

Marriage Matching with Search Friction: An Empirical Framework

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DSE2024, 6 Aug 2024

Introduction

- ▶ Interested in rationalizing the marriage distribution of 'who marries whom' by age.
- ▶ Develop an empirically tractable behavioral marriage matching model that allows for search friction in marriage.
- ▶ Propose an empirical version of the Shimer-Smith (2000) model.
- ▶ Empirically quantify the marital gains and search frictions across gender and age.
- ▶ What are relative importance of search friction and marital gains in marital decisions?

- ▶ Our equilibrium marriage matching model delivers a new closed form matching function to the matching problem with search friction.
- ▶ Develop an empirical strategy that separately identifies marital surpluses and search frictions.
 - ▶ To identify search friction, we augment cross-sectional data with marriage duration data. (A number of papers like ours uses this insight papers: notably Shin (2013), Ciscato (2023), Goussé, Jacquemet and Robin (2017))
- ▶ Apply our model to investigate how technology and social media has affected the marriage distribution in the United States from 2007/8 to 2017/18.
- ▶ Extends Choo and Siow(2006), Choo(2015), Chiappori, Salanié and Weiss (2017), and Galichon and Salanié (2022), among others.

Preview of Main Contributions

- (1) Develop a model delivers a new closed-form marriage matching function with search friction.
- (2) Propose an empirical strategy to separately identify marital gains and search friction by using cross-sectional data.
- (3) Apply our model to investigate how technology and social media has affected the marriage distribution in the United States from 2007/8 to 2017/18.

- ▶ The model delivers a new marriage matching function in the stationary equilibrium with closed form,

$$\mu = \mathcal{G}(v, s^m, s^f; \Pi, \rho), \quad (1)$$

- ▶ μ : equilibrium numbers of new marriages;
- ▶ v : endogenous equilibrium numbers of unsuccessful meetings;
- ▶ $s^m \equiv (s_k^m)_{k \in \mathcal{Z}}$ and $s^f \equiv (s_k^f)_{k \in \mathcal{Z}}$, : endogenous equilibrium numbers of available single men and women;
- ▶ Π : exogenous marital surplus parameters;
- ▶ ρ : exogenous search friction parameters.

Contributions - *Marriage Matching Function with Search Friction*

- ▶ Equation (1) needs to satisfy a set of accounting constraints in stationary equilibrium,

$$\mu_{i,j} + v_{i,j} = m_{i,j}, \quad \forall i, j, \quad (2)$$

$$s_i^m + \sum_{j \in \mathcal{Z}} \frac{\rho_{ij} s_i^m s_j^f}{\sqrt{S^m S^f}} \frac{\mu_{i,j}}{v_{i,j}} = l_i^m, \quad \forall i, \quad \text{and} \quad (3)$$

$$s_j^f + \sum_{i \in \mathcal{Z}} \frac{\rho_{ij} s_i^m s_j^f}{\sqrt{S^m S^f}} \frac{\mu_{i,j}}{v_{i,j}} = l_j^f, \quad \forall j. \quad (4)$$

- ▶ Separately identify marital gains and search friction by using cross-sectional data.
 - ▶ With only matching data, it faces the challenge to disentangle the marital preferences and search frictions as both could affect marriage outcomes.
 - ▶ Most existing papers use panel data to identify matching model with search frictions

- ▶ Apply our model to investigate how technology and social media has affected the marriage distribution in the United States from 2007/8 to 2017/18.
 - i) raised search cost among the young (21 to 31 years of age) while it lowered search cost among the old (older than 40).
 - ii) lowered marital surplus among the young (21 to 31 years of age) while it raised surplus among the old.

Road Map

- ▶ Model: setup, decision problems, solving the model
- ▶ Stationary equilibrium
- ▶ Identification
- ▶ Empirical application
- ▶ Conclusion

