Labor Market Matching, Wages, and Amenities

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Introduction

- Mortensen (2003) identifies four key forces that contribute to wage inequality
  1. worker productivity differences
  2. compensating differentials (Rosen 1986)
  3. endogenous matching in the presence of complementarities (Becker 1973)
  4. search frictions

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  - workers and firms are forward looking and employment decisions reflect all channels
- Quantifying the different forces requires a coherent model and sound identification

What we do

1. We extend the sequential auction model of Postel-Vinay and Robin (2002) with:
    - worker-firm interactions in production
    - worker-firm interactions in disutility of labor net of amenities
    - preference shocks associated with mobility (observed and priced)
    - firms choose wage contracts optimally
2. We prove nonparametric identification of main model primitives
    - proof delivers transparent sources of identification from mobility and wages
3. We apply the procedure to matched employer-employee data (Swedish)
    - wages are approximately monotonic in both firm and worker type
    - equilibrium sorting is strongly positive
    - match surplus is non-monotonic: workers do not agree on ranking of firms
    - most (69%) of wage variation is between worker types
    - within worker type the sources of wage dispersion differ greatly, i.e. compensation differentials accounts for more than 50% for the lowest and about 12% for highest

Related Literature

- Abowd, Kramarz, and Margolis (1999); Shimer and Smith (2000); Postel-Vinay and Robin (2002); Hagedorn, Law, and Manovskii (2017); Sorkin (2018); Taber and Vejlin (2020); Lamadon, Mogstad, and Setzler (2022)

- Lise and Postel-Vinay (2020); Lindenlaub and Postel-Vinay (2021); Arcidiacono, Gyetvai, Jardim, and Maurel (2022); Eeckhout and Kircher (2011); Lentz (2010); Lopes de Melo (2018); Lise, Meghir, and Robin (2016); Burdett and Mortensen (1998); Mortensen (2003); Hotz and Miller (1993); Arcidiacono and Miller (2011); An, Hu, and Shum (2013); Kasahara and Shimotsu (2009, 2012)

- Bonhomme, Lamadon, and Manresa (2019); Lentz, Piyapromdee, and Robin (2023); Arcidiacono, Gyetvai, Jardim, and Maurel (2022)

Model: Agents, preferences and technology

- x denotes a worker type, workers may be employed or unemployed, and may move
  - $c(x, y)$ denotes the disutility of labor net of amenities
  - $u(w)$ denotes the flow utility from a wage w
  - $b(x)$ denotes leisure and/or benefits and/or home production
  - ξ denotes a transitory shock for the worker, associated with changing state
- y denotes a firm type, firms have jobs that may be filled or vacant
  - $f(x, y)$ denotes match production, firm production is the sum over matches

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Model: Meeting technology and mobility shocks

New meetings take place in a frictional labor market (random search) Measures

- Workers meet vacancies with equilibrium probabilities λ_0 and λ_1
- Vacancies meet workers with probabilities $\lambda_0 L_0/V$ and $\lambda_1 L_1/V$
- Meetings are linked by an aggregate meeting function $M(L_0, L_1, V)$

A meeting is a draw of a job type y and a transitory mobility shock ξ

- ξ is drawn from G_0 (with negative support) when unemployed and G_1 (with full support) when employed
- ξ is associated with leaving current state (take job offer)
- ξ is observable (by worker, new firm, incumbent firm)
- ξ is transitory (consumed immediately upon a move)

- Existing matches separate with exogenous probability $\delta(x, y)$

Model: Values

Define the surplus to the worker in excess of unemployment, of a value W by

$$R = W - W_0(x) \geq 0$$

Define the maximum possible **worker surplus** in the match by

$$S(x, y) = \max\{R \quad | \quad \Pi_1(x, y, R) \geq \Pi_0(y)\}$$

where

- $\Pi_1(x, y, R)$ is the value to the firm having promised surplus R to the worker
- $\Pi_0(y)$ is the value of a vacancy
- $W_0(x)$ is the value to an unemployed worker

The Poaching Mechanism (2nd price auction)

- Upon meeting x and ξ are observed to all, while y and y' are private to the firms.

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  1. If $B' + \xi > \max\{B, \xi\}$, the poacher wins and must deliver the surplus $\max\{B, \xi\} - \xi$ to the worker, who collects a total of $\max\{B, \xi\}$.
  2. If $B \geq \max\{B' + \xi, \xi\} = B'^{+} + \xi$ (with $B'^{+} = \max\{B', 0\}$), the incumbent wins and must deliver at least $B'^{+} + \xi$ to the worker.
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- If a worker is unemployed, the mechanism proceeds as if $S = 0$.

Firm Problem, given promise $R \le S$ to worker x

$$\Pi_1(x,y,R) = \max_{\{w,R_0,R_1(B',\xi),B(\xi)\}} \Big\{ f(x,y) - w + \frac{\delta(x,y)}{1+r}\Pi_0(y) + \frac{\overline{\delta}(x,y)\overline{\lambda}_1}{1+r}\Pi_1(x,y,R_0)$$

$$+ \frac{\overline{\delta}(x,y)\lambda_1}{1+r}\iint \Big[\mathbf{1}_{\{B(\xi)\ge B'^{+}+\xi\}}\Pi_1(x,y,R_1(B',\xi))$$

$$+ \mathbf{1}_{\{B(\xi)< B'^{+}+\xi\}}\Pi_0(y)\Big]\, \mathrm{d}F_0(B'|x,\xi)\,\mathrm{d}G_1(\xi)\Big\},$$

subject to:

(PK) $$W_0(x) + R = u(w) - c(x,y) + \frac{\delta(x,y)}{1+r}W_0(x) + \frac{\overline{\delta}(x,y)\overline{\lambda}_1}{1+r}\left(W_0(x) + R_0\right)$$

$$+ \frac{\overline{\delta}(x,y)\lambda_1}{1+r}\iint \Big[\mathbf{1}_{\{B(\xi)\ge B'^{+}+\xi\}}\left(W_0(x) + R_1(B',\xi)\right)$$

$$+ \mathbf{1}_{\{B(\xi)< B'^{+}+\xi\}}\left(W_0(x) + \max\{B(\xi),\xi\}\right)\Big]\, \mathrm{d}F_0(B'|x,\xi)\,\mathrm{d}G_1(\xi),$$

(AC) $R_1(B',\xi) \ge B'^{+} + \xi$, (PC) $R_0 \ge 0$, and $\Pi_0(y) \le \Pi_1(x,y,R_0), \Pi_1(x,y,R_1)$.

