

# Lecture on Search and Bargaining in Decentralized Markets

---

Karam Kang  
University of Wisconsin-Madison

Econometric Society Summer School  
in Dynamic Structural Econometrics

August 8, 2024

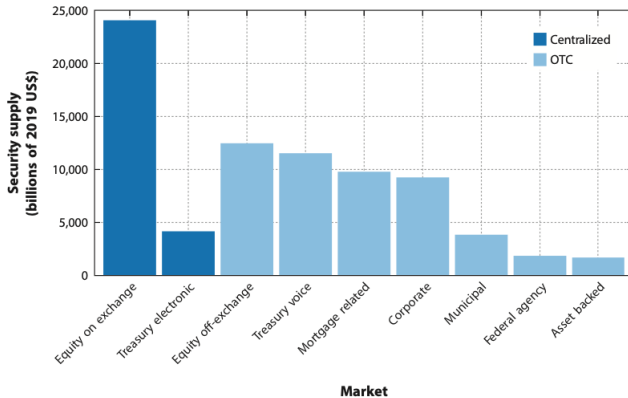
# Decentralized markets

- Labor markets
- Transportation (e.g., taxis, shipping in the international trade)
- Business-to-business transactions
- Various assets
  - o Consumer durable goods (e.g., cars and houses)
  - o Firm capital assets (e.g., plants and equipment)
  - o Bonds and derivatives

# Decentralized markets

- Labor markets
- Transportation (e.g., taxis, shipping in the international trade)
- Business-to-business transactions
- Various assets
  - o Consumer durable goods (e.g., cars and houses)
  - o Firm capital assets (e.g., plants and equipment)
  - o Bonds and derivatives
- Today's lecture focuses on decentralized *asset* markets when transaction data is available

# Many financial assets trade in over-the-counter (OTC) markets



**Figure 1**

Security supply outstanding in 2018, broken down between centralized and over-the-counter (OTC) markets.

## Key features of decentralized asset markets

- **Search and bargaining:** Traders must incur costs to search for trading partners and that, once a buyer and a seller meet, they must bargain to determine a price
- **Intermediaries:** In response to trading frictions, almost all decentralized markets have intermediaries

# Research questions

- What are the effects of search frictions on allocation, prices, market power of intermediaries, and welfare?
- What role do intermediaries play in the market?
- Is it a good idea to shift trades on a centralized platform? If so, how can we do it?
- ...

# Why does dynamics matter?

- Each asset trades several times during its lifetime
  - o Parties' trading decisions incorporate not only the expected cash flow that the asset generates, but also any cost that traders will incur in selling it at a later date
- Search process is dynamic
  - o Traders have options to delay trade and find some other counterparty in the market
- Learning-by-doing or network effects
  - o Intermediaries may benefit from trading over time
- Long-term relationship between investors and intermediaries

# Dynamic structural estimation of decentralized asset markets

- Many calibration papers; “Comparatively, the structural estimation of search models of OTC markets remains underdeveloped” (Weill (2020), p.767)
- Notable/recent papers using transaction data
  - Gavazza (2016) on the welfare impact of intermediaries in the secondary market for commercial aircraft
  - Brancaccio, Li & Schurhoff (2020) on dealers' learning by trading and implications of market transparency policies
  - Coen and Coen (2022) on the role of frequent traders in the sterling corporate bond market and policy implications
  - Pinter and Uslu (2022) on the welfare impact of search frictions in UK government and corporate bond market



*American Economic Review* 101 (June 2011): 1106–1143  
<http://www.aeaweb.org/articles.php?doi=10.1257/aer.101.4.1106>

## The Role of Trading Frictions in Real Asset Markets

By ALESSANDRO GAVAZZA\*

*This paper investigates how trading frictions vary with the thickness of the asset market by examining patterns of asset allocations and prices in commercial aircraft markets. The empirical analysis indicates that assets with a thinner market are less liquid—i.e., more difficult to sell. Thus, firms hold on longer to them amid profitability shocks. Hence, when markets for assets are thin, firms' average productivity and capacity utilization are lower, and the dispersions of productivity and of capacity utilization are higher. In turn, prices of assets with a thin market are lower and have a higher dispersion. (JEL A12, L11, L93)*

