Firm Dynamics in Trade

ES Summer Schools in Dynamic Structural Econometrics

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Introduction: Two broad sets of questions

- Firm-level dynamics and trade
  - Try to understand: exporter life cycle, entry, exit
  - Roles of technology, trade barriers, uncertainty, learning
- Aggregate outcomes shaped by firm-level dynamics
  - How do the welfare gains from trade liberalization depend on firm-level behavior?
  - Do models with firm-level dynamics help us understand the long- and short-run behavior of aggregate trade in response to changes in tariffs or over the business cycle?
  - What explains the delayed response of the trade balance to a change in the real exchange rate?

Introduction: Modeling approach

- Dynamic discrete-choice model of export market entry
- Embed the DDC model into two-country general equilibrium framework (capital accumulation, cross-country borrowing and lending,...)
- Use micro data to discipline parameters
  - Nested fixed point problem, Indirect inference, SMM
- Aggregate the decisions of heterogeneous firms to study "macro" questions

Outline

1. Data
- Criminally brief discussion of firm export dynamics
- Focus on entry, exit, and growth

2. Partial equilibrium: The canonical model
- Dynamic firm-choice problem

3. General equilibrium aggregation
- Embed PE model in general equilibrium

4. Discussion of computation, estimation

5. Trade liberalization example
- What is the payoff from all this work?

Two margins of growth

1. **Extensive margin:** Which firms export?
   - Measure as the export participation rate
   - Determined by starter and stopper rates

2. **Intensive margin:** Conditional on exporting, how much is exported?
   - Measure this as the *export intensity* of firm *i*

$$exs_{it} = \frac{\text{exports}_{it}}{\text{sales}_{it}}$$

   - Evolves over the exporter's life cycle

Decomposing aggregate trade

- Firms $i = 1 \dots n$ export. Firms $i = n+1 \dots N$ do not.
- Decompose aggregate export-sales ratio into three margins
  1. Extensive margin (first term on rhs)
  2. Intensive margin (second term on rhs)
  3. Exporter size premium (third term on rhs)

$$\frac{\sum_{i=1}^{n} exports_i}{\sum_{i=1}^{N} sales_i} = \frac{n}{N} \times \frac{n^{-1} \sum_{i=1}^{n} sales_i \times exs_i}{n^{-1} \sum_{i=1}^{n} sales_i} \times \frac{n^{-1} \sum_{i=1}^{n} sales_i}{N^{-1} \sum_{i=1}^{N} sales_i}$$

- Consider exports to the world, exports by destination country in paper

Decomposing aggregate trade

	All values are expressed as percentages					
	United States			Colombia 100+		
Panel A	1987	2007	log diff.	1983	2013	log diff.
Export/sales	6.3	11.6	61.1	5.2	13.9	97.7
Extensive	43.2	63.0	37.7	36.5	59.8	49.5
Intensive	9.9	15.5	44.9	10.8	20.3	62.8
Premium	148.0	119.5	−21.4	132.1	114.2	−14.6
Panel B						
Starter rate	10	–		6.9	13.8	
Stopper rate	17	–		11.9	10.1	

- Trade barriers fall $\rightarrow$ trade grows
- Extensive and intensive margins grow
- Newer, smaller exporters $\rightarrow$ size premium falls

Exporter life cycle

Export to total-sales ratio

	1	2	3	4	5	6	7	Long run
Starter	14.3	15.1	18.2	20.1	21.7	22.9	23.9	27.4
	–7	–6	–5	–4	–3	–2	–1	
Stopper	22.1	23.2	21.8	19.5	18.8	19.1	16.9	

- A new exporter grows by 50 percent in its first five years
- An exiting firm shrinks by about 30 percent in its last five years

Model: decision problem

- Consider a firm i making a decision to export: $x_{it} = \{0, 1\}$

$$V_t = \max_{\{x_{is}\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \frac{1}{1+r_s} x_{is} \left(\pi_{is}\left(\cdot\right) - f_{is}(\cdot)\right)$$

