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sec: 1

Course: ~~317~~ CSE 317

Ans to the Q NO 1

By LU Factorization of the matrix is

$$A = \begin{bmatrix} 3 & -7 & -2 & 2 \\ -3 & 5 & 1 & 0 \\ 6 & -4 & 0 & -5 \\ -9 & 5 & -5 & 12 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -7 & -2 & 2 \\ 0 & -2 & -1 & 2 \\ 0 & 10 & 4 & -9 \\ -9 & 5 & -5 & 12 \end{bmatrix} \begin{bmatrix} R_2 = R_2 + R_1 \\ R_3 = R_3 - 2R_1 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -7 & -2 & 2 \\ 0 & -2 & -1 & 2 \\ 0 & 10 & 4 & -9 \\ 0 & -16 & -11 & 18 \end{bmatrix} [R_4 = R_4 + 3R_1$$

$$A = \begin{bmatrix} 3 & -7 & -2 & 2 \\ 0 & -2 & -1 & 2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -3 & -1 \end{bmatrix} \begin{bmatrix} R_3 = R_3 + 5R_2 \\ R_4 = R_4 - 8R_2 \end{bmatrix}$$

$$A = \cancel{R_4 \to 3R_3} \begin{bmatrix} 3 & -7 & -2 & 2 \\ 0 & -2 & -1 & 2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix} [R_4' = R_4 - 3R_3]$$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -5 & 1 & 0 \\ -3 & 8 & 3 & 1 \end{bmatrix}$$

So the obtained matrix is the matrix (/

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -5 & 1 & 0 \\ -3 & 8 & 3 & 1 \end{bmatrix} \quad \text{and } U = \begin{bmatrix} 3 & -7 & -2 & 2 \\ 0 & -2 & -1 & 2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

~~and~~

By using the LU decomposition finding the solution

$Ax = b$, $b = \begin{bmatrix} -9 \\ 5 \\ 7 \\ 11 \end{bmatrix}$

we knew,

$A = LU \Rightarrow Ax = b \quad \therefore b = LUx$

$b = Ly$ Let. $[Ux = y]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -5 & 1 & 0 \\ -3 & 8 & 3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} -9 \\ 5 \\ 7 \\ 11 \end{bmatrix}$$

multiplying L and ~~b~~ y

$y_1 = \underline{-9}$

$\therefore -y_1 + y_2 = 5 \Rightarrow y_2 = -9 + 5 = \underline{-4}$

$\therefore 2y_1 - 5y_2 + y_3 = 7 \Rightarrow y_3 = \underline{5}$

$\therefore -3y_1 + 8y_2 + 3y_3 + y_4 = 11 \Rightarrow y_4 = \underline{1}$

$$y = \begin{bmatrix} -9 \\ -4 \\ 5 \\ 1 \end{bmatrix}$$

$$\therefore x = \begin{bmatrix} 3 \\ 4 \\ -6 \\ -1 \end{bmatrix} \text{(Ans:)}$$

Ans to the Q No 2

(a) Equation from the line L

The vector V, P to Q is given by $V = \langle 7-4, 6-5, 0-1 \rangle = \langle 3, 1, -1 \rangle$

$x = P + tV$

$\therefore x = \langle 4, 5, 1 \rangle + t\langle 3, 1, -1 \rangle$

Now point $R(8, 8, 8)$ in not on the line L,

$x = \langle 8, 8, 8 \rangle$ From equation

$\langle 8, 8, 8 \rangle \neq \langle 4, 5, 1 \rangle + t\langle 3, 1, -1 \rangle$

$x = \langle 4, 5, 1 \rangle + 1\langle 3, 1, -1 \rangle = \langle 7, 6, 0 \rangle$

So, $\langle 7, 6, 0 \rangle$ is another point L other than P and Q.