## Intermediation and Competition in Search Markets: An Empirical Case Study

---

Tobias Salz

*Massachusetts Institute of Technology*

Intermediaries in decentralized markets can affect buyer welfare both directly, by reducing expenses for buyers with high search cost, and indirectly, through a search externality that affects the prices paid by buyers who do not use intermediaries. I investigate the magnitude of these effects in New York City's trade-waste market, where buyers can search either by themselves or through a waste broker. Combining elements from the empirical search and procurement auction literatures, I construct and estimate a model for a decentralized market. Results from the model show that intermediaries improve welfare and benefit buyers in both the broker and the search markets.

## Search Frictions and Market Power in Negotiated-Price Markets

---

Jason Allen

*Bank of Canada*

Robert Clark

*Queen's University*

Jean-François Houde

*University of Wisconsin-Madison and National Bureau of Economic Research*

We provide a framework for empirical analysis of negotiated-price markets. Using mortgage market data and a search and negotiation model, we characterize the welfare impact of search frictions and quantify the role of search costs and brand loyalty for market power. Search frictions reduce consumer surplus by \$12/month/consumer, 28 percent of which can be associated with discrimination, 22 percent with inefficient matching, and 50 percent with search costs. Banks with large consumer bases have margins 70 percent higher than those with small consumer bases. The main source of this incumbency advantage is brand loyalty; however, price discrimination based on search frictions accounts for almost a third.

# Outline of the lecture

1. Models of decentralized asset trading
  - o Introduce a simple model and characterize the equilibrium
  - o Discuss (identification and) estimation of the model with transaction-level data
  - o Consider multiple dimensions to enrich the model (some dimensions for future research)
2. Application: My paper with Giulia Brancaccio, “Search Frictions and Product Design in the Municipal Bond Market”
  - o Brief overview of the paper: Research question, setting, and empirical approach
  - o Model and estimation

# Models of Decentralized Markets and Estimation Strategy

---

## Simple model: Setup

- Time is continuous, infinite-horizon
- A long-lived asset in fixed supply  $A \geq 0$
- A continuum of investors with a total measure normalized to 1
- An investor's flow utility:  $u(a, \nu)$ , depending on the asset holding  $a \in \mathbb{R}$  and her valuation characterized by  $\nu \in [\nu_L, \nu_H]$ , drawn from CDF  $F_\nu$
- With probability  $\alpha \in (0, 1)$ ,  $\nu$  can be redrawn (liquidity shock)
- Investors are randomly matched with other partners at rate  $\lambda$
- When they meet, they decide trade price  $p$  and quantity  $q$  via Nash bargaining with the buyer's bargaining power as  $\rho = 1/2$
- Investors discount payoffs at rate  $\delta$

## Simple model: Setup (Cont'd)

- Model primitives
  - $u(a, \nu)$ ,  $F_\nu$ ,  $\alpha$ ,  $\lambda$ ,  $\delta$ , and we set  $\rho = 1/2$
- Equilibrium objects
  - Trading rules  $p(a, \nu, a', \nu')$  and  $q(a, \nu, a', \nu')$
  - Equilibrium asset holding distribution  $\Phi(a, \nu)$
- Our focus for structural estimation:
  1. Given model primitives, is there an equilibrium? Is the equilibrium unique? How is an equilibrium characterized and solved?
  2. What do we observe about the equilibrium objects? Given what we observe, is there a (one-to-one) mapping to model primitives? If so, how do we build an estimator of the model primitives?

