

# Short- and Long-run dynamics in Electricity Markets using MIP tools

DSE Summer School

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# Outline

## I. Models of electricity markets

- Reducing complexity via dimension-reduction techniques
- Handling dynamics in the electricity sector

## II. Two teasers

- Gonzales, Ito, and Reguant (2023)
- Gowrisankaran, Langer, and Reguant (2024)

## III. Computational tools for short-run dynamics

- Clearing in electricity markets
- Modeling with JuMP (Julia)
- Models with complementarities

# I. Models of electricity markets

# Electricity markets are becoming a key aspect of decarbonizing our economies

- Need to reduce greenhouse gases (GHGs) to near zero.
- The electricity sector ( $\approx 35\text{-}40\%$  of CO<sub>2</sub> emissions) among the most active due to its larger potential to offer solutions.
- Several countries and states have an ambition of net-zero electricity by 2035.

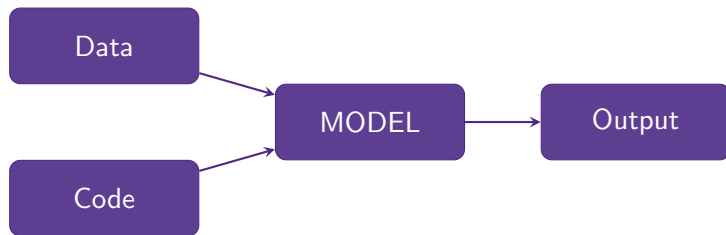
*How? Four (main) dimensions of change*

- Renewables + batteries
- Energy efficiency
- Demand response
- Electrification of other sectors, other forms of clean energy

# A straightforward problem statement but a highly complex system

- At its heart, all electricity market models have firms/technologies and information about demand (as a curve or fixed) to find the best allocation that ensures demand = supply (called **economic dispatch**).
- If the model takes into account discrete decisions about which power plants to turn on/off, it is called a **unit commitment problem** (more difficult to solve) to which we might add **low-frequency investment decisions**.
- Depending on the question at hand, the electricity markets in economic analysis are modeled abstracting away from many features.
  - ▶ E.g., big long-run policy questions like climate policy might be answered with a simplified version of the market.
- Depending on the question, some more detailed features need to be brought back.
  - ▶ E.g., transmission congestion regarding renewable expansion.

# Building models of electricity markets



- Model used to simulate impact of alternative configurations, profitability of investments, impacts of climate policies, etc.
- Does output for baseline match data? If not, do we need to expand code?
  - ▶ Not always, keep an eye on things that are important to our question and that we might not be matching well. A model is a simplification of a complex reality.

# Complexity of the model depends on the question at hand

## Common elements and options

- Supply side
  - ▶ Competitive (cost curves) or strategic (firms max profit)
  - ▶ At tech, firm, or plant level
  - ▶ With or without geography (transmission, usually with direct current approximation)
  - ▶ With or without startup costs (non-convexities)
- Demand side
  - ▶ Inelastic or responsive
  - ▶ Granular or aggregated

## Horizon and temporal linkages

- Level of aggregation
  - ▶ Hourly, daily, etc.
- Links between hours
  - ▶ Every hour independent from each other vs. temporal linkages (important for storage or startup costs)
- Horizon of choice
  - ▶ Day-to-day operations
  - ▶ Seasonal water storage
  - ▶ Capacity expansion model (investment)

# Dimension-reduction techniques can be a first step

- Electricity markets are highly complex.
- Electrical engineers often work with representative cases to make their contributions comparable, but they have limited empirical relevance.
- When analyzing one market in detail with historical data, analysis can become slow.
- Slow computations can lead to limited sample sizes (e.g., three months) or limited counterfactual/econometric analysis (e.g., no standard errors, limited policy analysis).
- Machine learning techniques can be used to reduce the size of the data.