(b) The Projection PR onto V,

$$Proj_V(PR) = \frac{PR \cdot V}{\|V\|^2} \cdot V$$

$PR = \langle 8-4, 8-5, 8-1 \rangle = \langle 4, 3, 7 \rangle$

$$Proj_V(PR) = \frac{\langle 4, 3, 7 \rangle \cdot \langle 3, 1, -1 \rangle}{\| 3, 1, -1 \|^2} \cdot \langle 3, 1, -1 \rangle$$

$$= \frac{37}{11} \cdot \langle 3, 1, -1 \rangle = \left\langle \frac{111}{11}, \frac{37}{11}, \frac{-37}{11} \right\rangle$$

(c) Distane of R to the line L

$$d = \frac{\|PR \times v\|}{\|v\|}$$

$$PR \times v = \begin{vmatrix} i & j & k \\ 4 & 3 & 7 \\ 3 & 1 & -1 \end{vmatrix} = \langle -10, 26, -13 \rangle$$

$$\therefore d = \frac{\|\langle -10, 26, -13 \rangle\|}{\|\langle 3, 1, -1 \rangle\|} = \frac{\sqrt{645}}{\sqrt{11}}$$

~~Rerpendnello~~

Perpendular PQ and PR

$$n = PQ \times PR$$

$$= \begin{vmatrix} i & j & k \\ 3 & 1 & -1 \\ 4 & 3 & 7 \end{vmatrix} = \langle 10, -17, 5 \rangle$$

(d) The Plane eauation

$$ax + by + cz = d$$

d is constend using point $P(4, 5, 1)$

$$\langle 10, -17, 5 \rangle \langle x-4, y-5, z-1 \rangle = 0$$

simplifics

$$10(x-4) - 17(y-5) + 5(z-1) = 0$$

$$\Rightarrow 10x - 40 - 17y + 85 + 5z - 5 = 0$$

$$\Rightarrow 10x - 17y + 5z = 60$$

$$10x - 17y + 5z = 60$$

(Ans)

Ans to the Q No 3

(a) Grousian Elemination on the matrix

$$B = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 0 \\ 2 & 1 & 7 \\ 3 & -2 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 5 & 5 \\ 0 & 4 & 4 \end{bmatrix} \left[\begin{array}{l} R_2' = R_2 + R_1 \\ R_3' = R_3 + (-2)R_1 \\ R_4' = R_4 + (-3)R_1 \end{array}\right]$$

$$= \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \left[\begin{array}{l} R_3' = R_3 + (-5)R_2 \\ R_4' = R_4 + (-4)R_2 \end{array}\right]$$

$\therefore$ Third and ~~forth~~ fourth row are indentical B are linearly dependent.

(b) From a linear combination

$$n_1 - 2n_2 + n_3 = 0$$

$$n_2 + n_3 = 0$$

$$\therefore n_2 = -n_3$$

$$\therefore n_2 = 1$$

$$n_3 = \boxed{-1}$$

Now

$$n_1 - 2n_2 + n_3 = 0$$

$$\Rightarrow n_1 - 2(-n_3) + n_3 = 0$$

$$\Rightarrow n_1 + n_3 = 0$$

$$n_1 = -n_3$$

$$n_1 + 2 + n_3 = 0$$

$$\therefore n_1 = 1$$

Linear combination first tow vector

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \text{(Ans)}$$

Ans to the Q No 4

(a)

S are orthogonal to each other

$a^T b = a_1 b_1 + a_2 b_2 + a_3 b_3 \rightarrow \boxed{\text{Formula}}$

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \quad c = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}, \quad = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

Now calculate ~~dot~~ dot Product for vectors,

$$a^T b = (1)(-1) + 0.4 + 1.1$$
$$= -1 + 0 + 1 = 0$$

$$a^T c = 1.2 + 0.1 + 1.(-2)$$
$$= 2 + 0 - 2$$
$$= 0$$

$$b^T c = (-1).2 + 4.1 + 1.(-2)$$
$$= -2 + 4 - 2$$
$$= 0$$

All dot Product are zero, so, S is orthogonal

(b) Gaussian Elimination:

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 8 \\ -1 & 4 & -1 & -96 \\ 1 & 1 & -2 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 1 & 8 \\ 0 & 4 & 0 & -88 \\ 0 & 1 & 3 & -8 \end{array}\right] \quad \left[\begin{array}{l} R_2' = R_2 - R_1 \\ R_3' = R_2 - R_1 \end{array}\right.$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 1 & 8 \\ 0 & 1 & 0 & -22 \\ 0 & 0 & 1 & -2 \end{array}\right] \quad \left[\begin{array}{l} R_2' = \frac{R_2}{4} \\ R_3' = R_3 - R_2 \\ R_3' = \frac{R_3'}{-3} \end{array}\right.$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & -22 \\ 0 & 0 & 1 & -2 \end{array}\right] \quad R_1' = R_1 - R_3$$

The system has a unique solution

All coeffecients are zero. vectors are linearly independent.

