Identification and Estimation of Continuous-Time Job Search Models with Preference Shocks

Peter Arcidiacono
Duke, NBER, IZA

Attila Gyetvai
Bank of Portugal

Arnaud Maurel
Duke, NBER, IZA

Ekaterina Jardim
Amazon

Motivation

- For many search models, establishing identification, and devising a tractable estimation procedure remains challenging.
- True in particular for nonstationary models, which arise in various settings:
  - Unemployment benefits expiring
  - Duration dependence in offer arrival rates
  - Aggregate productivity shocks
- Even in their most basic version (van den Berg, 1990): entail solving a non-linear second order differential equation at each iteration of the optimization procedure.

This paper

Bring together conditional choice probability (CCP) methods with continuous-time job search models:

- Incorporate preference shocks that affect the value of a particular job offer
  - Job offer at wage w will only be accepted probabilistically from the perspective of the econometrician
  - *Future* job offers at wage w will only be accepted probabilistically from the perspective of the worker too.
- Results in a tight connection between value functions and CCPs.
- Key difference with the dynamic discrete choice literature (Hotz and Miller, 1993): CCPs are not directly identified from the data.

This paper 2

- Derive *constructive* identification of the structural parameters for a class of job search models.
- Possible to estimate complex search models in a simple and tractable way.
- Application to Hungarian administrative data: non-stationarity plays a key role.

Related literatures

- **Identification of dynamic discrete choice models using CCPs**: Hotz and Miller (1993); Rust (1994); Magnac and Thesmar (2002); Arcidiacono et al. (2016); surveys by Aguirregabiria and Mira (2010); Arcidiacono and Ellickson (2011).
- **Empirical job search models**: Flinn and Heckman (1982); van den Berg (1990); surveys by Eckstein and van den Berg (2007); French and Taber (2011).
    - **Preference shocks in job search models**: Sorkin (2018); Llull and Miller (2018); Lentz et al. (2023); Lamadon et al. (2022).
- **Impact of UIB levels and duration on labor supply**: Card et al. (2007); Le Barbanchon et al. (2017); Johnston and Mas (2018).

General environment

Continuous time search model with off- and on-the-job search:

- Individuals are infinitely lived and discount the future at a rate $\rho > 0$.
- Job offers are characterized by a wage, w, and a job type, s (non-wage characteristics), both with finite support, Ω_w and Ω_s.
- Wage offer distributions: differ across employed and unemployed individuals, and by job type.
- Heterogeneous valuations of job offers via preference shocks, ε.
  - assumed to be drawn from a standard logistic distribution
  - logistic is important to obtain simple closed-form expressions for the structural parameters

Unemployed workers

- Job offers for type-s jobs accrue at a rate $\lambda^s(t)$, where t is unemployment duration.
- Conditional on receiving an offer from a type-s job, wages drawn from distribution $g_w^s(t)$.
- Receive utility $b(t)$ while unemployed.

Employed workers

Workers employed in a job (w, s) may experience three types of transitions:

- Get laid off at rate δ_0^s;
- Involuntary within firm wage and job type changes $\delta_{ww'}^{ss'}$, $(\delta_{ww}^{ss} = 0)$;
  - going to ignore these for the presentation
- Receive offers for job type s' given currently in job type s at rate $\lambda^{ss'}$;
  - Conditional on receiving an offer from a type-s' job, probability job pays w is $f_{w'}^{s'}$.
  - If an offer is accepted, pay a switching cost $c^{ss'}$ (assumed to be symmetric).

Workers receive utility $u_w + \phi^s$ when employed.

