Estimation of a Roy/Search/Compensating Differential Model of the Labor Market (ECMA, 2020)

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Four of the most important models for determining job choice and wage dynamics in labor economics are

- Roy model
- Search Model
- Compensating Differentials
- Human Capital

There is not much of a literature distinguishing between them.

Our goal is to write down a model incorporating all four of these and to use it to think about the relative contributions to earnings inequality

- We take "Roy Model" to mean heterogeneity in pre-market skill levels that varies across individuals and jobs. Workers choose occupation based on comparative advantage
- By "Compensating Differencials" we mean that that workers choose jobs that they "like." With identical skills and job choices may earn different wages because they choose different jobs. Requires:
 - workers care about job characteristics other than wages
 - workers have heterogeneity in these choices
 - We take these as given-we have in mind how much you enjoy the actual job not how much you enjoy your office or health benefits
- By "Search" we mean that workers can not always work at their preferred job but need to wait for an offer
- We incorporate human capital by allowing wages to go up with general experience (learning by doing)

The main goal of this work is to

- Write down models that incorporate Roy Model skill, search frictions, learning by doing and non-pecuniary tastes for jobs
- Establish non-parametric identification of (parts of) the model
- Estimate the model
- Use it to see how the various features contribute to Earnings Inequality

We are also not trying to write down the most complicated model of the labor market possible, rather we are trying to write down the simplest model that gets at the essence of the goals above

The Model

Our model has endogenous wage determination in which workers and firms bargain over wages

Determination of entry and exit of firms as well as provision of nonpecuniary aspects is not modeled here and taken as exogenous-thinking more about firm behavior would be an interesting and important extension

Highlights

- Finite number of types of jobs one can take (*J*)
- ψ_h level of human capital h = 0, ..., H
- Human capital stochastically evolves $(h \rightarrow h+1)$ with a Poisson arrival rate
- π_{ij} is the (net) productivity of worker i's human capital at firm type j
- ullet Wages paid in efficiency units of human capital so I am paid $R\psi_h$
- the flow value that individual i has for job j as

$$u_{ij}(R\psi_h)$$
.



We take time as continuous with

- δ_i : job destruction rate
- ullet λ_i^e : job arrival rate for a worker employed at another firm
- λ_i^n : job arrival rate for a non-employed worker
- λ_h : arrival rate of human capital accumulation (1.0 in practice)
- P* Probability of an offer immediately after job destroyed

Let $V_{ijh}(R)$ be the value function for worker i, with human capital ψ_h at firm j at wage R.

 V_{i0h} is value function for a nonemployed worker

 V_{i0h}^* is immediately after job destruction (incorporates probability of immediate offer)

Clearly we are abstracting from many things that can be added later

However, we think this gets at the core of what we care about

- The parameter λ_i^e picks up search frictions
- The distinction between $u_{ij}(R\psi_h)$ and $R\psi_h$ picks up compensating differentials
- Variation in wages across individuals but within jobs picks up Roy model heterogeneity (after accounting for bargaining)
- increase in h picks up on the job learning

Given the model there are many ways to decompose earnings inequality.

Wage determination

This is based on Cahuc, Postel-Vinay, and Robin (2006)

Main Components:

- Worker has bargaining power β
- Wage contracts lead to fixed hc rental rate unless the firm wants to offer a higher wage to respond to an outside offer
- Full information about worker tastes and productivity
- When worker gets offer from nonemployment
 - If there is surplus to the match the worker will work for the firm (i.e. there is a wage that the firm is willing to pay at which the worker prefers the job to nonemployment)
 - Worker and firms bargains over wage so wage is set to solve

$$V_{ijh}(R) = \beta V_{ijh}(\pi_{ij}) + (1 - \beta) V_{i0h}$$



- When employed worker gets an outside offer one of three things can happen
 - If surplus on new job is higher than surplus on older job, worker switches firm. New firm pays wage to solve

$$V_{i\ell h}(R) = \beta V_{i\ell h}(\pi_{i\ell}) + (1 - \beta) V_{ijh}(\pi_{ij})$$