(c) The dot product with v_1

$$c_1 v_1 \cdot v_1 + c_2 v_2 \cdot v_1 + c_3 v_3 \cdot v_1 = c_1 \|v_1\|^2$$

$$\Rightarrow v_1 (c_1 v_1 + c_2 v_2 + c_3 v_3) = v_1 \cdot y$$

[v_1 is orthogonal to v_2 and v_3]

$$\Rightarrow c_1 (v_1 \cdot v_1) = v_1 \cdot y$$

$$\Rightarrow c_1 \|v_1\|^2 = v_1 \cdot y$$

$$\Rightarrow c_1 (1^2 + 0^2 + 1^2) = (1)(8) + 0 \cdot (-9) + (1)(6)$$

$\Rightarrow c_1 . 2 = 8 + 0 + 6$

$\Rightarrow c_1 = \frac{8+6}{2} = \frac{14}{2} = 7$

(ii) Taking dot Product with v_2

$v_2 y = c_2 . v_2 \quad v_2$

$\Rightarrow v_2 y = c_2 \|v_2\|^2$

$\Rightarrow c_2 ((-1)^2 + 4^2 + 1^2) = (-1).8 + 4.(-9) + (1)$

$\Rightarrow c_2 = \frac{-38}{18} = -\frac{19}{9}$

dot Product with v_3

$v_1 . v_3 = v_2 . v_3 = 0$ (because they are orthagonal)

$v_3 y = c_3 v_3 . v_3$

$\Rightarrow c_3 \|v_3\|^2 = v_3 y$

$\Rightarrow c_3 . (2^2 + 1^2 + (-2)^2) = (2)(8) + (1)(-9) + (-2)(6)$

$\Rightarrow c_3 = \frac{-5}{9}$

Therefor $c_2 = c_3 = 0$, v_2 and v_3 do not contribute vector y is in orthagonal decomposition

(d) vector s ~~from~~ form of basis for R^3 vectors must be ~~to~~ linearly independent and must span R^3

vector in S,

$$V_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad V_2 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}, \quad V_3 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

Now,

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 4 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix} \quad \left[\begin{matrix} R_2' = R_1 \\ R_1' = R_2 \end{matrix} \right.$$

$$= \begin{bmatrix} -1 & 4 & 1 \\ 0 & 1 & 5 \\ 0 & -7 & 0 \end{bmatrix} \quad \left[\begin{matrix} R_3' = R_3 + 2R_1 \\ R_2' = \frac{R}{4} \end{matrix} \right.$$

$$= \begin{bmatrix} -1 & 4 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \left[\begin{matrix} R_3' = 1/2 * R_3 \\ R'' = R_3' - R_1 \end{matrix} \right.$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \left| \begin{matrix} R_1' = -1 R_1 \\ R_1'' = R_1' + 4 R_1 \end{matrix} \right.$$

we can conclude that thee vectors form a basis of R_3

(F)

the cordinates of y relative vector v_1, v_2, v_3

$$[y]_S = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

$$c_1 = \frac{v_1 \cdot y_1}{\|v_1\|^2}$$

$$= \frac{[1 . 0 . 1] \begin{bmatrix} 8 \\ -9 \\ 6 \end{bmatrix}}{\|v_1\|^2}$$

$$= \frac{1.8 + 0(-9) + (1)(6)}{1^2 + 0^2 + 1^2}$$

$$= \frac{14}{2} = 7$$

$$\therefore c_2 = \frac{v_2 y}{\|v_2\|^2}$$

$$= \frac{[-1 \ 4 \ 1] \begin{bmatrix} 8 \\ -9 \\ -6 \end{bmatrix}}{(-1)^2 + 4^2 + 1^2}$$

$$= \frac{(-1)(8) + 4(-9) + 1 \cdot 6}{1 + 4 + 1}$$

$$= \frac{-26}{18} = -\frac{13}{9}$$

$$c_3 = \frac{v_3 y}{\|v_3\|^2} = \frac{(2)(8) + 1 \cdot (-9) + (-2)(6)}{2^2 + 1^2 + (-2)^2} = \frac{11}{9}$$

coordinates of y relative to the vector v_1, v_2, v_3 are

$$[y]_S = \begin{bmatrix} 7 \\ -\frac{13}{9} \\ \frac{11}{9} \end{bmatrix} \text{(Ans)}$$