# Value function

- Investor's state:  $y \equiv (a, \nu)$
- Given the equilibrium distribution  $\Phi(y)$ , in steady state, an investor's value function is

$$\begin{aligned} \delta V(y) = & u(y) + \alpha \int_{\nu_L}^{\nu_H} \{V(a, \nu') - V(y)\} F_{\nu}(d\nu') \\ & + \lambda \int_{-\infty}^{\infty} \int_{\nu_L}^{\nu_H} (V(a + q(y, y'), \nu) - p(y, y') - V(y)) \Phi(da', d\nu') \end{aligned}$$

where

$$(p, q) = \arg \max_{p, q} [V(a + q, \nu) - V(y) - p]^{\frac{1}{2}} [V(a' - q, \nu') - V(y') + p]^{\frac{1}{2}}$$

s.t.

$$V(a' + q, \nu') - V(y') - p \geq 0; V(a - q, \nu) - V(y) + p \geq 0$$



## Nash bargaining price/quantity

Upon a meeting between  $y$  and  $y'$  investors,

$$(p, q) = \arg \max_{p, q} [V(a + q, \nu) - V(y) - p]^\rho [V(a' - q, \nu') - V(y') + p]^{(1-\rho)}$$

subject to participation conditions, where  $\rho = 1/2$

- $q$  maximizes the joint surplus:

$$q(y, y') = \arg \max_q \{ V(a + q, \nu) - V(y) + V(a' - q, \nu') - V(y') \}$$

- $p$  divides the total surplus according to the bargaining parameter  $\rho$

$$p = (1 - \rho) \{ V(a + q, \nu) - V(y) \} - \rho \{ V(a' - q, \nu') - V(y') \}$$

## Value function (revisited)

Steady-state investor's value function

$$\begin{aligned}\delta V(y) = & u(y) + \alpha \int_{\nu_L}^{\nu_H} \{V(a, \nu') - V(y)\} F_{\nu}(d\nu') \\ & + \lambda \int_{-\infty}^{\infty} \int_{\nu_L}^{\nu_H} (V(a + q(y, y')) - p(y, y') - V(y)) \Phi(da', d\nu')\end{aligned}$$

can now be written as:

$$\begin{aligned}\delta V(y) = & u(y) + \alpha \int_{\nu_L}^{\nu_H} \{V(a, \nu') - V(y)\} F_{\nu}(d\nu') \\ & + \lambda \int_{-\infty}^{\infty} \int_{\nu_L}^{\nu_H} \frac{1}{2} \max_q [V(a + q, \nu) - V(y) \\ & \quad + V(a - q', \nu') - V(y')] \Phi(da', d\nu')\end{aligned}$$

# Market clearing and the distribution of investor types

- $\Phi(a, \nu)$  is a joint CDF and market is cleared:

$$\int_{-\infty}^{\infty} \int_{\nu_L}^{\nu_H} \Phi(da, d\nu) = 1; \int_{-\infty}^{\infty} \int_{\nu_L}^{\nu_H} a \Phi(da, d\nu) = A$$

- Stationarity condition: At any  $(a^*, \nu^*) \in \mathbb{R} \times [\nu_L, \nu_H]$ ,

$$\begin{aligned} & -\alpha \Phi(a^*, \nu^*) (1 - F_\nu(\nu^*)) + \alpha \int_{-\infty}^{a^*} \int_{\delta^*}^{\delta_H} \Phi(da, d\nu) F_\nu(\nu^*) \\ & - \lambda \int_{-\infty}^{a^*} \int_{\nu_L}^{\nu^*} \left[ \int_{-\infty}^{\infty} \int_{\nu_L}^{\nu_H} \mathbb{I}_{\{q(a, \nu, a', \nu') > a^* - a\}} \Phi(a', \nu') \right] \Phi(da, d\nu) \\ & + \lambda \int_{a^*}^{\infty} \int_{\nu_L}^{\nu^*} \left[ \int_{-\infty}^{\infty} \int_{\nu_L}^{\nu_H} \mathbb{I}_{\{q(a, \nu, a', \nu') \leq a^* - a\}} \Phi(a', \nu') \right] \Phi(da, d\nu) = 0 \end{aligned}$$

# Existence and Uniqueness of Equilibrium

- Solving a bilateral trade model with unrestricted holdings is difficult
  - Trading rules + asset holding distribution must be pinned down simultaneously  $\rightarrow$  a complex fixed-point problem
  - Alternatives: (i) No heterogeneity in  $\nu$ , (ii) restriction on asset positions like  $\{0,1\}$
- Assuming that marginal utility of  $u(a,\nu)$  is linear and additively separable in  $(a,\nu)$  makes the problem easier

$$u(a,\nu) = \nu a - \kappa a^2$$

- Under this assumption, Theorem 1 in Uslu (2019) shows that a unique stationary equilibrium exists and characterizes the equilibrium

# Identification problem

- Recall the model primitives: Investors' flow utility  $u(\cdot, \cdot)$ ,  $F_\nu(\cdot)$ , liquidity shock frequency  $\alpha$ , meeting rate  $\lambda$ , discount rate  $\delta$
- We observe *all* transactions' price, quantity and timing
- Transaction timing data  $\Rightarrow \lambda$  is directly identified
  - o This is because trade always occurs upon meeting (which is the case in the model)
  - o If a meeting does not always end up with a trade, observed transaction frequency  $\neq \lambda$
- Discount rate assumed
- Remaining primitives:  $u(\cdot, \cdot)$ ,  $F_\nu(\cdot)$ ,  $\alpha$  (and bargaining parameter  $\rho$  which is assumed to be  $1/2$ )

## Estimation: A hypothetical case

Suppose  $(a, \nu)$  is observed for each investor/time

- $\alpha$  and  $F_\nu$  are directly identified
- So are equilibrium objects:  $\Phi(a, \nu)$ ,  $p(a, \nu, a', \nu')$  and  $q(a, \nu, a', \nu')$
- To estimate  $u(\cdot, \cdot)$ , we can match the model-predicted vs. observed prices and quantities
  1. Fix the parameters of  $u(\cdot, \cdot)$ ,  $\rho$ , say  $\psi$  at an initial guess,  $\psi_0$
  2. Given  $\Phi$ , solve for  $V(a, \nu; \psi_0)$  for all  $(a, \nu)$
  3. Given  $V(a, \nu; \psi_0)$ , solve for  $p, q$  functions
  4. Repeat until the solved  $p, q$  functions match with the observed functions

## Estimation: A hypothetical case

Suppose  $(a, \nu)$  is observed for each investor/time

- $\alpha$  and  $F_\nu$  are directly identified
- So are equilibrium objects:  $\Phi(a, \nu)$ ,  $p(a, \nu, a', \nu')$  and  $q(a, \nu, a', \nu')$
- To estimate  $u(\cdot, \cdot)$ , we can match the model-predicted vs. observed prices and quantities
  1. Fix the parameters of  $u(\cdot, \cdot)$ ,  $\rho$ , say  $\psi$  at an initial guess,  $\psi_0$
  2. Given  $\Phi$ , solve for  $V(a, \nu; \psi_0)$  for all  $(a, \nu)$
  3. Given  $V(a, \nu; \psi_0)$ , solve for  $p, q$  functions
  4. Repeat until the solved  $p, q$  functions match with the observed functions
- **WAIT!** The model will be most likely falsified by the data

## Estimation: A more realistic case

Suppose  $a$  is observed for each investor/time but  $\nu$  is NOT observed

- $\alpha$ ,  $F_\nu$ ,  $\Phi(a, \nu)$ , price/quantity functions are NOT observed
- We can still match the model-predicted vs. observed moments of prices, quantities, and asset holdings
  1. Fix the parameters of  $u(\cdot, \cdot)$ ,  $F_\nu$ ,  $\alpha$ , say  $\theta$  at an initial guess,  $\theta_0$
  2. Solve for  $V(a, \nu; \theta_0)$  and  $\Phi(a, \nu; \theta_0)$
  3. Given these objects, solve for  $p, q$  functions
  4. Repeat until the objective function based on the model-predicted vs. observed moments is maximized
- Identification?



## Enriching the model (1/2): Dealers

- Many decentralized markets involve intermediaries
- Typically all trades must be done with a dealer
- Dealers have access to inter-dealer market
  - Centralized inter-dealer market
  - Decentralized inter-dealer market
- Estimation strategy does not fundamentally change here

## Enriching the model (2/2): Endogenous search

- Suppose dealers choose meeting rate (or search intensity)  $\lambda$  at a cost
- FOC for the dealers to choose  $\lambda$  is used to identify/estimate the search cost
- Directly observing  $\lambda$  from data (given the modeling assumptions or from extra data) certainly helps

## Extra dimensions to consider

- Directed search (as opposed to random search) and alternative matching technologies
- Asymmetric information
- Multiple assets/portfolio choice

**Brancaccio & Kang (2023)**

---

# Research question

- Call for standardization in insurance, annuities, mortgage, ...
- Products with unique features are hard to evaluate
  - Product design may directly affect search costs
- Questions:
  - o Do producers benefit from designing overly complex products?
  - o If so, is there a role for regulation concerning product design?
- Context: US municipal bond market
- Whether or not standardization of municipal bonds can be beneficial has received a keen interest in policy communities

# Municipal bonds market

1. To finance a project, a government sells its bonds to an underwriter (syndicate) who then sells them to investors
  - o Underwriter can be selected by auctions or negotiations
  - o Bond design is determined at negotiations (if negotiated)
  - o Somewhat concentrated: Top 3 firms in a state cover 45%
  - o Sticky relationship: Top lead underwriter for an issuer issues 88% in volume (Chen et al, 2022)
2. Secondary markets provide liquidity after initial offering
  - o Investors' financial/tax circumstances and need for cash
  - o Underwriter can leverage information on who initially purchased its bonds and who owns them when acting as a dealer
    - this advantage is more pronounced when a bond is complex

# Municipal bond design

- Plain vanilla bond: face value, maturity & interest rate
- Often comes with **nonstandard provisions**
  - Optional redemption, sinking funds, floating/variable coupon rate, idiosyncratic coupon payment frequency, etc.
- Trade-off
  - **Flexibility in payment**: e.g. ability to refinance if rates fall
  - **Higher trading frictions** and interest costs
- SEC, Oct 2014: *“we should work to reduce the number of bespoke bond (...) if that would result in more liquidity”*

# What this paper does

1. IV to quantify the effects and distortions in bond complexity
  - o Underwriters' and government officials' rent-seeking behavior increases prevalence of complex bonds
  - o We document empirical evidence of trade-off regarding nonstandard provisions (flexibility vs. liquidity)
2. Build and estimate a model for bond design negotiation and decentralized trading
3. Study welfare impact of policies regulating bond design and reducing distortions



# Key findings from the structural analyses (1/2)

- Search frictions:
  - Average search cost is 10% of the gross profit
  - Underwriter's search cost is half of the average dealer's cost
  - Nonstandard bond provisions increase search costs and strengthen the underwriter's cost advantage
- Investor demand:
  - Complex bonds are niche products – increasing the dispersion of investor valuations, with little changes in the average
- Government preferences:
  - Government cost of paying back its debt is minimized with nonzero complexity
  - Officials' conflict of interest depends on local newspaper and repeated interactions between the issuer and the underwriter

## Key findings from the structural analyses (2/2)

What if the use of nonstandard provisions is limited, while the coupon rate is negotiated?

- Standardization would increase liquidity (+33%, median) and investors' overall surplus (+8%, median)
- Median change in coupon rate is -22.7 bp, leading to a saving of 9% of the total interest payments
  - Despite the interest savings, most local governments are unlikely to prefer this policy because they value flexibility
- Heterogeneity in coupon rate changes (IQR: [-99.6, +8.8] bp)
  - Interest savings are concentrated in low-income counties, especially those without revolving-door regulations
  - Mostly depends on the heterogeneity in underwriter's incentives to negotiate for a higher coupon rate

## 1. Bond design determined at origination

- o Official & underwriter negotiate complexity  $s$ , rate  $r$ , price  $F$
- o Underwriter purchases the bond at price  $F$

## 2. Trading subject to search frictions (based on Üslü 2019)

- o Captures underwriter's incentives for influencing bond design  $s$  to benefit as a dealer
- Heterogeneity: exogenous bond attributes
  - o Observed:  $x$  (e.g., maturity  $T$ , size  $A$ )
  - o Unobserved (to researcher):  $\xi$

# At origination

- **Underwriter's payoff**

$$\underbrace{V_U(s, r, x, \xi)}_{\text{from trading}} - F$$

- **Municipal government payoff**

$$F - c_0(s, x, \xi)A(1 + rT) + \theta_d s$$

- o  $A(1 + rT)$ : Principal and interest payment
- o  $c_0(s, x, \xi)$ : Marginal financing cost depends on attributes
- o  $\theta_d s$ : Issuer's dynamic incentives concerning bond design

# At origination

- **Underwriter's payoff**

$$\underbrace{V_U(s, r, x, \xi)}_{\text{from trading}} - F$$

- **Government official's payoff**

$$F - c_0(s, x, \xi)A(1 + rT) + \theta_d s + \psi(h, x)V_U(s, r, x, \xi)$$

- o  $A(1 + rT)$ : Principal and interest payment
  - o  $c_0(s, x, \xi)$ : Marginal financing cost depends on attributes
  - o  $\theta_d s$ : Issuer's dynamic incentives concerning bond design
  - o  $\psi(h, x)$ : Underwriter influence, varying by revolving-door ( $h$ )
- Nash bargaining  $\rightarrow$  bond design maximizes joint payoff

# Model overview (Reprise)

## 1. Bond design determined at origination

- o Official & underwriter negotiate complexity  $s$ , rate  $r$ , price  $F$
- o Underwriter purchases the bond at price  $F$

## 2. Trading subject to search frictions (based on Üslü 2019)

- o Captures underwriter's incentives for influencing bond design  $s$  to benefit as a dealer

## Modeling decentralized trades (1/4): Investors

- Large population of investors and dealers, each represented by a point in a interval with measure  $m_I(x)$  and  $m_D(x)$
- Investors meet dealers and trade in continuous time with finite horizon  $[0, T]$ , with discount rate  $\delta$
- Investors' flow utility from holding  $a \in \mathbb{R}$  units of the bond

$$\nu \log(r)a - \kappa_I(x, \xi)a^2$$

- **Heterogeneous** taste type:  $\nu$  drawn from  $F_{\nu|(\tau, s, x, \xi)}$
- **Liquidity shock**: At each instant, the investor draws a new type with probability  $a(x, \xi)$
- **Quadratic** utility in  $a$ :  $\kappa_I$  captures the opportunity cost of tying up the amount  $a$  in the bond (tractable; micro-founded from CARA utility)

## Modeling decentralized trades (2/4): Dealers

- Dealers' flow utility

$$\nu_D(x, \xi) \log(r)a - \kappa_D(x, \xi) a^2$$

- Payoff of holding  $a$  bonds at the end of maturity is  $\omega_I a$  for investors and  $\omega_D a$  for a dealer
- Dealers meet investors and other dealers to trade
  - Search frictions in inter-dealer market: Meeting rate among dealers is  $\lambda_D(x, \xi)$
  - Endogenous search frictions in dealer-to-investor trades: At each instant, meeting rate  $\lambda$  is chosen by each dealer at cost



## Modeling decentralized trades (3/4): Search frictions

- Every instant dealers choose meeting rate  $\lambda$  given search costs

$$\exp(\lambda) \times \phi_0(s, x_d, x, \xi) \underbrace{\exp(-\phi_1(s, x, \xi) \log(b+1))}_{\text{network effect}}$$

- Two components determine search costs
  - Base cost  $\phi_0$  depends on dealers' attributes  $x_d$
  - Network effects: “roledex model” of search
    - Easier to sell a bond to investors who have already traded it
    - Cost can decrease with client network  $b$
    - $b$  = cumulative trade by the dealer
- Underwriter cost advantage thanks to initial sales if  $\phi_1 > 0$

## Modeling decentralized trades (4/4): Bargaining

- Upon meeting, Nash bargaining determines trading price and quantity
  - Negotiated quantity maximizes the joint gains from trade
  - Price divides these gains, depending on the bargaining weights  $\rho_D$  (inter-dealer) and  $\rho(x, \xi)$  (dealer-to-investor)
  - Continuously distributed investor taste type + perfectly divisible bonds + short-selling is allowed  $\rightarrow$  All meetings end up with a trade