- If surplus on new job is sufficiently low that maximum wage on offered job would be turned down for based on current wage, nothing happens.
- If surplus on new job is higher than this, but lower than current job, wages are renegotiated. Current firm offers wage to solve

$$V_{ijh}(R) = \beta V_{ijh}(\pi_{ij}) + (1 - \beta) V_{i\ell h}(\pi_{i\ell})$$

When h < H,

$$\begin{split} &\left(\rho + \delta + \lambda_{h} + \Lambda_{ijh}^{e}(R)\right) V_{ijh}(R) \\ = & u_{ij}(R\psi_{h}) + \delta_{i} V_{i0h}^{*} + \lambda_{h} V_{ijh+1}(R) \\ &+ \sum_{\{\ell: V_{ijh}(R) < V_{i\ell h}(\pi_{i\ell}) \leq V_{ijh}(\pi_{ij})\}} \lambda_{\ell}^{e} \left[\beta V_{ijh}(\pi_{ij}) + (1-\beta) V_{i\ell h}(\pi_{i\ell})\right] \\ &+ \sum_{\{\ell: V_{i\ell h}(\pi_{i\ell}) > V_{ijh}(\pi_{ij})\}} \lambda_{\ell}^{e} \left[\beta V_{i\ell h}(\pi_{i\ell}) + (1-\beta) V_{ijh}(\pi_{ij})\right] \end{split}$$

with

$$\Lambda_{\mathit{ijh}}^{e}(R) \equiv \sum_{\{\ell: V_{\mathit{ijh}}(R) < V_{\mathit{i}\ell h}(\pi_{i\ell})\}} \lambda_{\ell}^{e}$$

(similar but simpler expression for V_{i0h} , V_{i0h}^* and $V_{iiH}(R)$).

Identification

A major goal of this work is to think carefully about identification of the parameters-and importantly what is not identified

Given the time, please trust me (or look at the paper) that almost everything is identified from

- rates of transition
- revealed preference
- panel data on wages for workers×firms

I want to focus on what is not identified

What is not identified

Problem 1: β

revealed preference reveals job ordering and wages in different states of the world, but not utility levels

As a result we can not identify β

However, we do know what happens if $\beta = 1$: that is maximum wage that a worker would ever receive at each job.

When we "normalize" the level of utility in our model as is standard in discrete choice models, β can be identified after this

It is important to keep in mind that β depends on this normalization, so it is odd to change model and hold β fixed

In our decompositions we get rid of monopsony power by sending $\beta=$ 1, we do this first

Problem 2: The selection problem

We can't observe all workers at all jobs, thus we can not say what their wage would be at all jobs

What you really need for full identification is extreme,

It is not just something that varies occupational choice holding wages constant-you need something like "identification at infinite"

This would allow you to identify for example the wages that Elon Musk would receive working at McDonalds

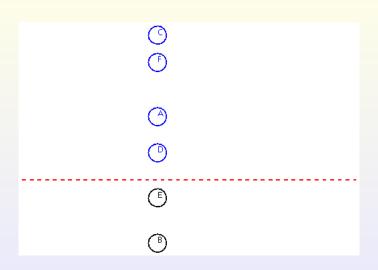
Its hard to imagine you could ever hope to identify this

At the same time, who cares?

It is hard to imagine any interesting counterfactual that would actually involve this

Our strategy here is to focus on identifying what can be identified

That is we will recognize this problem in the work-and as a result we will not simulate counter-factuals that involve these unidentified features



Econometric Specification

We assume that log productivity for worker *i* at firm *j* is

$$\log(\pi_{ij}) = \theta_i + \mu_j^w + v_{ij}^w$$

Flow value to firm is $\pi_{ij}\psi_h$

We observe log wages with i.i.d measurement error with variance σ_ξ^2

Workers utility is determined by

$$U_{ij}(R\psi_h) = \alpha \log(R\psi_h) + \mu_j^n + v_{ij}^n.$$

with θ_i independent of offered (μ_i^w, μ_i^u) independent of offered (v_{ij}^w, v_{ij}^n) .