Properties of the Optimal Contract

- Value is transferred according to

$$\frac{\partial \Pi_1(x, y, R)}{\partial R} = -\frac{1}{u'(w)}$$

- The wage can be written in closed form as a function of current worker surplus

$$w(x, y, R)$$

- The wage remains constant unless an outside offer triggers an increase

$w(x, y, R)$

Key properties

1. Mobility is based on match surpluses and the shock ξ, not current wage or R
2. Job-to-job transitions may happen even when $S(x,y') < S(x,y)$
3. Unemployed workers can start employment at $W > W_0(x)$
4. Wages are Markov conditional on types

These properties allow us to directly apply Bonhomme, Lamadon and Manresa (2019) to recover unobserved types in a first step.

- several quantities of the model are recovered directly
- doesn't require solving the model

Model Summary

Primitives

- Preferences: r, $u(w)$, $c(x, y)$, $b(x)$
- Production: $f(x, y)$
- Separation rates: $\delta(x, y)$
- Meeting probabilities: λ_0, λ_1
- Measures: $\ell(x)$, $n(y)$, and N
- Mobility shock distributions: G_0 and G_1

Endogenous outcomes

- Conditional mobility probabilities and wage distributions implied by $S(x, y)$, $W_0(x)$, $\Pi_0(y)$, $\Pi_1(x, y, R)$
- Measure of worker-firm matches $\ell_1(x, y)$

$\Pi_1(x, y, R)$ $\Pi_0(y)$ $W_0(x)$ $W_1(x, y, w)$ $\ell_1(x, y)$ λ Equilibrium $S(x, y)$ $w(x, y, R)$

Identification is constructive

Steps to map model to data:

1. Recover type-specific distributions, transitions, and wages
   - use BLM (2019) on fixed-T matched employer-employee data
2. Obtain surplus $S(x, y)$
   - integrate using observed probabilities from Step 1
3. Obtain G_0 and G_1
   - relate model employment transitions to those observed from Step 1
   - relate model wage distributions at transition to those from Step 1
4. Recover $\Pi_0(y)$, $f(x, y)$, and $\tilde{c}(x, y) = c(x, y) + b(x)$
   - use optimal contract to express $\Pi_1(x, y, R)$ in term of wage
   - evaluate $\Pi_1(x, y, R)$ at $R = S(x, y)$ to obtain $f(x, y)$
   - recover $\tilde{c}(x, y)$ from equilibrium wage equation.

Assumptions (Step 0)

1. The number of worker types and firm types are known
2. The discount rate r and flow utility $u(w)$ are known
3. $\tilde{c}(x, y) = c(x, y) + b(x)$ (includes disutility of labor, amenities and forgone home produciton)
4. G_1 has zero median and belongs to a parametric family $\mathcal{G}_1 = \{G_1(\xi; \theta)\}_\theta$, where θ is identified from observations in any bounded interval.
5. G_0 has support in $(-\infty; 0]$ and belongs to a parametric family $\mathcal{G}_0 = \{G_0(\xi; \theta)\}_\theta$, where θ is identified from observations in any bounded interval.

Step 1

Apply Bonhomme, Lamadon and Manresa (2019) results and estimation approach

- show that Markov assumptions are satisfied
- estimate type-specific distributions, transition rates and wages

This procedure transforms the original data, which is in terms of worker and firm names, into a new data set in terms of worker and firm types

- $\ell_0(x)$, $\mathbb{P}[\mathrm{UE}|x]$, $\mathbb{P}[y|x, \mathrm{UE}]$
- $\ell_1(x, y)$, $\mathbb{P}[\mathrm{JJ}|x, y]$, $\mathbb{P}[y'|x, y, \mathrm{JJ}]$, $\mathbb{P}[\mathrm{EU}|x, y]$
- $F_{\mathrm{UE}}(w|x, y_{t+1})$, $F_{\mathrm{JJ}}(w|x, y_t, y'_{t+1})$, and $F_{\mathrm{EE}}(w|x, y_t, w_t)$

From this point on we can treat x, y, and all of these objects as observed.

Step 2: Surpluses (the correct weighting of wages)

$$\frac{\partial R(x,y,w)}{\partial w} = \frac{(1+r)u'(w)}{r + \underbrace{\delta(x,y) + [1-\delta(x,y)]\,\lambda_1 \int \overline{G}_1\left[R(x,y,w) - S(x,y')^+\right]\frac{v(y')}{V}\,\mathrm{d}y'}_{\mathbb{P}[m_t \neq \mathrm{EE}|x,y,e_t=1] + \mathbb{P}[w_{t+1} > w_t, m_t = \mathrm{EE}|x,y,w_t=w]}},$$

The probability of any of these changes conditional on an (x,y) match is known.
By definition, $R(x,y,\underline{w}(x,y)) = 0$ and $R(x,y,\overline{w}(x,y)) = S(x,y)$.

It then follows that

$$R(x,y,w) = \int_{\underline{w}(x,y)}^{w} \frac{\partial R(x,y,w')}{\partial w}\,\mathrm{d}w',$$

This then identifies $S(x,y)$ as well as $w(x,y,R)$, which is the inverse function of $R(x,y,w)$.