Model setup - Model Environment

- ▶ Consider a stationary economy populated by overlapping generations of adults who live for Z periods.
- ▶ Certain numbers of age one males and females are born each period.
- ▶ (i, j) denote male and female age.
- ▶ l_i^m denotes the number of age i adult males, the vector $\mathbf{l}^m = (l_i^m)_{i \in \mathcal{Z}}$, the total male population is denoted by $L^m = \sum_{k \in \mathcal{Z}} l_k^m$. (l_j^f , \mathbf{l}^f and L^f)
- ▶ s_i^m denotes the number of age i single males the vector $\mathbf{s}^m = (s_k^m)_{k \in \mathcal{Z}}$. The total single male, $S^m = \sum_{i \in \mathcal{Z}} s_i^m$, (s_j^f , \mathbf{s}^f and S^f)
- ▶ $n(i, j)$ is the stock of (i, j) couples, the matrix $\mathbf{n} = (n(i, j))_{i, j \in \mathcal{Z}}$.
- ▶ \mathbf{l}^m and \mathbf{l}^f , are predetermined, \mathbf{s}^m , \mathbf{s}^f , and \mathbf{n} , are equilibrium quantities endogenously determined in the model

- ▶ The number of meetings between age i men and age j women,

$$m_{i,j} = \rho_{ij} \frac{s_i^m s_j^f}{S^m S^f} M(S^m, S^f), \quad (5)$$

which is the product of three component:

1. ρ_{ij} , the type-specific exogenous parameter capturing the search efficiency;
2. $\frac{s_i^m s_j^f}{S^m S^f}$, the fraction of the number of potential meetings between age i men and age j women to the total market-level potential meetings in one unit of time period;
3. $M(S^m, S^f)$, proportional to the total market-level meetings in one unit of time period, $M(S^m, S^f) = \sqrt{S^m S^f}$ following the literature.

Model setup - Search Technology Continues

- ▶ The rate that an age i man meets an age j woman is then given by:

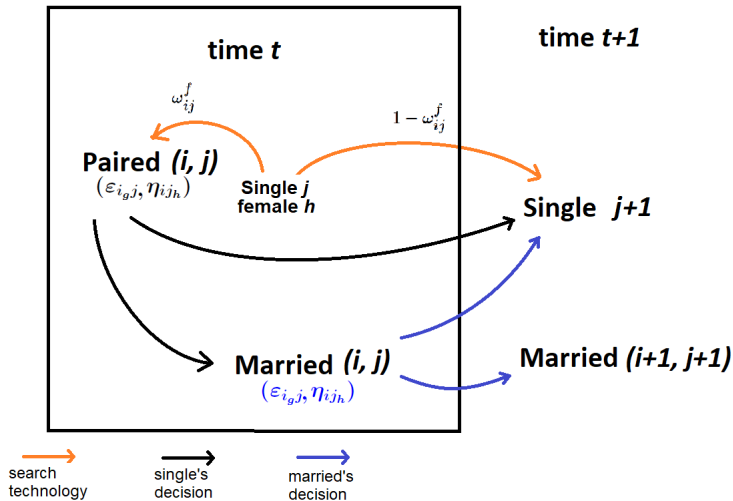
$$\omega_{ij}^m = \frac{m_{i,j}}{s_i^m} = \frac{\rho_{ij} s_j^f}{\sqrt{S^m S^f}}. \quad (6)$$

- ▶ A single individual is not guaranteed to meet someone of the opposite sex for sure. Let $\omega_{i0}^m = 1 - \sum_{j \in \mathcal{Z}} \omega_{ij}^m$ denotes the probability that a single age i man meets no one on the marriage market.
- ▶ Similarly, the rate that a single woman of age j meets a man of age i is given by:

$$\omega_{ij}^f = \frac{m_{i,j}}{s_j^f} = \frac{\rho_{ij} s_i^m}{\sqrt{S^m S^f}}, \quad (7)$$

- ▶ and $\omega_{0j}^f = 1 - \sum_{i \in \mathcal{Z}} \omega_{ij}^f$ denotes the probability that she meets no one.

Model setup - Time-line



Model Setup - Assumptions continues

Actions: Agents have binary actions. A married or paired couple of individuals g and h of (i, j) type have actions $a_{ijgh} \in \{0, 1\}$, where

- ▶ $a_{ijgh} = 1$ denotes the decision to marry (for paired couples) or remain married (for married couples), and
- ▶ $a_{ijgh} = 0$ denotes the decision to remain single (for paired couples) or divorce (for married couples).

Model does not differentiate between newly-weds and couples who got married in the previous periods and choose to remain married.