Flow utility for non-employment is

$$U_{i0} = \alpha E_{\theta} + \gamma_{\theta} (\theta_i - E_{\theta}) + \nu_{i0}^n$$

We assume that the arrival and destruction rates are not heterogeneous so we estimate the three parameters $(\delta, \lambda^n, \lambda^e)$.

Human capital evolves as

$$\log(\psi_h) = b_1 h + b_2 h^2 + b_3 h^3$$

We use a cubic spline so there are two free parameters and we choose the third to impose that

$$\frac{\partial \log(\psi_H)}{\partial h} = 0$$

We fix $\lambda_h=1$



We parameterize the model with parametric functional forms

$$\begin{aligned} \theta_i &\sim N\left(E_{\theta}, \sigma_{\theta}^2\right) \\ \log \delta_i &\sim N\left(d_0, \sigma_{\zeta}^2\right) \\ \log \nu_{i0}^n &\sim N\left(0, \sigma_{\nu}^2\right) \\ \xi_{it} &\sim N\left(0, \sigma_{\xi}^2\right) \\ (v_{ij}^n, v_{ij}^w) &\sim N(0, \Sigma_{\nu}) \\ \mu_j^n &= f_1\left[U_1(j) + f_3U_2(j)\right] \\ \mu_j^w &= f_2\left[f_3U_1(j) + U_3(j)\right] \end{aligned}$$

We normalize $var(v_{ij}^n) = 1$ and $cov(v_{ij}^n, v_{ij}^w) = 0$

This gives us 18 parameters

- Transitions: $d_0, \sigma_{\zeta}, \lambda^n, \lambda^e, P^*$
- General Ability: E_{θ} , σ_{θ}
- Measurement Error: σ_{ξ}
- Reservation Utility: $\gamma_{\theta}, \sigma_{\nu}$
- Idiosyncratic Tastes and Productivity: σ_{V^W} , α
- Firm Tastes and Productivity: f_1 , f_2 , f_3
- Human Capital: b₁, b₂
- Bargaining Process: β

Computation

While there is quite a bit going on in this model, solving it is very easy-almost closed form.

For a given person there are a finite number of state of the world:

- H levels of human capital
- J types of employers (and non-employment)

Utilities across jobs are all known

When we numerically solve the model we begin with the highest level of human capital, h = H. We first show to solve the model at the terminal case and then how to iterate for the lower values.

For simplicity I am going to ignore involuntary job to job transitions (i.e. assume $P^* = 0$)

Solve Model with h = H

$$\begin{split} &\left(\rho + \delta_{i} + \Lambda^{e}_{ijH}(R)\right) V_{ijH}(R) \\ = & u_{ij}(R\psi_{H}) + \delta_{i} V_{i0H} \\ &+ \sum_{\{\ell: V_{ijH}(w) < V_{i\ell H}(\pi_{i\ell}) \leq V_{ijH}(\pi_{ij})\}} \lambda^{e}_{\ell} \left[\beta V_{ijH}(\pi_{ij}) + (1 - \beta) V_{i\ell H}(\pi_{i\ell})\right] \\ &+ \sum_{\{\ell: V_{i\ell H}(\pi_{i\ell}) > V_{ijH}(\pi_{ij})\}} \lambda^{e}_{\ell} \left[\beta V_{i\ell H}(\pi_{i\ell}) + (1 - \beta) V_{ijH}(\pi_{ij})\right] \end{split}$$

$$(\rho + \Lambda_{i0H}^n) \ V_{i0H} = u_{i0H} + \sum_{\{\ell : V_{i\ell H}(\pi_{i\ell}) > V_{i0H}\}} \lambda_j^n \left[\beta \ V_{i\ell H}(\pi_{i\ell}) + (1-\beta) \ V_{i0H}\right].$$

$$\Lambda_{ijH}^{e}(R) \equiv \sum_{\{\ell: V_{ijH}(R) < V_{i\ell H}(\pi_{i\ell})\}} \lambda_{\ell}^{e}, \Lambda_{i0H}^{n} \equiv \sum_{\{\ell: V_{i\ell H}(\pi_{i\ell}) > V_{i0H}\}} \lambda_{\ell}^{n},$$