Step 3: $\lambda_0 G_0$, G_1, $v(y)/V$, λ_1, and $\delta(x,y)$

- The distribution of starting wages after a job-to-job transition is:

$$F_{\mathrm{JJ}}(w|x,y,y') = \frac{\overline{G}_1(\, S(x,y) - R(x,y',w)\,)}{\overline{G}_1(\, S(x,y) - S(x,y')\,)}.$$

- We know $F_{\mathrm{JJ}}(w|x,y,y')$ from Step 1
- We know $S(x,y)$ and $R(x,y',w)$ from Step 2
- We assumed $G_1(0) = 1/2$ (Assumption 4.a)

$G_1(\xi)$ is nonparameterically identified for $\xi \in [-S(x,y), 0]$

- We need to be paramtertic outside this range (Assumption 4.b), i.e, symmetry plus an assumption about the tail.

Step 3: $\lambda_0 G_0$, G_1, $v(y)/V$, λ_1, and $\delta(x,y)$

The probability of separating to unemployment is

$$\mathbb{P}[\mathrm{EU}|x,y] = \delta(x,y) + [1-\delta(x,y)]\,\lambda_1 \left(\int (1-\phi(x,y'))\,\frac{v(y')}{V}\,\mathrm{d}y' \right) \overline{G}_1(S(x,y))$$

The job-to-job transition probabilities for worker x from firm y to y' are

$$\mathbb{P}(y'|x,y,\mathrm{JJ}) = [1-\delta(x,y)]\,\lambda_1 \frac{v(y')}{V}\,\overline{G}_1(S(x,y)-S(x,y')),$$

Relative probability of moving from y to y' and from y to y'', and adding up:

$$\frac{\mathbb{P}(y'|x,y,\mathrm{JJ})}{\mathbb{P}(y''|x,y,\mathrm{JJ})} = \frac{v(y')}{v(y'')}\,\frac{\overline{G}_1(S(x,y)-S(x,y'))}{\overline{G}_1(S(x,y)-S(x,y''))}, \qquad \text{and} \quad \int \frac{v(y)}{V}\,\mathrm{d}y = 1.$$

We have identified $\frac{v(y)}{V}$, $\delta(x,y)$, and λ_1

Step 3: $\lambda_0 G_0$, G_1, $v(y)/V$, λ_1, and $\delta(x,y)$

The probability moving from unemployment to firm type y is

$$\mathbb{P}[y|x, \mathrm{UE}] = \frac{v(y)}{V} \lambda_0 \overline{G}_0(-S(x,y))$$

The distribution of starting wages from unemployment is

$$F_{\mathrm{UE}}(w|x,y) = \frac{\lambda_0 \overline{G}_0\left(-R(x,y,w)\right)}{\lambda_0 \overline{G}_0\left(-S(x,y)\right)}.$$

$\lambda_0 G_0(\xi)$ is nonparametrically identified for $\xi \in [-\overline{S}, 0]$

- We need to be parametric for $\xi \leq -\overline{S}$ (required for counterfactuals that increase $\overline{S}$)

Step 4: Production $f(x,y)$ and vacancies V

At the maximum wage in an (x,y) match we have that $\Pi_1(x,y,S(x,y)) = \Pi_0(y)$:

$$f(x,y) = w\left(x,y,S(x,y)\right) + \frac{r}{1+r}\Pi_0(y),$$

Using the optimal contract $\partial\Pi_1(x,y,R)/\partial R = -1/u'(w(x,y,R))$ to write:

$$r\,V\Pi_0(y) = \lambda_0 \iint_{-S(x,y)}^{0} \frac{\overline{G}_0(\xi)}{u'\left(w(x,y,-\xi)\right)}\ell_0(x)\,\mathrm{d}\xi\,\mathrm{d}x$$

$$+\lambda_1 \iiint_{-S(x,y)}^{0} \frac{\overline{G}(\xi+S(x,y'))}{u'\left(w(x,y,-\xi)\right)}\left(1-\delta(x,y')\right)\ell_1(x,y')\,\mathrm{d}\xi\,\mathrm{d}y'\,\mathrm{d}x\,.$$

The constant V can be recovered by matching the labor share of value added:

$$\text{labor share} = \mathbb{E}\left[w_{it}\right]/\mathbb{E}\left[w(x_i,y_{j(i,t)},S(x_i,y_{j(i,t)})) + \frac{1}{V}\frac{rV}{1+r}\Pi_0(y)\right].$$

Step 5: Dis-utility of work $\tilde{c}(x, y)$

Evaluating the wage equation at the maximal (x, y)-wage we have

$$(1+r)u\left(\overline{w}(x,y)\right) = (1+r)\,\tilde{c}(x,y) + \left[r + \delta(x,y)\right] S(x,y) - \left[1 - \delta(x,y)\right] \lambda_1 \int_{S(x,y)}^{\infty} \overline{G}_1(\xi)\,\mathrm{d}\xi\,.$$

$\tilde{c}(x, y)$ is identified.

Estimation

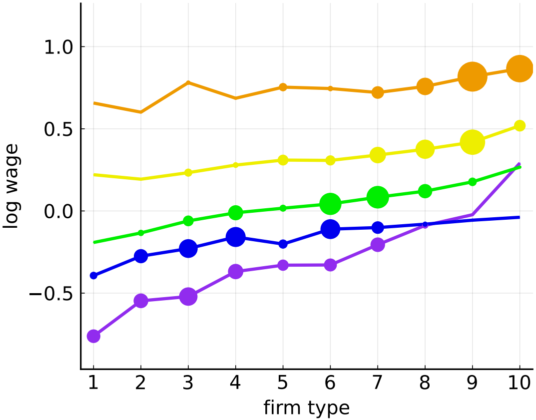
Step 1

- discrete types: 10 firm and 5 worker

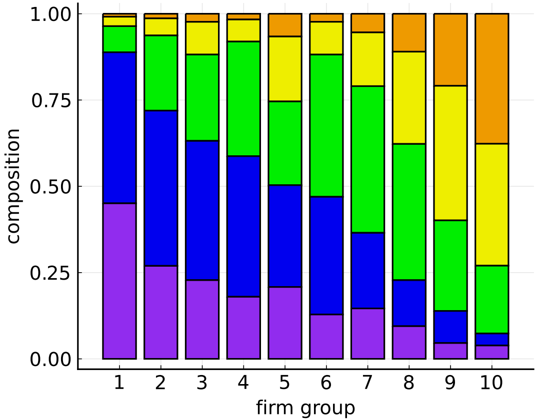
Step 2

- G_1 is a logistic with scale parameter ρ_1 to be estimated
- G_0 is a logistic, truncated above by zero, with scale parameter ρ_0 to be estimated
- $u(w) = \log(w)$
- r set to 5 percent annual

