Physics

Semester 1 2017

Cxo05

April 25, 2017

1 Simple Harmonic Motion

Simple harmonic motion is the motion of an object whose restoring force is directly proportional to its displacement.

$$\text{Angular Frequency } \omega = 2\pi f$$

$$f = \frac{1}{T}$$

1.1 Horizontal Spring Mass System

$$F_x = -kx$$

$$F = ma$$

$$ma = -kx$$

$$a = \frac{F_x}{m} = -\frac{k}{m}x$$

1.2 Energy in Simple Harmonic Motion

We know that the object has both kinetic energy and potential energy from the spring.

$$K.E. = \frac{1}{2}mv_x^2 \text{ and } P.E. = \frac{1}{2}kx^2$$

Since energy is conserved, we combine the two equations.

$$T.E. = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$$

Solving for v_x,

$$v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2} = \omega\sqrt{A^2 - x^2}$$

Maximum velocity occurs at $x = 0$,

$$v_{max} = \sqrt{\frac{k}{m}}A$$

1.3 Vertical Oscillations

When a mass is hung on an unstretched vertical spring with spring constant k, the string extends by ΔL.

$$k\Delta L = mg$$

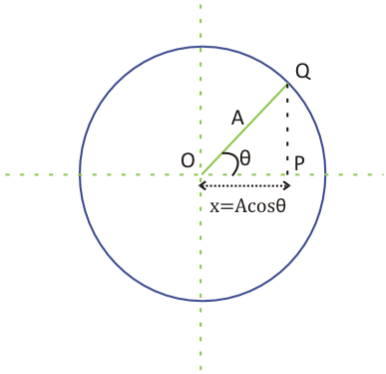
For springs in parallel,

$$k_T = k_1 + k_2$$

For springs in series,

$$k_T = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$

1.4 Circle of Reference



Point Q of moves in a counter clockwise direction. The constant angular velocity of Q is equals to the angular frequency of P's SHM.

$$\omega = \Delta\phi/\Delta t$$

From the figure, we can see that $x = A\cos\theta$. Note that $\phi = \theta$. If Q is at the extreme right end of the diameter at $t = 0$, then $\phi = 0$.

$$\phi = \omega t$$

$$x = A\cos\omega t$$

The velocity of Q is obtained by taking the derivative of $x = A\cos\omega t$ with respect to t.

$$v_x = -\omega A\sin\omega t$$

$$a_Q = \omega^2 A$$

$$a_x = -\omega^2 A\cos\omega t$$

Combining the above and $x = A\cos\omega t$,

$$a_x = -\omega^2 x$$

We are now able to express the frequency of an object with respect to k and m.

$$\omega^2 = \frac{k}{m}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

We are hence able to simplify some equations.

$$a_x = -\frac{k}{m}x$$

$$v_x = \pm\omega\sqrt{A^2 - x^2}$$

IMPORTANT When $x_0 \neq A$, $x = A\cos(\omega t + \phi_0)$, where ϕ_0 is the phase constant, found by setting $t = 0$.

1.5 The simple pendulum

The restoring force of a pendulum is the component of force tangent to the circular path at that point.

$$F = -mg\sin\theta$$

With small angle approximation however, $\sin\theta$ is almost equals to θ in radians.

$$F = -mg\theta = -mg\frac{x}{L}$$

The constant mg/L thus represents the force constant k.

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{L}}$$

1.6 Damped and forced oscillations

Damping causes a decrease in amplitude. An additional force acting on the object that varies with time in a period way is a driving force.

2 Waves

There are two types, transverse and longitudinal waves and can be represented by snapshot graphs and history graphs.
Snapshot graphs follows all particles at one single time, $\Delta y, x$.
History graphs follows only one particle but at all times, $\Delta y, t$.

2.1 Periodic mechanical waves

The very important formula,

$$v = \lambda f$$

Each wave advances by one wavelength λ during each period T. Particles one wavelength apart move in phase with each other.