# Simplifying the data

- Key idea is to identify “representative hours” with some “weights” for how important each hour or location is.
- These representative hours can then be used in the model (together with the weights) to ensure that the model is representative (but runs much faster).
- *Note:* The hourly clustering is easiest, but it treats each hour as independent. Depending on the problem, clustering days or weeks might be better.
  - ▶ E.g., for a short-term battery problem, need to look at battery behavior for at least three days; for hydro, very difficult to cluster due to seasonal rains and long-term storage.

# Clustering of different dimensions

- Dimension reduction techniques can be used in many ways to reduce the computational demands of electricity market models.
- Today: application simplifies the time dimension.
- Other examples:
  - ▶ Types of consumers with highly dimensional smart-meter data (e.g., see Cahana et al, 2022).
  - ▶ Geographical granularity to simplify nodal market data (e.g., see Mercadal, 2021; Gonzales, Ito and Reguant, 2023).
  - ▶ Types of production units to simplify technologies in the model.
  - ▶ State space for long-run dynamics (e.g., see Gowrisankaran, Langer, and Reguant, 2024).

# Dynamics in electricity

Several dimensions involve dynamics:

- Startup of power plants (Reguant 2014; Cullen, 2015).
- Allocation of hydro resources (Crampes and Moreaux, 2001; Bushnell, 2003).
- Batteries (Karaduman, 2021; Butters, Dorsey and Gowrisankaran, 2022).
- Divestitures (Linn and MacCormack, 2019; Gowrisankaran, Langer, and Reguant, 2024).
- Renewable entry (Gonzales, Ito, and Reguant, 2023).

Implementation in each of the papers can be widely different from a technical perspective!

# Careful choices in dynamics are also necessary

In any sector, important to decide how dynamics are modeled:

- Level of aggregation: hourly, monthly, annual?
- One-shot vs. multiple periods?
- Stationary infinite horizon vs. finite horizon?
- Relevant dynamic variables vs. those that can be simplified?
- Strategic vs. competitive vs. social planner under dynamics? When are the last two equivalent?

# Breaking up short- and long-run

Several dimensions involve dynamics:

- Oftentimes modeling can be broken down into two nests:
  - ▶ Short-run hourly nest (with or without dynamics)
  - ▶ Long-run investment nest (with or without dynamics)

**Note:** Recursive vs. finite horizon formulation a question in both depending on complexity of state space, relevant horizon, and degree of “recursiveness”.

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# Overview of papers' methods

## Short-run Hourly Models

## Long-run Models

GIR (2023)

- Weekly/monthly models
- Water and ramping dynamics
- Stylized transmission model

- Static long-run zero profit condition
- (Fancier model for revisions but ultimately simple!)

GLR (2024)

- Annual “representative” model
- Ramping dynamics
- Utilization/total cost annual decision

- Three-year decisions on investments
- Some uncertainty on commodity prices
- Finite horizon with final recursive state

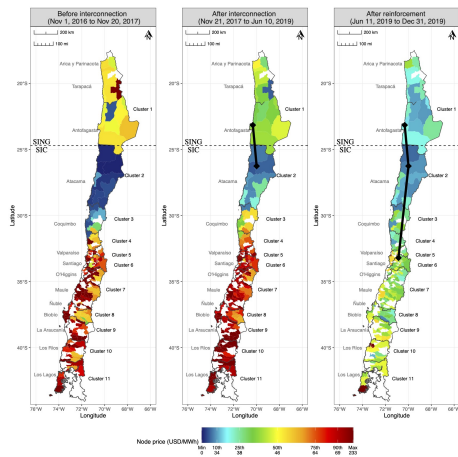


No uncertainty



# Gonzales, Ito, and Reguant (2023)

- Gonzales, Ito, and Reguant (2022) quantify the value of transmission infrastructure in Chile.
- **Question:** What is the cost benefit of the expansion project?
- **Tools:** event study + structural model of the Chilean electricity market.
- **Findings:**
  - ▶ We highlight the dynamic benefits of grid expansion, enabling increased renewable expansion.
  - ▶ The cost of transmission can be quickly recovered, even when ignoring the added climate change benefits.