- Fixed export costs: $f_{it}\left(\epsilon_{it,} x_{it-1}, x_{it-2}, ..., x_{it-k}\right)$ depend on random variable and experience

- Flow profits: $\pi\left(x_{it}, z_{it}, d_{it}\right)$
  - z_{it} = variables related to productive efficiency
  - d_{it} = variables related to foreign demand for firm i's variety
  - Assumes constant returns to scale, otherwise $z_{it}\left(s_{it}, d_{it}\right)$ where s_{it} is sales at home

Model: foreign demand

- Firm i, charging price p_{it}, sells

$$d_{it}(p_{it}) = \omega_{it} \left(p_{it} \frac{\tau_t \xi_t \tilde{\xi}_{it}}{P_t} \right)^{-\theta} D_t$$

- Common factors: market size (D_t), real exchange rate (P_t), ad-valorem tariff (τ_t), iceberg trade costs (ξ_t)
- Idiosyncratic factors: demand shifter (ω_{it}) and $\left(\tilde{\xi}_{it}\right)$ e.g., shipping/distribution technology
  - Two idiosyncratic factors redundant, combine into ξ_{it}
  - No congestion effects on distribution
- CES framework is common

Fixed costs

- $f(\epsilon_{it}, x_{it-1})$: only $t-1$ export status matters
- $f(\epsilon_{it}, 1) < f(\epsilon_{it}, 0)$: cost of entering exceeds continuation cost
- Paying the entry cost lowers iceberg cost from $\xi = \infty$ to $\xi < \infty$
- A richer model in which fixed costs depend on experience requires tracking longer history

Uncertainty

- Microeconomic $(z, \xi, f(\epsilon_{it}, x_{it-1}))$
  - Let z, ξ follow AR1 process $\left(\rho_z, \sigma_z^2, \rho_\xi, \sigma_\xi^2\right)$
  - Let stochastic component follow $\epsilon_{it} \sim \log \text{Normal}\left(0, \sigma_\epsilon^2\right)$
- Macroeconomic
  - Processes for exchange rate (P_t) & demand (D_t) depend on equilibrium concept
  - In partial equilibrium (P, D) are exogenous processes
  - In general equilibrium, (P, D) depend on shocks and transmission (can be highly non-linear)
  - For tariffs no standard

Bellman Equation

- The firm solves a standard discrete-choice problem

$$V_t\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right) = \max\left\{V_t^0\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right), V_t^1\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right)\right\}$$

- To solve this problem we will need to know
  - A firm's survival probability (δ_{it})
  - The interest rate (r_t)
- The *t*s capture non-stationary functions from aggregate shocks
  - Most partial equilibrium models assume stationarity

Bellman Equation

- Value of not exporting

$$
\begin{aligned}
V_t^0\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right) &= \pi_t\left(0, z_{it}, \xi_{it}\right) \\
&\quad + \delta_{it} \underset{z,\xi,f}{\mathrm{E}} \frac{1}{1+r_{t+1}} V_{t+1}\left(0, z_{it+1}, \xi_{it+1}, f_{it+1}\right)
\end{aligned}
$$

- Value of exporting

$$
\begin{aligned}
V_t^1\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right) &= \pi_t\left(1, z_{it}, \xi_{it}\right) - f_{it}(\epsilon_{it}, x_{it-1}) \\
&\quad + \delta_{it} \underset{z,\xi,f}{\mathrm{E}} \frac{1}{1+r_{t+1}} V_{t+1}\left(1, z_{it+1}, \xi_{it+1}, f_{it+1}\right)
\end{aligned}
$$

- Focus on a stationary environment for now (drop ts)

Decision Rules

- Assume 1) f is deterministic (i.e. $\sigma_\epsilon = 0$) and 2) export and domestic profit increasing in z

