A Hidden Markov Model of Wages and Employment Mobility with Worker and Firm Heterogeneity

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Introduction

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- A framework for the study of the determinants of wages and job mobility. Builds on Lentz, Piyapromdee, and Robin (2023).
- Identification of latent firm and worker types. Worker types dynamic.
- Speaks to for example human capital dynamics applications like Bagger, Fontaine, Postel-Vinay, and Robin (2014), Lise and Postel-Vinay (2020), Taber and Vejlin (2020), Lentz and Roys (2024).
- Preliminary results on Italian data. Proof of concept before move to (much higher quality) Danish data.
- Contemplating adoption of VEM estimation approach to the firm classification problem.

Model

Matches and agent heterogeneity

- A job is a match between a worker and a firm.
- A worker is characterized by (k, x),
  - latent type $k \in \{1, ..., K\}$.
  - observable, time varying characteristics, x. (Likely only tenure)
- A firm is characterized by latent type $\ell \in \{1, ..., L\}$.
- At any given time, a worker can be matched with at most one firm or be non-employed.
- A firm can be matched with many workers.

Match Wages

- Match wages are log-normally distributed.
- Specifically, log wage, w, is distributed according to,

$$f_{k\ell}(w|x) = \frac{1}{\sigma_{k\ell}(x)}\varphi\left(\frac{w-\mu_{k\ell}(x)}{\sigma_{k\ell}(x)}\right).$$

  - $\mu_{k\ell}(x)$ is a k-worker's average log-wage when matched with an ℓ-firm.
  - $\sigma_{k\ell}(x)$ is the standard deviations of the noise innovations.
  - $\varphi(\cdot)$ is the Gaussian kernel.

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- The worker accepts the outside offer with probability,

$$P_{k\ell\ell'}(x) = \frac{\gamma_{k\ell'}(x)}{\gamma_{k\ell}(x) + \gamma_{k\ell'}(x)},$$

  where $\gamma_{k\ell}(x) = \exp\big(V_{k\ell}(x)/\nu_{kx}\big)$ reflects the worker's valuation of a match with an ℓ-firm.
  - *Random utility interpretation:* At point of choice, each match is subject to iid Gumbel distributed value addition. ν_{kx} variance parameter.
  - *Random mobility cost interpretation:* At point of choice, mobility cost realization drawn from logit distribution. ν_{kx} variance parameter.
  - Sorkin (2018) proposes a restricted version.
  - Logit choice in search also in Lentz, Maibom, and Moen (2023) and Arcidiacono, et al. (2023)

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- $M_{k\ell\ell'}(x) = \lambda_{k\ell'}(x) P_{k\ell\ell'}(x)$ for $\ell, \ell' > 0$.

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- $M_{k\ell\ell'}(x) = \lambda_{k\ell'}(x) P_{k\ell\ell'}(x)$ for $\ell, \ell' > 0$.
- Employed k-worker is laid off into non-employment with probability $\delta_{k\ell}(x) = M_{k\ell 0}(x)$.
- Non-employed k-worker finds a job with an ℓ-firm with probability $\varphi_{k\ell}(x) = M_{k0\ell}(x)$.

Worker type transitions

- A worker's type k follows a hidden Markov process.
- Each spell-year a type (k, x) worker matched with type ℓ draws a type realization from $A\left(k' \mid k, x, \ell\right)$.

Initialization

- Initial worker type distribution, π^w
- Initial firm type distribution, π^f.
- Initial match distribution, $m(k, \ell)$.

Data and Estimation

Data

- Italian register data, 1982-2001.
- Data on monthly wages, worker and employer IDs.
- Observable worker characteristics: potential experience, tenure in the job, age, sex.
- As in Lentz, Piyapromdee, and Robin (2023), more observable characteristics can be included in analysis with use of for instance Danish data.

Worker and firm classification

- Adopt a finite mixture approach as for example Bonhomme, Lamadon and Manresa (2019)
- Worker latent types $k = 1, \ldots, K$; firm latent types $\ell = 1 \ldots, L$.