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Order the firm types from lowest to highest in terms of

$$u_{ij}\left(\pi_{ij}\psi_H\right)$$

and notice that will also order the value functions $V_{ijH}(\pi_{ij})$ and thus the order of jobs people will choose.

There turns out to be a very useful notational change: For all j, define V_{ijH}^* implicitly by

$$V_{ijH}(\pi_{ij}\psi_H) = V_{ijH}^* + \frac{\delta_i V_{i0H}}{\rho + \delta_i}$$

To solve the model we start with the most preferred job (j = J)

$$(\rho + \delta_i) V_{iJH}(\pi_{iJ}) = u_{iJ}(\pi_{iJ}\psi_H) + \delta_i V_{i0H}$$

SO

$$V_{iJH}^* = \frac{u_{iJ}(\pi_{iJ}\psi_H)}{\rho + \delta_i}$$

Using algebraic manipulation we obtain the following equation

$$V_{ijH}^* = V_{ij+1H}^* - \frac{u_{ij+1}(\pi_{ij+1}\psi_H) - u_{ij}(\pi_{ij}\psi_H)}{\rho + \delta_i + \Lambda_{ijH}^e(\pi_{ij+1})\beta}$$

which allows to go backward from V_{ij+1H}^* to V_{ijH}^* and thus we can solve for all of the V_{ijH}^* in closed form.

Next consider the nonemployment equation.

Let ℓ_0 be the lowest utility firm from which the worker would except a job. First simplify the equations above to write

$$(\rho + \beta \Lambda_{i0H}^n) V_{i0H} = u_{i0H} + \sum_{j=\ell_0}^J \lambda_j^n \beta \left(V_{ijH} + \frac{\delta V_{i0H}}{\rho + \delta} \right)$$

For any ℓ_0 we can solve the equation above for $V_{i0H}(\ell_0)$.

We solve for the value of ℓ_0 that satisfies

$$V_{i\ell_0-1H}(\pi_{i\ell_0-1}\psi_H) < V_{i0H}(\ell_0) \le V_{i\ell_0H}(\pi_{i\ell}\psi_H).$$

Given this, to see how to calculate rental rates note that for someone hired out of Nonemployment to a firm of type j we solve for the value of R that solves

$$\begin{split} &\left(\rho + \delta_{i} + \Lambda_{ijh}^{e}(R)\right) \left[\beta V_{ijH}\left(\pi_{ij}\right) + (1 - \beta) V_{i0H}\right] \\ &= u_{ij}(R\psi_{H}) + \left(\sum_{\ell=\ell_{0}}^{j} \lambda_{\ell}^{e} \left[\beta V_{ijH}(\pi_{ij}) + (1 - \beta) V_{i\ell H}(\pi_{i\ell})\right]\right) \\ &+ \left(\sum_{\ell=j+1}^{J} \lambda_{\ell}^{e} \left[\beta V_{i\ell H}(\pi_{i\ell}) + (1 - \beta) V_{ijH}(\pi_{ij})\right]\right) + \delta_{i} V_{i0H}. \end{split}$$

With log utility this is closed form.