Step 1 (data only): Wages and Sorting Patterns[1]



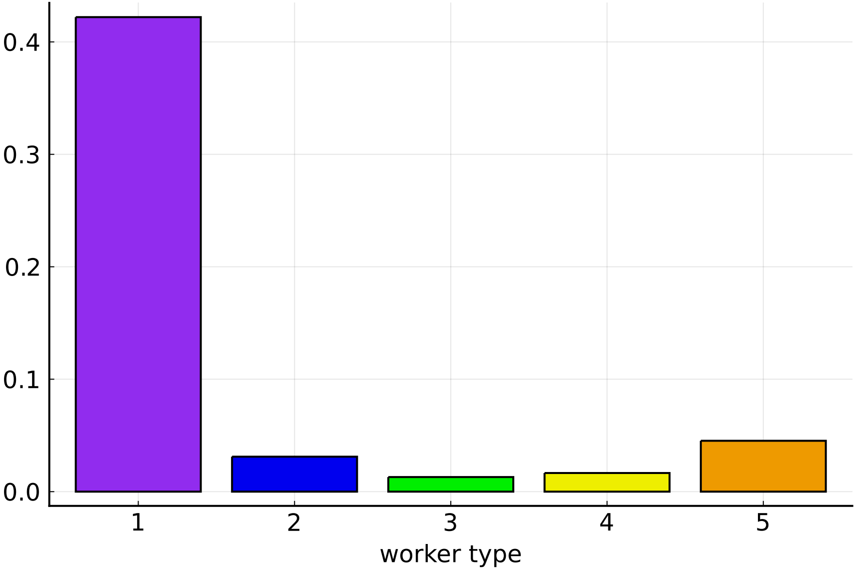
Mean wage by (x,y)-type



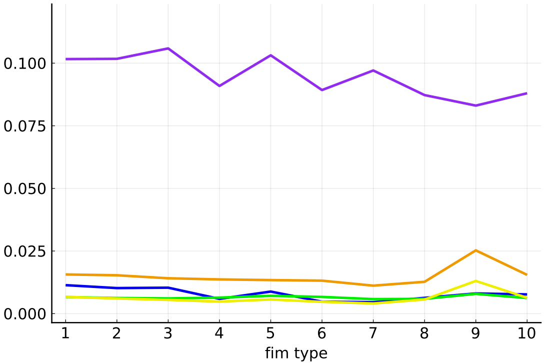
$\ell_1(x,y) / \int \ell_1(x,y)\,\mathrm{d}x$

[1] Worker and Firms are ordered by mean wage in all figures.