Value function for the unemployed

- Consider some small time interval Δt.
- The value of being unemployed can be expressed as:

$$V_0(t) = b(t)\Delta t + \frac{\Delta t}{1+\rho\Delta t}\sum_w \sum_w \lambda^s(t) g_w^s(t) \mathbb{E}_\varepsilon \max\left\{V_w^s + \varepsilon, V_0(t+\Delta t)\right\}$$
$$+ \frac{1 - \sum_s \lambda^s(t)\Delta t}{1+\rho\Delta t} V_0(t+\Delta t).$$

which can be rewritten as:

$$\rho V_0(t) = b(t)(1+\rho\Delta t) + \sum_s \sum_w \lambda^s(t) g_w^s(t) \mathbb{E}_\varepsilon \max\left\{V_w^s - V_0(t+\Delta t) + \varepsilon, 0\right\}$$
$$+ \frac{V_0(t+\Delta t) - V_0(t)}{\Delta t}$$

Value function for the unemployed

Letting $\Delta t \to 0$ and denoting by $\dot{V}_0(t)$ the derivative of $V_0(t)$ with respect to unemployment duration, we obtain the differential equation:

$$\rho V_0(t) = b(t) + \sum_s \sum_w \lambda^s(t) g_w^s(t) \mathbb{E}_\varepsilon \max\left\{V_w^s - V_0(t) + \varepsilon, 0\right\} + \dot{V}_0(t)$$

Value function for employed

- The value function associated with holding a job at wage w is given by:

$$V_w^s = u_w^s + \delta^s V_0(0) + \sum_{s'} \sum_{w'} \lambda^{ss'} f_{w'}^{s'} \mathbb{E}_\varepsilon \max \left\{ V_{w'}^{s'} - c^{ss'} + \varepsilon, V_w^s \right\}$$

- Expressing the future value term relative to not taking the new job and collecting terms yields:

$$V_w^s = \frac{u_w^s + \delta^s V_0(0) + \sum_{s'} \sum_{w'} \lambda^{ss'} f_{w'}^{s'} \mathbb{E}_\varepsilon \max \left\{ V_{w'}^{s'} - V_w^s - c^{ss'} + \varepsilon, 0 \right\}}{\rho + \delta^s}$$

Choice probabilities

- Given the distributional assumption on the preference shock, the probability of accepting an offer w at job type s conditional on unemployment duration t is:

$$p_w^s(t) = \frac{\exp(V_w^s)}{\exp(V_0(t)) + \exp(V_w^s)} = \frac{\exp(V_w^s - V_0(t))}{1 + \exp(V_w^s - V_0(t))}$$

- The probability of taking an offer of w' conditional being employed at wage w is:

$$p_{ww'}^{ss'} = \frac{\exp(V_{w'}^{s'} - c^{ss'})}{\exp(V_w^s) + \exp(V_{w'}^{s'} - c^{ss'})}$$

- These relationships can then be used to express the future value terms as functions of the CCP's rather than the value functions themselves

Value functions and CCPs

- The value function for unemployment can be expressed as:

$$\begin{aligned}\rho V_0(t) =& b(t) + \sum_s \sum_w \lambda^s g_w^s(t) \mathbb{E}_\varepsilon \max\{V_w^s - V_0(t) + \varepsilon, 0\} + \dot{V}_0(t) \\ =& b(t) + \sum_s \sum_w \lambda^s(t) g_w^s(t) \log\left[1 + \exp(V_w - V_0(t))\right] + \dot{V}_0(t) \\ =& b(t) - \sum_s \sum_w \lambda^s(t) g_w^s(t) \log\left[1 - p_w^s\right] + \dot{V}_0(t)\end{aligned}$$

- Similar substitutions for the value function for employment produce:

$$\begin{aligned}(\delta + \rho) V_w^s =& u_w^s + \delta^s V_0(0) + \sum_{s'} \sum_{w'} \lambda^{ss'} g_w^s(t) \mathbb{E}_\varepsilon \max\left\{V_{w'}^{s'} - V_w^s - c^{ss'} + \varepsilon, 0\right\} \\ =& u_w^s + \delta^s V_0(0) - \sum_{s'} \sum_{w'} \lambda^{ss'} g_w^s(t) \log\left[1 - p_{ww'}^{ss'}\right]\end{aligned}$$