Exogenous Parameters:

- ▶ $\beta \in (0, 1)$, represents the discount factor,
- ▶ $\theta \in (0, 1)$ is the Nash bargaining solution, the bargaining power of men

Model Setup - Assumptions

- ▶ Assume that preferences over partners and the evolution of the state variables satisfy two assumptions:
- ▶ **Additive Separability (AS)** in utilities
The sum of one-period utilities from marriage for incumbent or paired couples is additively separable in the mean utilities, Π_{ij} , and the sum of idiosyncratic shocks, $\varepsilon_{igj,1} + \eta_{ijh,1}$.
- ▶ **Conditional Independence (CI)**: the unobserved shocks are independent across periods.

Model Setup - Assumptions continues

PREFERENCES:

- ▶ Π_{ij} denote (i, j) type couple per-period systematic marital gain.
- ▶ **Endogenous** per-period net utility that male g (or female h) receives from action a_{ijgh} : $u(a_{ijgh}, (i, j), \varepsilon_{i_g j}, \eta_{ij_h})$ (or $w(a_{ijgh}, (i, j), \varepsilon_{i_g j}, \eta_{ij_h})$).
- ▶ When couple marries, $a_{ijgh} = 1$, the aggregate marital utilities is,

$$\begin{aligned} & u(a_{ijgh} = 1, (i, j), \varepsilon_{i_g j}, \eta_{ij_h}) + w(a_{ijgh} = 1, (i, j), \varepsilon_{i_g j}, \eta_{ij_h}) \\ & = \Pi_{ij} + \varepsilon_{i_g j, 1} + \eta_{ij_h, 1}. \end{aligned}$$

- ▶ If the meeting was unsuccessful or the married couple decides to divorce, $a_{ijgh} = 0$.
- ▶ Per-period systematic gain from remaining single or divorcing is normalized to zero.

$$\begin{aligned} & u(a_{ijgh} = 0, (i, j), \varepsilon_{i_g j}, \eta_{ij_h}) = \varepsilon_{i_g j, 0}, \text{ and} \\ & w(a_{ijgh} = 0, (i, j), \varepsilon_{i_g j}, \eta_{ij_h}) = \eta_{ij_h, 0}. \end{aligned}$$

DP: Married (or Paired) Individuals

- ▶ (i, j) type couple (g, h) makes a binary decision a that maximizes their life-cycle expected discounted utility. $U((i, j), \varepsilon_{ijg}, \eta_{ijh})$.
- ▶ This value function takes the form,

$$\begin{aligned} U((i, j), \varepsilon_{ijg}, \eta_{ijh}) = & \max \left\{ u(a = 1, (i, j), \varepsilon_{ijg}, \eta_{ijh}) \right. \\ & + \beta \mathbb{E} [U((i + 1, j + 1), \varepsilon'_{i'j'}, \eta'_{i'j_h}) | (i, j), \varepsilon_{ijg}, \eta_{ijh}, a = 1], \\ & u(a = 0, (i, j), \varepsilon_{ijg}, \eta_{ijh}) \\ & \left. + \beta \mathbb{E} [U((i + 1, 0), \varepsilon'_{i+1g0}, (\eta'_{0k_{h'}})_k) | (i, j), \varepsilon_{ijg}, \eta_{ijh}, a = 0] \right\}. \end{aligned}$$

DP: Single Individuals

- ▶ The value function for the single male g with state $((i, 0), \varepsilon_{i_g 0})$ is then given by the Bellman equation,

$$\begin{aligned} U((i, 0), \varepsilon_{i_g 0}, (\eta_{0j_h}^i)_j) &= \sum_k \omega_{ik}^m U((i, k), \varepsilon_{i_g 0}^k, \eta_{0k_h}^i) \\ &+ \omega_{i0}^m \beta \mathbb{E}[U((i+1, 0), \varepsilon'_{i+1_g 0}, (\eta'_{0j_h})_j | (i, 0), \varepsilon_{i_g 0}, (\eta_{0j_h}^i)_j)]. \end{aligned}$$

Solving the model

- Assume that $(\varepsilon_{ij,1} + \eta_{ij,1})$ and $\varepsilon_{ij,0}/\theta$ are independently drawn from Type I Extreme Value distribution, the probability that male g of couple type (i,j) remains married this period is given by,

$$\mathcal{P}_{ij,1} = \frac{\exp[\Pi_{ij} + \beta C_{i+1,j+1} \cdot \mathbb{I}(i < Z, j < Z)]}{1 + \exp[\Pi_{ij} + \beta C_{i+1,j+1} \cdot \mathbb{I}(i < Z, j < Z)]}.$$