Similarly for worker hired at j with an outside offer from k with k < j we solve for the value of R for which

$$\begin{split} &\left(\rho + \delta_{i} + \Lambda_{ijh}^{e}(R)\right) \left[\beta V_{ijH}\left(\pi_{ij}\right) + (1 - \beta) V_{ikH}\left(\pi_{ik}\right)\right] \\ &= u_{ij}(R\psi_{H}) + \left(\sum_{\ell=k+1}^{j} \lambda_{\ell}^{e} \left[\beta V_{ijH}(\pi_{ij}) + (1 - \beta) V_{i\ell H}(\pi_{i\ell})\right]\right) \\ &+ \left(\sum_{\ell=j+1}^{J} \lambda_{\ell}^{e} \left[\beta V_{i\ell H}(\pi_{i\ell}) + (1 - \beta) V_{ijH}(\pi_{ij})\right]\right) + \delta_{i} V_{i0H}. \end{split}$$

Solve model for h < H

Basically works the same way just with

$$u_{ij}(\pi_{ij}\psi_h) + \lambda_h V_{ijh+1}(\pi_{ij})$$

instead of $u_{ij}(\pi_{ij}\psi_h)$

This makes calculation of *R* more complicated, but still is linear and can come directly in closed form.

Data

Matched worker/firm data from Denmark.

We observe the exact dates you hold the jobs and once a year we observe the wage.

To be in our sample you need:

- 1985-2003
- done with schooling
- younger than 56

For each parameter we will choose a statistic in the data to help identify it

In doing this we will use the identification discussion to guide us

In the identification model we first discussed how to identify turnover and then wages. We will use this strategy as well in thinking of an auxiliary model.

Auxiliary Details

Most are straight forward, three of note

- we use revealed preference for compensating differentials
- we use other people's revealed preference for common component of utilty
- we use coefficient on tenure² in spell fixed effect log wage regression to identify bargaining parameter

Identification Map

identification map			
Counterfactual	Structural Parameter	Auxiliary Parameter	
Learning	<i>b</i> ₁	Coefficient on Experience	
by Doing	b_2	Coefficient on Experience ²	
Monopsony	β	Coefficient on Tenure ²	
Premarket	$\sigma_{ heta}$	Variance Between Person	
Skills	$\sigma_{m{V}^p}$	Variance Between Jobs	
Nonpecuniary Tastes	α	Fraction Wage Drops at JtJ	
	f_{u}	$E(\widetilde{S}_{i\ell j}\widetilde{h}_{-i\ell j})$	
	$f_{u,p}$	$E(\widetilde{w}_{it}\widetilde{h}_{-it})$	
Search Frictions	λ^e	Average Length Job	
	λ^n	Average Length Nonnemp. Spell	
Measurement Error	σ_{ξ}	Variance Within Job	
	f_p	$E\left(\widetilde{w}_{it}\widetilde{w}_{-it}\right)$	
	μ_{δ}	Average Length Employment Spell	
	σ_{δ}	Var Employment Duration	
	extstyle ext	Sample Mean <i>w_{it}</i>	
	P *	Involuntary Job-to-Job	
	$\sigma_{ u}$	Var Nonemployment Duration	
	$\gamma_{ heta}$	$Cov(\overline{w}, non-empl dur)$	

Fit

Aux Parameter	Data (Std. Err.)	Model
Avg Length Emp. Spell	377 (0.193)	377
Avg Length Nonnemp. Spell	91.4 (0.094)	91.2
Avg Length Job Spell	106 (0.102)	106
Sample Mean <i>w_{it}</i>	4.50 (0.000)	4.50
Between Persons×100	8.03 (0.012)	8.00
Between Jobs×100	2.87 (0.006)	2.88
Within Job×100	1.49 (0.003)	1.49
$E(\widetilde{w}_{i\ell j}\widetilde{w}_{-i\ell j}) \times 100$	0.77 (0.004)	0.77
$E(\widetilde{r}_{-i\ell j}\widetilde{w}_{i\ell j}) \times 100$	0.69 (0.005)	0.69
$cov(\widetilde{r}_{-i\ell j},\widetilde{\widetilde{S}}_{i\ell j}) imes 100$	8.18 (0.013)	8.21
Fraction Wage Drops	0.400 (0.000)	0.392
Coeff Exper×100	2.48 (0.006)	2.47
Coeff Exper ² × 1000	-0.291 (0.001)	-0.292
Coef Tenure ² × 1000	-0.460 (0.003)	-0.460
Var(Nonemployment)	16000 (50.39)	16150
$Cov(\overline{w}_i, Non-employment)$	-3.42 (0.030)	-3.43
Var(Employment Dur)	102000 (72.82)	102666
Invol Job to Job	0.205 (0.011)	0.205