Likelihood given firm classification

- The complete likelihood of worker i's history and that s/he is type k is

$$\mathcal{L}_i(k_1, \ldots, k_{S_i}; \beta, F) = \frac{m_{k\ell_{i1}}(x)\pi_k^w \pi_{\ell_{i1}}^f}{\#(\ell_{i1}|F)} \prod_{s=1}^{S_i} \left[f(w_{is}|k_s, l_{is}, x_{is}) M(\neg|k_s, l_{is}, x_{is})^{d_{is}-D_{is}} \right] \times$$
$$\prod_{s=1}^{S_i-1} \left[\left(\frac{M(l_{i,s+1}|k_s, l_{is}, x_{is})}{\#(\ell_{is}|F)} \right)^{D_{is}} A(k_{s+1}|k_s, l_{i,s+1}, x_{i,s+1}) \right],$$

where
- $D_{is} = 1$ if employment transition (E-U, U-E and E-E).
- d_{is} is duration of spell in spell-year s.
- $F = (\ell_1, \ldots, \ell_J)$ is firm classification.
- $\ell_{is} = \ell_{j_{is}}$, where j_{is} is firm matched to worker i at time s. $\ell_{is} = 0$ if unmatched.
- β are the model parameters.

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$$\prod_{s=1}^{S_i-1}\left[\left(\frac{M(l_{i,s+1}|k_s,l_{is},x_{is})}{\#(\ell_{is}|F)}\right)^{D_{is}}A(k_{s+1}|k_s,l_{i,s+1},x_{i,s+1})\right],$$

  where
  - $D_{is}=1$ if employment transition (E-U, U-E and E-E).
  - d_{is} is duration of spell in spell-year s.
  - $F=(\ell_1,\ldots,\ell_J)$ is firm classification.
  - $\ell_{is}=\ell_{j_{is}}$, where j_{is} is firm matched to worker i at time s. $\ell_{is}=0$ if unmatched.
  - β are the model parameters.
- The marginal likelihood is $\mathcal{L}_i(\beta,F)=\sum_k\mathcal{L}_i(k;\beta,F)$.

Likelihood Maximization by EM algorithm

- Rabiner (1989) is a nice tutorial on estimation of HMM's by the EM algorithm.
- M-step: Iteration $m+1$ model parameters, $\beta^{(m+1)}$, are obtained as the maximand of the expected complete log likelihood, $\beta^{(m+1)} = \arg\max_\beta \sum_{i=1}^{I} E_{\beta^{(m)}} \ln \mathcal{L}_i(k_1, \ldots, k_{S_i}; \beta, F)$.
  - The M-step ends up requiring two objects:

$$p_{is}^{(m)}(k) = \Pr(k_{is} = k | X_i, \beta^{(m)})$$
$$p_{is}^{(m)}(k, k') = \Pr(k_{is} = k, k_{i,(s+1)} = k' | X_i, \beta^{(m)})$$

- E-step: For given model parameters, $\beta^{(m)}$ determine individual i posterior to allow determination of expectation, $E_{\beta^{(m)}}$.
- The E step for hidden Markov models is referred to as the Baum-Welch algorithm.

Baum-Welch

- Forward step: $F_{is}(k|\beta^{(m)}) = \Pr(X_{i,1:s}, k_{is} = k|\beta^{(m)})$ by recursion,

$$F_{is}(k) = \left(\sum_{k'=1}^{K} F_{i,(s-1)}(k')\Pr(k_{is} = k|k_{i,(s-1)} = k', X_{i,(s-1)})\right)\Pr(X_{i,s}|k_{is} = k),$$

  and appropriate initialization of $F_{i1}(k)$.
- Backward step: $B_{is}(k|\beta^{(m)}) = \Pr(X_{i,(s+1):S_i}|X_{i,1:s}, k_{is} = k, \beta^{(m)})$, by recursion,

$$B_{is}(k) = \sum_{k'=1}^{K}\left(\Pr(k_{i,(s+1)} = k'|k_{is} = k)\Pr(X_{i,(s+1)}|k_{i,(s+1)} = k')B_{i,(s+1)}(k')\right),$$

  with intialization $B_{i,S_i}(k) = 1$.

Baum-Welch posteriors

- The forward and backward pass coefficients deliver the posteriors,

$$
\begin{aligned}
p_{is}(k) &= \Pr(k_{is} = k | X_{1:S_i}) \\
&= \frac{\Pr(X_{i,1:S_i}, k_{is} = k)}{\sum_{k'=1}^{K} \Pr(X_{i,1:S_i}, k_{is} = k)} \\
&= \frac{F_{is}(k) B_{is}(k)}{\sum_{k'=1}^{K} F_{is}(k') B_{is}(k')} \\
p_{is}(k, k') &= \Pr(k_{is} = k, k_{i,(s+1)} = k' | X_{1:S_i}) \\
&= \frac{\Pr(X_{i,1:S_i}, k_{is} = k, k_{i,(s+1)} = k')}{\sum_{k,k'} \Pr(X_{i,1:S_i}, k_{is} = k, k_{i,(s+1)} = k')} \\
&= \frac{F_{is}(k) \Pr(X_{i,(s+1)} | k_{i,(s+1)} = k') \Pr(k_{i,(s+1)} = k' | k_{is} = k) B_{i,(s+1)}(k')}{\sum_{k'=1}^{K} F_{is}(k') B_{is}(k')}
\end{aligned}
$$