- The corresponding integrated value function for an age $i < Z$ male married to an age j female is given by,

$$\mathbb{U}_{i,j} = \theta[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{U}_{i+1,0} \cdot \mathbb{I}(i < Z),$$

- and the integrated value function for an age $i < Z$ single male is,

$$\mathbb{U}_{i,0} = \sum_k \omega_{ik}^m \theta[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{U}_{i+1,0} \cdot \mathbb{I}(i < Z).$$

- ▶ Similarly the integrated value function for an age $j \in \mathcal{Z}$, married female is given by,

$$\mathbb{W}_{i,j} = (1 - \theta)[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{W}_{i+1,0} \cdot \mathbb{I}(j < Z),$$

- ▶ and the corresponding integrated value function for a single age $j \in \mathcal{Z}$ female is given by

$$\mathbb{W}_{0,j} = \sum_k \omega_{kj}^f (1 - \theta)[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{W}_{0,j+1} \cdot \mathbb{I}(j < Z).$$

Solving the Model: Dynamic Matching Function with Search Friction

- ▶ $\mu_{i,j}$ the flow of new (i,j) marriages, and $v_{i,j}$ the flow of unsuccessful paired (i,j) meetings. Using this to estimate the probabilities $\hat{P}_{ij,1} = \frac{\mu_{i,j}}{m_{i,j}}$ and $\hat{P}_{ij,0} = \frac{v_{i,j}}{m_{i,j}}$, substituting them into the log-odds ratio, we obtain the equilibrium matching function equation

$$\mu_{i,j} = \begin{cases} \exp(\kappa) \exp(\Pi_{ij}) v_{i,j} \left(\frac{v_{i+1,j+1}}{m_{i+1,j+1}} \right)^{-\beta} \\ \quad \times \prod_{k=1}^Z \left(\frac{v_{i+1,k}}{m_{i+1,k}} \right)^{\beta \theta \omega_{i+1k}^m} \left(\frac{v_{k,j+1}}{m_{k,j+1}} \right)^{\beta(1-\theta) \omega_{kj+1}^f}, & \text{if } i < Z, j < Z, \\ \exp(\Pi_{ij}) v_{i,j}, & \text{if } i = Z \text{ or } j = Z, \end{cases}$$

- ▶ where the term $\kappa \equiv \beta c - \beta \theta c \sum_{k=1}^Z \omega_{i+1k}^m - \beta(1-\theta)c \sum_{k=1}^Z \omega_{kj+1}^f$.

Stationary equilibrium

- ▶ Consider a stationary economy populated by overlapping generations of adults who live for Z periods.
- ▶ constant number of age one males and females are born to the economy each period.
- ▶ inflow and outflow of married couples of each type must exactly balance each other.
- ▶ the outflow or dissolution of (i, j) -type marriage is denoted by $n(i, j)(1 - \mathcal{P}_{ij,1})$, where $n(i, j)$ is the stock of (i, j) -type couples, and $(1 - \mathcal{P}_{ij,1})$ is the dissolution or divorce probability for an (i, j)

$$m_{i,j}\mathcal{P}_{ij,1} = n(i, j)(1 - \mathcal{P}_{ij,1}).$$

- ▶ the stationary steady state stock of (i, j) marriages,

$$n(i, j) = m_{i,j} \frac{\mathcal{P}_{ij,1}}{(1 - \mathcal{P}_{ij,1})} = \frac{\rho_{ij} \varrho s_i^m s_j^f}{\sqrt{S^m S^f}} \frac{\mathcal{P}_{ij,1}}{(1 - \mathcal{P}_{ij,1})}. \quad (8)$$

- ▶ the following accounting balance conditions for each gender and type $i, j \in \mathcal{Z}$,

$$l_i^m = s_i^m + \sum_{j \in \mathcal{Z}} n(i, j), \quad (9)$$

$$l_j^f = s_j^f + \sum_{i \in \mathcal{Z}} n(i, j). \quad (10)$$

Solving the Model: continues

A stationary marriage market equilibrium with search friction is defined by the tuple $(\mathbf{s}^m, \mathbf{s}^f, \mathbf{n}, \mathbf{U}, \mathbf{W}, \mathcal{P})$, which comprises two vectors indicating the quantities of single males and females $(\mathbf{s}^m, \mathbf{s}^f)$, a matrix representing the stocks of marriages \mathbf{n} , two vectors that encapsulate the expected values for males and females within unions (\mathbf{U}, \mathbf{W}) , and a matrix detailing the probabilities of opting for marriage \mathcal{P} . In this equilibrium, the vectors \mathbf{s}^m and \mathbf{s}^f are solutions to the fixed-point equations defined by

$$l_i^m = s_i^m + \sum_{j \in \mathcal{Z}} \frac{\rho_{ij} \varrho s_i^m s_j^f}{\sqrt{S^m S^f}} \frac{\mathcal{P}_{ij,1}}{(1 - \mathcal{P}_{ij,1})}, \text{ and}$$
$$l_j^f = s_j^f + \sum_{i \in \mathcal{Z}} \frac{\rho_{ij} \varrho s_i^m s_j^f}{\sqrt{S^m S^f}} \frac{\mathcal{P}_{ij,1}}{(1 - \mathcal{P}_{ij,1})},$$

. The matrix \mathbf{n} is given by the equation

$$n(i, j) = m_{i,j} \frac{\mathcal{P}_{ij,1}}{(1 - \mathcal{P}_{ij,1})} = \frac{\rho_{ij} \varrho s_i^m s_j^f}{\sqrt{S^m S^f}} \frac{\mathcal{P}_{ij,1}}{(1 - \mathcal{P}_{ij,1})}.$$

The value function matrices, \mathbf{U} and \mathbf{W} , are determined by equations

$$\mathbb{U}_{i,j} = \theta[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{U}_{i+1,0} \cdot \mathbb{I}(i < Z),$$

$$\mathbb{U}_{i,0} = \sum_k \omega_{ik}^m \theta[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{U}_{i+1,0} \cdot \mathbb{I}(i < Z).$$

$$\mathbb{W}_{i,j} = (1 - \theta)[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{W}_{i+1,0} \cdot \mathbb{I}(j < Z),$$

$$\mathbb{W}_{0,j} = \sum_k \omega_{kj}^f (1 - \theta)[c - \ln \mathcal{P}_{ij,0}] + \beta \mathbb{W}_{0,j+1} \cdot \mathbb{I}(j < Z),$$

and the probabilities within \mathcal{P} are defined by equation

$$\mathcal{P}_{ij,1} = \frac{\exp[\Pi_{ij} + \beta C_{i+1,j+1} \cdot \mathbb{I}(i < Z, j < Z)]}{1 + \exp[\Pi_{ij} + \beta C_{i+1,j+1} \cdot \mathbb{I}(i < Z, j < Z)]}.$$

Identification

- ▶ Model primitives: search friction parameters ρ and the marital preference parameters Π .
- ▶ Observables: the matching distribution, $\hat{\nu} = (\hat{\mu}, \hat{s}^m, \hat{s}^f)$, and the divorce rates $\hat{\delta} = (\delta_{i,j})_{i,j \in \mathcal{Z}}$.
- ▶ Identification challenge: disentangle marital gains and search friction.
- ▶ Key idea: duration data identify the marital gains from marriages and then use observed matches identify search friction.

- Recall :

$$\mu_{ij} = m_{i,j} \mathcal{P}_{ij,1} = \frac{\rho_{i,j} s_i^m s_j^f}{\sqrt{S^m S^f}} \mathcal{P}_{ij,1},$$

- Assumption: married couples and paired couples face the same decision problems:

$$\hat{\mathcal{P}}_{ij,1} = 1 - \hat{\delta}_{i,j} \quad \text{and} \quad \hat{\mathcal{P}}_{ij,0} = \hat{\delta}_{i,j}. \quad (11)$$

- This allows us to identify

$$\hat{\rho}_{i,j} = \frac{\hat{\mu}_{i,j}}{1 - \hat{\delta}_{i,j}} \frac{\sqrt{\hat{S}^m \hat{S}^f}}{\hat{s}_i^m \hat{s}_j^f},$$

- ▶ With $\hat{\rho}_{i,j}$, we can identify the search probabilities,

$$\hat{\omega}_{ij}^m = \frac{\hat{\rho}_{ij} \hat{S}_j^f}{\sqrt{\hat{S}^m \hat{S}^f}}, \quad \text{and} \quad \hat{\omega}_{ij}^f = \frac{\hat{\rho}_{ij} \hat{S}_i^m}{\sqrt{\hat{S}^m \hat{S}^f}}.$$

and

$$\hat{\kappa} = \beta c - \beta \theta c \sum_{k=1}^Z \hat{\omega}_{i+1k}^m - \beta(1-\theta)c \sum_{k=1}^Z \hat{\omega}_{kj+1}^f, \quad (12)$$

- ▶ Rearranging the log-odd ratio gives us the identification equation of $\Pi_{i,j}$,

$$\hat{\Pi}_{ij} = \begin{cases} -\hat{\kappa} + \log \hat{\mathcal{P}}_{ij,1} - \log \hat{\mathcal{P}}_{ij,0} + \beta \log \hat{\mathcal{P}}_{i+1j+1,0} \\ \quad - \beta \sum_{k=1}^Z \log [\hat{\mathcal{P}}_{i+1k,0}^{\hat{\omega}_{i+1k}^m \theta} \hat{\mathcal{P}}_{kj+1,0}^{\hat{\omega}_{kj+1}^f (1-\theta)}], & \text{if } i < Z, j < Z \\ \log \hat{\mathcal{P}}_{ij,1} - \log \hat{\mathcal{P}}_{ij,0}, & \text{if } i = Z \text{ or } j = Z \end{cases}$$

Empirical Application: Data Summary

► U.S. American Community Survey Data, 2007 and 2017

A. Available singles and stock of marrieds

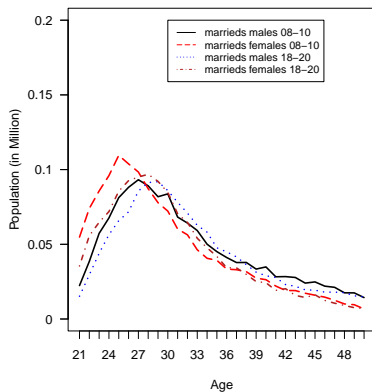
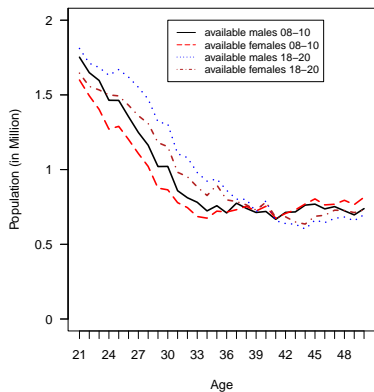
	2007	2017	Δ
Available men (million)	28.60	31.32	9.51%
Available women (million)	27.00	29.49	9.22%
Average age of single men	32.97	32.33	
Average age of single women	33.61	32.85	
Stock of marrieds (million)	26.86	23.57	-12.25%
Average age of married men	39.01	39.12	
Average age of married women	37.27	37.42	

B. New marrieds and divorces

	2008-10	2018-20	Δ
New marrieds (million)	1.40	1.33	-5.00%
Average age of newly married men	32.15	32.30	
Average age of newly married women	30.17	30.62	
Divorces (million)	1.64	1.14	-30.49%
Average age of divorced men	38.20	38.89	
Average age of divorced men	36.56	37.30	

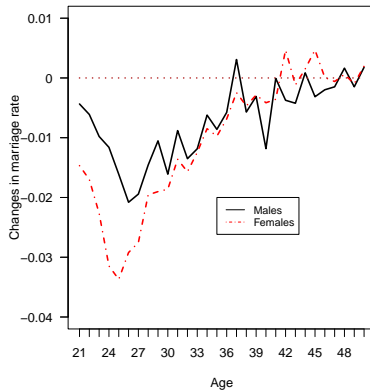
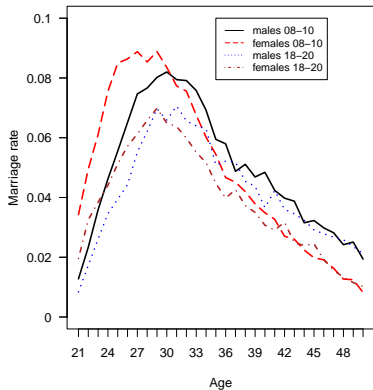
Empirical Application: continue