## Equilibrium in the trading market (1/3): Value functions and meeting rate

Dealer's value function:

$$\begin{aligned}\dot{V}(\tau; u) = & -\delta V(\tau; u) + v_D(a) + \lambda_D \int_{u'} \left\{ V(\tau; a + q_D(\tau; u, u'), b, \phi_0) - V(\tau, u) \right. \\ & \left. - p_D(\tau; u, u') \right\} d\Phi_D(\tau; du') + \max_{\lambda} \left[ \lambda \int_y \left\{ V(\tau; a - q_I(\tau; u, y), b+1, \phi_0) \right. \right. \\ & \left. \left. - V(\tau; u) + p_I(\tau; u, y) \right\} d\Phi_I(\tau; dy) - \phi_0 \exp(-\phi_1 \log(b+1)) \exp(\lambda) \right]\end{aligned}$$

Equilibrium meeting rate: Based on the FOC,

$$\begin{aligned}\lambda(\tau; u) = & \log \left( \frac{1}{\phi_0 \exp(-\phi_1 \log(b+1))} \int_y \left\{ V(\tau; a - q_I(\tau; u, y), b+1, \phi_0) \right. \right. \\ & \left. \left. - V(\tau; u) + p_I(\tau; u, y) \right\} d\Phi_I(\tau; dy) \right)\end{aligned}$$

## Equilibrium in the trading market (2/3): Transactions

Investor's value function:

$$\begin{aligned}\dot{W}(\tau; y) = & -\delta W(\tau; y) + v_I(y) + \alpha \int [W(\tau; a, \nu') - W(\tau; y)] f(\nu' | \tau) d\nu' \\ & + \frac{m_D}{m_I} \int_u \lambda(\tau; u) \left\{ W(\tau; a + q_I(\tau; u, y)) - W(\tau; y) - p_I(\tau; u, y) \right\} \Phi_D(\tau; du)\end{aligned}$$

Equilibrium quantity and price for dealer-to-investor trades:

$$q_I(\tau; u, y') = \arg \max_q \left\{ W(\tau; a' + q, \nu') - W(\tau; y') + V(\tau; a - q, b + 1, \phi_0) - V(\tau; u) \right\}$$

$$p_I(\tau; u, y') = \rho \max_q \left\{ W(\tau; a' + q, \nu') - W(\tau; y') - V(\tau; a - q, b + 1, \phi_0) + V(\tau; u) \right\}$$

## Equilibrium in the trading market (3/3): States

Equilibrium path of the investor state distribution:

$$\begin{aligned}-\dot{\Phi}_I(\tau; u) &= -\alpha \Phi_I(\tau; a, \nu) [1 - F(\nu|\tau)] + \alpha \int_{-\infty}^a \int_{\nu}^{\infty} \Phi_I(\tau; da, d\nu') F(\nu'|\tau) \\ &\quad - \int_{-\infty}^{\nu} \int_{-\infty}^a \int_u \lambda(\tau; u) \mathbb{I}_{\{\tilde{a} + q_I(\tau; u, \tilde{a}, \tilde{\nu}) > a\}} \Phi_D(\tau; du) \Phi_I(\tau; d\tilde{a}, d\tilde{\nu}) \\ &\quad + \int_{-\infty}^{\nu} \int_a^{\infty} \int_u \lambda(\tau; u) \mathbb{I}_{\{\tilde{a} + q_I(\tau; u, \tilde{a}, \tilde{\nu}) \leq a\}} \Phi_D(\tau; du) \Phi(\tau; d\tilde{a}, d\tilde{\nu})\end{aligned}$$

- This recursive characterization of the state distribution is essential to ensure that the model is computationally tractable