2.2 Wave speeds

The speed of a transverse wave in a rope under tension is proportional to the square root of the tension, divided by the mass per unit length.

$$v = \sqrt{\frac{F_T}{\mu}}$$

2.3 Mathematical description of a wave

Suppose that the displacement of a particle at the left end at $x = 0$.

$$y = A\sin\omega t$$

The wave travels from $x = 0$ to some point x to the right of the origin in an amount of time given by x/v, where v is the wave speed. So the motion of point x at time t is the same as the motion of point $x = 0$ at the earlier time $(t - x/c)$.

$$y(x,t) = A\sin\omega\left(t - \frac{x}{v}\right)$$

$$y(x,t) = A\sin\omega\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Thus at $x = 0$,

$$y = A \sin 2\pi \frac{t}{T}$$

Wave number k.

$$k = \frac{2\pi}{\lambda}$$

Phase difference ϕ_0 in waves.

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$$

When the particle is moving in the positive x direction, $kx - \omega t$ and in negative x direction, $kx + \omega t$.

$$y(x,t) = A \sin(kx - \omega t + \phi_0)$$

2.4 Reflections and superposition

2.4.1 Wave reflection

Waves reflecting on a fixed end. When the pulse arrives, the rope exerts an upward force on the wall and the wall exerts a downward force on the rope. The pulse thus inverts as it reflects.
Waves reflecting on a free end. When the pulse arrives, no force acts on the wave, thus, it reflects without inverting.

2.4.2 Wave overlap

When pulses overlap, the displacement of the string at any point is the vector sum of the displacements due to the individual pulses.

2.5 Standing waves and normal modes

2.5.1 Standing waves

Nodes, points on the wave that do not move. Anti-nodes, points midway between nodes, with the greatest amplitude. Because it does not seem to move along the string, it is called a standing wave.
Open ends of a pipe is always a displacement anti-node. Closed end of a pipe is always a displacement node.

2.5.2 Normal modes

Wavelengths for standing waves in strings/Closed-Closed tubes/Open-Open tubes. Largest wavelength is when $m_{mode} = 1$ and only has one anti-node.

$$NN = \frac{\lambda_m}{2} = \frac{L}{m}$$

$$f_1 = \frac{v}{2L}$$

$$f_m = m\frac{v}{2L}$$

$m = 1$ Fundamental frequency
$m = 2$ Second Harmonic, First overtone
...
For Open-Close tubes however, the fundamental mode has only one quarter of a wavelength in a tube of length L, hence the $m = 1$ wavelength is $\lambda_1 = 4L$.

$$AN = \frac{\lambda_m}{4} = \frac{L}{m}$$

$$f_m = m\frac{v}{4L} = mf_1$$

Where m = 1,3,5,7 ...

Fundamental frequency on a vibrating string.

$$f_1 = \frac{1}{2L}\sqrt{\frac{F_T}{\mu}}$$

2.6 Beats

If two waves at first are in phase, they will interfere constructively and a large amplitude resultant wave occurs. When the two waves move, they become progressively out of phase until they interfere destructively and a low amplitude resultant wave occurs.

2.7 The Doppler effect

When a source of sound is moving away from the receiver, it has a lower apparent frequency. When a source of sound is moving towards the receiver, it has a high apparent frequency.

2.8 Power and Intensity of wave

If a wave has power P,

$$I = \frac{P}{Area}$$

If a source of spherical waves radiates uniformly in all directions,

$$I = \frac{P_{source}}{4\pi r^2}$$

$$\frac{I_1}{I_2} = \frac{{r_2}^2}{{r_1}^2}$$

3 Lens

Image is always virtual when outgoing rays don't actually come from the image.

- When object on same side as reflecting surface, distance is positive.
- When image is on same side as reflecting surface, the distance is positive.

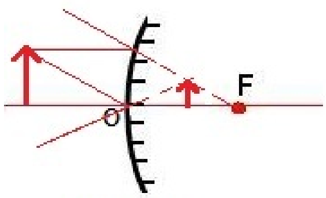
3.1 Plane mirrors

Object and image will have the same size and orientation when using plane mirrors.

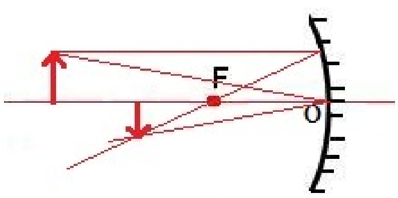
$$s = -s'$$

Image formed is always upright but reversed.