Firm classification

- For the preliminary results today, I work just with a simple k-means clustering on centroid that includes 10 wage CDF moments and 2 moments that capture worker inflow and outflow intensities.
- We are seeing promise in a VEM approach to the problem.

VEM

- Let X be observed data. $Z = (Z^w, Z^f)$ is missing data (indicator variables on types).
- Denote by $\ln \mathcal{L}(X)$ the log-likelihood of the data.
- For any distribution $R(Z)$ (and using $\mathcal{L}$ also for probabilities, when appropriate).

$$\begin{aligned}\ln \mathcal{L}(X) &= \sum_Z R(Z) \ln \mathcal{L}(X) = \sum_Z R(Z) \ln \left(\frac{\mathcal{L}(X, Z)}{\mathcal{L}(Z \mid X)} \right) \\ &= \sum_Z R(Z) \ln \mathcal{L}(X, Z) - \sum_Z R(Z) \ln \mathcal{L}(Z \mid X) \\ &\geq \sum_Z R(Z) \ln \mathcal{L}(X, Z) - \sum_Z R(Z) \ln R(Z),\end{aligned}$$

  where inequality obtains from Gibbs, $-\sum_Z R(Z) \ln R(Z) \leq -\sum_Z R(Z) \ln \mathcal{L}(Z \mid X)$.
- If $\mathcal{L}(Z \mid X)$ is tractable, then EM algorithm is available to maximize likelihood. Uses $R^*(Z) = \mathcal{L}(Z \mid X)$
- VEM: Choose feasible $R(Z)$ so as minimize distance to $\mathcal{L}(Z \mid X)$. Equivalently,

$$R^* = \arg\max_{R \in \mathcal{R}} \sum_Z R(Z) \ln \mathcal{L}(Z \mid X) - \sum_Z R(Z) \ln R(Z).$$

Firm prior independence

- $\mathcal{L}(Z \mid X) = \mathcal{L}(Z^W \mid Z^f, X)\, \mathcal{L}(Z^f \mid X)$.
- In our setup, intractability arises because of posterior dependence of firm types through their common workers.
- Zhao, Hao, Zhu (2024): VEM on bipartite degree-corrected stochastic latent block model.
- Maintain $p_i = (p_{ik}(s)) \in [0,1]^{K \times S_i}$ to describe the worker classification, Z^w.
- Similarly for firms, let $\tau_j = \left(\tau_{j\ell}\right) \in [0,1]^L$, where $\tau_{j\ell} = \Pr_{R^f}\left\{Z^f_{j\ell} = 1\right\}$.
- We will approximate $\mathcal{L}(Z^W \mid Z^f, X)\, \mathcal{L}(Z^f \mid X)$ by $R(Z) = R^W(Z^W) R^f(Z^f)$, combined with an assumption of independence between firm priors,

$$R^f(Z^f) = \prod_{j=1}^{J} \prod_{\ell=1}^{L} \tau_{j\ell}^{Z^f_{j\ell}}.$$

M-step

- Model parameters maximize expected log likelihood, $\sum_{Z^f} R^f(Z^f) \sum_{Z^w} R^w(Z^w) \ln \mathcal{L}(X, Z^f, Z^w)$.
- As before, first order conditions involve $p_{is}(k)$, $p_{is}(k, k')$. Now, also $\tau_{j\ell}$.

Optimal classifications

- Worker and firm E steps set classifications by,

$$
\begin{aligned}
R^* &= \arg\max_{R \in \mathcal{R}} \sum_Z R(Z) \ln \mathcal{L}(Z \mid X) - \sum_Z R(Z) \ln R(Z) \\
&= \arg\max_{R^w, R^f} \Big[\sum_{Z^f} R^f(Z^f) \sum_{Z^w} R^w(Z^w) \ln \mathcal{L}(X, Z^f, Z^w) \\
&\quad - \sum_{Z^w} R^w(Z^W) \ln R^w(Z^W) - \sum_{Z^f} R^f(Z^f) \ln R^f(Z^f). \Big]
\end{aligned}
$$