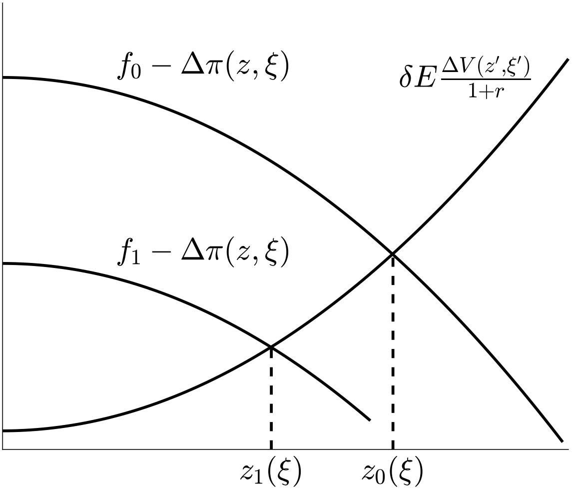
- Optimal policy is a cutoff rule $z_m(\xi)$ s.t. $x_{it} = 1$ iff $z \geq z_m(\xi)$

$$
\begin{aligned}
f_m - \left[\pi\left(1, z_m(\xi), \xi\right) - \pi\left(0, z_m(\xi), \xi\right)\right] &= \frac{\delta}{1+r} E \left[\begin{array}{c} V^1\left(z', \xi', f_1\right) \\ -V^0\left(z', \xi', f_0\right) \end{array} \right] \\
f_m - \Delta\pi\left(z_m(\xi), \xi\right) &= \frac{\delta}{1+r} E\left[\Delta V\left(z', \xi', f_1, f_0\right)\right]
\end{aligned}
$$

- The LHS is the current cost of exporting net of increased profits

- The RHS is the future benefit (increase in market value of the firm)

Breakevens



$f_0 - \Delta\pi(z,\xi)$
$\delta E \frac{\Delta V(z',\xi')}{1+r}$
$f_1 - \Delta\pi(z,\xi)$
$z_1(\xi)$
$z_0(\xi)$

Properties

- Crucial outcome of dynamic decision: $z_1(\xi) < z_0(\xi)$
  - Harder to break into exporting than to stay
- Consistent with data, we have
  - **Exporter hysteresis:** Firms continue exporting after conditions deteriorate
  - **Low exit rate:** Exporters will delay exiting to avoid paying the entry cost again
  - **Export Premium:** Exporters are larger than nonexporters
    - Increasing in the average fixed cost
    - Falling in the difference in fixed costs

Distributions

- The cutoff thresholds and the process for (z, ξ) determine the measure of firm types $\mu(z, \xi, f)$
- $\mu(z, \xi, f_0)$ [$\mu(z, \xi, f_1)$] denotes the beginning of period non-exporters [exporters]
- The measures of current nonexporters and exporters

$$N_N = \int_{\xi} \int_{0}^{z_0(\xi)} \mu(z, \xi, f_0) + \int_{\xi} \int_{0}^{z_1(\xi)} \mu(z, \xi, f_1)$$

$$N_X = \int_{\xi} \int_{z_0(\xi)}^{\infty} \mu(z, \xi, f_0) + \int_{\xi} \int_{z_1(\xi)}^{\infty} \mu(z, \xi, f_1)$$

- The export participation share is $N_X / (N_N + N_X)$

Laws of motion

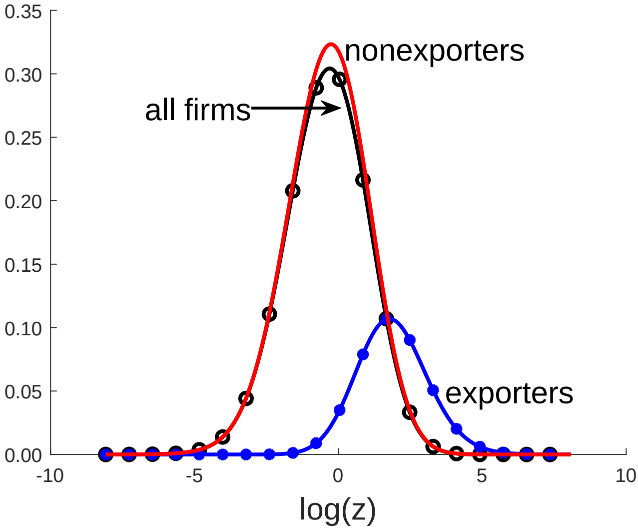
$$N_X' = \delta_{X,X} \Pr(\text{continue})\, N_X + \delta_{N,X} \Pr(\text{start})\, N_N$$

