Dynamic Auctions with Budget-Constrained Bidders: Evidence from the Online Advertising Market

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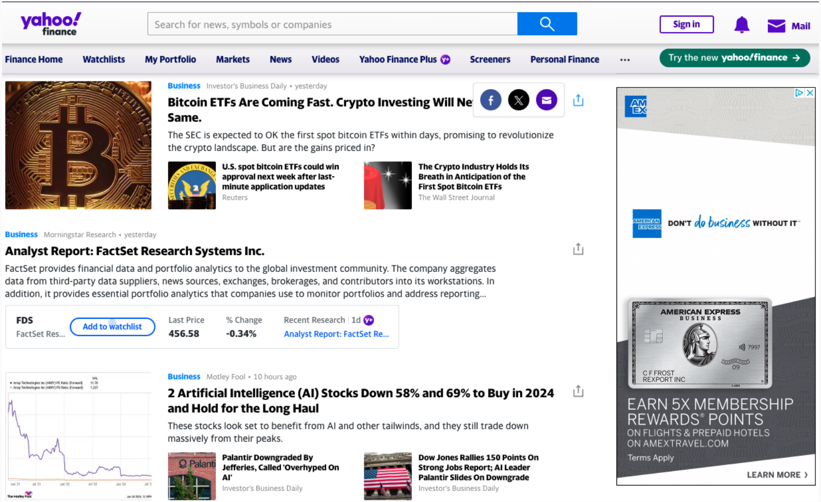
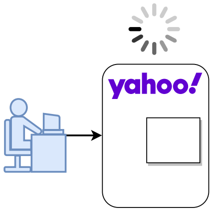
Motivation

- Auctions are used in many real-world contexts
    - Price discovery and buyer competition
- Led to extensive theoretical and empirical research
    - focusing on bidders facing only one auction
- However, firms and consumers often face auctions sequentially
- Dynamics become important if financially constrained
    - Little empirical attention
- Ex: Financial markets, energy markets, eBay, online ad market

Online Display Ads



Online Advertising Auctions (Real-Time Bidding)



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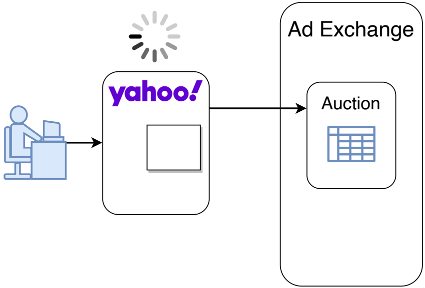

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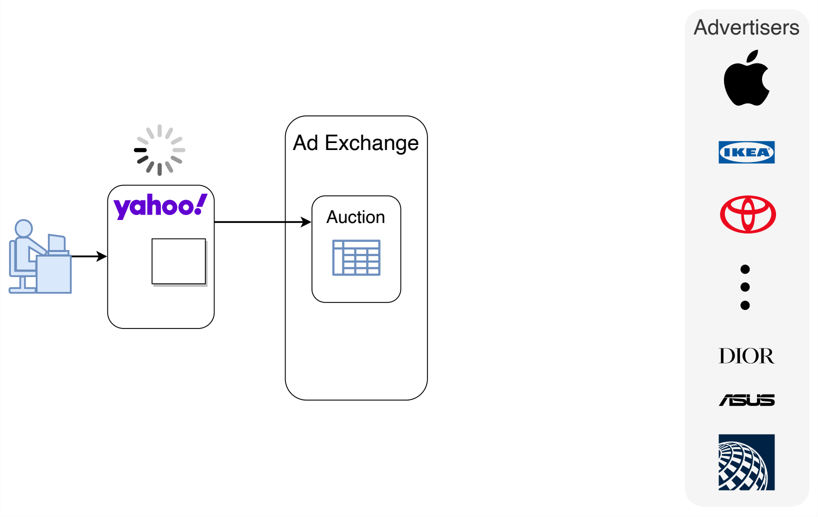