# Static impacts: Event study effects of the line

$$c_t = \alpha_1 I_t + \alpha_2 R_t + \alpha_3 c_t^* + \alpha_4 X_t + \theta_m + u_t$$

- Our method uses insights from Cicala (2022)
  - ▶  $c_t$  is the observed cost
  - ▶  $c_t^*$  is the nationwide merit-order cost (least-possible dispatch cost under full trade in Chile)
  - ▶  $I_t = 1$  after the interconnection;  $R_t = 1$  after the reinforcement
  - ▶  $X_t$  is a set of control variables;  $\theta_t$  is month fixed effects
  - ▶  $\alpha_1$  and  $\alpha_2$  are the impacts of interconnection and reinforcement

## Static impacts: Event study effects of the line

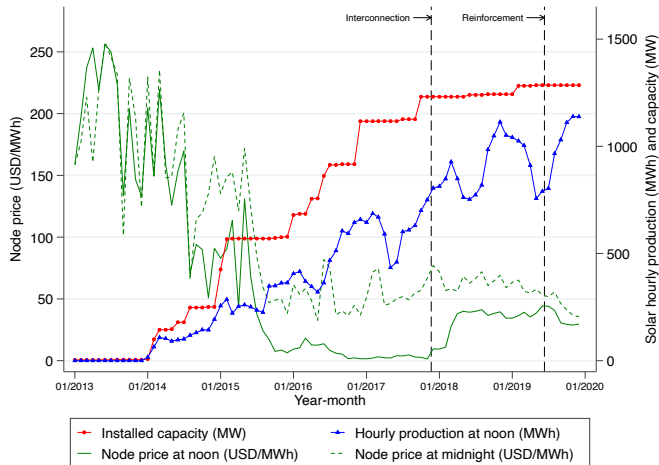
	Hour 12		All hours	
1(After the interconnection)	-2.42	(0.26)	-2.07	(0.17)
1(After the reinforcement)	-0.96	(0.58)	-0.61	(0.37)
Nationwide merit-order cost	1.12	(0.03)	1.03	(0.01)
Coal price [USD/ton]	-0.03	(0.01)	-0.01	(0.01)
Natural gas price [USD/m <sup>3</sup> ]	-10.36	(4.33)	-0.65	(3.09)
Hydro availability	0.43	(0.14)	0.00	(0.00)
Scheduled demand (GWh)	-0.51	(0.13)	-0.01	(0.00)
Sum of effects	-3.38		-2.68	
Mean of dependent variable	35.44		38.63	
Month FE	Yes		Yes	
Sample size	1033		1033	
R <sup>2</sup>	0.94		0.97	

# Does this static event study analysis get the full impact?

- Our theory based on the timing of investment effects:
  - ▶ Yes if solar investment occurs **simultaneously** with integration
  - ▶ No if solar investment occurs in **anticipation** of integration

# Solar investment occurred in anticipation of integration

- Investment occurred in the anticipation of the profitable environment
- [→] Static analysis does not capture the full impact of market integration
- [→] We address this challenge with a structural model

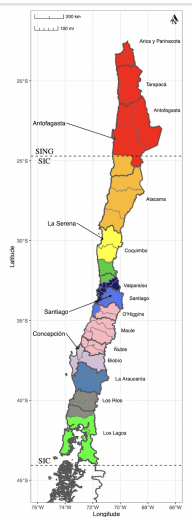


# Buidling a model to get at the full effect

- Impacts of the grid can be static and dynamic:
  - ▶ Production benefits: more solar can be sent to the demand centers, prices in solar regions go up.
  - ▶ Investment benefits: more solar power is built.
- We highlight that an event study is likely to capture only the first kind of effects (e.g., around time of expansion).
- We build a model of the Chilean electricity market to quantify the benefits of market integration including its investment effects.