► Observed available singles and newly marrieds by age



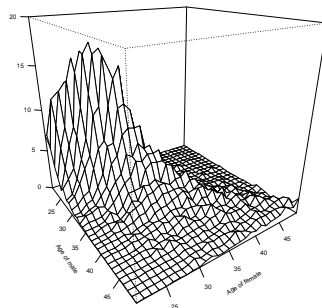
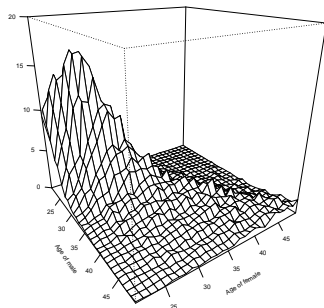
Empirical Application: continue

► Observed marriage rates and their changes by age



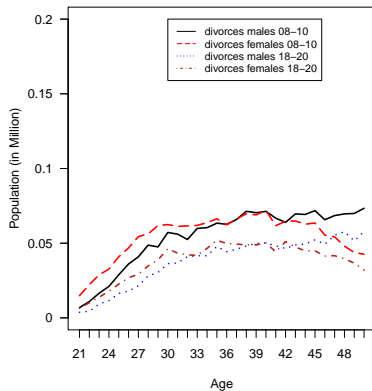
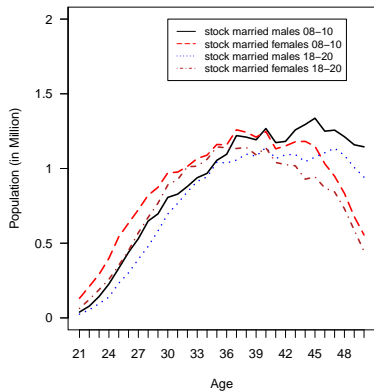
Empirical Application: continue

- Observed surface of observed μ_{ij} in thousand



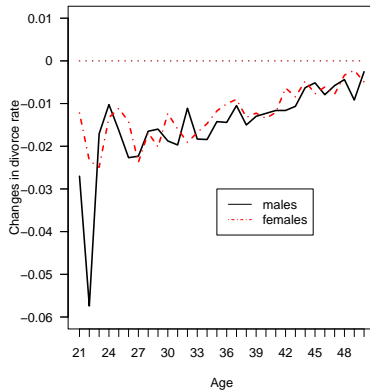
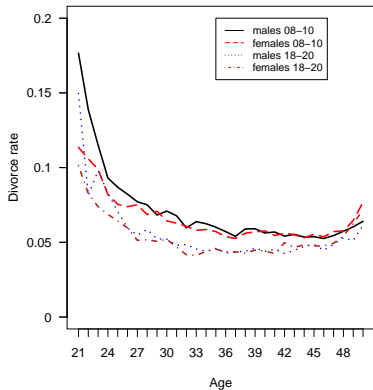
Empirical Application: continue

► Observed stock of marrieds and divorces



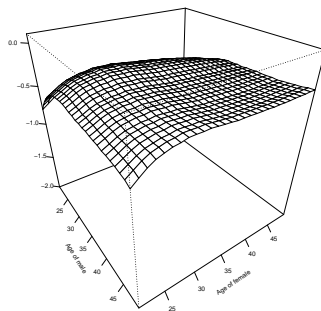
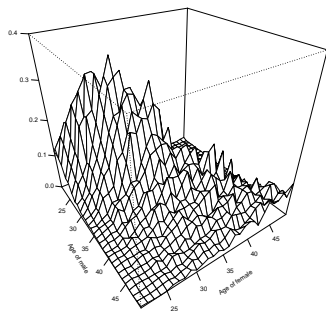
Empirical Application: *continue*

► Observed divorce rates and their changes by age



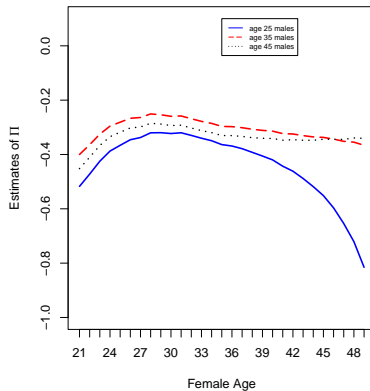
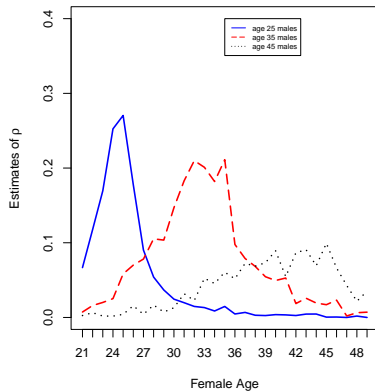
Empirical Application: continue