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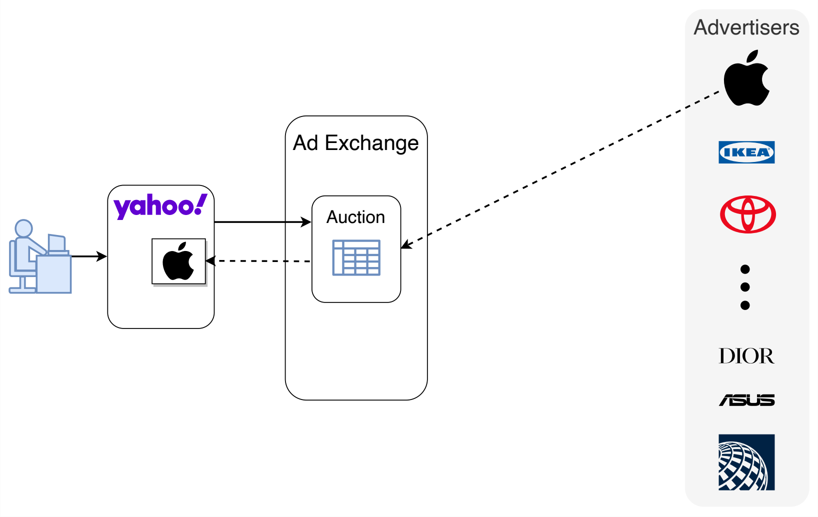

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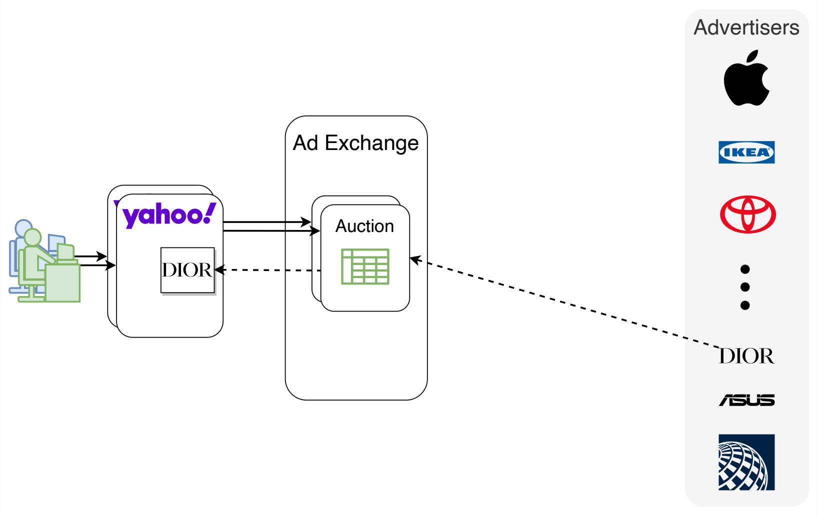

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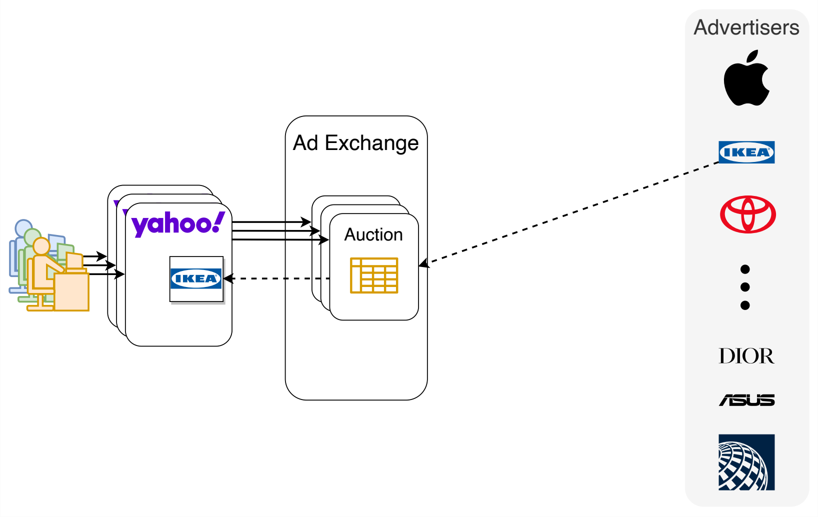

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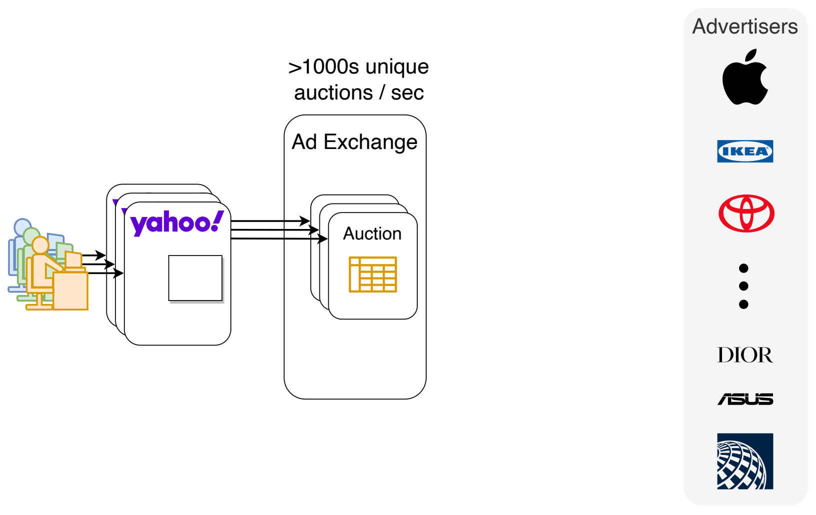