$$N_N' = \delta_{X,N} [1 - \Pr(\text{continue})]\, N_X + \delta_{NN} [1 - \Pr(\text{start})]\, N_N + N_E$$

- A more careful exposition would focus on

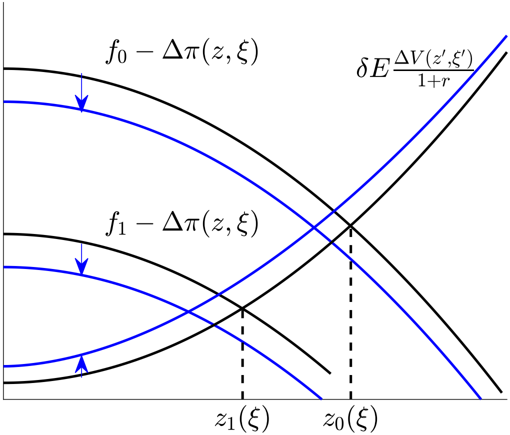
$$\mu'(z, \xi, f) = T(\mu(z, \xi, f))$$

Distributions



Comparative static: permanent decrease in tariff rate

- Increasing entry and decreasing exit today and in the future
- Through law of motion trade will increase gradually



General equilibrium

- Embed the firm-decision problem into general equilibrium
- Two symmetric countries (Foreign variables with asterisks)
  - With symmetric countries and policy, trade is balanced
  - Asymmetric countries or unilateral liberalization generate international capital flows
- Also: free entry of firms, physical capital, intermediate goods

- Why general equilibrium?
  - Account for feedback through prices
  - Feedback typically dampens effects vis-à-vis partial equilibrium

Heterogeneous firms

- Differentiated varieties, monopolistic competition
- The differentiated-variety production function

$$y_i = \left(zk_i^{\alpha}\ell_i^{1-\alpha}\right)^{1-\alpha_x} x^{\alpha_x}$$

  - k is physical capital, ℓ is labor
  - x intermediate good (a composite of varieties)
  - z follows an AR(1) process

Final good production

- Constant returns to scale, perfect competition
- Firm (a good) state is $s = (z, \xi, f)$

$$Y_t = \left[\int y_{Ht}(s)^{\frac{\theta-1}{\theta}} \mu_t(s)\, ds + \int y_{Ft}(s)^{\frac{\theta-1}{\theta}} \mu_t^*(s)\, ds \right]^{\frac{\theta}{\theta-1}}$$

- Final good used for consumption, physical capital investment, and intermediate goods (x)

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t + X_t$$

Household's Problem

$$\max_{C_t, K_{t+1}} \; E \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

$$\text{s.t. } \; C_t + K_{t+1} = w_t L_t + (1 + r_t - \delta_k) K_t + T_t + \Pi_t \qquad t = 0, 1, \ldots$$

$$C_t > 0, \;\; K_0 = \overline{K}$$

- L is the household's labor endowment ($L = 1$)
- r is the rental rate of capital
- w is the wage
- T is the lump-sum rebate of tariff revenue
- Π is the profit earned by domestic firms

Estimation overview

- Estimation target is United States in the early 1990s
- Assume the United States is in a stationary equilibrium
- Break the parameters into two sets
  1. Ones chosen without solving for the model’s equilibrium
  2. Ones that require solving for the model’s equilibrium