Worker E step

- Problem becomes,

$$\max_{R^w} \sum_{Z^w} R^w(Z^w) \ln \left[\exp\left(\sum_{Z^f} R^f(Z^f) \ln \mathcal{L}(X, Z^f, Z^w) \right) \right] - \sum_{Z^w} R^w(Z^W) \ln R^w(Z^W).$$

- Delivers that optimal worker priors are independent across workers and,

$$R_i^w(Z_i^w) = \frac{\exp \sum_{Z^f} R^f(Z^f) \ln \mathcal{L}_i(X_i, Z^f, Z_i^w)}{\sum_{Z^w} \exp \sum_{Z^f} R^f(Z^f) \ln \mathcal{L}_i(X_i, Z^f, Z_i^w)}.$$

- Analogous to standard worker E step, but now involves integrations over the firm classification.
- Given the assumption of firm prior independence, the integrated log likelihood $\sum_{Z^f} R^f(Z^f) \ln \mathcal{L}_i(X_i, Z^f, Z_i^w)$ can be calculated with same Baum-Welch approach. Only, probability of each emission $\Pr(X_{i,s} | k_{is} = k)$ now comes to involve an integration over the involved firm type priors.

Firm E step

- The firm prior independence assumption implies first order conditions for $\tau_{j\ell}$ directly from the expected log likelihood,

$$\ln \tau_{j\ell} \propto \frac{\partial \sum_{Z^f} R^f(Z^f) \sum_{Z^w} R^w(Z^w) \ln \mathcal{L}(X, Z^f, Z^w)}{\partial \tau_{j\ell}}.$$

- Has straightforward analytical expression.
- Solve by your favorite sparse non-linear equation solver.

Preliminary Results

Types

- $K = 12$ and $L = 6$.
- Firm type classification by k-means. Wage CDF, worker inflow and outflow statistics.
- Invariant observed characteristics: $z \in 1, \ldots, 8$. (Age) by (sex). Age: (<30), (30-35), (35-45), (>45). Used in initial worker type realization distribution.
- Wages and mobility patterns depend only on worker type.
- Wage types assigned through the linear projection:

$$\mu_{k\ell} = \bar{\mu} + a_k + b_\ell + \tilde{\mu}_{k\ell}.$$

- a_k worker wage type.
- b_ℓ firm wage type.

Transition matrix restriction

- Restrict to allow non-zero coefficients only on diagonal and 2 subdiagonals.

Wage decomposition and correlations

Year	Var(μ)	Var(a)/ Var(μ)	Var($\tilde{\mu}$)/ Var(μ)	Var(b)/ Var(μ)	Cor(a, b)
1982	0.0578	0.4114	0.2910	0.2883	0.0281
1983	0.0584	0.4145	0.2812	0.2774	0.0492
1984	0.0584	0.4163	0.2750	0.2789	0.0558
1985	0.0586	0.4228	0.2614	0.2713	0.0767
1986	0.0589	0.4287	0.2534	0.2642	0.0999
1987	0.0599	0.4362	0.2384	0.2522	0.1413
1988	0.0611	0.4336	0.2249	0.2412	0.1828
1989	0.0620	0.4388	0.2172	0.2316	0.2041
1990	0.0629	0.4357	0.2105	0.2227	0.2350
1991	0.0633	0.4364	0.2071	0.2187	0.2508
1992	0.0633	0.4378	0.2054	0.2164	0.2541
1993	0.0634	0.4345	0.2080	0.2147	0.2483
1994	0.0639	0.4289	0.2113	0.2145	0.2506
1995	0.0644	0.4264	0.2126	0.2110	0.2540
1996	0.0648	0.4251	0.2150	0.2077	0.2517
1997	0.0648	0.4278	0.2157	0.2081	0.2506
1998	0.0649	0.4317	0.2187	0.2060	0.2432
1999	0.0660	0.4273	0.2189	0.2014	0.2520
2000	0.0666	0.4211	0.2219	0.2008	0.2491
2001	0.0666	0.4226	0.2189	0.2066	0.2403

Transition matrix $\Pr(\tilde{a}_k|a_k)$, non-employment

$a_k \setminus \tilde{a}_k$	0.090	0.019	0.177	0.469	0.210	0.043	−0.090	−0.189	−0.235	−0.231	−0.269	−0.170
0.090	0.836	0.164										
0.019	0.005	0.836	0.159									
0.177		0.066	0.918	0.016								
0.469			0.190	0.467	0.344							
0.210				0.004	0.979	0.017						
0.043					0.572	0.355	0.072					
−0.090						0.007	0.647	0.346				
−0.189							0.028	0.950	0.023			
−0.235								0.233	0.757	0.010		
−0.231									0.041	0.704	0.255	
−0.269										0.005	0.990	0.005
−0.170											0.278	0.722

Transition matrix $\Pr(\tilde{a}_k|a_k)$, Employment, $\ell = 1$.