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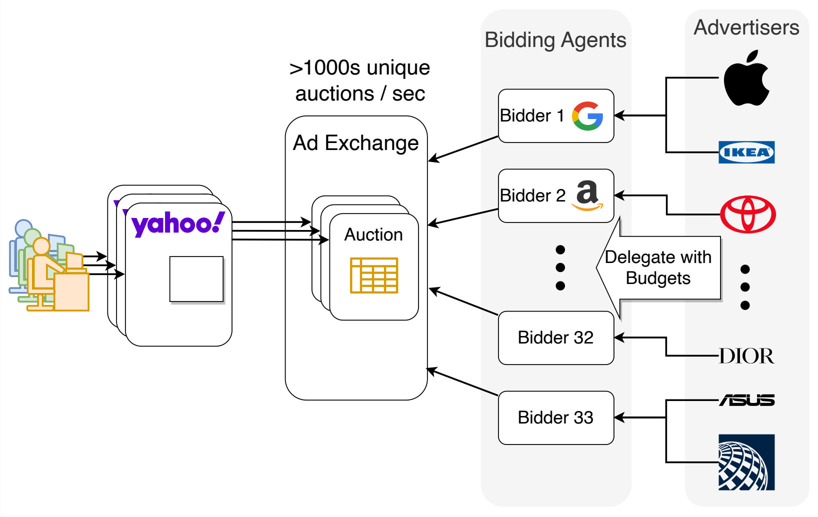

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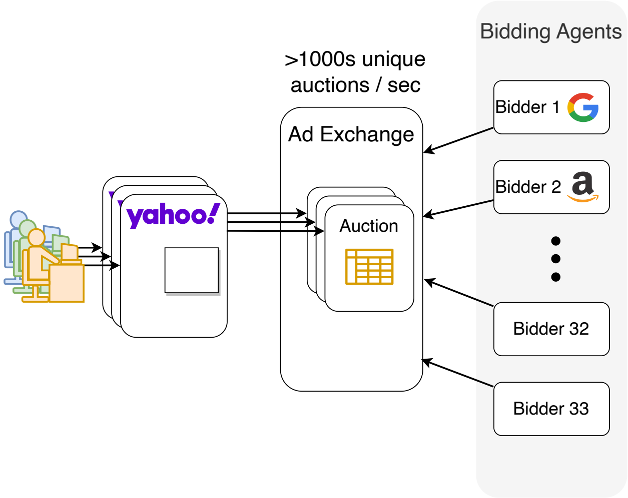


Objective:

Dynamically allocate advertising budgets across auctions

Research Questions

Dynamic auctions with budget-constrained bidders

- How do bidders dynamically compete against each other?
- How does the auction format affect the dynamic competition?

This paper

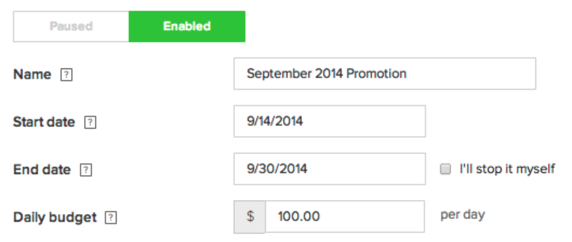
- First empirical analysis of dynamic auctions with budget constraints
  - using a novel proprietary dataset of online ad auctions
- Novel structural framework of dynamic auctions with budget-constrained bidders
- Quantify how dynamic constraints shape bidders' strategic behavior
  - Substantial impact on participation and bid decisions
  - Heterogeneity in budgets $\Rightarrow$ Heterogeneity in dynamic behavior
- Counterfactual simulation to analyze mechanism design
  - Ongoing debate: First-price auction vs Second-price auction
  - First-price auction better for bidders with smaller budgets

Roadmap

1. Background
2. Descriptive Evidence
3. Structural Model
4. Estimation / Results
5. Counterfactuals

Background: Budgets and Bidding Agents

- Advertisers set up ad campaigns with bidding agents
- Campaign settings: Goal, target audience, length, and budget
- Generally enforced through daily budget constraints



- Bidding agents strategize while adhering to daily budgets

Data

- Bid-level data on ad auctions hosted on Yahoo ad exchange
  - Current industry standard: First-price auctions
- One week data from the second quarter of 2021
- 16 websites owned by Yahoo (Finance, News, etc)
- 33 bidding agents and 71,011 advertisers.