1. External parameters

- Not estimated, but common in the literature
  - $\theta = 5$, $\sigma = 1$
- "Estimated" (target)
  - $\tau = 0.10$ (average tariff rate = 10%)
  - $\beta = 0.96$ (real interest rate = 4%)
  - $\delta_k = 0.1$ (physical capital depreciation rate = 10%)
  - $\alpha = 0.3$ (capital share in income = 30%)
  - $\alpha_x = 0.80$ ($\frac{\text{gross output}}{\text{value added}} = 2.8$)

2. Internal parameters

$$z' = \rho_z z + \epsilon_z \qquad \epsilon_z \sim \mathrm{N}(0, \sigma_z)$$

$$\xi' = \rho_\xi \xi + \epsilon_\xi \qquad \epsilon_\xi \sim \mathrm{N}(0, \sigma_\xi)$$

- Discretize z into many values
- Discretize ξ into two values $\xi_H > \xi_L$, with symmetric transition prob ρ_ξ
- Overall, we need to estimate:

$$\gamma = \{\chi_0, \chi_1, \rho_z, \sigma_z, \mu_e, f_0, f_1, \xi_L, \xi_H, \rho_\xi\}$$

To do so, we need to solve the model.

Steady-state algorithm (for given parameter values)

0. Initial set up

- Construct grids; Discretize continuous stochastic processes

1. Guess (K, C, M), solve decision problem

- Value/policy function iteration to convergence
- Key output: Policy functions over export entry/exit

2. Compute stationary distribution

- Iterate on l.o.m. using policy functions from step 1

3. Use stationary distribution to construct aggregates

4. Check for equilibrium and return to step 1 as needed

- Capital market clearing
- Free entry condition
- Consumer budget constraint

A complication

- The export entry cutoffs z_0, z_1 are generally between nodes
- Thus small changes in parameters can lead to discrete change in the mass of firms making the choice
- Can lead to instability in convergence or parameter estimation
- A solution: interpolate and randomize
  - Find the cutoffs using the value functions
  - Assume firms are distributed uniformly between the nodes and then let the decision rule be based on the share of firms that meet the threshold

Estimation algorithm

0. Determine a set of moments from the data m^d

1. Guess parameter vector γ

2. Solve for steady state (previous algorithm)

3. Simulate panel of data, compute moments from model $m(\gamma)$

4. Compute e. Is it minimized? If not return to step 1

$$e = \left[m^d - m(\gamma)\right]' W \left[m^d - m(\gamma)\right]$$

- Code $e = f(\gamma, m^d)$
- Pass $f()$ to your favorite minimizer (if exactly identified, NL solver)
- A good initial guess of γ helps a lot (easier said than done)

Improvements to the algorithms

- The steady-state model solution can be broken up
  - Solve value function and compute stationary distribution first
  - Then solve for entry and aggregate quantities
  - This is just a more complex Hopenhayn model
- The estimation can be broken up into steps, too
  - Some parameters can be determined without knowing the aggregates
- Computing moments
  1. Simulate a panel: can be slow, but data are from finite samples
  2. Use decision rules and ergodic distributions: faster, but lose small sample aspect

Choosing moments

$$\gamma = \{\chi_0, \chi_1, \rho_z, \sigma_z, \mu_e, f_0, f_1, \xi_L, \xi_H, \rho_\xi\}$$

- Need informative moments—this can be the hard part
  - Experiment with the model, compute derivatives
  - Not typically a one-to-one mapping of moments and parameters

- $f_0, f_1 \rightarrow$ export part., stopper rate, exporter premium
- $\xi_L, \xi_H, \rho_\xi \rightarrow$ ***exs$_{it}$***: overall, new exporters, 5-year exporters
- $\rho_z, \sigma_z \rightarrow$ firm size distribution, autocorrelation of sales
- $\chi_0, \chi_1, \mu_e \rightarrow$ entrants' labor share, shutdown labor share

Estimation results: Export technology

Parameter		Calibrated value
Export entry cost	f_0/f_e	0.038
Export continuation cost	f_1/f_0	0.715
High variable trade cost	ξ_H	1.72
Low variable trade cost	ξ_L	1.07
Exporter-type persistence	ρ_ξ	0.92