# A structural model to study a dynamic effect on investment

- We divide the Chilean market to five regional markets with interconnections between regions (now expanding to 11)
- Model solves constrained optimization to find optimal dispatch that minimizes generation cost
- Constraints:
  - 1 Hourly demand = (hourly supply - transmission loss)
  - 2 Supply function is based on plant-level hourly cost data
  - 3 Demand is based on node-level hourly demand data
  - 4 Transmission capacity between regions:
    - ▶ Actual transmission capacity in each time period
    - ▶ Counterfactual: As if Chile did not integrate markets



# The structural model solves this constrained optimization

$$\begin{aligned} & \underset{q_{it} \geq 0}{\text{Min}} \quad C_t = \sum_{i \in I} c_{it} q_{it}, \\ \text{s.t.} \quad & \sum_{i \in I} q_{it} - L_t = D_t, \quad q_{it} \leq k_i, \quad f_r \leq F_r. \end{aligned} \tag{1}$$

## ■ Variables:

- ▶  $C_t$ : total system-wise generation cost at time  $t \in T$
- ▶  $c_{it}$ : marginal cost of generation for plant  $i \in I$  at time  $t$
- ▶  $q_{it}$ : dispatched quantify of generation at plant  $i$
- ▶  $L_t$ : Transmission loss of electricity
- ▶  $D_t$ : total demand
- ▶  $k_i$ : the plant's capacity of generation
- ▶  $f_r$ : inter-regional trade flow with transmission capacity  $F_r$



## Dynamic responses are solved as a zero-profit condition

$$E \left[ \sum_{t \in T} \left( \frac{p_{it}(k_i) q_{it}(k_i)}{(1+r)^t} \right) \right] = \rho k_i \quad (2)$$

■ where:

- ▶ NPV of profit (left hand side) = Investment cost (right hand side)
- ▶  $\rho$ : solar investment cost per generation capacity (USD/MW)
- ▶  $k_i$ : generation capacity (MW) for plant  $i$
- ▶  $p_{it}$ : market clearing price at time  $t$
- ▶  $q_{it}$ : dispatched quantify of generation at plant  $i$
- ▶  $r$ : discount rate

■ This allows us to solve for the profitable level of entry for each scenario

# We calibrate the model with detailed market data

- Network model

- ▶ k-means clustering of province prices into 5 zones, observed flows between clusters to set transmission.

- Supply curve:

- ▶ based on observed production and/or observed reported costs.

- Demand:

- ▶ based on nodal level data, aggregated to clusters.

- Solar potential:

- ▶ based on days without transmission congestion.

- Cost of solar:

- ▶ based on zero profit condition.

# The cost and benefit of the transmission investments

- Cost of the interconnection and reinforcement

- ▶ \$860 million and \$1,000 million (Raby, 2016; Isa-Interchile, 2022)

- Benefit—we focus on three benefit measures

- ▶ Changes in consumer surplus
- ▶ Changes in net solar revenue (= revenue – investment cost)
- ▶ Changes in environmental externalities

# Cost-benefit results

**Table:** Cost-Benefit Analysis of Transmission Investments

	(1)	(2)
<b>Modelling assumptions</b>		
Investment effect due to lack of integration	No	Yes
<b>Benefits from market integration (million USD/year)</b>		
Savings in consumer cost	176.3	287.6
Savings in generation cost	73.4	218.7
Savings from reduced environmental externality	-161.4	249.4
Increase in solar revenue	110.7	183.5
<b>Costs from market integration (million USD)</b>		
Construction cost of transmission lines	1860	1860
Cost of additional solar investment	0	2522
<b>Years to have benefits exceed costs</b>		
With discount rate = 0	14.8	6.1
With discount rate = 5.83%	> 25	7.2
With discount rate = 10%	> 25	8.4
<b>Internal rate of return</b>		
Lifespan of transmission lines = 50 years	6.95%	19.67%
Lifespan of transmission lines = 100 years	7.23%	19.67%

# Assessing the cost-benefit

- With the model, we can compute the benefits of the line, with and without investment effects.
- We find that investment effects are key to justify the cost of the line.
- The line was also very attractive from a consumer welfare perspective, even at 5.83% discount rate (Chile's official rate).
- Political economy makes renewable expansion “easy” in Chile.
- How to reduce political economy challenges in other jurisdictions?