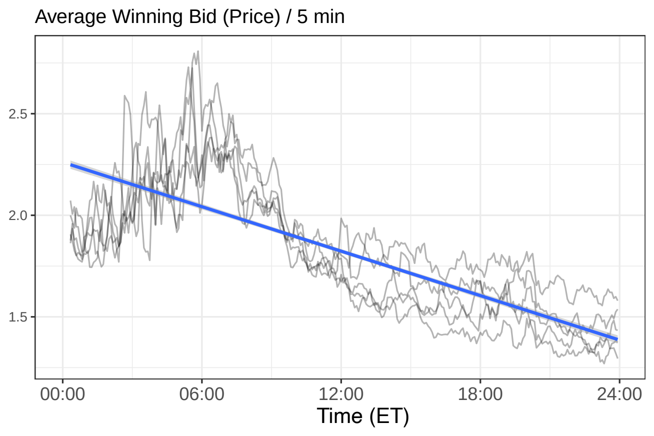
Roadmap

1. Background
2. Descriptive Evidence
    - Empirical patterns consistent with intertemporal budget constraints
3. Structural Model
4. Estimation / Results
5. Counterfactuals

Observation 1: Declining Price

- Entry rate and bid level decline within each day

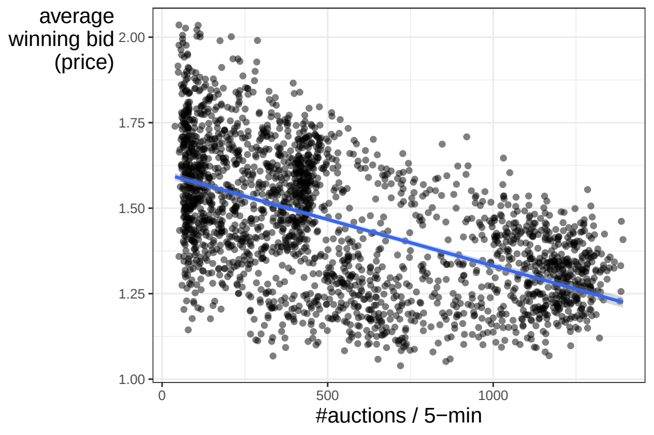
  $\Longrightarrow$ declining average price (winning bid)



- Daily budgets are renewed at 12am (ET) for most advertisers

Observation 2: Bidders' respond to freq. of auctions

- When the freq. of auctions is higher, bidders enter auctions at a lower rate and submit lower bids
- 10% $\uparrow$ in freq. of auctions $\implies$ 1.3% $\downarrow$ in price
  - while controlling for auction characteristics and hour FE



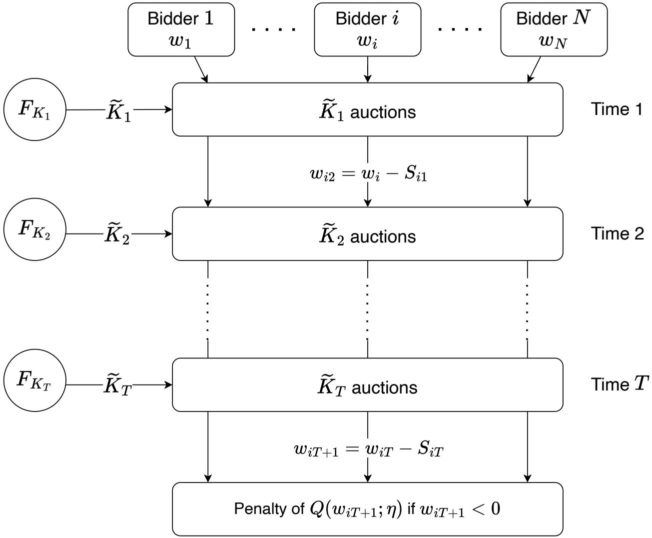
Review of Stylized Facts

- Consistent patterns connected to budget constraints:
  1. Declining average price within each day
  2. Inverse relationship between the freq. of auctions and competition
- Suggests classical implications may not apply
- Need a new model to conduct analysis

Roadmap

1. Background
2. Descriptive Evidence
3. Structural Model
    - Recover bidders' budgets & valuations for ad opportunities
4. Estimation / Results
5. Counterfactuals