$a_k \setminus \tilde{a}_k$	0.090	0.019	0.177	0.469	0.210	0.043	−0.090	−0.189	−0.235	−0.231	−0.269	−0.170
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0.177		0.066	0.918	0.016								
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−0.235								0.233	0.757	0.010		
−0.231									0.041	0.704	0.255	
−0.269										0.005	0.990	0.005
−0.170											0.278	0.722

Transition matrix $\Pr(\tilde{a}_k|a_k)$, Employment, $\ell = 2$

$a_k \setminus \tilde{a}_k$	0.090	0.019	0.177	0.469	0.210	0.043	−0.090	−0.189	−0.235	−0.231	−0.269	−0.170
0.090	0.865	0.135										
0.019	0.053	0.926	0.021									
0.177		0.026	0.944	0.030								
0.469			0.032	0.841	0.127							
0.210				0.116	0.814	0.070						
0.043					0.097	0.735	0.168					
−0.090						0.121	0.811	0.069				
−0.189							0.194	0.701	0.105			
−0.235								0.159	0.704	0.137		
−0.231									0.095	0.884	0.021	
−0.269										0.057	0.726	0.217
−0.170											0.083	0.917

Transition matrix $\Pr(\tilde{a}_k|a_k)$, Employment, $\ell = 3$

$a_k \setminus \tilde{a}_k$	0.090	0.019	0.177	0.469	0.210	0.043	−0.090	−0.189	−0.235	−0.231	−0.269	−0.170
0.090	0.805	0.195										
0.019	0.123	0.842	0.035									
0.177		0.015	0.940	0.044								
0.469			0.143	0.819	0.038							
0.210				0.158	0.794	0.048						
0.043					0.184	0.774	0.043					
−0.090						0.247	0.705	0.048				
−0.189							0.226	0.671	0.103			
−0.235								0.339	0.597	0.064		
−0.231									0.205	0.720	0.075	
−0.269										0.104	0.753	0.143
−0.170											0.164	0.836

Transition matrix $\Pr(\tilde{a}_k|a_k)$, Employment, $\ell = 4$

$a_k \setminus \tilde{a}_k$	0.090	0.019	0.177	0.469	0.210	0.043	−0.090	−0.189	−0.235	−0.231	−0.269	−0.170
0.090	0.858	0.142										
0.019	0.212	0.494	0.295									
0.177		0.200	0.669	0.131								
0.469			0.012	0.962	0.026							
0.210				0.103	0.860	0.037						
0.043					0.138	0.824	0.038					
−0.090						0.176	0.771	0.052				
−0.189							0.137	0.779	0.084			
−0.235								0.123	0.817	0.059		
−0.231									0.149	0.790	0.060	
−0.269										0.161	0.763	0.076
−0.170											0.160	0.840

Transition matrix $\Pr(\tilde{a}_k|a_k)$, Employment, $\ell = 4$

$a_k \setminus \tilde{a}_k$	0.090	0.019	0.177	0.469	0.210	0.043	−0.090	−0.189	−0.235	−0.231	−0.269	−0.170
0.090	0.785	0.215										
0.019	0.050	0.881	0.069									
0.177		0.202	0.752	0.045								
0.469			0.034	0.900	0.066							
0.210				0.097	0.864	0.039						
0.043					0.117	0.853	0.030					
−0.090						0.138	0.766	0.097				
−0.189							0.250	0.584	0.166			
−0.235								0.401	0.439	0.161		
−0.231									0.099	0.827	0.074	
−0.269										0.182	0.691	0.126
−0.170											0.081	0.919

Transition matrix $\Pr(\tilde{a}_k|a_k)$, Employment, $\ell = 5$

$a_k \setminus \tilde{a}_k$	0.090	0.019	0.177	0.469	0.210	0.043	−0.090	−0.189	−0.235	−0.231	−0.269	−0.170
0.090	0.911	0.088										
0.019	0.025	0.901	0.075									
0.177		0.064	0.912	0.024								
0.469			0.009	0.944	0.047							
0.210				0.107	0.855	0.038						
0.043					0.107	0.849	0.044					
−0.090						0.106	0.817	0.076				
−0.189							0.427	0.180	0.393			
−0.235								0.095	0.764	0.141		
−0.231									0.183	0.775	0.042	
−0.269										0.236	0.503	0.260
−0.170											0.087	0.913

