phi \text{ is a scalar function.}$$

- For surface integration we need a parameterised surface so that our $\boldsymbol{x} = \mathbf{x}(u,v)$, now we define 2 VERY IMPORTANTelements:

$$\mathbf{t}_u = \frac{\partial \mathbf{x}}{\partial u}$$
$$\mathbf{t}_v = \frac{\partial \mathbf{x}}{\partial v}$$

these elements will come up in multiple areas, for now we will use them to define the unit normal:

$$\hat{\mathbf{n}} = \frac{\mathbf{t}_u \times \mathbf{t}_v}{|\mathbf{t}_u \times \mathbf{t}_v|}$$

as well as 2 different surface elements, the scalar element :

$$\mathrm{d}S = |\mathbf{t}_u \times \mathbf{t}_v|\, dudv$$

and the vector element:

$$\mathrm{d}\boldsymbol{S} = \hat{\mathbf{n}}\mathrm{d}S = (\mathbf{t}_u \times \mathbf{t}_v)dudv$$

These are used when integrating a surface with a scalar or vector function respectively:

$$\iint\limits_S f(x,y)dS = \iint\limits_R f(u,v)\,|\mathbf{t}_u \times \mathbf{t}_v|\, dudv$$
$$\iint\limits_S \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{S} = \iint\limits_S \boldsymbol{F} \cdot (\mathbf{t}_u \times \mathbf{t}_v)dudv$$

(4) VECTOR INTEGRAL THEOREMS (IMPORTANT, HAS COME UP ON BOTH AVAILABLE PAPERS MULTIPLE TIMES):

- The divergence theorem helps convert volume integrals into surface integrals. It states that, for a vector field $\boldsymbol{F}$ that is continuously differentiable throughout a volume V,

$$\iiint\limits_V \boldsymbol{\nabla} \cdot \boldsymbol{F} dV = \oiint\limits_S \boldsymbol{F} \cdot d\boldsymbol{S}$$

where S is the surface enclosing the volume V.

- Stokes's theorem converts surface integrals to line integrals. It states for a vector field $\boldsymbol{F}$ that is continuously differentiable on a surface S,

$$\iint\limits_S (\boldsymbol{\nabla} \times \boldsymbol{F}) \cdot d\boldsymbol{S} = \oint\limits_C \boldsymbol{F} \cdot d\boldsymbol{S}$$

where the closed curve C is the boundary of S

- A couple notes to help remember: Divergence theorem starts with div$\boldsymbol{F}$, and Stokes's begins with curl$\boldsymbol{F}$ (I remember this because Stokes begins with an S which is a *curl*-y letter). Di$\boldsymbol{V}$ergence theorem deals with volumes while $\boldsymbol{S}$tokes's deals with surfaces.

(5) Curvilinear Co-Ordinates:

- Most of this section in the notes is NOT REQUIRED KNOWLEDGE, the only things to memorise here are a curviliniear co-ordinate system are mappings new co-ordinate systems similar to polars, there are important elements:

$$\begin{aligned} h_u &= |\mathbf{t}_u| \\ \mathbf{e}_u &= \frac{\mathbf{t}_u}{h_u} \\ h_v &= |\mathbf{t}_v| \\ \mathbf{e}_v &= \frac{\mathbf{t}_v}{h_v} \\ h_w &= |\mathbf{t}_w| \\ \mathbf{e}_w &= \frac{\mathbf{t}_w}{h_w} \end{aligned}$$

- the h_i's are called scale factors, the $\mathbf{e}_i$'s are called the unit vectors.
- A system with where $\mathbf{e}_i \cdot \mathbf{e}_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$ is called an orthogonal system and the vectors form an orthonormal basis.
- A system where $\mathbf{e}_i \cdot (\mathbf{e}_j \times \mathbf{e}_k) = \begin{cases} 0 & \text{if any of } i, j \text{ or } k \text{ are equal} \\ 1 & \text{if } (i,j,k) = (1,2,3), (2,3,1), (3,1,2) \\ -1 & \text{if } (i,j,k) = (1,3,2), (3,2,1), (2,1,3) \end{cases}$ is called right handed. I am aware this notation is kind of weird but the best way to think it through is to label i, j, k as $1, 2, 3$ and do different permutations to make sure.
- This is to my knowledge all that is required to know for Curvilinears as the rest SHOULD be given on the exam, I missed the lectures these where covered in so take that with a pinch of salt.

(6) General advice/motivation.

These past papers are tough. This module is tough. You are smart enough to do this. You are a second year Maths student at a pretty high up Russel Group Uni, you made it through first year, you can make it through this. Believe in yourself because I believe in you. Get plenty of rest before the exam, eat a decent meal before going to bed, eat something for breakfast on your way to the exam. Stratergies I use to help me revise/remember things:

- Find silly rules: Something silly and simple will stick with you more than something long and complex. Association help memories form.
- Listening to catchy music: My general stratergy is to listen to songs that stick in my head while revising and then relisten to them when I'm walking to the exam, if the songs are stuck in my head it helps me remember the things I was revising while listening to those songs.
- Explaining your answers: If you're with someone or someone is stuck, try explaining your answer to them. Generally this makes sure you understand what you're talking about and also spreads the knowledge (This is what I do most, and why I'm doing this doc, teaching really helps me remember). If you don't like working with other people around try an old programmers trick called the "Rubber Duck Method", when you finish a question go step by step and explain it to an inanimate object (typically a rubber duck as the name sugests), this basically does the same thing.
- Try make your study environment similar to an exam hall: Wear similar clothes, work in slightly duller light, sit on a less comfy chair, study around the same time as the exam will be, take off a coat etc. etc.

These are not guaranteed "Do these to pass an exam", but simply suggestions to help you with revision. Take them with a grain of salt and stay calm. Do whatever works best for you and good luck on Tuesday!