- Gowrisankaran, Langer, and Reguant (2024) quantify the interaction between regulatory distortions and the phase out of coal generators.
- **Question:** What is the scope for regulatory changes to speed-up phase-out of coal towards more efficient levels?
- **Tools:** structural model of regulatory distortions with dynamic investment.
- **Findings:**
  - ▶ We quantify that regulatory distortions delay coal phase-out.
  - ▶ Tweaking regulatory parameters has limited benefits compared to more direct policies (e.g., taxing carbon).

# Conceptual Model of Regulatory Incentives

- 1 Regulator uses prudence standards to limit incentive for over-investment.
  - ▶ For coal, utility demonstrates prudence by using it to meet load.
  - ▶ This limits capital but doesn't fully correct the AJ incentive.
- 2 Utility still doesn't have the incentive to generate with the lowest cost technologies.
  - ▶ Regulator therefore sets a maximum rate of return that is decreasing with  $TVC$ .
  - ▶ Incentivize utility (but imperfectly) to use lowest cost technology.
- 3 If a new technology suddenly becomes available:
  - ▶ AJ incentive implies that utility keeps too much of the legacy technology.
  - ▶ Prudence incentive leads to over-use of the legacy technology.
  - ▶ This may slow an energy transition.

# Model of Maximum Rate of Return and Rate Base

- In each year,  $y$ , regulator allows a maximum rate of return,  $\bar{s}_y$ , on the rate base,  $B_y$ , of:

$$\bar{s}_y = \left( \frac{TVC_y}{CostBasis} \right)^{-\gamma}$$

- ▶ Incentivizes low costs for  $\gamma > 0$ .
- ▶ *CostBasis*: observable fuel and import costs before the energy transition.
  - ▶ Used to capture unavoidable costs, such as transmission costs.
- The utility earns this rate of return on its rate base,  $B_y$ :

$$B_y = \alpha \left[ K_y^{CCNG} + \alpha^{NGT} K_y^{NGT} + \alpha^{COAL} \left( \frac{\exp(\mu_1 + \mu_2 U_y)}{1 + \exp(\mu_1 + \mu_2 U_y)} \right) K_y^{COAL} \right].$$

- ▶ We model coal usage  $U_y = \bar{Q}^{COAL} / K^{COAL}$  as affecting the rate base.
- Regulator sets consumer rates such that  $Revenues_y = TVC_y + \bar{s}_y \times B_y$ .



# Long-Run Retirement and Investment Decisions

- A utility facing this regulatory framework makes investment and retirement decisions every 3-year period,  $t$ , over 30 years, with 95% annual discount factor.
- Utilities choose coal retirement  $x_t^{COAL} \leq 0$  and CCNG investment,  $x_t^{CCNG} \geq 0$ .
- Investment costs build on Ryan (2012) and Fowlie, Reguant, and Ryan (2016):

$$\delta_0^f \mathbb{1}\{x_t^f \neq 0\} + x_t^f (\delta_1^f + x_t^f \delta_2^f + \sigma^f \varepsilon_t^f).$$

- Unobservable component is on linear marginal cost term:
  - ▶ Allows for a non-singleton density of  $x_t^f$  (Kalouptsi, 2018; Caoui, 2023).
  - ▶ Each  $\varepsilon_t^f$  is distributed standard normal and observed before the  $x_t^f$  choice.

# State and Timing for Investment/Retirement Decisions

- Investment and retirement decisions depend on:

- 1 Natural gas fuel price  $p_t^{NG}$ , which follows an exogenous AR(1) process.
- 2 Coal and CCNG capacity, which evolve endogenously.
- 3 Heat rates, coal fuel prices, demand, import supply curves, NGT capacity.