- Cheap to create an exporter compared to creating a new plant
- Big difference between being a good and bad exporter
- Export type is persistent
- Exporter intensity driven by share of good and bad exporters
- Big picture: Large gain to becoming a good exporter. Not easy to do.
  $\rightarrow$ high stopper rates; low export participation

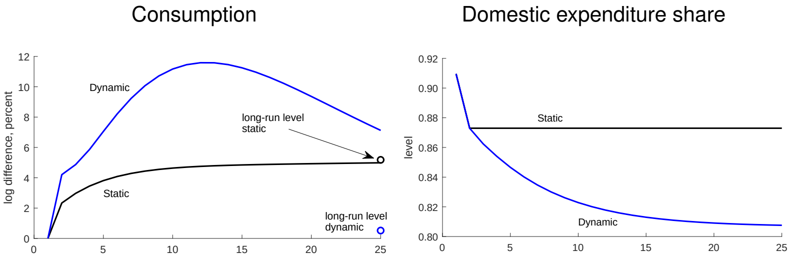
A tariff liberalization

- Start in stationary equilibrium
- Surprise cut in tariffs to $\tau = 0$ in both countries
- Perfect foresight for rest of time
- Compute transition path between steady states
  - Compute the two steady states
  - Assume reach new steady state in T periods
  - Guess path of capital prices
  - Compute value functions, distributions for $t = 0 \ldots T$
  - Iterate backwards on value funcs, forward on distributions

Two versions of the model

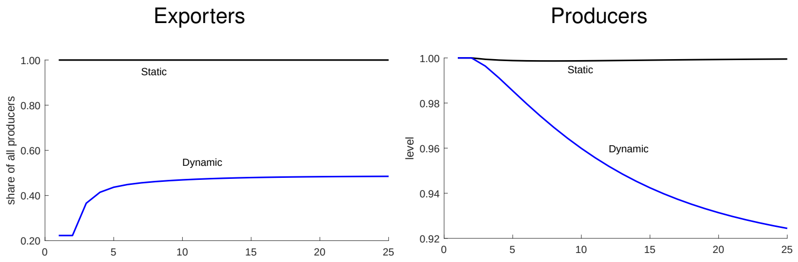
1. The dynamic firm model
2. The static firm model
   - Static model except for capital accumulation
   - Set $f_0 = f_1 = 0$; every firm exports
     - No extensive margin dynamics
     - No forward-looking decision
   - Set $\xi_H = \xi_L = 1.62$ (match agg. export-sales ratio)
     - No intensive margin dynamics
   - This is essentially Krugman (1980) with heterogeneous productivity

Tariff liberalization



- Consumption
  - Dynamic: Consumption overshoots its long-run level
  - Static: Neoclassical dynamics
- Aggregate trade share (1-domestic share)
  - Dynamic: Slow growth (int. + ext. margins)
  - Static: Instant adjustment (only int. margin)

Tariff liberalization



- Export participation
  - Dynamic: Exporters grow gradually, driving aggregate dynamics
- Number of producers
  - Dynamic: Decreasing
  - Static: Small dip, then recovery

Trade liberalization with firm dynamics

- Initial equilibrium has too many firms
  - Imports are relatively expensive
  - Value variety, so create domestic firms
- Liberalization
  - Buy cheaper varieties from abroad
  - Need fewer domestic firms
  - Consume resources that would have gone to firm creation $\rightarrow$ overshooting in consumption
  - Takes time to build of exporters; aggregate trade grows slowly

Welfare

	Static exporters	Dynamic exporters
Long-run trade elast.	4.00	9.22
ΔC_{ss}	5.18	0.48
Δ Welfare	4.62	6.66
Δ Welfare/ΔC_{ss}	0.89	13.81

- Static elasticity is $\theta - 1$
- Dynamic elasticity captures exten. margin and better export tech.
- Static model delivers higher steady-state consumption…
- …but Dynamic model has higher welfare (from overshooting)

- Static “sufficient-statistic” approach is not a good approximation to the dynamic model