Dynamic Auctions with Budget-Constrained Bidders



In our application,

$N = 33$

$T = 24$

Model Implications

- Analyze the best-response problem under an oblivious equilibrium concept (Weintraub et al., 2008)
- Auctions are strategically linked together unlike standard models
- Intertemporal budget constraints introduce a dynamic tradeoff
- Entry and bid strategies depend on
  - frequency of auctions K_t
  - remaining budget w_{it}
  - current and future competitiveness

Roadmap

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Estimation Method

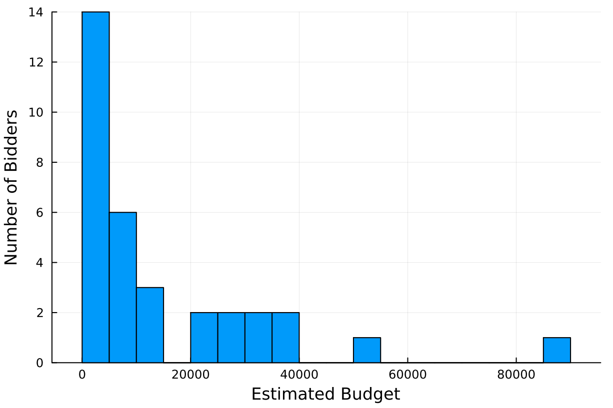
Using our novel dataset from Yahoo, we take a two-step approach to avoid equilibrium computation[1]:

1. Estimate each bidder's belief over other players' behavior
2. MLE to estimate the model primitives using the structural model
    - Inner loop: solving the best-response problem via backward induction

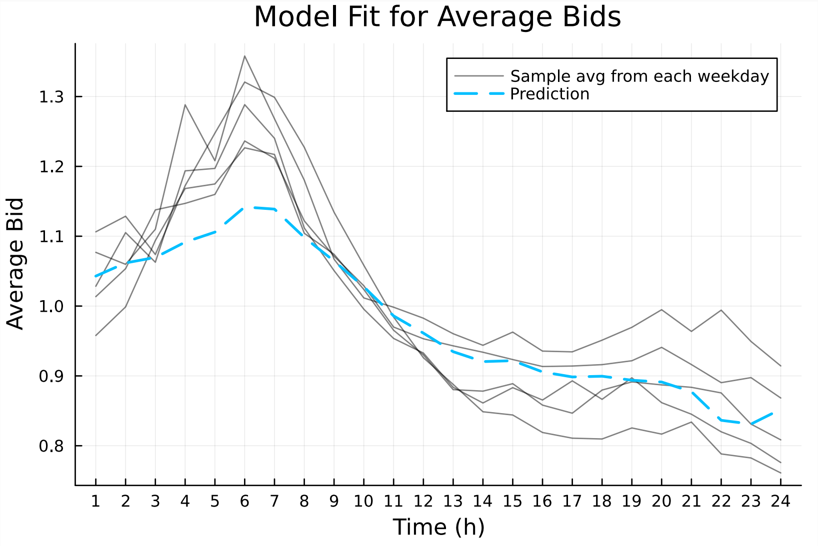
[1] Bajari et al. (2007); Aguirregabiria and Mira (2007)

Estimation Results

- Statistically significant evidence that budget constraints matter
  - Spending exceeds budget approximately 26% of the time, but amounts to around 8% of budget plot
- Significant heterogeneity in $(w_i)_{i=1}^N$
  - $\Rightarrow$ heterogeneity in entry & bid behavior plot



Model Fit



Model Fit for Average Bids
Sample avg from each weekday
Prediction
Average Bid
1.3
1.2
1.1
1.0
0.9
0.8
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
Time (h)

Roadmap

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First-price vs Second-price auction

- CF to generate insights for mechanism design
- We use our equilibrium solver to compare FPA and SPA
  - SPA is no longer strategy-proof
  - No prior welfare comparison
- Motivated by an institutional shift that happened after 2018
  - Sellers and buyers demanded more transparency from ad exchanges
- Conventional analysis:
  - FPA = SPA in revenue and welfare considerations
- In dynamic auctions with budget-constrained bidders, we find significant difference in welfare outcomes

FPA vs SPA: Dynamics

- Price variance is lower in FPA than in SPA
  - Similar to the standard case (Krishna, 2009)
- Bidders with smaller budgets are more aggressive under FPA
- Larger bidders are pressured to spend rapidly
  - In later periods, they become tighter constrained
- Smaller bidders capitalize on the reduction in competition
  - Better off under FPA than SPA
- Total welfare and revenue are both marginally higher under FPA

FPA vs SPA: Implications

- Policy
  - FPA $>$ SPA for smaller firms
  - FPA encourages smaller bidders to be more competitive against larger bidders
- Dynamic mechanism design with financial constraints
  - Price volatility determines dynamic competition
  - Not specific to daily budget constraints

Conclusion

- Propose a novel structural framework of dynamic auctions with budget-constrained bidders
- Empirically analyze how budget constraints shape dynamic competition in the online advertising market
  - First empirical analysis
- Find significant impact from budget constraints which is heterogeneous across bidders
- First-price auction better at encouraging smaller bidders to be more competitive
  - driven by the difference in price volatility

Thank you!

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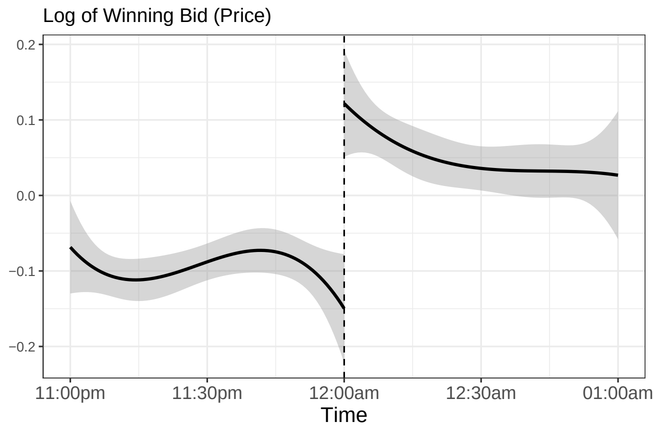
Summary Statistics

variable	n	mean	std	min	median	max
Bid	8,856,603	1.000	1.682	0.061	0.577	369.070
# entrants	1,229,300	7.205	4.732	1.000	7.000	25.000
Win bid	1,229,300	2.294	3.441	0.061	1.182	369.070

- Note: avg bid normalized to 1 in sample (confidentiality)

Observation 2: Price jumps when budgets are renewed

- There is a consistent jump at midnight
- Reg. Discontinuity: Price jumps by $\sim 40\%$ on average
  - while controlling a rich set of auction characteristics



Stage Game

- State variables: K_t, $(w_{it})_{i=1}^N$
- K_t first-price auctions
  - Private entry cost: $C_{ikt} \overset{iid}{\sim} F_C$
  - Private valuation: $X_{ikt} \overset{iid}{\sim} F_X$
- Strategies chosen before C_{ikt} and X_{ikt} are realized
  - Entry threshold strategy: $\overline{c}_{it} \geq 0$
  - Bid strategy: $x \in \mathbb{R} \mapsto b_{it}(x) \in \mathbb{R}$
  - If $C_{ikt} \leq \overline{c}_{it}$, submits $b_{it}(X_{ikt})$
- Stage payoffs

$$\sum_{\text{auctions won}} (X_{ikt} - b_{it}(X_{ikt})) - \sum_{\text{auctions entered}} C_{ikt}$$

- Payment

$$S_{it} = \sum_{\text{auctions won}} b_{it}(X_{ikt})$$

Solution Concept

- Bidders have little info on their rivals' spending
- Large-market equilibrium: Each bidder tracks only (K_t, w_{it})
  - similar to oblivious equilibrium (Weintraub et al., 2008)
- Eqb. object: Prob. of winning, CDF of the highest competing bid

$$\begin{aligned}\Psi_t(b \mid K_t) &= \Pr\left(\max_{j\neq i} B_{jkt} \leq b \mid K_t\right) \\ &= E_{(w_{jt})_{j\neq i}}\left[\underbrace{\Pr\left(\max_{j\neq i} B_{jkt} \leq b \mid K_t, (w_{jt})_{j\neq i}\right)}_{\text{Determined by } \left(\bar{c}_t(K_t, w_{jt}), b_t(\cdot|K_t, w_{jt})\right)}\right]\end{aligned}$$

Bidding Problem

- Take $\Pr(\text{win}) = \Psi_t(\cdot \mid K_t)$ as given
- Best-response entry and bid strategies solved via backward induction

$$V_t(K_t, w_{it}) = \max_{\bar{c}, b(\cdot)} \underbrace{K_t F_C(\bar{c})}_{\substack{\text{Expected} \\ \#\text{Entry}}} \left(\underbrace{E\left[\Psi_t(b(X) \mid K_t)(X - b(X))\right]}_{\text{Expected Surplus}} - \underbrace{E[C \mid C \le \bar{c}]}_{\substack{\text{Expected} \\ \text{Entry Cost}}} \right)$$
$$+ \underbrace{E\left[EV_{t+1}(w_{it} - S_{it}) \mid b(\cdot), \bar{c}\right]}_{\text{Continuation Value}}$$

where

$$EV_{t+1}(w) = E_{K_{t+1}}[V_{t+1}(K_{t+1}, w)]$$
$$EV_{T+1}(w) = -Q(w; \eta)$$

- If no constraint ($\eta = 0$), static auction with entry
  - Li and Zheng (2009)

First-order conditions

Model Primitives

Using our novel dataset of online ad auctions from Yahoo, we estimate:

1. F_{K_t} : Time-variant distribution of #auctions
2. F_C : Distribution of entry costs
3. F_X : Distribution of valuations for ad opportunities
4. η : Parameter in penalty $Q(w_{iT+1}; \eta) = \eta w_{iT+1}^2 \mathbb{1}_{\{w_{iT+1}<0\}}$
5. $(w_i)_{i=1}^N$: Bidders' budgets

identification

Why Daily Budgets?

- Advertisers impose them on bidding agents
- They may want them for a few reasons
  1. Preventing overspending from an error
  2. Ensuring consistent exposure
  3. Convenience for accounting/billing
- Our structural model could be used to assess a cf. change

◂ background

◂ figure decline

◂ figure jump

◂ review

Why Declining Price?

- Similar theoretical findings in sequential auctions with unit-demand bidders with independent valuations[2]
- Large heterogeneity in ad opportunities
- When bidders find a consumer that matches well with their ad, they have delay costs.
    1. they may face worse objects later
    2. they are not guaranteed to win later

◂ figure

◂ review

[2]Bernhardt and Scoones (1994); Engelbrecht-Wiggans (1994); Gale and Hausch (1994)

First-Order Conditions

- Entry threshold FOC

$$\overline{c} = \underbrace{E\left[\Psi_t(b(X) \mid K_t)(X - b(X))\right]}_{\text{Static Threshold}} + \underbrace{\frac{1}{K_t f_C(t)} \frac{\partial}{\partial \overline{c}} E\left[EV_{t+1}(w_{it} - S_{it}) \mid b(\cdot), \overline{c}\right]}_{\text{Dynamic Tradeoff}}$$

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- Bid strategy FOC

$$E\Bigg[\underbrace{\left(X - \frac{\Psi_t(b(X) \mid K_t)}{\Psi_t'(b(X) \mid K_t)} - b(X)\right)}_{\text{Static FOC}} \Psi_t'(b(X) \mid K_t) \nabla_\gamma b(X)\Bigg]$$

$$+ \underbrace{\frac{1}{K_t F_C(\overline{c})} \nabla_\gamma E\left[EV_{t+1}(w_{it} - S_{it}) \mid b(\cdot), \overline{c}\right]}_{\text{Dynamic Tradeoff}} = 0$$

back

Identification of η and $(w_i)_{i=1}^N$

- Key assumption: $X_{ikt} \perp \left(K_t, (S_{is})_{s=1}^{t-1}\right)$
- Intuition: capture the observed correlation by the dynamic tradeoffs
- Plausible since valuations are computed based on click/sale probability and value from such events.

moment condition

◂ back

Moment Condition

- For the correct parameters θ, we have

$$B_{ikt} = b_t \left(X_{ikt} \mid K_t, w_i - \sum_{s=1}^{t-1} S_{is}; \theta \right)$$

- $X_{ikt} \perp \left(K_t, (S_{is})_{s=1}^{t-1} \right)$ implies

$$E\left[b_t^{-1}\left(B_{ikt} \mid K_t, w_i - \sum_{s=1}^{t-1} S_{is}; \theta \right) - E[X_{ikt} \mid \theta] \mid K_t, (S_{is})_{s=1}^{t-1} \right] = 0$$

back

Estimated structural parameters

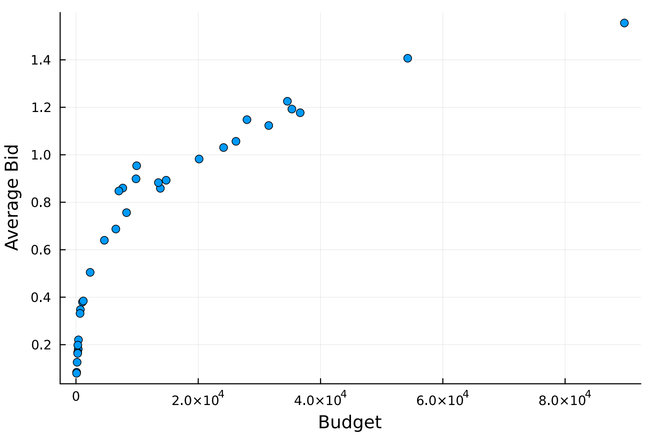
Parameters	Estimate	SE
μ_C	-11.3776	0.0091
σ_C	7.2533	0.0062
μ_X	0.9046	0.0007
σ_X	1.0950	0.0006
η	0.6457	0.0084

- $C \sim \text{TruncatedNormal}(\mu_C, \sigma_C)$
- $X \sim \text{LogNormal}(\mu_X, \sigma_X)$

back

Average bid per bidder

- Similar pattern for entry probability



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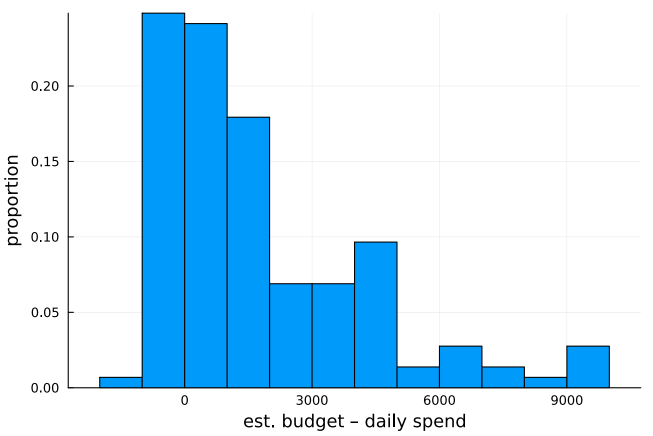
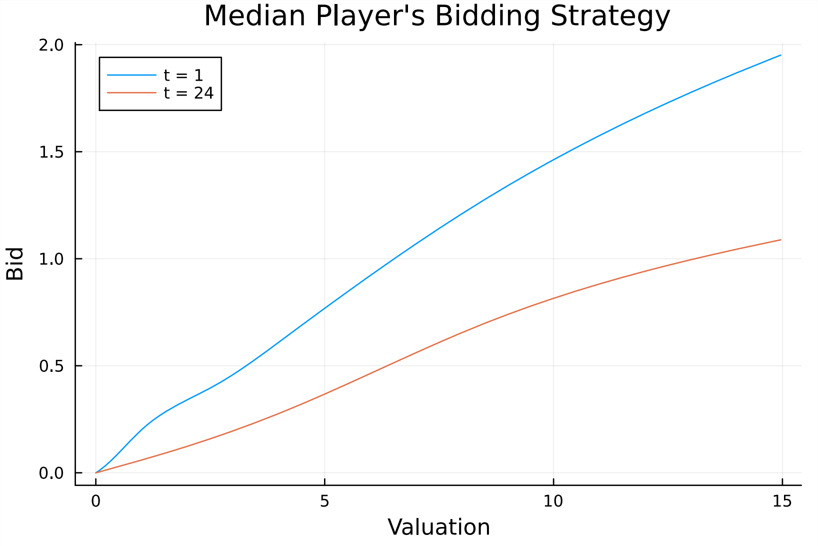
Distribution of daily remaining budget



◂ back

Bidding Strategy ($t = 1$ vs $t = 24$)



back

Quantify impact of intertemporal budget constraints

- Avg. markdown $\frac{\text{valuation}-\text{bid}}{\text{valuation}}$ is 83%
- Simulate a bidder's behavior after removing its budget constraint
  - by setting $\eta = 0$
  - CF avg. markdown is 59%
- Decomposition of markdown:

$$83\% = 59\%(\text{static markdown}) + 24\%(\text{dynamic markdown})$$

- Similarly, dynamic incentives decrease entry rate by 25 percentage points from the CF counterpart

CF Basic Results

Auction Format	First Price	Second Price
Price Average	\$2.364	\$2.362
Price Variance	1.1246	3.565
Expected Total Revenue	\$480,427.33	\$480,073.49
Expected Total Bidder Surplus	\$1,191,000	\$1,185,000

back

Intertemporal budget constraints lead to risk aversion

- Dynamic budget constraints lead bidders to exhibit risk aversion.
  - Concave continuation value: $EV_t(w_{it} - S_{it})$
- Echoes well-documented findings in finance that firms exhibit risk aversion when they have financial constraints (Froot et al., 1993; Opler et al., 1999).
  - Concave value function (Milne and Robertson, 1996; Holt, 2003; Rochet and Villeneuve, 2005).

back

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