Did Harold Zuercher Have Time Separable Preferences?

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Motivation

- The traditional time additive utility function:

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}[u(c_t)]$$

  - widely adopted due to mathematical and computational convenience.
- Impose several restrictions:
  - both willingness to substitute across time and willingness to substitute across states are captured by u.
  - indifference to the timing of the resolution of uncertainty.

Motivation

- Models with non-separable time preferences such as Epstein and Zin (1989) allow for a clean separation of time and risk preferences.
- This generality makes their use popular in many fields such as macroeconomics and finance.
  - explain the equity premium puzzle (e.g., Mehra and Prescott (1985) and Bansal and Yaron (2004)).
- The importance of non-separable preferences has been known in the dynamic discrete choice literature.

Motivation

Rust (1994) points out:

- *"A number of experiments have indicated that human decision-making under uncertainty may not always be consistent with the von Neumann-Morgenstern axioms.*
- *In addition, expected-utility models imply that agents are indifferent about the timing of the resolution of uncertain events, whereas human decision-makers seem to have definite preferences over the time at which uncertainty is resolved."*

Our Paper

- We develop theoretical results and empirical tools for analyzing DDC models with non-separable time preferences.
- Our model introduces several novel features:
  - flexibly captures risk and time preferences.
  - allows non-trivial preferences over when uncertainty is resolved.
  - nests the standard time-separable expected utility model.
- An application to the bus engine replacement data in Rust (1987): using separable time preferences leads to biased estimates in a systematic way.

Non-separable Time Preferences

- Recall standard models with separable time preferences:

$$\mathbb{E}_c[(1-\beta)u(c) + \beta \mathbb{E}_{v|c}[v]]$$

Risk is <u>additively separable</u> across time.

- Consider non-separable time preferences:

$$\mathbb{E}_c[\phi\big((1-\beta)u(c) + \beta \phi^{-1}(\mathbb{E}_{v|c}[v])\big)]$$

ϕ is a strictly increasing aggregator function, its curvature captures the agent's attitudes towards how risk is resolved across different time periods.

Parametric Special Cases

- Linear ϕ: reduce to standard separable models.
- CRRA Epstein-Zin:

$$u(c) = c^{1-\rho}, \quad \phi(z) = z^{\frac{1-\alpha}{1-\rho}}.$$

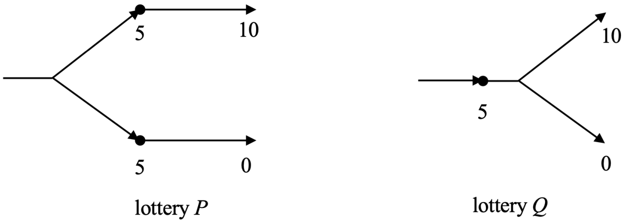
- CARA Epstein-Zin:

$$u\left(c\right) = \rho^{-1}\left(1 - e^{-\rho c}\right), \quad \phi\left(z\right) = \alpha^{-1}\left(1 - \left(1 - \rho z\right)^{\frac{\alpha}{\rho}}\right).$$

- α and ρ characterize risk and intertemporal preferences. When $\alpha = \rho$ in either case, ϕ becomes linear.

Early vs. Late Resolution of Uncertainty

- ϕ allows for greater flexibility in modeling agent's non-trivial preferences over when risk is resolved.



- The two lotteries are the same in the probabilities and outcomes: they differ only in the timing (the uncertainty about period 2 outcome is resolved earlier in P).
- CRRA and CARA: the agent prefers early resolution of uncertainty (i.e., prefers P to Q) iff $\rho \leq \alpha$.

Empirical Model

- Per-period consumption: $c(d_t, x_t, \varepsilon_t) = \pi(d_t, x_t) + \varepsilon_t(d_t)$.
  - d_t: choice
  - x_t: observable state
  - $\varepsilon_t = (\varepsilon_t(1), \varepsilon_t(2), \cdots, \varepsilon_t(J)) \sim G$: indiosyncratic shocks.
- Bellman equation:

$$V(x_t, \varepsilon_t) = \max_d \phi\Big((1-\beta)\, u\big(c\,(d, x_t, \varepsilon_t)\,\big) + \beta\phi^{-1}\big(\mathbb{E}[V(x_{t+1})]\big)\Big).$$

- Ex-ante value: $V(x_{t+1}) = \int V(x_{t+1}, \varepsilon_{t+1}) dG(\varepsilon_{t+1})$.

Value Function Iteration Operator

$$T(V)(x_t) = \int \max_{d} \left\{ \phi\bigg((1-\beta)\, u\big(c\,(d, x_t, \varepsilon_t)\,\big) + \beta\phi^{-1}\big(\mathbb{E}[V(x_{t+1})]\big) \bigg) \right\} dG(\varepsilon_t)$$

- The ex-ante value V is obtained as a solution of the functional equation $T(V) = V$.
- Existence? Uniqueness?

Existence of the Fixed Point

Theorem 1

Suppose v^* *and* v_* *are finite and let* $\mathcal{V}$ *be the set of functions bounded by some* $\underline{v} \leq v_*$ *and* $\bar{v} \geq v^*$*. Then* $T : \mathcal{V} \to \mathcal{V}$ *has a fixed point. Moreover,* $\lim_n T^n(\underline{v})$ *and* $\lim_n T^n(\bar{v})$ *are its smallest and largest fixed points respectively.*

- The proof makes use of lattice theory and Tarski's fixed point theorem.
- Provide a way to compute the smallest and largest value functions by iterating T.
- Week conditions needed for specific parameterizations:
  - CRRA Epstein-Zin: $\mathbb{E}[\varepsilon]$ is finite.
  - CARA Epstein-Zin: $\mathbb{E}\left[e^{-t\varepsilon}\right]$ is finite for all $t \in \mathbb{R}$.

  These conditions are easily satisfied for normal or type I extreme value distributions.

Contraction Mapping Theorem

Theorem 2

T is a contraction mapping if

$$M := \mathbb{E}\left[\max_{d \in D} \sup_{\pi \in [\underline{\pi}, \bar{\pi}], z \in [\underline{v}, \bar{v}]} \psi'_{u(\pi+\varepsilon_d)}(z)\right] < 1$$

where $\psi_y(z) := \phi((1-\beta)y + \beta\phi^{-1}(z))$.

- Simplified formula for specific parameterizations:
  - Linear ϕ (separable time preferences): $\beta < 1$
  - CRRA Epstein-Zin: $\rho \leq \alpha$ and $\beta^{\frac{1-\alpha}{1-\rho}} < 1$
  - CARA Epstein-Zin: $\rho \geq \alpha$ and $\beta^{\frac{\alpha}{\rho}} < 1$

Estimation

- Nested Fixed Point algorithm.
  - Let $\theta = (\alpha, \rho, \theta_\pi)$ be the vector of structural primitives.

$$LL(\theta) = \sum_{i}^{N} \log\left(\prod_{t=1}^{T} \underbrace{Pr(d_{it}|x_{it};\theta)}_{\text{CCP}} \underbrace{Pr(x_{it}|x_{it-1}, d_{it-1})}_{\text{state transition}} \right)$$

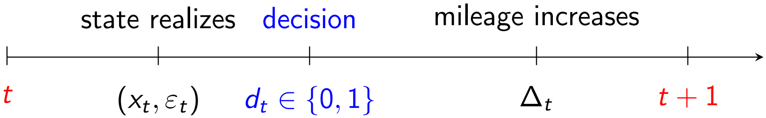
  - Outer loop: search for θ to maximize the LL.
  - Inner loop: given θ, solve for V via iteration.
  - No closed-form solutions for CCP and V (simulation methods needed).

Engine Replacement Model Revisited

- We apply our model to the bus engine replacement decisions originally studied in Rust (1987).
  - 104 buses managed by Harold Zuercher.
  - 10 years of monthly data on bus mileage and engine replacements.

- We extend the model to allow
  - the agent to have non-separable time preferences
  - the agent to be risk averse
  - the agent to earn revenue from operating the bus

Timeline



- x_t: accumulated mileage observed at the beginning of t.
- ε_t: unobserved payoff shock.
- Δ_t: incremental mileage realized within period t.

$$\pi(\Delta_t, d_t, x_t) = \begin{cases} \underbrace{\theta_d \Delta_t}_{\text{revenue}} - \underbrace{RC}_{\text{replacement cost}} & \text{if } d_t = 1 \\ \theta_d \Delta_t - \underbrace{\theta_x x_t}_{\text{maintenance cost}} & \text{if } d_t = 0 \end{cases}$$

Value Functions

- Apply CARA parameterization (to allow negative payoff).
- Choice-specific value functions

$$v(d_t, x_t, \varepsilon_t) = \frac{1}{\alpha}\Bigg[1 - E_{\Delta_t|x_t,d_t}\Bigg\{(1-\beta)\exp\Bigg(-\rho\underbrace{(\pi(\Delta_t, d_t, x_t) + \sigma\varepsilon_t(d_t))}_{\text{current period payoff}}\Bigg)$$

$$+\beta\Bigg(1 - \alpha\underbrace{V(x_{t+1})}_{\text{ex-ante value}}\Bigg)^{\frac{\rho}{\alpha}}\Bigg\}^{\frac{\alpha}{\rho}}\Bigg],$$

where $x_{t+1} = (1 - d_t)x_t + \Delta_t$.

Different Model Specifications

- We estimate four model specifications
  (1) non-separable time preferences (no restrictions on α and ρ);
  (2) separable time preferences with risk aversion ($\alpha = \rho$);
  (3) Rust model with revenue (separable and risk neutral);
  (4) original Rust model (fix $\theta_d = 0$).
- Fix $RC = 8$ (this is the number reported by HZ, see Rust (1987) Table 3), $\beta = 0.9$, and $\varepsilon \sim N(0, 1)$. Estimate $\theta = (\alpha, \rho, \theta_d, \theta_x, \sigma)$.

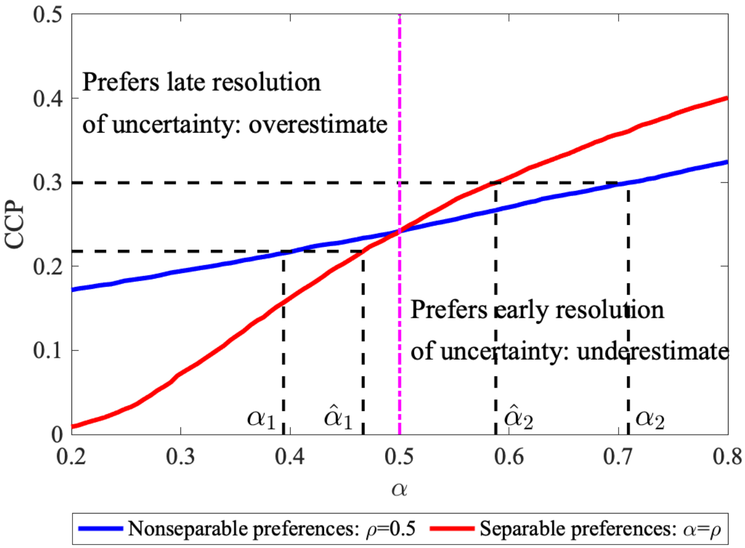
Results

	(1) Nonseparable	(2) Separable	
θ_d	0.0526	0.1019	diff estimates of θ_d
	(0.0073)	(0.0829)	
θ_x	0.1077	0.0329	diff estimates of θ_x
	(0.0122)	(0.0022)	
σ	1.6070	1.5436	
	(0.0491)	(0.0623)	
α	0.1023	0.1457	over estimate α
	(0.5087)	(0.0095)	
ρ	0.5555		prefers late resolution
	(0.0200)		of uncertainty
LL	-299.4404	-300.8139	Reject separable pref