Parameter Estimates

Parameter	Description	Estimate	Standard Error
E_{θ}	Mean worker productivity	4.26	(0.001)
$\sigma_{ heta}$	Std. dev. of worker productivity	0.217	(0.001)
σ_{V^p}	Std. dev. of match productivity	0.211	(0.001)
α	Weight on log wage	3.575	(0.043)
β	Bargaining power	0.844	(800.0)
P^*	Probability of immediate offer upon job destruction	0.394	(0.019)
λ^n	Non-employment job offer arrival rate	0.989	(0.002)
λ^e	Employment job offer arrival rate	2.079	(0.011)
μ_{δ}	Mean of log job destruction distribution	-2.96	(0.026)
σ_{δ}	Std. dev. of log job destruction distribution	2.262	(0.009)
$b_1 \times 100$	Coefficient on linear term (human capital)	0.262	(0.103)
$b_2 \times 100$	Coefficient on quadratic term (human capital)	0.087	(0.006)
σ_{ξ}	Std. dev. of measurement error	0.139	(0.001)
f_u	Firm utility parameter	2.163	(0.169)
f_p	Firm productivity parameter	0.142	(0.003)
$f_{u,p} \times 100$	Firm utility×productivity parameter	0.467	(0.532)
$\sigma_{ u}$	Std. dev. of idiosyncratic non-employment utility	0.351	(0.012)
γ_{θ}	Worker ability contribution to flow utility	-0.282	(0.030)

Counterfactual Decomposition

In terms of the experiments:

- Eliminating human capital means people get maximum human capital immediately
- Eliminating monopsony power-set $\beta = 1$
- Eliminating pre-market skills means setting wages to the mean log wage by firm type (holding preferences for jobs constant)
- Eliminating Search means setting

$$\lambda^e \to \infty$$

 Eliminating nonpecuniary aspects means preference over wage only (conditional on preferring job to nonemployment)

Decompositions

(A)		(B)	
Total	0.104	Total	0.104
No Learning by Doing	0.096	No Learning by Doing	0.096
No Monopsony	0.093	No Monopsony	0.093
No Premarket Skill (CA)	0.050	No Search Frictions	0.086
No Premarket Skill	0.008	No Premarket Skill (CA)	0.049
No Search Frictions	0.007	No Premarket Skill	0.007
(C)		(D)	
Total	0.104	Total	0.104
No Learning by Doing	0.096	No Learning by Doing	0.096
No Monopsony	0.093	No Monopsony	0.093
No Non-pecuniary	0.087	No Non-pecuniary	0.087
No Premarket Skill (CA)	0.048	No Search Frictions	0.061
No Premarket Skill at All	0.006	No Premarket Skill (CA)	0.047

Understanding importance of Search

Workers are searching for four different things:

- firm specific wage
- firm specific nenpecuniary aspects
- individual x firm type productivity match
- individual×firm type utility match

In our different simulations above we changed which of these are being searched for and it matters a lot

Simulation	Searching For	Importance
(A)	1,2,4	1%
(B)	1,2,3,4	7%
(C)	1	6%
(D)	1,3	26%

Compensating Differentials and Search Frictions

Just because these are not the major contributors to earnings inequality does not mean they aren't important

Search Frictions

- Obviously important for turnover and unemployment-in our model they drive both
- Wages would be about 0.22 log points higher without search frictions (about 0.10 is the negotiation)

Non-pecuniary Tastes

- On turnover, roughly 1/3 of competing offers would change if people only cared about wages
- people earn on average 0.20 log points less to take jobs they like

Utility Decomposition

Another way to show the importance of search and compensating differentials is to do a "variance in utility" decomposition rather than just wages