Ans to the Q. NO 5

(a) $(a+b)^T (a-b) = \|a\|^2 - \|b\|^2$

(b) $\|a+b\|^2 + \|a-b\|^2 = 2(\|a\|^2 + \|b\|^2)$

(a) sol: $a^T(a-b) + b^T(a-b)$

$= a^Ta - a^Tb + b^Ta - b^Tb$

$= a^Ta - a^Tb + a^Tb - b^Tb$

$= a^Ta - a^Tb$

$= \|a\|^2 - \|b\|^2$

b) sol b: $\|a+b\|^2 + \|a-b\|^2$

$= (a+b)^T(a+b) + (a-b)^T(a-b)$

$= a^T(a+b) + b^T(a+b) + a^T(a-b) - b^T(a-b)$

$= a^Ta + \cancel{a^Tb} + \cancel{b^Ta} + b^Tb + a^Ta - \cancel{a^Tb} + \cancel{b^Ta}$

$= 2a^Ta + 2b^Tb$

$= 2\|a\|^2 + 2\|b\|^2$ (Prove) Ans

Ans to the Q NO 6

sol: Given that $F(x) = \|x-e\|^2 - \|x-d\|^2$

$= (x-e)^T(x-e) - (x-d)^T(x-d)$

$= (x^Tx - e^Tx - x^Te + e^Te) - (x^Tx - d^Tx - x^Td + d^Td)$

$= x^Tx - 2e^Tx + e^Te - x^Tx + 2d^Tx - d^Td$

$= 2(d-e)^Tx + \|e\|^2 - \|d\|^2$

f can expressed as $f(x) = a^Tx + b$. Linear in fuctions $\|e\| = \|d\|$ for this

Ans to the Q No 7

Sol: quartic Polynomial, $P(x) = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + c_5 x^4$

$P(0) = 0,\ P'(0) = 0,\ P(1) = 1,\ P'(1) = 0$

Equations are:

$$c_1 = 0$$
$$c_2 = 0$$

$$c_1 + c_2 + c_3 + c_4 + c_5 = 1$$

$$c_2 + 2c_3 + 3c_4 + 4c_5 = 0$$

$Ac = b$. form can be written as

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix},\quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

(Ans)

Ans to the Q No 8

Sol: The interpolation condition

$$\frac{c_1 + c_2 t_i + c_3 t_i^2}{1 + d_1 t_i + d_2 t_i^2} = y_i,\quad i = 1 \dots k$$

~~$\Rightarrow c_1 + c_2 t_i + c_3 t_i^2 = y_2 + y_i d_1$~~

$\Rightarrow c_1 + c_2 t_i + c_3 t_i^2 = y_i d_1 t_i + y_2 d_2 t_i^2$

$\Rightarrow c_1 + c_2 t_i + c_3 t_i^2 - y_i d_i t_i - y_2 d_2 t_i^2 = y_i$

$$A\theta = b$$

$$\begin{bmatrix} 1 & t_1 & t_1^2 & y_1 t_1 & y_1 t_1^2 \\ 1 & t_2 & t_2^2 & y_2 t_2 & y_2 t_2^2 \\ \vdots \\ 1 & t_k & t_k^2 & y_k t_k & y_k t_k^2 \end{bmatrix} \qquad b = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix}$$

Ans to the Q No 9

(a)

given that Data = {(1,2) (2,3) (3,4) (4,5) (5,6)} contains 5 data points and assume the initial cluster ~~center~~ center
= {(1,1) (5,5)}

Ex: $\boxed{\text{Iteration 1}}$

cluster 1: {(1,2) (2,3) (3,4)}
cluster 2: {(4,5) (5,6)}

center 1: (2,3)
center 2: (4.5, 5.5) (Ans)

$\boxed{\text{Iteration 2}}$

cluster 1: {(1,2) ,(2,3)}
cluster 2: {(3,4) (4,5), (5,6)}

center 1: (1.5, 2.5)
center 2: (4,5) (Ans)

Ans to the Q No 10

ⓐ To ~~deat~~ determine if the vector y is in the span of the first two coloums of the matrix