- ▶ Estimated ρ_{ij} and Π_{ij} in 2008-10



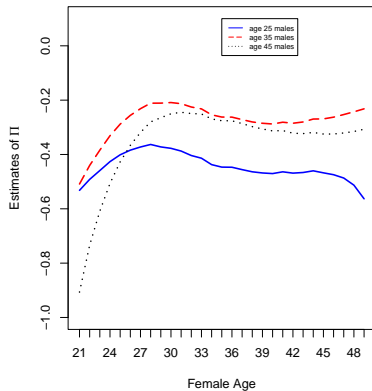
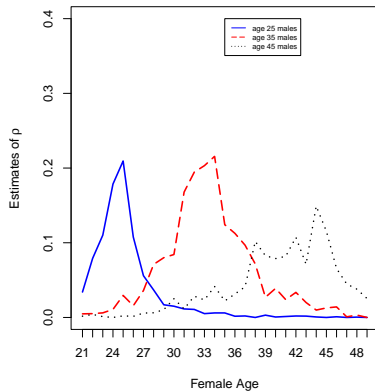
Empirical Application: continue

- Estimated ρ_{ij} and Π_{ij} in 2008-10 for ages 25, 35, 45 males



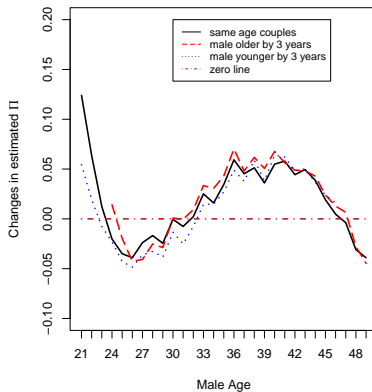
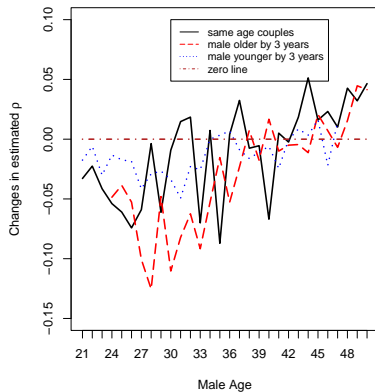
Empirical Application: continue

- Estimated ρ_{ij} and Π_{ij} in 2018-20 for ages 25, 35, 45 males



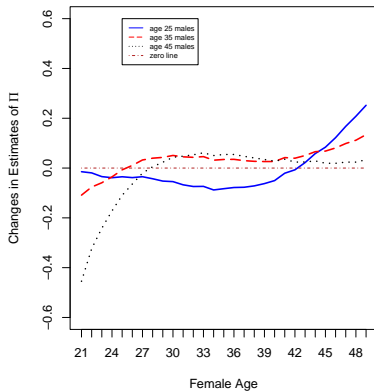
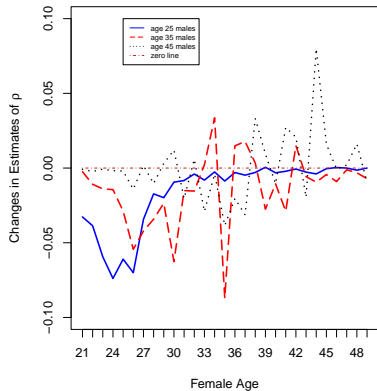
Empirical Application: continue

► Changes in $\hat{\rho}_{ij}$ and $\hat{\Pi}_{ij}$



Empirical Application: continue

- Changes in $\hat{\rho}_{ij}$ and $\hat{\Pi}_{ij}$ for ages 25, 35, 45 males



Conclusion

- ▶ Propose a empirical model of marriage matching with search frictions.
- ▶ Rationalizes a new marriage matching function with search friction.
- ▶ Develop a empirical strategy to separately identify marital gains and search frictions.
- ▶ Applied our model to investigate how advancement in social media and internet penetration has affected marital gains and search cost from 2007/8 to 2017/18.
- ▶ Preliminary results showed that these technological advancement
 - i) raised search cost among the young (21 to 31 years of age) while it lowered search cost among the old (older than 40).
 - ii) lowered marital surplus among the young (21 to 31 years of age) while it raised surplus among the old.