Results

	(1) Nonseparable	(2) Separable	(3) Rust - rev	(4) Rust-orig
θ_d	0.0526	0.1019	0.0001	
	(0.0073)	(0.0829)	(0.0351)	
θ_x	0.1077	0.0329	0.0208	0.0208
	(0.0122)	(0.0022)	(0.0009)	(0.0014)
σ	1.6070	1.5436	1.4883	1.4770
	(0.0491)	(0.0623)	(0.0542)	(0.0566)
α	0.1023	0.1457		
	(0.5087)	(0.0095)		
ρ	0.5555			
	(0.0200)			
LL	-299.4404	-300.8139	-301.4402	-301.6273

Biased Estimates



Counterfactual

- The agent has uncertainty about the incremental mileage:
  - affects current-period revenue
  - affects future maintenance costs
- A subsidy program:
  - helps the agent smooth revenue across periods.
  - the agent receives C each period even if the incremental mileage is 0.
  - similar to the "capacity payment".

Counterfactual

- Agents with separable preferences: $C = \$120$ (almost doubles!)
- Agents with nonseparable preferences: $C = \$61.6$
- Intuition: when the agent prefers late resolution of uncertainty, the original setting (uncertainty in revenue) is less unfavorable.
- Misspecifying agents' preferences as time-separable when it is in fact not leads to misleading policy recommendations.

Main Takeaways

- It's well known that non-separable time preferences is more realistic but challenging to incorporate in DDC models.
- What we've shown:
  - nice theoretical properties: existence, contraction.
  - estimation is fairly straightforward: simulated nested fixed point.
  - applicable to many empirical settings.
  - ignoring nonseparable time preferences may severely bias the estimates and distort policy implications.

Related Literature

- Theory works:
  - Kreps and Porteus (1978), Epstein and Zin (1989, 1991).
  - Lu and Saito (2020): a model of dynamic stochastic choice that incorporates Epstein and Zin.
- Dynamic discrete choice:
  - Rust (1987) and many others.
  - No works on dynamic discrete choice that incorporates non-separable preferences.

Discussion on Identification

- What data variations are needed to empirically distinguish nonseparable preferences from separable preferences?

- A two-states problem: $s_t \in \{0, 1\}$ and $\Delta_t \in \{0, 1\}$.

$$Pr(\Delta_t = 1 | d_t = 1, s_t) = 1,$$

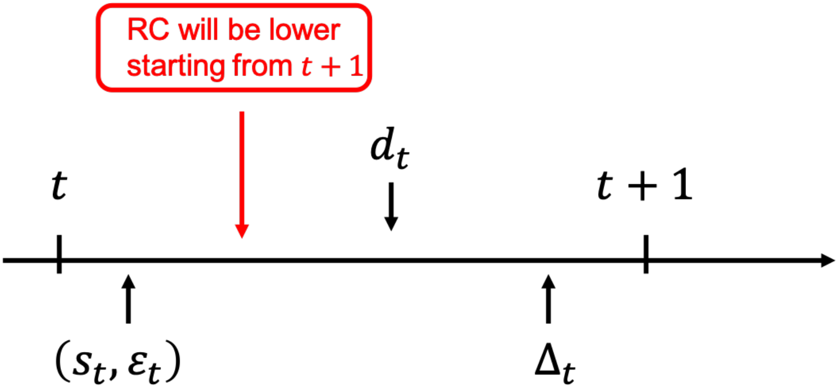
$$Pr(\Delta_t = 1 | d_t = 0, s_t = 0) = 1,$$

$$Pr(\Delta_t = 1 | d_t = 0, s_t = 1) = p, \quad p \in (0, 1)$$

- Future state:

$$s_{t+1} = \max\{(1 - d)s_t + \Delta_t, 1\}.$$

Exogenous Shifters



- This information has <u>no direct effect</u> on the agent's current payoff, but will change the <u>future value</u> $V(1)$.
- Separable preferences: the changes in V for both options cancel out, CCP remains the same.

A Graphical Illustration