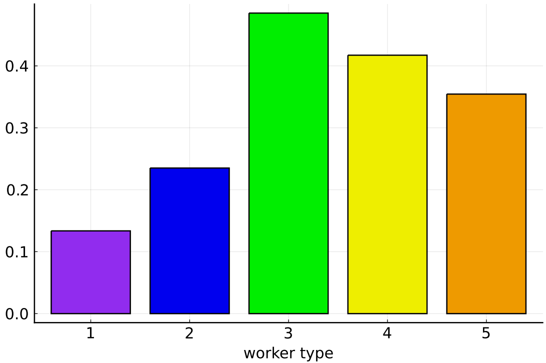
Step 1 (data only): Unemployment Rates



Step 1 (data only): EU and UE transitions

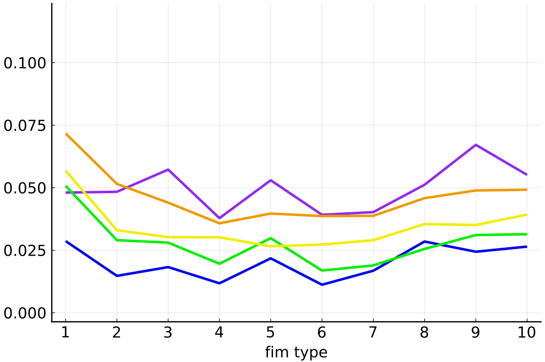


Separations to unemployment

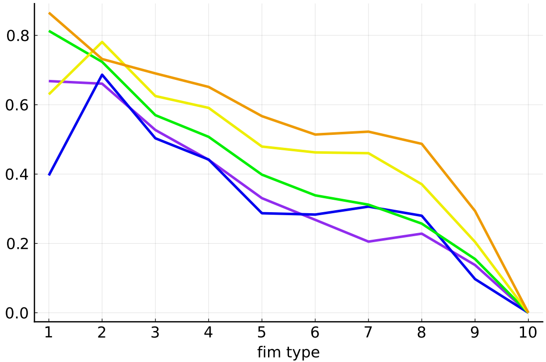


Job finding rate

Step 1 (data only): Job-to-job separation

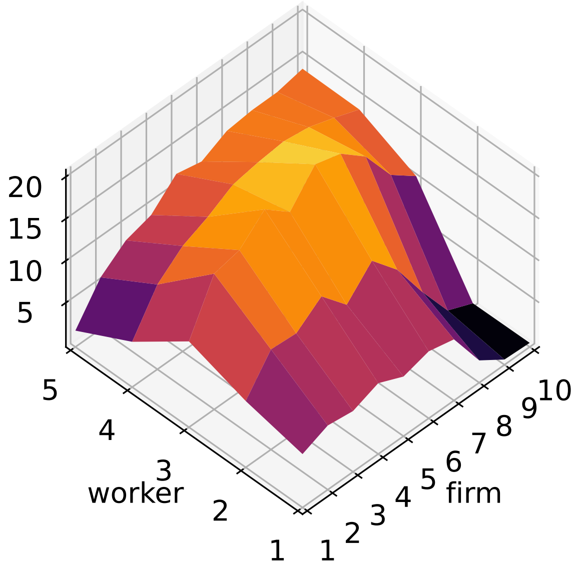


Probability of job change

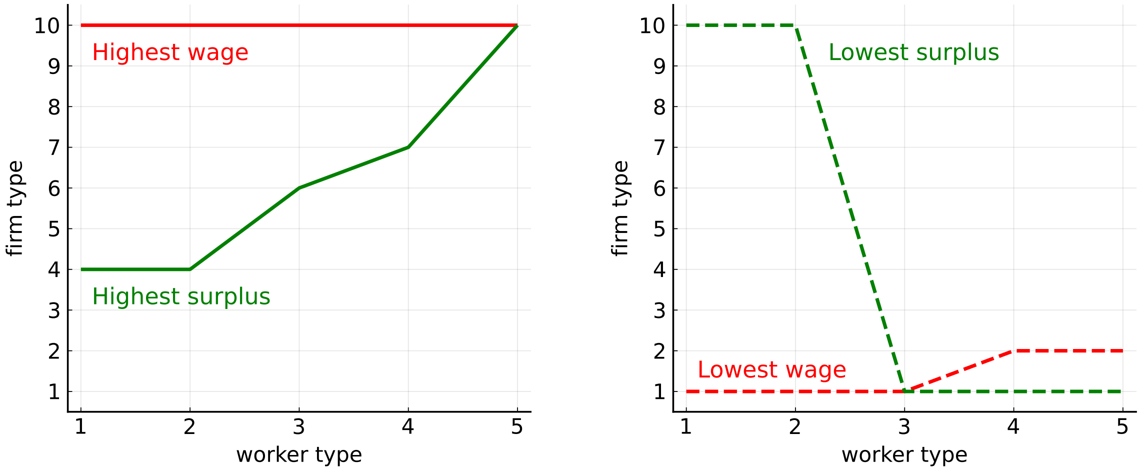


Probability of moving to a higher y conditional on moving

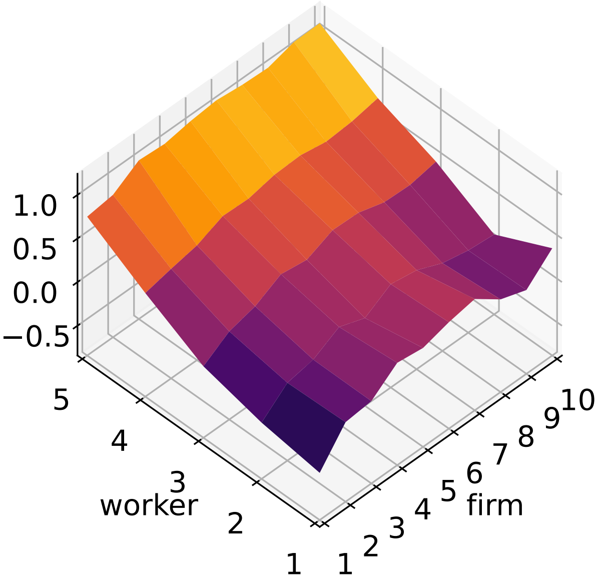
Surplus $S(x, y)$



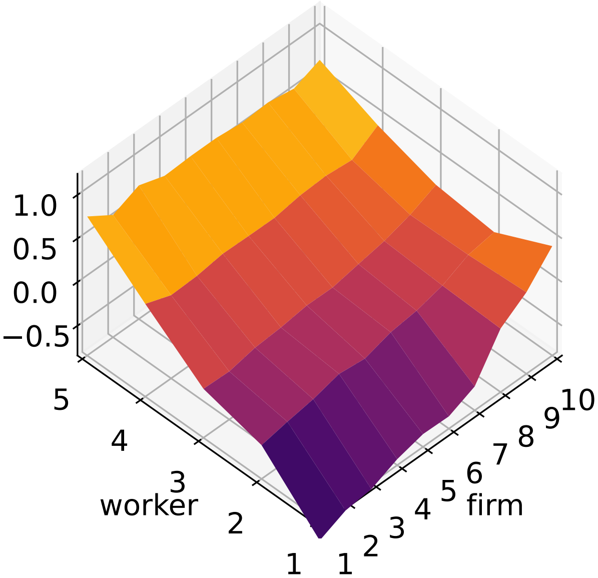
Compare: Surplus vs Wage Ranking



Production function $\log f(x, y)$



Disutility of labor $\tilde{c}(x, y) = c(x, y) + b(x)$



More

- Model fit to wage dynamics Wage Dynamics
- Quantitative role of preference shocks Role of preference shocks
- Stationary distribution: model and data Distributions: Model and Data

Sources of Wage Variation

$$\underbrace{Var(\log w)}_{\text{total}} = \underbrace{\mathbb{E}[Var(\log w|x)]}_{\text{within worker: } 31\%} + \underbrace{Var(\mathbb{E}[\log w|x])}_{\text{between worker: } 69\%}$$

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$$\underbrace{Var(\log w|x)}_{\text{within worker}} = \underbrace{\mathbb{E}[Var(\log w|R,x)|x]}_{\text{within } R} + \underbrace{Var(\mu_R|x)}_{\text{between R}}$$

where $\mu_R = \mathbb{E}[\log w|R,x]$

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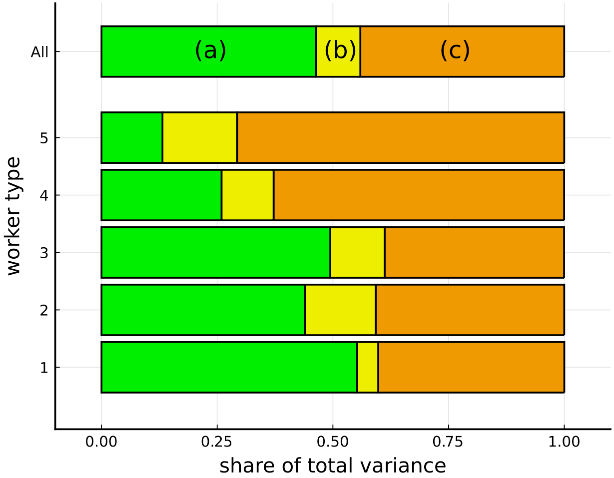
$$= \underbrace{\mathbb{E}[Var(\mathbb{E}[\log w|R,y,x]|R,x)|x]}_{\text{Compensating Differential (Rosen+PVR)}} + \underbrace{Var(\mathbb{E}(\mu_R|y,x)|x)}_{\text{Search Friction (BM)}} + \underbrace{\mathbb{E}[Var\,[\mu_R|y,x]\,|x}_{\text{Search Friction (PVR)}}$$

