Mathematics
Semester 2 2017

Cxo05

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1 Methods of Proof

Proving stuff, trivial right? Probably not. Presentation plays a large part on whether you get the full mark or not.

1.1 Definitions

1.1.1 Statements

A statement is a sentence that is true or false but not both.

1.1.2 Conditional statements

Conditional statements are statements where the truth of a statement is conditioned on the truth of another statement.

$$p \to q$$

The above denotes the statement "If p, then q". This statement is false when p is true and q is false; otherwise it is true.

1.1.3 Converse of a statement

The converse of $p \to q$ is $q \to p$. These statements are **NOT** logically equivalent.

1.1.4 Contrapositive of a statement

The contrapositive of $p \to q$ is $\sim q \to \sim p$. These statements are logically equivalent, thus the original statement is true when its contrapositive is true.

1.1.5 Inverse of a statement

The inverse of $p \to q$ is $\sim p \to \sim q$. These statements are **NOT** logically equivalent.

1.2 Proof by contradiction

1. Suppose that the statement to be proved is false.
2. Show that the assumption leads to a contradiction.

Since a state cannot be both true and false, the statement being false leads to a contradiction, therefore it must be true.

$$r : p \to q$$

$$\sim r :\sim (p \to q)$$

$$\sim r : p \wedge \sim q$$

1.3 Proof by mathematical induction

1. Let P_n be the statement you want to prove. Add the range of n as well.
2. Show that P_n is true for the base case of n.
3. Let k be an arbitrary integer such that its range is the same as the range of n and suppose that the statement P_k is true.
4. Expand the LHS of P_{k+1} to include our assumption of P_k.
5. Continue doing math until you reach the RHS of P_{k+1}
6. Thus, $P_k \rightarrow P_{k+1}$
7. Hence, by mathematical induction, P_n is true for all integers n, within its specified range.

2 Differentiation III

This thus concludes the summary for Year 4 Mathematics Semester 2 2017.