What is observed in the data

1. $h^{ss'}_{ww'}$, the hazard rate of moving from a job of type s with wage w to a job of type s' with wage w'
    - Count the number of times in the data move from job type s and wage w to job type s' and wage w' and divide by the total time spent in job type s at wage w
2. $h^{s}_{w}(t)$, the hazard rate out of unemployment given a duration t to a job that pays w at job type s
3. δ^{s}, the hazard rate of moving from employment to unemployment from job type s

To keep the notation simple and help with intuition, going to focus on the case **without** job types but all the arguments go through with them

EE hazards

The hazard of moving from w to w' is given by:

$$h_{ww'} = \lambda f_{w'} p_{ww'}$$

- the offer arrival rate (λ)
- times the probability that the offer pays w' ($f_{w'}$)
- times the probability of accepting w' given current wage w ($p_{ww'}$)

Identification of offered wage distribution

Note that $p_{ww} = p_{w'w'}$ for all $\{w, w'\}$ implying:

$$\frac{h_{ww}}{h_{w'w'}} = \frac{f_w}{f_{w'}}$$

Summing over w' and rearranging terms yields:

$$f_w = \frac{h_{ww}}{\sum_{w'} h_{w'w'}}$$

- clearly goes through with job types
- doesn't depend on the logistic distribuiton

Identification of λ

The logistic distribution of the preference shocks yields the following relationship between the value functions and the CCP's:

$$\ln\left(\frac{p_{ww'}}{1-p_{ww'}}\right) = V_{w'} - V_w - c$$

implying:

$$\ln\left(\frac{p_{ww'}}{1-p_{ww'}}\right) + \ln\left(\frac{p_{w'w}}{1-p_{w'w}}\right) = -2c$$

Identification of λ cont.

Using the fact that:

$$p_{ww'} = \frac{h_{ww'}}{\lambda f_{w'}}$$

we obtain:

$$\ln\left(\frac{h_{ww'}}{\lambda f_{w'} - h_{ww'}}\right) + \ln\left(\frac{h_{w'w}}{\lambda f_{w} - h_{w'w}}\right) = -2c$$

It follows that the following equality holds for any given triplet $\{w, w', \tilde{w}\}$:

$$\left(\frac{h_{ww'}}{\lambda f_{w'} - h_{ww'}}\right) \ln\left(\frac{h_{w'w}}{\lambda f_{w} - h_{w'w}}\right) = \left(\frac{h_{w\tilde{w}}}{\lambda f_{\tilde{w}} - h_{w\tilde{w}}}\right)\left(\frac{h_{\tilde{w}w}}{\lambda f_{w} - h_{\tilde{w}w}}\right)$$

Identification of λ cont. 2

Looks like a quadratic, but the part that doesn't have a λ after cross-multiplying is zero...

Solving for λ yields:

$$\lambda = \frac{[f_{\tilde{w}} h_{\tilde{w}w} + f_w h_{w\tilde{w}}] h_{ww'} h_{w'w} - [f_{w'} h_{w'w} + f_w h_{ww'}] h_{w\tilde{w}} h_{\tilde{w}w}}{f_{\tilde{w}} f_w h_{ww'} h_{w'w} - f_{w'} f_w h_{w\tilde{w}} h_{\tilde{w}w}}$$

Identification of $p_{ww'}$ and c

We can then recover $p_{ww'}$ using:

$$p_{ww'} = \frac{h_{ww'}}{\lambda f_{w'}}$$

and c using:

$$\ln\left(\frac{h_{ww'}}{\lambda f_{w'} - h_{ww'}}\right) + \ln\left(\frac{h_{w'w}}{\lambda f_{w} - h_{w'w}}\right) = -2c$$

Identification of u_w

- Consider the log odds of choosing to accept a job offering w' when the current job pays w.
- Expressing the Emax term with respect to the value of the new job, yields:

$$\ln\left(\frac{p_{ww'}}{1-p_{ww'}}\right)=\frac{u_{w'}-u_w+\lambda\sum_{\tilde{w}}\left[\ln\left(p_{w\tilde{w}}\right)-\ln\left(p_{w'\tilde{w}}\right)\right]f_{\tilde{w}}}{\rho+\delta+\lambda}-c$$

  where everything is known except for $u_{w'}$ and u_w.
- The flow utility u_w is then identified up to a constant.