Transition matrix $\Pr(\tilde{a}_k|a_k)$, Employment, $\ell = 6$

$a_k \setminus \tilde{a}_k$	0.090	0.019	0.177	0.469	0.210	0.043	−0.090	−0.189	−0.235	−0.231	−0.269	−0.170
0.090	0.922	0.078										
0.019	0.181	0.749	0.070									
0.177		0.012	0.976	0.012								
0.469			0.121	0.865	0.014							
0.210				0.153	0.818	0.029						
0.043					0.198	0.773	0.030					
−0.090						0.241	0.718	0.041				
−0.189							0.190	0.755	0.056			
−0.235								0.187	0.708	0.105		
−0.231									0.073	0.879	0.048	
−0.269										0.398	0.452	0.150
−0.170											0.171	0.829

Worker wage type change - immediate firm type treatment

$a_k \setminus b_\ell$		-0.2828	-0.1047	-0.0475	0.0628	0.1588	0.1803
-0.2693	0.0006	0.0237	0.0348	0.0194	0.0136	0.0181	0.0300
-0.2349	0.0107	0.0078	0.0048	0.0189	0.0059	0.0158	0.0089
-0.2314	−0.0098	−0.0011	−0.0023	−0.0032	−0.0028	−0.0036	−0.0021
-0.1890	0.0017	0.0144	0.0243	0.0172	0.0098	0.0177	0.0163
-0.1702	−0.0276	−0.0083	−0.0086	−0.0080	−0.0159	−0.0162	−0.0169
-0.0897	−0.0335	0.0092	0.0066	0.0088	0.0183	0.0280	0.0281
0.0191	0.0256	0.0071	0.0136	0.0145	0.0617	0.0143	0.0240
0.0435	0.0858	−0.0062	0.0121	0.0154	0.0179	0.0250	0.0290
0.0905	−0.0117	−0.0096	−0.0063	−0.0154	−0.0101	−0.0139	−0.0056
0.1775	−0.0057	0.0048	−0.0032	−0.0188	0.0066	0.0105	0.0017
0.2103	−0.0019	0.0183	0.0214	0.0186	0.0204	0.0329	0.0347
0.4687	−0.1441	−0.0420	−0.0147	−0.0269	−0.0101	−0.0514	−0.0388
Avg	−0.0091	0.0015	0.0069	0.0034	0.0096	0.0064	0.0091

Worker wage type change - firm type treatment 5 yrs out

$a_k \setminus b_\ell$		-0.2828	-0.1047	-0.0475	0.0628	0.1588	0.1803
-0.2693	0.0079	0.0139	0.0203	0.0276	0.0201	0.0250	0.0292
-0.2349	0.0436	0.0389	0.0389	0.0473	0.0419	0.0492	0.0210
-0.2314	-0.0133	0.0025	0.0081	-0.0004	0.0039	0.0060	-0.0123
-0.1890	0.0113	0.0250	0.0487	0.0423	0.0295	0.0463	0.0363
-0.1702	-0.0817	-0.0804	-0.0760	-0.0207	-0.0352	-0.0303	-0.0302
-0.0897	-0.0314	-0.0091	0.0206	0.0268	0.0439	0.0652	0.0624
0.0191	0.1023	0.0931	0.0970	0.0985	0.1110	0.0809	0.0808
0.0435	0.1095	-0.0079	0.0538	0.0591	0.0649	0.0756	0.0791
0.0905	-0.0029	-0.0018	-0.0048	0.0054	-0.0019	-0.0021	-0.0032
0.1775	-0.0029	0.0026	-0.0023	-0.0110	0.0158	0.0136	0.0030
0.2103	-0.0005	0.0061	0.0230	0.0233	0.0250	0.0421	0.0507
0.4687	-0.2122	-0.1705	-0.1428	-0.1466	-0.1199	-0.1359	-0.1038
Avg	-0.0058	-0.0073	0.0070	0.0126	0.0166	0.0196	0.0177

Concluding thoughts

- Not shown: Estimator captures truth on simulate data quite well.
- Still early days.
- Much work to be done on proper restrictions on type transition matrix.
- Possibly identify type transition matrices by observed worker type (say, education).
- VEM firm classification.