$y = c_1 \cdot$ collumn $1 + c_2 \cdot$ collumn

collumn 1 $= \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, Collumn 2 $= \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$

$y = \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}$

$c_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + c_2 \cdot \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix}$

System eauation,

$2c_1 - 2c_2 = 5$

$c_1 - c_2 = 4$

$2c_2 = 2$

$c_2 = 1$, $c_1 = 4 + 1 = 5$

$\therefore 2 \times 5 - 2 \times 1 = 5$

$\Rightarrow 10 - 2 \neq 5$

$\therefore 8 \neq 5$

y is not span of first two colloumns ~~of~~ of matrix

(b)

~~Leaf~~ To find the normal equation or for the least square regression Problem for this data we have to set up the system of equation by using the given data.

Lets r denote the coefficients of the regression linear or a, b and c. the normal equation deried the equation

$$x^T x \beta = x^T y$$

x is a matrix the vector of ~~cof~~ coefficents are y is the vector tangent output in this case the matrix First 2 collumns given matrix and y is the third column.

$$x = \begin{bmatrix} 2 & -2 \\ 1 & -1 \\ 0 & 2 \end{bmatrix} \quad y = \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}$$

The normal equation are:

$$x^T x = \begin{bmatrix} 2 & -2 \\ -2 & 9 \end{bmatrix}$$

$$x^T y = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

normal equation is

$$\begin{bmatrix} 2 & -2 \\ -2 & 9 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

solving the system of equation of find the values of B_1 and B_2

Inverse matrix

$$\beta = (x^T x)^{-1} x^T y$$

$$\beta = \begin{bmatrix} 2 & -2 \\ -2 & 9 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

calculate the inverse matrix $(x^T x)$ and multiply it by $x^T y$ find value β_1 and β_2 equation, $y = x\beta$

$$\therefore x^T x = x^T y$$

$$x = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix} \quad y = \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}$$

$$x^T x = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ -2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & -3 \\ 3 & 5 & -3 \\ -3 & -3 & 9 \end{bmatrix}$$

$$x^T y = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 0 \\ -2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 11 \\ 12 \\ -2 \end{bmatrix}$$

Least square regression Problem are:

$$\begin{bmatrix} 3 & 3 & -3 \\ 3 & 5 & -3 \\ -3 & -3 & 9 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 11 \\ 12 \\ -2 \end{bmatrix}$$

(c) Normal equation in Gaussian Elimation

$$\left[\begin{array}{ccc|c} 3 & 3 & -3 & 11 \\ 3 & 5 & -3 & 12 \\ -3 & -3 & 9 & -2 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 3 & 3 & -3 & 11 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 6 & 9 \end{array}\right] \quad [R_3 = R_3 - R_1]$$

$\therefore$

$$3x_1 + 3x_2 - 3x_3 = 11$$

$$2x_2 = 1$$

$$x_2 = 1/2$$

$$\therefore 6x_3 = 9 \therefore x_3 = 9/6 = \frac{3}{2}$$

$$\therefore 3x_1 + 3x.\frac{1}{2} - 3 \times \frac{3}{2} = 11$$

$$3x_1 = 11 - \frac{3}{2} + \frac{9}{2}$$

$$x_1 = \frac{11-6}{3} = \frac{5}{3}$$

so

$$\begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5/3 \\ 1/2 \\ 3/2 \end{bmatrix}$$

by solving normal equation using Gaussian Elimination.

Ans to the Q NO 01

Given that,

(a) student ID, Level of Preparation (P), Assignment, midterm exam (m), final Exam (F)

x_1 = student, x_2 = Level, x_3 = Assignments

x_4 = midterm

output final exam

x_2 use 2 binary during variable for value at y

d_1	d_2	
0	0	→ bad
1	0	→ medium
0	1	→ Good

equation is $\underline{x}\,\underline{B} = y$

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ 101 & 00 & 70 & 40 \\ 201 & 10 & 90 & 90 \\ 301 & 01 & 60 & 80 \\ 401 & 01 & 80 & 60 \\ 501 & 00 & 50 & 70 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix}}_{\beta} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}}_{y}$$

$$y = \begin{bmatrix} 40 \\ 80 \\ 60 \\ 60 \\ 50 \end{bmatrix}$$

$\therefore$ Equation $= (\underline{x}^T \underline{x}) B$

$= x^T y$

Ans: