People- or Place-Based Policies to Tackle Disadvantage? Evidence from Matched Family-School-Neighborhood Data

Lucienne N.Y. Disch
University of Pennsylvania

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Introduction

Motivation

- Standardized **test scores vary** a lot across neighborhoods and schools
  $\rightarrow$ **proxy for human capital**, with critical implications for **inequality**
- Test scores are **highly predictive for future** (labor market) outcomes
  $\rightarrow$ especially **important** for children from disadvantaged families

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- **Understanding sources** of test scores dispersion is crucial
  $\rightarrow$ allows for more **targeted people- or place-based policies**
- **But many unknowns** with respect to **producing human capital**
  - Family vs School vs Neighborhood?
  - What are complementary family-school-neighborhood ties?
  - Sorting vs Treatment?

This Paper

- **New strategy** to **disentangle sorting from treatment** under unobserved heterogeneity and three-sided interactions
- Exploit **institutional features** of Wake County's network structure and **spatial data** to get child's school and neighborhood identifiers over time

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    - Composition of family types in different neighborhoods and schools

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    - Composition of family types in different neighborhoods and schools
- Counterfactual analyses: (i) school quality $\uparrow$ and (ii) random reallocations

Preview of Findings

- **Family** is **crucial**
  - Being in **good places** is **beneficial** for own school performance, but cannot fully compensate for family "deficits"
- **Families sort**
  - Neighborhood sorting more significant (due to their heterogeneity)
- Improving school quality increases test scores especially around median
  - School quality has limited effect if neighborhoods factors ignored

The Model

Environment

- *Environment* consists of J neighborhoods, C schools, and N families with one child $\rightarrow$ each set divided into finite groups
- *Heterogeneity* across families/schools/neighborhoods characterized by their latent type/category/class
  - $\boldsymbol{\alpha_i \in \{1, \ldots, L\}}$ is **type of child i**, where L discrete and known
  - $\mathbf{s_{it}}$: **category of school $\mathbf{c_{it}}$** that child i attends at time t
  - $\mathbf{k_{it}}$: **class of neighborhood $\mathbf{j_{it}}$** where child i lives at time t
- **Mobility** *between states* (a neighborhood and/or school at time t and another neighborhood and/or school at t+1) is denoted $\boldsymbol{ks \rightarrow k's'}$
- Child i receives test scores Y_{it} at time t

Key Model Assumption and Timing

- Condition underlying the decomposition of test scores distributions into match-specific components:

A1 (conditional independence of test scores given child type).

Given $\alpha_i = l$ (on top of states), the random variables Y_{i1} and Y_{i2} are conditionally independent.

- Timing
  - In period 1:
    - α_i is drawn from a distribution that depends on k and s
    - Y_{i1} is drawn from a distribution that depends on α_i, k, s
  - In period 2:
    - transition probability may depend on α_i, k, s, k', s'
    - type of second state may depend on α_i and first state

Objective

Main purpose is to recover:

1. Distributions of test scores for children of type α in neighborhoods of class k and schools of category s
   - ▷ **complementarities**
2. Composition of type-α families in class-k neighborhoods and category-s schools
   - ▷ **sorting patterns**

Model nests two stratified mixture models

First Stratified Mixture Model aka Empirical Equation I

Bivariate distribution of test scores for movers as mixture problem:

$$Pr[Y_{i1} \leq y, Y_{i2} \leq y' | ks \to k's'] = \sum_{\alpha=1}^{L} Pr[Y_{i1} \leq y, Y_{i2} \leq y' | ks \to k's', \alpha_i = l] \pi_{ks \to k's'}(\alpha)$$

- $F_{ks\alpha}(Y_1)$: cdf of test scores in period 1, in neighborhood class k and school category s, for child type α
- $F^m_{k's'\alpha}(Y_2)$: cdf of test scores in period 2 for type α-transitioners to $k's'$
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$$= \sum_{\alpha=1}^{L} F_{ks\alpha}(Y_1) \cdot F^{m}_{k's'\alpha}(Y_2) \cdot \pi_{ks \rightarrow k's'}(\alpha) \qquad \text{(Eq. I)}$$

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Second Stratified Mixture Model aka Empirical Equation II

Second key equation considers cross-section in period 1:

$$Pr[Y_{i1} \leq y|ks] = \sum_{\alpha=1}^{L} F_{ks\alpha}(Y_1) \cdot q_{ks}(\alpha) \qquad \text{(Eq. II)}$$

- $F_{ks\alpha}(Y_1)$: cdf of test scores in period 1, in k and s, for child type α
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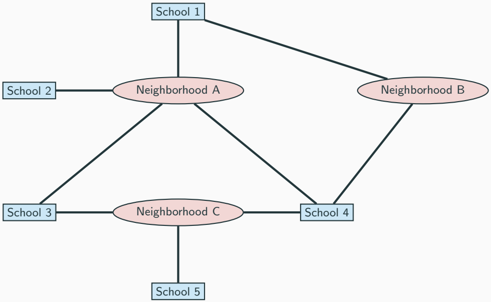
Next:

- map the model to the data and
- provide conditions under which all parameters appearing in the two stratified mixture models are identified

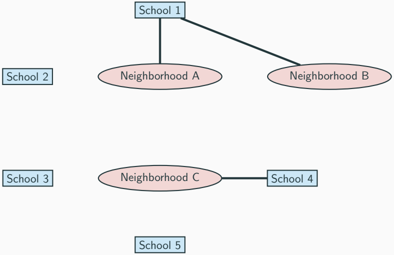
Mapping the Model to the Data

Data & Institutional Background: NC School System

- Relationship between family-, school-, and neighborhood heterogeneity and test scores is tested with data from **North Carolina Education Research Data Center** (sample period: 2010–2017; pooled 2 periods)
  - matched with **geospatial data** from **Wake County**
  - **standardized tests**: end-of-grade tests in both mathematics and reading for all children in public schools in grades 3-8
  - very suitable **network structure**:



Changes in Base Schools for some Neighborhoods



(a) Pre-Rezoning

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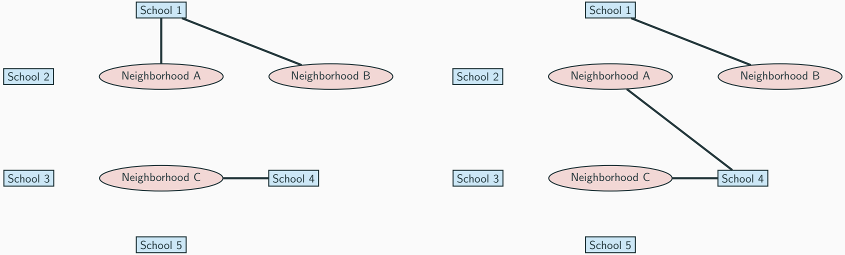
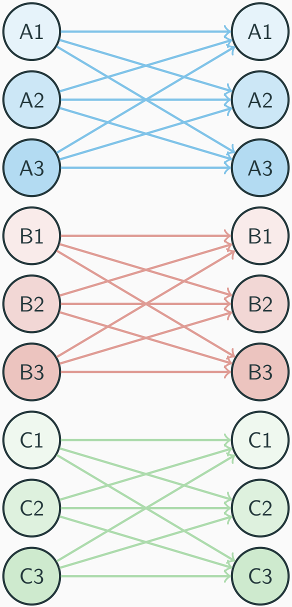


(a) Pre-Rezoning **(b)** Post-Rezoning

Figure 1: Changes in school zones altering the network structure

Model Identification

Graph Connectivity



Asymmetry of Transitions between States and Rank Condition

- $\pi_{ks \rightarrow k's'}(\alpha)$ from mixture model requires state-dependent transitions

  ▷ non-random mobility

  ▷ asymmetry in child type composition of transitioners

- set of distributions must be linearly independent $\rightarrow$ matrices formed from these distributions have full rank

  ▷ model parameters (latent types and their distributions) can be uniquely determined from the observed data

Assumptions key for identification of complementarities!

Model Estimation

Model Estimation

- Estimate latent types of places (dimension reduction into states):
    1. **Cluster Neighborhoods into Classes** with Distributional k-Means
        - take stayers' distributions of test scores across J neighborhoods $\rightarrow$ partition into K classes
        - impose class assignments on movers $\rightarrow$ heterogeneity of neighborhoods is at class level

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  3. **Aggregate into (neighborhood class, school category)-states**
     $k \in \{A, B, C\}$ and $s \in \{1, 2, 3\}$ generate nine distinct pairs:

$$state_{it}(k, s) = \begin{cases} A1 & \text{if } k_{it} = A \text{ and } s_{it} = 1 \\ A2 & \text{if } k_{it} = A \text{ and } s_{it} = 2 \\ A3 & \text{if } k_{it} = A \text{ and } s_{it} = 3 \\ B1 & \text{if } k_{it} = B \text{ and } s_{it} = 1 \\ B2 & \text{if } k_{it} = B \text{ and } s_{it} = 2 \\ B3 & \text{if } k_{it} = B \text{ and } s_{it} = 3 \\ C1 & \text{if } k_{it} = C \text{ and } s_{it} = 1 \\ C2 & \text{if } k_{it} = C \text{ and } s_{it} = 2 \\ C3 & \text{if } k_{it} = C \text{ and } s_{it} = 3 \end{cases}$$

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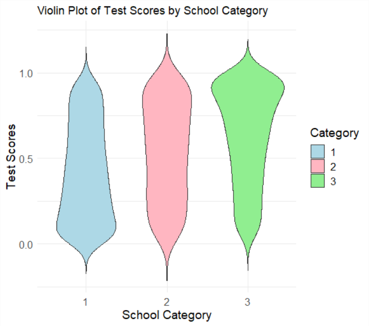
- Estimate latent child types:

  ▷ Estimate the two finite mixture models (separately) with EM algorithm

Results

Clustering Results of the Distributional K-Means

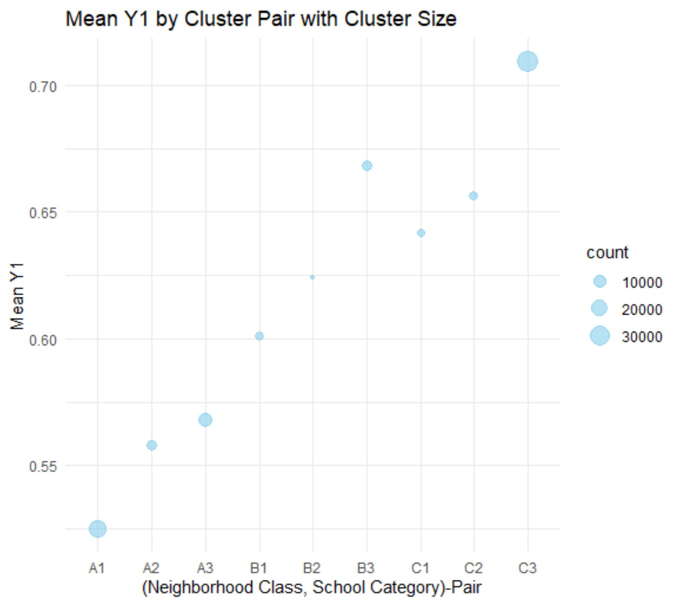
(a) Neighborhood Classes



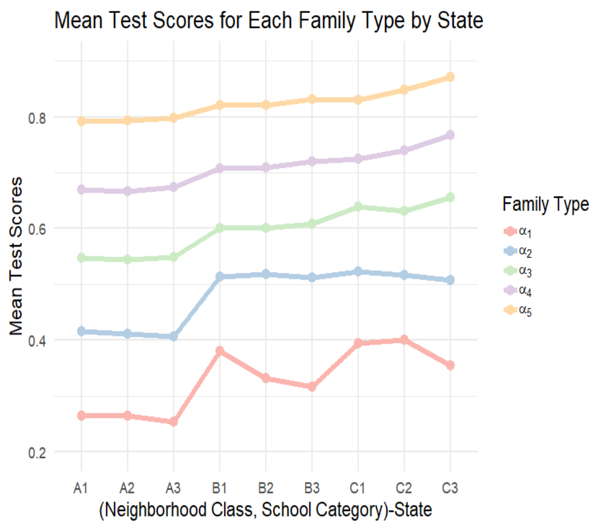
(b) School Categories

Figure 3: Violin Plots of Test Scores by Cluster

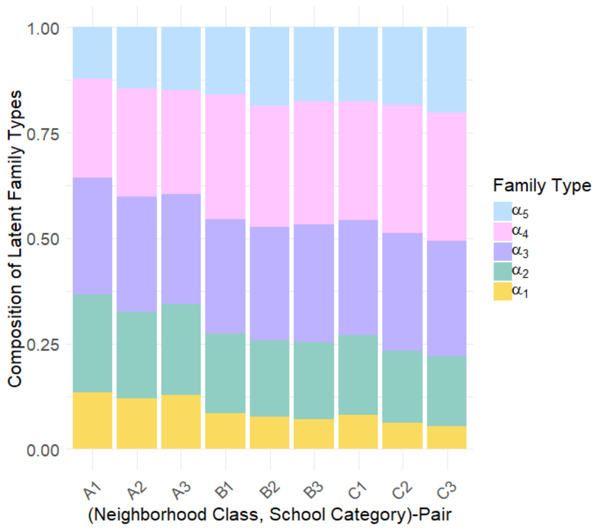
Mean and Size of Resulting Cluster-Pairs



Results: Complementarities



Results: Sorting

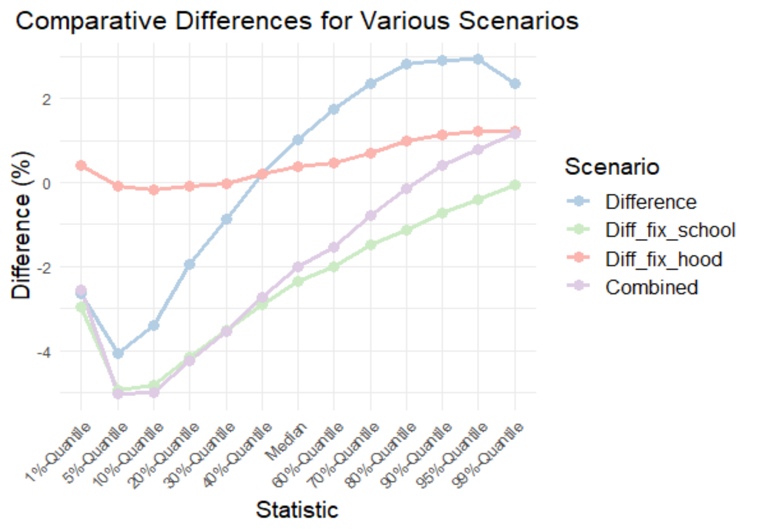


Counterfactual Analyses

School Quality Improvement

Statistic	$\Delta_{cat1 \rightarrow cat2}$	$\Delta_{cat1 \rightarrow cat3}$
Mean	0.8 %	1.3 %
1%−Quantile	0.4 %	0.0 %
5%−Quantile	0.9 %	0.8 %
10%−Quantile	1.2 %	1.3 %
20%−Quantile	1.3 %	1.6 %
30%−Quantile	1.2 %	1.9 %
40%−Quantile	1.0 %	1.8 %
Median	0.9 %	1.7 %
60%−Quantile	0.8 %	1.5 %
70%−Quantile	0.7 %	1.3 %
80%−Quantile	0.4 %	1.0 %
90%−Quantile	0.4 %	1.0 %
95%−Quantile	0.1 %	0.7 %
99%−Quantile	0.4 %	1.0 %
Variance	−0.001	0.000

Random Reallocation Exercises



Conclusion

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- Considered all three sources of variation in test scores unified
- **Family is key** for academic performance
  - Policies that provide support and resources can empower families
- **Neighborhoods play a crucial role**
  - Positive complementarities for low test score performers when residing in area with high test scores
  - Policies aimed at improving neighborhood environments or housing mobility programs can have significant impact on education

Thank You!

brunnerl@sas.upenn.edu

Appendix

Related Literature

- Families/schools/neighborhoods as determinants for children's outcomes
  Cunha-Heckman (2007, 2008); Chetty-Hendren (2016), Chyn-Katz (2021); Agostinelli et al. (2024)
  - ▷ **Contribution:** Quantification in a comprehensive way

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    - imposes restrictive forms of complementarity → at odds with sorting models where neighborhoods are background contexts
      Becker, 1974; Durlauf, 2004
  - **Solution:** extend Bonhomme-Lamadon-Manresa (2019)

  ▷ **Contribution:** Adding a third dimension (such as geography)

Derivation of First Stratified Mixture Model

Decomposition of joint conditional probability of a *transitioning* child i:

$Pr[Y_{i1} \leq y, Y_{i2} \leq y' | ks \rightarrow k's']$

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Dimension Reduction (Estimate Latent Types of Places)

Clustering Neighborhoods into Classes:

- consider **stayers'** distributions of test scores across neighborhoods and solve k-means problem to partition the J neighborhoods into K classes:

$$\min_{V,\mathbf{k}} \sum_{j=1}^{J} w_j \sum_{i=1}^{N} \left\| \hat{F}_j(y_i) - v_{k_j}(y_i) \right\|^2,$$

  where:

  - $\hat{F}_j(y_i)$: value of the y_i-th percentile for the j-th neighborhood
  - $v_{k_j}(y_i)$: y_i-th percentile of cluster center of j-th neighborhood
    - $\mathbf{k} = \{k_1, ..., k_J\}$ is the cluster assignment vector
    - $V = \{\mathbf{v_1}, ..., \mathbf{v_K}\}$ is the set of cluster centers
  - w_j: weight for the j-th neighborhood, representing the size

- impose class assignments on movers' neighborhood identifiers

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Clustering Schools into Categories

- in a similar vein

Aggregation: (Neighborhood Class, School Category)-States

- combine neighborhood classes and school categories into pairs to get (neighborhood class, school category)-states
  - e.g., three neighborhood classes, $k \in \{A, B, C\}$, and three school categories, $s \in \{1, 2, 3\}$, generate nine distinct states:

$$
state_{it}(k, s) = \begin{cases} A1 & \text{if } k_{it} = A \text{ and } s_{it} = 1 \\ A2 & \text{if } k_{it} = A \text{ and } s_{it} = 2 \\ A3 & \text{if } k_{it} = A \text{ and } s_{it} = 3 \\ B1 & \text{if } k_{it} = B \text{ and } s_{it} = 1 \\ B2 & \text{if } k_{it} = B \text{ and } s_{it} = 2 \\ B3 & \text{if } k_{it} = B \text{ and } s_{it} = 3 \\ C1 & \text{if } k_{it} = C \text{ and } s_{it} = 1 \\ C2 & \text{if } k_{it} = C \text{ and } s_{it} = 2 \\ C3 & \text{if } k_{it} = C \text{ and } s_{it} = 3 \end{cases}
$$

- (latent) heterogeneity of neighborhoods/schools is at class/category level

Estimation of Finite Mixture Models (Latent Child Types)

- Idea: given (neighborhood class, school category)-states ("fixed effects"), recover child types ("random effects") $\rightarrow$ "correlated random effects"
- Estimate the two finite mixture models (separately) with EM algorithm
- Specified that families belong to L latent types $\rightarrow$ model is parametric given family, school and neighborhood heterogeneity
- E.g., for Eq. I, where q represents state and α is discrete, let test scores densities be normal with (q,α)-specific means and variances:

$$P(Y_1, Y_2|\mu_{Y_1}, \mu_{Y_2}, \sigma^2_{Y_1}, \sigma^2_{Y_2}, \pi, q \rightarrow q') = \sum_{\alpha=1}^{L} \pi_{\alpha,q\rightarrow q'} \mathcal{N}(Y_1|\mu_{Y_1\alpha q}, \sigma^2_{Y_2\alpha q'}) \times \mathcal{N}(Y_2|\mu_{Y_2\alpha q'}, \sigma^2_{Y_2\alpha q'}),$$

- Corresponding log-likelihood function:

$$\sum_{i=1}^{N_q} \sum_{q=1}^{Q} \sum_{q'=1}^{Q} \mathbb{1}\{\hat{q}_{i1} = q\} \mathbb{1}\{\hat{q}_{i2} = q'\} \ln \left(\sum_{\alpha=1}^{L} \pi_{qq'}(\alpha; \theta_p) f_{q\alpha}(Y_{i1}; \theta_f) f^{q}_{q'\alpha}(Y_{i2}; \theta_{fq}) \right)$$

- Decomposes distribution of test scores into match-specific components (parameters) $\rightarrow$ reveals type-specific family-school-neighborhood effects

Random Reallocation Exercises

Statistic	Difference (SE)	Diff fix school (SE)	Diff fix hood (SE)
Mean	0.44% (0.0001)	−2.52% (0.0010)	0.42% (0.0018)
1%−Quantile	−2.65% (0.0003)	−2.97% (0.0004)	0.41% (0.0018)
5%−Quantile	−4.08% (0.0003)	−4.94% (0.0007)	−0.09% (0.0025)
10%−Quantile	−3.41% (0.0002)	−4.83% (0.0010)	−0.18% (0.0025)
20%−Quantile	−1.96% (0.0002)	−4.16% (0.0011)	−0.09% (0.0023)
30%−Quantile	−0.87% (0.0002)	−3.52% (0.0012)	−0.04% (0.0022)
40%−Quantile	0.21% (0.0002)	−2.91% (0.0012)	0.19% (0.0021)
Median	1.03% (0.0002)	−2.37% (0.0012)	0.37% (0.0019)
60%−Quantile	1.74% (0.0002)	−2.00% (0.0011)	0.47% (0.0018)
70%−Quantile	2.36% (0.0002)	−1.48% (0.0010)	0.69% (0.0017)
80%−Quantile	2.83% (0.0001)	−1.13% (0.0010)	0.99% (0.0015)
90%−Quantile	2.92% (0.0002)	−0.72% (0.0008)	1.14% (0.0012)
95%−Quantile	2.93% (0.0002)	−0.42% (0.0007)	1.21% (0.0009)
99%−Quantile	2.36% (0.0003)	−0.07% (0.0006)	1.22% (0.0007)
Variance	0.0068 (0.0000)	0.0041 (0.0000)	0.0014 (0.0001)