- Timing within each period is:

- 1 Utility learns  $p_t^{NG}$  and makes its investment/retirement decisions
- 2 Earns period profits,  $\pi^*(K^{COAL}, K^{CCNG}, p^{NG})$  from operations decisions
- 3 Realizes its retirements and investments

# Hourly Operations Decisions

- Every hour,  $h$ , of year,  $y$ , the utility meets load with generation or imports:

$$\pi^*(K^{COAL}, K^{CCNG}, p^{NG}) = \max_{\vec{q}_y} \overbrace{\left( \frac{TVC(p^{NG}, \vec{q}_y)}{CostBasis} \right)^{-\gamma}}^{\text{Rate of return}} \overbrace{B(\vec{q}_y, K^{COAL}, K^{CCNG})}^{\text{Rate base}}$$

- ▶ subject to meeting load and capacity constraints.
- Total variable costs  $TVC_y$  include import, fuel, startup/ramping, and O&M costs.
- Hours are connected via ramping costs, rate of return, and annual coal usage.
  - ▶ We solve for the optimum with a full-information finite horizon model.

# Structural Estimation

- 1 Estimate import supply curves following Bushnell, Mansur, and Saravia (2008).
- 2 Estimate most structural parameters from utilities' hourly operations decisions:
  - ▶ Use indirect inference: GMM nested fixed-point approach
  - ▶ Finds parameters to match data correlations similar to reduced form evidence.
- 3 Estimate investment and retirement costs from dynamic decisions.
  - ▶ Also GMM full solution nested fixed-point approach.
  - ▶ Annual operating profits at each state are inputs to Bellman equation.
  - ▶ Moments capture differences between model and data investment/retirement.
  - ▶ Apply Gowrisankaran and Schmidt-Dengler (2024) algorithm:
    - ▶ Idea: find  $\varepsilon^f$  cutoffs for chosen investment levels while eliminating others.

## Coefficient Estimates for Investment/Retirement Decisions

Parameter	Notation	Value	Std. Dev.
Fixed cost of coal retirement $\times 1e2$	$\delta_0^{COAL}$	-0.446	(9.79)
Linear coal cost per MW	$\delta_1^{COAL}$	3.196	(0.44)
Quadratic coal cost per MW / $1e3$	$\delta_2^{COAL}$	0.117	(0.02)
Coal shock standard deviation per MW	$\sigma^{COAL}$	-0.430	(0.02)
Fixed cost of CCNG investment $\times 1e2$	$\delta_0^{CCNG}$	-0.509	(0.01)
Linear CCNG cost per MW	$\delta_1^{CCNG}$	6.487	(0.08)
Quadratic CCNG cost per MW / $1e3$	$\delta_2^{CCNG}$	0.270	(0.05)
CCNG shock standard deviation per MW	$\sigma^{CCNG}$	-1.671	(0.06)

Note: All values in millions of 2006 dollars.

# Counterfactuals examine value of regulatory changes

- Modifying the relevance of cost minimization incentives ( $\gamma$ , TVC penalty).
- Reducing the incentive to overutilize coal ( $\mu_1, \mu_2$ ).
- Results:
  - ▶ Firms shift use between coal and gas.
  - ▶ Firms still maintain too much coal capacity and over-invest in gas, challenging the next energy transition.

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# Tools today

- Model of electricity market
  - ▶ Static, long-run investment
  - ▶ Using mixed-integer programming tools in Julia
- Clustering to simplify data
  - ▶ To reduce problem size
  - ▶ Using k-means

## Our example today

- Our example today will clear the market based on data from California (CAISO, see Reguant (2019)).
- We then need to solve for the objective function.

$$\begin{aligned} \max_q \quad & S(q) - C(q) \\ \text{s.t.} \quad & \text{demand} = \text{supply}, \\ & \text{other constraints.} \end{aligned}$$

- We solve for the quantities that maximize the gross surplus  $S$  minus the costs of generation  $C$ .
- Implicitly or explicitly, there is a price to electricity consumption.
- One can also clear for investment in the same optimization (social planner/competitive).