Recall that

$$U_{ij}(R\psi_h) = \alpha log(R\psi_h) + \mu_j^n + v_{ij}^n.$$

To get this in the same units as log wages we can rescale simply by dividing by $\boldsymbol{\alpha}$

$$\widetilde{U}_{ij}(R\psi_h) = log(R\psi_h) + \left(\frac{\mu_j^n + v_{ij}^n}{\alpha}\right).$$

We then do things exactly as the decomposition above for comparison

Utility Decomposition

(A)		(B)	
Total	0.234	Total	0.234
No Learning by Doing	0.219	No Learning by Doing	0.219
No Monopsony	0.210	No Monopsony	0.210
No Premarket Skill (CA)	0.177	No Search Frictions	0.081
No Premarket Skill	0.143	No Premarket Skill (C)	0.088
No Search Frictions	0.047	No Premarket Skill	0.047
(C)		(D)	
Total	0.234	Total	0.234
No Learning by Doing	0.219	No Learning by Doing	0.219
No Monopsony	0.210	No Monopsony	0.210
No Nonpecuniary	0.087	No Nonpecuniary	0.087
No Premarket Skill (CA)	0.048	No Search Frictions	0.061
No Premarket Skill	0.006	No Premarket Skill (CA)	0.047

Conclusion

We have written down a framework that incorporates Human Capital, The Roy Model, Search, and Compensating Differentials.

While all four are important-Roy inequality is the most important for overall earnings inequality

This framework can be built on with many new features added

Employment Dynamics Parameters (before wages):

- d₀: Mean length of employment spells
- σ_{ζ} : Var. length of employment spells
- λ^n : Length of non-employment spells
- λ^e : Length of Job spells
- σ_{ν} : Var. nonemployment Duration
- P*: Fraction job-to-job that are involuntary
- $f_1 : E(\widetilde{S}_{i\ell j}\widetilde{h}_{-i\ell j})$

For auxiliary parameters involving Wages we have 11 parameters left:

$$E_{\theta}, \sigma_{\theta}, \sigma_{\xi}, var\left(v_{ij}^{w}\right), \alpha, f_{2}, f_{3}, b_{1}, b_{2}, \beta, \gamma_{\theta}$$

For $\sigma_{\theta}^2, \sigma_{\xi}^2, var\left(v_{ij}^w\right)$: We use the decomposition

$$\frac{\sum \left(w_{it}^{m} - \overline{w}\right)^{2}}{\sum T_{i\ell j}} = \frac{\sum \left(w_{i\ell jt}^{m} - \overline{w_{i\ell j}}\right)^{2}}{\sum T_{i\ell j}} + \frac{\sum \left(\overline{w_{i\ell j}} - \overline{w_{i}}\right)^{2}}{\sum T_{i\ell j}} + \frac{\sum \left(\overline{w_{i}} - \overline{w}\right)^{2}}{\sum T_{i\ell j}}$$

That is we use each of the three expressions on the right hand side

For the next 5 we use the moments:

- E_{θ} :Sample mean of w_{it}
- f_2 :E $(\widetilde{w}_{it}\widetilde{w}_{-it})$
- f_3 :E($\widetilde{w}_{it}\widetilde{h}_{-it}$)
- \bullet α : Fraction wage drops
- γ_{θ} : Cov(\overline{w} , non-empl dur)

That leaves three parameters: b_1 , b_2 , and β

We use a worker× match fixed effect regression:

$$\mathbf{w}_{i\ell jt} = \beta_{i\ell j} + \beta_1 \mathbf{E}_{i\ell jt} + \beta_2 \mathbf{E}_{i\ell jt}^2 + \beta_3 \mathsf{T} \mathbf{E}_{i\ell jt}^2 + \epsilon_{i\ell jt}$$

(the level of tenure and experience are perfectly collinear within a job spell)

Idea is that β determines the rate at which wages grow on the job.

From λ_e we know the rate at which new jobs come

The idea is that β_3 picks up the magnitude

