Marriage Matching with Search Friction: An Empirical Framework

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Introduction

- Interested in rationalizing the marriage distribution of 'who marries whom' by age.
- Develop an empirically tractable behavioral marriage matching model that allows for search friction in marriage.
- Propose an empirical version of the Shimer-Smith (2000) model.
- Empirically quantify the marital gains and search frictions across gender and age.
- What are relative importance of search friction and marital gains in marital decisions?

Introduction continues

- Our equilibrium marriage matching model delivers a new closed form matching function to the matching problem with search friction.
- Develop an empirical strategy that separately identifies marital surpluses and search frictions with cross-sectional data.
  - To identify search friction, we utilize the marriage duration (divorce) data. (A number of papers like ours uses this insight papers: notably Shin (2013), Ciscato (2023), Goussé, Jacquemet and Robin (2017))
- Apply our model to investigate how technology and social media has affected the marriage distribution in the United States from 2007/8 to 2017/18.
- Extends Choo and Siow(2006), Choo(2015), Chiappori, Salanié and Weiss (2017), and Galichon and Salanié (2022), among others.

Preview of Main Contributions

(1) Develop a model delivers a new closed-form marriage matching function with search friction.

(2) Propose an empirical strategy to separately identify marital gains and search frictions by using cross-sectional data.

(3) Apply our model to investigate how technology and social media has affected the marriage distribution in the United States from 2007/8 to 2017/18.

Contributions - *Marriage Matching Function with Search Friction*

- The model delivers a new closed-form marriage matching function in the stationary equilibrium,

$$\boldsymbol{\mu} = \mathcal{G}(\boldsymbol{\upsilon}, \boldsymbol{s}^m, \boldsymbol{s}^f; \boldsymbol{\Pi}, \boldsymbol{\rho}), \tag{1}$$

  - $\boldsymbol{\mu}$: equilibrium numbers of new marriages;
  - $\boldsymbol{\upsilon}$: endogenous equilibrium numbers of unsuccessful meetings;
  - $\boldsymbol{s}^m \equiv (s_k^m)_{k \in \mathcal{Z}}$ and $\boldsymbol{s}^f \equiv (s_k^f)_{k \in \mathcal{Z}}$, : endogenous equilibrium numbers of available single men and women;
  - $\boldsymbol{\Pi}$: exogenous marital surplus parameters;
  - $\boldsymbol{\rho}$: exogenous search friction parameters.

Contributions - *Identification*

- Separately identify marital gains and search frictions by using cross-sectional data.
  - With only matching data, it is challenging to disentangle the marital preferences and search frictions as both could affect marriage outcomes.
  - Most existing papers use panel data to identify matching model with search frictions

Contributions - *Empirical Application*

- Apply our model to investigate how technology and social media has affected the marriage distribution in the United States from 2007/8 to 2017/18.
  i) raised search cost among the young (21 to 31 years of age) while it lowered search cost among the old (older than 40).
  ii) lowered marital surplus among the young (21 to 31 years of age) while it raised surplus among the old.

Road Map

- Model: setup, decision problems, solving the model
- Stationary equilibrium
- Identification
- Empirical application
- Conclusion

Model setup - Model Environment

- Consider a stationary economy populated by overlapping generational adults who live for Z periods.
- Certain numbers of age one adult males and females are born each period.
- (i,j) denote male and female age.
- l_i^m denotes the population of age i adult males,
  the vector is denoted by $\boldsymbol{l}^m = (l_i^m)_{i\in\mathcal{Z}}$,
  the total male population is denoted by $L^m = \sum_{k\in\mathcal{Z}} l_k^m$. ($l_j^f$, $\boldsymbol{l}^f$ and L^f)
- s_i^m denotes the number of age i single males the vector $\boldsymbol{s}^m = (s_k^m)_{k\in\mathcal{Z}}$.
  The total single male, $S^m = \sum_{i\in\mathcal{Z}} s_i^m$, ($s_j^f$, $\boldsymbol{s}^f$ and S^f)
- $n(i,j)$ is the stock of (i,j) couples, the matrix $\boldsymbol{n} = (n(i,j)_{i,j\in\mathcal{Z}})$.
- $\boldsymbol{l}^m$ and $\boldsymbol{l}^f$, are predetermined, $\boldsymbol{s}^m$, $\boldsymbol{s}^f$, and $\boldsymbol{n}$, are equilibrium quantities endogenously determined in the model

Model setup - Search Technology

- The number of meetings between age i men and age j women,

$$m_{i,j} = \rho_{ij} \frac{s_i^m s_j^f}{S^m S^f} M(S^m, S^f), \tag{2}$$

which is the product of three components:

1. ρ_{ij}, the type-specific exogenous parameter capturing the search efficiency;
2. $\frac{s_i^m s_j^f}{S^m S^f}$, the fraction of the number of potential meetings between age i men and age j women to the total market-level potential meetings in one unit of time period;
3. $M(S^m, S^f)$, proportional to the total market-level meetings in one unit of time period, $M(S^m, S^f) = \sqrt{S^m S^f}$ following the literature.

Model setup - Search Technology: continues

- The rate that an age i man meets an age j woman is then given by:

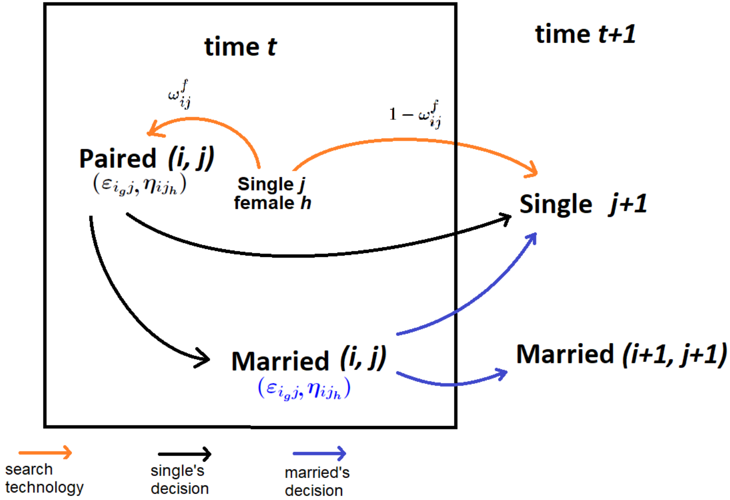
$$\omega_{ij}^m = \frac{m_{i,j}}{s_i^m} = \frac{\rho_{ij} s_j^f}{\sqrt{S^m S^f}}. \quad (3)$$

- A single individual is not guaranteed to meet someone of the opposite sex for sure. Let $\omega_{i0}^m = 1 - \sum_{j \in \mathcal{Z}} \omega_{ij}^m$ denotes the probability that a single age i man meets no one on the marriage market.
- Similarly, the rate that a single woman of age j meets a man of age i is given by:

$$\omega_{ij}^f = \frac{m_{i,j}}{s_j^f} = \frac{\rho_{ij} s_i^m}{\sqrt{S^m S^f}}, \quad (4)$$

- and $\omega_{0j}^f = 1 - \sum_{i \in \mathcal{Z}} \omega_{ij}^f$ denotes the probability that she meets no one.

Model setup - Time-line



Model Setup - Assumptions

Actions: Agents have binary actions. A married or paired couple of individuals g and h of (i, j) type have actions $a_{ijgh} \in \{0, 1\}$, where

- $a_{ijgh} = 1$ denotes the decision to marry (for paired couples) or remain married (for married couples), and
- $a_{ijgh} = 0$ denotes the decision to remain single (for paired couples) or divorce (for married couples).

Model does not differentiate between newly-paired partners and couples who got married in the previous periods and choose to remain married.

Exogenous Parameters:

- $\beta \in (0, 1)$, represents the discount factor,
- $\theta \in (0, 1)$ is the Nash bargaining solution, the bargaining power of men

Preferences:

- Π_{ij} denote (i,j) type couple per-period systematic marital gain.
- Endogenous per-period net utility that male g (or female h) receives from action a_{ijgh}: $u(a_{ijgh},(i,j),\varepsilon_{i_g j},\boldsymbol{\eta}_{ij_h})$ (or $w(a_{ijgh},(i,j),\varepsilon_{i_g j},\boldsymbol{\eta}_{ij_h})$).
- When couple marries, $a_{ijgh}=1$, the aggregate marital utilities is,

$$u(a_{ijgh}=1,(i,j),\varepsilon_{i_g j},\boldsymbol{\eta}_{ij_h})+w(a_{ijgh}=1,(i,j),\varepsilon_{i_g j},\boldsymbol{\eta}_{ij_h})$$
$$=\Pi_{ij}+\varepsilon_{i_g j,1}+\eta_{ij_h,1}.$$

- If the meeting was unsuccessful or the married couple decides to divorce, $a_{ijgh}=0$.
- Per-period systematic gain from remaining single or divorcing is normalized to zero.

$$u(a_{ijgh}=0,(i,j),\varepsilon_{i_g j},\boldsymbol{\eta}_{ij_h})=\varepsilon_{i_g j,0},\ \text{and}$$
$$w(a_{ijgh}=0,(i,j),\varepsilon_{i_g j},\boldsymbol{\eta}_{ij_h})=\eta_{ij_h,0}.$$

Model Setup - Assumptions continue

- Assume that preferences and the evolution of the state variables satisfy two assumptions:
- **Additive Separability (AS)** in utilities
  The sum of one-period utilities from marriage for incumbent or paired couples is additively separable in the mean utilities, Π_{ij}, and the sum of idiosyncratic shocks, $\varepsilon_{i_g j,1} + \eta_{ij_h,1}$.
- **Conditional Independence (CI):** the unobserved shocks are independent across periods.

DP: Married (or Paired) Individuals

- (i, j) type couple (g, h) makes a binary decision a that maximizes their life-cycle expected discounted utility. $U((i, j), \varepsilon_{i_g j}, \eta_{ij_h})$.
- This value function for age i man conditional on marrying or pairing with an age j woman takes the form,

$$
\begin{aligned}
U((i,j), \varepsilon_{i_g j}, \eta_{ij_h}) = \quad & \max \Big\{ u(a=1, (i,j), \varepsilon_{i_g j}, \eta_{ij_h}) \\
& + \beta \mathbb{E}\big[U((i+1, j+1), \varepsilon'_{i'_g j'}, \eta'_{i' j'_h}) | (i,j), \varepsilon_{i_g j}, \eta_{ij_h}, a=1 \big], \\
& u(a=0, (i,j), \varepsilon_{i_g j}, \eta_{ij_h}) \\
& + \beta \mathbb{E}\big[U((i+1, 0), \varepsilon'_{i+1_g 0}, (\eta'_{0k_{h'}})_k) | (i,j), \varepsilon_{i_g j}, \eta_{ij_h}, a=0 \big] \Big\}.
\end{aligned}
$$

DP: Single Individuals

- The value function for the single male g with state $((i,0),\varepsilon_{i_g 0})$ is then given by the Bellman equation,

$$U((i,0),\varepsilon_{i_g 0},(\boldsymbol{\eta}^{i}_{0j_h})_j) = \sum_k \omega^m_{ik} U((i,k),\varepsilon^{k}_{i_g 0},\boldsymbol{\eta}^{i}_{0k_h})$$
$$+\,\omega^m_{i0}\beta\mathbb{E}\big[U((i+1,0),\varepsilon'_{i+1_g 0},(\boldsymbol{\eta}'_{0j_{h'}})_j|(i,0),\varepsilon_{i_g 0},(\boldsymbol{\eta}^{i}_{0j_h})_j)\big].$$

Solving the model

- Assume that $(\varepsilon_{i_g j,1} + \eta_{ij_h,1})$ and $\varepsilon_{i_g j,0}/\theta$ are independently drawn from Type I Extreme Value distribution, the probability that male g of couple type (i, j) remains married this period is given by,

$$\mathcal{P}_{ij,1} = \frac{\exp[\Pi_{ij} + \beta C_{i+1,j+1} \cdot \mathbb{I}(i < Z, j < Z)]}{1 + \exp[\Pi_{ij} + \beta C_{i+1,j+1} \cdot \mathbb{I}(i < Z, j < Z)]}.$$

- The corresponding integrated value function for an age $i < Z$ male married to an age j female is given by,

$$\mathbb{U}_{i,j} = \theta[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{U}_{i+1,0} \cdot \mathbb{I}(i < Z),$$

- and the integrated value function for an age $i < Z$ single male is,

$$\mathbb{U}_{i,0} = \sum_k \omega^m_{ik} \theta[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{U}_{i+1,0} \cdot \mathbb{I}(i < Z).$$

Solving the model - continues

- Similarly the integrated value function for an age $j \in \mathcal{Z}$, married female is given by,

$$\mathbb{W}_{i,j} = (1-\theta)[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{W}_{i+1,0} \cdot \mathbb{I}(j < Z),$$

- and the corresponding integrated value function for a single age $j \in \mathcal{Z}$ female is given by

$$\mathbb{W}_{0,j} = \sum_k \omega^f_{kj}(1-\theta)[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{W}_{0,j+1} \cdot \mathbb{I}(j < Z).$$

Solving the Model: Dynamic Matching Function with Search Friction

- $\mu_{i,j}$ the flow of new (i,j) marriages, and $\upsilon_{i,j}$ the flow of unsuccessful paired (i,j) meetings. Using this to estimate the probabilities $\hat{\mathcal{P}}_{ij,1} = \frac{\mu_{i,j}}{m_{i,j}}$ and $\hat{\mathcal{P}}_{ij,0} = \frac{\upsilon_{i,j}}{m_{i,j}}$, substituting them into the log-odds ratio, we obtain the equilibrium matching function equation

$$\mu_{i,j} = \begin{cases} \exp(\kappa)\exp(\Pi_{ij})\upsilon_{i,j}\left(\frac{\upsilon_{i+\mathbf{1},j+\mathbf{1}}}{m_{i+\mathbf{1},j+\mathbf{1}}}\right)^{-\beta} \\ \qquad \times \prod_{k=1}^{Z}\left(\frac{\upsilon_{i+\mathbf{1},k}}{m_{i+\mathbf{1},k}}\right)^{\beta\theta\omega^m_{i+\mathbf{1}k}}\left(\frac{\upsilon_{k,j+\mathbf{1}}}{m_{k,j+\mathbf{1}}}\right)^{\beta(\mathbf{1}-\theta)\omega^f_{kj+\mathbf{1}}}, & \text{if } \ i<Z, j<Z, \\ \exp(\Pi_{ij})\upsilon_{i,j}, & \text{if } \ i=Z \text{ or } j=Z, \end{cases}$$

- where the term $\kappa \equiv \beta c - \beta\theta c \sum_{k=1}^{Z}\omega^m_{i+1k} - \beta(1-\theta)c\sum_{k=1}^{Z}\omega^f_{kj+1}$.

Stationary equilibrium

- In the stationary equilibrium, the inflow and outflow of married couples of each type must exactly balance each other.
- The inflow of (i, j)-type marriage is given by $m_{i,j}\mathcal{P}_{ij,1}$, where $m(i, j)$ is the number of (i, j)-type meetings, and $\mathcal{P}_{ij,1}$ is the marriage probability for an (i, j) paired partner.
- The outflow of (i, j)-type marriage is given by $n(i, j)(1 - \mathcal{P}_{ij,1})$, where $n(i, j)$ is the stock of (i, j)-type couples, and $(1 - \mathcal{P}_{ij,1})$ is the dissolution or divorce probability for an (i, j).

$$m_{i,j}\mathcal{P}_{ij,1} = n(i, j)(1 - \mathcal{P}_{ij,1}).$$

Stationary equilibrium: continue

- The stationary steady state stock of (i, j) marriages,

$$n(i,j) = m_{i,j} \frac{\mathcal{P}_{ij,1}}{(1-\mathcal{P}_{ij,1})} = \frac{\rho_{ij} s_i^m s_j^f}{\sqrt{S^m S^f}} \frac{\mathcal{P}_{ij,1}}{(1-\mathcal{P}_{ij,1})}. \tag{5}$$

- The following accounting balance conditions for each gender and type $i, j \in \mathcal{Z}$,

$$l_i^m = s_i^m + \sum_{j \in \mathcal{Z}} n(i,j), \tag{6}$$

$$l_j^f = s_j^f + \sum_{i \in \mathcal{Z}} n(i,j). \tag{7}$$

Stationary equilibrium: continues

A stationary marriage market equilibrium with search friction is defined by the tuple $(\boldsymbol{s}^m, \boldsymbol{s}^f, \boldsymbol{n}, \boldsymbol{U}, \boldsymbol{W}, \boldsymbol{\mathcal{P}})$, which comprises two vectors indicating the quantities of single males and females $(\boldsymbol{s}^m, \boldsymbol{s}^f)$, a matrix representing the stocks of marriages $\boldsymbol{n}$, two vectors that encapsulate the expected values for males and females within unions $(\boldsymbol{U}, \boldsymbol{W})$, and a matrix detailing the probabilities of opting for marriage $\boldsymbol{\mathcal{P}}$. In this equilibrium, the vectors $\boldsymbol{s}^m$ and $\boldsymbol{s}^f$ are solutions to the fixed-point equations defined by

$$l_i^m = s_i^m + \sum_{j\in\mathcal{Z}} \frac{\rho_{ij} s_i^m s_j^f}{\sqrt{S^m S^f}} \frac{\mathcal{P}_{ij,\mathbf{1}}}{(1-\mathcal{P}_{ij,\mathbf{1}})}, \text{ and}$$
$$l_j^f = s_j^f + \sum_{i\in\mathcal{Z}} \frac{\rho_{ij} s_i^m s_j^f}{\sqrt{S^m S^f}} \frac{\mathcal{P}_{ij,\mathbf{1}}}{(1-\mathcal{P}_{ij,\mathbf{1}})},$$

. The matrix $\boldsymbol{n}$ is given by the equation

$$n(i,j) = m_{i,j} \frac{\mathcal{P}_{ij,1}}{(1-\mathcal{P}_{ij,1})} = \frac{\rho_{ij} s_i^m s_j^f}{\sqrt{S^m S^f}} \frac{\mathcal{P}_{ij,1}}{(1-\mathcal{P}_{ij,1})}.$$

The value function matrices, $\boldsymbol{U}$ and $\boldsymbol{W}$, are determined by equations

$$\begin{aligned}
\mathbb{U}_{i,j} &= \theta[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{U}_{i+1,0} \cdot \mathbb{I}(i < Z), \\
\mathbb{U}_{i,0} &= \sum_k \omega^m_{ik} \theta[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{U}_{i+1,0} \cdot \mathbb{I}(i < Z). \\
\mathbb{W}_{i,j} &= (1-\theta)[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{W}_{i+1,0} \cdot \mathbb{I}(j < Z), \\
\mathbb{W}_{0,j} &= \sum_k \omega^f_{kj} (1-\theta)[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{W}_{0,j+1} \cdot \mathbb{I}(j < Z),
\end{aligned}$$

and the probabilities within $\boldsymbol{\mathcal{P}}$ are defined by equation

$$\mathcal{P}_{ij,1} = \frac{\exp[\Pi_{ij} + \beta C_{i+1,j+1} \cdot \mathbb{I}(i < Z, j < Z)]}{1 + \exp[\Pi_{ij} + \beta C_{i+1,j+1} \cdot \mathbb{I}(i < Z, j < Z)]}.$$

Identification

- Model primitives: search friction parameters $\boldsymbol{\rho}$ and the marital preference parameters $\boldsymbol{\Pi}$.
- Observables: the matching distribution, $\hat{\boldsymbol{\nu}} = (\hat{\boldsymbol{\mu}}, \hat{\mathbf{s}}^m, \hat{\mathbf{s}}^f)$, and the divorce rates $\hat{\boldsymbol{\delta}} = (\delta_{i,j})_{i,j\in\mathcal{Z}}$.
- Key idea: duration data identify the marital gains from marriages and then use observed matches identify search friction.

Identification continues

- Recall :

$$\mu_{ij} = m_{i,j}\mathcal{P}_{ij,1} = \frac{\rho_{i,j} s_i^m s_j^f}{\sqrt{S^m S^f}} \mathcal{P}_{ij,1},$$

- Assumption: married couples and paired couples face the same decision problems:

$$\hat{\mathcal{P}}_{ij,1} = 1 - \hat{\delta}_{i,j} \quad \text{and} \quad \hat{\mathcal{P}}_{ij,0} = \hat{\delta}_{i,j}. \tag{8}$$

- This allows us to identify

$$\hat{\rho}_{i,j} = \frac{\hat{\mu}_{i,j}}{1 - \hat{\delta}_{i,j}} \frac{\sqrt{\hat{S}^m \hat{S}^f}}{\hat{s}_i^m \hat{s}_j^f},$$

- With $\hat{\rho}_{i,j}$, we can identify the search probabilities,

$$\hat{\omega}_{ij}^{m} = \frac{\hat{\rho}_{ij}\hat{s}_j^f}{\sqrt{\hat{S}^m \hat{S}^f}}, \quad \text{and} \quad \hat{\omega}_{ij}^{f} = \frac{\hat{\rho}_{ij}\hat{s}_i^m}{\sqrt{\hat{S}^m \hat{S}^f}}.$$

and

$$\hat{\kappa} = \beta c - \beta\theta c \sum_{k=1}^{Z} \hat{\omega}_{i+1k}^{m} - \beta(1-\theta)c \sum_{k=1}^{Z} \hat{\omega}_{kj+1}^{f}, \tag{9}$$

- Rearranging the log-odd ratio gives us the identification equation of $\Pi_{i,j}$,

$$\hat{\Pi}_{ij} = \begin{cases} -\hat{\kappa} + \log \hat{\mathcal{P}}_{ij,1} - \log \hat{\mathcal{P}}_{ij,0} + \beta \log \hat{\mathcal{P}}_{i+1j+1,0} & \\ \quad -\beta \sum_{k=1}^{Z} \log[\hat{\mathcal{P}}_{i+1k,0}^{\hat{\omega}_{i+1k}^{m}\theta} \hat{\mathcal{P}}_{kj+1,0}^{\hat{\omega}_{kj+1}^{f}(1-\theta)}], & \text{if } \ i < Z, j < Z \\ \log \hat{\mathcal{P}}_{ij,1} - \log \hat{\mathcal{P}}_{ij,0}, & \text{if } \ i = Z \text{ or } j = \end{cases}$$

Empirical Application: Data Summary

- U.S. American Community Survey Data, 2007 and 2017

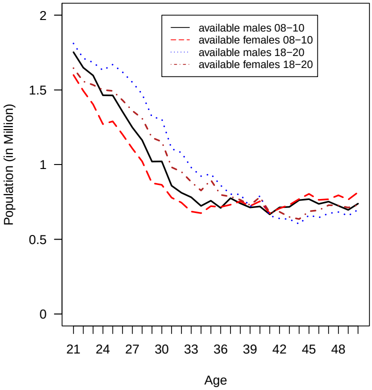
A. Available singles and stock of marrieds

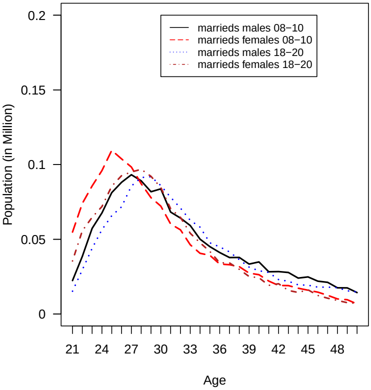
	2007	2017	Δ
Available men (million)	28.60	31.32	9.51%
Available women (million)	27.00	29.49	9.22%
Average age of single men	32.97	32.33	
Average age of single women	33.61	32.85	
Stock of marrieds (million)	26.86	23.57	-12.25%
Average age of married men	39.01	39.12	
Average age of married women	37.27	37.42	

B. New marrieds and divorces

	2008-10	2018-20	Δ
New marrieds (million)	1.40	1.33	-5.00%
Average age of newly married men	32.15	32.30	
Average age of newly married women	30.17	30.62	
Divorces (million)	1.64	1.14	-30.49%
Average age of divorced men	38.20	38.89	
Average age of divorced men	36.56	37.30	

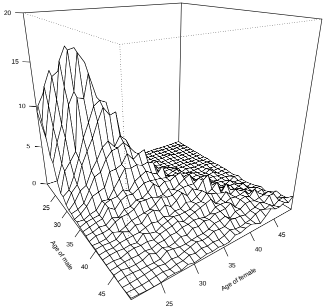
- Observed available singles and newly marrieds by age

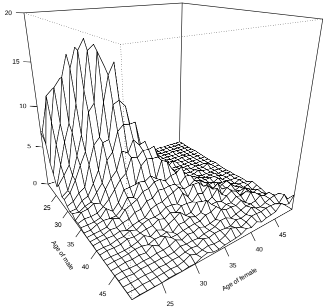




Empirical Application: continue

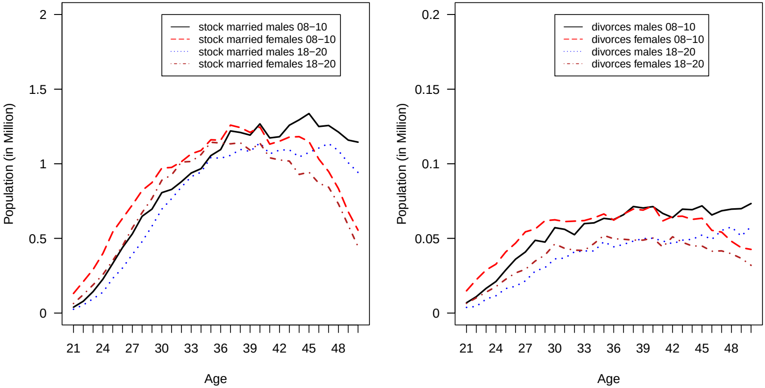
- Surface of observed μ_{ij} in thousand





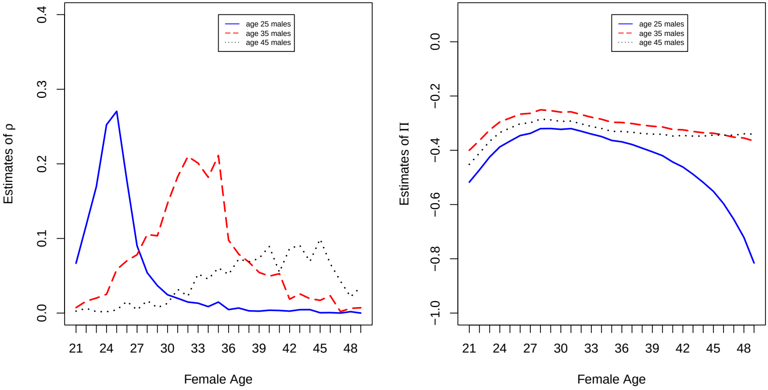
Empirical Application: continue

- Observed stock of marrieds and divorces



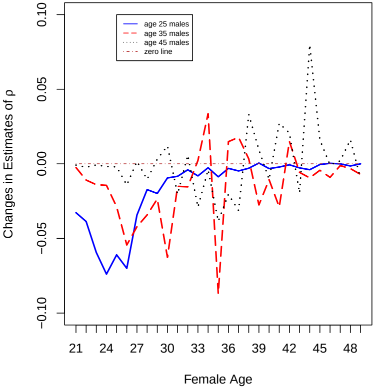
Empirical Application: continue

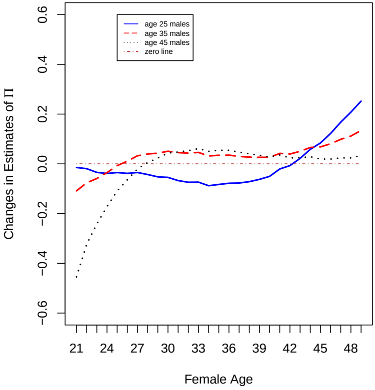
- Estimated ρ_{ij} and Π_{ij} in 2008-10 for ages 25, 35, 45 males



Empirical Application: continue

- Changes in $\widehat{\rho_{ij}}$ and $\widehat{\Pi_{ij}}$ for ages 25, 35, 45 males





Conclusion

- Propose an empirical model of marriage matching with search frictions.
- Rationalizes a new marriage matching function with search friction.
- Develop an empirical strategy to separately identify marital gains and search frictions.
- Applied our model to investigate how advancement in social media has affected marital gains and search cost from 2007/8 to 2017/18.
- Preliminary results showed that these technological advancement

i) raised search cost among the young (21 to 31 years of age) while it lowered search cost among the old (older than 40).

ii) lowered marital surplus among the young (21 to 31 years of age) while it raised surplus among the old.