where $\mu_R = \mathbb{E}[\log w|R,x]$

Contributions to Within-Worker Wage Variance



(a) compensating differential (b) BM search friction (c) PVR search friction

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$$\underbrace{Var(\log w|x)}_{\text{within worker}} = \underbrace{\mathbb{E}[Var(\log w|R,x)|x]}_{\text{within } R} + \underbrace{Var(\mu_R|x)}_{\text{between R}}$$

$$= \underbrace{\mathbb{E}[Var(\mathbb{E}[\log w|R,y,x]|R,x)|x]}_{\text{Compensating Differential (Rosen+PVR)}} + \underbrace{Var(\mathbb{E}(\mu_R|y,x)|x)}_{\text{Search Friction (BM)}} + \underbrace{\mathbb{E}[Var\,[\mu_R|y,x]\,|x}_{\text{Search Friction (PVR)}}$$

$$\underbrace{Var(\mathbb{E}[\log w|x])}_{\text{between worker: 69\%}} = \underbrace{\mathbb{E}[Var(\mu_x|y)]}_{\text{within firm, between worker: 51\%}} + \underbrace{Var(\mathbb{E}[\mu_x|y])}_{\text{between firm, sorting } 18\%} .$$

where $\mu_R = \mathbb{E}[\log w|R,x]$ and $\mu_x = E[\log w|x]$.

The End

Quantitative role of preference shocks

- Preference shocks account for 19% of mobility (81% of moves would be the same without the preference shock)

- Preference shocks account for 14% of wage variance (fix realized shock to 0 for contacts when employed)

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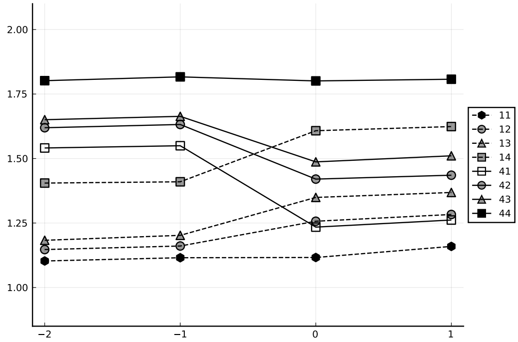
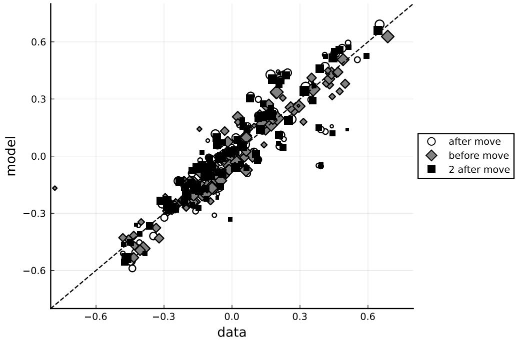
Model Fit to Wage Dynamics



(a) Movers event study

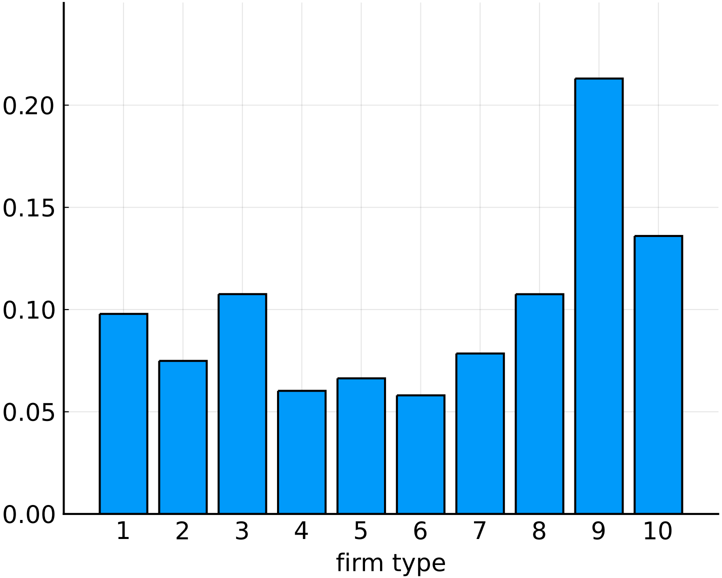


(b) Fit around the move

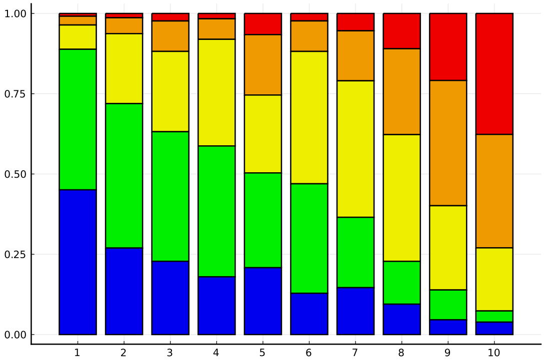
Figure: Event Study plot

Notes: Figure a) reproduces the event study plot from Card et al. (2013), it plots yearly earnings before and after a move for different firms, grouped by quartiles of earnings. Figure b) shows the fit of the model for annual earnings before and after a move for all pairs of firm types. Back