Identification of $g_w(t)$

- Note that the difference in log odds between accepting a job that pays w versus w' out of unemployment is:

$$\ln\left(\frac{p_w(t)}{1-p_w(t)}\right) - \ln\left(\frac{p_{w'}(t)}{1-p_{w'}(t)}\right) = V_w - V_{w'}$$

  where $V_w - V_{w'}$ is known
- We can then substitute in for $p_w(t)$ with:

$$p_w(t) = \frac{h_w(t)}{\lambda(t)g_w(t)}$$

Identification of $g_w(t)$ 2

- Denoting $\kappa_{ww'} = V_w - V_{w'}$ and solving for $1/\lambda_0(t)$ yields:

$$\frac{1}{\lambda_0(t)} = \frac{g_w(t)h_{w'}(t)\exp(\kappa_{ww'}) - g_{w'}(t)h_w(t)}{h_w(t)h_{w'}(t)\exp(\kappa_{ww'} - 1)} \tag{1}$$

- We can get some intuition for what is happening in (1) by decomposing the hazards as $h_w(t) = h(t)g^*_w(t)$ where $g^*_w(t)$ is the accepted wage distribution out of unemployment at t.
- Substituting in and simplifying yields:

$$\frac{1}{\lambda_0(t)} = \frac{g_w(t)\exp(\kappa_{ww'})}{g^*_w(t)h(t)\exp(\kappa_{ww'} - 1)} - \frac{g_{w'}(t)}{g^*_{w'}(t)h(t)\exp(\kappa_{ww'} - 1)} \tag{2}$$

so that the (unknown) offered wages are markdowns of the accepted wages.

Identification of $g_w(t)$ 3

- We can difference out the $1/\lambda(t)$ term using a different pairs of wages yielding a linear system with $W-1$ unknowns at each t
- Lots of equation with the different pairs but only $W-2$ are non-redundant
- Need to place some structure on the offered wage distribution out of unemployment
- One example: restrict the cdf of offered wages out of unemployment over time such that $G_w(t) = G_w(1)^{\alpha(t)}$ where $0 \leq \alpha(t) \leq 1$ and we get $\alpha(t)$ to be falling over time

Identification of $\lambda(t)$ and $p_w(t)$

Identification of $\lambda(t)$ and $p_w(t)$ immediately follow from:

$$\lambda(t) = \frac{h_w(t)h_{w'}(t)\exp(\kappa_{ww'} - 1)}{g_w(t)h_{w'}(t)\exp(\kappa_{ww'}) - g_{w'}(t)h_w(t)}$$

$$p_w(t) = \frac{h_w(t)}{\lambda(t)g_w(t)}$$

Identification of $V_0(t)$

To recover $V_0(t)$, we express the log odds of taking a job at w and normalize the value of working relative to staying at the same job:

$$\ln\left(\frac{p_w(t)}{1-p_w(t)}\right) = \frac{u_w - \delta V_0(0) - \lambda \sum_{w'} \ln(1-p_{ww'}) f_{w'}}{\rho+\delta} - V_0(t)$$

Evaluating at $V_0(0)$ and rearranging yields:

$$V_0(0) = \frac{1}{\rho}\left[u_w - \lambda \sum_{w'} \ln\left(1-p_{ww'}\right) f_{w'}\right] - \frac{\rho+\delta}{\rho} \ln\left(\frac{p_w(0)}{1-p_w(0)}\right)$$

Identification of $V_0(t)$ immediately follows and we can recover $\dot{V}_0(t)$ by differentiating