Initial conditions for the investor state distribution: Investors do not hold the asset at the beginning of the trading

$$\Phi_I(T; a, \nu) = \mathbb{I}_{\{a \geq 0\}} F_{\nu|\tau}(\nu|T)$$

## Equilibrium bond design: “Too much” complexity?

$$\max_{(s,r)} -c_0(s,x,\xi)A(1+rT) + \theta_d s + (1 + \psi(h,x))V_U(s,r,x,\xi)$$

- Underwriter value  $V_U$  does not fully incorporate investor surplus and dealers' search costs
- Why would underwriter benefit from complex bonds?
  1. Intermediaries might benefit from increasing search frictions
    - o Increase costs, but also market power
  2. Vertically integrated underwriter can “raise rivals' costs”
    - o Exclusive initial sales  $\rightarrow$  large client network ahead of others
    - o Complex bonds might strengthen network effects
- Underwriter's influence on officials *magnifies* distortion

- Primitives to recover
  - Dealers' and investors' preferences
  - Search costs and bargaining parameters
  - Government officials' preferences
- Observables: For each bond
  - Trading prices, quantities, and timing
  - Dealer's state (inventory and experience)
  - Bond attributes  $(x, s, r)$  and regulation  $h$

## Estimation strategy

1. For each bond  $i$ , use trading data to estimate search cost and investor preference parameters,  $\theta_i$
2. Use estimates  $\hat{\theta}_i$  to recover the impact of attributes on search costs and preferences
  - o Recall  $\theta_i = \theta(s_i, x_i, \xi_i)$
  - o IV approach based on revolving-door regulations
3. Estimate government preferences ( $\psi(h, x)$ ,  $c_0(x, s, \xi)$  and  $\theta_d$ ) by employing GMM based on FOC for  $(s, r)$



## Estimation procedure: Step 1, primitives for each bond

- Investors' and dealers' preferences
  - Investor taste type dist.  $F_\nu$  parameters
  - Investor inventory cost  $\kappa_I$ ; liquidity shock prob.  $\alpha$
  - Dealers' flow utility parameters  $\kappa_D$  and  $\nu_D$
- Search costs and bargaining
  - Initial search cost type dist.  $F_{\phi_0}$  parameters
  - Network parameter  $\phi_1$
  - Inter-dealer meeting rate  $\lambda_D$
  - Dealer-to-investor bargaining parameter  $\rho$
- Denote the above parameters as  $\theta_i$  for each bond  $i$
- Note: We assume to know  $m_I/m_I$ ,  $\rho_D = 0.5$ ,  $\delta = 0.5$ ,  $\omega_D = 0.75$  and  $\omega_I = 1$

## Estimation procedure: Step 1, building the estimator

- Estimate the dealer state distribution using a Kernel estimator from the observed states of dealers
- Minimize an objective function over  $\theta_i$ 
  - Given  $\theta_i$ , solve for the equilibrium using a nested fixed point algorithm (Rust, 1987)
  - Squared differences btw simulated vs. observed moments based on (1) the average inter-dealer trading price and quantity and their covariance with the dealer's inventory; (2) the average trading price and quantity for dealer-to-investor transactions, their variance, and their covariance with the dealer's inventory and trading network
  - Log-likelihood of the timing of transactions for each dealer, conditional on the dealer's state

## Discussion on identification

- No formal proof of identification
- Some (potentially) compelling arguments:
  - Gavazza (2006, p.1775) *“Specifically, although the model is highly nonlinear, so that (almost) all parameters affect all outcomes, the identification of some parameters relies on some key moments in the data”*
  - For two dealers with the same geographic specialization but different levels of inventory, high  $\kappa$  would make the after-trade inventories close to each other
  - $\nu_D$  determines the price difference between inter-dealer and dealer-to-investor trades
  - The over-time and across-dealer variation in the dealers' inventory and its correlation with trading prices is useful for identifying the bargaining parameter

## **Closing Remark and References**

---

# Takeaways

- Transaction-level data in decentralized markets offers us opportunities to study the role of search frictions and intermediaries
- Many open questions; lots of theory models on search and bargaining (and relative paucity of structural empirical work)
- Depending on questions and setting, choose your empirical approach
- All references are in the “papers” folder