# Solving the model with JuMP

- JuMP makes the formulation of electricity dispatch models relatively seamless.
- One code to express the model, one can then call several solvers depending on the needs.
- I will give you a “hint” of what JuMP can do.
- Example of highly configurable electricity expansion model based on Julia + JuMP:
  - ▶ <https://github.com/GenXProject/GenX>
  - ▶ <https://netzeroamerica.princeton.edu/>

# Ingredients to a mathematical model

- Parameters/Inputs
- Variables
- Constraints
- Objective function
- Sense of the objective function
- The solver we want to use

Note: In mathematical programming, the terms 'variables' and 'parameters' are used the opposite way as in econometrics! Variables: what we are trying to solve. Parameters: what we already have, the inputs.

- There is an **array of optimization resources** that are tailored to be particularly efficient in certain problems.
- Developed/used more in engineering and operations research.
- Examples:
  - ▶ Quadratic programs
  - ▶ Linear programs with integer variables
  - ▶ Nonlinear programs with integer variables
  - ▶ Programs with complementary conditions

# Building blocks of the model

following Bushnell (2010)

- Model with perfect competition and free entry.
- Continuous investment in different technologies.
- Equivalent to least-cost **social planner outcome**.
- Entry of each technologies occurs until revenues of the marginal unit equal levelized costs of investment and operating costs.
- Assess long-run generation mix (coal, CCGT, peaking gas).
- Focus on thermal generation.

# Model equations solution

The model equations are as follows

- **Demand** (we will assume this to be linear)

$$Q_t(p_t) = a_t - f(p_t)$$

- **Quantity**

$$q_{it} \geq 0 \perp p_t - c_i - \psi_{it} \leq 0 \quad \forall i, t$$

- **Shadow**

$$\psi_{it} \geq 0 \perp q_{it} - K_i \leq 0 \quad \forall i, t$$

- **Zero Profit**

$$K_i \geq 0 \perp F_i - \sum_t \psi_{it} \leq 0 \quad \forall i$$

The model is a complementarity problem. To solve these problems one can use special software or do it “brute force”.

# Complementary conditions formulation

- We can think of each complementarity condition as the product of two variables.
- We want to minimize the objective function and make sure it is zero subject to the constraints of  $z$  and  $w$  being non-negative:

$$\begin{aligned} \min \quad & z'w \\ \text{s.t.} \quad & z \geq 0, w \geq 0 \end{aligned}$$

- We need to check objective function is zero.
- Special solvers are tailored to solve these problems, such as PATH.



# Mixed-integer programming

- We refer to mixed-integer programming for problems that have both discrete and continuous variables that we are trying to solve for.
- In the last 10-15 years, this type of problems has become easier to solve.
- Electricity markets are an important application, as there are many discrete decisions:
  - ▶ Should we use a power plant or not?
  - ▶ Is a technology “marginal” or not?
  - ▶ Is a transmission line at capacity or not?
  - ▶ For piece-wise linear functions, at which side of the function should we be?
  - ▶ Etc.

# Mixed-integer formulation

- Mixed integer programs can be used very generally to express constraints or model discrete decisions.
- We can also use “tricks” to mimic Khun-Tucker conditions.
- For our complementarity-equivalent problem, we have:

$$\begin{aligned} z &\geq 0, w \geq 0 \\ z &\leq M \cdot u, w \leq M \cdot (1 - u) \end{aligned}$$

- $u$  is a binary variable that sets condition on or off.
- Either  $z$  is zero or  $w$  is zero.
- $M$  is a large number (convention to call it ‘M’, literally a big number).

# Interested in these topics?

- See materials at <https://mreguant.github.io/em-course>.