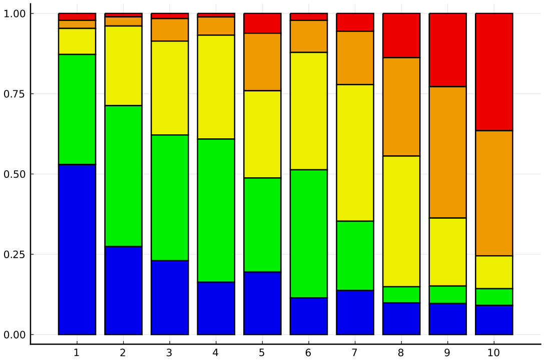
Vacancy distribution $v(y)$



Implied stationary distribution and the data



Model implied Stationary Distribution



Cross-section in Data

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The Value of a Vacancy

The annuity value for a firm with a vacancy of type y' is:

$$r\Pi_0(y) = \max_{B_0(x,\xi),B_1(x,\xi)} \frac{\lambda_0 L_0}{V} \iint \mathbf{1}_{\{B_0(x,\xi)+\xi>0\}} \left[\Pi_1(x,y,-\xi) - \Pi_0(y)\right] \frac{\ell_0(x)}{L_0} \, \mathrm{d}G_0(\xi) \, \mathrm{d}x$$
$$+ \lambda_1 \iiint \mathbf{1}_{\{B_1(x,\xi)+\xi>B'\}} \left[\Pi_1(x,y,B'-\xi) - \Pi_0(y)\right] \mathrm{d}F_1(B',x|\xi) \, \mathrm{d}G_1(\xi).$$

Model Summary

Worker Values

$$rW_0(x) = (1 + r)b(x).$$

$$\begin{aligned} W_1(x, y, w) = u(w) - c(x, y) + \frac{\delta(x, y)}{1 + r} W_0(x) + \frac{\overline{\delta}(x, y)\overline{\lambda}_1}{1 + r} \left(W_0(x) + R_0\right) \\ + \frac{\overline{\delta}(x, y)\lambda_1}{1 + r} \iint \Big[\mathbf{1}_{\{B(\xi) \geq B'^{+} + \xi\}} \big(W_0(x) + R_1(B', \xi)\big) \\ + \mathbf{1}_{\{B(\xi) < B'^{+} + \xi\}} \left(W_0(x) + \max\left\{B(\xi), \xi\right\}\right) \Big] \mathrm{d}F_0(B' | x, \xi)\, \mathrm{d}G_1(\xi). \end{aligned}$$

Model Summary

Stationary Distribution of Matches

In a stationary truth telling equilibrium the flows out of and into $\ell_1(x,y)$ are equal:

$$\ell_1(x,y)\left(\delta(x,y)+\overline{\delta}(x,y)\lambda_1\int\overline{G}_1\big(S(x,y){-}S(x,y')\big)v(y')\,\mathrm{d}y'\right)$$
$$=\lambda_0\ell_0(x)\frac{v(y)}{V}\overline{G}_0(-S(x,y))+\lambda_1\frac{v(y)}{V}\int\overline{G}_1\big(S(x,y'){-}S(x,y)\big)\overline{\delta}(x,y)\ell_1(x,y')\,\mathrm{d}y'.$$

With truth-telling the distribution of retention bids faced by firms are given by:

$$F_0(B'|x,\xi)=\int_{\mathbf{1}\{S(x,y')\leq B'\}}\frac{v(y')}{V}dy',$$
$$F_1(B',x|\xi)=\int_{\mathbf{1}\{S(x,y')\leq B'\}}\overline{\delta}(x,y')\frac{\ell_1(x,y')}{L_1}dy',$$

which simply states that incumbent firms draw outside bids from vacancies, vacancies draw from the cross-section of employed workers, and all firms bid truthfully.

Model Summary

A *stationary search equilibrium with sequential auctions*

is characterized by meeting probabilities λ_0 and λ_1; employment measure $\ell_1(x,y)$; bid distributions $F_0(B'|\xi,x), F_1(B'|\xi,x)$; firm value functions $\Pi_0(y), \Pi_1(x,y,R)$ and their respective policies such that:

1. The meeting probabilities λ_0, λ_1 are consistent with the meeting technology .
2. Taking F_0, F_1 as given, $\Pi_1(x,y,R)$ and $\Pi_0(y)$ solve the firm problem, which takes into account the mobility decisions of workers.
3. The policies of $\Pi_1(x,y,R)$ are $\Pi_0(y)$ are truth-telling: for firm y employing worker x, $B(\xi) = S(x,y)$ and for vacancy y, $B_0(x,\xi) = B_1(x,\xi) = S(x,y)$.
4. $F_0(B'|\xi,x), F_1(B'|\xi,x)$ are generated by the stationary distribution $\ell_1(x,y)$.

Model Summary

Equilibrium meeting probabilities

Let $M(L, V)$ be the number of meetings per period, where $L = L_0 + \kappa L_1$

Then

- $\lambda_0 = M/L$ is the probability an unemployed worker meets a vacancy
- $\lambda_1 = \kappa M/L$ is the probability an employed worker meets a vacancy
- $\lambda_0 L_0/V$ is the probability a vacancy meets an unemployed worker
- $\lambda_1 L_1/V$ is the probability a vacancy meets an employed worker

Model Summary

Equilibrium wage equation

$$
\begin{aligned}
u(w(x,y,R)) = c(x,y) + \frac{r+\delta(x,y)}{1+r}R + \frac{r}{1+r}W_0(x) \\
- \frac{\lambda_1(1-\delta(x,y))}{1+r} \int \Bigg[\int_{R-S(x,y')}^{S(x,y)-S(x,y')} \left(S(x,y') + \xi - R\right) g(\xi)\,\mathrm{d}\xi \\
+ \int_{S(x,y)-S(x,y')}^{\infty} \left(\max\{\xi, S(x,y)\} - R\right) g(\xi)\,\mathrm{d}\xi \Bigg] \frac{v(y')}{V}\,\mathrm{d}y'.
\end{aligned}
$$