Identification of $b(t)$

Identification of $b(t)$ also follows immediately:

$$b(t) = \rho V_0(t) + \lambda(t) \sum_w \ln\left(1 - p_w(t)\right) g_w(t) - \dot{V}_o(t)$$

Data

- Use administrative data from Hungary, focusing on Budapest
- Labor market information on half the Hungarian population; those born on January 1st 1927 and every second day thereafter
- Know primary firm on the 15th of the month
- See both labor income for the whole month as well as how much the primary firm paid
- Coupled with information on days worked, get close to the exact date a job change occurred
- Focus on males aged 25-50 over the time period January, 2003 to November 2007
- For the unemployed, restrict to 2004 to 2005

Hungarian Unemployment Benefits

- Focus on this time period because Hungary's unemployment system was stable over it
- Roughly speaking:
  1. Eligible for *unemployment benefits* for up to 270 days
  2. Once those are exhausted, eligible for *unemployment assistance* for another 180 days (longer if older than 45)
  3. Unemployment benefits pay more than unemployment assistance

Data Processing

1. Equally-spaced deciles (either 10 or 50) with the exception of the first bin
2. First bin includes those making between 75% and 107% of the minimum wage
3. Treat each year separately due to minimum wage changes and an inordinate amount of wage increases at 1st of the year

Job to Job Transitions

Current wage \ Accepted wage	1	2	3	4	5	6	7	8	9	10
1	19,728	4,240	2,597	2,278	2,003	1,643	1,112	928	712	499
2	3,996	2,845	1,547	1,121	828	747	476	365	284	172
3	2,092	1,329	2,273	1,411	969	701	478	358	221	137
4	1,531	759	1,066	2,009	1,283	1,031	615	462	260	145
5	1,315	501	619	1,080	2,082	1,297	760	561	300	206
6	921	413	436	582	999	2,109	1,433	870	443	263
7	715	308	318	364	539	877	2,046	1,295	753	314
8	568	239	194	289	365	418	734	1,850	1,426	603
9	414	166	166	200	305	290	405	747	2,234	1,658
10	336	132	150	187	228	266	302	411	898	4,591

Unemployment to Job Transitions

(a) All transitions

	Overall	**By unemployment duration (days)**				
		1–30	*31–60*	*61–90*	*91–180*	*181–269*
Mean U duration (days)	111.5	20.5	46.1	75.7	130.7	220.0
Mean acc. wage (HUF)	2,715	3,102	2,898	2,712	2,649	2,444
Share $\underline{w}$	30.6	20.3	22.8	29.3	34.2	38.6

Estimation

Step 0 Pre-classify firms

- 3 firms: small, low CD, high CD

Step 1 Estimate reduced form hazards out of particular wages and job types, transitions over firm types conditional on current wages, accepted wages, initial firm types, and initial wages

- 3 unobserved worker types
- $h_{ww'}^{ss'} = h_w^s p_w^{ss'*} f_{ww'}^{ss'*}$
- impose structure on the reduced form such that:
  - hazards out of wages are monotonically decreasing in current wages conditional on current firm type
  - offered wage distribution always worse than accepted wage distribution
- recover conditional type probabilities and offered wage distributions

Estimation

Step 2 Estimate remaining E-side parameters: $\lambda^{ss'}$ and parameters of the utility function

Step 3 Estimate U-side parameters sequentially:

1. offered wage distribution out of unemployment $g_w^s(t)$
2. $\lambda^s(t)$
3. $V_0(t)$ and $\dot{V}_0(t)$
4. unemployed benefits $b(t)$

Estimation Results (still working...)

Employed side:

- Heterogeneity in offer arrival rates by firm and worker type
  - but current firm type has little effect on offer arrival rates
- Worker types with a preference for a particular firm type more likely to get offers from that firm type

Unemployed side:

- Substantial decreases in both offer arrival rates and offered wages over time