3.2 Reflections at a spherical surface



Convex mirror



Concave mirror

F is the focal point. Center of curvature C is between F and $2F$. R is the distance between C and the surface. f is the distance between F and the surface.

3.2.1 Focal length

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} = \frac{1}{f}$$

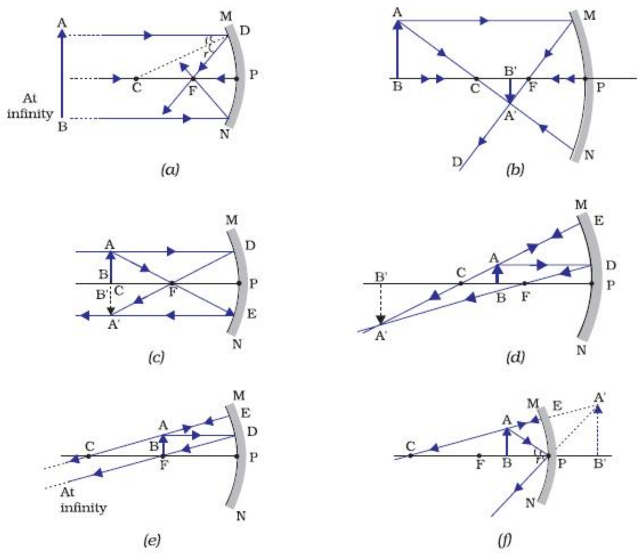
When C is on the same side as the outgoing ray, R is positive.

3.2.2 Lateral magnification

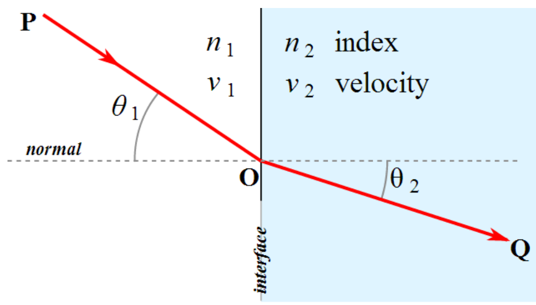
$$m = \frac{y'}{y} = -\frac{s'}{s}$$

IMPORTANT Both equations can be used for both concave and convex mirrors and lens.

3.2.3 Drawing ray diagrams



3.3 Refraction



$$n_1 sin\theta_1 = n_2 sin\theta_2$$

Total internal reflection occurs when the angle of incidence is more than or equals to critical angle. To find critical angle, we use Snell's law.

$$\theta_c = sin^{-1}\left(\frac{n_2}{n_1}\right)$$

3.3.1 Refraction at a spherical surface

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

$$m = \frac{n_a s'}{n_b s}$$

At a plane surface where $R = \infty$.

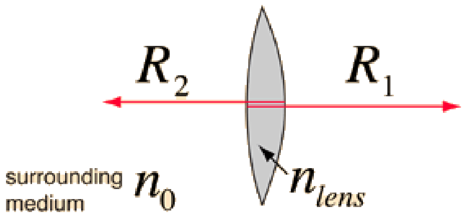
$$\frac{n_a}{s} + \frac{n_b}{s'} = 0$$

3.4 Thin lens

Equation for focal length and lateral magnification can be applied.
IMPORTANT When half the lens is covered, the image is blurred.

3.4.1 Lens-maker equation

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



Sign conventions for R:

- $+/-$ for both sides convex
- $+/+$ for left side convex, right side concave
- $-/+$ for both sides concave
- $-/-$ for left side concave, right side convex

3.5 Multiple lens

When two lens are placed right next to each other, in contact.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

3.5.1 Microscope

A microscope consists of two lens, the eyepiece with the angular magnification M_2 and the objective with lateral magnification m_1.

$$M = m_1 M_2 = \frac{(25cm)(D_{iobjective})}{f_1 f_2}$$

3.5.2 Telescope

The telescope works in a similar way like the microscope.

$$M = -\frac{f_1}{f_2}$$

Where f_1 is eyepiece and f_2 is objective.

This thus concludes the summary for Year 4 Physics Semester 1 2017.