Model Summary | Properties of Optimal Contract | Step 3

Surplus equation

The highest wage a firm will offer is given by:

$$\overline{w}(x,y) = f(x,y) - \frac{r}{1+r}\Pi_0(y).$$

At this wage, the worker is receiving all the surplus, and we can write:

$$\begin{aligned}(r+\delta(x,y))\,S(x,y) = (1+r)\left[u(\overline{w}(x,y)) - c(x,y)\right] \\ - rW_0(x) + \lambda_1(1-\delta(x,y))\int_{S(x,y)}^{\infty}\overline{G}_1(\xi)\ \mathrm{d}\xi.\end{aligned}$$

Model Summary Step 3

Equilibrium Values

Using the equilibrium offers by firms we have

$$rW_0(x) = (1+r)b(x) + \lambda_0 \int_0^{\infty} \overline{G}_0(\xi)\,\mathrm{d}\xi.$$

Using the equilibrium offers and properties of the optimal contract we have

$$\begin{aligned} r\Pi_0(y) = {} & \frac{\lambda_0 L_0}{V} \int_{S(x,y)>0} \int_{-S(x,y)}^{0} \frac{\overline{G}_0(\xi)}{u'\left(w(x,y,-\xi)\right)} \frac{\ell_0(x)}{L_0}\,\mathrm{d}\xi\,\mathrm{d}x \\ & + \frac{\lambda_1 L_1}{V} \iint_{S(x,y)>0} \int_{-S(x,y)}^{0} \frac{\overline{G}_1(\xi + S(x,y'))}{u'\left(w(x,y,-\xi)\right)} (1-\delta(x,y)) \frac{\ell_1(x,y')}{L_1}\,\mathrm{d}\xi\,\mathrm{d}x\,\mathrm{d}y', \end{aligned}$$

Step 4

Model: ... types

- $\ell(x)$ denotes the *exogenous* measure of type-x workers, with total measure 1
- $\ell_0(x)$ denotes the measure of unemployed workers of type x
- $L_0 = \int \ell_0(x)\,\mathrm{d}x$ denotes the total number of unemployed searchers

- $n(y)$ denotes the *exogenous* measure of type-y jobs, with total measure N
- $v(y)$ denotes the measure of type-y vacancies
- $V = \int v(y)\,\mathrm{d}y$ denotes the total number of vacancies

- $\ell_1(x, y)$ denotes the measure of matches of type (x, y)
- $\ell(x) = \ell_0(x) + \int \ell_1(x, y)\,\mathrm{d}y$ and $n(y) = v(y) + \int \ell_1(x, y)\,\mathrm{d}x$
- $L_1 = \iint \ell_1(x, y)\,\mathrm{d}x\,\mathrm{d}y$

back

Abowd, J. M., F. Kramarz, and D. N. Margolis (1999): "High wage workers and high wage firms," *Econometrica*, 67, 251–333.

An, Y., Y. Hu, and M. Shum (2013): "Identifiability and inference of hidden Markov models," Tech. rep., Technical report.

Arcidiacono, P., A. Gyetvai, E. Jardim, and A. Maurel (2022): "Identification and Estimation of Continuous-Time Job Search Models with Preference Shocks," Unpublished.

Arcidiacono, P. and R. A. Miller (2011): "Conditional choice probability estimation of dynamic discrete choice models with unobserved heterogeneity," *Econometrica*, 79, 1823–1867.

Bonhomme, S., T. Lamadon, and E. Manresa (2019): "A distributional framework for matched employer employee data," *Econometrica*, 87, 699–739.

Burdett, K. and D. T. Mortensen (1998): "Wage differentials, employer size, and unemployment," *International Economic Review*, 257–273.

Card, D., J. Heining, and P. Kline (2013): "Workplace heterogeneity and the rise of West German wage inequality," *The Quarterly journal of economics*, 128, 967–1015.

Eeckhout, J. and P. Kircher (2011): "Identifying sorting—in theory," *The Review of Economic Studies*, 78, 872–906.

Hagedorn, M., T. H. Law, and I. Manovskii (2017): "Identifying equilibrium models of labor market sorting," *Econometrica*, 85, 29–65.

Hotz, V. J. and R. A. Miller (1993): "Conditional choice probabilities and the estimation of dynamic models," *The Review of Economic Studies*, 60, 497–529.

Kasahara, H. and K. Shimotsu (2009): "Nonparametric identification of finite mixture models of dynamic discrete choices," *Econometrica*, 77, 135–175.

——— (2012): "Sequential estimation of structural models with a fixed point constraint," *Econometrica*, 80, 2303–2319.

Lamadon, T., M. Mogstad, and B. Setzler (2022): "Imperfect competition, compensating differentials, and rent sharing in the US labor market," *American Economic Review*, 112, 169–212.

Lentz, R. (2010): "Sorting by search intensity," *Journal of Economic Theory*, 145, 1436–1452.

Lentz, R., S. Piyapromdee, and J.-M. Robin (2023): "On worker and firm heterogeneity in wages and employment mobility: Evidence from danish register data," *Econometrica*.

Lindenlaub, I. and F. Postel-Vinay (2021): "The Worker-Job Surplus," Working Paper 28402, National Bureau of Economic Research.

Lise, J., C. Meghir, and J.-M. Robin (2016): "Matching, sorting and wages," *Review of Economic Dynamics*, 19, 63–87.

Lise, J. and F. Postel-Vinay (2020): "Multidimensional skills, sorting, and human capital accumulation," *American Economic Review*, 110, 2328–76.

Lopes de Melo, R. (2018): "Firm wage differentials and labor market sorting: Reconciling theory and evidence," *Journal of Political Economy*, 126, 313–346.

Mortensen, D. (2003): *Wage dispersion: why are similar workers paid differently?*, MIT press.

Postel-Vinay, F. and J.-M. Robin (2002): "Equilibrium wage dispersion with worker and employer heterogeneity," *Econometrica*, 70, 2295–2350.

Shimer, R. and L. Smith (2000): "Assortative Matching and Search," *Econometrica*, 68, 343–369.

Sorkin, I. (2018): "Ranking firms using revealed preference," *The Quarterly Journal of Economics*, 133, 1331–1393.